

# Numerical study of parameters influence over the dynamics of a piezo-magneto-elastic energy harvesting device

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**NUMERICO** – Nucleus of Modeling and Experimentation with Computers  
[numerico.ime.uerj.br](http://numerico.ime.uerj.br)

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São José dos Campos - SP, Brazil



## 1 Introduction

## 2 Nonlinear Dynamics

## 3 Numerical Experiments

## 4 Final Remarks

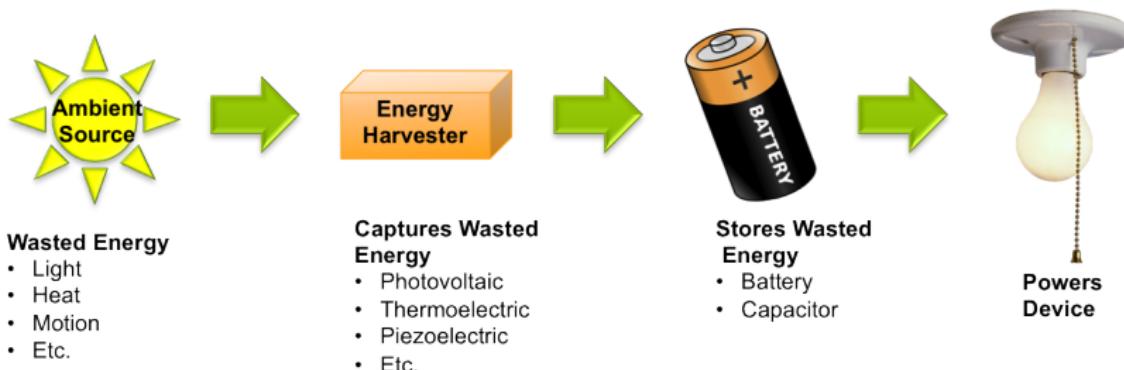


# Section 1

## Introduction



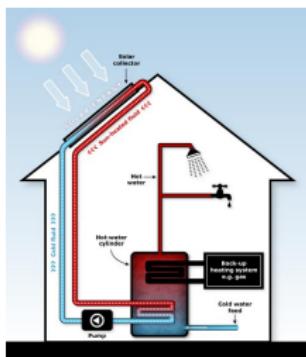
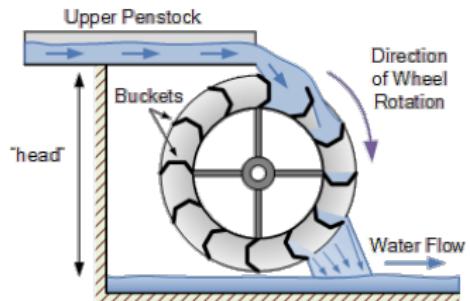
# Energy Harvesting



- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

\*Picture from <http://hiddenjoules.com/intro/what/>

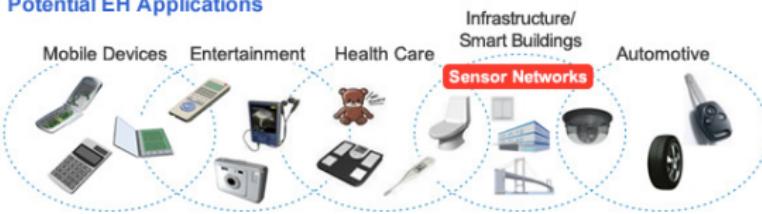
# Classical Technologies in Energy Harvesting



\*Pictures obtained from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

# Emergent Technologies in Energy Harvesting

## Potential EH Applications



Energy  
Storage



Alkaline Batteries



Lithium Batteries



Super capacitors



Solar  
(Photovoltaic)



Industrial  
Solar Solns.



Heat -  
Thermo



Vibration -  
Magnetic  
Induction

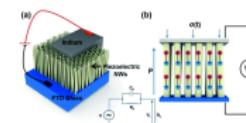


Piezoelectric

## Commodity Products



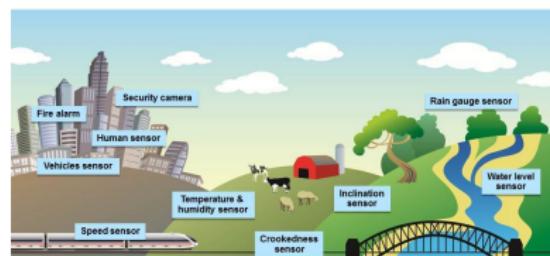
## Emerging Technologies



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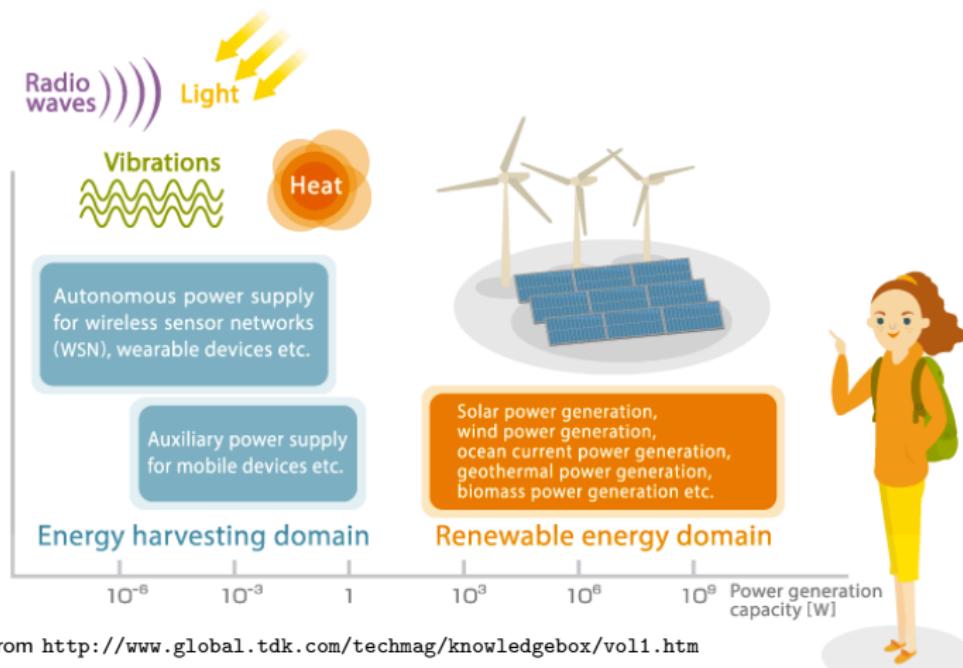


Spansion Energy Harvesting Technology Can Power the IoT



# Energy Scale for Modern Harvesting Devices

- Power generation capacity and main applications of energy harvesting



\*Picture from <http://www.global.tdk.com/techmag/knowledgebox/vol1.htm>

# Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
  - Time series
  - Bifurcation diagrams
  - Basis of attractions
- Propose strategies to enhance the recovered energy
  - Nonlinear optimization
  - Control of chaos
- Model the underlying uncertainties and study their influence
  - System parameters variability
  - Noise in system excitation



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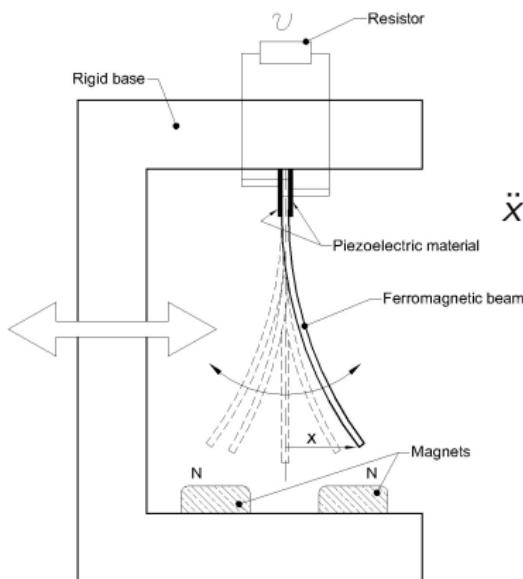


## Section 2

### Nonlinear Dynamics



# Bi-stable energy harvesting device



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1-x^2) - \chi v = f \cos \Omega t$$

$$\dot{v} + \lambda v + \kappa \dot{x} = 0$$

$$x(0) = x_0, \quad \dot{x}(0) = \dot{x}_0, \quad v(0) = v_0$$



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

## Section 3

### Numerical Experiments

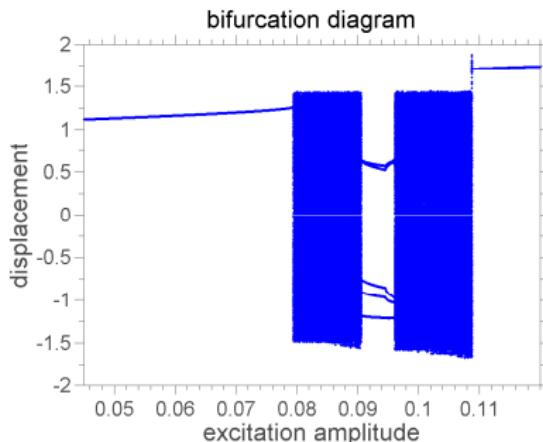


# Nonlinear dynamics animation

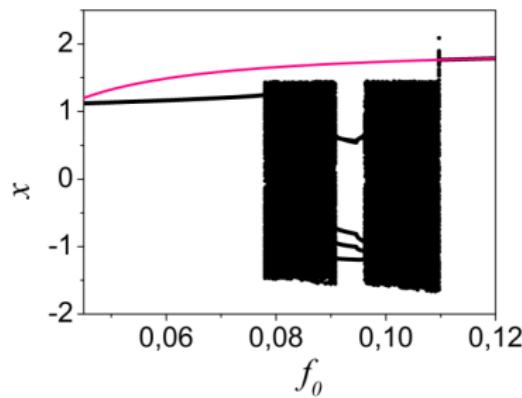


# Computational code verification

- Fixed  $\Omega = 0.8$
- Initial condition:  $(x_0, \dot{x}_0, v_0) = (1\ 0\ 0)$



(a) Computed diagram



(b) Reference diagram

T. Leite, A. S. de Paula, A. T. Fabro and M. Savi, A numerical analysis of the electrical output response of a nonlinear piezoelectric oscillator subjected to a harmonic and random excitation, *XXXVII Iberian Latin American Congress on Computational Methods in Engineering, Brasilia, DF, Brazil, 2016*.

# Voltage vs excitation amplitude ( $\Omega = 0.8$ )

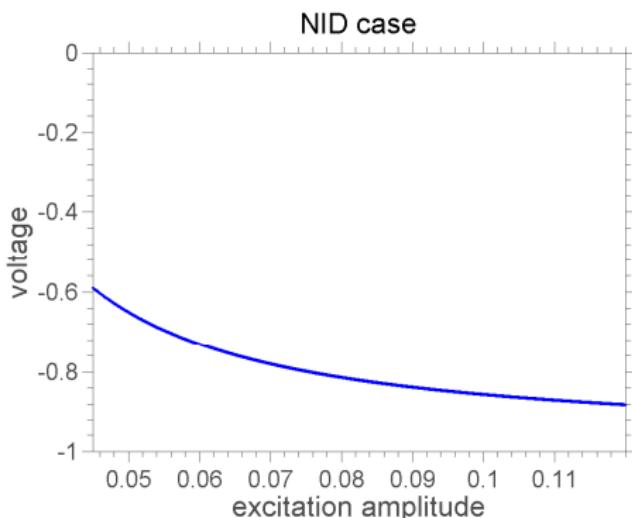


Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$

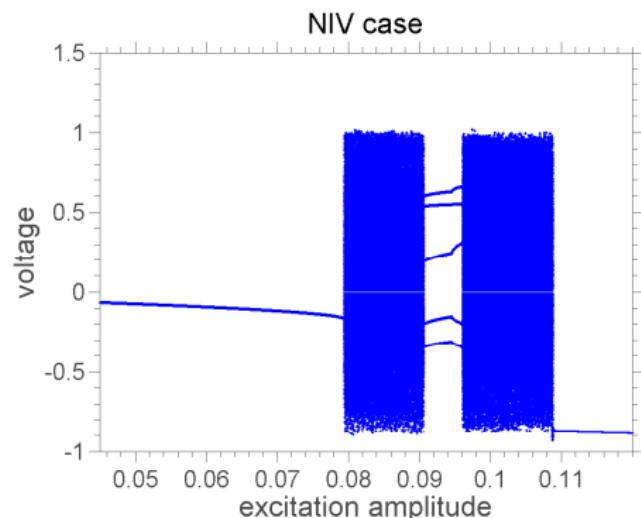


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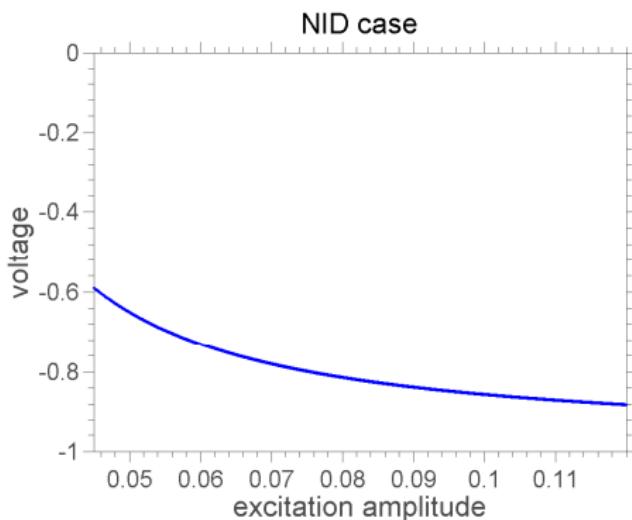
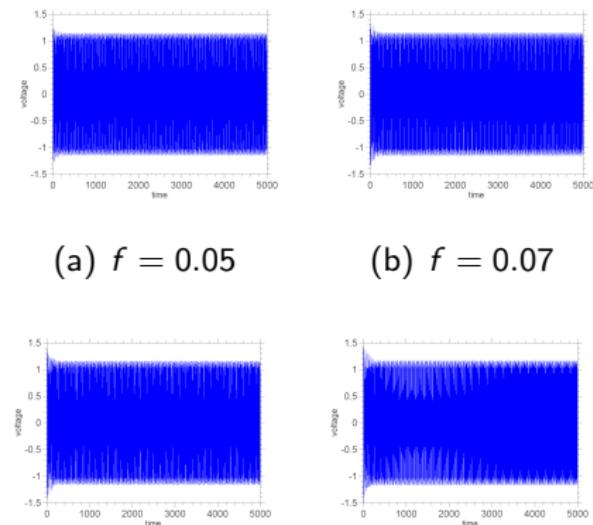
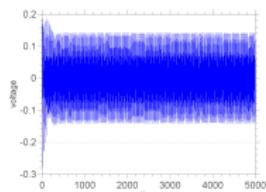
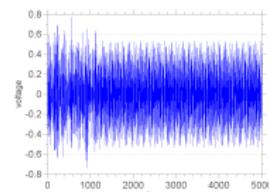
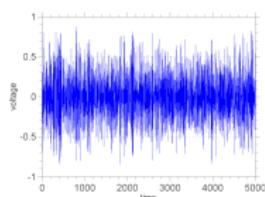
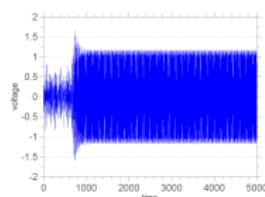
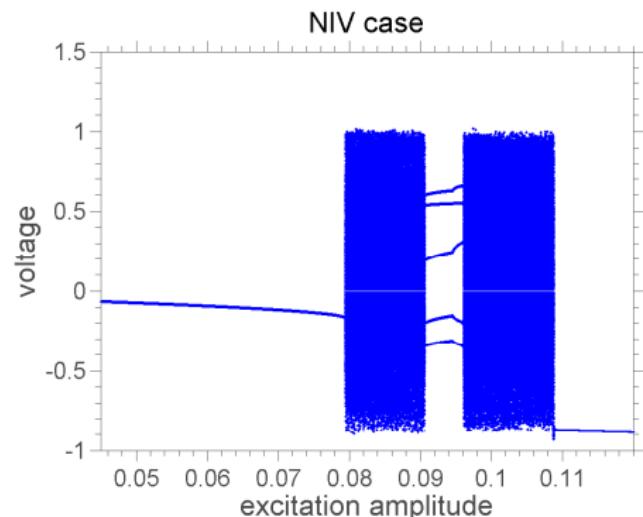


Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$



# Voltage vs excitation amplitude ( $\Omega = 0.8$ )

(e)  $f = 0.060$ (f)  $f = 0.092$ (g)  $f = 0.100$ (h)  $f = 0.116$ Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$

# Voltage vs excitation frequency ( $f = 0.083$ )

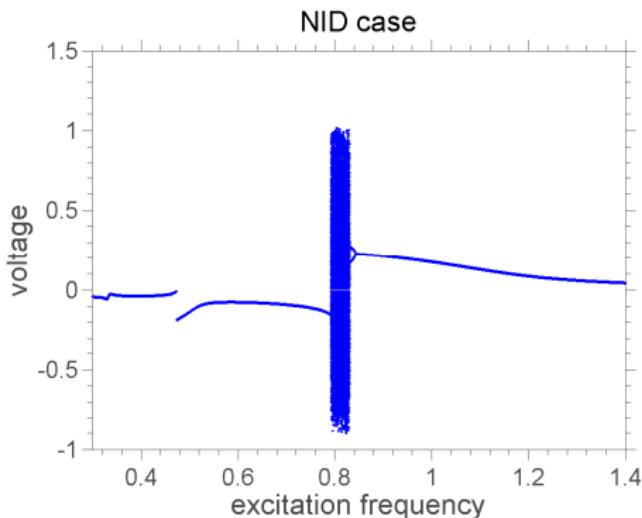


Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$

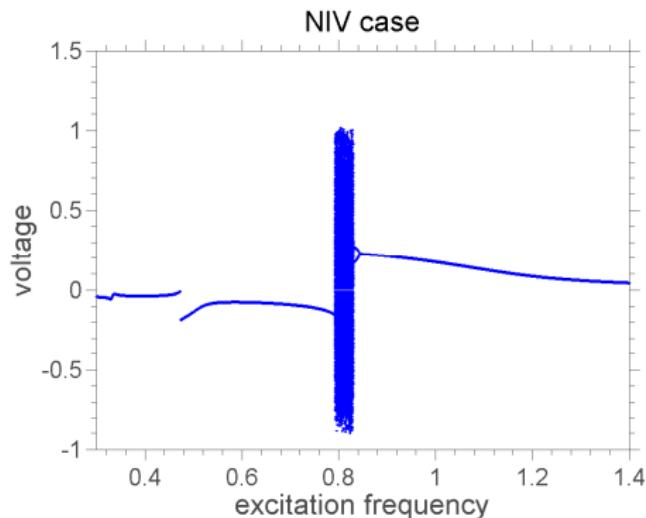


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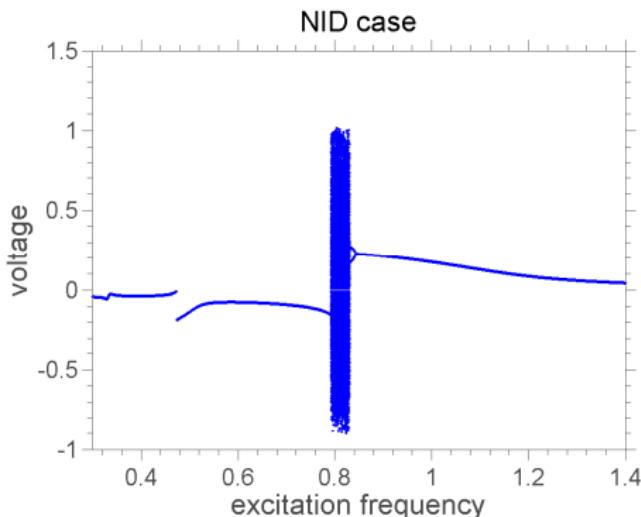
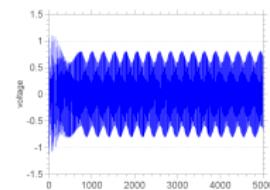
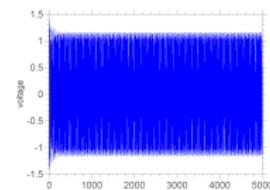


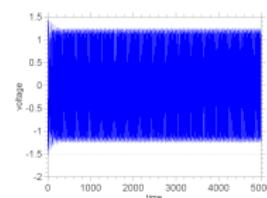
Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$



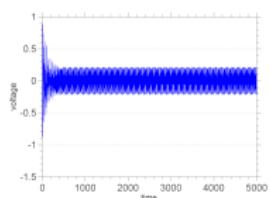
(a)  $\Omega = 0.600$



(b)  $\Omega = 0.800$

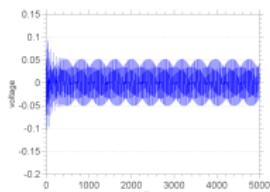
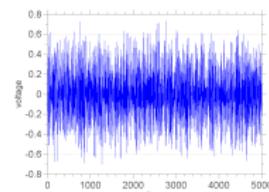
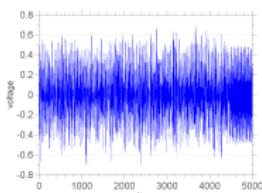
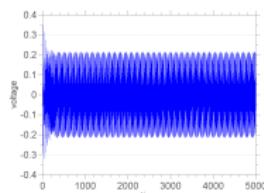
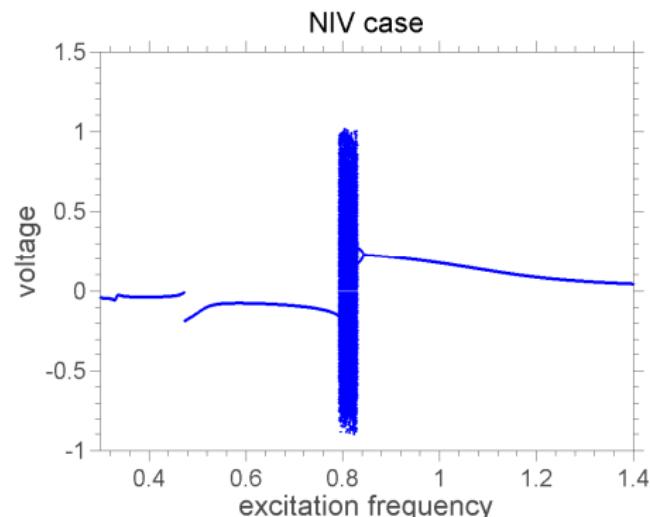


(c)  $\Omega = 0.840$

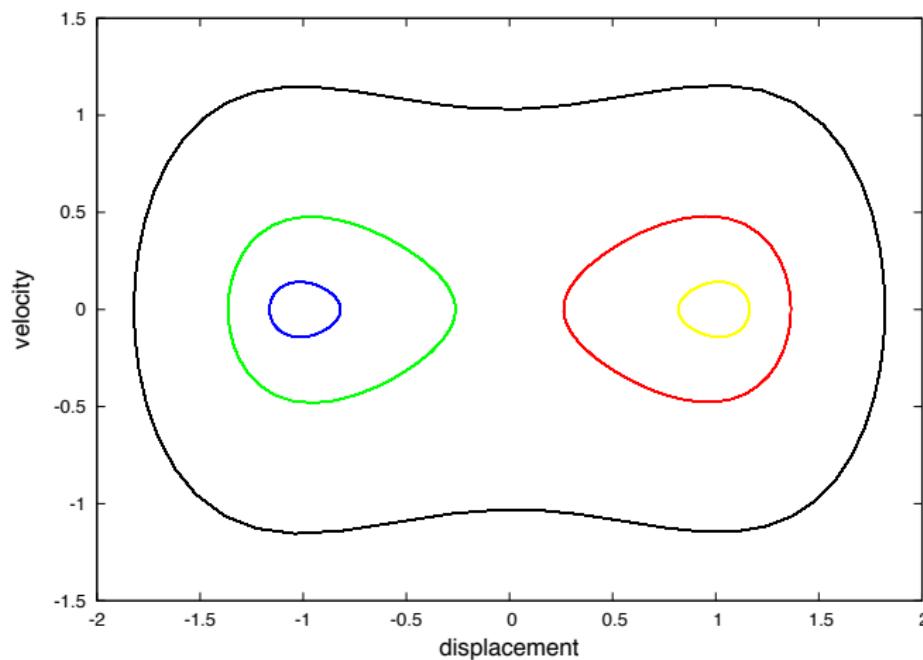


(d)  $\Omega = 1.200$

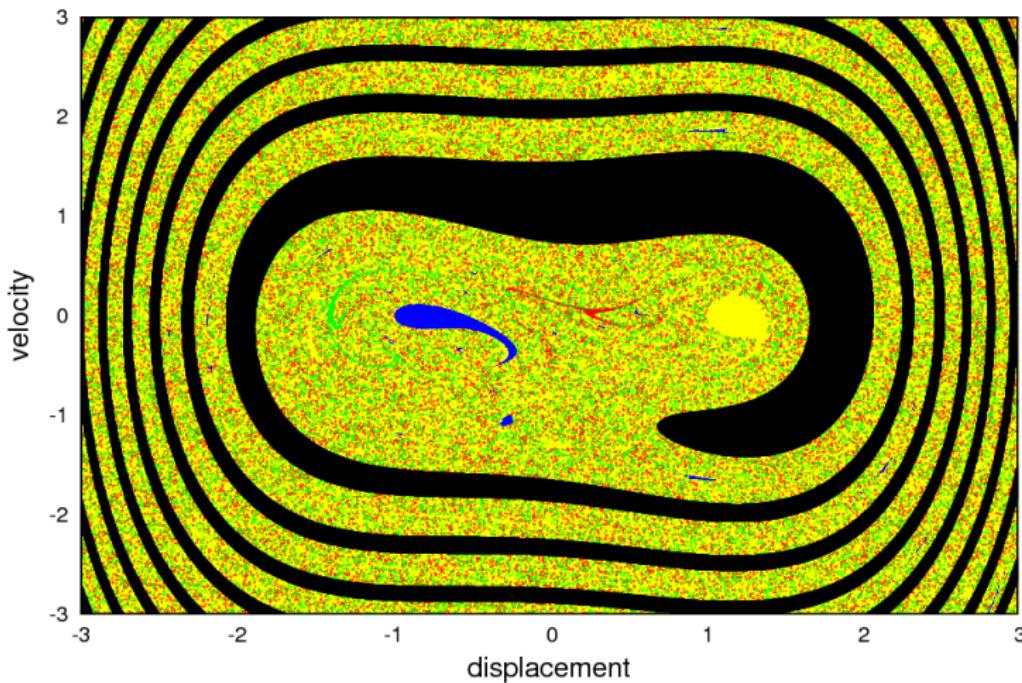
# Voltage vs excitation frequency ( $f = 0.083$ )

(e)  $\Omega = 0.400$ (f)  $\Omega = 0.800$ (g)  $\Omega = 0.840$ (h)  $\Omega = 1.200$ Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$

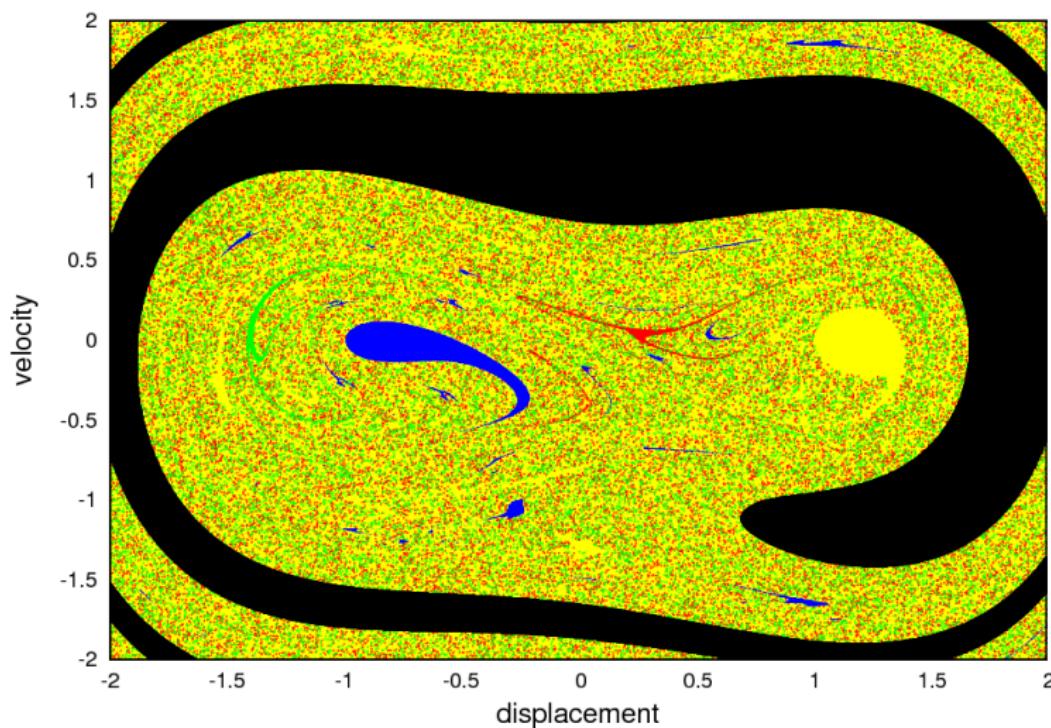
# Basins of Attraction ( $f = 0.12$ and $\Omega = 0.8$ )



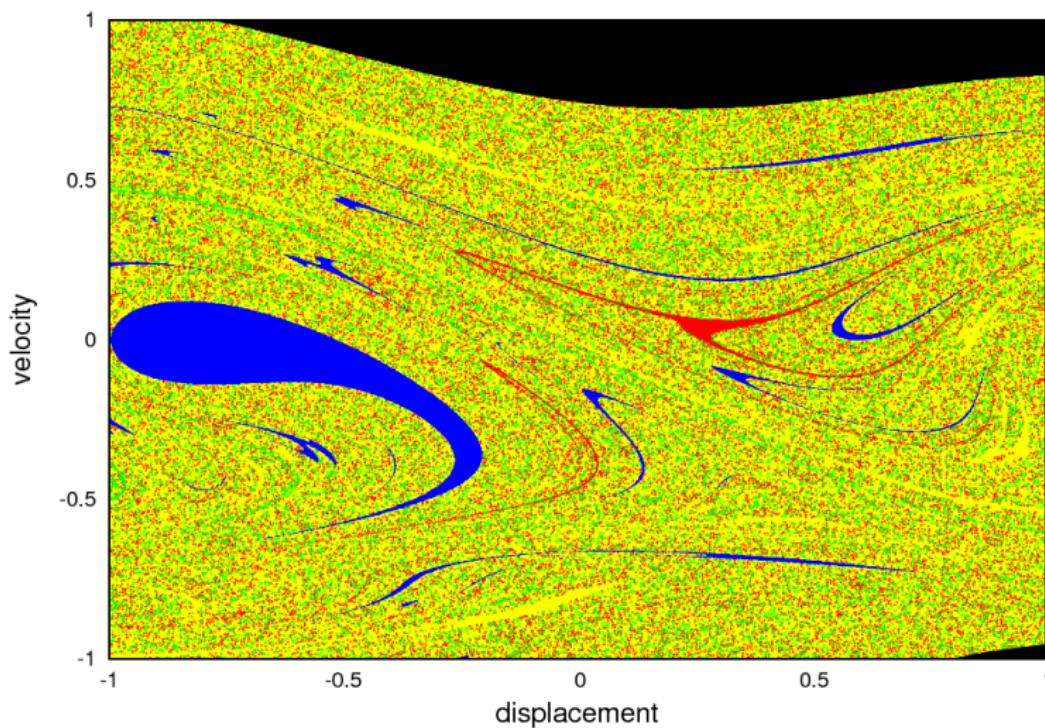
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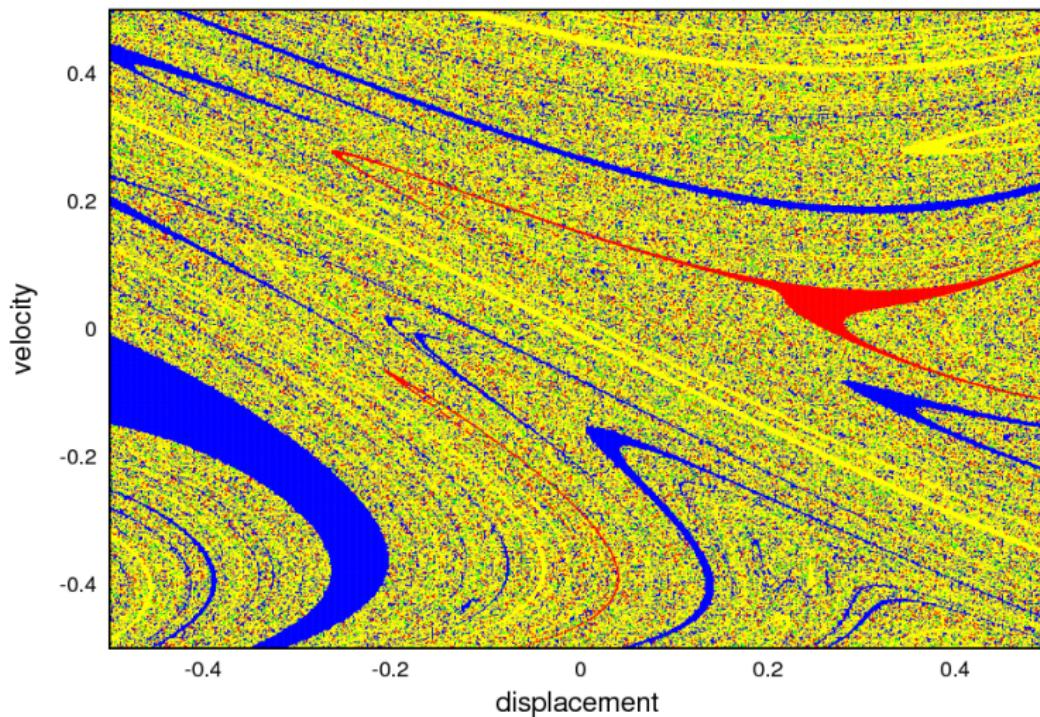
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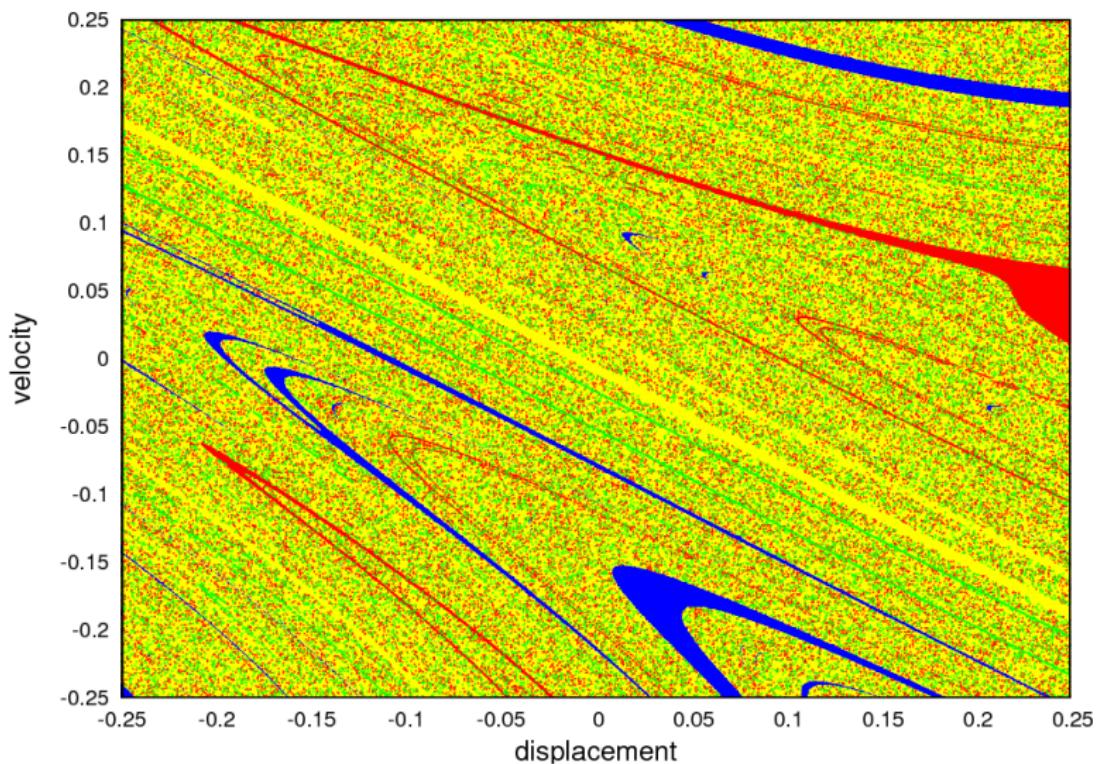
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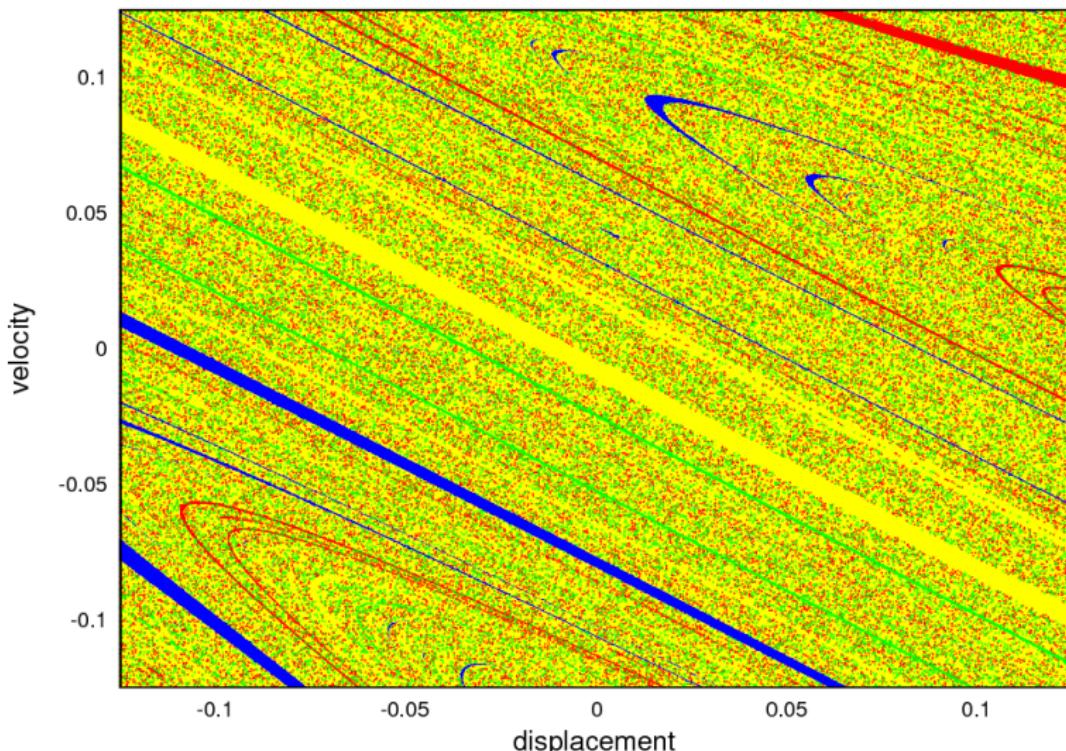
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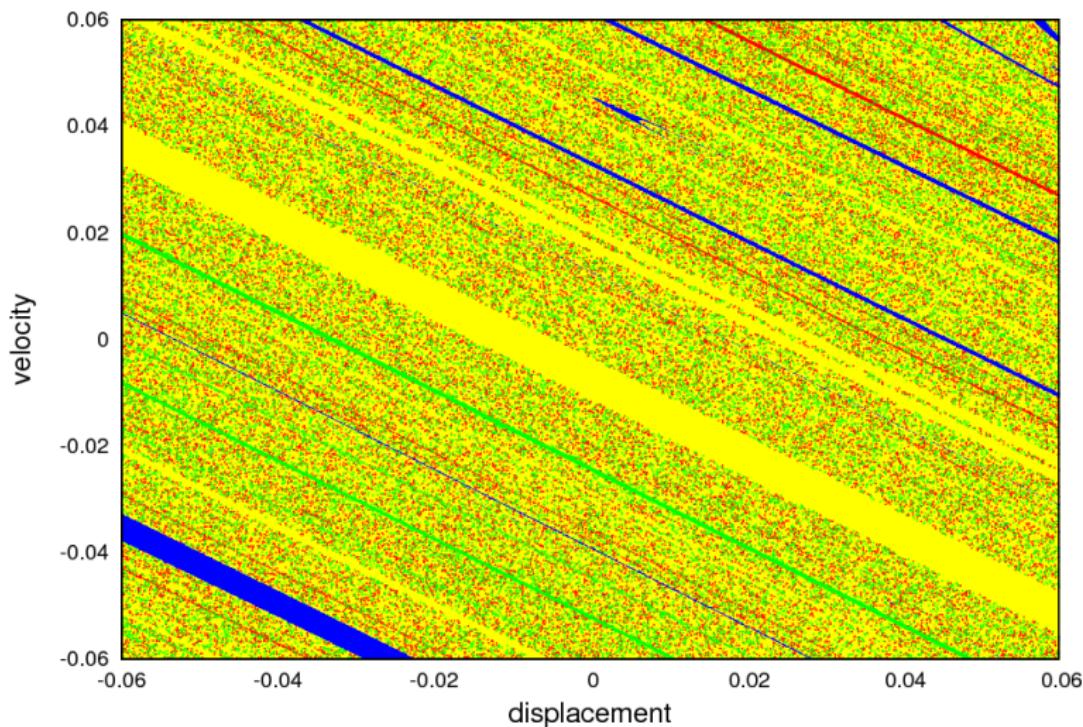
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# Basins of Attraction ( $f = 0.12$ and $\Omega = 0.8$ )



# Basins of Attraction ( $f = 0.12$ and $\Omega = 0.8$ )



## Section 4

### Final Remarks



# Final remarks

## Contributions:

- Detailed investigation of the system nonlinear dynamics
  - Bifurcation in system response due to external excitation
  - Sensitivity to initial conditions

## Ongoing research:

- Modeling of system parameters uncertainties
- Control of chaos to improve the system efficiency



# Acknowledgments

Academic discussion:

- Prof<sup>a</sup>. Aline de Paula (UnB)
- Prof. Adriano Fabro (UnB)
- Mr. Tiago Pereira (UnB)

Financial support:

- CNPq
- CAPES
- FAPERJ



# Thank you for your attention!

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A. Cunha Jr,

**Enhancing the performance of a bi-stable energy harvesting device via cross-entropy method.**  
(under review) <https://hal.archives-ouvertes.fr/hal-01531845>



J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr,

**Numerically exploring the nonlinear dynamics of a piezo-magneto-elastic energy harvesting device.**  
(in preparation)

# Physical system parameters

parameter	value
$\xi$	0.01
$\chi$	0.05
$f$	0.083
$\Omega$	0.8
$\lambda$	0.05
$\kappa$	0.5

# Displacement vs excitation amplitude ( $\Omega = 0.8$ )

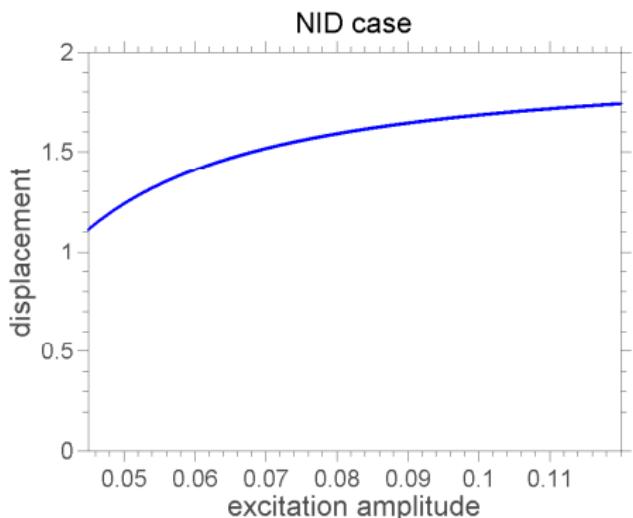


Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$

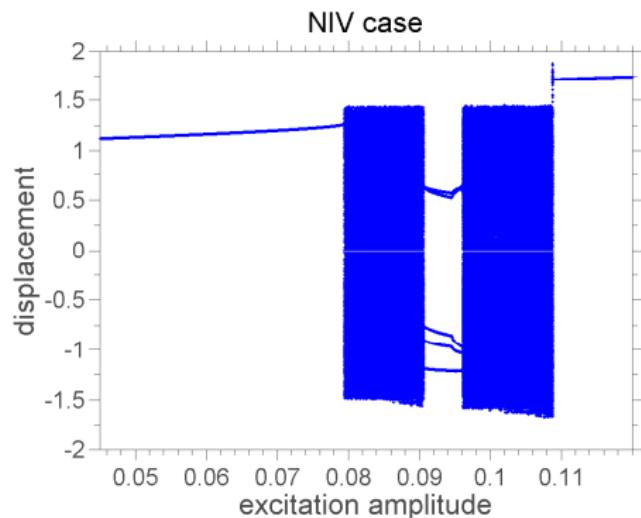


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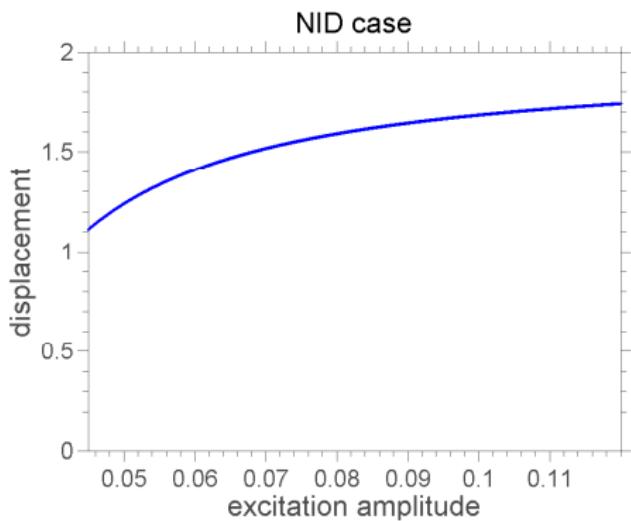
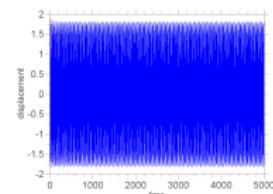
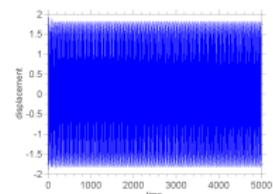


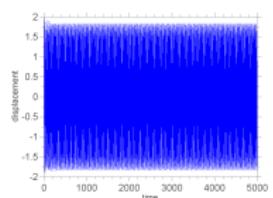
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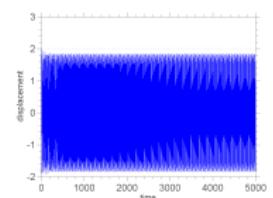
(a)  $f = 0.05$



(b)  $f = 0.07$

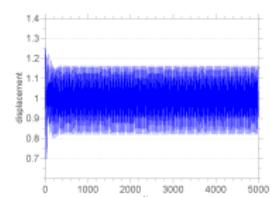


(c)  $f = 0.09$

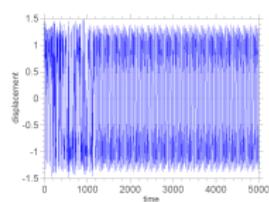


(d)  $f = 0.11$

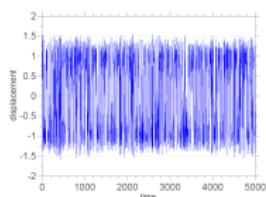
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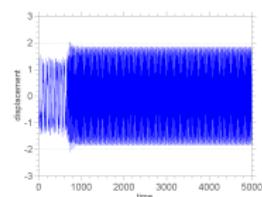
(e)  $f = 0.060$



(f)  $f = 0.092$



(g)  $f = 0.100$



(h)  $f = 0.116$

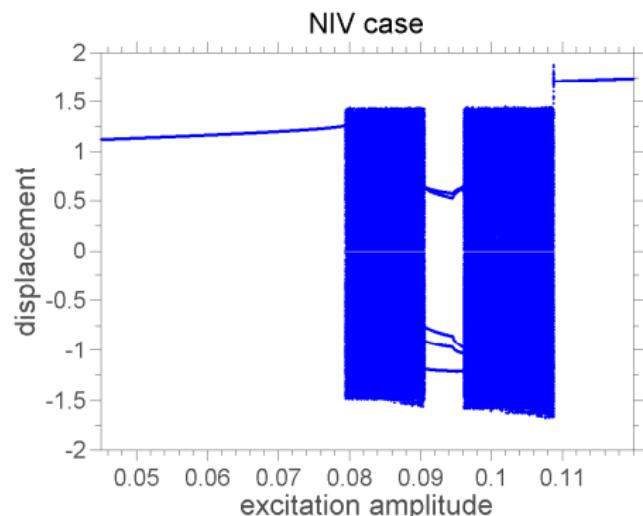


Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$

# Displacement vs excitation frequency ( $f = 0.083$ )

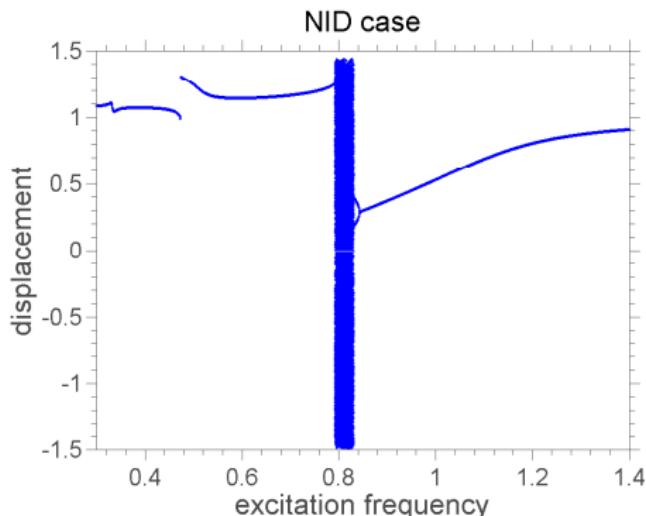


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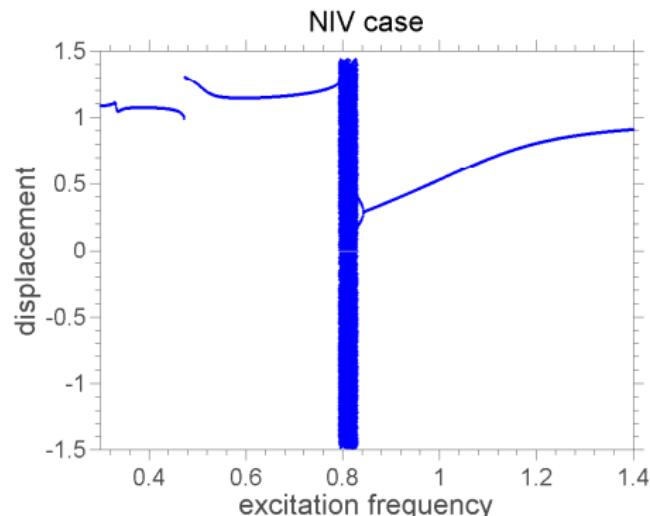


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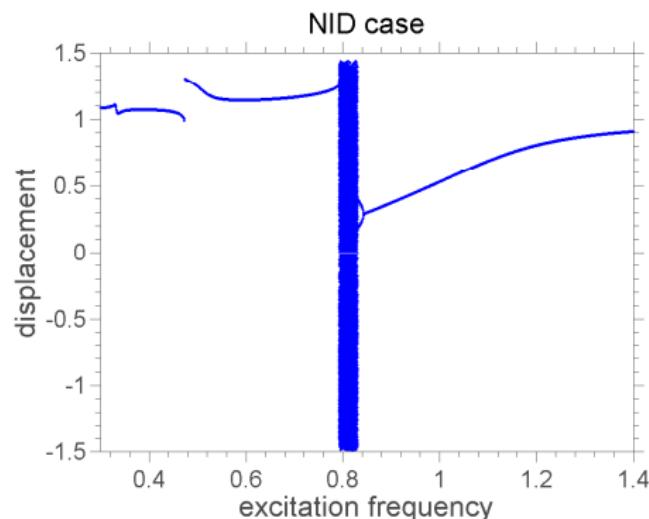
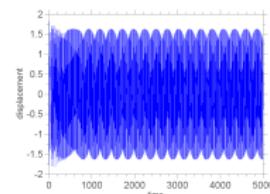
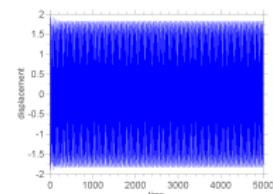


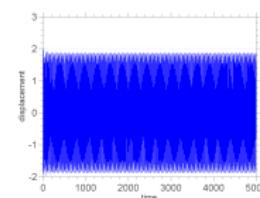
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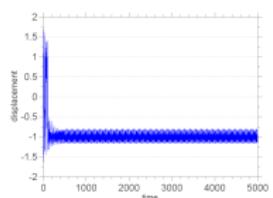
(a)  $\Omega = 0.600$



(b)  $\Omega = 0.800$

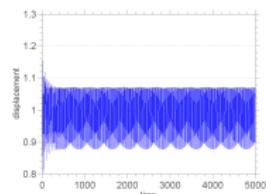
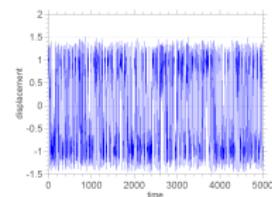
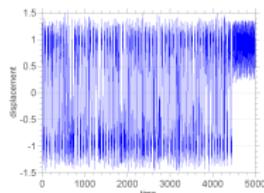
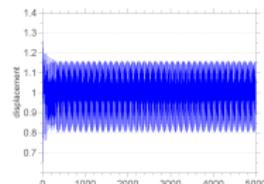
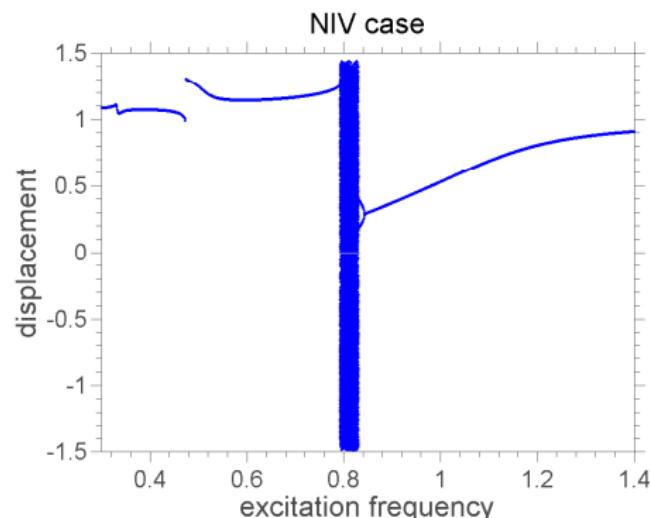


(c)  $\Omega = 0.840$



(d)  $\Omega = 1.200$

# Displacement vs excitation frequency ( $f = 0.083$ )

(e)  $\Omega = 0.400$ (f)  $\Omega = 0.800$ (g)  $\Omega = 0.840$ (h)  $\Omega = 1.200$ Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$

# Velocity vs excitation amplitude ( $\Omega = 0.8$ )

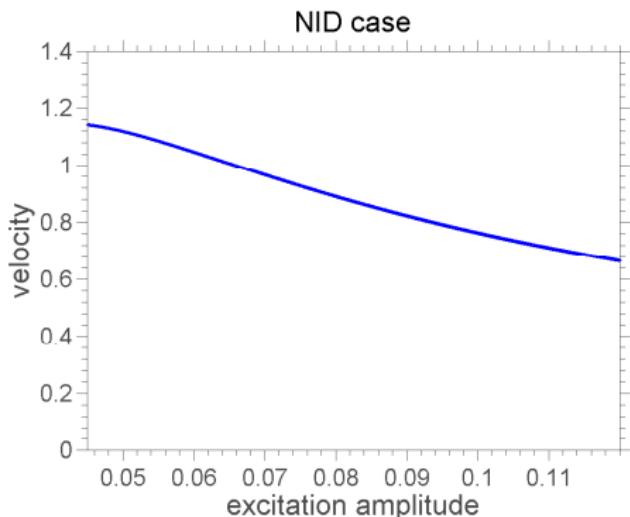


Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$

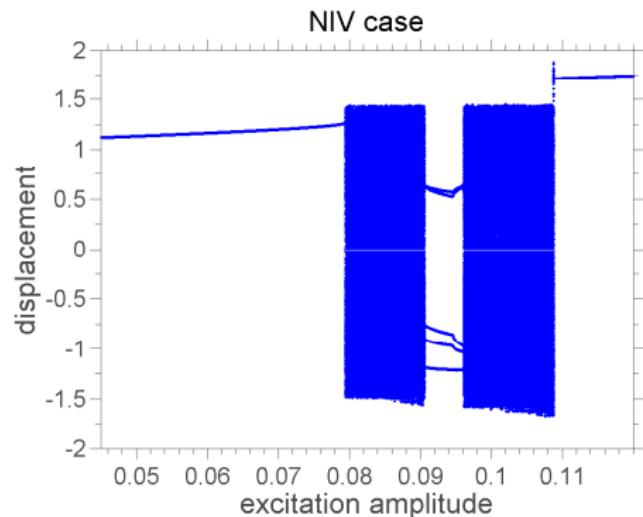


Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$

# Velocity vs excitation amplitude ( $\Omega = 0.8$ )

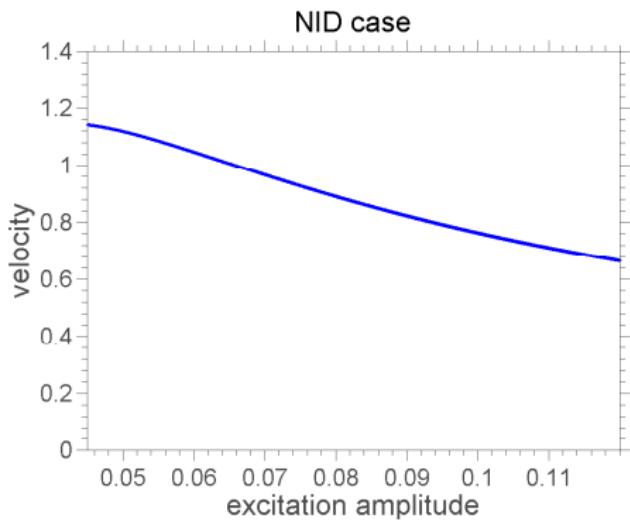
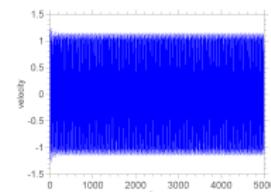
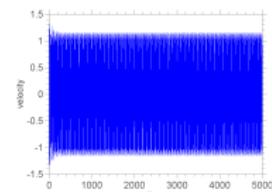


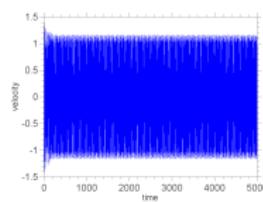
Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$



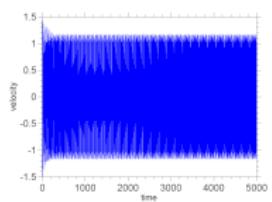
(a)  $f = 0.05$



(b)  $f = 0.07$

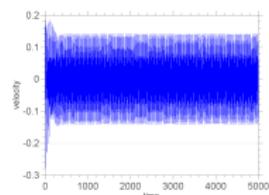
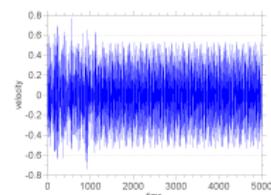
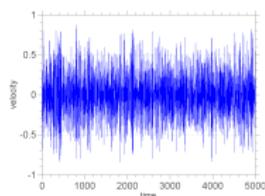
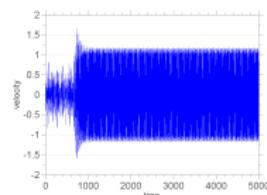
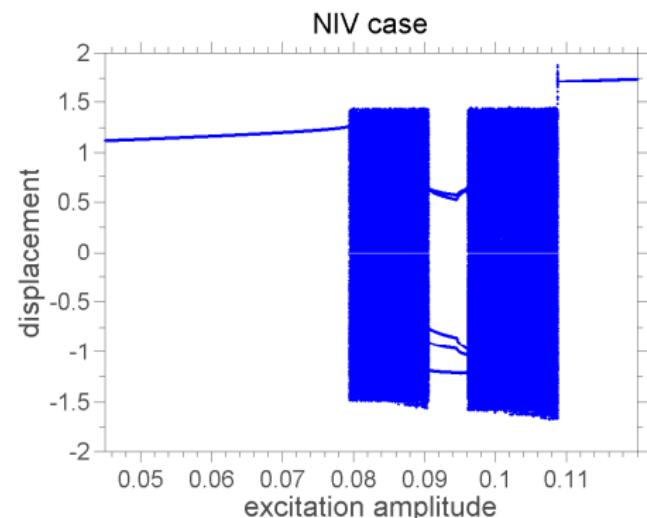


(c)  $f = 0.09$



(d)  $f = 0.11$

# Velocity vs excitation amplitude ( $\Omega = 0.8$ )

(e)  $f = 0.060$ (f)  $f = 0.092$ (g)  $f = 0.100$ (h)  $f = 0.116$ Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$

# Velocity vs excitation frequency ( $f = 0.083$ )

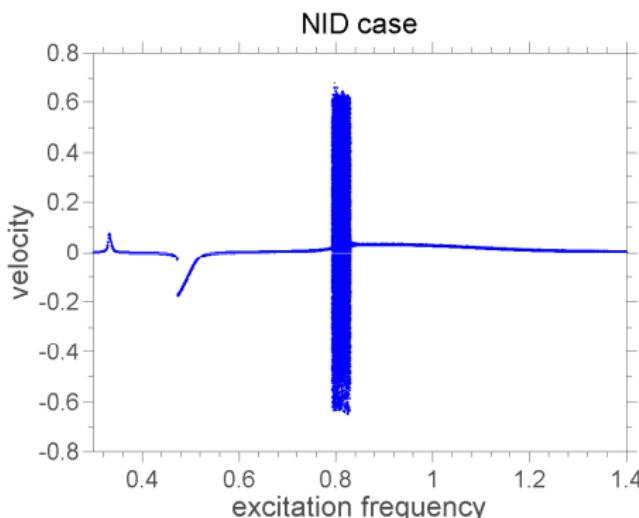


Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$

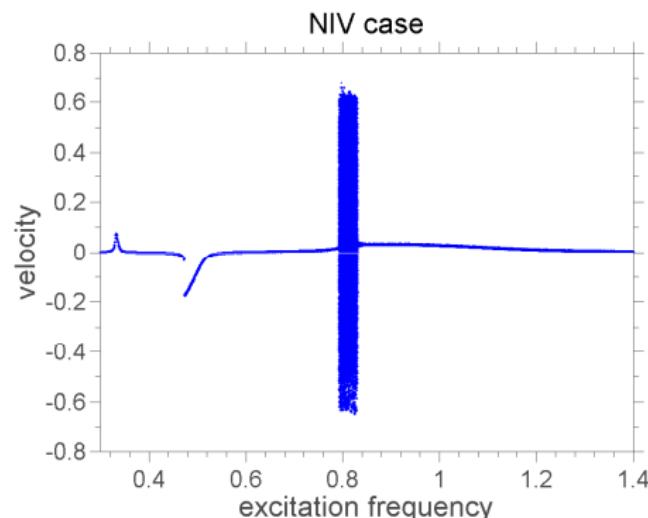


Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$

# Velocity vs excitation frequency ( $f = 0.083$ )

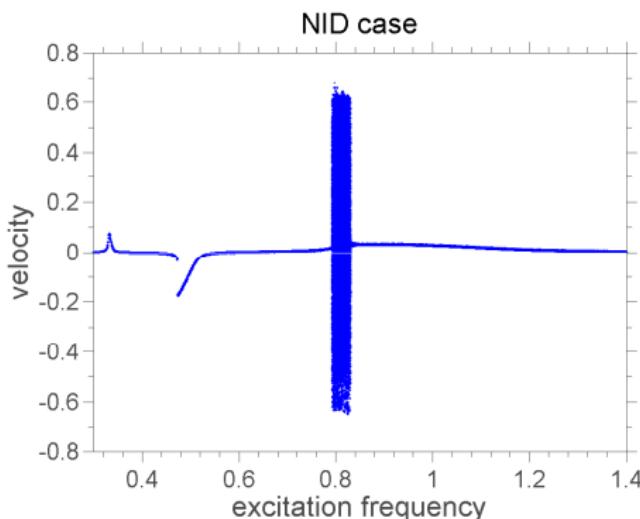
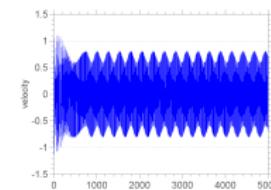
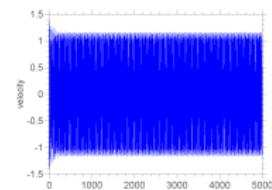


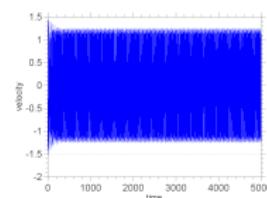
Figure:  $(x_0, \dot{x}_0, v_0) = (0 \ 1 \ 1)$



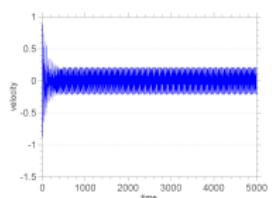
(a)  $\Omega = 0.600$



(b)  $\Omega = 0.800$

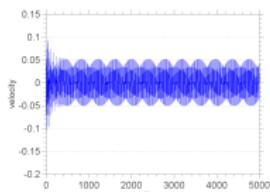
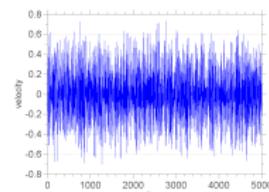
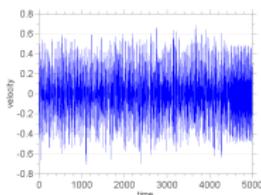
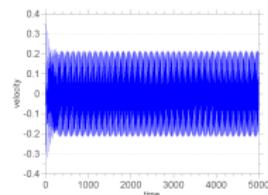
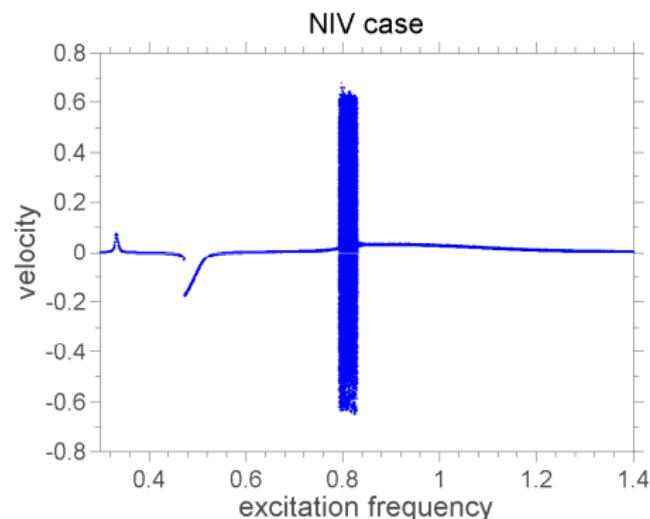


(c)  $\Omega = 0.840$



(d)  $\Omega = 1.200$

# Velocity vs excitation frequency ( $f = 0.083$ )

(e)  $\Omega = 0.400$ (f)  $\Omega = 0.800$ (g)  $\Omega = 0.840$ (h)  $\Omega = 1.200$ Figure:  $(x_0, \dot{x}_0, v_0) = (1 \ 0 \ 1)$