The nonlinear dynamics of a bistable energy harvesting system with colored noise disturbances

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NUMERICO – **Nu**cleus of **M**odeling and **E**xpe**ri**mentation with **Co**mputers

http://numerico.ime.uerj.br

In collaboration with: Vinicius Lopes (UERJ)
João Peterson (UERJ)

CCIS 2019
March 19–22, 2019
Georgia Tech, Atlanta, USA

Outline

Section 1

Introduction

April 4,
2005

March 13,
2013

L. Gammaitoni, **Fundamentals on energy**, NiPS Summer School 2018, University of Perugia.


IS-1
ST. JUDE MEDICAL
Sylmar, CA USA
VICTORY™ SR
5610 SSIR
S/N 1723182

Wikimedia Commons, **"File:St Jude Medical pacemaker in hand.jpg — Wikimedia Commons, the free media repository"**, 2014.

What's common in both cases?

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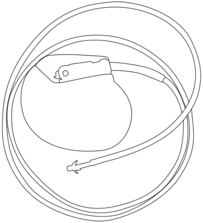
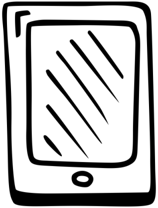
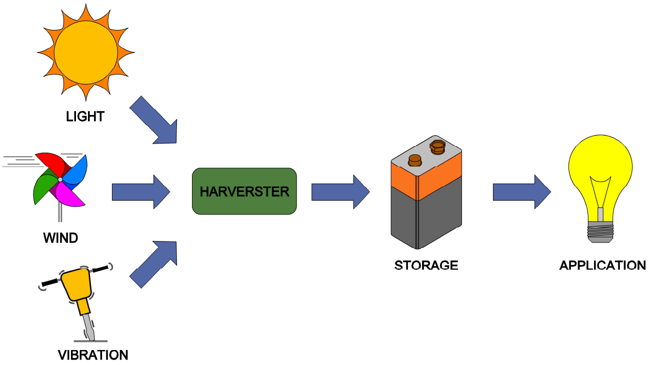
Electronic devices demanding energy!



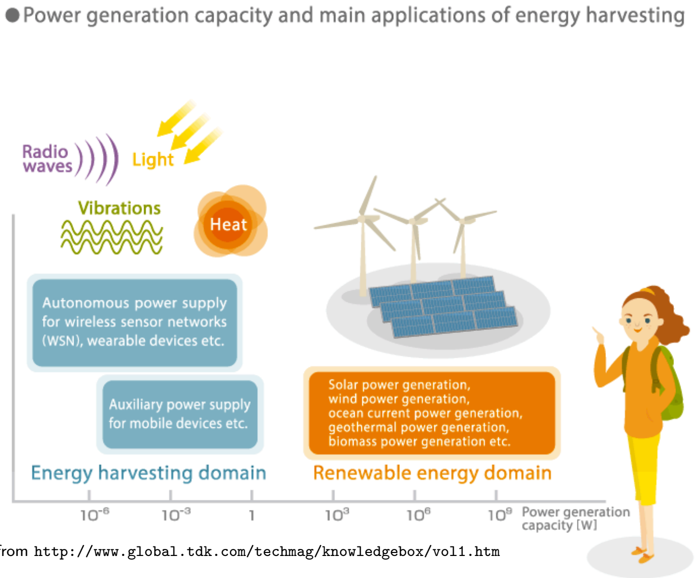


Energy Harvesting concept



- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

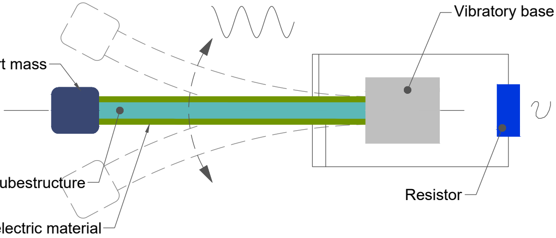
Energy scale for modern Energy Harvesting technologies



*Picture from `http://www.global.tdk.com/techmag/knowledgebox/vol1.htm`

Vibration based Energy Harvesting

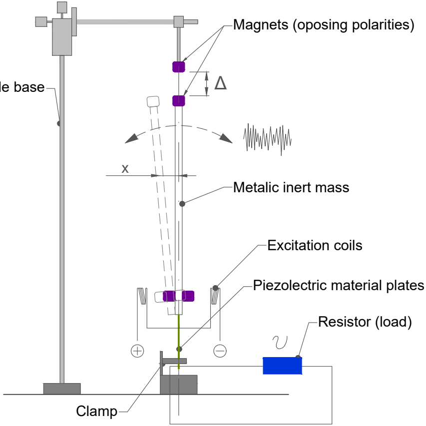
Monostable system driven by regular signal



S. Roundy, P. K. Wright and J. Rabaey, A study of low level bibrations as a power source for wireless sensor nodes. **Computer Communications**, 26: 1131-1144, 2003.

Vibration based Energy Harvesting

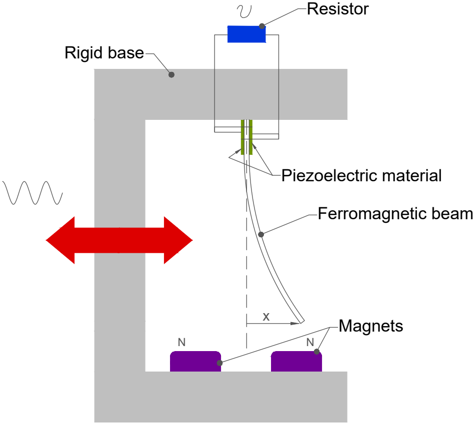
Bistable system driven by a noisy signal



F. Cottone, H. Vocca and L. Gammaitoni, Nonlinear energy harvesting. **Physical Review Letters**, 102: 080601, 2009.

Vibration based Energy Harvesting

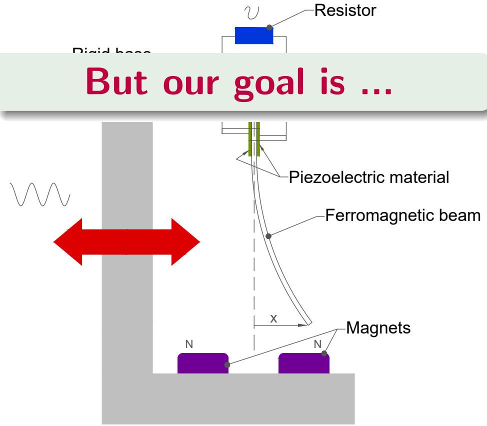
Bistable system driven by regular signal



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Vibration based Energy Harvesting

Bistable system driven by regular signal

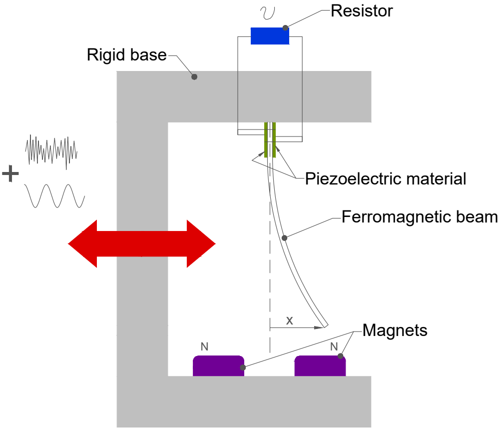


But our goal is ...

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Vibration based Energy Harvesting

Bistable system driven by regular and noisy signals



V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr, **On the nonlinear stochastic dynamics of piezo-magneto-elastic energy harvester driven by colored noise**, (in preparation) 2019.

Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
  - Time series
  - Poincaré sections
  - Bifurcation diagrams
  - Basis of attractions
  - Test 0-1 for chaos
- Model the underlying uncertainties and study their influence
  - System parameters variability
  - Noise in system excitation
- Propose strategies to enhance the recovered energy
  - Nonlinear optimization
  - Control of chaos

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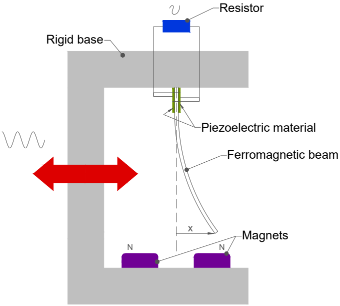
Section 2

Nonlinear Dynamics

Bistable harvester driven by regular signal



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t$$

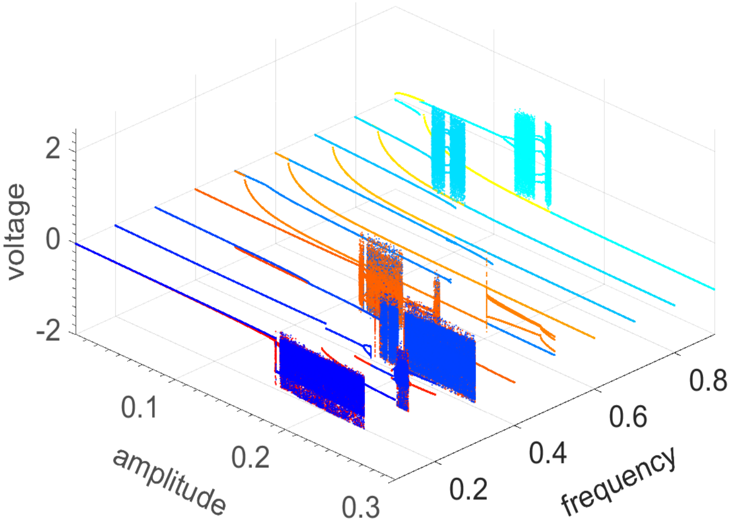
$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0, \;\; \dot{x}(0) = \dot{x}_0, \;\; \upsilon(0) = \upsilon_0$$

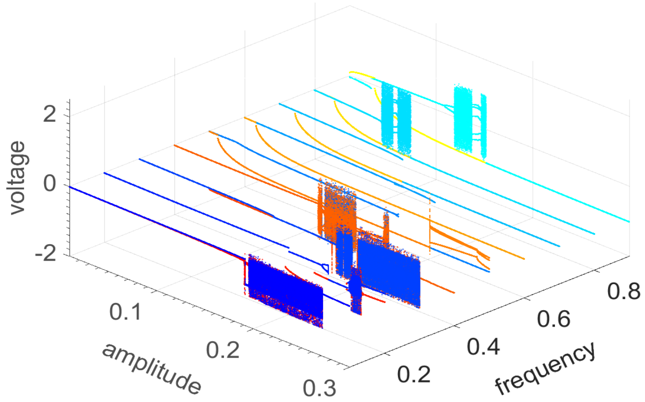
A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Nonlinear dynamics animation

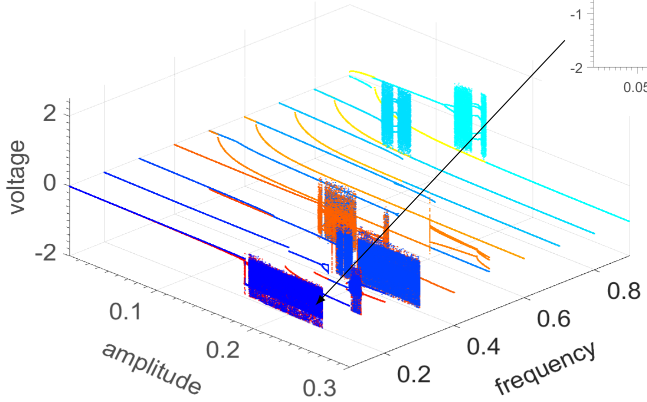
Global overview of force amplitude effect

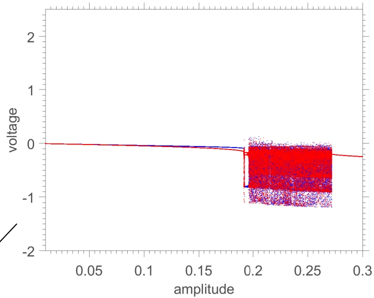


Bifurcation diagrams: voltage vs amplitude



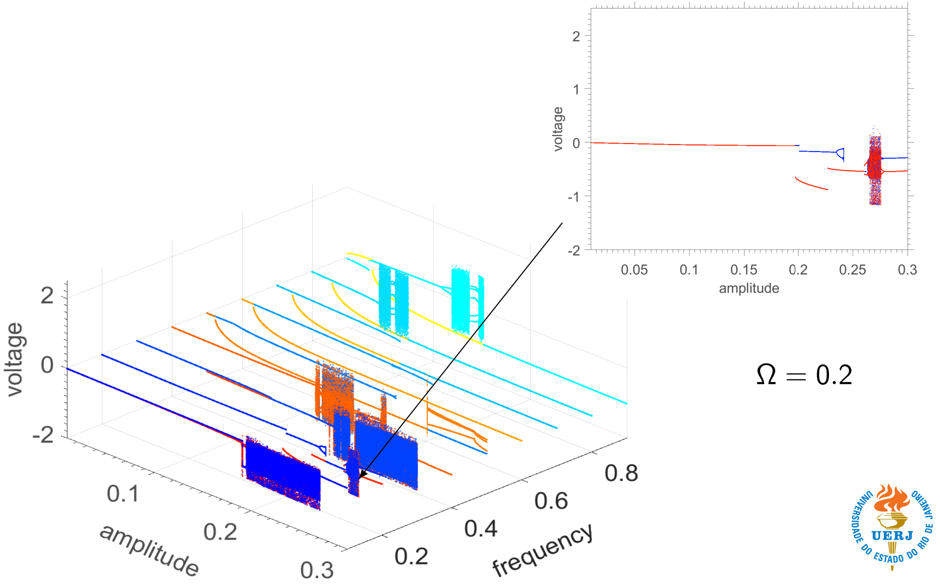
Bifurcation diagrams: voltage vs amplitude





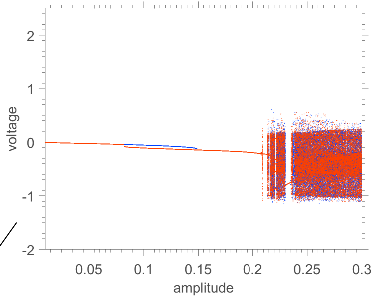
$$\Omega = 0.1$$

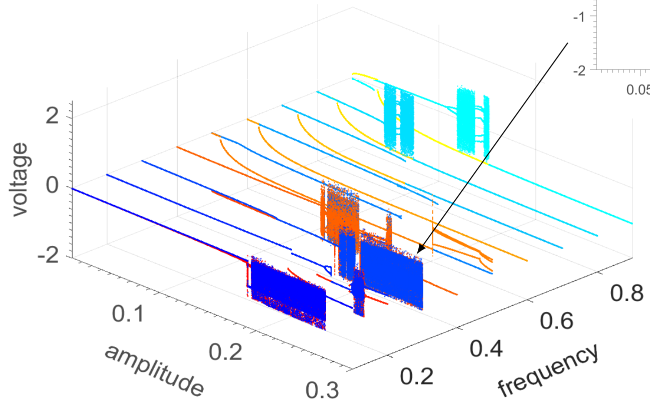
Bifurcation diagrams: voltage vs amplitude



$\Omega = 0.2$

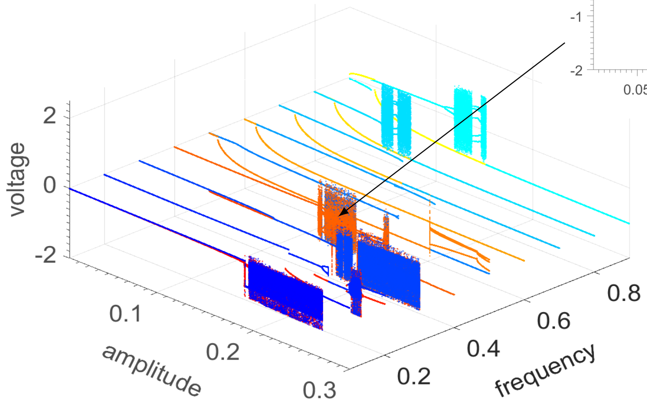
Bifurcation diagrams: voltage vs amplitude

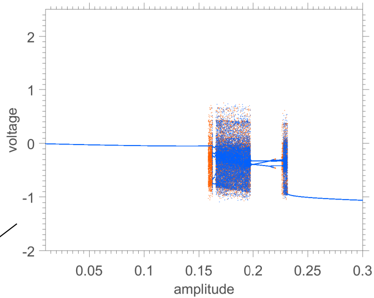




$\Omega = 0.3$

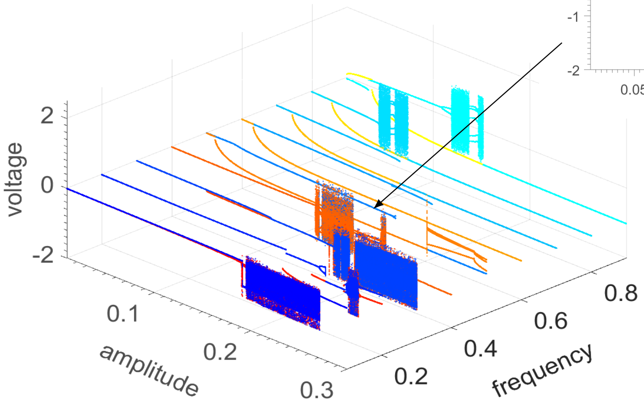
Bifurcation diagrams: voltage vs amplitude

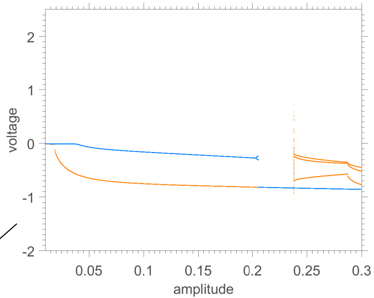




$\Omega = 0.4$

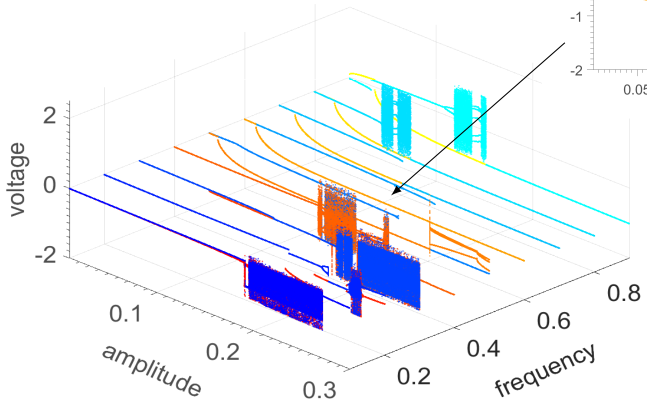
Bifurcation diagrams: voltage vs amplitude

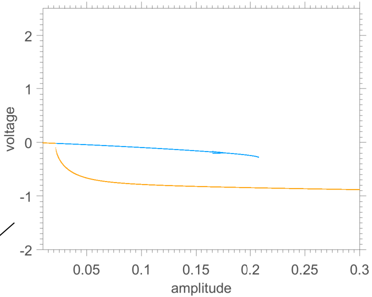




$\Omega = 0.5$

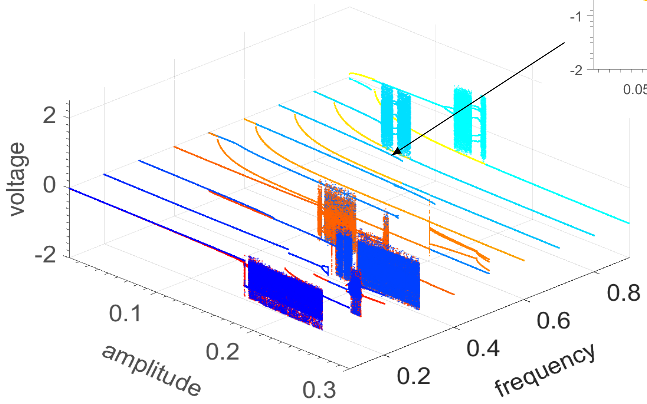
Bifurcation diagrams: voltage vs amplitude

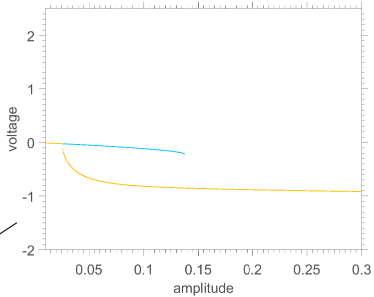




$\Omega = 0.6$

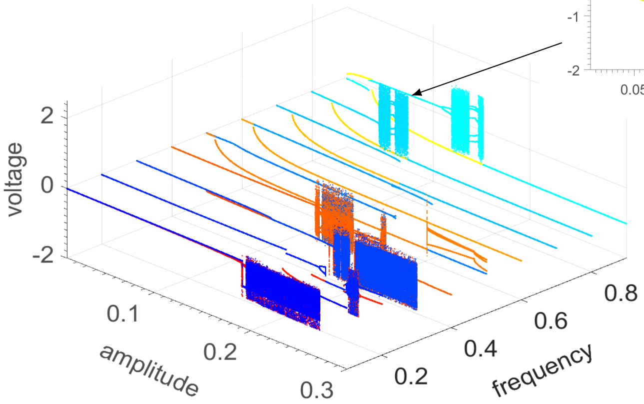
Bifurcation diagrams: voltage vs amplitude

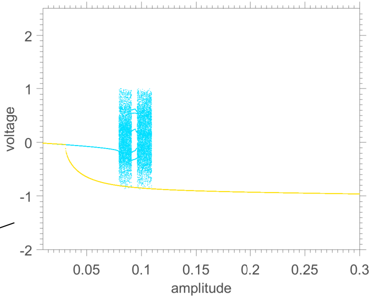




$\Omega = 0.7$

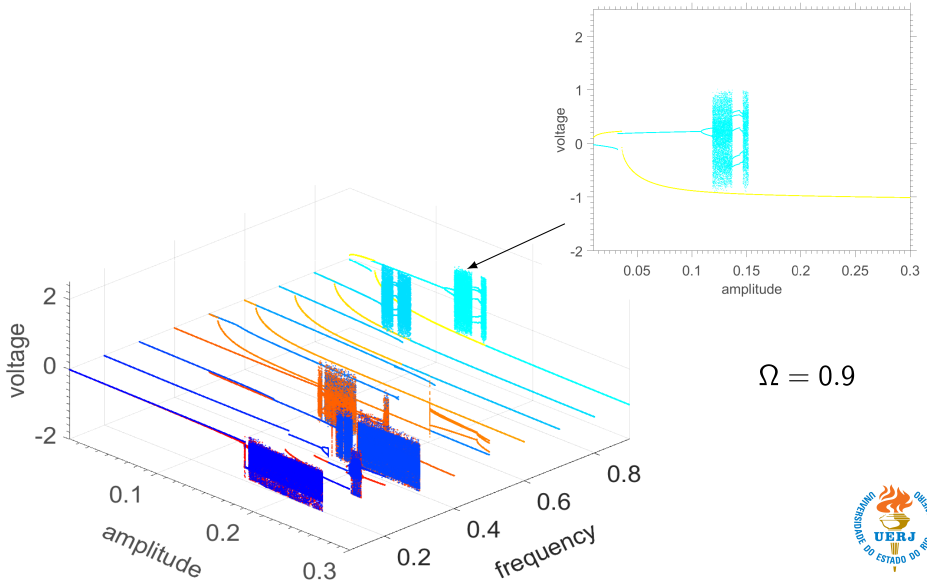
Bifurcation diagrams: voltage vs amplitude





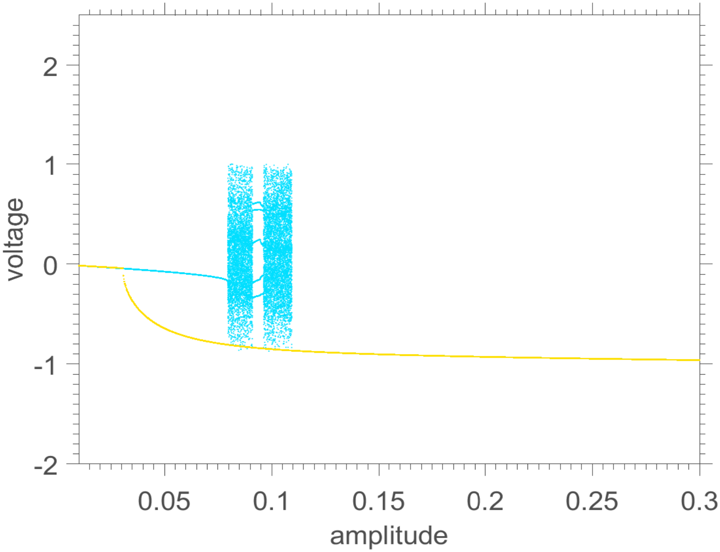
$\Omega = 0.8$

Bifurcation diagrams: voltage vs amplitude

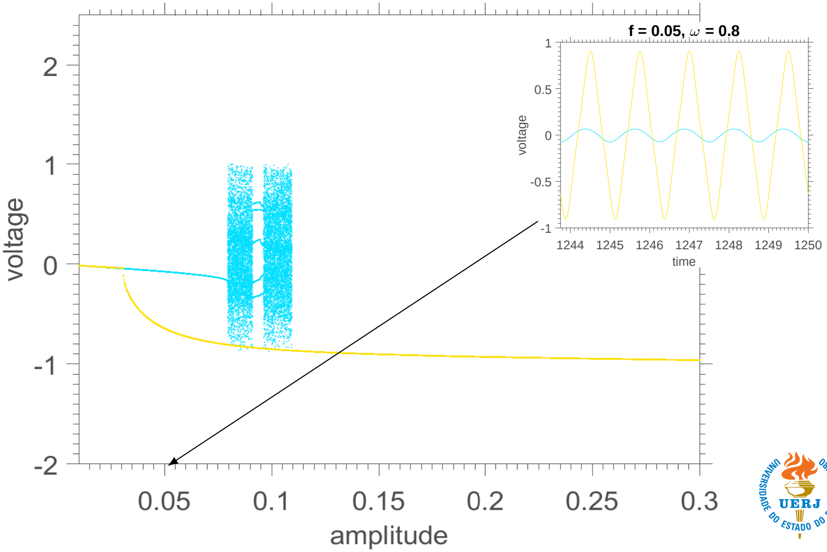


$\Omega = 0.9$

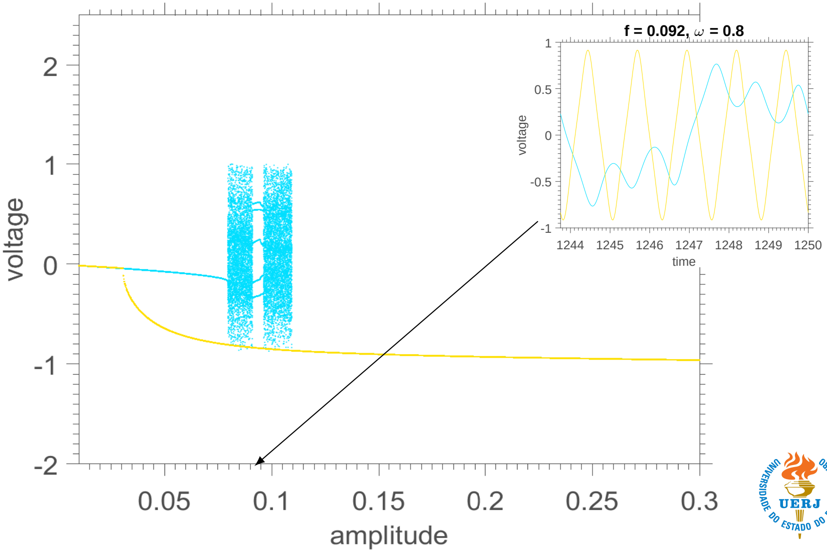
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



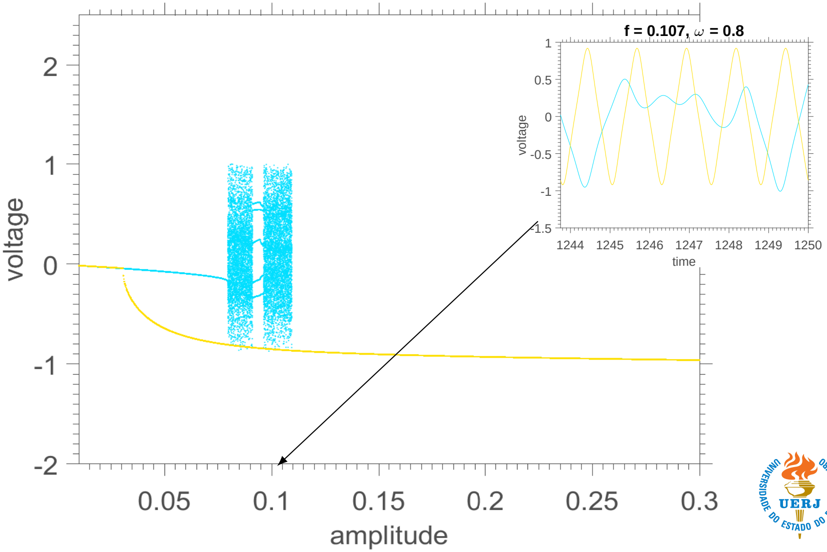
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Basins of attraction

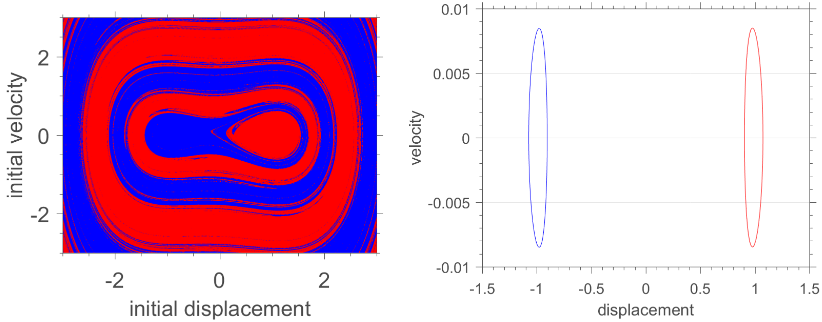


Figure: $f = 0.083$ and $\Omega = 0.1$

Basins of attraction

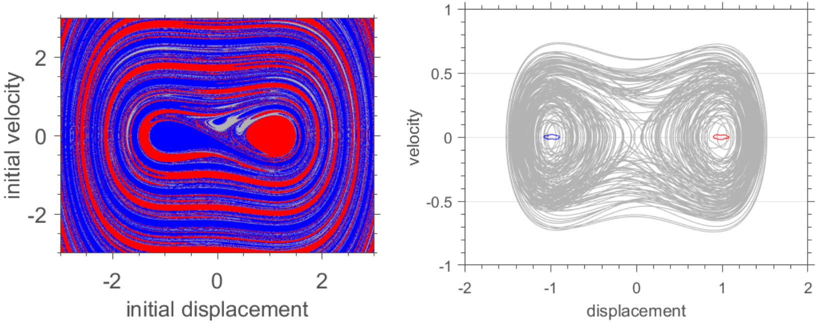
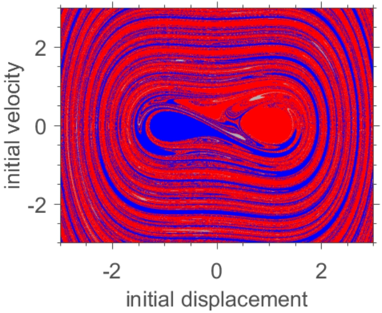


Figure: $f = 0.083$ and $\Omega = 0.2$

Basins of attraction



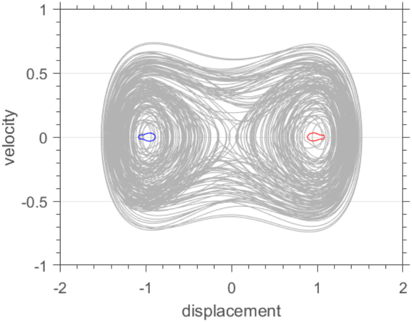


Figure: $f = 0.083$ and $\Omega = 0.3$

Basins of attraction

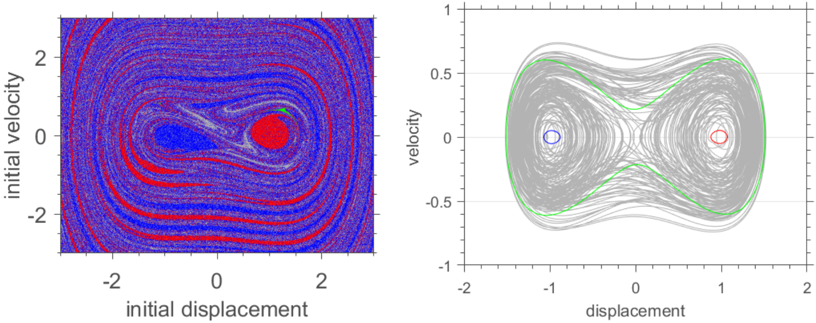


Figure: $f = 0.083$ and $\Omega = 0.4$

Basins of attraction

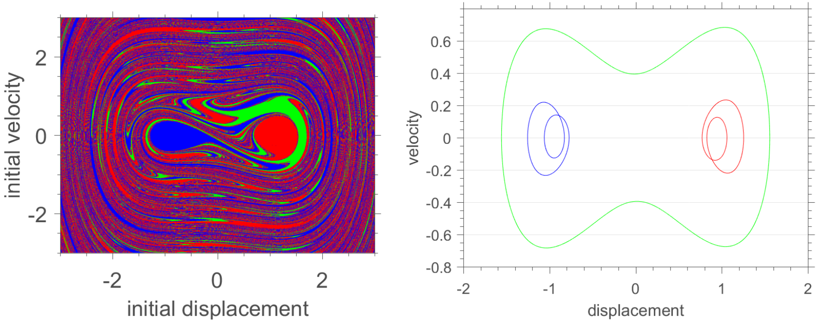


Figure: $f = 0.083$ and $\Omega = 0.5$

Basins of attraction

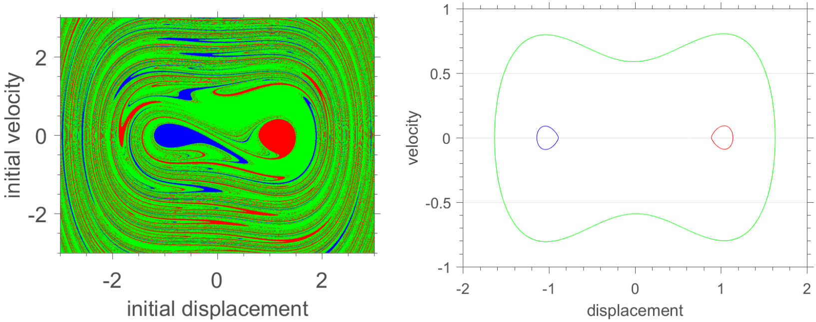


Figure: $f = 0.083$ and $\Omega = 0.6$

Basins of attraction

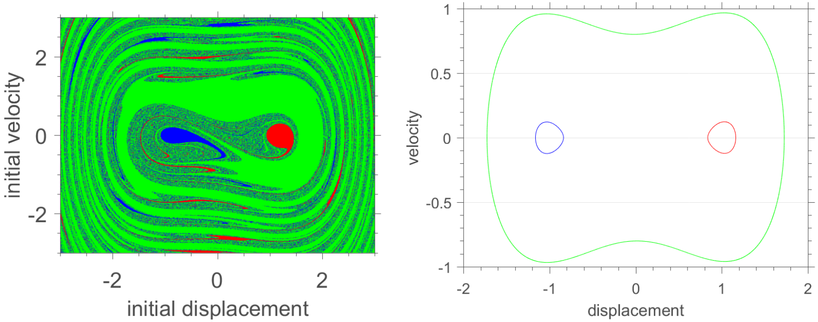
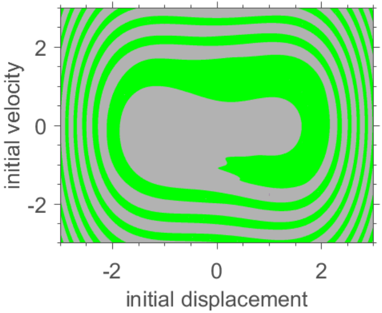


Figure: $f = 0.083$ and $\Omega = 0.7$

Basins of attraction



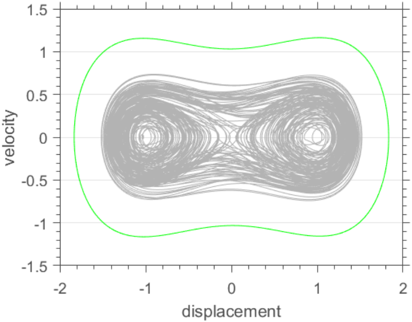


Figure: $f = 0.083$ and $\Omega = 0.8$

Basins of attraction

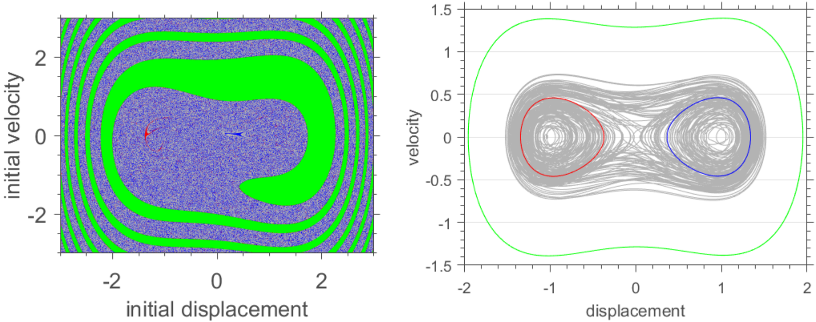
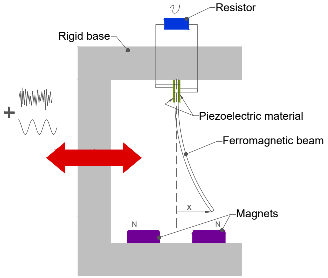


Figure: $f = 0.083$ and $\Omega = 0.9$

Section 3

Stochastic Dynamics

Bistable harvester driven by regular and noisy signals



$$\ddot{x}+2\xi\dot{x}-\frac{1}{2}x(1-x^2)-\chi\upsilon = f\cos\Omega t + \text{“noise”}$$

$$\dot{\upsilon}+\lambda\upsilon+\kappa\dot{x}=0$$

$$x(0)=x_0,\ \dot{x}(0)=\dot{x}_0,\ \upsilon(0)=\upsilon_0$$

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Nonlinear stochastic dynamical system

- external excitation N_t:
  - zero-mean stationary Gaussian process
  - covariance function:

$$\mathtt{cov}_{N_t}(t_1, t_2) = \sigma \, \exp\left(-\frac{|t_2 - t_1|}{\tau_{corr}}\right), \quad \frac{\sigma^2}{\tau_{corr}} = constant$$

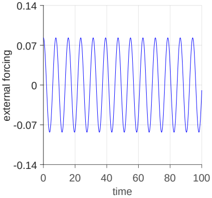
- stochastic evolution law:

$$\ddot{X} + 2\xi\dot{X} - \frac{1}{2}x(1 - X^2) - \chi V = f \cos \Omega t + N_t$$

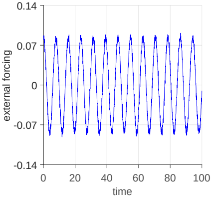
$$\dot{V} + \lambda V + \kappa \dot{X} = 0$$

$$X(0) = x_0, \;\; \dot{X}(0) = \dot{x}_0, \;\; V(0) = v_0$$

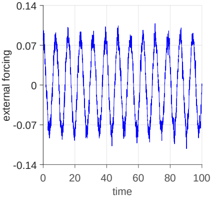
Realizations of random external force



(a) 1% of noise



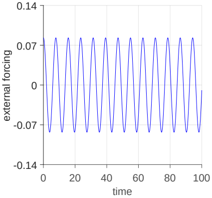
(b) 25% of noise



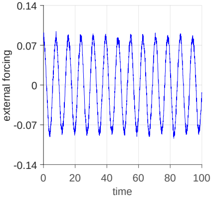
(c) 50% of noise

$$\sigma^2/\tau_{cor} = 1$$

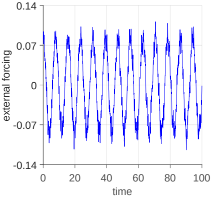
Realizations of random external force



(a) 1% of noise



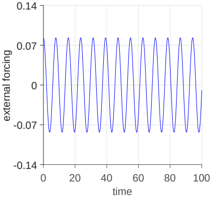
(b) 25% of noise



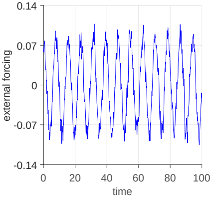
(c) 50% of noise

$$\sigma^2/\tau_{cor} = 0.5$$

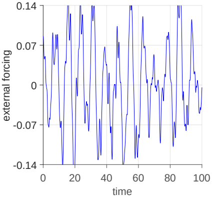
Realizations of random external force



(a) 1% of noise



(b) 25% of noise

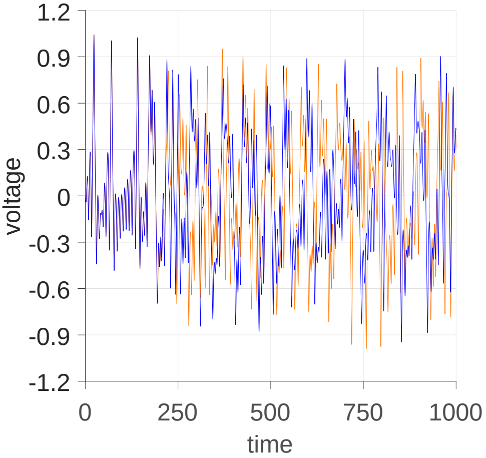


(c) 50% of noise

$$\sigma^2/\tau_{cor} = 0.1$$

Nonlinear stochastic dynamics animation

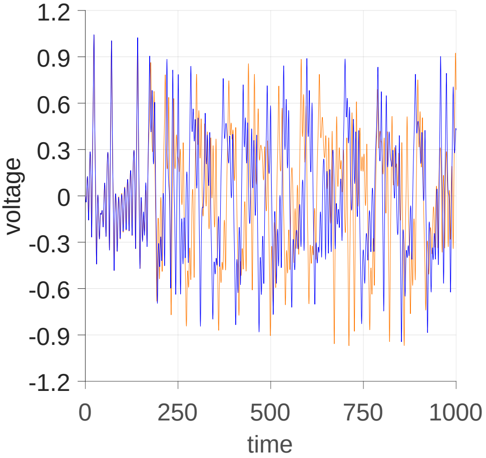
Typical voltage time series (1% of noise)



(a) $\sigma^2/\tau_{cor} = 1$

$f = 0.115$, $\Omega = 0.8$

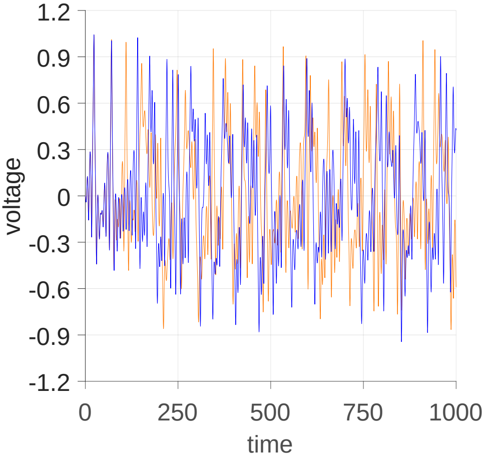
Typical voltage time series (1% of noise)



(b) $\sigma^2/\tau_{cor} = 0.5$

$f = 0.115$, $\Omega = 0.8$

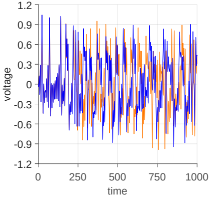
Typical voltage time series (1% of noise)



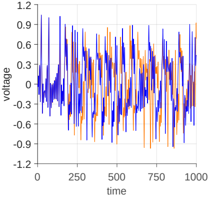
(c) $\sigma^2/\tau_{cor} = 0.1$

$f = 0.115,\ \Omega = 0.8$

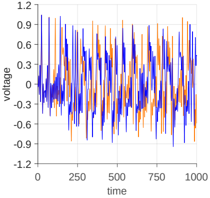
Typical voltage time series (1% of noise)



(a) $\sigma^2/\tau_{cor} = 1$



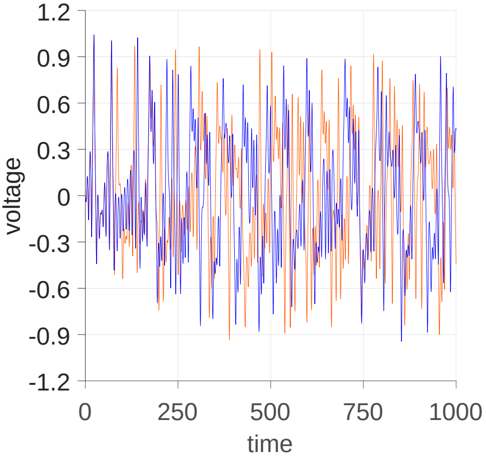
(b) $\sigma^2/\tau_{cor} = 0.5$



(c) $\sigma^2/\tau_{cor} = 0.1$

$$f = 0.115, \ \Omega = 0.8$$

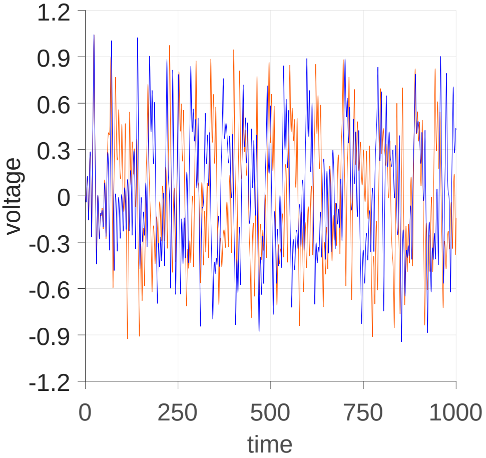
Typical voltage time series (25% of noise)



(a) $\sigma^2/\tau_{cor} = 1$

$f = 0.115,\ \Omega = 0.8$

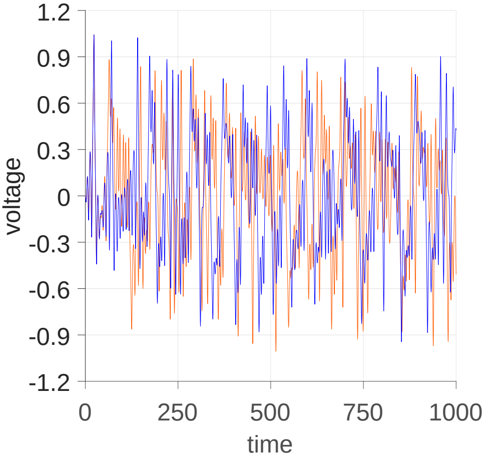
Typical voltage time series (25% of noise)



(b) $\sigma^2/\tau_{cor} = 0.5$

$f = 0.115$, $\Omega = 0.8$

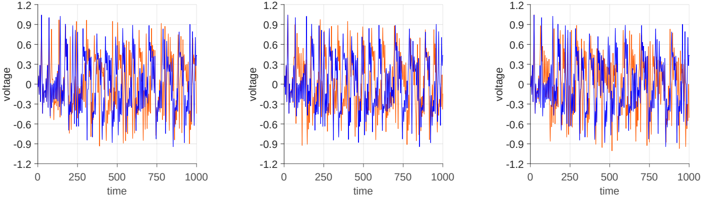
Typical voltage time series (25% of noise)



(c) $\sigma^2/\tau_{cor} = 0.1$

$f = 0.115$, $\Omega = 0.8$

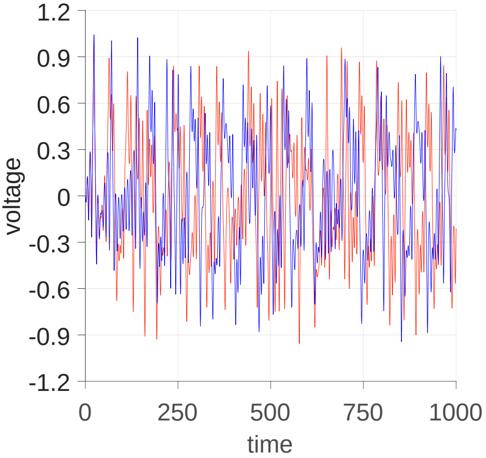
Typical voltage time series (25% of noise)



(a) $\sigma^2/\tau_{cor} = 1$ (b) $\sigma^2/\tau_{cor} = 0.5$ (c) $\sigma^2/\tau_{cor} = 0.1$

$$f = 0.115, \; \Omega = 0.8$$

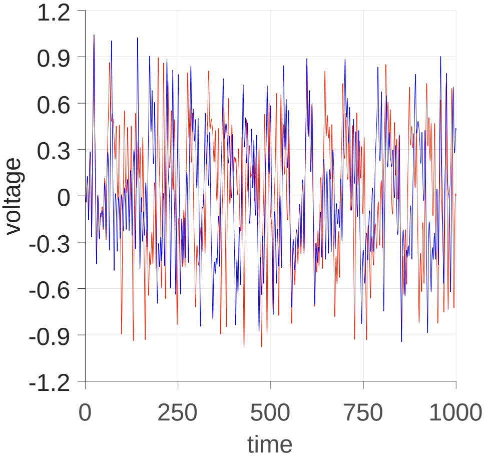
Typical voltage time series (50% of noise)



(a) $\sigma^2/\tau_{cor} = 1$

$f = 0.115,\ \Omega = 0.8$

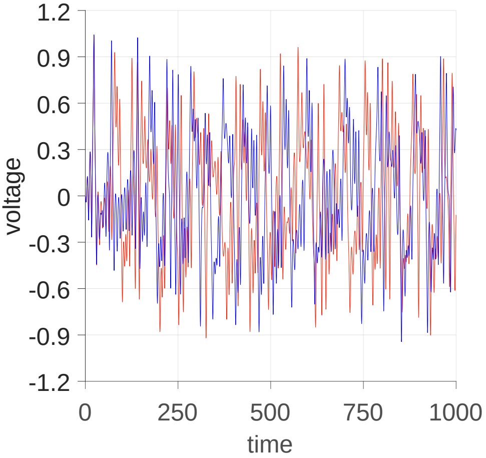
Typical voltage time series (50% of noise)



(b) $\sigma^2/\tau_{cor} = 0.5$

$f = 0.115$, $\Omega = 0.8$

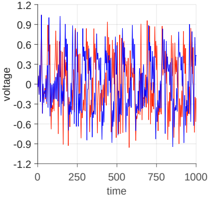
Typical voltage time series (50% of noise)



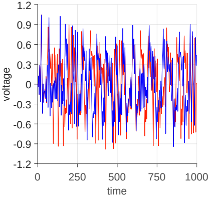
(c) $\sigma^2/\tau_{cor} = 0.1$

$f = 0.115$, $\Omega = 0.8$

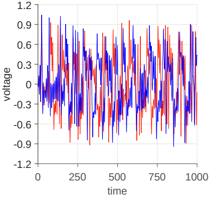
Typical voltage time series (50% of noise)



(a) $\sigma^2/\tau_{cor} = 1$



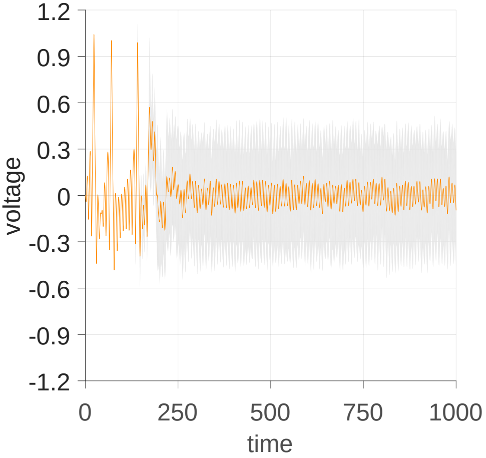
(b) $\sigma^2/\tau_{cor} = 0.5$



(c) $\sigma^2/\tau_{cor} = 0.1$

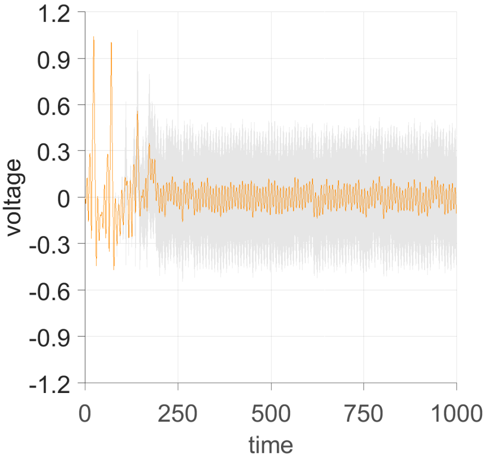
$$f = 0.115,\ \Omega = 0.8$$

Low-order statistics (1% of noise)



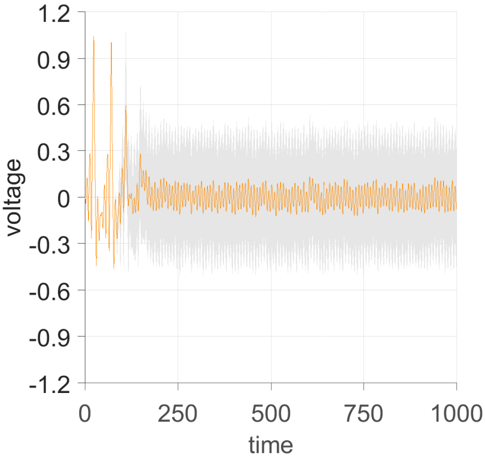
(a) $\sigma^2/\tau_{cor} = 1$

Low-order statistics (1% of noise)



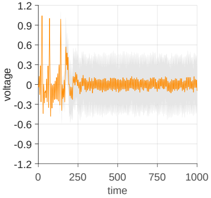
(b) $\sigma^2/\tau_{cor} = 0.5$

Low-order statistics (1% of noise)

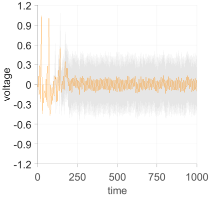


(c) $\sigma^2/\tau_{cor} = 0.1$

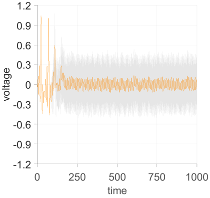
Low-order statistics (1% of noise)



(a) $\sigma^2/\tau_{cor} = 1$

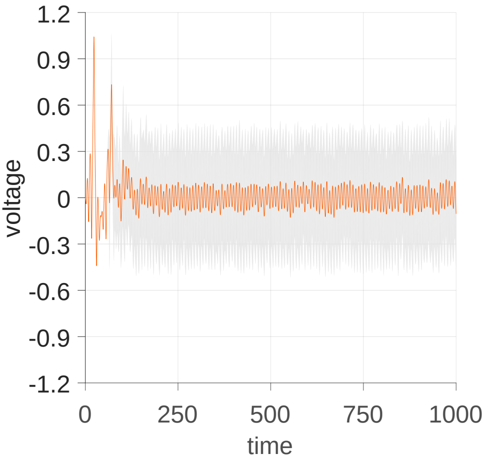


(b) $\sigma^2/\tau_{cor} = 0.5$



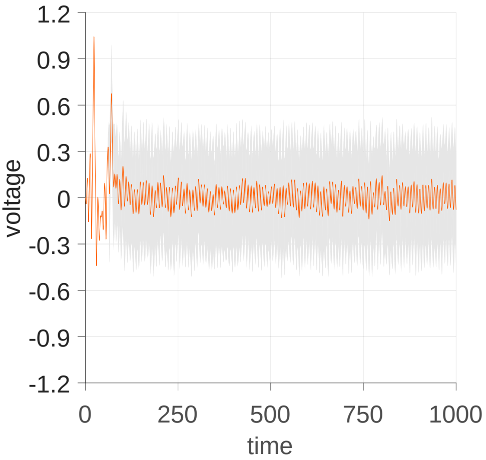
(c) $\sigma^2/\tau_{cor} = 0.1$

Low-order statistics (25% of noise)



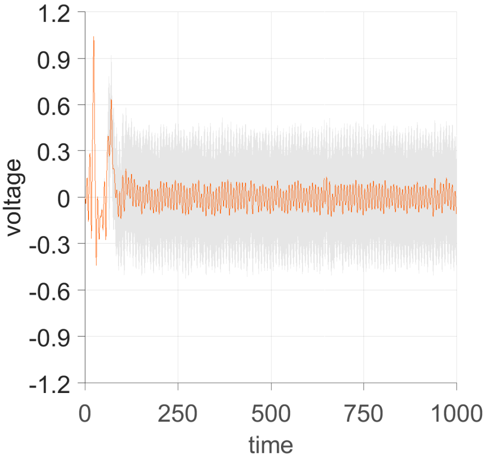
(a) $\sigma^2/\tau_{cor} = 1$

Low-order statistics (25% of noise)



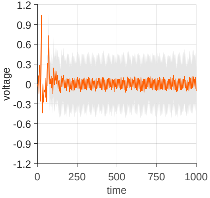
(b) $\sigma^2/\tau_{cor} = 0.5$

Low-order statistics (25% of noise)

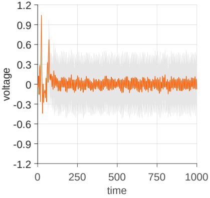


(c) $\sigma^2/\tau_{cor} = 0.1$

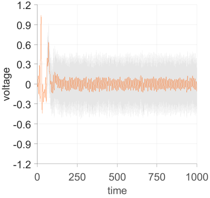
Low-order statistics (25% of noise)



(a) $\sigma^2/\tau_{cor} = 1$

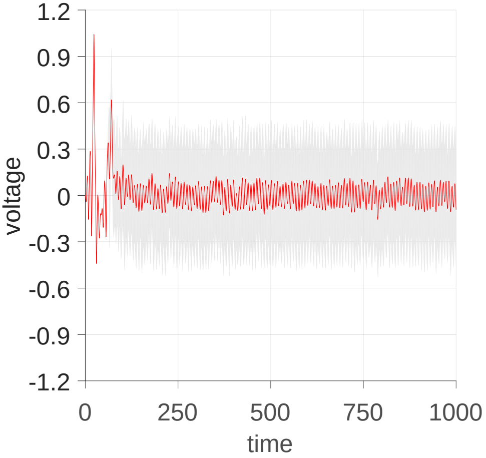


(b) $\sigma^2/\tau_{cor} = 0.5$



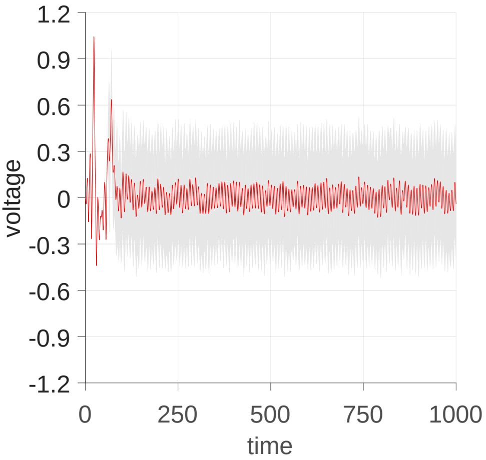
(c) $\sigma^2/\tau_{cor} = 0.1$

Low-order statistics (50% of noise)



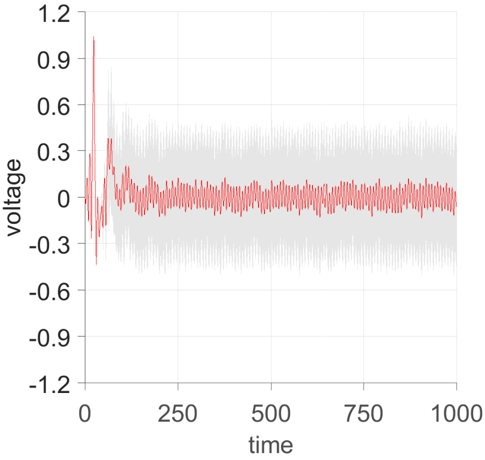
(a) $\sigma^2/\tau_{cor} = 1$

Low-order statistics (50% of noise)



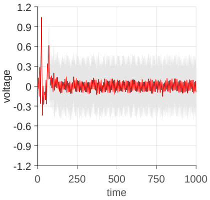
(b) $\sigma^2/\tau_{cor} = 0.5$

Low-order statistics (50% of noise)

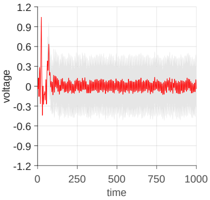


(c) $\sigma^2/\tau_{cor} = 0.1$

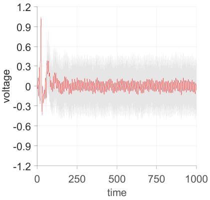
Low-order statistics (50% of noise)



(a) $\sigma^2/\tau_{cor} = 1$

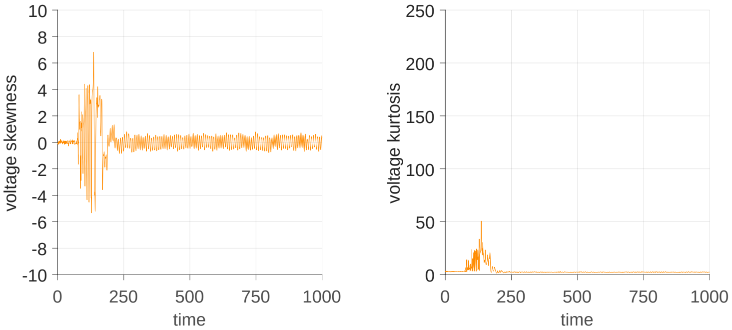


(b) $\sigma^2/\tau_{cor} = 0.5$



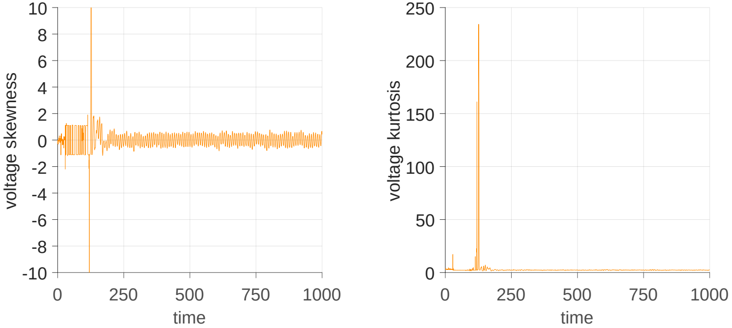
(c) $\sigma^2/\tau_{cor} = 0.1$

High-order statistics (1% of noise)



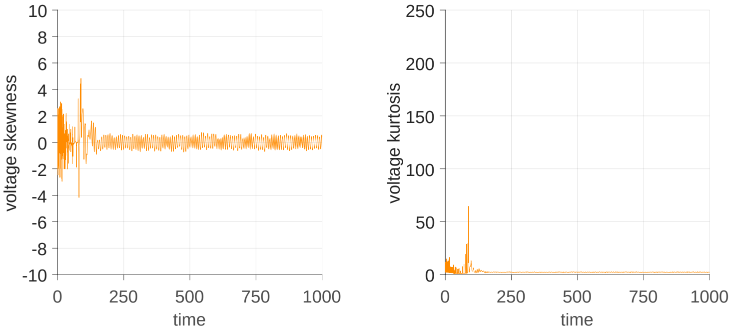
(a) $\sigma^2/\tau_{cor} = 1$

High-order statistics (1% of noise)



(b) $\sigma^2/\tau_{cor} = 0.5$

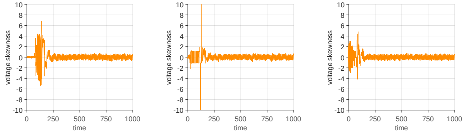
High-order statistics (1% of noise)



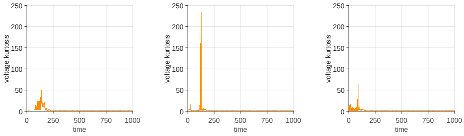
(c) $\sigma^2/\tau_{cor} = 0.1$

High-order statistics (1% of noise)

$$\sigma^2/\tau_{cor} \in \{1,\ 0.5,\ 0.1\}$$

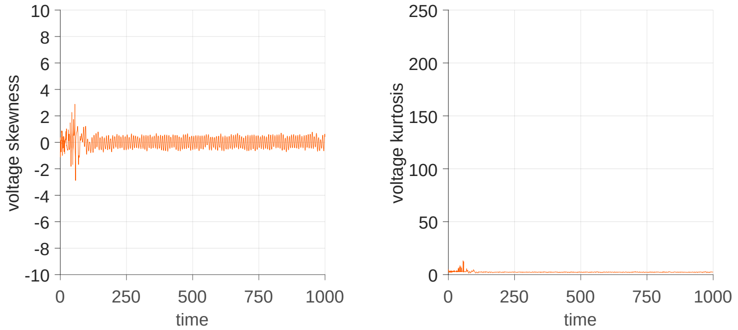


sknewness



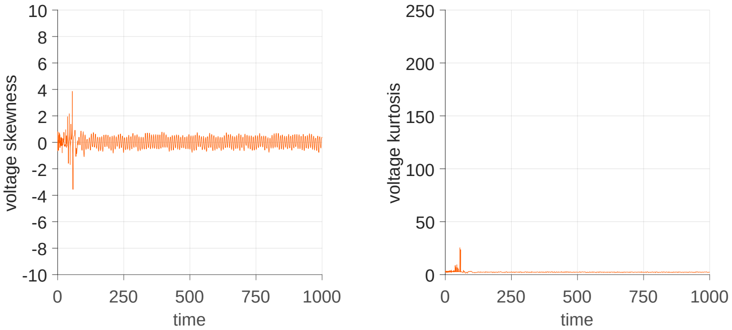
kurtosis

High-order statistics (25% of noise)



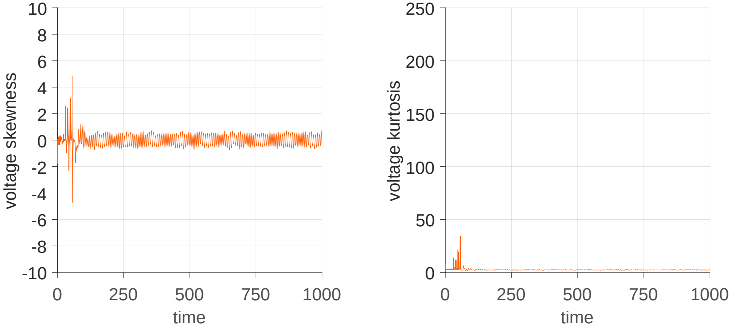
(a) $\sigma^2/\tau_{cor} = 1$

High-order statistics (25% of noise)



(b) $\sigma^2/\tau_{cor} = 0.5$

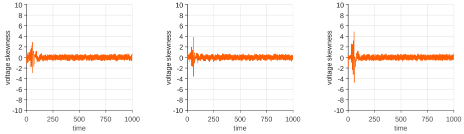
High-order statistics (25% of noise)



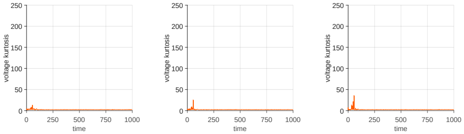
(c) $\sigma^2/\tau_{cor} = 0.1$

High-order statistics (25% of noise)

$$\sigma^2/\tau_{cor} \in \{1,\ 0.5,\ 0.1\}$$

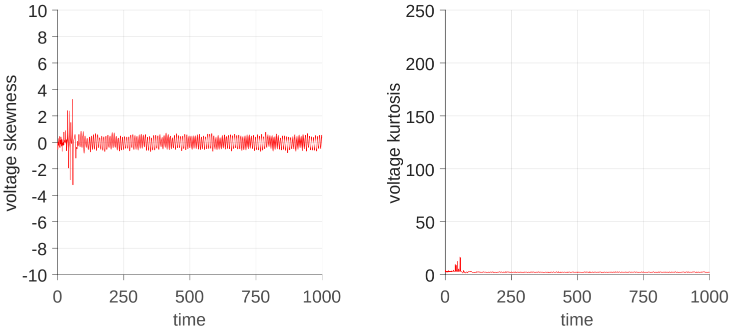


sknewness



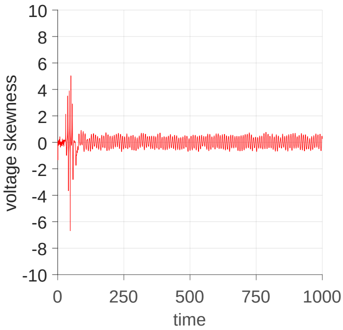
kurtosis

High-order statistics (50% of noise)



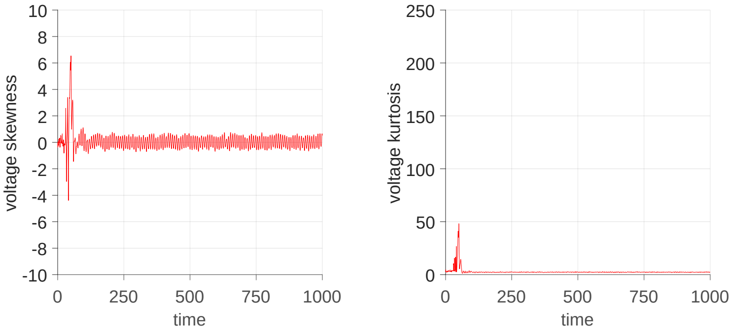
(a) $\sigma^2/\tau_{cor} = 1$

High-order statistics (50% of noise)



(b) $\sigma^2/\tau_{cor} = 0.5$

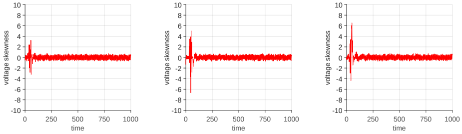
High-order statistics (50% of noise)



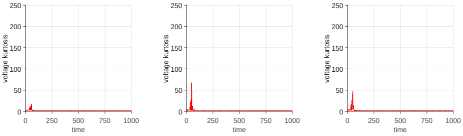
(c) $\sigma^2/\tau_{cor} = 0.1$

High-order statistics (50% of noise)

$$\sigma^2/\tau_{cor} \in \{1,\ 0.5,\ 0.1\}$$



sknewness



kurtosis

Section 4

Final Remarks

Final remarks

Contributions:

- Investigation of the system deterministic dynamics
  - bifurcation analsysis
  - basis of attractions exploration
- Investigation of the system stochastic dynamics
  - colored noise disturbance
  - intensity and correlation time effects

Ongoing research:

- Improve system efficiency via control of chaos
- Construction of an experimental apparatus

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- Profª. Aline de Paula (UnB)
- Prof. Adriano Fabro (UnB)
- Mr. Tiago Pereira (UnB)

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CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

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Thank you for your attention!

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V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr,
Nonlinear characterization of a bistable energy harvester dynamical system, 2019.
(under review) https://hal.archives-ouvertes.fr/hal-02054683

J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr,
Numerically exploring the nonlinear dynamics of a piezo-magneto-elastic energy harvesting device, 2019.
(under review) https://hal.archives-ouvertes.fr/hal-02013382

A. Cunha Jr,
Enhancing the performance of a bi-stable energy harvesting device via cross-entropy method, 2017.
(under review) https://hal.archives-ouvertes.fr/hal-01531845

Physical system parameters

parameter	value
ξ	0.01
χ	0.05
f	0.083
Ω	0.8
λ	0.05
κ	0.5

Computational representation of the noise

Karhunen-Loève decomposition:

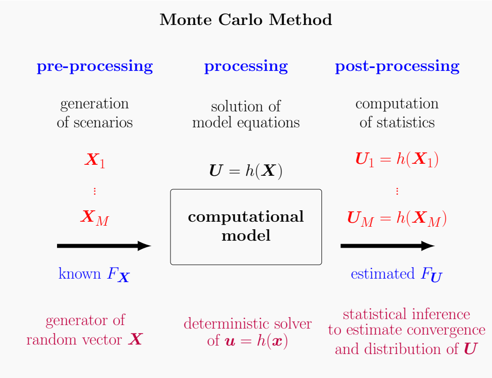
$$N_t = \mathbb{E}\left\{N_t\right\} + \sum_{n=1}^{\infty} \sigma_N \sqrt{\lambda_n}\, \varphi_n(t)\, Y_n$$

$$\int_{\mathbb{R}} \text{cov}_N(t_1, t_2)\, \varphi_n(t_2)\, dt_1 = \lambda_n\, \varphi_n(t_1), \quad t_1 \in \mathbb{R}$$

$$\mathbb{E}\left\{Y_n\right\} = 0 \qquad \text{and} \qquad \mathbb{E}\left\{Y_n Y_m\right\} = \delta_{mn}$$

D. Xiu, **Numerical Methods for Stochastic Computations: A Spectral Method Approach**, Princeton University Press, 2010.

Propagation of uncertainties: Monte Carlo method



A. Cunha Jr, R. Nasser, R. Sampaio, H. Lopes, and K. Breitman, *Uncertainty quantification through Monte Carlo method in a cloud computing setting.* **Computer Physics Communications**, 185: 1355–1363, 2014.