OPTIMIZING TRUSS STRUCTURES WITH NATURAL FREQUENCY CONSTRAINTS USING THE CROSS-ENTROPY METHOD

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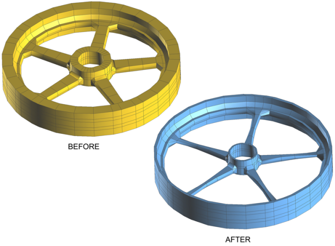
Evolution wheel

http://langevo.blogspot.com/2016/01/germanic-wheels-non-linear-evolution.html
https://simulatemore.mscsoftware.com/automated-structural-optimization-in-msc-nastran/

Evolution wheel



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Introduction

Some challenges:

- Gradient based methods are not possible in some cases
- Metaheuristics may be an alternative, but may have high computational cost or may be prohibitive.
- Finding a metaheuristic is a research challenge
- Cross-entropy method (CE) has been used successful in combinatorial optimization and estimation of rare events in the last two decades

Research proposal:

- Propose a Cross-entropy framework for structural optimization and investigate its accuracy and efficiency

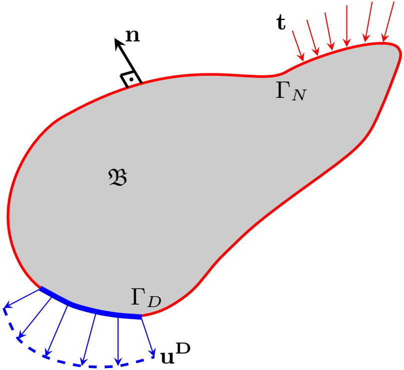
Generic formulation of optimization problem

Find
$$\mathbf{x}^\star = \underset{\mathbf{x}}{\operatorname{argmax}}\, \mathcal{J}(\mathbf{x}) \ ,$$

such that
$$p_i(\mathbf{x}) = 0 \quad i = 1, 2, \ldots, M \ ,$$
$$q_j(\mathbf{x}) \leq 0 \quad j = 1, 2, \ldots, N \ .$$

- $\mathcal{J}$: objective function
- $\mathbf{x}$: vector with the design variables
- Dual relationship: $\min \mathcal{J}(\mathbf{x}) = \max[-\mathcal{J}(\mathbf{x})]$

Optimization Framework



Balance of linear *momentum*:

$$\nabla \cdot \boldsymbol{\sigma}(\boldsymbol{u}) = \mathbf{0}$$

Balance angular *momentum*:

$$\boldsymbol{\sigma}(\boldsymbol{u}) = \boldsymbol{\sigma}^T(\boldsymbol{u})$$

Kinematic relationship:

$$\epsilon(\boldsymbol{u}) = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)$$

Constitutive relationship:

$$\boldsymbol{\sigma}(\boldsymbol{u}) = \boldsymbol{\mathcal{C}} : \epsilon(\boldsymbol{u})$$

- Boundary conditions

$$\boldsymbol{\sigma}(\boldsymbol{u}) \cdot \boldsymbol{n} = \boldsymbol{t} \text{ in } \Gamma_N$$

$$\boldsymbol{u} = \boldsymbol{u}^{\boldsymbol{D}} \text{ in } \Gamma_D$$

- Natural frequencies

$$\omega \geq \omega^*$$

  - where natural frequencies are obtained from

$$[\boldsymbol{K}]\phi = \omega^2[\boldsymbol{M}]\phi \ \left(\textit{eigenvalue problem}\right)$$

$[\boldsymbol{K}] \rightarrow$ stiffness matrix
$[\boldsymbol{M}] \rightarrow$ mass matrix

Optimization statement

Find $\boldsymbol{x}^\star$ which minimize

$$\mathcal{J}(\boldsymbol{x}) = \int_{\mathfrak{Truss}} \rho(\boldsymbol{x})\, dV \qquad \text{(mass of structure)}$$

such that

$$\omega_1 \geq \omega_1^*, \; \cdots \;, \omega_k \geq \omega_k^* \qquad \text{(natural frequency)}$$

with

$$\mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max} \qquad \text{(design limits)}$$

Key Idea: "Transform" the optimization problem into a rare-event estimation problem.

- Hypothesis: there is single maximum

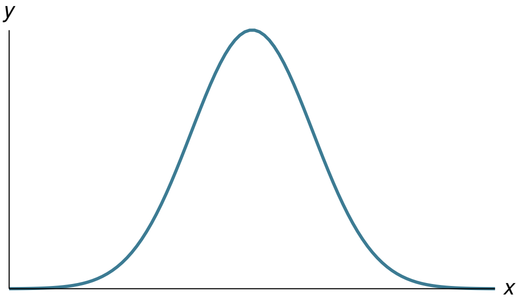
$$\gamma^* = \mathcal{J}(\boldsymbol{x}^*) = \max \mathcal{J}(\boldsymbol{x})$$

- Penalized formulation

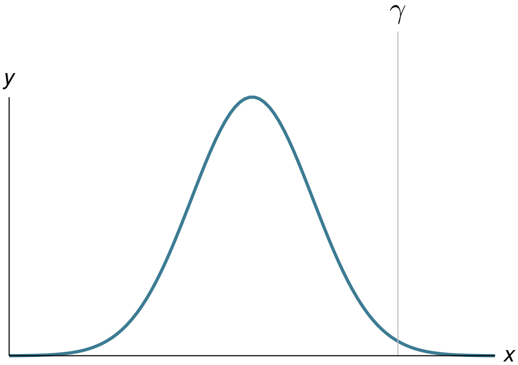
Find $\mathbf{x}^\star$ which

$$\boldsymbol{x}^\star = \operatorname{argmax} \left\{ \mathcal{J}(\mathbf{x}) + \sum_{i=1}^{K} \nu_i \, \max\left\{0, q_i(\boldsymbol{X})\right\} \right\}$$

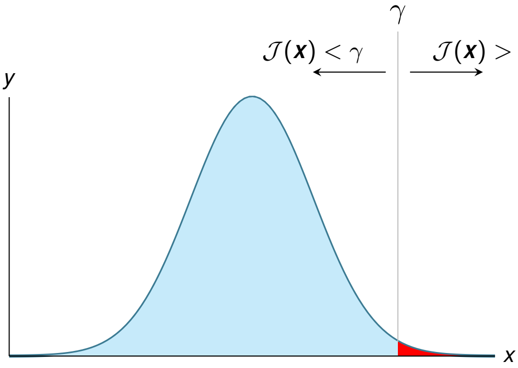
$\nu_i < 0$ measures the importance (cost) of the ith penalty


y
x

Rare-event probability estimation



Rare-event probability estimation



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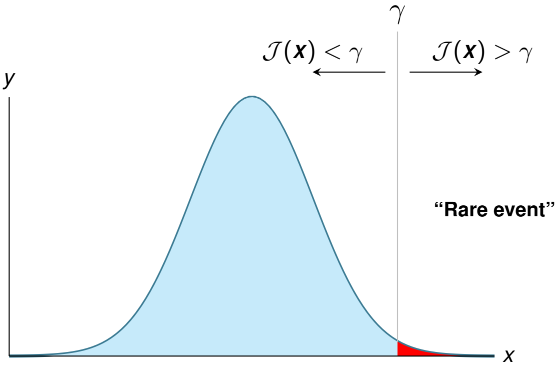

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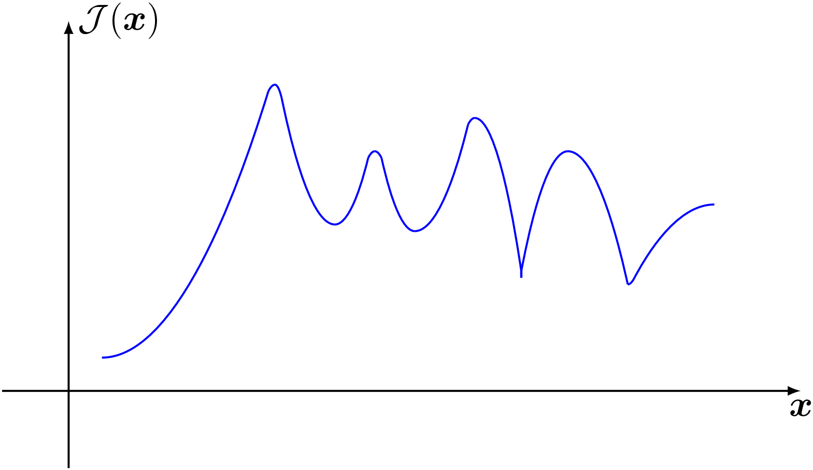
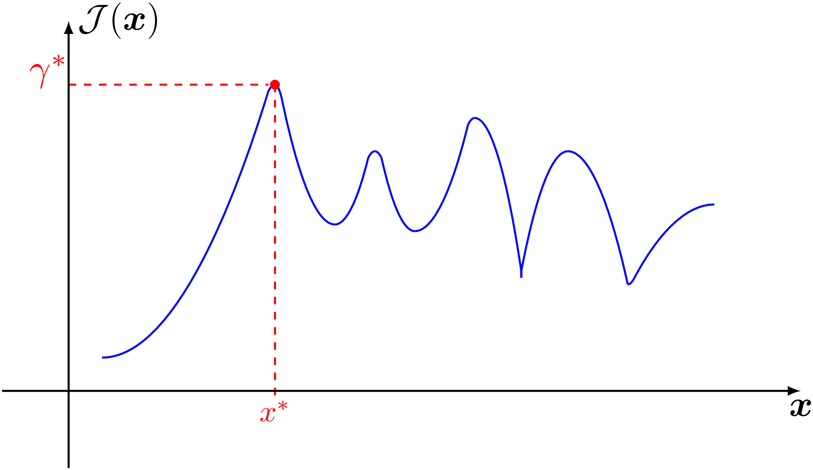
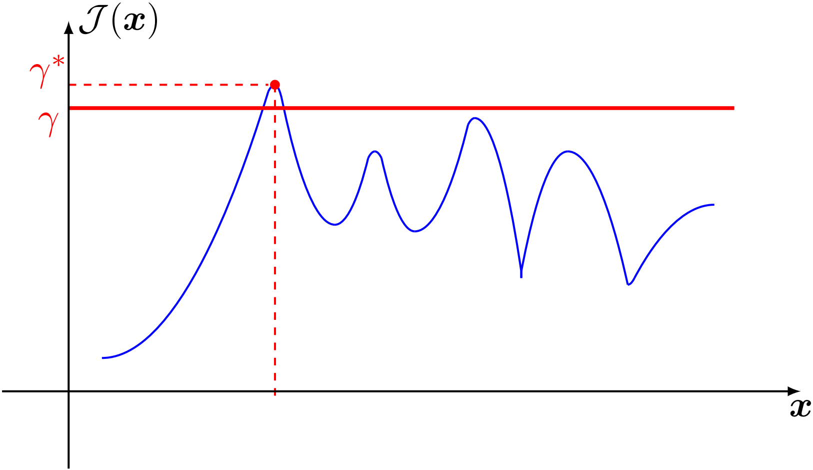


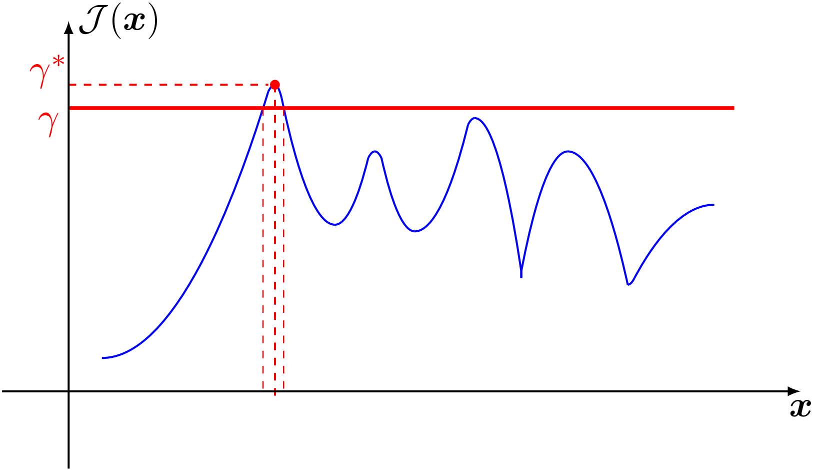


$\mathcal{J}(\boldsymbol{x})$
γ^*
x^*
$\boldsymbol{x}$

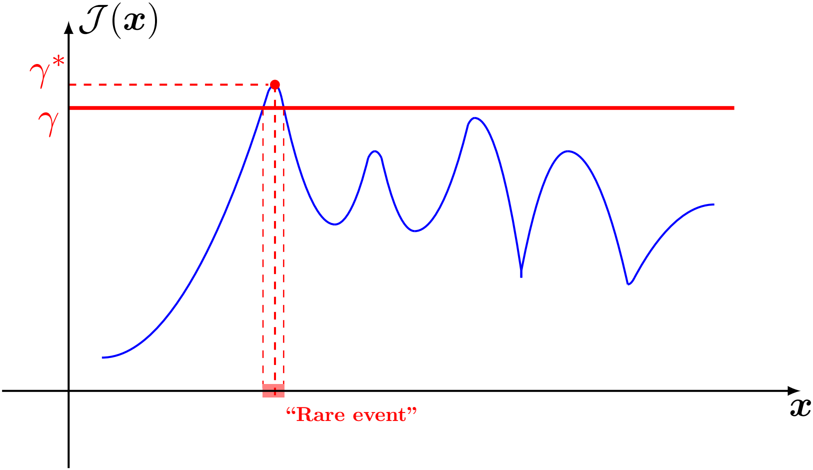


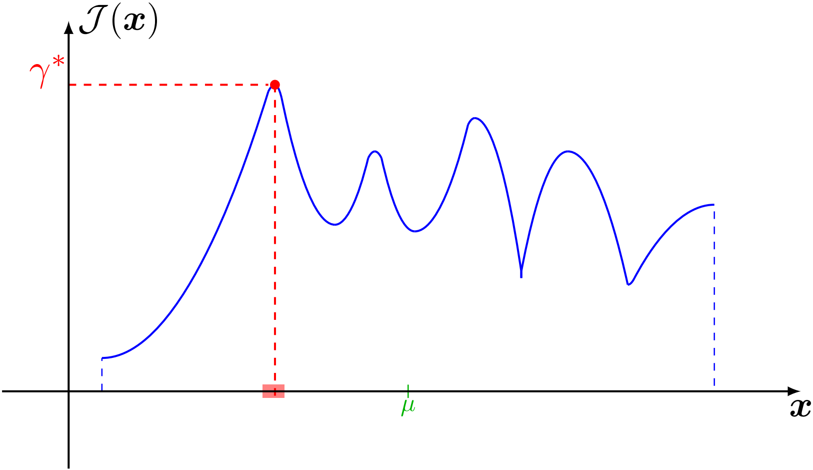
$\mathcal{J}(\boldsymbol{x})$
γ^*
γ
$\boldsymbol{x}$

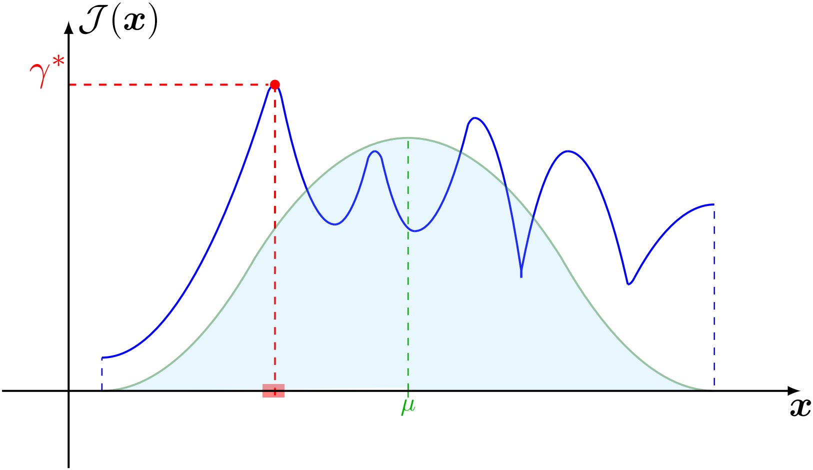
Rare-event probability estimation



Rare-event probability estimation

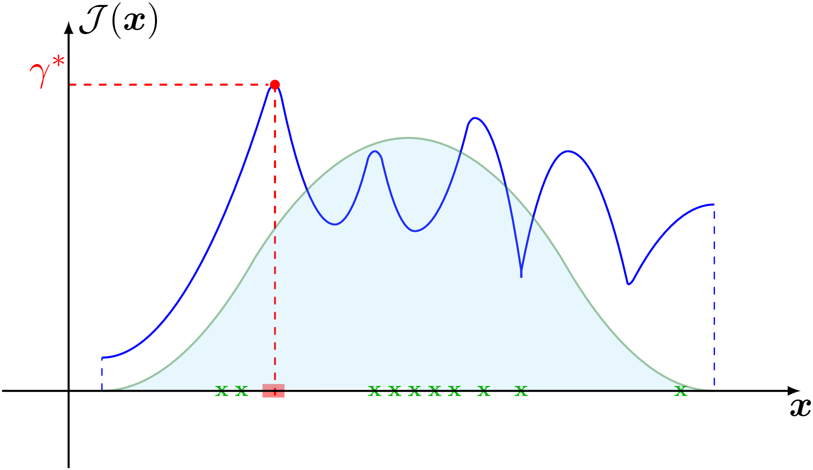


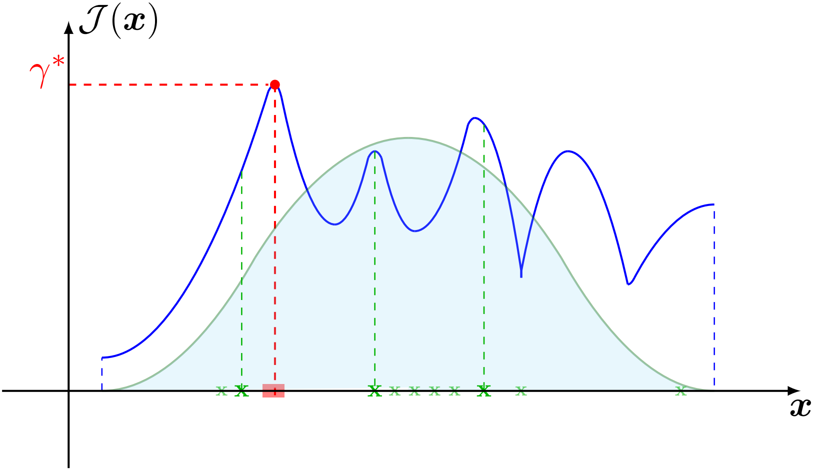

$\mathcal{J}(\boldsymbol{x})$
γ^*
μ
$\boldsymbol{x}$



$\mathcal{J}(\boldsymbol{x})$
γ^*
μ
$\boldsymbol{x}$

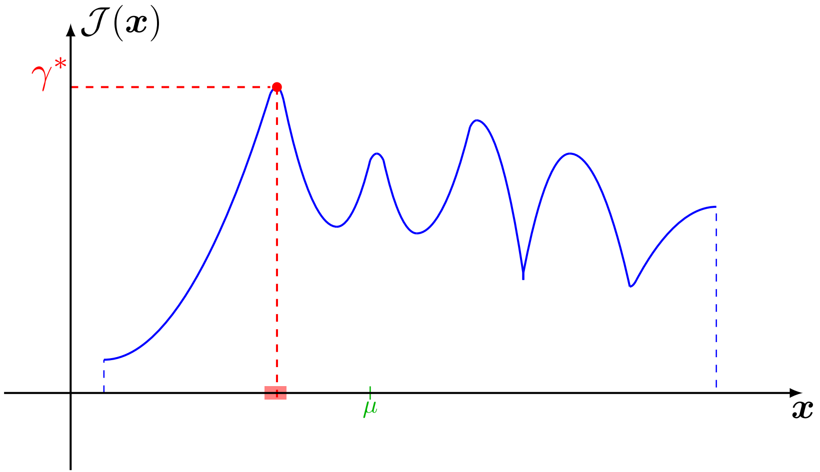
Rare-event probability estimation

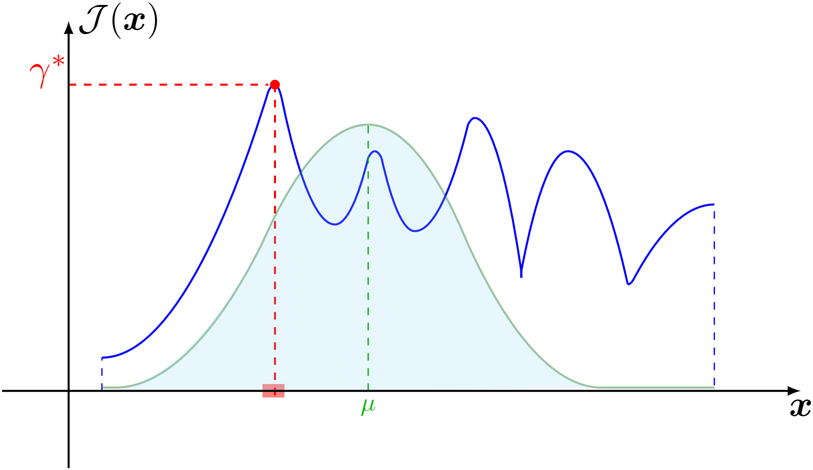


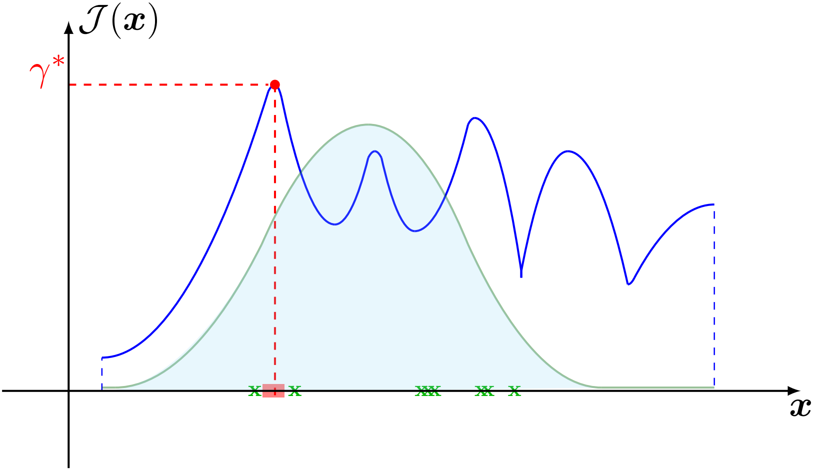


$\mathcal{J}(\boldsymbol{x})$
γ^*
$\boldsymbol{x}$

Rare-event probability estimation




$\mathcal{J}(\boldsymbol{x})$
γ^*
μ
$\boldsymbol{x}$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γ^*
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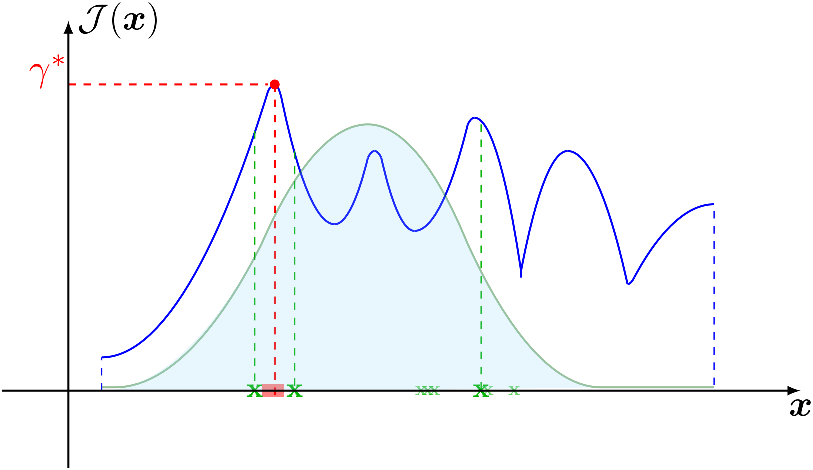

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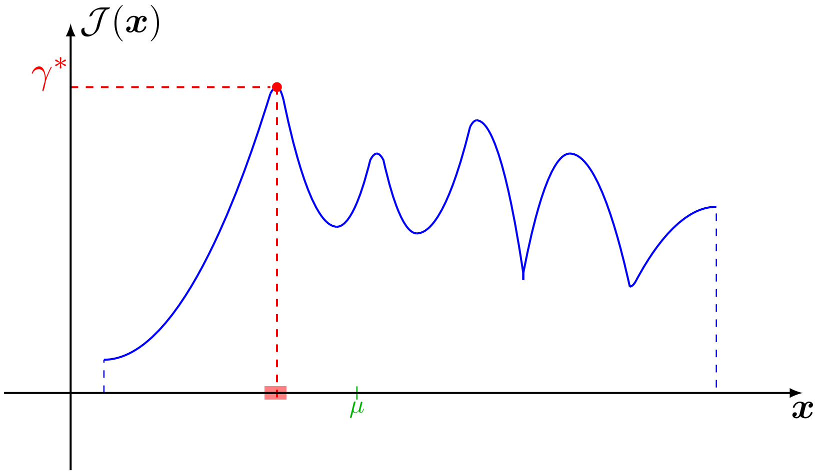
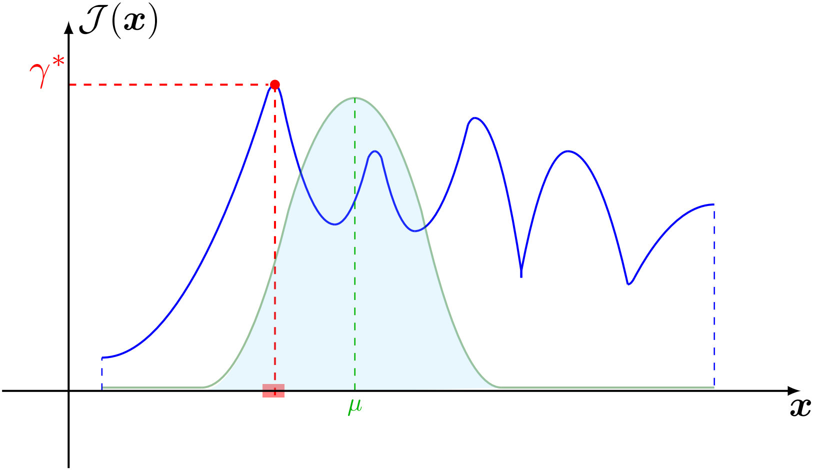
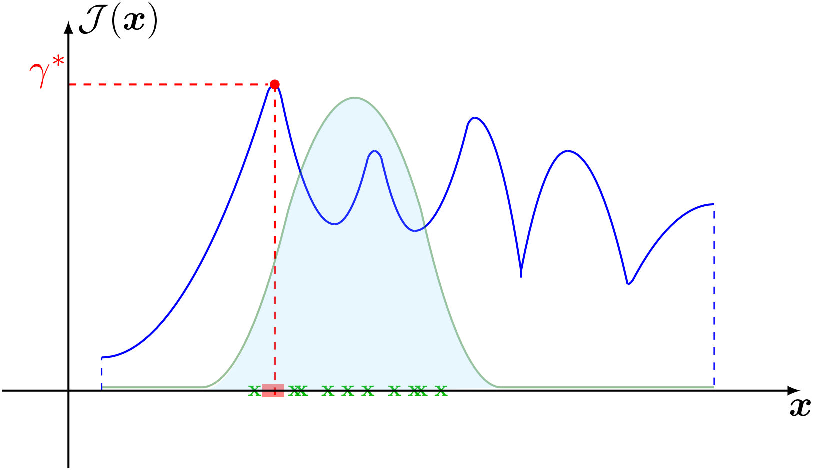


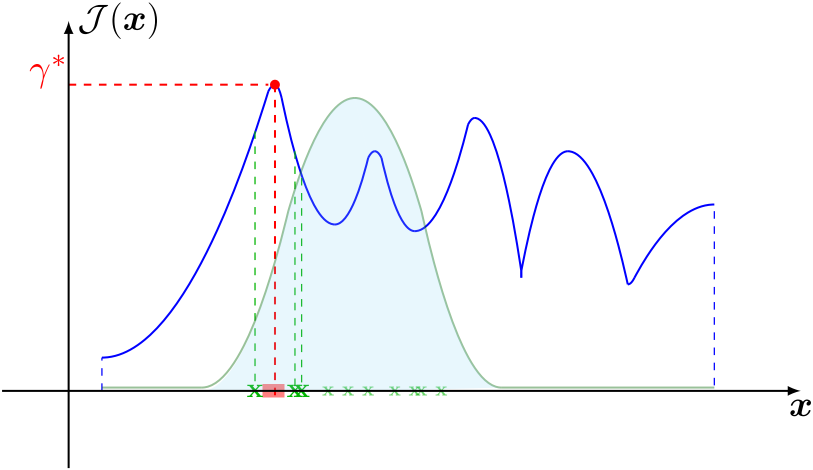


$\mathcal{J}(\boldsymbol{x})$
γ^*
μ
$\boldsymbol{x}$

Rare-event probability estimation



Rare-event probability estimation



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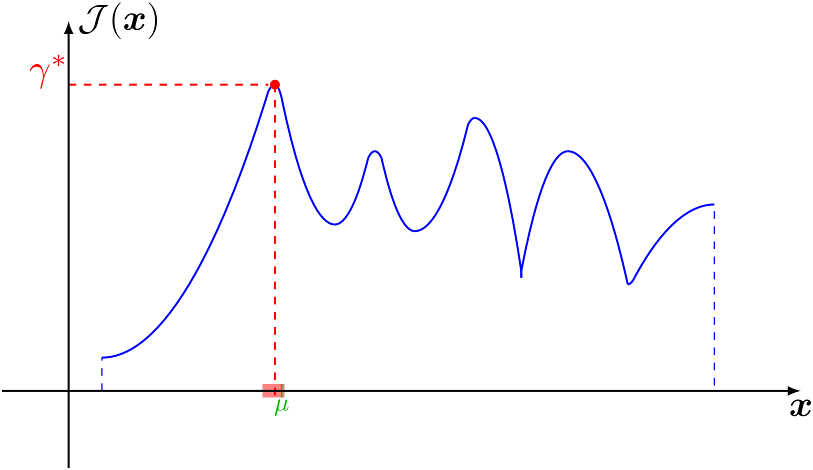
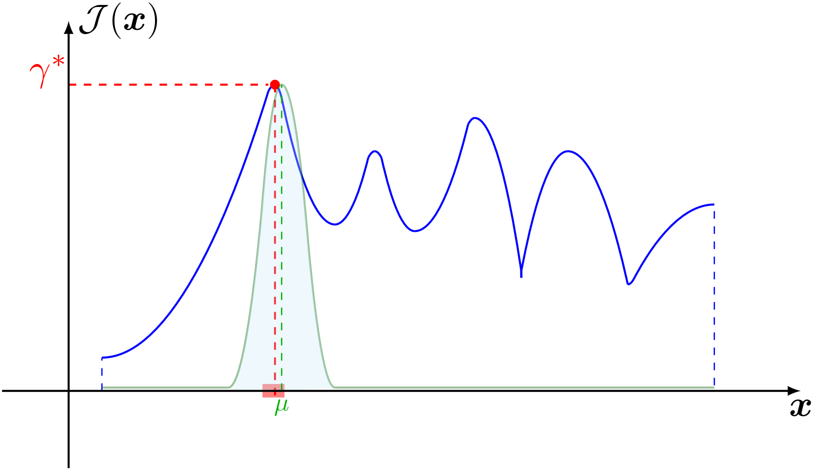
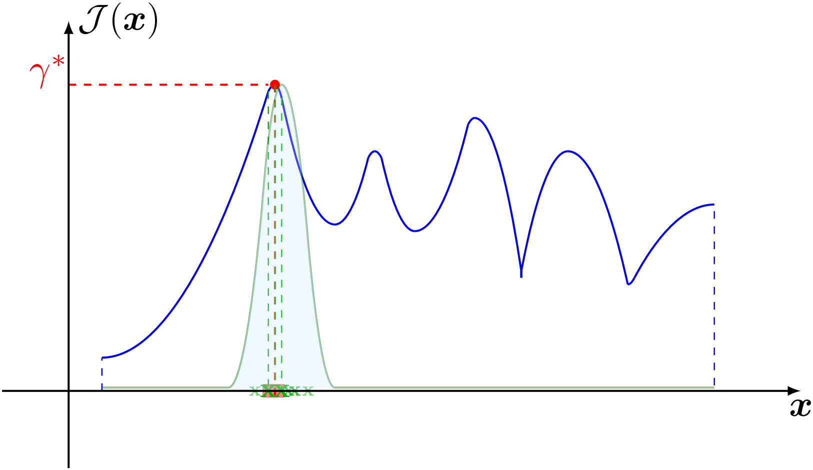
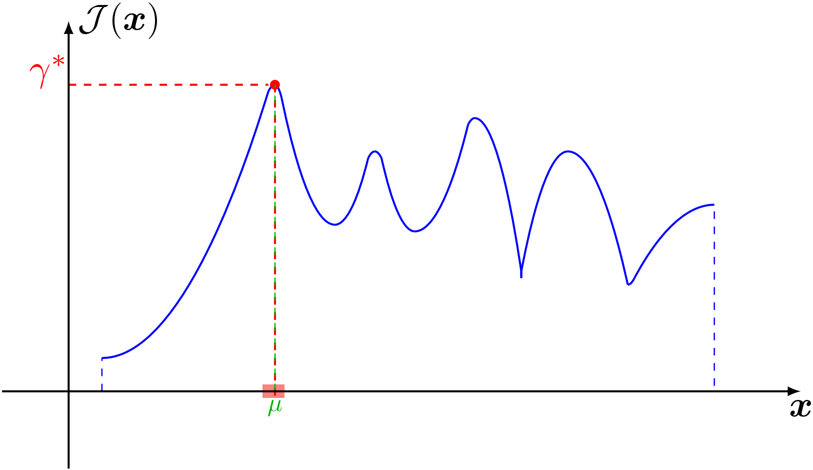


Rare-event probability estimation

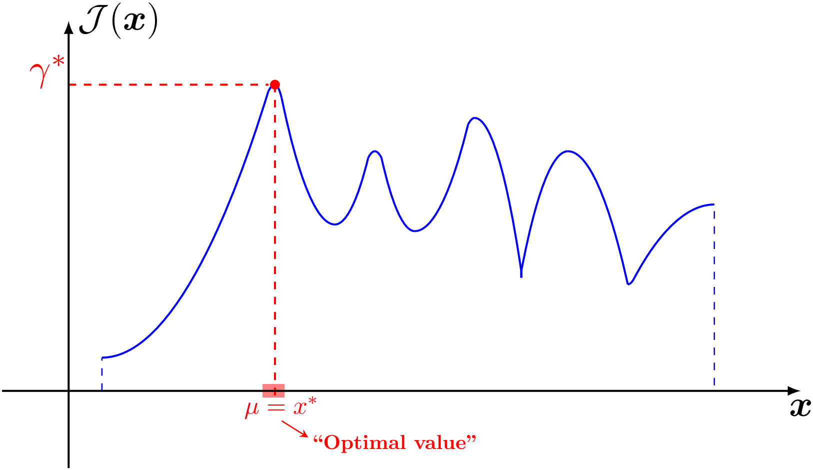




$\mathcal{J}(\boldsymbol{x})$
γ^*
$\boldsymbol{x}$


$\mathcal{J}(\boldsymbol{x})$
γ^*
μ
$\boldsymbol{x}$

Rare-event probability estimation



Cross-entropy framework

Given a random design vector $\mathbf{X} \sim f\left(\mathbf{x};\mathbf{v}\right)$ and fixed reference level $\gamma \approx \gamma^\star = \max \mathcal{J}(\mathbf{x})$ one has that $\mathcal{J}(\mathbf{X}) \geq \gamma$ is a rare-event.

Cross-Entropy Method:

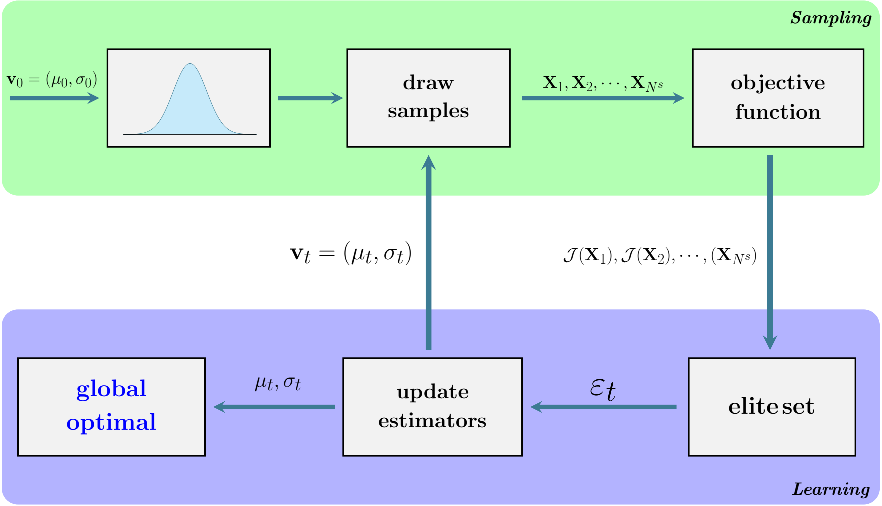
Generates an "optimal sequence" of estimators $\left(\widehat{\gamma}_t, \widehat{\mathbf{v}}_t\right)$ such that

$$\widehat{\gamma}_t \xrightarrow{a.s.} \gamma^\star \text{ and } f\left(\mathbf{x}, \widehat{\mathbf{v}}_t\right) \xrightarrow{a.s.} \delta\left(\mathbf{x} - \mathbf{x}^\star\right)$$

Optimal: "minimize KL divergence between $\delta\left(\mathbf{x} - \mathbf{x}^\star\right)$ and $f\left(\cdot\,,\ \mathbf{v}\right)$"

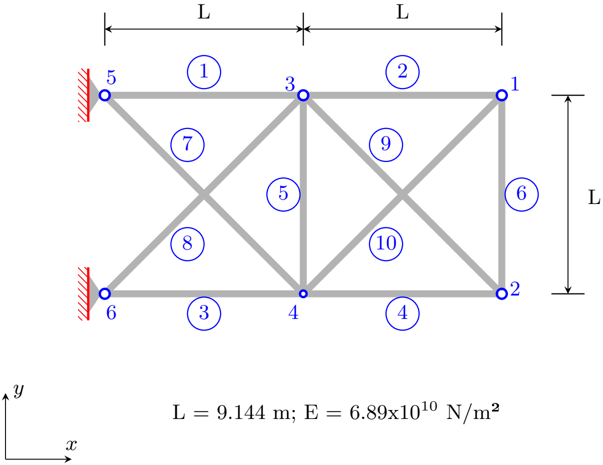
R. Y. Rubinstein and Dirk P. Kroese, **Simulation and the Monte Carlo Method**, Wiley, 3rd Edition, 2017.

Cross-entropy schematic



Numerical Results

- Design variables: 10 A_e



$$L = 9.144 \text{ m};\ E = 6.89\text{x}10^{10}\ \text{N/m}^2$$

R. T. Haftka and Z. Gürdal, **Element of Structural Optimization**, 3rd ed., Kluwer Publishers, Dordrecht, Netherlands, 1992.

Natural frequency constraint

$$\min_{\boldsymbol{x}} m = \sum_{e=1}^{10} \rho A_e L_e \qquad \boldsymbol{x} = \{A_e\}$$

s.t

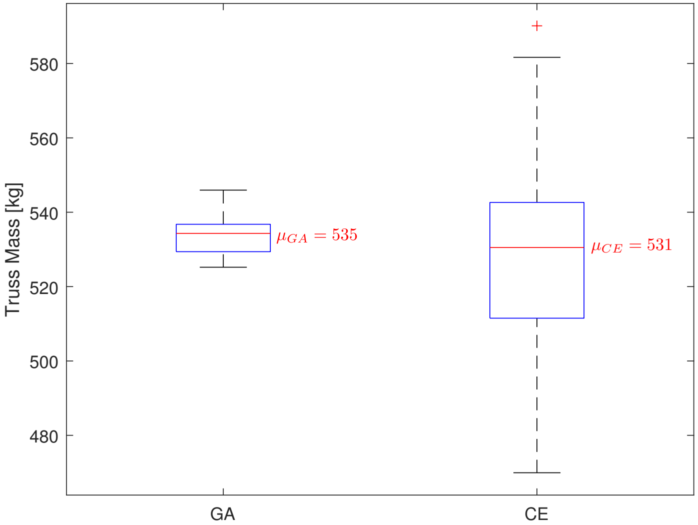
$\omega_1 \geq 7\ Hz$, $\omega_2 \geq 15\ Hz$ and $\omega_3 \geq 20\ Hz$

with

$$65.4\ mm^2 \leq A_e \leq 5000\ mm^2$$

$$m_{ad} = 454\ kg$$

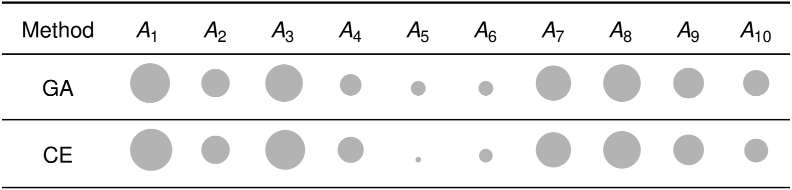
V. Ho-Huu and T. Vo-Duy and T. Luu-Van and L. Le-Anh and T. Nguyen-Thoi, **Optimal design of truss structures with frequency constraints using improved differential evolution algorithm based on an adaptive mutation scheme**, Automation in Construction,68:81-94, 2016.


Truss Mass [kg]
580
560
540
520
500
480
GA
CE
$\mu_{GA} = 535$
$\mu_{CE} = 531$

Natural frequency results

Method	ω_1	ω_2	ω_3
GA	7.0	16.6	20.0
CE	7.0	16.2	20.0

*Natural frequencies in *Hz*



Natural frequency results

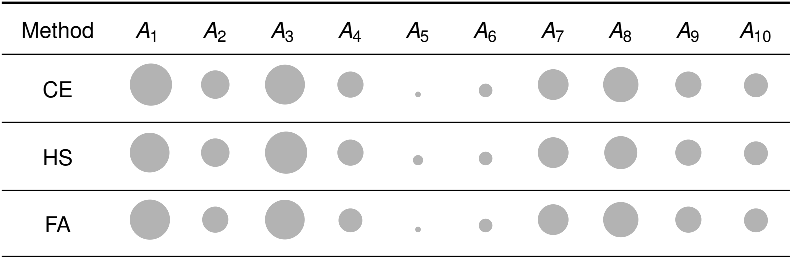
Method	mass (kg)
CE	531
HS	535
FA	531

Method	ω_1	ω_2	ω_3
CE	7.0	16.2	20.0
HS	7.0	16.7	20.1
FA	7.0	16.1	20.0

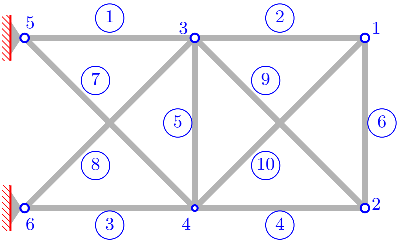
*Natural frequencies in *Hz*

Leticia F. F. Miguel and Leandro F. F. Miguel, **Shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms**, Expert Systems with Applications,10:9458–9467, 2012.

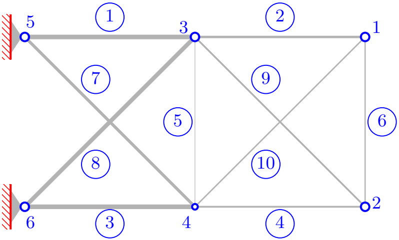
Natural frequency results



Initial model:



Optimized model:



Final Remarks

Conclusions:

- CE is a metaheuristic which may be used in structural optimization
- The results indicate that CE performs favorably compared to GA, HS and FA
- In summary, CE demonstrates strong performance and efficiency in the considered structural optimization problem

Future directions:

- Improve the CE for structural optimization

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27th ABCM International Congress of Mechanical Engineering
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VALE

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CAPES

Thank for your attention!

Questions?!

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