On the efficiency of a bi-stable energy harvesting device driven by a random excitation

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NUMERICO – **Nu**cleus of **M**odeling and **E**xpe**ri**mentation with **Co**mputers

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Vinícius Lopes (UERJ)

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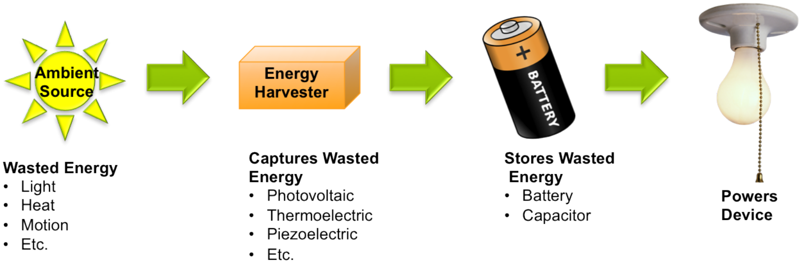
Sofia, Bulgaria

Section 1

Introduction

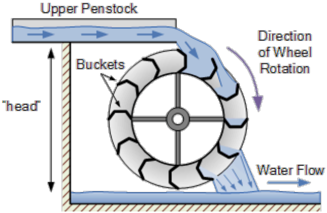
Energy Harvesting

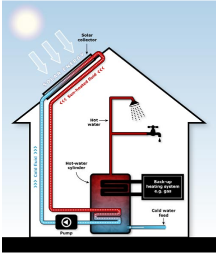


- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

*Picture from http://hiddenjoules.com/intro/what/

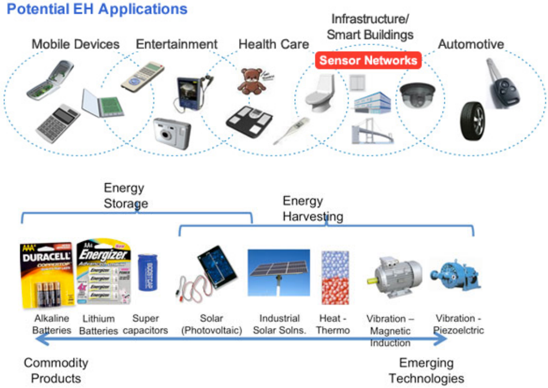
Classical Technologies in Energy Harvesting

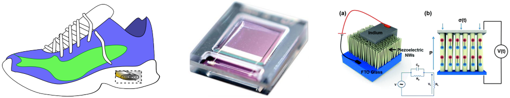


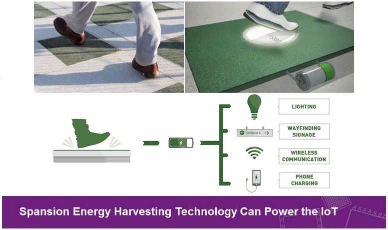


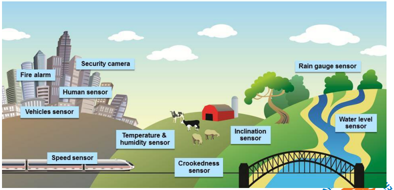
*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

Emergent Technologies in Energy Harvesting



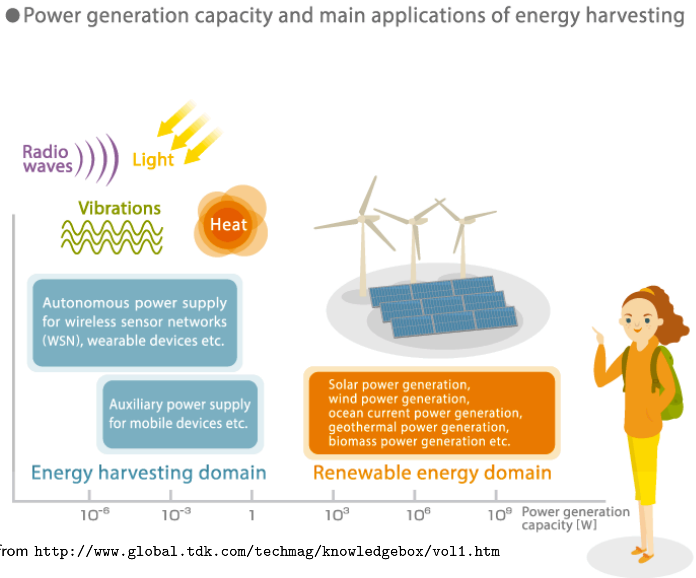






*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

Energy Scale for Modern Harvesting Devices



*Picture from `http://www.global.tdk.com/techmag/knowledgebox/vol1.htm`

Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
- Propose strategies to enhance the recovered energy
- Model the underlying uncertainties and study their influence

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- Investigate in detail the underlying nonlinear dynamics
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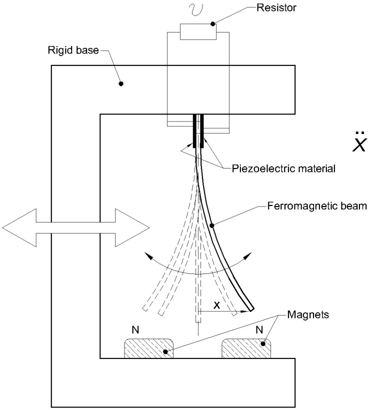
Section 2

Nonlinear Dynamics

Bi-stable energy harvesting device



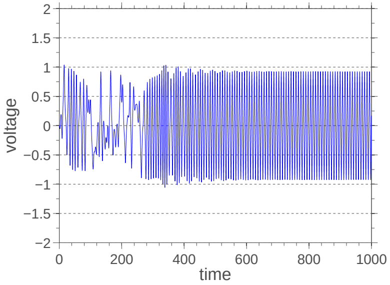
$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

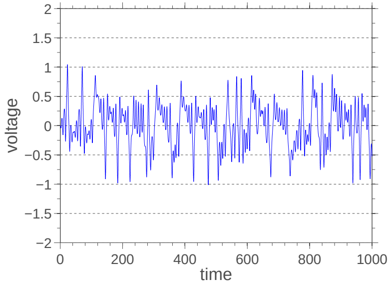
$$x(0) = x_0, \;\; \dot{x}(0) = \dot{x}_0, \;\; \upsilon(0) = \upsilon_0$$

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Nonlinear dynamics: time series

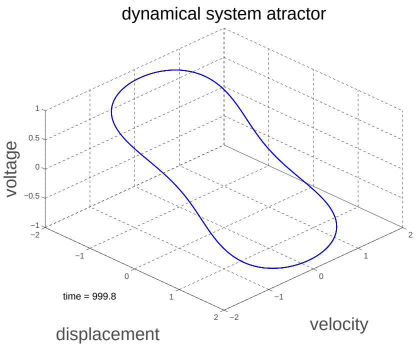


(a) f = 0.115

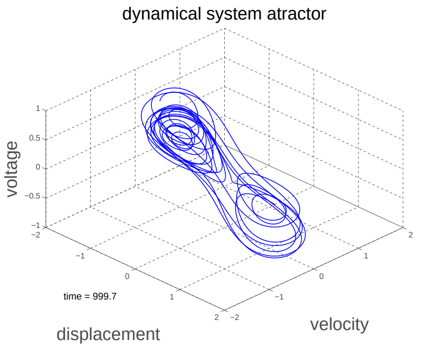


(b) f = 0.083

Nonlinear dynamics: system atractor

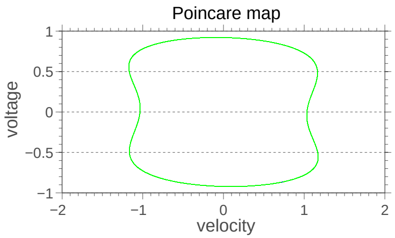


(a) $f = 0.115$

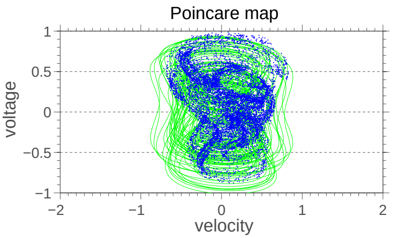


(b) $f = 0.083$

Nonlinear dynamics: Poincaré maps

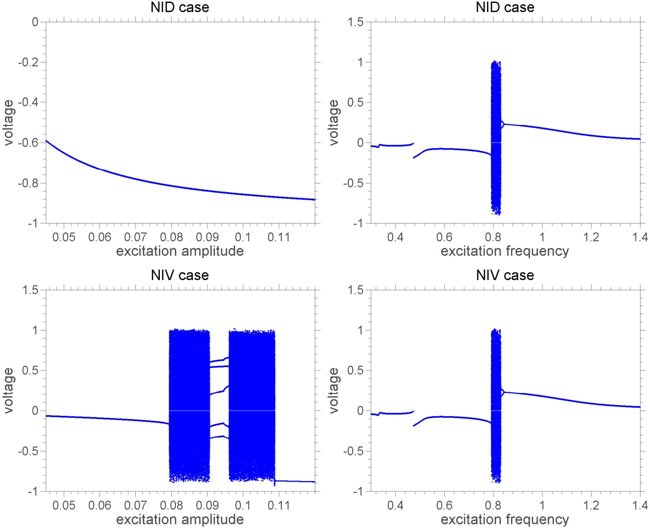


(a) f = 0.115

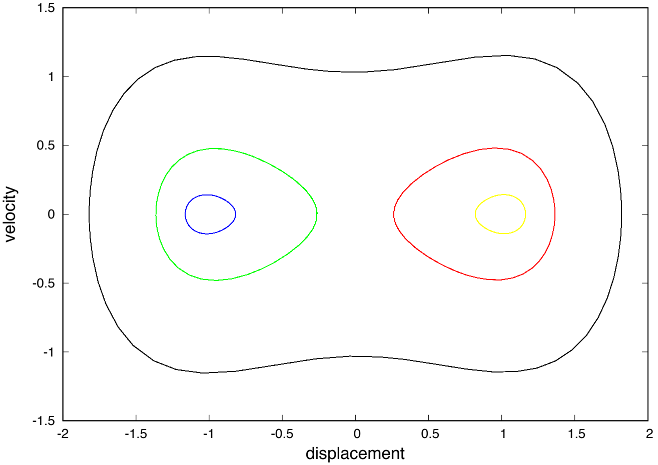


(b) f = 0.083

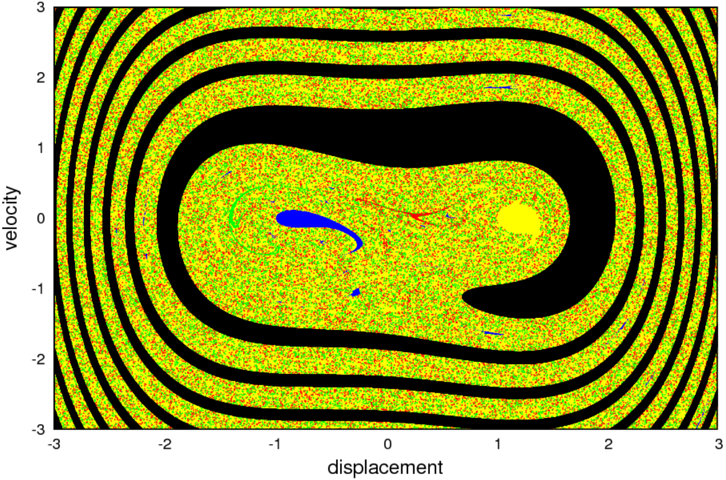
Nonlinear dynamics: biffucation diagrams



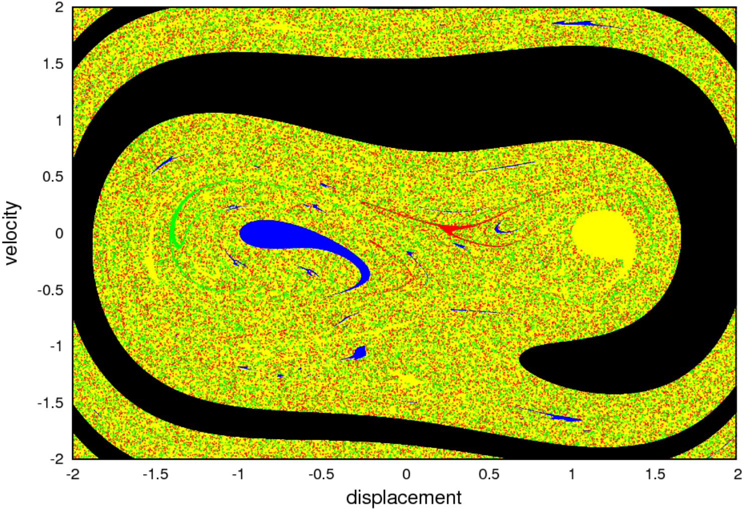
Nonlinear dynamics: basins of attraction



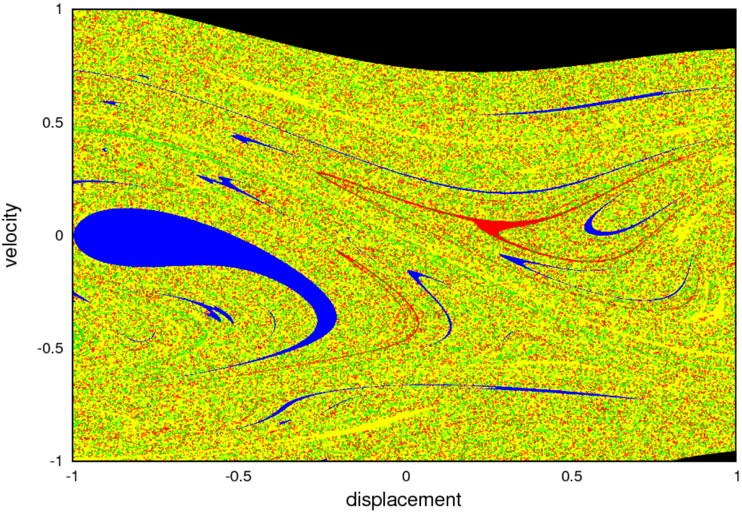
Nonlinear dynamics: basins of attraction



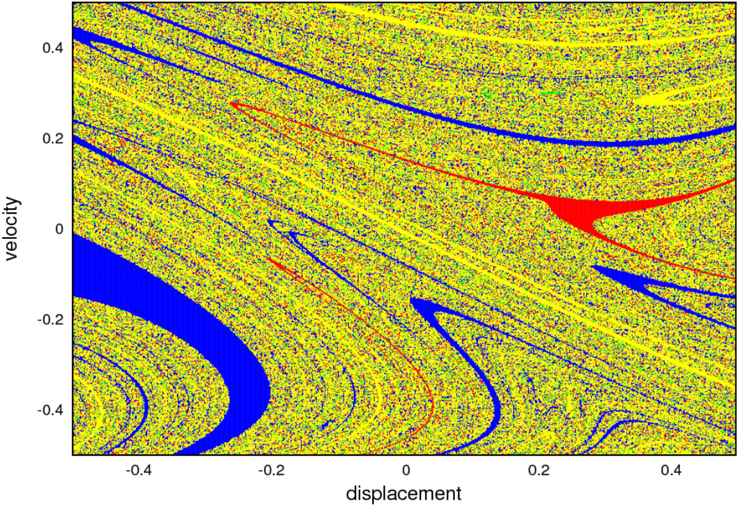
Nonlinear dynamics: basins of attraction



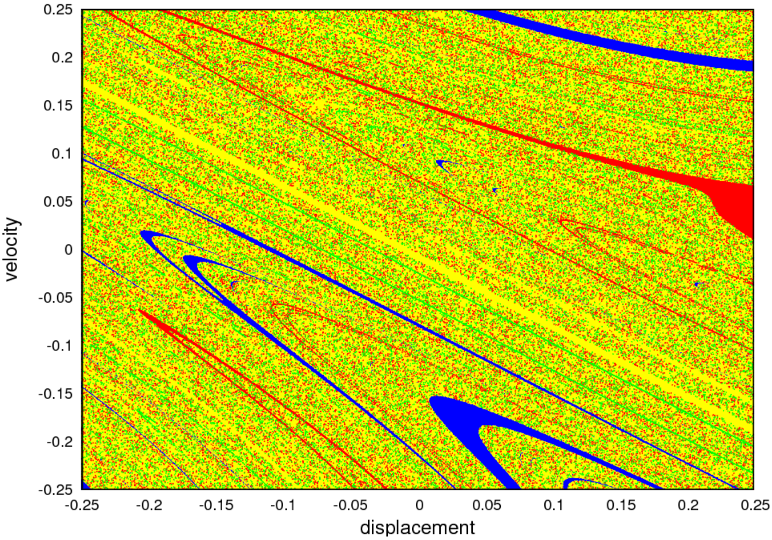
Nonlinear dynamics: basins of attraction



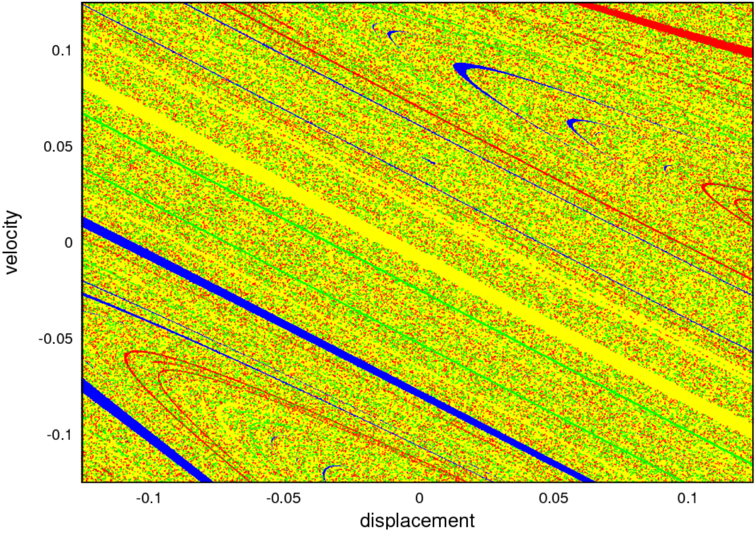
Nonlinear dynamics: basins of attraction



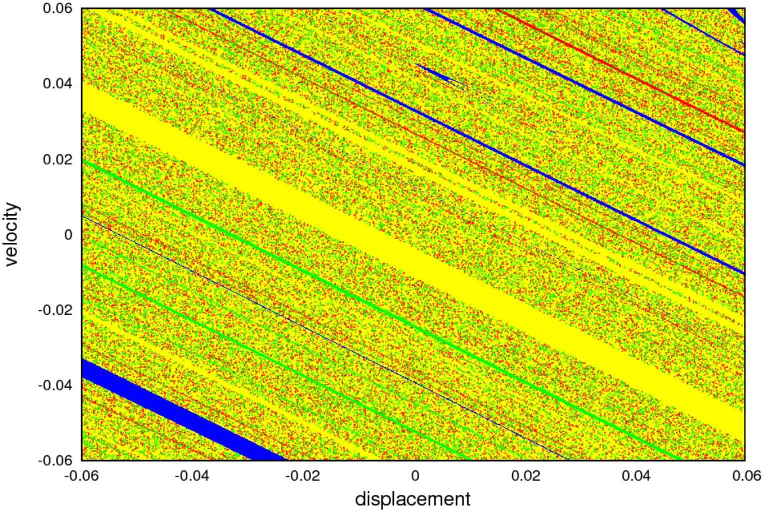
Nonlinear dynamics: basins of attraction



Nonlinear dynamics: basins of attraction



Nonlinear dynamics: basins of attraction



Section 3

Stochastic Modeling

Probabilistic model for external forcing

- Nonlinear dynamical system:

$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1-x^2) - \chi\upsilon = f\cos\Omega t$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0,\ \dot{x}(0) = \dot{x}_0,\ \upsilon(0) = \upsilon_0$$

- noise (zero-mean stationary random process):

$$\mathbb{N}_t = \left\{\mathbb{N}(t), t \geq 0\right\}$$

- noise covariance function:

$$\mathrm{cov}_{\mathbb{N}_t}(t_1, t_2) = \exp\left(-\frac{(t_2 - t_1)}{t_{corr}}\right)$$

Probabilistic model for external forcing

- Nonlinear dynamical system:

$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1-x^2) - \chi v = f\cos\Omega t + \text{“noise”}$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

$$x(0) = x_0,\ \dot{x}(0) = \dot{x}_0,\ v(0) = v_0$$

- noise (zero-mean stationary random process):

$$\mathbb{N}_t = \left\{\mathbb{N}(t), t \geq 0\right\}$$

- noise covariance function:

$$\text{cov}_{\mathbb{N}_t}(t_1, t_2) = \exp\left(-\frac{(t_2 - t_1)}{t_{corr}}\right)$$

Computational representation of the noise

Truncated Karhunen-Loève expansion:

$$\mathbb{N}_t \approx \sum_{n=1}^{N_{KL}} \sqrt{\lambda_n}\, \varphi_n(t)\, \mathtt{Y}_n$$

$$\int_{\mathbb{R}} \mathrm{cov}_{\mathbb{N}_t}(t_1, t_2)\, \varphi_n(t_2)\, dt_1 = \lambda_n\, \varphi_n(t_1), \qquad t_1 \in \mathbb{R}$$

$$\mu_{\mathtt{Y}_n} = 0 \qquad \text{and} \qquad \mathbb{E}\left\{\mathtt{Y}_n \mathtt{Y}_m\right\} = \delta_{mn}$$

D. Xiu, **Numerical Methods for Stochastic Computations: A Spectral Method Approach**, Princeton University Press, 2010.

R. Ghanem, P. Spanos, **Stochastic Finite Elements: A Spectral Approach**, Dover Publications, 2003.

Realization of random external force

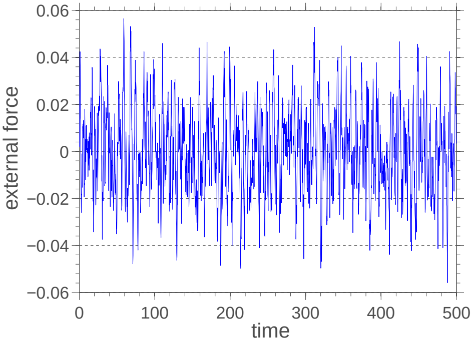


Figure: Time series of random forcing generated by KL expansion.

Stochastic nonlinear dynamical system

Nonlinear dynamics evolves (almost sure) according to

$$\ddot{\mathbb{X}}_t + 2\xi\dot{\mathbb{X}}_t - \frac{1}{2}\mathbb{X}_t(1 - \mathbb{X}_t^2) - \chi\mathbb{\Upsilon}_t = f\,\cos\Omega\,t + \mathbb{N}_t \quad a.s.$$

$$\dot{\mathbb{\Upsilon}}_t + \lambda\mathbb{\Upsilon}_t + \kappa\dot{\mathbb{X}}_t = 0 \quad a.s.$$

$$\mathbb{X}_0 = x_0,\ \dot{\mathbb{X}}_0 = \dot{x}_0,\ \mathbb{\Upsilon}_0 = \upsilon_0 \quad a.s.$$

where

$$\mathbb{X}_t = \left\{\mathbb{X}(t), t \geq 0\right\} \quad \text{and} \quad \mathbb{\Upsilon}_t = \left\{\mathbb{\Upsilon}(t), t \geq 0\right\}$$

are random processes.

Propagation of uncertainties: Monte Carlo method

Monte Carlo Method

pre-processing	**processing**	**post-processing**
generation of scenarios	solution of model equations	computation of statistics
$\boldsymbol{X}_1$ ⋮ $\boldsymbol{X}_M$	$\boldsymbol{U} = h(\boldsymbol{X})$	$\boldsymbol{U}_1 = h(\boldsymbol{X}_1)$ ⋮ $\boldsymbol{U}_M = h(\boldsymbol{X}_M)$
$\longrightarrow$	$\boxed{\textbf{computational model}}$	$\longrightarrow$
known $F_{\boldsymbol{X}}$		estimated $F_{\boldsymbol{U}}$
generator of random vector $\boldsymbol{X}$	deterministic solver of $\boldsymbol{u} = h(\boldsymbol{x})$	statistical inference to estimate convergence and distribution of $\boldsymbol{U}$

A. Cunha Jr, R. Nasser, R. Sampaio, H. Lopes, and K. Breitman, *Uncertainty quantification through Monte Carlo method in a cloud computing setting.* **Computer Physics Communications**, 185: 1355–1363, 2014.

Section 4

Numerical Experiments

Nonlinear dynamics animation (without noise)

Nonlinear dynamics animation (with noise)

System dynamics: typical realization time series

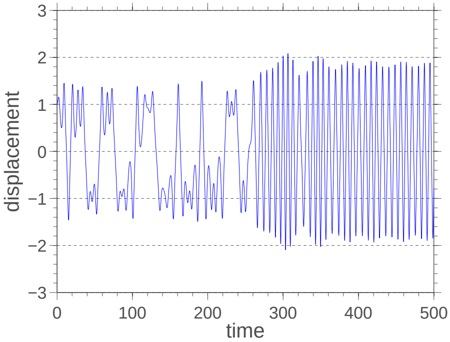


Figure: Time series of displacement.

System dynamics: typical realization time series

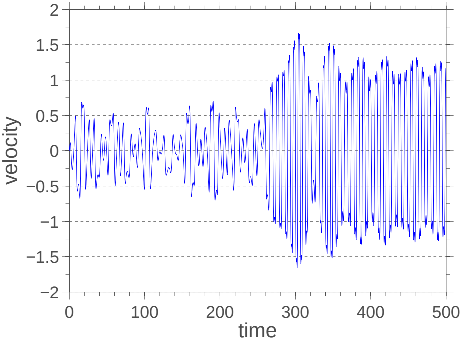


Figure: Time series of velocity.

System dynamics: typical realization time series

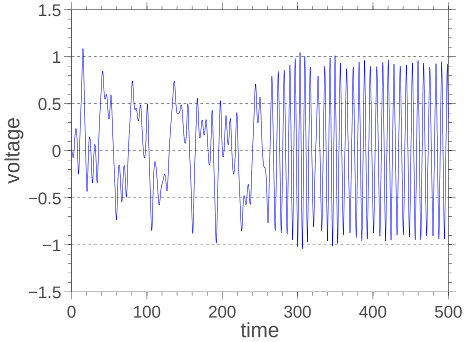
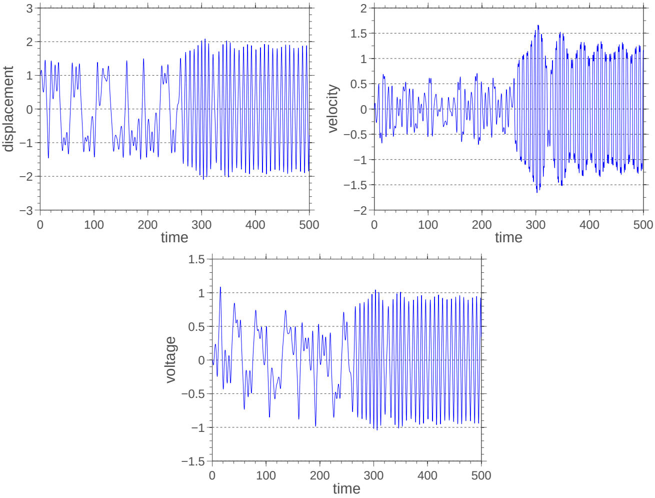


Figure: Time series of voltage.

System dynamics: typical realization time series



System dynamics: phase space

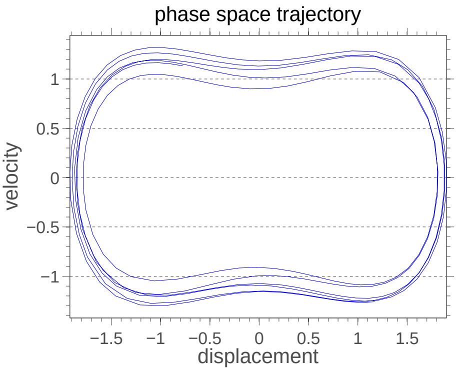


Figure: Projection of system trajectory into displacement–velocity plane.

System dynamics: phase space

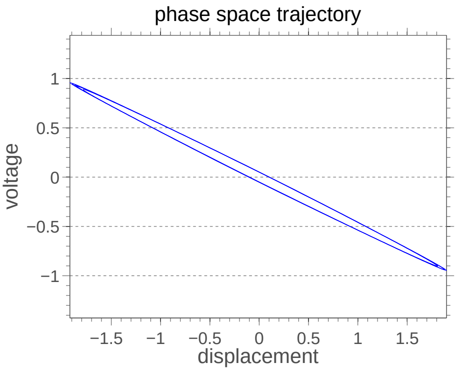


Figure: Projection of system trajectory into displacement–voltage plane.

System dynamics: phase space

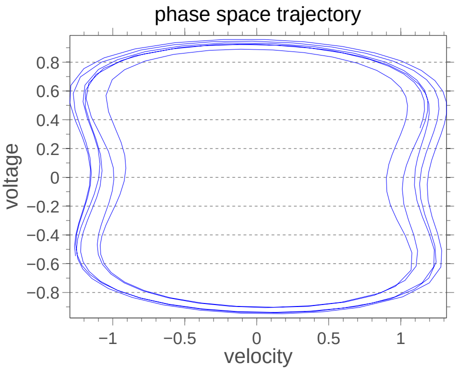
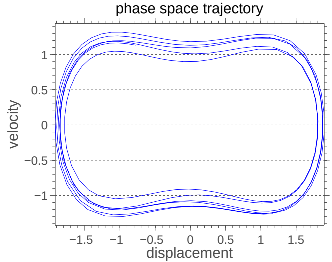
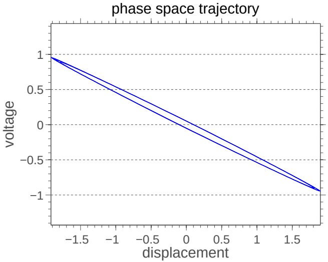
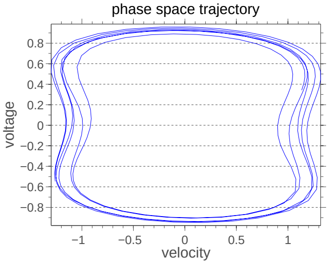


Figure: Projection of system trajectory into velocity–voltage plane.

System dynamics: phase space







System dynamics: attractor

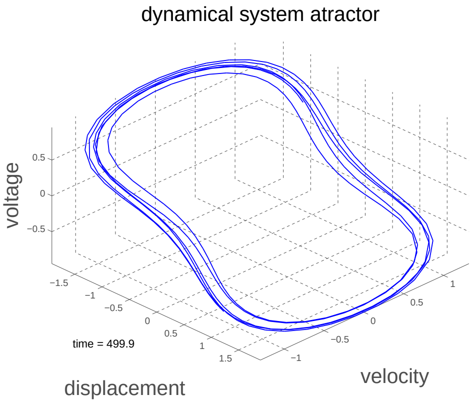


Figure: System dynamics trajectory in phase space.

System stochastic dynamics: 1st order statistics

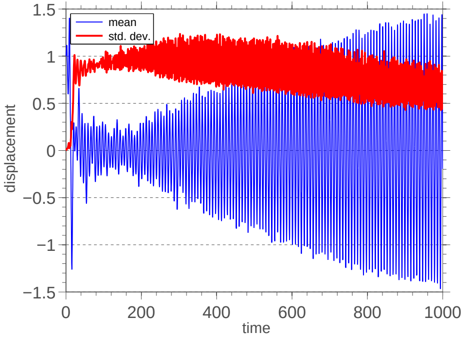


Figure: Time series of displacement 1st order statistics.

System stochastic dynamics: 1st order statistics

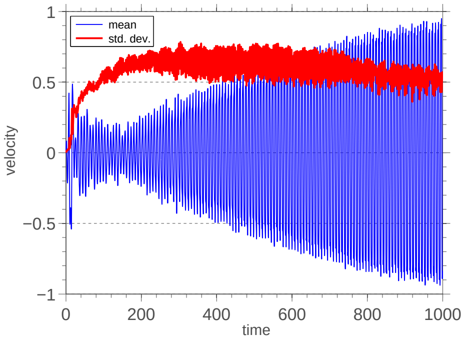


Figure: Time series of velocity 1st order statistics.

System stochastic dynamics: 1st order statistics

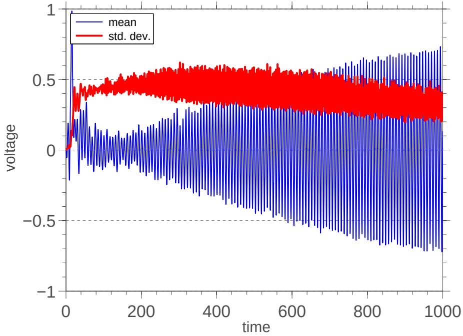


Figure: Time series of voltage 1st order statistics.

System stochastic dynamics: 1st order statistics

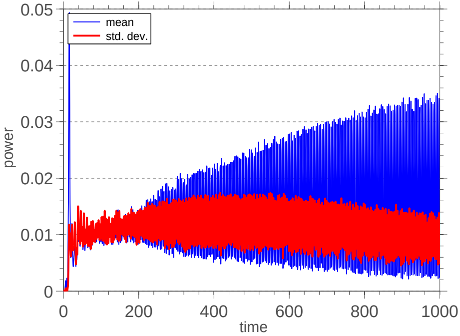
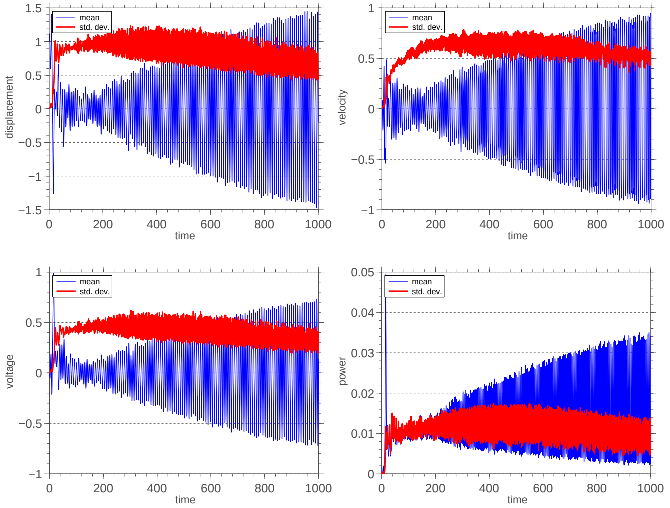


Figure: Time series of power 1st order statistics.

System stochastic dynamics: 1st order statistics



System stochastic dynamics: confidence band

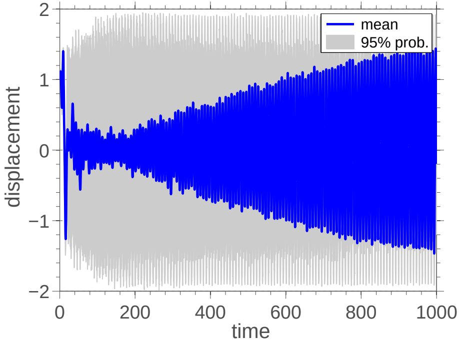


Figure: Time series of displacement 1st order statistics.

System stochastic dynamics: confidence band

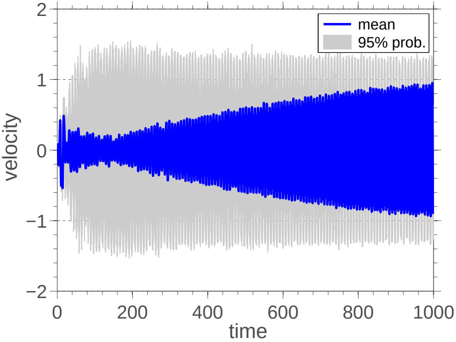


Figure: Time series of velocity 1st order statistics.

System stochastic dynamics: confidence band

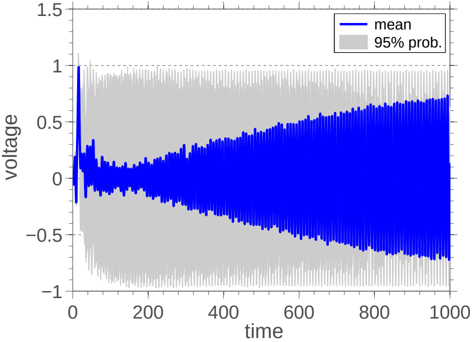


Figure: Time series of voltage 1st order statistics.

System stochastic dynamics: confidence band

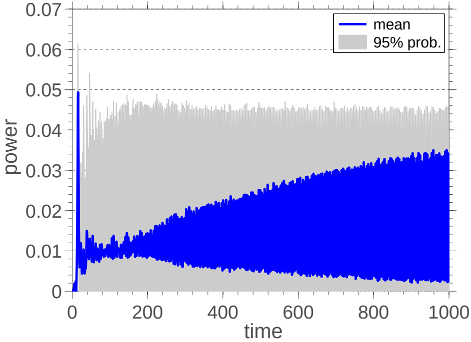
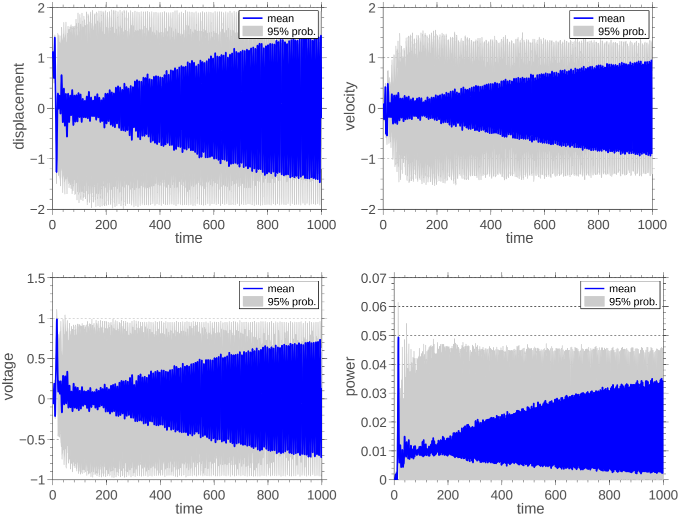
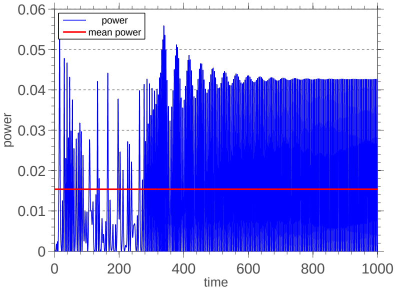


Figure: Time series of power 1st order statistics.

System stochastic dynamics: confidence band



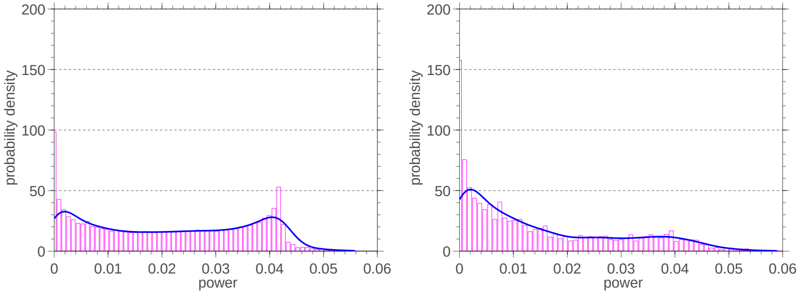
Efficiency of the nonlinear system



(a) without noise (b) with noise

Figure: Time series of circuit output power .

Temporal distribution of output power



(a) without noise (b) with noise

Figure: Temporal distribution of a single realization of the output power.

Evolution of output power distribution

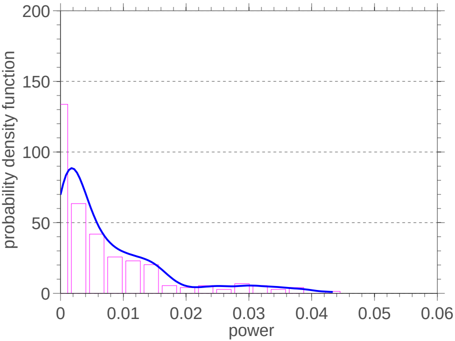


Figure: Distribution of output power at $t = 250$.

Evolution of output power distribution

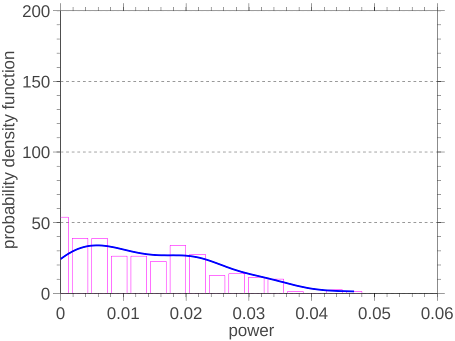


Figure: Distribution of output power at $t = 500$.

Evolution of output power distribution

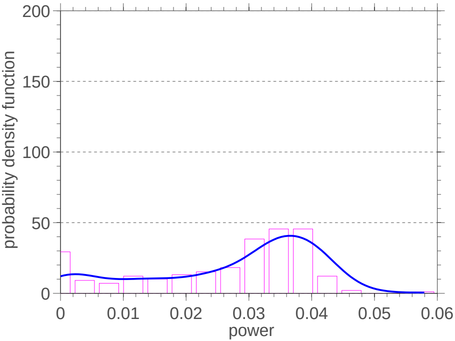


Figure: Distribution of output power at $t = 750$.

Evolution of output power distribution

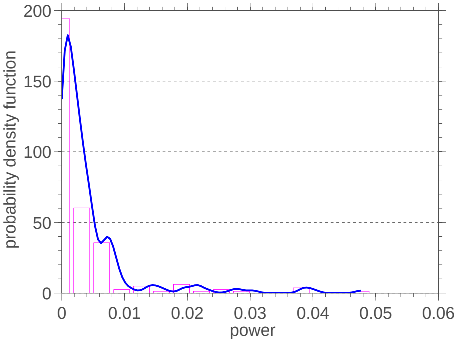
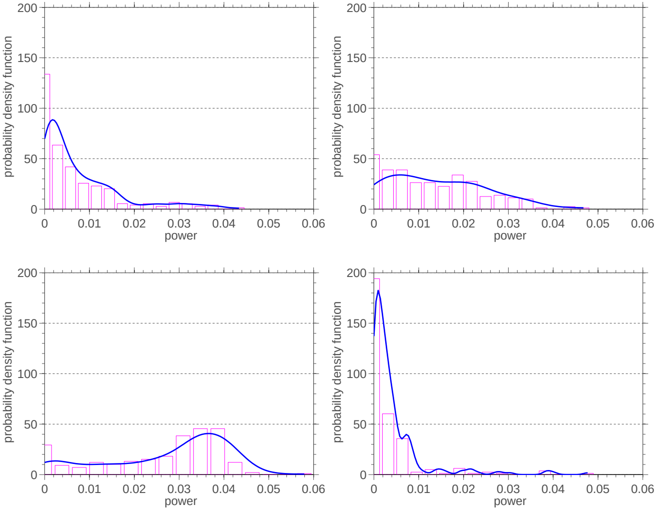
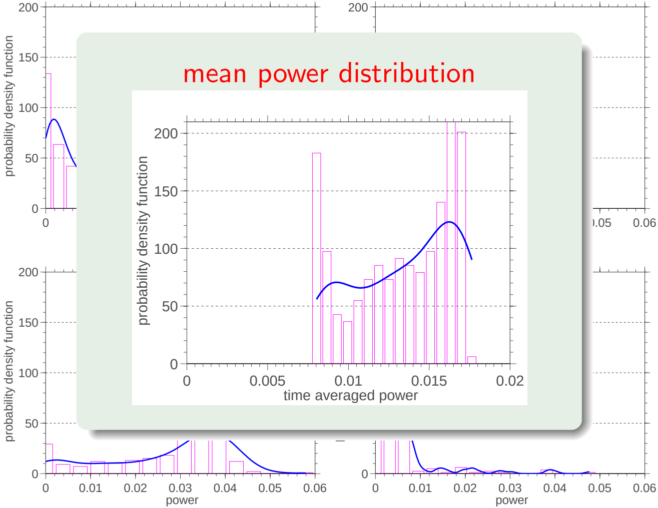


Figure: Distribution of output power at $t = 1000$.

Evolution of output power distribution



Evolution of output power distribution



Section 5

Final Remarks

Concluding remarks

Contribution and conclusions:

- Construction of a parametric probabilistic model of uncertainties for the nonlinear dynamics of a bi-stable energy harvester
- Noise disturbes the harvester dynamic behaviour
- Noise reduces the efficiency of the bi-stable harvester

Ongoing research:

- Study the effect of correlation time on the efficiency
- Implementation of strategies of chaos control to improve the bi-stable harvester efficiency

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Thank you for your attention!

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Physical system parameters

parameter	value
ξ	0.01
χ	0.05
f	0.115
Ω	0.8
λ	0.05
κ	0.5
x_0	1.0
$\dot{x}_0$	0.0
v_0	0.0
t_{corr}	1.0
σ_{N_t}	1.0