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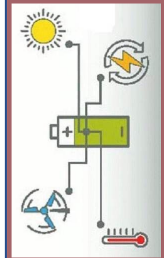

EH DIALOG

POLITECHNIKA LUBELSKA

# Exploring the behavior of a bistable energy harvester via global sensitivity analysis


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# Vibration Energy Harvester

Vibration Energy harvesters

Challenge: Create an efficient system over wide frequency bandwidth.

# Vibration Energy Harvester

Vibration Energy harvesters

Challenge: Create an efficient system over wide frequency bandwidth.

Some proposals:

- tuning the resonance frequency (can requires energy)
- multiple degrees of freedom
- multi modal devices
- non-linearity application (monostable, bistable and so on)

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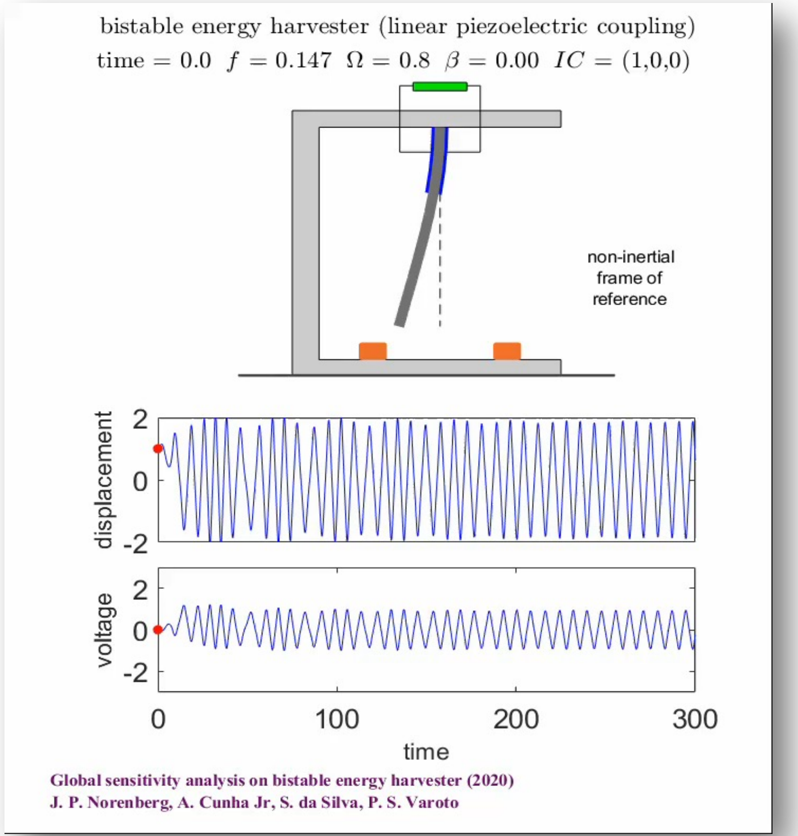
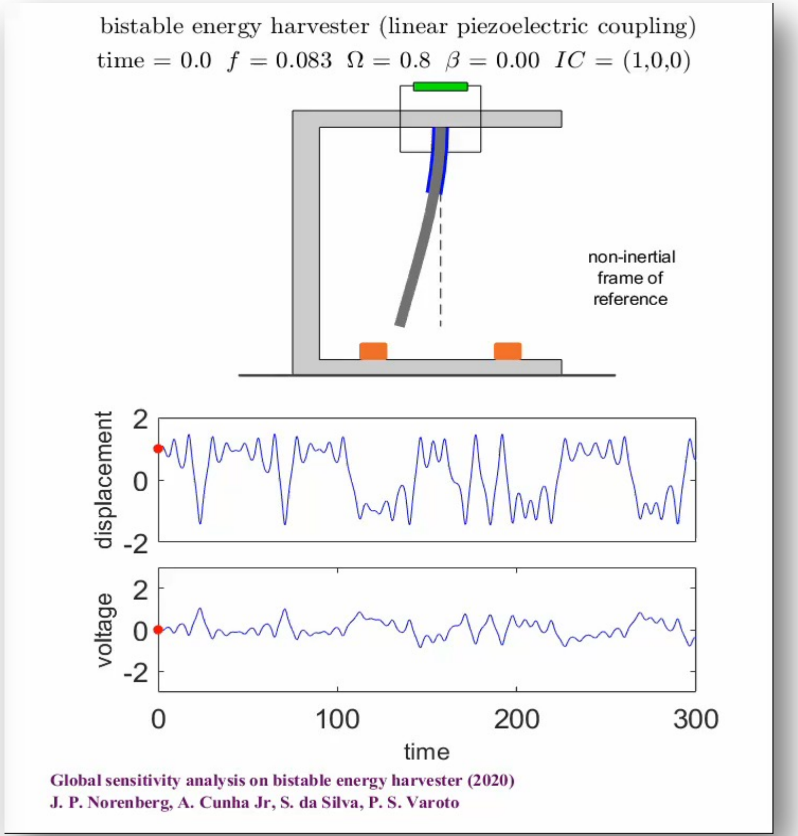
# Sensitivity by excitation



# Sensitivity by uncertainty parameter

Manufacturing and conditions process:

- Geometry
- Propriets material
- External Excitation

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What about:

Sensitivity by uncertainty parameters in Power Harvesting?

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What about:

Sensitivity by uncertainty parameters in Power Harvesting?

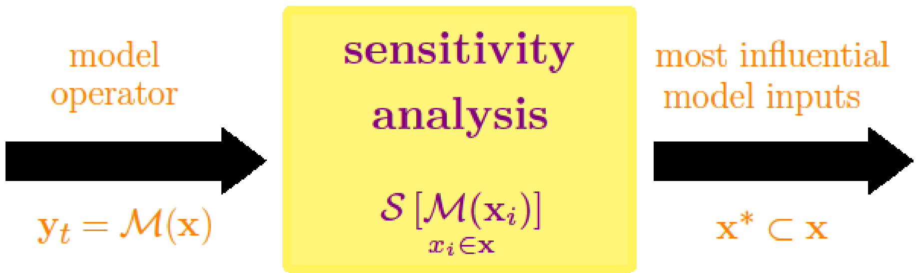
Objetive:

Identify the most sensitivity parameter of bistable energy harvester over different dynamics behaviors

# Outline

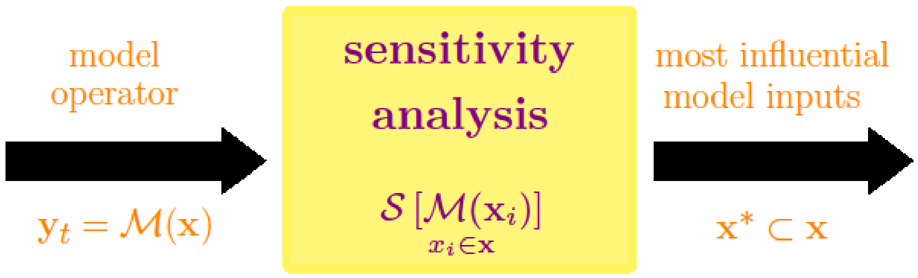
1. Sensitivity Analysis
2. Metamodeling by PCE
3. Bistable Energy Harvester
4. Results
5. Final Remarks

# Sensitivity Analysis




I. M. Sobol Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. Mathematics and Computers in Simulation, 55(1-3): 271-280, 2001.

# Sensitivity Analysis




Main contributions:

- Simpler probabilistic model constructions
- Decision making
- Nontrivial insight into the behavior
- Important step for robustness and optimization problems

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# Sensitivity Analysis

Mathematical Model

***Hoeffding-Sobol* decomposition:**

$$Y = \mathcal{M}(\boldsymbol{X}) \Rightarrow \boxed{Y = \mathcal{M}_0 + \sum_{i=1}^{k} \mathcal{M}_i + \sum_{i<1}^{k} \mathcal{M}_{ij} + \cdots + \mathcal{M}_{1\ldots k}}$$

- $\mathcal{M}_0 = \mathbb{E}\{Y\}$
- $\mathcal{M}_i(X_i) = \mathbb{E}\{Y|X_i\} - \mathcal{M}_0$
- $\mathcal{M}_{ij}(X_i, X_j) = \mathbb{E}\{Y|X_i, X_j\} - \mathcal{M}_i - \mathcal{M}_j - \mathcal{M}_0$
- $\vdots$

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# Sensitivity Analysis

*Variance*-Decomposition:

$$\sum_{i}^{k} S_i + \sum_{i<j}^{k} S_{ij} + \cdots + S_{12\cdots k} = 1$$

- $S_i = \dfrac{Var[\mathcal{M}_i(X_i)]}{Var[\mathcal{M}(X)]}$ → First-order Sobol' indices
- $S_{ij} = \dfrac{Var[\mathcal{M}_{ij}(X_i, X_j)]}{Var[\mathcal{M}(X)]}$ → Second-order Sobol' indices
- $\vdots$

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# Polynomial Chaos Expansion (PCE)

Metamodeling by PCE:

$$Y = \mathcal{M}_0(X) \approx \sum_{\alpha \in \mathcal{A}} y_\alpha \Phi_\alpha(X)$$

Where:

- $y_\alpha$: deterministic coeffiecients (to be determined)
- $\Phi_\alpha$: multivariate orthogonal polynomial bases

B. Sudret. Global sensitivity analysis using polynomial chaos expansions. Reliability Engineering and System Safety, 93, 964-979, 2008.

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# Post-processing techniques PCE

Mean:

$$\mu^{PC} = \mathcal{Y}_0$$

Variance:

$$\sigma^{PC} = \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} \mathcal{Y}_\alpha^2$$

Sobol Indices:

$$S_i = \sum_{\substack{\alpha \in \mathcal{A}_i \\ \alpha \neq 0}} \mathcal{Y}_\alpha^2 \Big/ \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} \mathcal{Y}_\alpha^2$$ and $$S_{ij} = \sum_{\substack{\alpha \in \mathcal{A}_{ij} \\ \alpha \neq 0}} \mathcal{Y}_\alpha^2 \Big/ \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} \mathcal{Y}_\alpha^2$$

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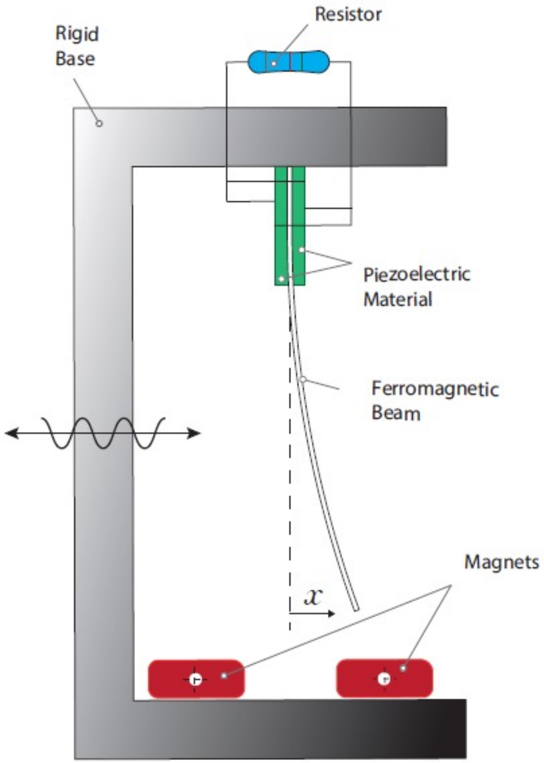
# Bistable Energy Harvester




$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}(1 - x^2) - \chi v = f\cos\Omega t$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

+ initial conditions

Average power (quantity of interest)

$$\boxed{P_{avg} = \frac{1}{T}\int_{ti}^{ti+T} \lambda v(t)^2\,dt}$$

A. Erturk, J. Hoffman, D.J., Inman, D.J. A piezomagnetoelastic structure for broadband vibration energy harvesting. Appl. Phys. Lett., 94, 254102, 2009

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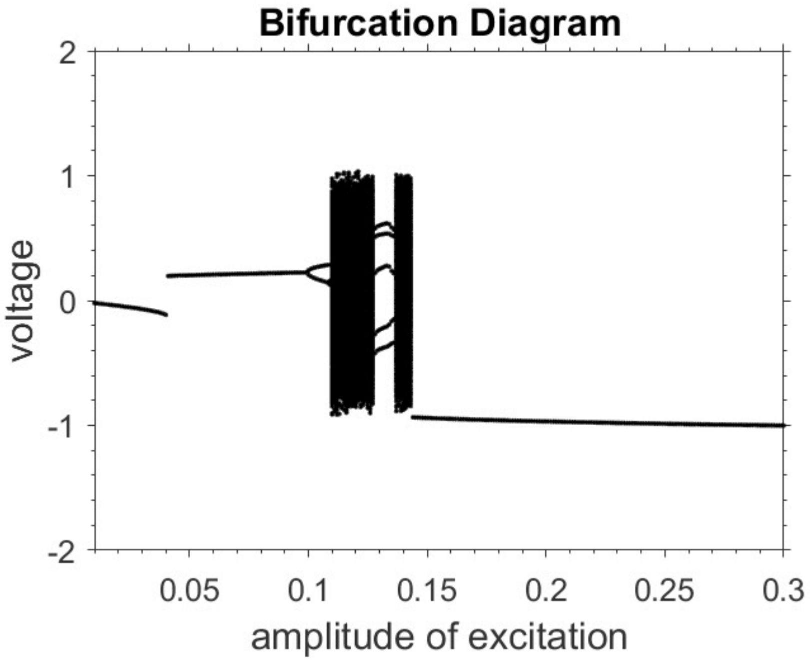
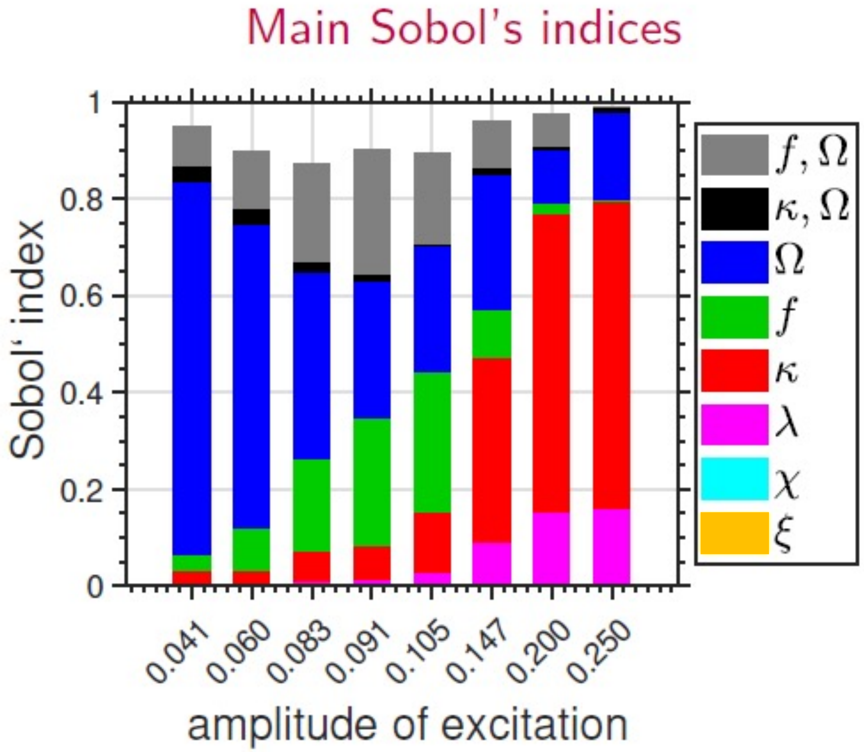
# Results



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# Conclusions

Some conclusions about uncertainty sensitivity of a bistable energy harvester:

- Model probabilistic can be reduced.
- Frequency and amplitude of excitation and Piezoelectric propriets are most influence.
- Sensitivity depends on dynamic stability.

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## Thank you very much

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