Evaluation of Fractional-Order Sliding Mode Control Applied to an Energy Harvesting System

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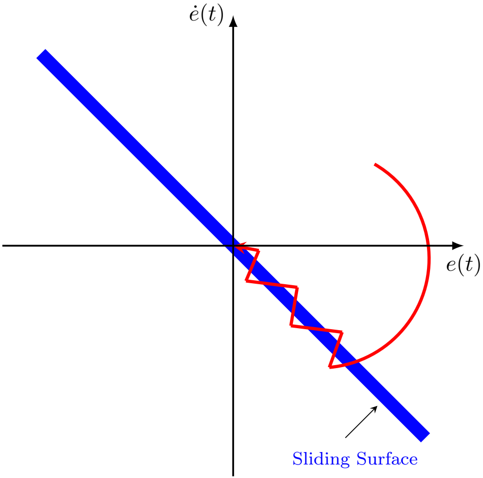
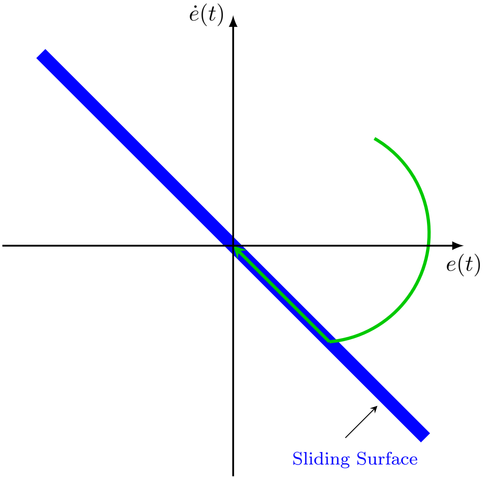
SLIDING MODE CONTROL



FRACTIONAL-ORDER
SLIDING MODE CONTROL

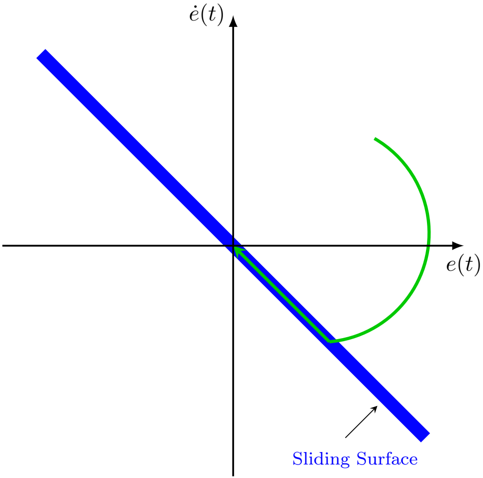


CROSS ENTROPY METHOD

+

FRACTIONAL-ORDER

SLIDING MODE CONTROL



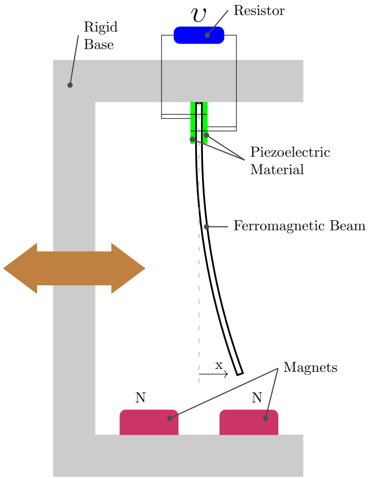
Research objective

This research has the following objective:

- To compare a classic Sliding Mode Control (SMC) and a Fractional-Order Sliding Mode Control (FOSMC), focusing on the energy consumption performance of controllers.;

Outline

Bistable Energy Harvester



Nonlinear Bistable Vibration Energy Harvester, proposed by Erturk et al (2009).

Equation of motion (2-DOF):

$$\ddot{x} + 2\,\xi\,\dot{x} - \frac{1}{2}\,x\left(1 - x^2\right) - \chi\,v = F + u$$

$$+\dot{v} + \Lambda\,v + \kappa\,\dot{x} = 0$$

$+$ inicial conditions

v - voltage across the resistor

x - displacement of the beam

t - time variable

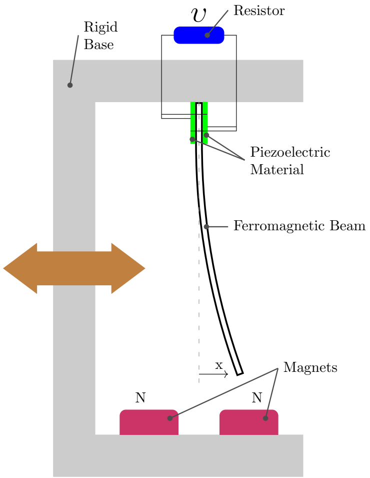
χ - mechanical coupling facto

κ - electrical coupling factor

Λ - inverse characteristic time

u - control signal

F - natural system excitation (harmonic or random)



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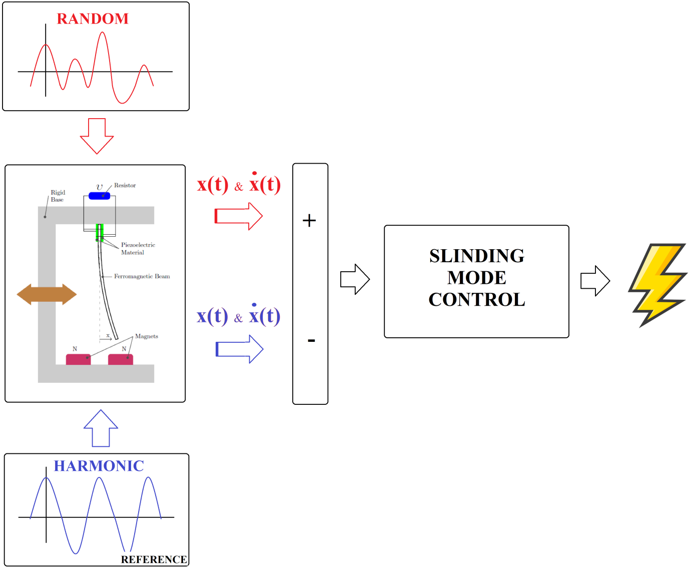
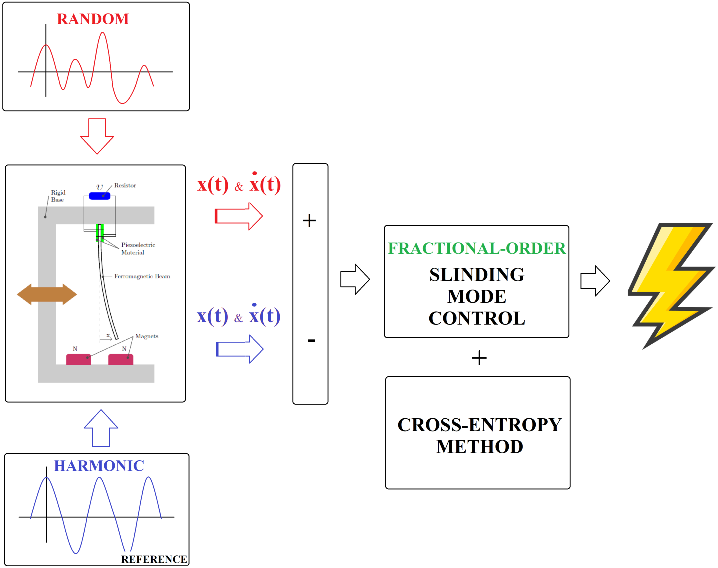
Average Power:

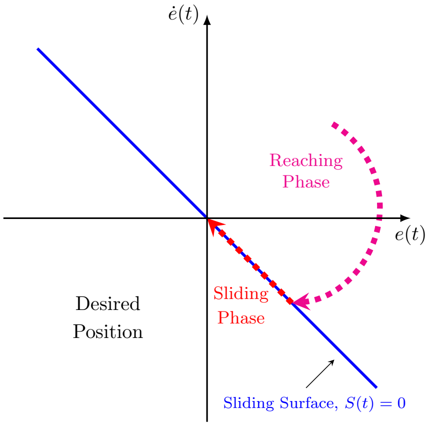
$$P_{avg} = \frac{1}{T}\int_0^T \Lambda\,v\left(t\right)^2 dt$$

System to be controlled




RANDOM
Rigid Base
Resistor
Piezoelectric Material
Ferromagnetic Beam
Magnets
N
N
HARMONIC
REFERENCE
x(t) & ẋ(t)
x(t) & ẋ(t)
+
-
FRACTIONAL-ORDER
SLINDING MODE CONTROL
+
CROSS-ENTROPY METHOD

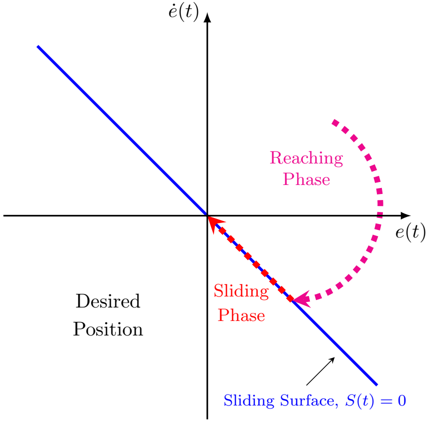
Fractional-Order Sliding Mode Control



Sliding surface of the **SMC** control (second-order system):

$$S(t) = \lambda_1 e(t) + \frac{d\,e(t)}{dt}$$

λ_1 - sliding gain
$e(t)$ - tracking error



Sliding surface of the **FOSMC** control (second-order system):

$$S(t) = \lambda_1\, e(t) + \lambda_2\, \frac{d\, e(t)}{dt} + \lambda_3\, \mathcal{D}^{\alpha} e(t)$$

$\lambda_1, \lambda_2, \lambda_3$ - sliding gains
$e(t)$ - tracking error
$\mathcal{D}^{\alpha}$ - fractional derivative operator.

Definitions: fractional integral

Several definitions are possible!

- Riemann-Liouville

$$\mathcal{I}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau$$

- Hadamard

$$\mathcal{I}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t \left(\log \frac{t}{\tau}\right)^{\alpha-1} f(\tau) \frac{d\tau}{\tau}$$

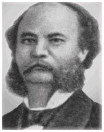
*Pictures from Wikipedia

D.Ortigueira and J.A.Tenreiro Machado, What is a fractional derivative? **Journal of Computational Physics**, 293:4-13 2015.

Several definitions are also possible!


- Riemann-Liouville

$$\mathcal{D}^{\alpha} f(t) = \frac{d^n}{dt^n} \mathcal{I}^{n-\alpha} f(t)$$

- Grünwald-Letnikov

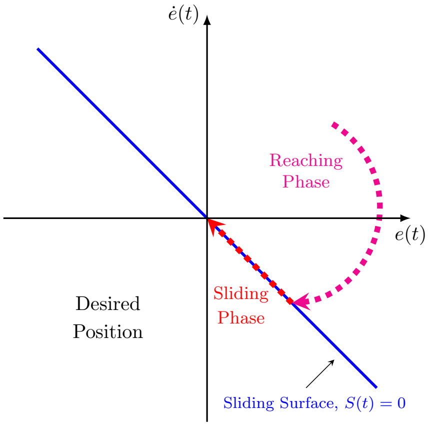
$$\mathcal{D}^{\alpha} f(t) = \lim_{N \to \infty} \left[\frac{\left(\frac{t-a}{N}\right)^{-\alpha}}{\Gamma(-\alpha)} \sum_{j=0}^{N-1} \frac{\Gamma(j-\alpha)}{\Gamma(j+1)} f\left(t - j\left(\frac{t-a}{N}\right)\right)\right]$$

- Caputo

$$\mathcal{D}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-u)^{(n-\alpha-1)} f^{(n)}(u) du$$

*Pictures from Wikipedia and Research Gate

D.Ortigueira and J.A.Tenreiro Machado, What is a fractional derivative? **Journal of Computational Physics**, 293:4-13 2015.



Control signal (SMC and FOSMC):

$$u(t) = u_{eq}(t) + u_{sw}(t)$$

Lyapunov function candidate:

$$V(S) = \frac{1}{2} S^2$$

Equivalent control u_{eq}: $\dot{V}(S) = S\,\dot{S} = 0$

Switching control u_{sw}: $\dot{V}(S) = S\,\dot{S} < 0$

$$u_{sw} = -k \frac{S}{||S|| + \epsilon}$$

SMC

$$S(t) = \lambda_1 e(t) + \frac{d\ e(t)}{dt}$$

$$\dot{V}(S) = S\,\dot{S} = S\,(\lambda_1 \dot{e}(t) + \ddot{e}(t)) = 0$$

$$u_{eq_{IO}} = \ddot{r} + 2\,\xi\,\dot{x} - \chi\,v - \frac{1}{2}\,x(1 - x^2) - F - \lambda_1\,\dot{e}$$

FOSMC

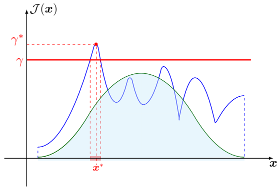
$$S(t) = \lambda_1\,e(t) + \lambda_2\,\frac{d\ e(t)}{dt} + \lambda_3\,\mathcal{D}^{\alpha} e(t)$$

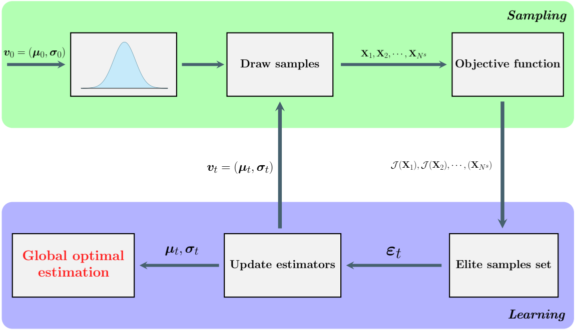
$$\dot{V}(S) = S\,\dot{S} = S\,(\lambda_1\,\dot{e}(t) + \lambda_2\,\frac{d^2\ e(t)}{dt^2} + \lambda_3\,\mathcal{D}^{\alpha+1} e(t)) = 0$$

$$u_{eq_{FO}} = \ddot{r} + 2\,\xi\,\dot{x} - \chi\,v - \frac{1}{2}\,x(1 - x^2) - F - \frac{\lambda_1}{\lambda_2}\,\dot{e} - \frac{\lambda_3}{\lambda_2}\,\mathcal{D}^{\alpha+1} e$$

Cross-Entropy method

“Transform” the optimization problem into a rare-event estimation problem.





$$\mathcal{P}\{\mathcal{J}(x) \geq \gamma\} \approx 0 \quad \text{for} \quad \gamma \approx \gamma^*$$

$\mathcal{J}(x) \geq \gamma$ is a rare event

Cunha, A. Enhancing the performance of a bistable energy harvesting device via the cross-entropy method. Nonlinear Dyn 103, 137–155 (2021).
https://doi.org/10.1007/s11071-020-06109-0

Control Effort:

$$ISU = \frac{1}{T} \int_0^T \Lambda\, u(t)^2 dt$$

Average Power:

$$P_{avg} = \frac{1}{T} \int_0^T \Lambda\, v(t)^2 dt$$

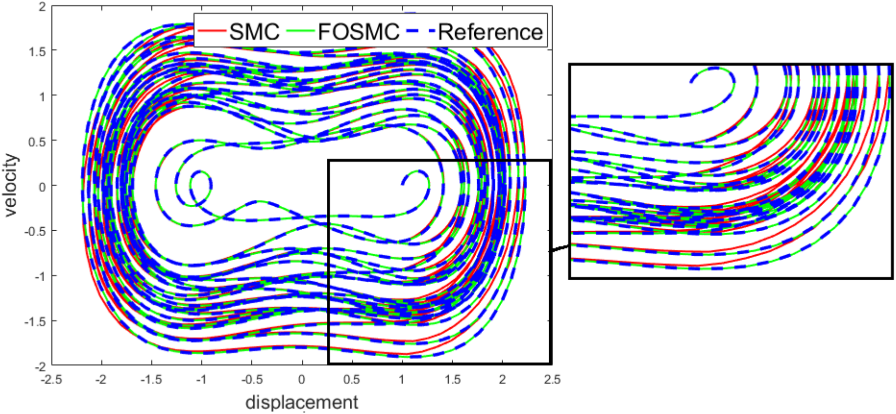
Minimize $\mathcal{F}$ via Cross-Entropy optimization

$$\mathcal{F} = \frac{ISU}{P_{avg}}$$

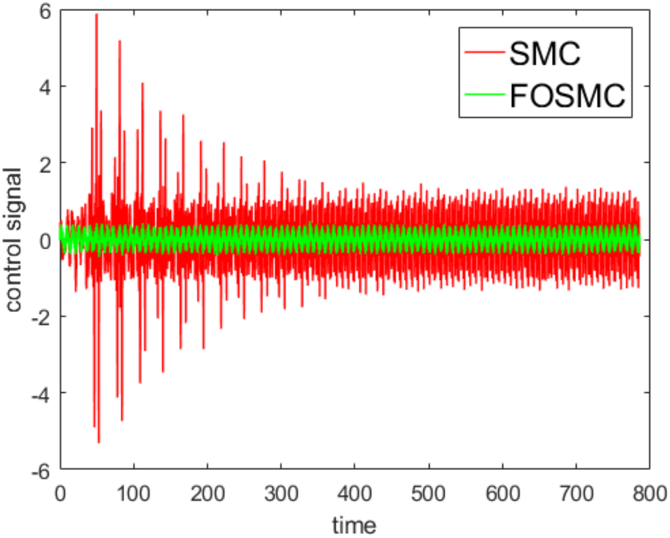
Results

parameters	SMC	FOSMC	
ϵ	0.1	0.1	
k	13.07	17.27	
λ_1	3.21	11.39	
λ_2	-	0.56	
λ_3	-	10.74	
α	-	1.65	
performance			
ISU	378.37	25.11	**-93.36%**
P_{avg}	0.0183	0.0183	
$\mathcal{F}$	20697	1374.6	

Phase-plane graphic



Control Signal comparative



Final Remarks

Conclusion:

- The FOSMC can obtain configurations that reduce the control energy expenditure, making the control signal up to 93% more economical (less control effort), and still maintaining the same generated energy.
- In addition, the Cross-Entropy optimization method proved to be effective to find the optimal performance, even with increasing parameters, being a method with potential to deal with fractional controllers.

Acknowledgments

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CAPES

Thank you for your attention!

Questions?!

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