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Abstract In agricultural industry, the process of orchards spraying is of extreme importance to avoid losses and reduction of quality in the products. In orchards spraying process an equipment called sprayer tower is used. It consists of a reservoir and fans mounted over an articulated tower, which is supported by a vehicle suspension. Due to soil irregularities this equipment is subject to random loads, which may hamper the proper dispersion of the spraying fluid. This work presents the construction of a consistent stochastic model of uncertainties to describe the non-linear dynamics of an orchard sprayer tower. In this model, the mechanical system is described by as a multi-body with three degrees of freedom, and random loadings as a harmonic random process. Uncertainties are taken into account through a parametric probabilistic approach, where maximum entropy principle is used to specify random parameters distributions. The propagation of uncertainties through the model is computed via Monte Carlo method.

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1 Introduction

The spraying process of orchards has extreme importance in fruit growing, not only to prevent the economic damages associated with the loss of a production, but also to ensure the quality of the fruit that will arrive the final consumer. This process uses an equipment, called *sprayer tower*, which is illustrated in Figure 1. This equipment is composed by a vehicle suspension and a support tower, equipped with several fans, and in a typical operating condition it vibrates nonlinearly [8, 3].

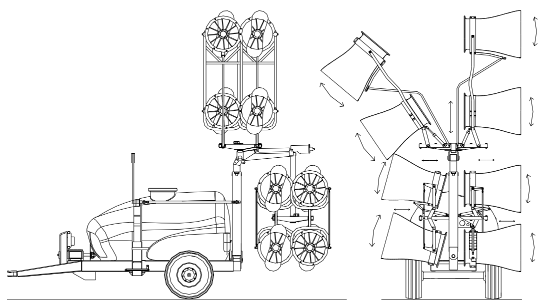


Fig. 1 Sketch of the orchard sprayer tower (courtesy of Máquinas Agrícolas Jacto S/A).

Understanding the dynamics of this equipment is essential to its design and also to discover operating conditions that may be harmful to the spraying process. In this sense, this work aims to model the nonlinear dynamics of the sprayer tower, taking into account the deterministic and stochastic aspects. In particular, it is of great interest to predict the maximum amplitude of the lateral vibration of the structure, and verify if the uncertainties in the soil-induced loadings are capable of generating undesirable levels of oscillations.

The rest of this chapter is organized as follows. In section 2 it is presented the deterministic model and analysis for the mechanical system. A stochastic model to take into account the uncertainties associated to the model parameters, and the corresponding stochastic simulations are shown in section 3. Finally, in section 4, final remarks are highlighted.

2 Deterministic analysis

In the modeling process developed here, the mechanical system is considered as the multibody system illustrated in Figure 2. The masses of the chassis and the tank are assumed to be concentrated at the bottom of the double pendulum, as a point mass denoted by m_1. On the other hand, the point mass m_2, at the top of the double pendulum, takes into account the masses of the fans. The point of articulation between the moving suspension and the tower is denoted by P and its distance to the suspension center of gravity is L_1. The junction P has torsional stiffness k_T, and damping torsional coefficient c_T. The tower has length L_2, and is considered to be massless. The left wheel of the vehicle suspension is represented by a pair spring/damper with constant respectively given by k_1 and c_1 located at a distance B_1 from suspension center line, and it is subject to a vertical displacement y_{e1}. Similarly, the right wheel is represented by a pair spring/damper characterized by k_2 and c_2, it is B_2 away from suspension center line, and displaces vertically y_{e2}. The moments of inertia of the suspension and of the tower, with respect to their centers of gravity, are respectively denoted by I_1 and I_2. The acceleration of gravity is denoted by g. Finally, introducing the inertial frame of reference XY, the vertical displacement of the suspension is measured by y_1, while its rotation is computed by ϕ_1, and the rotation of the tower is denoted by ϕ_2. Therefore, this model, which was developed by [7, 8], has three degrees of freedom: y_1, ϕ_1 and ϕ_2.

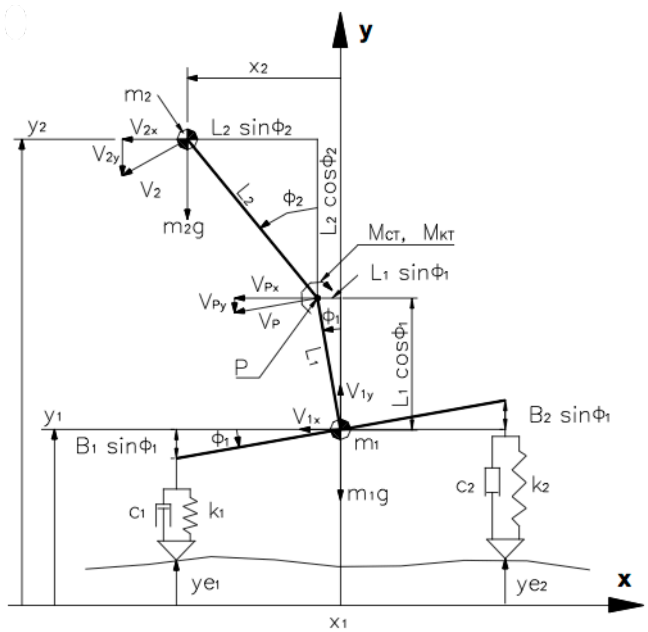


Fig. 2 Schematic representation of the mechanical-mathematical model: an inverted double pendulum, mounted on a moving suspension.

It can be deduced from the geometry of Figure 2 that tower horizontal (lateral) displacement is given by

$$x_2 = -L_1 \sin\phi_1 - L_2 \sin\phi_2. \tag{1}$$

Typically, the sprayer tower moves on an irregular terrain during its operation, which induces oscillatory displacements (loads) in the tires. In order to reproduce a typical load (induced by soil) the left and right tires displacements are respectively assumed to be periodic functions in time, out of phase, with the same amplitude, and a single frequencial component,

$$y_{e1}(t) = A\,\sin(\omega t)\,, \;\; \text{and} \;\; y_{e2}(t) = A\,\sin(\omega t + \rho)\,, \tag{2}$$

where A and ω respectively denotes the amplitude and frequency of the tires displacements, and ρ is the phase shift between the two tires.

Employing a Lagrangian formalism to obtain the nonlinear dynamical system associated to the mechanical system, the following set of ordinary differential equations is obtained

$$[M]\begin{pmatrix}\ddot{y}_1\\ \ddot{\phi}_1\\ \ddot{\phi}_2\end{pmatrix} + [N]\begin{pmatrix}\dot{y}_1^2\\ \dot{\phi}_1^2\\ \dot{\phi}_2^2\end{pmatrix} + [C]\begin{pmatrix}\dot{y}_1\\ \dot{\phi}_1\\ \dot{\phi}_2\end{pmatrix} + [K]\begin{pmatrix}y_1\\ \phi_1\\ \phi_2\end{pmatrix} = \mathbf{g} - \mathbf{h}, \tag{3}$$

where $[M]$, $[N]$, $[C]$ and $[N]$ are 3×3 (configuration dependent) real matrices, respectively, defined by

$$[M] = \begin{bmatrix} m_1+m_2 & -m_2\,L_1\,\sin\phi_1 & -m_2\,L_2\,\sin\phi_1 \\ -m_2\,L_1\,\sin\phi_1 & I_1+m_2\,L_1^2 & m_2\,L_1\,L_2\,\cos(\phi_2-\phi_1) \\ -m_2\,L_2\,\sin\phi_1 & m_2\,L_1\,L_2\,\cos(\phi_2-\phi_1) & I_2+m_2\,L_2^2 \end{bmatrix}, \tag{4}$$

$$[N] = \begin{bmatrix} 0 & -m_2\,L_1\,\cos\phi_1 & -m_2\,L_2\,\cos\phi_2 \\ 0 & 0 & -m_2\,L_1\,L_2\,\sin(\phi_2-\phi_1) \\ 0 & -m_2\,L_1\,L_2\,\sin(\phi_2-\phi_1) & 0 \end{bmatrix}, \tag{5}$$

$$[C] = \begin{bmatrix} c_1+c_2 & (c_2\,B_2-c_1\,B_1)\cos\phi_1 & 0 \\ (c_2\,B_2-c_1\,B1)\cos\phi_1 & c_T+(c_1\,B_1^2+c_2\,B_2^2)\cos^2\phi_1 & -c_T \\ 0 & -c_T & c_T \end{bmatrix}, \tag{6}$$

$$[K] = \begin{bmatrix} k_1+k_2 & 0 & 0 \\ (k_2\,B_2-k_1\,B1)\cos\phi_1 & k_T & -k_T \\ 0 & -k_T & k_T \end{bmatrix}, \tag{7}$$

and let $\mathbf{g}$, and $\mathbf{h}$ be (configuration dependent) vectors in $\mathbb{R}^3$, respectively, defined by

$$\mathbf{g} = \begin{pmatrix} (k_2\,B_2 - k_1\,B_1)\sin\phi_1 + (m_1 + m_2)g \\ (k_1\,B_1^2 + k_2\,B_2^2)\sin\phi_1\,\cos\phi_1 - m_2\,g\,L_1\,\sin\phi_1 \\ -m_2\,g\,L_2\,\sin\phi_2 \end{pmatrix}, \tag{8}$$

and

$$\mathbf{h} = \begin{pmatrix} k_1\,y_{e1} + k_2\,y_{e2} + c_1\,\dot{y}_{e1} + c_2\,\dot{y}_{e2} \\ -B_1\,\cos\phi_1\,(k_1\,y_{e1} + c_1\,\dot{y}_{e1}) + B_2\,\cos\phi_1\,(k_2\,y_{e2} + c_2\,\dot{y}_{e2}) \\ 0 \end{pmatrix}. \tag{9}$$

Considering the static equilibrium configuration as initial condition, the resulting nonlinear initial value problem is integrated using a Runge-Kutta method [1]. The evolution of this nonlinear dynamic system is investigated in the interval $[t_0, t_f] = [0, 30]\,s$, adopting for the physical and geometrical parameters the nominal (deterministic) values shown in Table 1.

Table 1 Nominal parameters used in the simulations of the mechanical system.

parameter	value	unit
m_1	6500	kg
m_2	800	kg
L_1	0.2	m
L_2	2.4	m
I_1	6850	$kg\,m^2$
I_2	6250	$kg\,m^2$
k_1	465×10^3	N/m
k_2	465×10^3	N/m
c_1	5.6×10^3	$N/m/s$
c_2	5.6×10^3	$N/m/s$
B_1	0.85	m
B_2	0.85	m
k_T	45×10^3	N/rad
c_T	50×10^3	$Nm/rad/s$
A	0.15	m
w	2π	rad/s
ρ	$\pi/4$	rad

The time series corresponding to the tower horizontal (lateral) dynamics x_2 can be seen in Figure 3, while the corresponding phase space trajectories projections (in $\mathbb{R}^3$ and $\mathbb{R}^2$) are presented in Figure 4.

From a qualitative point of view, the simulation results shown in Figures 3 and 4 allow one to see that, after a transient regime of approximately $5\,s$, the sprayer tower dynamics accumulate into a limit cycle. Hence, for any practical purpose, the permanent behavior is periodic. In addition, from the quantitative point of view, these same results show that the tower can oscillate with amplitude bigger then

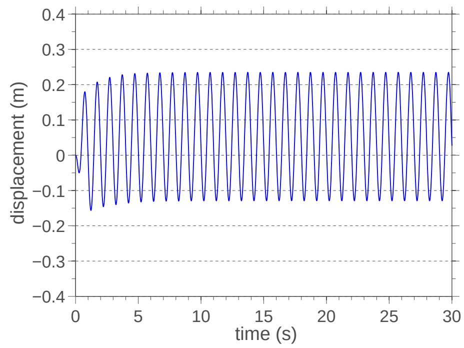


Fig. 3 Time series of tower horizontal dynamics: x_2.

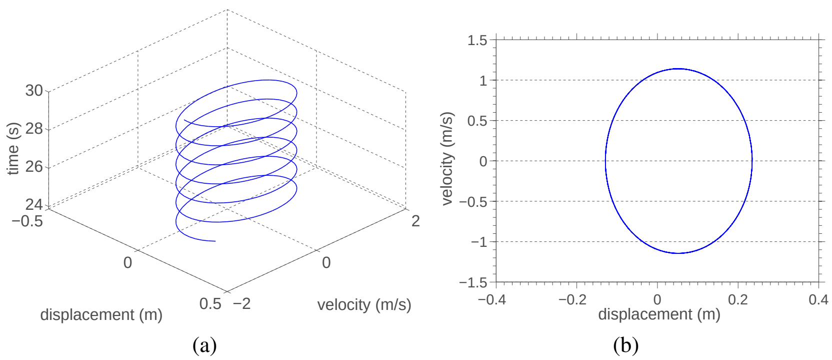


Fig. 4 Projections of horizontal dynamics phase space trajectory. (a) x_2 atractor in $\mathbb{R}^3$; (b) x_2 atractor in $\mathbb{R}^2$.

$0.2\,m$, which is the value of B_1 and B_2. This shows that the sprayer tower can reach a critical level of horizontal (lateral) vibration, which can be harmful to the spraying process.

This analysis used a deterministic model for dynamics, where amplitude and frequency of external loading are assumed to be known. However, in practice, amplitude and frequency of external excitation are not known with precision, which induces uncertainties to the dynamic response of the structure. Taking into account the effect of these uncertainties on the model response is essential for a robust design, being the purpose of the next section.

3 Stochastic analysis

Consider a probability space $(\Theta, \Sigma, \mathbb{P})$, where Θ is a sample space, Σ is a σ-field over Θ, and $\mathbb{P} : \Sigma \to [0,1]$ is a probability measure. In this probabilistic space, the amplitude A and the frequency ω are respectively modeled by the independent random variables $\mathbb{A} : \Sigma \to \mathbb{R}$ and $\mathbb{w} : \Sigma \to \mathbb{R}$.

To specify the distribution of these random parameters, based only on theoretical information known about them, the maximum entropy principle is employed [5, 10]. For $\mathbb{A}$, which is a positive parameter, it is assumed that: (i) the support of the probability density function (PDF) is the positive real line, i.e., $\texttt{Supp}\, p_{\mathbb{A}} = (0, +\infty)$; (ii) the mean value is known, i.e. $\mathbb{E}\{\mathbb{A}\} = \mu_{\mathbb{A}} \in (0, +\infty)$; and (iii) $\mathbb{A}^{-1}$ is a second order random variable, so that $\mathbb{E}\{\ln \mathbb{A}\} = q, \; |q| < +\infty$. Besides that, for $\mathbb{w}$, that is also a positive parameter, the only known information is assumed to be the support $\texttt{Supp}\, p_{\mathbb{w}} = [\omega_1, \omega_2] \subset (0, +\infty)$.

Consequently, the distributions which maximize the entropy have the following PDFs

$$p_{\mathbb{A}}(a) = \mathbb{1}_{(0,+\infty)}(a)\, \frac{1}{\mu_{\mathbb{A}}} \left(\frac{1}{\delta_{\mathbb{A}}^2}\right)^{\left(\frac{1}{\delta_{\mathbb{A}}^2}\right)} \frac{1}{\Gamma(1/\delta_{\mathbb{A}}^2)} \left(\frac{a}{\mu_{\mathbb{A}}}\right)^{\left(\frac{1}{\delta_{\mathbb{A}}^2}-1\right)} \exp\left(-\frac{a}{\delta_{\mathbb{A}}^2 \mu_{\mathbb{A}}}\right), \quad (10)$$

and

$$p_{\mathbb{w}}(\omega) = \mathbb{1}_{[\omega_1,\omega_2]}(\omega)\, \frac{1}{\omega_2 - \omega_1}, \quad (11)$$

which correspond, respectively, to the gamma and uniform distributions. In the above equations $\mathbb{1}_X(x)$ denotes the indicator function of the set X, and $0 \leq \delta_{\mathbb{A}} = \sigma_{\mathbb{A}}/\mu_{\mathbb{A}} < 1/\sqrt{2}$ is a dispersion parameter, being $\sigma_{\mathbb{A}}$ the standard deviation of $\mathbb{A}$.

The stochastic simulations reported here adopted, for the random variables $\mathbb{A}$ and $\mathbb{w}$, the following parameters $\mu_{\mathbb{A}} = 0.15\,m$, $\sigma_{\mathbb{A}} = 0.2 \times \mu_{\mathbb{A}}$, and $[\omega_1, \omega_2] = [0, 2] \times 2\pi\, rad/s$.

Due to the randomness of $\mathbb{A}$ and $\mathbb{w}$, the tire displacements are now described by the following random processes

$$\mathbb{y}_{e1}(t) = \mathbb{A} \sin(\mathbb{w} t)\,, \;\text{ and }\; \mathbb{y}_{e2}(t) = \mathbb{A} \sin(\mathbb{w} t + \rho)\,. \quad (12)$$

Therefore, the dynamics of the mechanical system evolves (almost sure) according to the following system of stochastic differential equations

$$[\mathbb{M}] \begin{pmatrix} \ddot{\mathbb{y}}_1 \\ \ddot{\mathbb{\phi}}_1 \\ \ddot{\mathbb{\phi}}_2 \end{pmatrix} + [\mathbb{N}] \begin{pmatrix} \dot{\mathbb{y}}_1^2 \\ \dot{\mathbb{\phi}}_1^2 \\ \dot{\mathbb{\phi}}_2^2 \end{pmatrix} + [\mathbb{C}] \begin{pmatrix} \dot{\mathbb{y}}_1 \\ \dot{\mathbb{\phi}}_1 \\ \dot{\mathbb{\phi}}_2 \end{pmatrix} + [\mathbb{K}] \begin{pmatrix} \mathbb{y}_1 \\ \mathbb{\phi}_1 \\ \mathbb{\phi}_2 \end{pmatrix} = \mathbb{g} - \mathbb{h}, \;\; a.s. \quad (13)$$

where the real-valued random matrices/vectors $[\mathbb{M}]$, $[\mathbb{N}]$, $[\mathbb{C}]$, $[\mathbb{K}]$, $\mathbb{g}$ and $\mathbb{h}$ are stochastic versions of the matrices/vectors $[M]$, $[N]$, $[C]$, $[K]$, $\mathbf{g}$ and $\mathbf{h}$.

Monte Carlo (MC) method [6, 4] is employed to compute the propagation of uncertainties of the random parameters through the nonlinear dynamics. In this method, realizations of the random parameters are generated. Each one defines a new deterministic nonlinear dynamical system, which is integrated using the procedure described in section 2. Then, statistics of the generated data is calculated to access the stochastic nonlinear dynamics.

The map $n_s \in \mathbb{N} \mapsto \texttt{conv}(n_s) \in \mathbb{R}$, used to evaluate the convergence of MC simulation, is defined by

$$\texttt{conv}(n_s) = \left(\frac{1}{n_s} \sum_{n=1}^{n_s} \int_{t=t_0}^{t_f} \left(\mathbb{y}_1(t,\theta_n)^2 + \mathbb{\phi}_1(t,\theta_n)^2 + \mathbb{\phi}_2(t,\theta_n)^2 \right) \, dt \right)^{1/2}, \qquad (14)$$

where n_s is the number of MC realizations. See [9] for further details.

As can be seen in Figure 5, which shows the evolution of $\texttt{conv}(n_s)$ as a function of n_s, for $n_s = 1024$ the metric value is stationary. So, all MC simulations reported in this work use $n_s = 1024$ to address the stochastic dynamics.

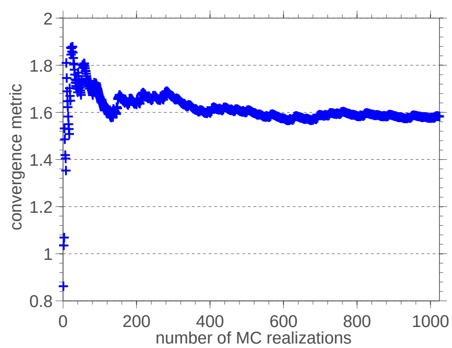


Fig. 5 Illustration of MC convergence metric as function of the number of realizations.

Realizations of tower horizontal (lateral) dynamics x_2 time series can be seen in Figure 3, as well as the corresponding 95% of probability confidence band. A wide variability in x_2 can be observed. This fact may also be noted in Figure 7, which shows the evolution of the sample mean and standard deviation of x_2. Note that, in all the interval of analysis, the standard deviation is bigger than the mean value, which indicates a significant spreading of the realizations with respect to the mean.

In Figure 8 are presented estimations of the (normalized[1]) probability density function (PDF) of the tower horizontal vibration, for different instants of time. In all cases it is possible to observe asymmetries with respect to mean and multimodal behavior. In Figure 9 the reader finds the time average of the tower horizontal dynamics PDF, which reflects the multimodal characteristic observed in the instants of time analyzed in Figure 8.

[1] In this context, the meaning of normalized is zero mean and unity standard deviation.

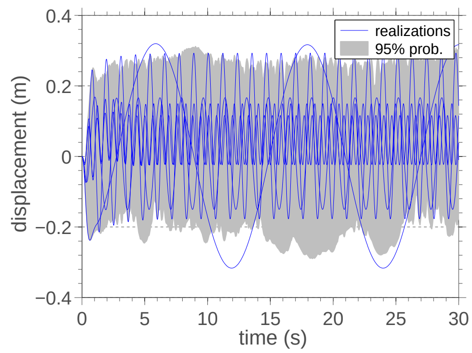


Fig. 6 Confidence envelope and some realizations for tower horizontal displacement.

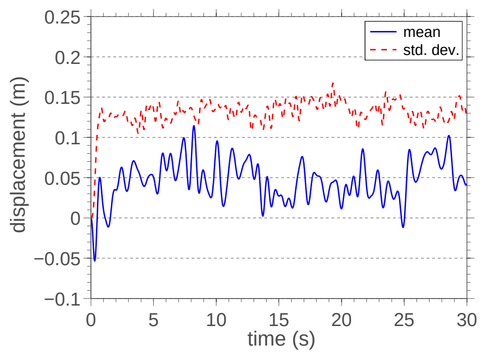


Fig. 7 Sample mean/standard deviation for tower horizontal displacement.

By way of reference, a lateral vibration with an amplitude level greater than 10% of the B_1 value will be considered high, i.e.,

$$\text{large vibration} = \left\{ \mathbb{x}_2(t) > 10\% \text{ of } B_1 \right\}. \tag{15}$$

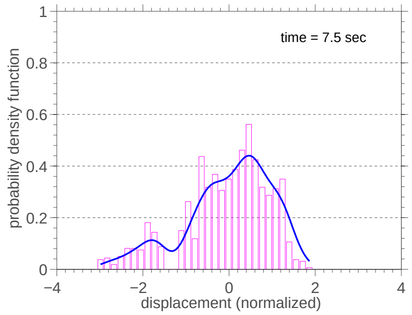
For any instant t, it is of interest to determine the value of

$$\mathbb{P}\left\{ \mathbb{x}_2(t) > 10\% \text{ of } B_1 \right\} = 1 - \mathbb{P}\left\{ \mathbb{x}_2(t) \leq 10\% \text{ of } B_1 \right\}, \tag{16}$$

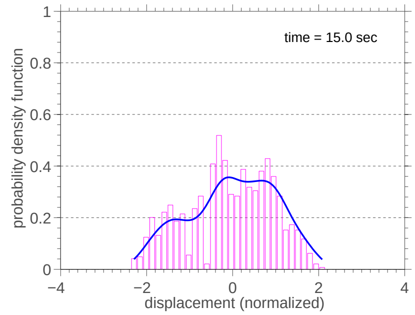
where

$$\mathbb{P}\left\{ \mathbb{x}_2(t) \leq 10\% \text{ of } B_1 \right\} = \int_{-\infty}^{B_1/10} dF_{\mathbb{x}_2(t)}(x_2). \tag{17}$$

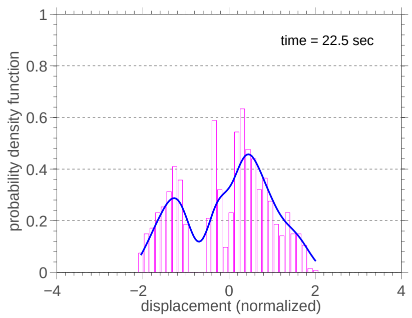
In Figure 10 the reader can see the value of $\mathbb{P}\left\{ \mathbb{x}_2(t) > 10\% \text{ of } B_1 \right\}$ as function of time. Note that, for almost all the instants, the probability of an unwanted level



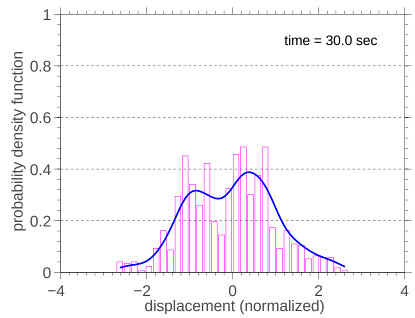
(a) $t = 7.5\,s$



(b) $t = 15.0\,s$



(c) $t = 22.5\,s$



(d) $t = 30.0\,s$

Fig. 8 Probability density function of tower horizontal dynamics (at different instants).

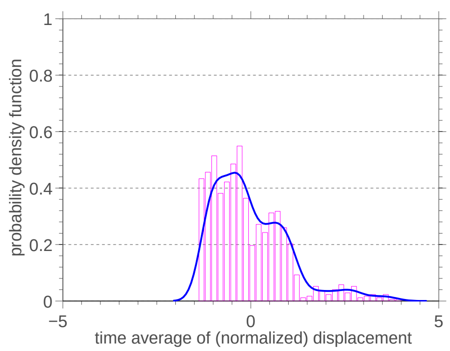


Fig. 9 Time average of tower horizontal dynamics probability density function.

of vibration may be significative values, being this value almost always greater than 40%.

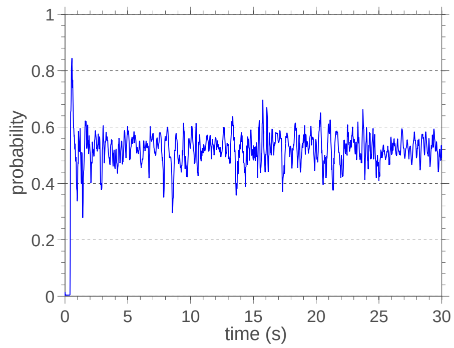


Fig. 10 Evolution of the probability of large horizontal vibrations.

4 Final remarks

This work presented the study of the nonlinear dynamics of an orchard tower sprayer subjected to random excitations due to soil irregularities. Random loadings were taken into account through a parametric probabilistic approach, where the external loading was modeled as a harmonic random process, with random parameters distributions specified by maximum entropy principle. Monte Carlo simulations of the stochastic dynamics reveal a wide range of possible responses for the mechanical system, and show the possibility of large lateral vibrations being developed during the sprayer operation.

Preliminary results of this work were presented in [2], and deeper analysis of this problem, including a more complex stochastic loading, can be found in [3].

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