An optimal fractional LQR-based control approach applied to a cart-pendulum system

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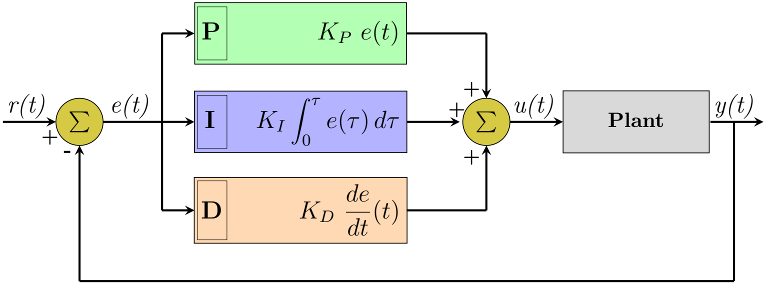
Classical PID-controller

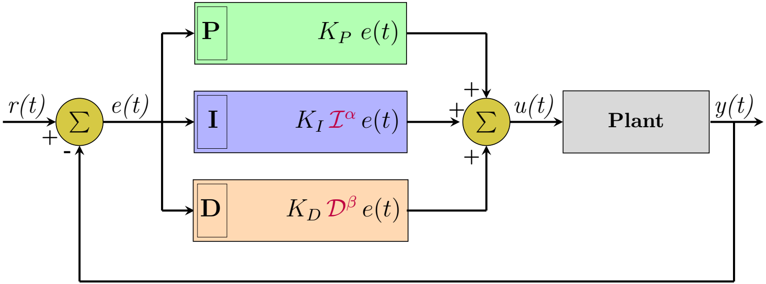


Control signal based on integer-order operators:

$$u(t) = K_P\, e(t) + K_I \int e(\tau)\, d\tau + K_D \frac{d}{dt} e(t)$$

$\boxed{\text{Tunning parameters: } K_P,\ K_I,\ K_D.}$

Fractional-order PID-controller



Control signal based on fractional-order operators:

$$u(t) = K_P\, e(t) + K_I\, \mathcal{I}^\alpha\, e(t) + K_D\, \mathcal{D}^\beta\, e(t)$$

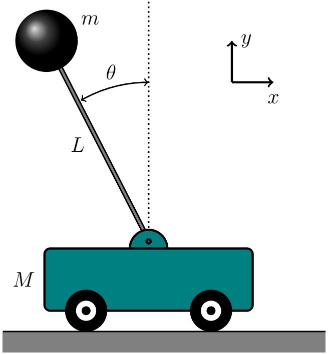
Tunning parameters: $K_P,\ K_I,\ K_D,\ \alpha,\ \beta.$

Research objectives

This research has the following objective:

- Evaluate the performance of a fractional-order controller in comparison with a integer-order controller;
- To compare the optimization methods Genetic Algorithm and Cross-Entropy to obtain the one that best performs together with the fractional controller.

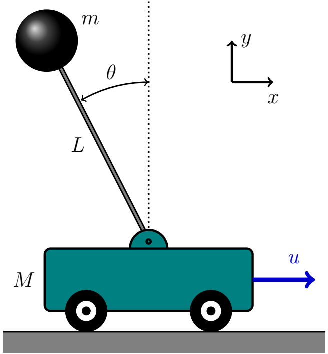
Inverted cart-pendulum system



$$m\,L\,\cos\theta(t)\,\ddot{x}(t) + (J + m\,L^2)\,\ddot{\theta}(t) - m\,g\,L\,\sin\theta(t) = 0$$

$$(m + M)\,\ddot{x}(t) + m\,L\,\cos\theta(t)\,\ddot{\theta}(t) - m\,L\,\sin\theta(t)\,\dot{\theta}^2(t) = 0$$

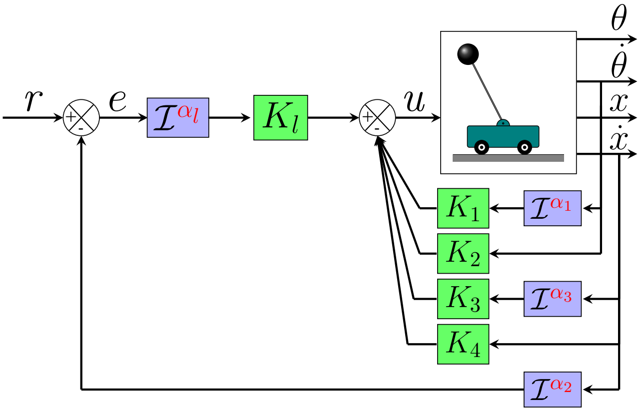
Inverted cart-pendulum system



$$m\,L\,\cos\theta(t)\,\ddot{x}(t) + (J + m\,L^2)\,\ddot{\theta}(t) - m\,g\,L\,\sin\theta(t) = 0$$

$$(m + M)\,\ddot{x}(t) + m\,L\,\cos\theta(t)\,\ddot{\theta}(t) - m\,L\,\sin\theta(t)\,\dot{\theta}^2(t) = u$$

Proposed fractional LQR-based controller



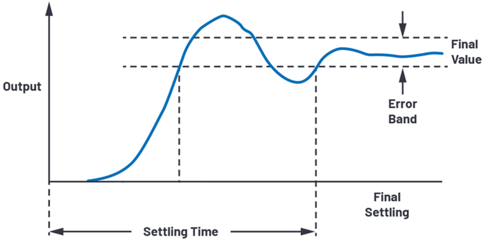
$\mathcal{I}^{\alpha}$ has an additional parameter to tune the controller, if $\alpha \in \mathbb{Z}$ (integer-order) and if $\alpha \notin \mathbb{Z}$ (fractional-order).

Controller performance index

ISE - Integrated Square Error $\rightarrow ISE = \int_0^t e(t)^2 dt$

ISU - Integrated Square Control Signal $\rightarrow ISU = \int_0^t u(t)^2 dt$

ST - Settling Time $\rightarrow$



*Pictures obtained from Google Images, several sources.

LQR-based fitness function

- Evaluating the indexes separately $\Rightarrow$ **high computational cost**
- Assembled all indexes into a single objective function
- Linear quadratic regulator (LQR) method $\Rightarrow$ **Quadratic cost function with state variables and control signal**

Minimize

$$\mathcal{F} = \int_0^{\tau} \left(w_1 \frac{x^2(t)}{ISE_{x_I}} + w_2 \frac{\theta^2(t)}{ISE_{\theta_I}} + w_3 \frac{u^2(t)}{ISU_{u_I}} + w_4 \frac{\dot{x}^2(t)}{ISE_{\dot{x}_I}} + w_5 \frac{\dot{\theta}^2(t)}{ISE_{\dot{\theta}_I}} \right) dt$$

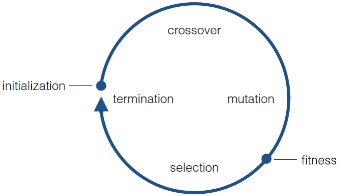
such that

$$P = 100\,(\max(0, ST_\theta - ST_\theta^*))^2 + 100\,(\max(0, ST_x - ST_x^*))^2$$

Applied optimization methods

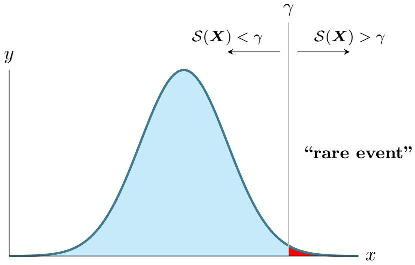
- **Genetic algorithms (GA)**

Method based on the biological concept of natural evolution.



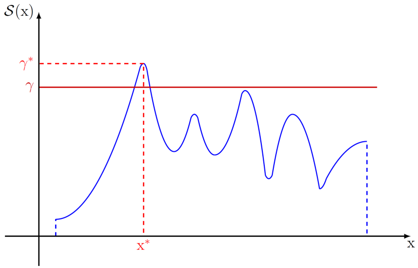
- **Cross-Entropy method (CE)**

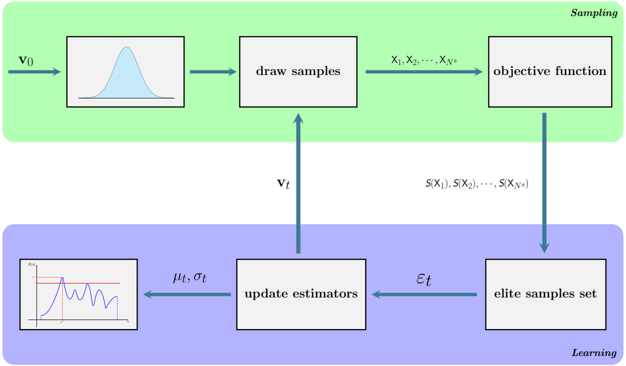
"Transform" the optimization problem into a rare-event estimation problem.



Rubinstein, R. Y., Kroese, D. P. **Simulation and the Monte Carlo Method**. Wiley, 3rd edition (2017).

Cross-Entropy (CE) method

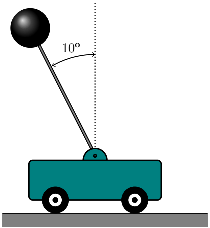




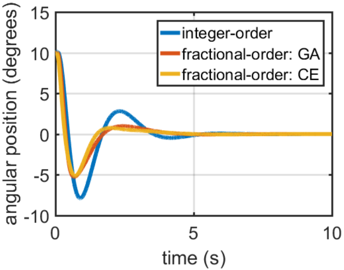
$$\mathcal{P}\{\mathcal{S}(\mathrm{x}) \geq \gamma\} \approx 0 \quad \text{for} \quad \gamma \approx \gamma^*$$

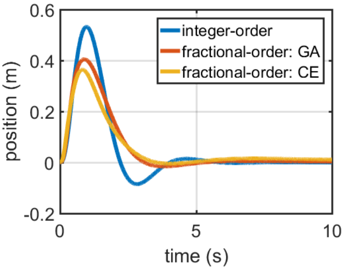
$\mathcal{S}(\mathrm{x}) \geq \gamma$ is a rare event

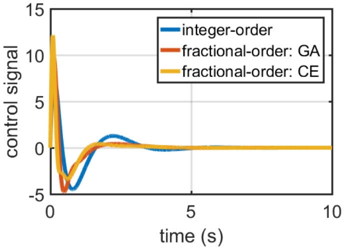
Tunning strategy comparison

initial conditions	controller	tunning methods
	integer-order	Pole Placement method
	fractional-order	Genetic algorithms method
		Cross-Entropy method

Numerical experiments





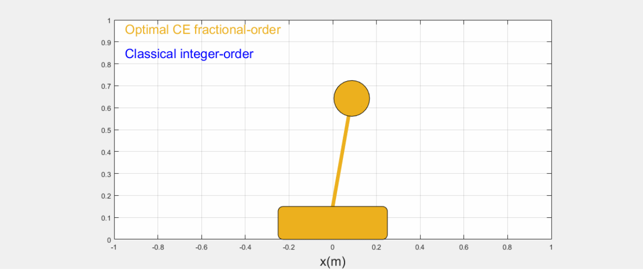


Numerical experiments

Results compared to integer controller		
performance index	fractional controller (GA)	fractional controller (CE)
ISE θ	-45%	-55%
ISE $\dot{\theta}$	-11%	-17%
ISE x	-29%	-47%
ISE $\dot{x}$	-46%	-57%
ISU	-6%	-9%
ST θ	-19%	-19%
ST x	-3%	1%
fitness function	-27%	-37%

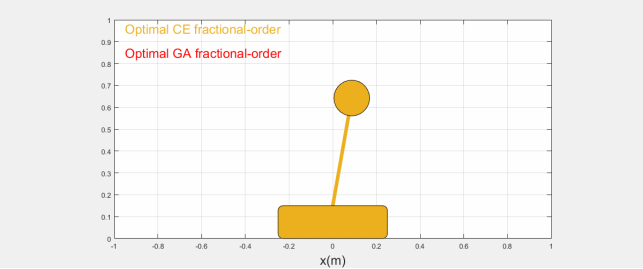
methods performance	**GA**	**CE**
function evaluation	29520	4400

Controllers in action



Optimal CE fractional-order
vs
Classical integer-order

Optimal CE fractional-order
vs
Optimal GA fractional-order



Final Remarks

- As seen in the literature, fractional-order controller can enhance the control system performance
  - Improve all observed performance indices at the same time
  - Makes the control more efficient and faster with less error and requiring less effort of control
- However, the novelty is the use of the Cross-Entropy method to improve the efficiency of the fractional controller (optimization with many variables)
  - Presenting better processing performance (faster) than the GA method
  - Finding better answers that the GA method was unable to find

Acknowledgments

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CAPES

Thank you for your attention!

Questions?!

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