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TECHNICAL PAPER

An optimizationless stochastic volterra series approach for nonlinear model identification

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Abstract
Volterra series is a widely used tool for identifying physical systems with polynomial nonlinearities. In this approach, the Volterra kernels expanded using Kautz functions can be identified using several techniques to optimize the filters' poles. This methodology is very efficient when the system observations are not subject to high noise-induced variabilities (uncertainties). However, this optimization procedure may not be effective when the uncertainty level is increased since the optimal value might be susceptible to small perturbations. Seeking to overcome this weakness, the present work proposes a new stochastic method of identification based on the Volterra series, which does not solve an optimization problem. In this new approach, the Volterra kernels are described as stochastic processes. The parameters of Kautz filters are considered independent random variables so that their probability distribution captures the variabilities. The effectiveness of the new technique is tested experimentally in a nonlinear mechanical system. The results show that the identified stochastic Volterra kernels can reproduce the nonlinear dynamics characteristics and the data variability.

Keywords Stochastic Volterra series · Uncertain systems · Nonlinear systems · Stochastic models

1 Introduction

Systems identification is an essential topic of research related to dynamics control since the prediction of complex systems behavior, subjected to nonlinearities and uncertainties, is an everyday task in engineering sciences [1]. This problem is relatively simple and well solved when the linearity hypothesis is applicable. However, a lot of real systems may exhibit nonlinear phenomena, depending on factors such as (i) operational conditions, (ii) constitutive relationships of materials, and (iii) application of excitations and loads [2].

The literature presents several methods that can be used to identify systems subjected to nonlinear phenomena, such as Hilbert transform, NARMAX Models, High-Order Frequency Response Functions (HOFRFs), and Restoring Force Surface (RFS) method [3]. The use of Volterra series expanded in orthonormal bases-mainly the Kautz functions-is also frequently reported in the literature to describe nonlinear systems, with satisfactory results in oscillatory phenomena with polynomial nonlinearities [4–6]. The major difficulty for the use of Kautz functions based Volterra series is related to the definition of the basis parameters, which requires the solution of a nonlinear optimization problem [7, 8], a task that is computationally expensive and very sensitive to the initial guess for the parameters. To make this task even more difficult, every real system is subject to uncertainties, which manifest themselves via (i) measurements noise in the system observations (experimental data); (ii) variabilities of the real system concerning its simple configuration (due to geometric imperfections, manufacturing

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irregularities, environmental conditions, etc.); or (iii) lack of knowledge about the model structure (ignorance about the system physics) [9]. These uncertainties make the solution to the optimization problem mentioned above even more complex and unreliable. Nevertheless, to the best of the authors' knowledge, few papers in the literature consider the approach of the Volterra series (with Kautz bases) applied to the identification of systems taking into account the variabilities (uncertainties) present in the measured data [10, 11]. In this way, to conduct a careful identification process and, consequently, obtain an accurate response prediction, it is essential to consider the effect of these uncertainties.

Some initiatives in this sense can be found in the literature, such as [12], which presents the formulation of orthonormal bases in robust control, considering parametric uncertainties in linear systems. Additionally, some expansions to represent nonlinear Volterra kernels in an uncertain scenery are shown with generalized orthonormal basis functions (GOBF) in [13] and Kautz functions in [14, 15]. In this latter work, Rosa et al. [15] consider Volterra kernels as uncertain objects, and an optimization procedure is employed to determine the Kautz bases parameters to describe the kernel's limits and minimize the uncertainties influence. In [10, 11], the authors considered the stochastic version of the Volterra series, implemented using random Kautz functions. However, an optimization procedure to determine the pole values that define the Kautz functions was performed for each Monte Carlo run. This optimization problem is non-convex, which requires global search algorithms, e.g., genetic algorithms (very computationally costly) since the use of gradient-based optimizers becomes highly dependent on the initial guess. This procedure can be very costly when considering a high number of MC runs, making the development of new methodologies essential for the practical use of the stochastic model.

This paper proposes to address the problem of nonlinear system identification in the presence of uncertainties-for instance, noise in experimental observations-using a stochastic version of Kautz functions based on the Volterra series, without the use of an optimization procedure to determine exact values for the Kautz parameters. In this context, the Volterra series is defined in a parametric probabilistic framework. A stochastic model of uncertainties for the nonlinear system under analysis is identified, where model parameters are described by independent random variables (instead of deterministic values), and the system response is treated as a random process (rather than a deterministic function of time). One advantage of this approach is that it allows the construction of confidence bands for the Volterra kernels, i. e., it provides more robust predictions for the model output once it considers potential variabilities of the system response. Furthermore, the main advantage of the proposed methodology is that it does not require the optimization of deterministic Kautz poles values, as they are considered to be random variables. A deterministic optimal value becomes nonrepresentative in situations such as described above because a single-pole value may not represent all the data variation in the measured signals, and an optimization considering data variation with many measured signals can be very costly.

The proposed identification methodology uses a parametric probabilistic approach to address model parameters uncertainties, where Kautz poles are treated as independent random variables. Their probability distributions are obtained conservatively utilizing the maximum entropy principle [16], taking into account only known information about these parameters. Statistics about the nonlinear system response are obtained using the Monte Carlo method [17, 18]. The proposed methodology's effectiveness is tested by identifying a clamped-free beam undergoing large displacements. Measurements were performed on different days to induce variations in the data related to uncertainties in boundary conditions, sensor positions, screws tightening, etc. The results obtained with the new methodology's application have shown that the identified random Kautz poles allow the stochastic model to predict the variability of the system response with a certain level of confidence, ensuring some robustness to the model output concerning the underlying uncertainties. Such robustness cannot be obtained with an optimum pole-based deterministic model.

2 Stochastic identification via Volterra series

2.1 System uncertainties

Uncertainties are classified in the literature as being of two types, aleatory or epistemic. The first type is intrinsic to scenarios with variabilities, such as those described in Sect. 1, and cannot be eliminated, only better characterized. The second type, epistemic uncertainties, is only due to the lack of information. By increasing knowledge about a particular system, these uncertainties can be mitigated [9, 19].

The methodology adopted in this work only considers the aleatory uncertainties, also known as data uncertainties. Therefore, the uncertainties are materialized in variations in the model parameters. In this sense, the authors assume that the Volterra series can produce a reliable representation of nonlinearities present in the system's dynamics. The epistemic uncertainties must be considered in future works.

Additionally, the model parameters subjected to uncertainties are described as random variables or random processes, defined on the probability space $(\Theta, \Sigma, \mathbb{P})$, where Θ is a sample space, Σ is a σ-algebra over Θ, and $\mathbb{P}$ is a probability measure. It is assumed that any random variable

$\theta \in \Theta \mapsto \mathbb{V}(\theta) \in \mathbb{R}$ in this probabilistic setting, with probability distribution $P_{\mathbb{V}}(dv)$ on $\mathbb{R}$, admits a probability density function (PDF) $v \mapsto p_{\mathbb{V}}(v)$ with respect to dv [10].

2.2 Stochastic Volterra series

In this paper, a stochastic version of the Volterra series is employed to identify the nonlinear system of interest. In this sense, the discrete-time Volterra series describes the random system output as

$$\mathbb{y}(\theta,k) = \sum_{\eta=1}^{\infty} \sum_{n_1=0}^{N_1-1} \dots \sum_{n_\eta=0}^{N_\eta-1} \mathbb{H}_\eta(\theta,n_1,\dots,n_\eta) \prod_{i=1}^{\eta} u(k-n_i), \quad (1)$$

where $k \in \mathbb{Z} \mapsto u(k)$ is a deterministic input signal, the random process $(\theta,k) \in \Theta \times \mathbb{Z} \mapsto \mathbb{y}(\theta,k)$ is the system response and $(\theta,n_1,\dots,n_\eta) \in \Theta \times \mathbb{Z}^\eta \mapsto \mathbb{H}_\eta(\theta,n_1,\dots,n_\eta)$ represents the random version of the η-order Volterra kernel. The representation in the form of higher-order convolutions allows one to directly split the system output into a sum of linear and nonlinear contributions (that are random processes in this case).

The main drawback is related to the difficulty of convergence of the series using a large number of terms. However, the problem is reduced expanding the Volterra kernels into an orthonormal basis (this work employs the use of Kautz functions [20, 21]). Once the system response varies during the measuring process, it is natural to model the Kautz functions as random processes, since their definition are associated with the dynamics of the system response $\mathbb{y}(\theta,k)$ and depend on the parameters of damping ratio and natural frequency, which are subjected to uncertainties. Considering the random Kautz orthonormal basis expansion, the random output is written as

$$\mathbb{y}(\theta,k) \approx \sum_{\eta=1}^{\infty} \sum_{i_1=1}^{J_1} \dots \sum_{i_\eta=1}^{J_\eta} \mathbb{B}_\eta\left(\theta,i_1,\dots,i_\eta\right) \prod_{j=1}^{\eta} \mathbb{l}_{i_j}(\theta,k), \quad (2)$$

where $J_1,\dots,J_\eta$ are the number of samples in each orthonormal projections of the Volterra kernels, the random process $(\theta,i_1,\dots,i_\eta) \in \Theta \times \mathbb{Z}^\eta \mapsto \mathbb{B}_\eta(\theta,i_1,\dots,i_\eta)$ represents the η-order random Volterra kernel, expanded in the orthonormal basis, and the random process $(\theta,k) \in \Theta \times \mathbb{Z} \mapsto \mathbb{l}_{i_j}(\theta,k)$ is a simple filtering of the deterministic input signal $u(k)$ by the random Kautz function

$$\mathbb{l}_{i_j}(\theta,k) = \sum_{n_i=0}^{K-1} \Psi_{i_j}(\theta,n_i)u(k-n_i), \quad (3)$$

where $K = \max\{J_1,\dots,J_\eta\}$ and $(\theta,n_j) \in \Theta \times \mathbb{Z} \mapsto \Psi_{i_j}(\theta,n_j)$ represents the random version of the i_j-th Kautz filter. More information on the identification approach based on deterministic Volterra series can be found in [4]. Details about deterministic Kautz functions are given in Sect. 2.3. The reader is also encouraged to see [12].

Finally, the coefficients of the kernels can be estimated considering Monte Carlo (MC) simulations and the least squares method

$$\boldsymbol{\Phi} = (\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}^{\mathrm{T}}\mathbf{Y}, \quad (4)$$

where, considering each stochastic realization θ and the series truncated on the third-order kernel, the matrix $\boldsymbol{\Gamma}$ can be completed with the input signal filtered $\mathbb{l}_{i_j}(\theta,k)$

$$\boldsymbol{\Gamma} = \left[\begin{matrix} \mathbb{l}_{1_1}(\theta,1) & \dots & \mathbb{l}_{J_{11}}(\theta,1) & \mathbb{l}_{1_2}(\theta,1)\mathbb{l}_{1_2}(\theta,1) & \mathbb{l}_{1_2}(\theta,1)\mathbb{l}_{2_2}(\theta,1) & \dots & \mathbb{l}_{J_{22}}(\theta,1)\mathbb{l}_{J_{22}}(\theta,1) \\ \mathbb{l}_{1_1}(\theta,2) & \dots & \mathbb{l}_{J_{11}}(\theta,2) & \mathbb{l}_{1_2}(\theta,2)\mathbb{l}_{1_2}(\theta,2) & \mathbb{l}_{1_2}(\theta,1)\mathbb{l}_{2_2}(\theta,2) & \dots & \mathbb{l}_{J_{22}}(\theta,2)\mathbb{l}_{J_{22}}(\theta,2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbb{l}_{1_1}(\theta,n_s) & \dots & \mathbb{l}_{J_{11}}(\theta,n_s) & \mathbb{l}_{1_2}(\theta,n_s)\mathbb{l}_{1_2}(\theta,n_s) & \mathbb{l}_{1_2}(\theta,n_s)\mathbb{l}_{2_2}(\theta,n_s) & \dots & \mathbb{l}_{J_{22}}(\theta,n_s)\mathbb{l}_{J_{22}}(\theta,n_s) \end{matrix}\right.$$
$$\left.\begin{matrix} \mathbb{l}_{1_3}(\theta,1)\mathbb{l}_{1_3}(\theta,1)\mathbb{l}_{1_3}(\theta,1) & \mathbb{l}_{1_3}(\theta,1)\mathbb{l}_{1_3}(\theta,1)\mathbb{l}_{2_3}(\theta,1) & \dots & \mathbb{l}_{J_{33}}(\theta,1)\mathbb{l}_{J_{33}}(\theta,1)\mathbb{l}_{J_{33}}(\theta,1) \\ \mathbb{l}_{1_3}(\theta,2)\mathbb{l}_{1_3}(\theta,2)\mathbb{l}_{1_3}(\theta,2) & \mathbb{l}_{1_3}(\theta,2)\mathbb{l}_{1_3}(\theta,2)\mathbb{l}_{2_3}(\theta,2) & \dots & \mathbb{l}_{J_{33}}(\theta,2)\mathbb{l}_{J_{33}}(\theta,2)\mathbb{l}_{J_{33}}(\theta,2) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbb{l}_{1_3}(\theta,n_s)\mathbb{l}_{1_3}(\theta,n_s)\mathbb{l}_{1_3}(\theta,n_s) & \mathbb{l}_{1_3}(\theta,n_s)\mathbb{l}_{1_3}(\theta,n_s)\mathbb{l}_{2_3}(\theta,n_s) & \dots & \mathbb{l}_{J_{33}}(\theta,n_s)\mathbb{l}_{J_{33}}(\theta,n_s)\mathbb{l}_{J_{33}}(\theta,n_s) \end{matrix}\right] \quad (5)$$

where n_s represents the number of points of the training data. The vector $\mathbf{Y}$ can be completed with the experimental output signal $\mathbb{y}(\theta,k)$

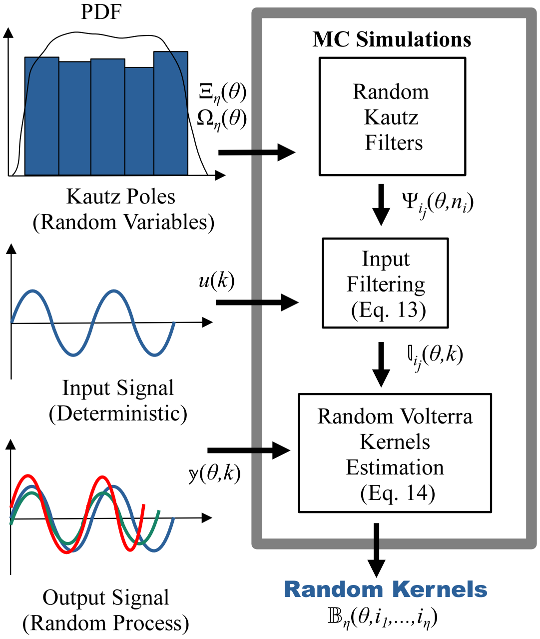


Fig. 1 Description of the random kernels identification methodology

$$\mathbf{Y} = \begin{Bmatrix} y(\theta,1) \\ y(\theta,2) \\ \vdots \\ y(\theta,n_s) \end{Bmatrix}, \tag{6}$$

and,

$$\mathbf{\Phi} = \begin{Bmatrix} \mathbb{B}_1(\theta,1) \\ \vdots \\ \mathbb{B}_1(\theta,J_1) \\ \mathbb{B}_2(\theta,1,1) \\ \mathbb{B}_2(\theta,1,2) \\ \vdots \\ \mathbb{B}_2(\theta,J_2,J_2) \\ \mathbb{B}_3(\theta,1,1,1) \\ \mathbb{B}_3(\theta,1,1,2) \\ \vdots \\ \mathbb{B}_3(\theta,J_3,J_3,J_3) \end{Bmatrix}, \tag{7}$$

has the terms of the orthonormal kernels $\mathbb{B}_\eta$ to each realization θ. The procedure is repeated until the convergence is achieved. Figure 1 shows a flowchart of the stochastic Volterra kernels identification.

2.3 Deterministic Kautz functions

To better understand the work, it is helpful to know the deterministic form of Kautz functions. The generalized form of Kautz functions is written as [7]

$$\psi_{2j-1}(z) = \frac{\sqrt{1-d^2}\sqrt{1-c^2}}{z^2+d(c-1)z-c}\left[\frac{-cz^2+d(c-1)z+1}{z^2+d(c-1)z-c}\right]^{j-1}, \tag{8}$$

$$\psi_{2j}(z) = \psi_{2j-1}(z)\frac{z-d}{\sqrt{1-d^2}}, \tag{9}$$

being the values of c and d respectively defined by

$$c = -\beta_{2\eta-1}\beta_{2\eta}. \tag{10}$$

$$d = \frac{\beta_{2\eta-1}+\beta_{2\eta}}{1+\beta_{2\eta-1}\beta_{2\eta}}, \tag{11}$$

The functions $\beta_{2\eta-1}$ and $\beta_{2\eta}$ are the Kautz poles

$$\beta_{2\eta} = -\xi_{2\eta}\omega_{2\eta} - j\omega_{2\eta}\sqrt{1-\xi_{2\eta}^2}, \tag{12}$$

$$\beta_{2\eta-1} = -\xi_{2\eta-1}\omega_{2\eta-1} + j\omega_{2\eta-1}\sqrt{1-\xi_{2\eta-1}^2}, \tag{13}$$

where the parameters ω_η and ξ_η are the natural frequency and damping ratio of the system, and η represents the number of the kernel. For a stable system, one has $\|\beta_{2\eta-1}\|,\|\beta_{2\eta}\| < 1$. In identification processes, it is common use some optimization methodology to find ω_η and ξ_η [7].

2.4 The stochastic Kautz parameters

In an uncertain framework, the definition of Kautz functions is difficult, and an optimization procedure is usually implemented to estimate the Kautz pole's parameters. However, the optimization problem can generate some errors in an uncertain scenario, even more, if we consider a deterministic estimation, besides being very costly. Therefore, to construct a robust method, it is necessary to provide a statistic certification (the reliability envelope for the model parameters). Such certification can be obtained using a stochastic model, where probability distributions are identified for the model parameters instead of deterministic scalar values. As mentioned before, the Kautz poles' parameters depend on the system dynamics. As we consider the system output a random process that varies in each experimental realization, it is natural to consider that the poles' parameters vary. Therefore, the parameters ω_η and ξ_η for each kernel are considered as random variables $\theta \in \Theta \mapsto \Xi_\eta(\theta) \in \mathbb{R}$, $\theta \in \Theta \mapsto \Omega_\eta(\theta) \in \mathbb{R}$. The random character of the poles will allow the construction of the random Kautz functions described in the last section.

The critical step of the methodology is related to the definition of the distributions of Ω_η and Ξ_η. It is assumed that the only information known about each random variable is the support $[a, b]$. In this sense, the maximum entropy principle is employed to construct a consistent probabilistic model for the random Kautz parameters [16]. Thus, the probability density function that maximizes the entropy is a uniform distribution,

$$p(x) = \mathbb{1}_{[a,b]}(x)\frac{1}{b-a}\,, \tag{14}$$

where $p(x)$ is the probability density function of a uniformly distributed random variable, a and b are the inferior and superior limits to the distribution. Then, to each kernel, the Kautz parameters Ω_η and Ξ_η distributions are estimated between these limits. Finally, the Kautz filters random processes are computed through MC simulations [17, 18] and then used to filter the system's deterministic input data in the Volterra kernels identification process. In this paper, the Kautz parameter limits are defined based on the system response variation because the Kautz poles are associated with the system's dynamics. To be more clear, thought the estimation of natural frequencies and damping ratios of the system considering the operation in the linear regime of motion.

3 Effectiveness test for the new methodology

3.1 System of interest

To test the efficacy of the new identification methodology, a nonlinear system composed of a magneto-elastic beam, subject to large displacement, is considered. The experimental setup that emulates this behavior is shown in Fig. 2. It is the same setup used in [6, 10, 22], composed by an $300 \times 18 \times 3$ (mm) aluminum beam with steel mass connected in the free end to cause a magnetic interaction between the beam and a magnet. A MODAL SHOP shaker (Model Number: K2004E01) is attached 50 mm from the clamped and used to excite the structure considering different levels of voltage amplitude 0.01 V (low level), 0.10 V (medium level), and 0.15 V (high level). The system presents nonlinear behavior for large displacement amplitudes when a high level of excitation amplitude is applied. A vibrometer *laser Polytec* (Model: OFV-525/-5000-S) and a *Dytran* load cell (Model: 1022V) are used to measure the velocity in the free end of the beam and the force excitation, respectively. The magnet positioned next to the free end of the beam interacts with the steel mass and generates a nonlinear hardening behavior [10].

3.2 Definition of the Kautz functions

The uncertain system response is used to establish the limits of Kautz parameters without any optimization procedure. The natural frequency and damping ratio of the system are determined based on experimental data measured on different days and used to establish the limits of Kautz parameters distribution defined in Sect. 2.4. To find the number of Kautz functions for each kernel, the accuracy of the model is observed. The Volterra series is truncated in the third-order component because the cubic effect can describe the polynomial nonlinearity related to the system in the study, the effect of cubic stiffness. Table 1 shows the parameters found to the Kautz functions for each kernel.

The limits are determined based on the variation of the system's physical parameters, natural frequency, and damping ratio of the measured data. The PDFs of the parameters are obtained considering the function shown in Eq. (14) and 250 samples; then, the Kautz functions are obtained via MC simulations. This number of samples is determined based on a MC convergence test applied in the identified kernels. Figure 3 shows the Kautz poles to the three kernels represented in the z plain, considering the dispersion of the parameters Ω_η and Ξ_η. It can be seen that the probabilistic distribution allows a sweep in a region of the plane, which assures the model the ability to describe the variation of the uncertain response of the system.

With these parameters, it is possible to evaluate the stochastic Kautz functions (Ψ_η) and use the estimated random functions in the process of Volterra kernels identification, which is described in the following section.

3.3 Volterra kernels identification

To identify the uncertain Volterra kernels, 195 samples of experimental data, measured on 5 different days, were used. The identification was performed in two steps [6]. First, a low-level chirp was applied in the structure (0.01 V), varying the excitation frequency between 10 and 50 Hz, in the first mode bandwidth to identify the linear kernel. After that, a high-level chirp was applied (0.15 V) with the same frequency range to identify the second and third kernels. The stochastic Kautz functions described in Sect. 2.4 were used in the process of kernels estimation combining the functions with the experimental system output measured on different days. Their statistics were obtained via MC simulations within a total of 1250 samples. Figure 4 presents the mean value for the first kernel, represented in the time domain, with the respective envelope for a 99% confidence band. Figures 5 and 6 show the mean value of the main diagonal of the second and third kernels, also represented in the time domain, with the respective 99% confidence bands. The dispersion for the second- and third-order kernels is larger than for the first-order ones. The second-order kernel has a large variability because the system response is approximately symmetric, and its contribution to the response is low, making it difficult to estimate in an uncertain scenery.

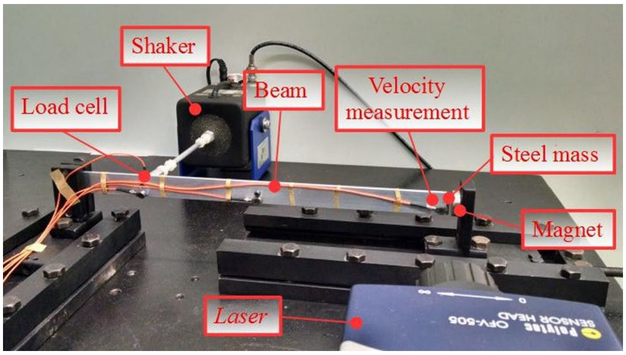


Fig. 2 Nonlinear system to be identified [22]

Table 1 Kautz functions parameters

Parameter	1st kernel	2nd kernel	3rd kernel
Kautz functions	2	2	4
Ω_{inf} (rad/s)	145.5	145.5	145.5
Ω_{sup} (rad/s)	147.5	147.5	147.5
Ξ_{inf} (%)	1.5	1.5	1.5
Ξ_{sup} (%)	2.0	2.0	2.0

The large dispersion observed on the impulse response of the third-order kernel is associated with the number of random Kautz functions used, representing a higher propagation of uncertainties related to the Kautz poles. On the other hand, fewer functions do not allow the model to describe the system behavior adequately.

3.4 Model validation

Once the stochastic model is identified, it is important to validate the model and observe if it can describe the system behavior characteristics. First, the same chirp signal used in the identification process is applied, considering

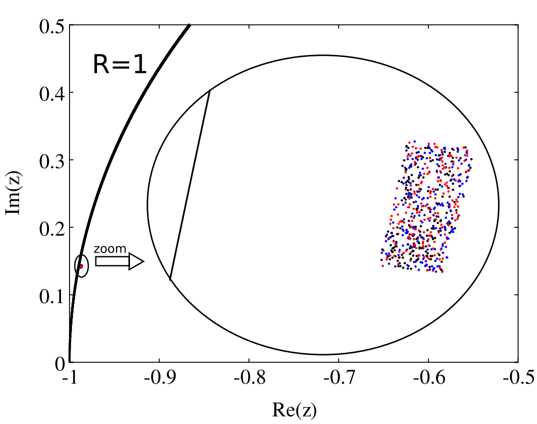


Fig. 3 Kautz poles represented in discrete domain (z). Blue circle—first, red circle—second, black circle—third kernel (colour figure online)

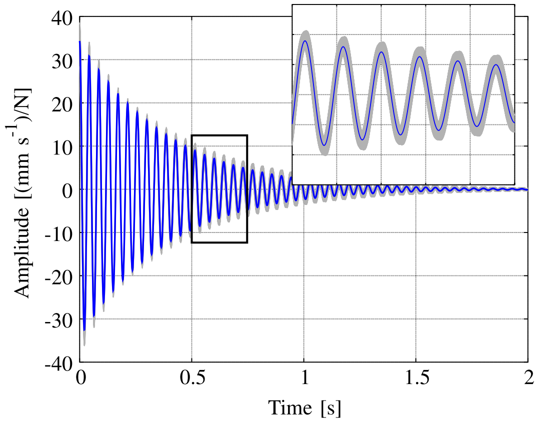


Fig. 4 First kernel represented in time domain with 99% confidence band. Continuous line—mean value

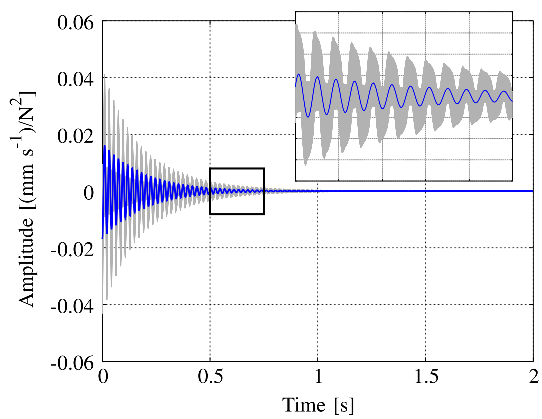


Fig. 5 Main diagonal of second Volterra kernel represented in time domain with 99% confidence band. Continuous line—mean value

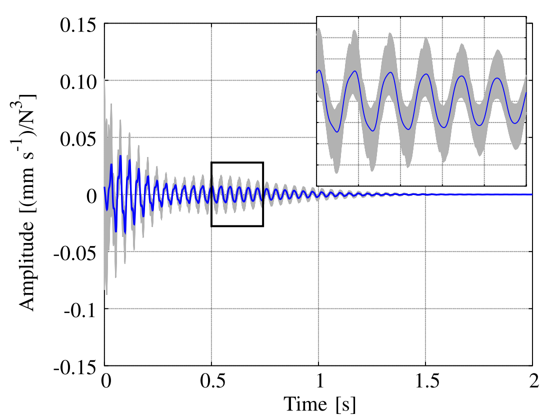
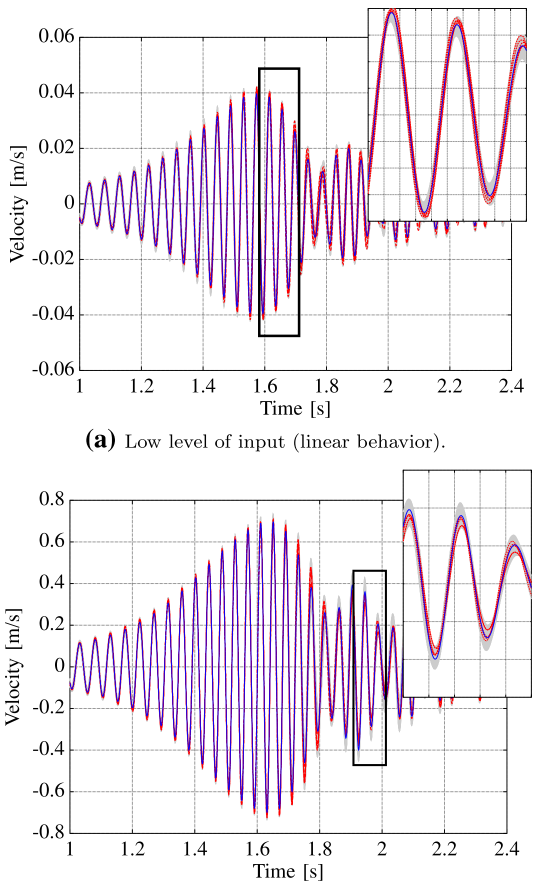


Fig. 6 Main diagonal of third Volterra kernel represented in time domain with 99% confidence band. Continuous line—mean value



(a) Low level of input (linear behavior).

(b) High level of input (nonlinear behavior).

Fig. 7 Volterra model response with 99% of confidence bands. Blue line—mean model, red line—new experimental data (colour figure online)

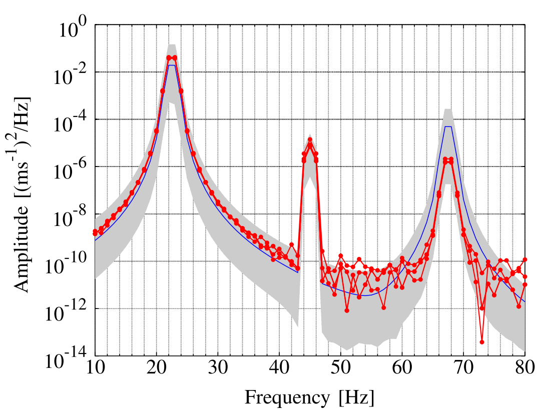


Fig. 8 Volterra model response with 99% of confidence bands for a single frequency sine input. Blue line—mean model, red line—new experimental data (colour figure online)

two amplitude levels. It is important to note that the signals used in this process are new samples of the experimental data (that were not used in the identification process), with similar dynamical characteristics. Figure 7 shows the model prediction with 99% of confidence bands, compared with experimental data measured on different days, considering linear and nonlinear regimes of motion, respectively. The results are satisfactory because the model can describe the system behavior in linear and nonlinear components and the data variation simultaneously (the experimental dispersion is lower than the confidence bands). The mean of the model output cannot describe the variability of the experimental data, which confirms the importance of constructing a stochastic model.

Finally, it is important to validate the model using an input signal with different characteristics than the ones used in the identification process. As the system is nonlinear, its behavior is strongly input-signal dependent. Therefore, a sine signal with a fixed frequency is used as input to validate the model. This signal has a high level of amplitude (0.15 V) and frequency close to the system's natural frequency ($\approx$ 23 Hz), inducing the nonlinear behavior. Figure 8 shows the results in the frequency domain to help the visualization of the frequency components. The presence of multiple harmonics in the response shows that the system behavior is nonlinear in this condition, and the model can describe this behavior in all-important frequency components. The large dispersion in the model response is a consequence of the number of Kautz functions used. However, as previously commented, fewer functions in nonlinear kernels do not allow the model to describe the system's nonlinearities.

4 Conclusion

Identifying nonlinear systems subject to uncertainties is challenging, especially when a gray-box model that depends directly on the input/output signals, like the Volterra series, is used. In this scenario, the optimization of Kautz poles can be costly and considered irrelevant once a deterministic value of the pole cannot describe the fluctuations necessary in Kautz functions to approximate the variation in system response.

This work presented a methodology for nonlinear system identification based on a stochastic version of the Volterra series (expanded using Kautz functions), where model parameters are described as independent random variables and the model output as a random process. In this methodology, the Kautz poles are determined based on the uncertain output signals of the system and the maximum entropy principle. The results show that this stochastic version of the Volterra series can describe the system response considering the experimental data variations obtained from measurements performed on different days. The stochastic model obtained is robust to uncertainties and can be adequately validated.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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