Structural Optimization using Cross-Entropy Method

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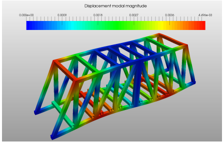
Outline

Structural Optimization

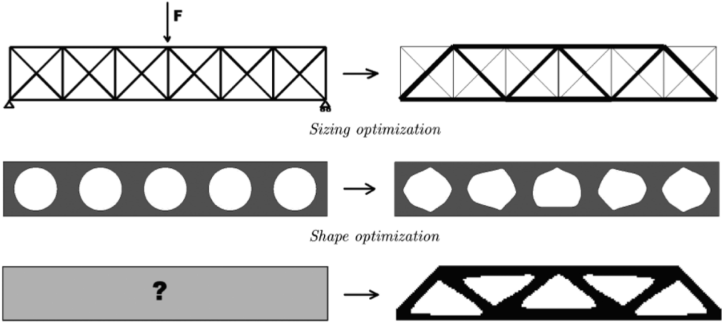
- Typical Objectives
  - Mass reduction (weight)
  - Change the Natural Frequency (avoid Resonance)
  - Improve Layout
  - Improve Construction
  - Improve Assembly
  - Reduce internal stresses
  - Reduce material used
  - **Reduce Cost**

- Applications
  - Automobile Industry
  - Aerospace Industry
  - Construction Sector

  etc.



Different problems in structural optimization



Sizing optimization

Shape optimization

Topology optimization

M. P. Bendsøe and O. Sigmund, **Topology Optimization: Theory, Methods and Applications**, Springer, 2003.

Challenges and objectives

Some challenges:

- Derivative or gradient based methods are not possible in some cases
- Metaheuristics can be great computational cost or until prohibitive

Research objectives:

- Propose a Cross-Entropy framework for structural optimization
- Investigate its accuracy and efficiency

Formulation of the structural optimization problem

Find $\mathbf{x}^\star$ which minimize

$$m(\mathbf{x}) = \int_B \rho(\mathbf{x}) dV,$$

(mass of structure)

such that

$$\mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max},$$

(design limits)

$$\sigma(\mathbf{x}) \leq S_y,$$

(yield strength)

$$\sigma(\mathbf{x}) \leq \sigma_E.$$

(buckling)

Generic optimization framework

Find $\mathbf{x}^\star$ which maximize

$$\mathcal{F}(\mathbf{x})$$

such that

$$\mathcal{G}_m(\mathbf{x}) \leq 0, \;\; m = 1, \cdots, M$$

(original formulation)

Find $\mathbf{x}^\star$ which maximize

$$\mathcal{S} = \mathcal{F}(\mathbf{x}) + \sum_{m=1}^{M} H_m \max\left\{0, \mathcal{G}_m(\mathbf{x})\right\}$$

(penalized formulation)

Cross-entropy framework

Key Idea: "Transform" the optimization problem into a rare-event estimation problem.

Given a random design vector $\mathbf{X} \sim f(\mathbf{x}; \mathbf{v})$ and fixed reference level $\gamma \approx \gamma^\star = \max \mathcal{S}(\mathbf{x}^\star)$ one has that $\mathcal{S}(\mathbf{X}) \geq \gamma$ is a rare-event.

Cross-Entropy Method:

Generates an "optimal sequence" of estimators $(\widehat{\gamma}_t, \widehat{\mathbf{v}}_t)$ such that

$$\widehat{\gamma}_t \xrightarrow{a.s.} \gamma^\star \text{ and } f(\mathbf{x}, \widehat{\mathbf{v}}_t) \xrightarrow{a.s.} \delta(\mathbf{x} - \mathbf{x}^\star)$$

Optimal: "minimize KL divergence between $\delta(\mathbf{x} - \mathbf{x}^\star)$ and $f(\cdot\,, \mathbf{v})$"

R. Y. Rubinstein and Dirk P. Kroese, **Simulation and the Monte Carlo Method**, Wiley, 3rd Edition, 2017.

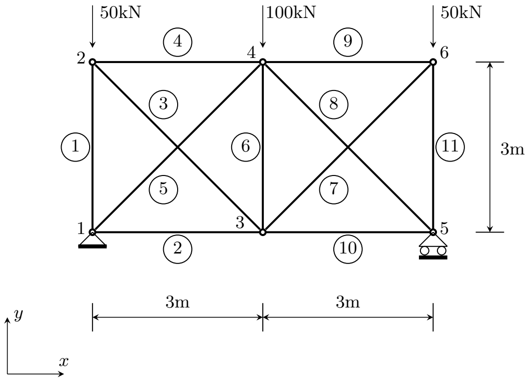
Cross-entropy algorithm

1. Define N, N^e, t_{max}, $t = 0$, $f\left(\cdot, \mathbf{v}\right)$ and $\widehat{\mathbf{v}}_0$
2. Update level $t = t + 1$
3. Generate $\mathbf{X}_1, \cdots, \mathbf{X}_N$ (iid) samples from $f\left(\cdot, \widehat{\mathbf{v}}_{t-1}\right)$
4. Evaluate performance function $\mathcal{S}(\mathbf{X}_n)$ at samples $\mathbf{X}_1, \cdots, \mathbf{X}_N$ and sort the results $\mathcal{S}_{(1)} \leq \cdots \leq \mathcal{S}_{(N)}$
5. Update estimators $\widehat{\gamma}_t$ and $\widehat{\mathbf{v}}_t$
6. Repeat ② — ⑤ while stopping criterion is not met

R. Y. Rubinstein and Dirk P. Kroese, **Simulation and the Monte Carlo Method**, Wiley, 3rd Edition, 2017.

Structural Models

- Truss 1



A. J. M. Ferreira, **MATLAB Codes for Finite Element Analysis Solids and Structures**, Springer, 2009.

Results Truss 1 without buckling

Method	mass (kg)	$d_i^\star$ (mm)	$t^\star$ (mm)	Func Evaluate	CPU time* (sec)
SQP	78	20.0	3.5	5	0.3
GA	78	20.0	3.5	2657	5.0
CE	78	20.0	3.5	175	0.4

*Dell Inspiron i15 7559-A30 "Core i7" 2.8 GHz 16GB 1600 MHz DDR3L

Cross-Entropy method

- NCE = 25
- $\rho = 10\%$
- tol $= 10^{-4}$
- Stopped after t = 7 iterations

- $20mm \leq d_i \leq 100mm$ and $3mm \leq t \leq 20mm$

Results Truss 1 with buckling

Method	mass (kg)	$d_i^\star$ (mm)	$t^\star$ (mm)	Func Evaluate	CPU time* (sec)
SQP	288	56.1	5.0	21	0.3
GA	293	55.9	5.1	5250	13.0
CE	288	56.1	5.0	100	0.4

*Dell Inspiron i15 7559-A30 "Core i7" 2.8 GHz 16GB 1600 MHz DDR3L

Cross-Entropy method

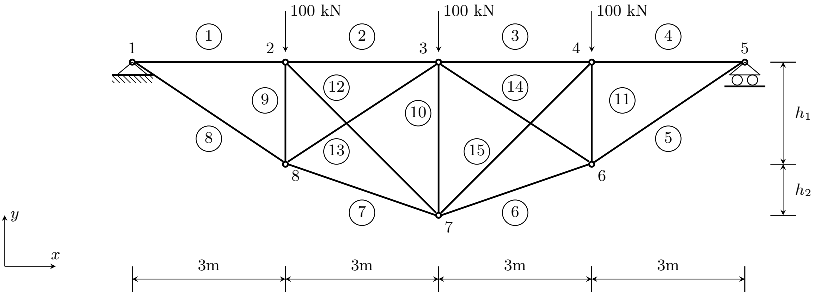
- NCE = 25
- $\rho = 10\%$
- tol $= 10^{-4}$
- Stopped after t = 4 iterations

- $50mm \leq d_i \leq 100mm$ and $5mm \leq t \leq 20mm$

Truss 1 *(video)*

Structural Models

- Truss 2



S. Kalanta, J. Atkočiūnas, T. Ulitinas and A. Grigusevičius, Optimization of bridge trusses height and bars cross-sections, In *The Baltic Journal of Road and Bridge Engineering*, 7(2):112-119, 2012.

Results Truss 2 with buckling

Method	mass (kg)	$d_i^\star$ (mm)	$t^\star$ (mm)	$h_1^\star$ (m)	$h_2^\star$ (m)	Func Evaluate	CPU time* (sec)
SQP	815	68.5	10.0	1.32	0.10	56	0.6
GA	856	71.2	10.5	1.01	0.10	2625	20.0
CE	852	69.1	10.3	1.2	0.49	625	1.2

*Dell Inspiron i15 7559-A30 "Core i7" 2.8 GHz 16GB 1600 MHz DDR3L

Cross-Entropy method

- NCE $= 25$
- $\rho = 10\%$
- tol $= 10^{-4}$
- Stopped after t $= 25$ iterations

- $50mm \leq d_i \leq 100mm$, $10mm \leq t \leq 20mm$, $1000mm \leq h_1 \leq 2000mm$ and $100mm \leq h_2 \leq 1000mm$

Concluding remarks

Contributions:

- Cross-Entropy new framework for structural optimization

Conclusions:

- CE presents accuracy comparable to SQP and GA
- CE presents an efficiently more or less comparable to SQP
- CE is much faster than GA

Future directions:

- Explore CE framework for optimization of 3D structures
- Test CE framework in topology optimization

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Thank you for your attention!

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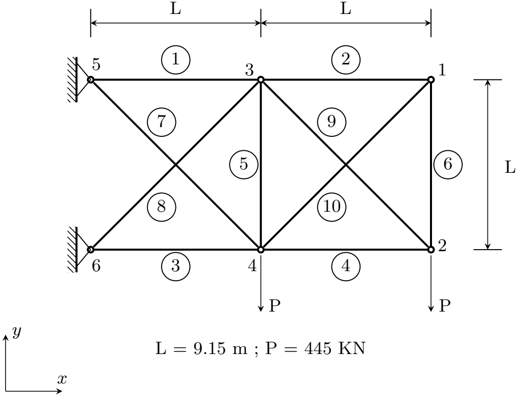
- S. Kalanta, J. Atkočiūnas, T. Ulitinas and A. Grigusevičius, Optimization of bridge trusses height and bars cross-sections, In *The Baltic Journal of Road and Bridge Engineering*, 7(2):112-119, 2012.

Annex

Truss 3



L = 9.15 m ; P = 445 KN

R. T. Haftka and Z. Gürdal, **Element of Structural Optimization**, 3rd Edition, Kluwer Publishers 1992.

Results Truss 3 with buckling

Method	mass (kg)	Func Evaluate	CPU time* (sec)
SQP	2557	98	1.3
GA	2886	9472	31.0
CE	2795	2500	4.9

*Dell Inspiron i15 7559-A30 "Core i7" 2.8 GHz 16GB 1600 MHz DDR3L

Cross-Entropy method

- NCE = 50
- $\rho = 10\%$
- tol $= 10^{-4}$
- Stopped after t = 50 iterations

Truss 3 SQP

bar	1	2	3	4	5	6	7	8	9	10
d_i(mm)	188.9	150.0	192.5	150.0	150.0	150.0	194.3	187.2	166.8	150.0
σ (MPa)	89	22	-94	-35	18	22	68	-61	45	-31
σ_e (MPa)	130	85	134	85	85	85	68	64	52	43

- $t = 15mm$ and $150mm \leq d_i \leq 300mm$

Truss 3 GA

bar	1	2	3	4	5	6	7	8	9	10
d_i(mm)	198.8	174.3	207.1	209.0	183.6	173.9	234.9	184.2	170.6	187.0
σ (MPa)	89	21	-91	-25	13	21	61	-58	42	-27
σ_e (MPa)	142	112	154	156	123	111	97	62	54	64

- $t = 15mm$ and $150mm \leq d_i \leq 300mm$

Truss 3 CE

bar	1	2	3	4	5	6	7	8	9	10
d_i(mm)	195.8	162.6	176.9	189.2	224.1	161.5	197.9	196.2	176.9	177.7
σ (MPa)	89	21	-99	-28	15	21	64	-62	42	-27
σ_e (MPa)	138	98	115	130	178	97	71	69	57	58

- $t = 15mm$ and $150mm \leq d_i \leq 300mm$