
# The cross-entropy method for optimization of energy harvesting systems

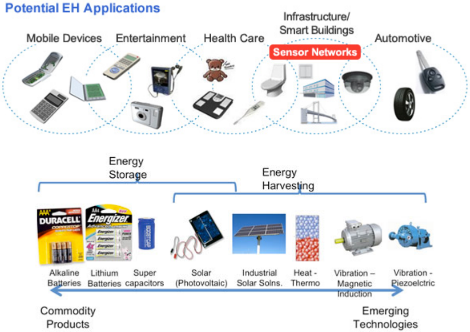
Americo Cunha Jr

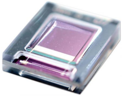
Rio de Janeiro State University – UERJ

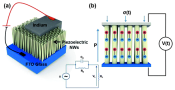
ONLINE, FEBRUARY 16 - 19, 2021

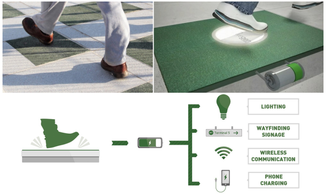
# Energy Harvesting Systems



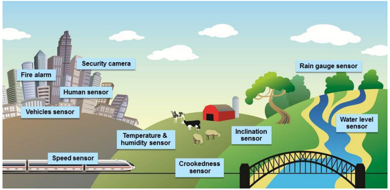





Spansion Energy Harvesting Technology Can Power the IoT

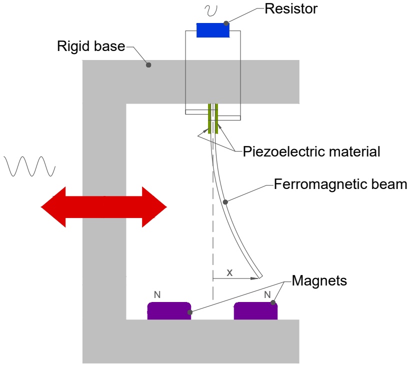


# Research objectives

- Propose a strategy of design to enhance the recovered energy
  - Formulate a nonlinear non-convex optimization problem
  - Use the cross-entropy method to obtain an efficient solution

# Bistable vibration harvester






$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1-x^2) - \chi v = f\cos(\Omega t)$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

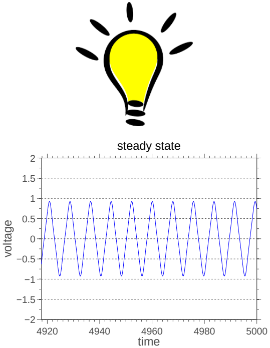
+ initial conditions

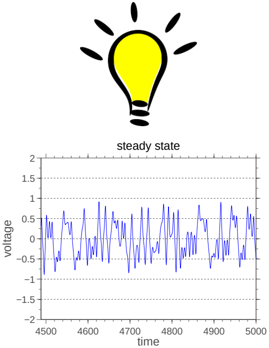
Mean output power:

$$P = \frac{1}{T}\int_t^{t+T} \lambda v^2(\tau)\,d\tau$$

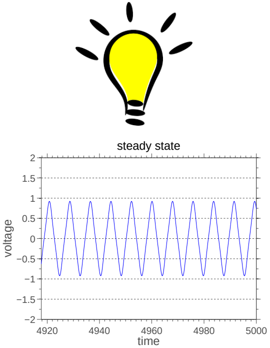
A. Erturk et al., *A piezomagnetoelastic structure for broadband vibration energy harvesting.* **Applied Physics Letters**, 94: 254102, 2009.

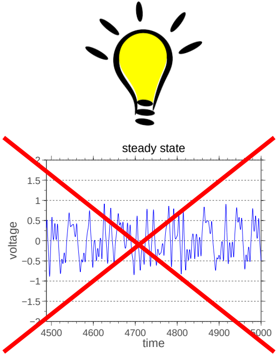
# For practical use of the electrical energy ...



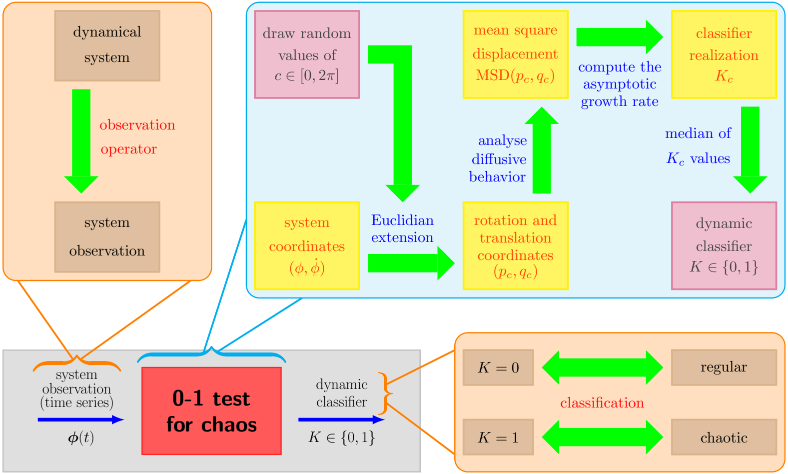
# For practical use of the electrical energy ...




... irregular voltage is undesirable!

# The 0-1 test for chaos



# Optimization framework

- $\boldsymbol{x}$ — design variables vector
- $\mathcal{S}(\boldsymbol{x})$ — mean power
- $\mathcal{G}(\boldsymbol{x})$ — 0-1 test for chaos classifier

Constrained formulation:

$$\max \mathcal{S}(\boldsymbol{x}) \qquad \text{s.t.} \qquad \mathcal{G}(\boldsymbol{x}) = 0 \quad \text{and} \quad \boldsymbol{x}_{min} \leq \boldsymbol{x} \leq \boldsymbol{x}_{max}$$

Penalized formulation:

$$\boldsymbol{x}^{\star} = \arg\max \{ \mathcal{S}(\boldsymbol{x}) + H \max(0, \mathcal{G}(\boldsymbol{x})) \}$$

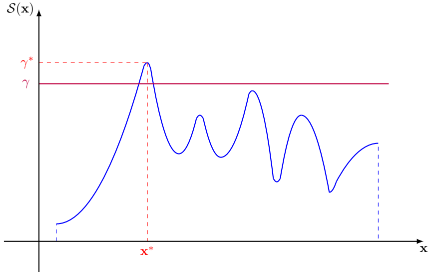
Peculiarities:

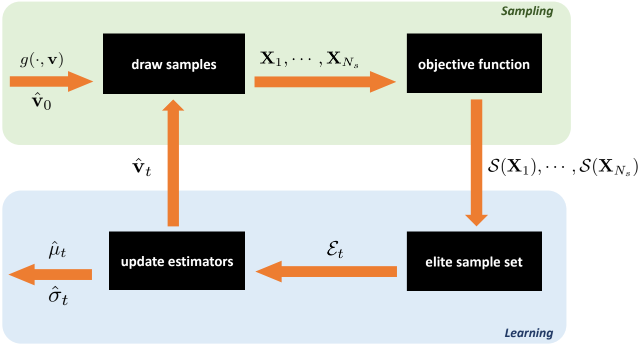
- Test 0-1 for chaos constraint is a discontinuous function of $\boldsymbol{x}$
- Gradient-based methods are not applicable
- Evolutionary algorithms can be used (but we prefer not!)

# Cross-entropy method



Transform the optimization problem into a rare-event estimation problem




$$\mathcal{P}\{\mathcal{S}(\mathrm{x}) \geq \gamma\} \approx 0 \quad \text{for} \quad \gamma \approx \gamma^*$$

$\mathcal{S}(\mathrm{x}) \geq \gamma$ is a rare event

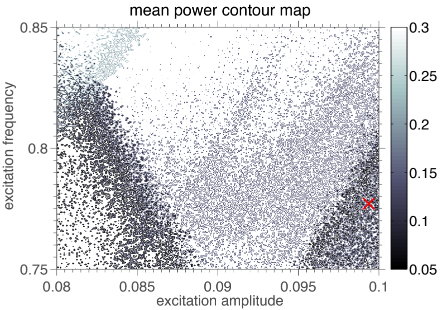
# Reference solution

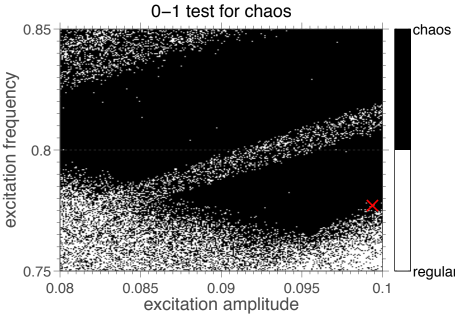


- Direct search on a numerical grid
- Design variables: $f$ and $\Omega$
- Feasible domain: $\mathcal{D} = \{0.08 \leq f \leq 0.1 \ \text{ and } \ 0.75 \leq \Omega \leq 0.85\}$
- Grid resolution: $256 \times 256$ points
- Function evaluations: 65 536
- CPU time:[1] $\approx$ 4 hours

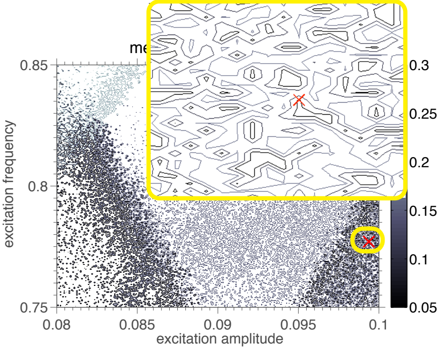
[1] Dell Inspiron Core i7-3632QM 2.20 GHz RAM 12GB

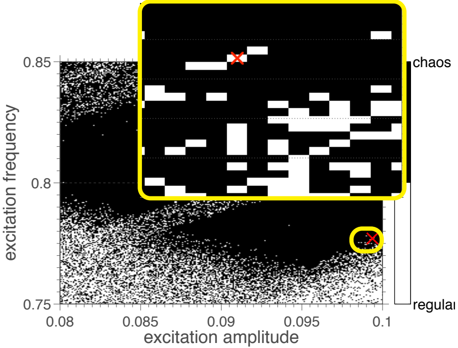
# Reference solution: mean power



# Reference solution: mean power



# Cross-entropy solution



- Design variables: $f$ and $\Omega$
- Feasible domain: $\mathcal{D} = \{0.08 \leq f \leq 0.1 \;\text{ and }\; 0.75 \leq \Omega \leq 0.85\}$
- Number of CE samples: 50
- Percentage of elite samples: 10%
- CE samples distribution: Truncated Gaussian
- Convergence criterium: $||\boldsymbol{\sigma}||_\infty < 1 \times 10^{-3}$
- Function evaluations: 1 300
- CPU time:[2] $\approx$ 5 minutes

[2] Dell Inspiron Core i7-3632QM 2.20 GHz RAM 12GB

# Cross-entropy animation

# Cross-entropy performance



| samples | levels | CPU time[3] | speed-up | function evaluation |
|---|---|---|---|---|
| reference | — | $\sim$ 3.6h | — | 65 536 |
| 25 | 19 | $\sim$ 2 min | $\sim$ 120 | 475 |
| 50 | 26 | $\sim$ 5 min | $\sim$ 45 | 1 300 |
| 75 | 30 | $\sim$ 8 min | $\sim$ 25 | 2 250 |
| 100 | 28 | $\sim$ 10 min | $\sim$ 20 | 2 800 |

[3] Dell Inspiron Core i7-3632QM 2.20 GHz RAM 12GB

# Noisy and high-dimensional



**Noisily external forcing:** $\boldsymbol{x} = (f, \Omega)$

Direct search:
$P_{max} = 0.0173$
$\boldsymbol{x}^* = (0.0998, 0.7763)$
$\approx$ 4 hours

Cross-entropy:
$P_{max} = 0.0170$
$\boldsymbol{x}^* = (0.0991, 0.7675)$
$\approx$ 4 minutes

- robustness to noise

**Moderate high-dimensional case:** $\boldsymbol{x} = (\xi, \chi, \lambda, \kappa)$

Direct search:
$P_{max} = 0.1761$
$\boldsymbol{x}^* =$
$(0.0340, 0.0600, 0.2000, 1.5000)$
$\approx$ 4 hours

Cross-entropy:
$P_{max} = 0.1612$
$\boldsymbol{x}^* =$
$(0.0237, 0.1053, 0.1953, 1.4923)$
$\approx$ 35 minutes

- good performance

# Concluding remarks



Contributions:

- Formulation of a nonlinear non-convex optimization problem to enhance power recovered by a bistable energy harvesting system
- Efficient solution of this optimization problem by means of cross-entropy method

Conclusions:

- The CE method is a power technique to deal with non-convex optimization problems in dynamical systems, in particular, for energy harvesting systems
- It is simple, robust, efficient, generalizable and extensible.

Future direction:

- Parallelization of the CE optimization algorithm

# Acknowledgments


Academic discussion:

- Prof. Welington Oliveira (MINES Paris-Tech)
- Prof. Luca Gammaitoni (University of Perugia)

Financial support:

FAPERJ
Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

# Thank you for your attention!

`americo.cunha@uerj.br`

`www.americocunha.org`

@AmericoCunhaJr @AmericoCunhaJr

A. Cunha Jr, *Enhancing the performance of a bistable energy harvesting device via the cross-entropy method.* **Nonlinear Dynamics**, vol. 103(1), pp. 137-155, 2021.