Bifurcation analysis and control of chaos on bistable piezoelectric energy harvesting systems

Americo Cunha Jr

Collaborators: Vinicius Lopes, Leonardo de la Roca, João Peterson, Marcelo Pereira, José Geraldo Telles, Elbert Macau

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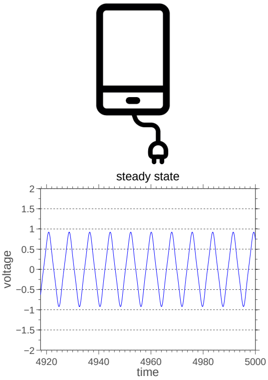
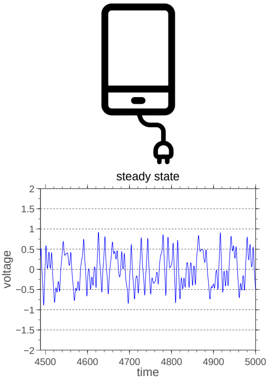
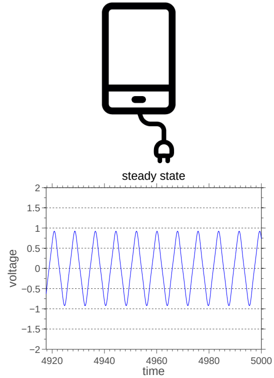
Where do you want to charge your phone?

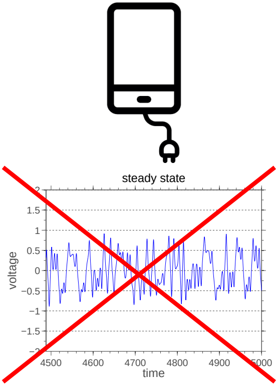




*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

Where do you want to charge your phone?





In some cases irregular voltage is undesirable!

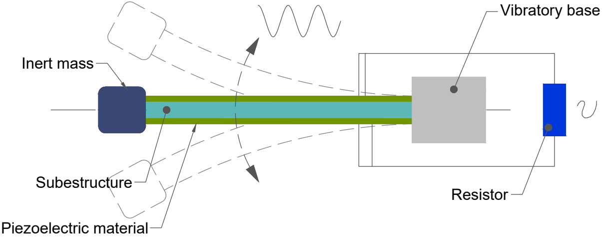
*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

Section 1

Introduction

Linear Vibratory Harvester

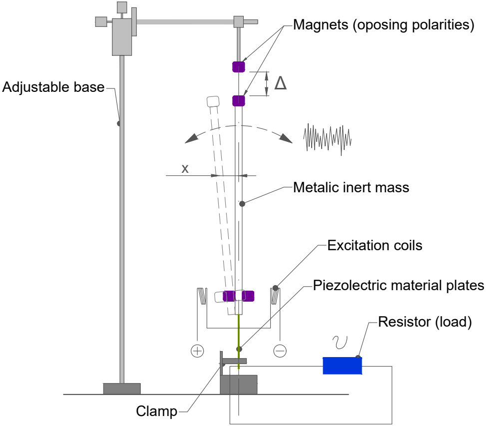
Monostable system driven by regular signal



S. Roundy, P. K. Wright and J. Rabaey, A study of low level bibrations as a power source for wireless sensor nodes. **Computer Communications**, 26: 1131-1144, 2003.

Nonlinear Vibratory Harvester

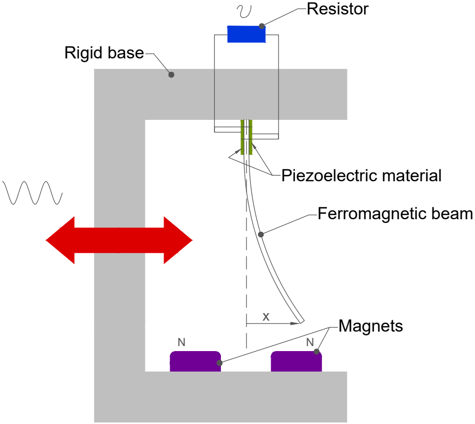
Bistable system driven by a noisy signal



F. Cottone, H. Vocca and L. Gammaitoni, Nonlinear energy harvesting. **Physical Review Letters**, 102: 080601, 2009.

Nonlinear Vibratory Harvester

Bistable system driven by regular signal



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Research objectives

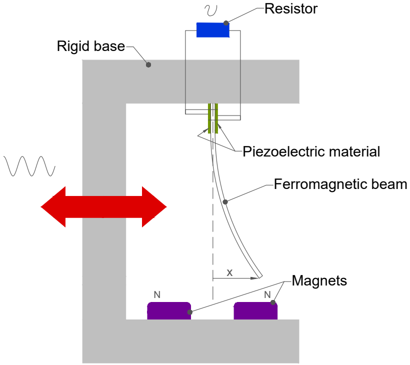
This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
  - Bifurcation diagrams
  - Basis of attractions
- Propose strategies to enhance the recovered energy
  - Control of chaos

Section 2

Dynamical System

Bistable harvester driven by regular signal



$$\ddot{x}+2\xi\dot{x}-\frac{1}{2}x(1-x^2)-\chi\upsilon = f\cos\Omega t$$

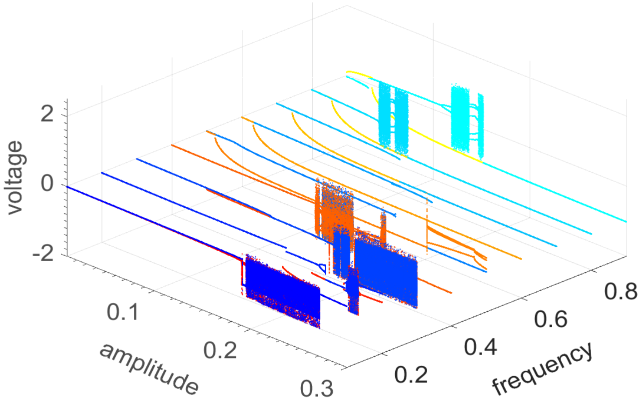
$$\dot{\upsilon}+\lambda\upsilon+\kappa\dot{x}=0$$

$$x(0)=x_0,\ \dot{x}(0)=\dot{x}_0,\ \upsilon(0)=\upsilon_0$$

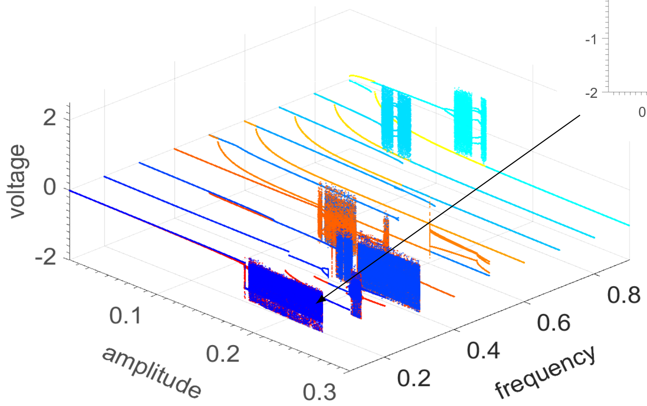
A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

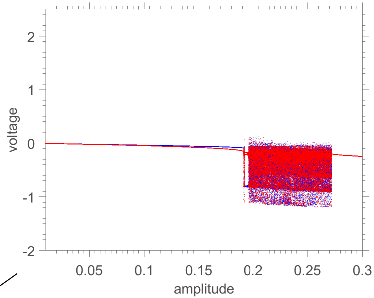
Nonlinear dynamics animation

Bifurcation diagrams: voltage $\times$ amplitude



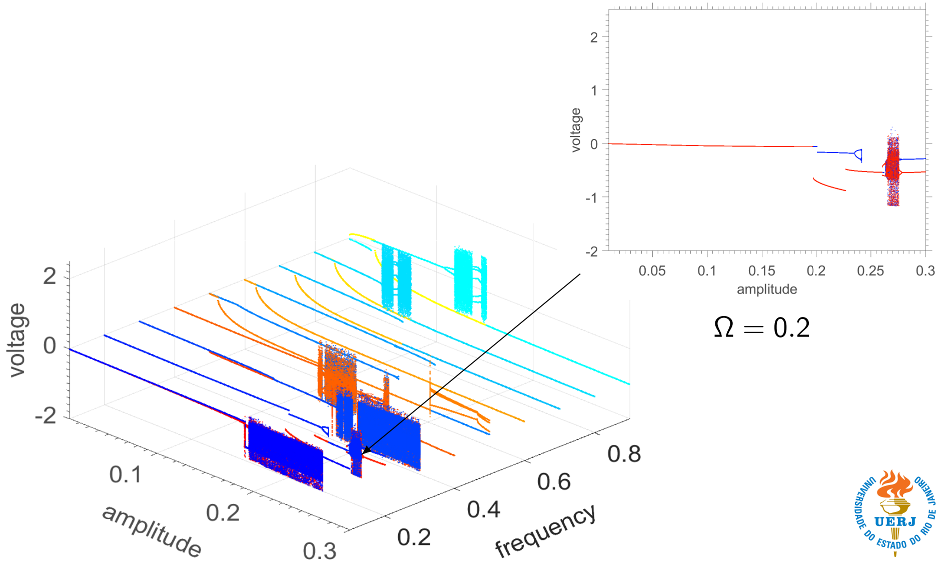
Bifurcation diagrams: voltage $\times$ amplitude





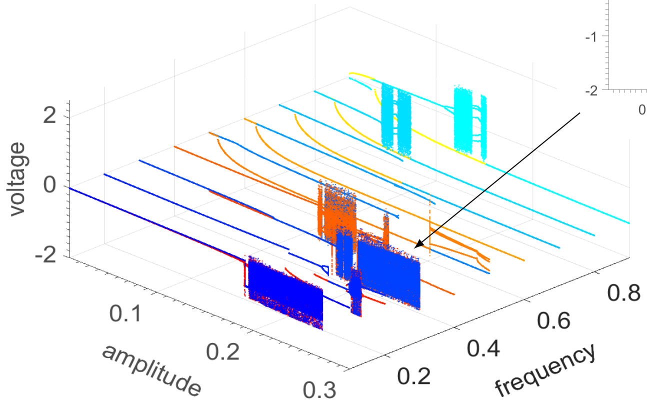
$\Omega = 0.1$

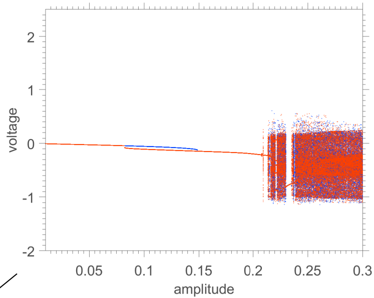
Bifurcation diagrams: voltage × amplitude



$\Omega = 0.2$

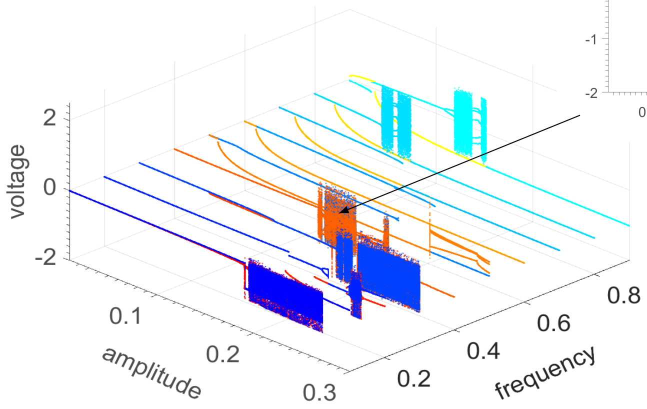
Bifurcation diagrams: voltage $\times$ amplitude

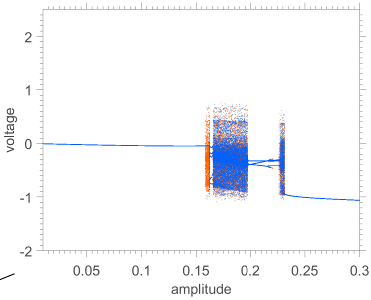




$\Omega = 0.3$

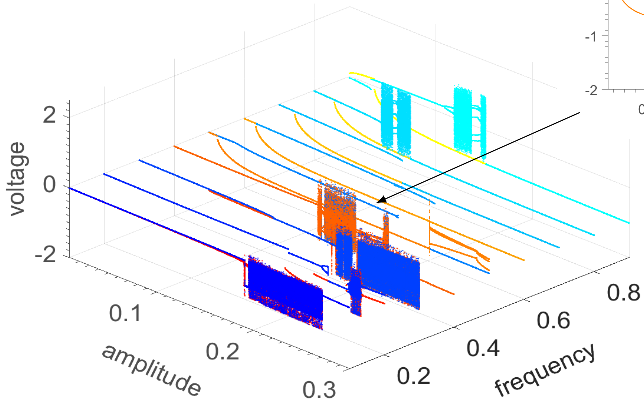
Bifurcation diagrams: voltage × amplitude

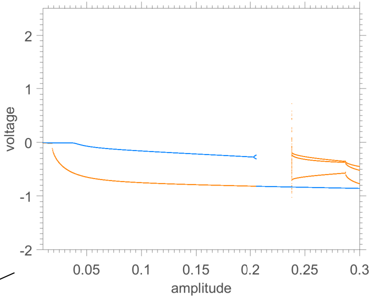




$\Omega = 0.4$

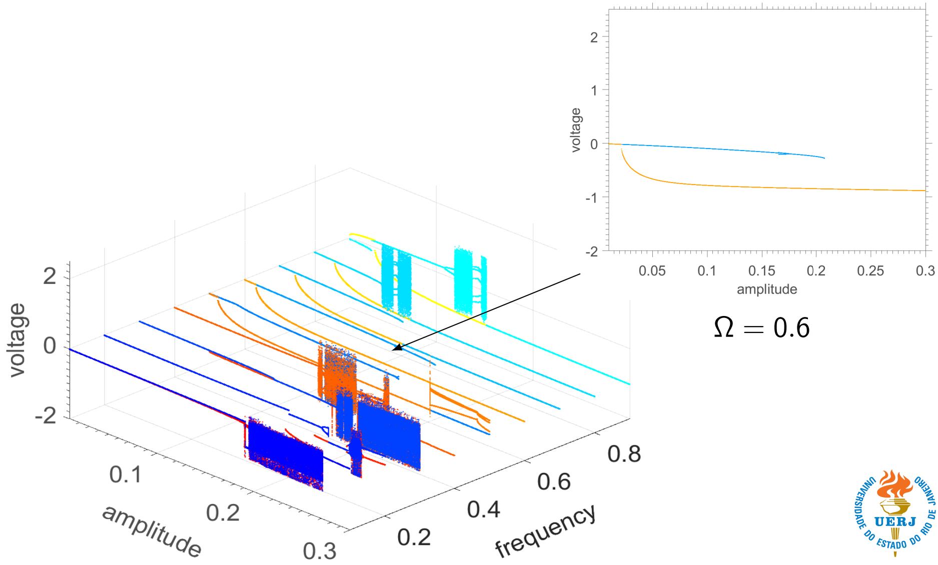
Bifurcation diagrams: voltage $\times$ amplitude





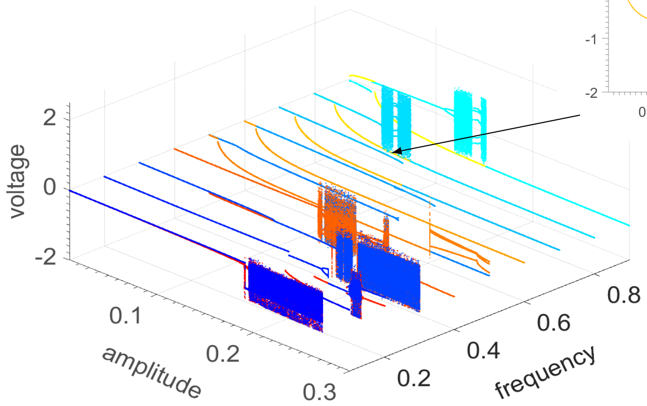
$\Omega = 0.5$

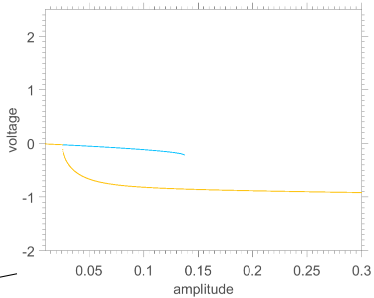
Bifurcation diagrams: voltage × amplitude



$\Omega = 0.6$

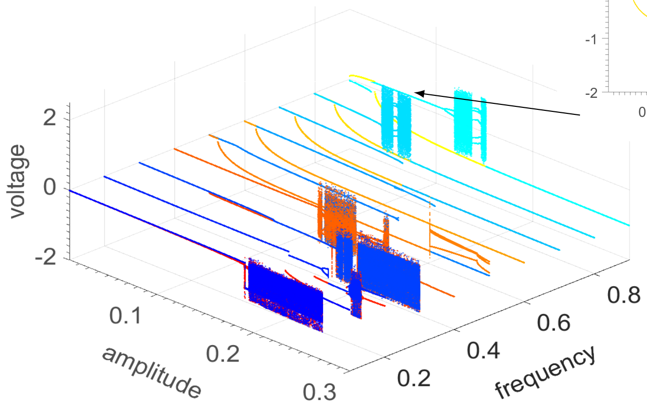
Bifurcation diagrams: voltage × amplitude

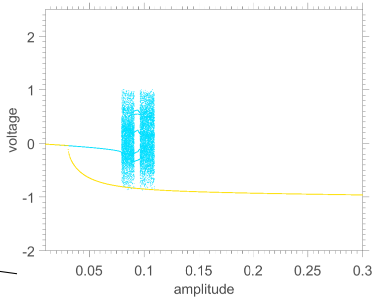




$\Omega = 0.7$

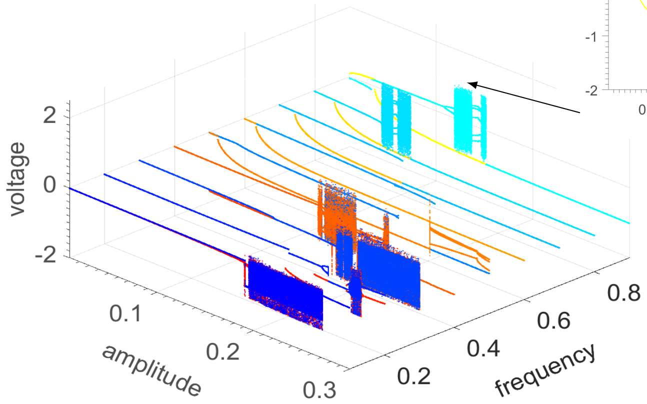
Bifurcation diagrams: voltage $\times$ amplitude

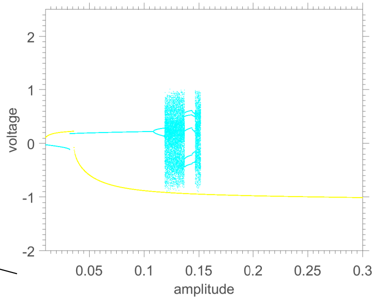




$\Omega = 0.8$

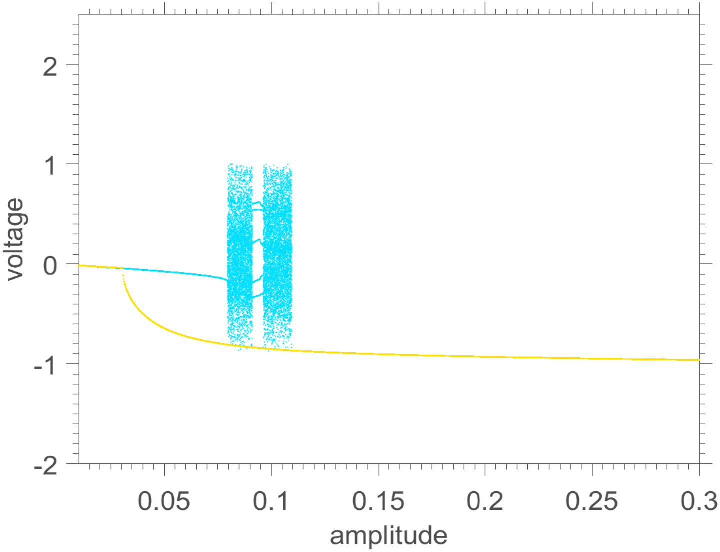
Bifurcation diagrams: voltage $\times$ amplitude



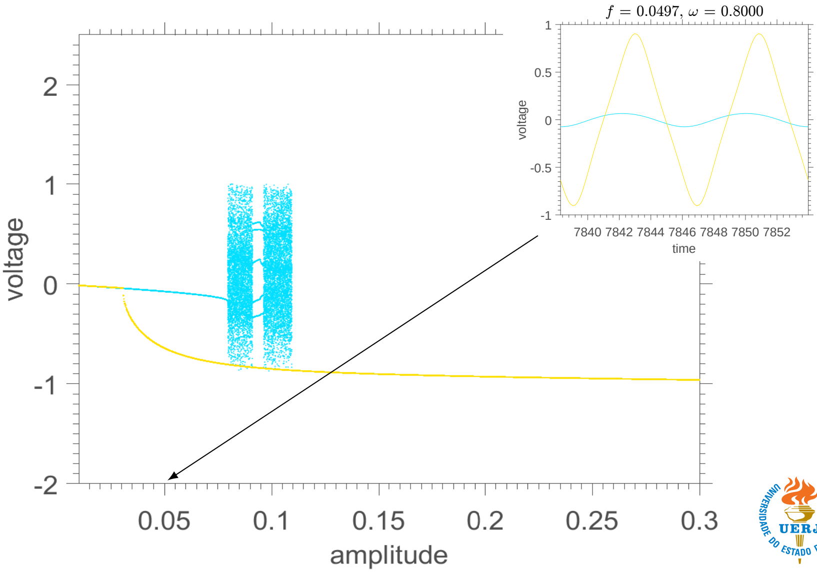


$\Omega = 0.9$

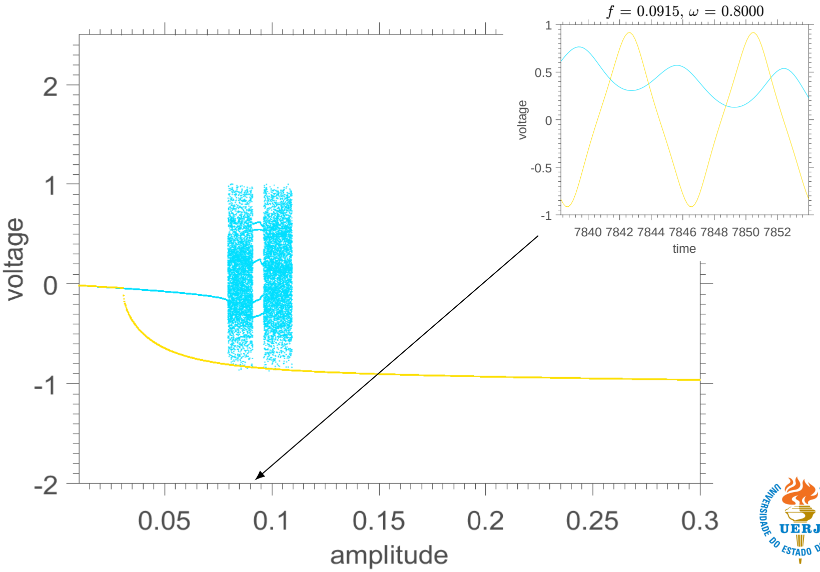
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



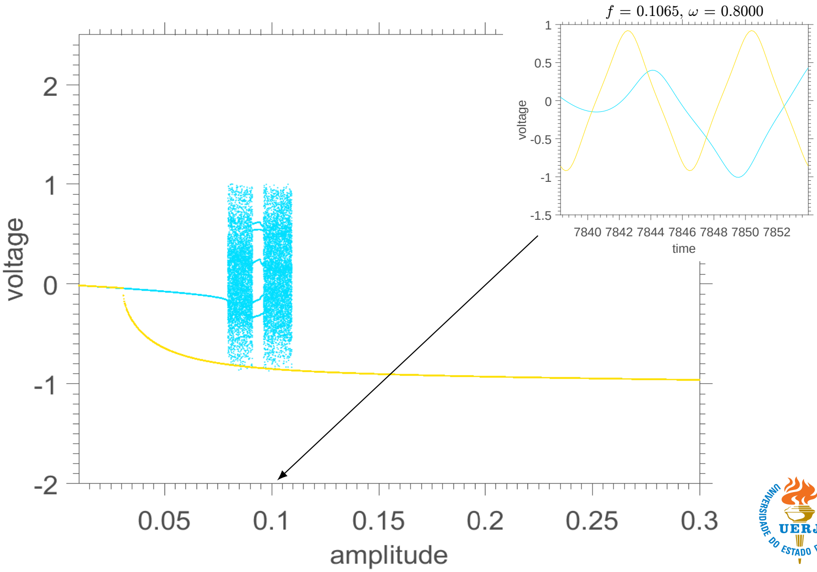
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Basins of attraction

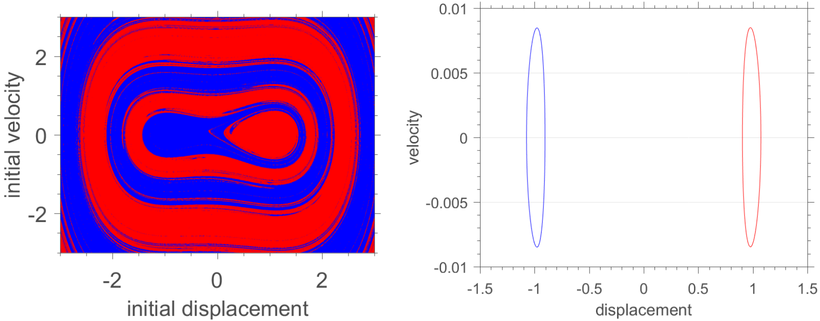


Figure: $f = 0.083$ and $\Omega = 0.1$

Basins of attraction

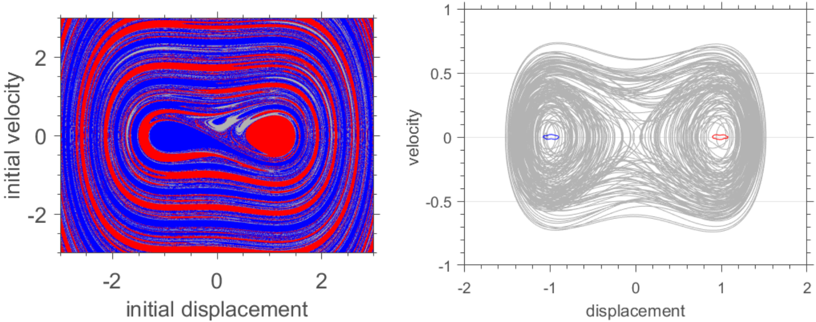


Figure: $f = 0.083$ and $\Omega = 0.2$

Basins of attraction

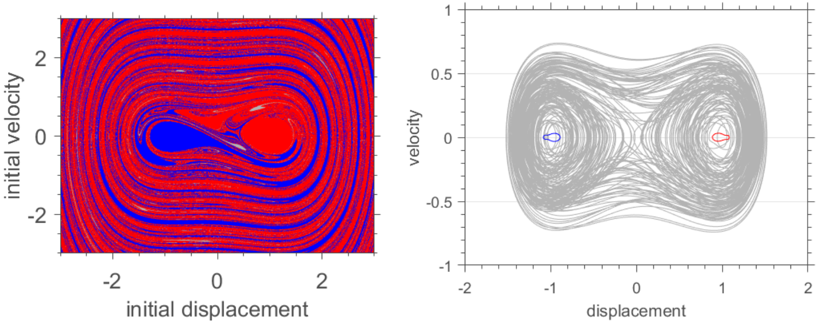


Figure: $f = 0.083$ and $\Omega = 0.3$

Basins of attraction

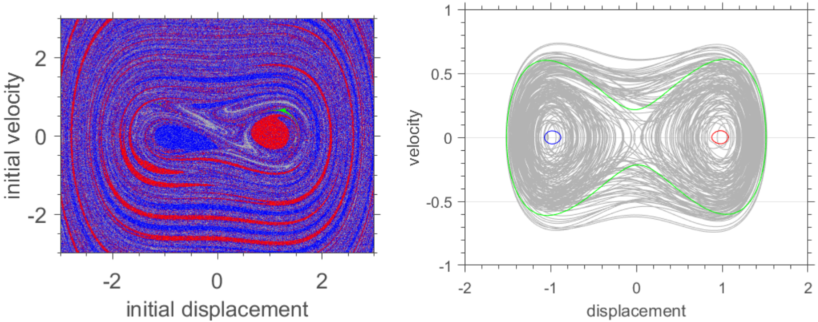


Figure: $f = 0.083$ and $\Omega = 0.4$

Basins of attraction

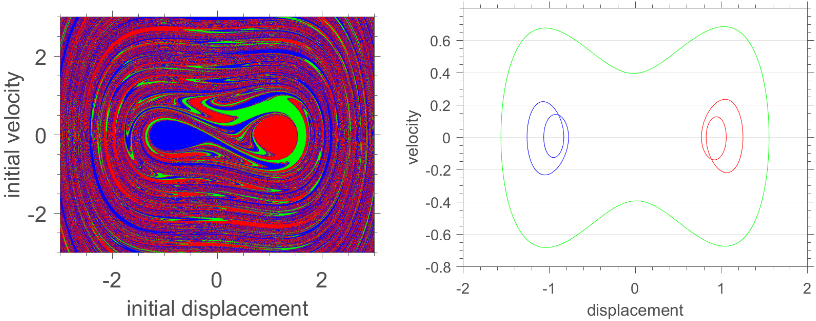


Figure: $f = 0.083$ and $\Omega = 0.5$

Basins of attraction

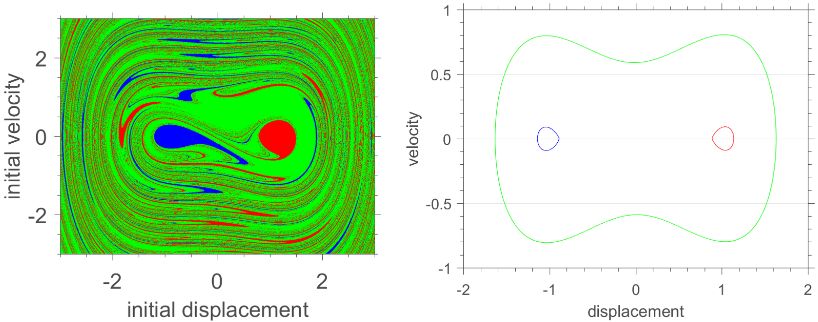


Figure: $f = 0.083$ and $\Omega = 0.6$

Basins of attraction

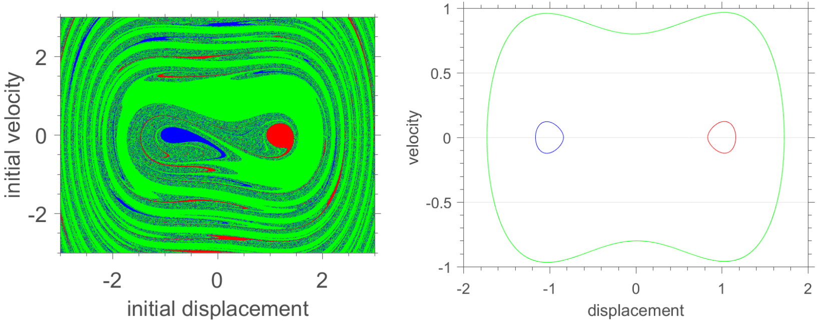


Figure: $f = 0.083$ and $\Omega = 0.7$

Basins of attraction

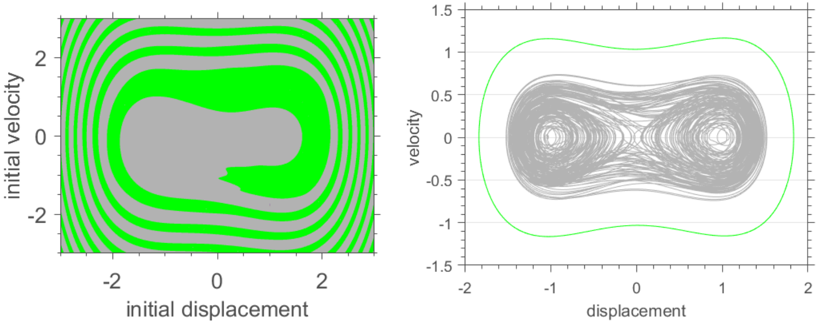


Figure: $f = 0.083$ and $\Omega = 0.8$

Basins of attraction

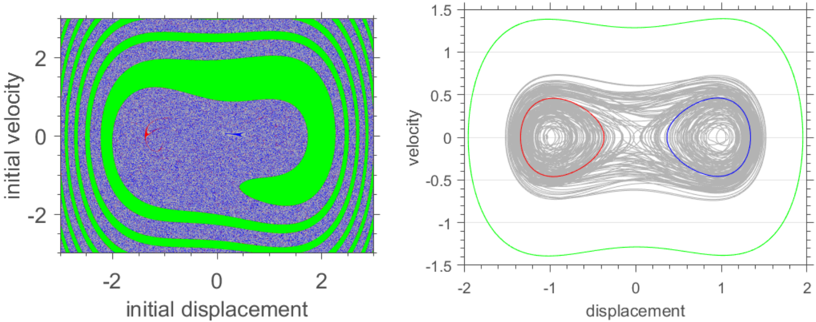
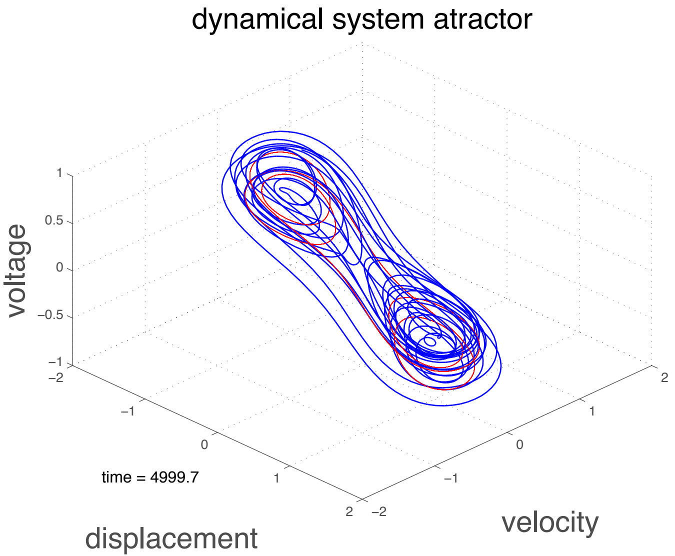


Figure: $f = 0.083$ and $\Omega = 0.9$

Section 3

Controlling Chaos

UPO embedded into a chaotic atractor



How to explore these unstable periodic orbits?

$\Longrightarrow$ **Techniques for control of chaos (known UPO is required)**

OGY:

- Control performed by a sequence of (discrete) small "kicks" that forces the system trajectory to stay in the target orbit

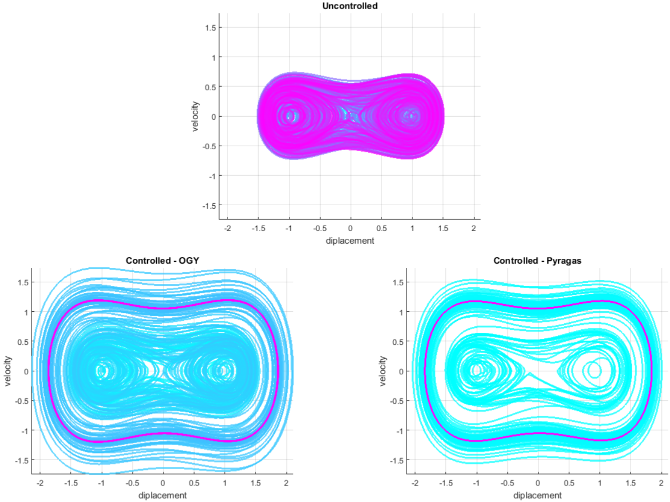
E. Ott, C. Grebogi, J. Yorke, Controlling chaos, **Physical Review Letters**, 64:1196, 1990.

Pyragas:

- Control performed by a (continuous) low intensity signal which is almost zero if the system evolves close to the target orbit, and increases when it starts to drift way

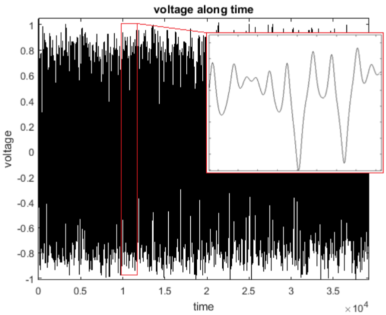
K. Pyragas, Continuous control of chaos by self-controlling feedback, **Physics Letters A**, 170:421, 1992.

Stabilization via control of chaos ($f = 0.083$, $\Omega = 0.8$)

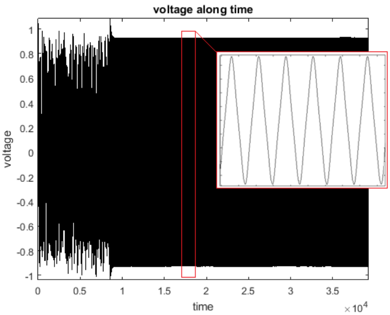


*The colors represent the evolution of the system, from blue to pink, as the time progresses.

Stabilization via control of chaos ($f = 0.083$, $\Omega = 0.8$)

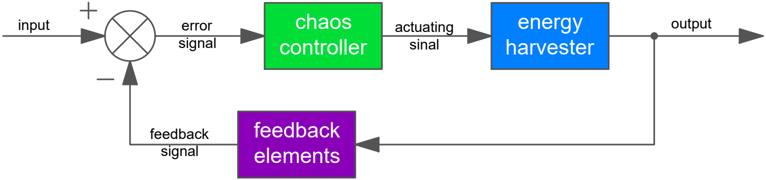


uncontrolled dynamics



OGY controlled dynamics

Feedback control law



E. Ott, C. Grebogi, J. Yorke, Controlling chaos, **Physical Review Letters**, 64:1196, 1990.

OGY control of chaos

- Poincaré map:

$$y_{n+1} = g(y_n,\, p)$$

- Linearization around a deviation from the target orbit:

$$y_{n+1} - y^* = \partial_y\, g(y^*,\, p^*)\,(y_n - y^*) + \partial_p\, g(y^*,\, p^*)\,(p - p^*)$$

- Goal:

$$\left\| y_{n+1} - y^* \right\| \to 0$$

- Controller project:

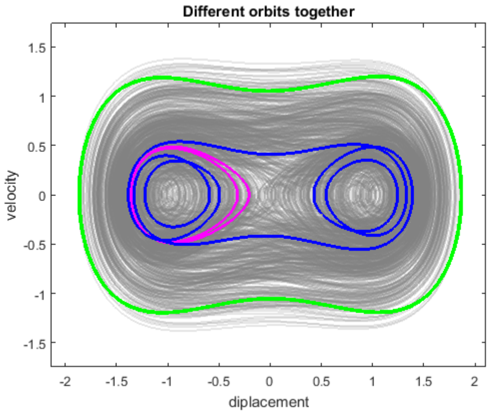
$$p - p^* = -K\,(y_n - y^*), \qquad \|K\| \leq \frac{1 - \left\|\partial_y\, g(y^*,\, p^*)\right\|}{\left\|\partial_p\, g(y^*,\, p^*)\right\|}$$

E. Ott, C. Grebogi, J. Yorke, `Controlling chaos`, **Physical Review Letters**, 64:1196, 1990.

T. Kapitaniak, **Chaos for Engineers: theory, applications and control**, Springer, 2nd Ed, 2000.

Orbits - $f = 0.090$, $\Omega = 0.8$

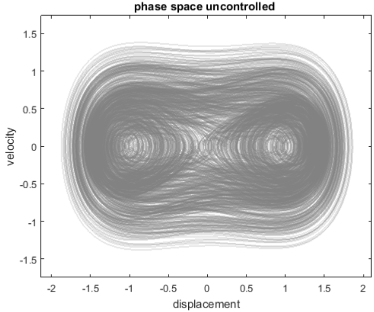
Phase space with controlled and uncontrolled orbits.



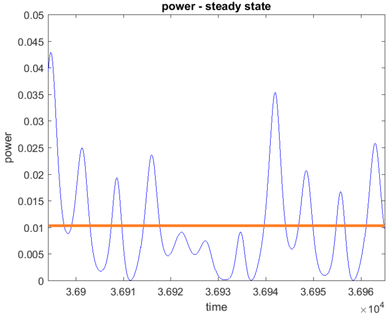
Periodic orbits within the chaotic attractor

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

uncontrolled dynamics



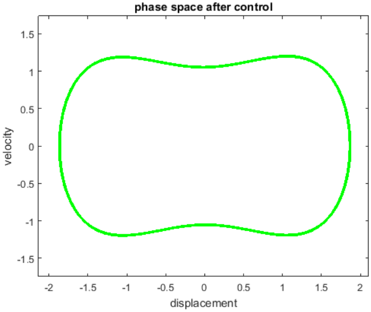
system trajectoty



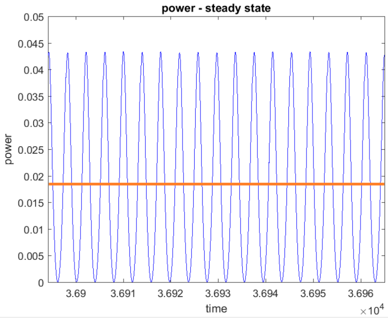
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 1 orbit



system trajectoty

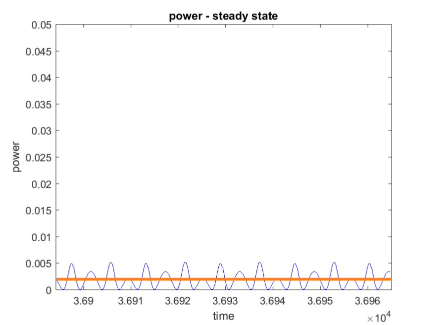


output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 2 orbit

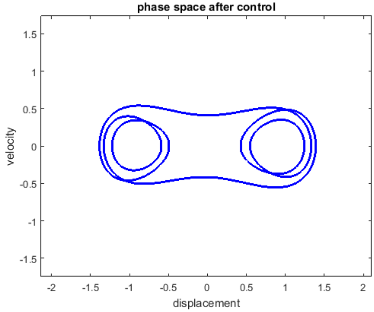
system trajectoty



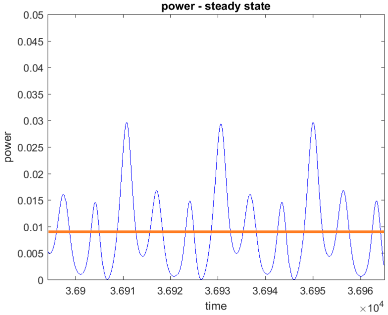
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 5 orbit



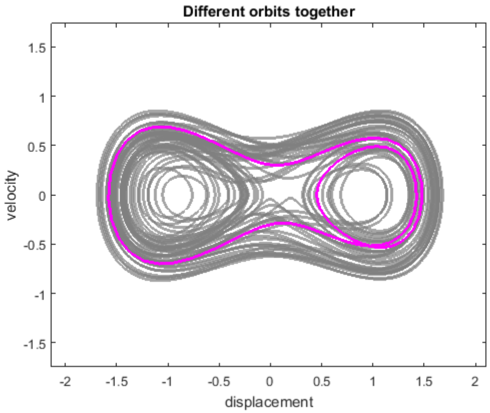
system trajectoty



output power

Orbits - $f = 0.115$, $\Omega = 0.3$

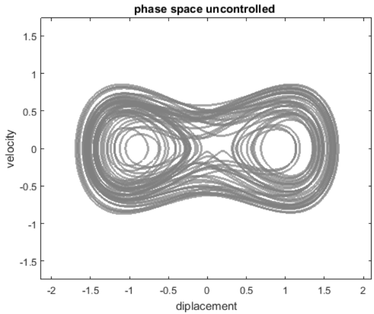
Phase space with controlled and uncontrolled orbits.



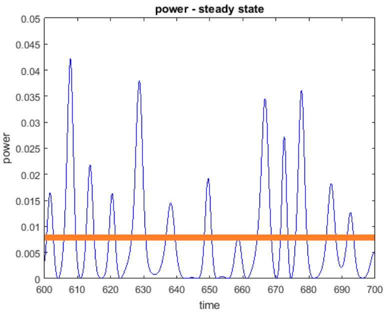
Periodic orbit within the chaotic attractor

OGY control with different orbits ($f = 0.115$, $\Omega = 0.3$)

uncontrolled dynamics



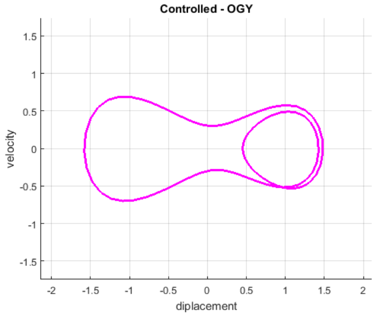
system trajectoty



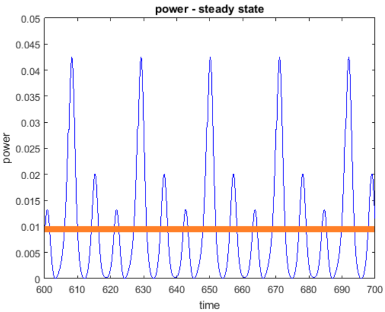
output power

OGY control with different orbits ($f = 0.115$, $\Omega = 0.3$)

controlled dynamics in a period 2 orbit



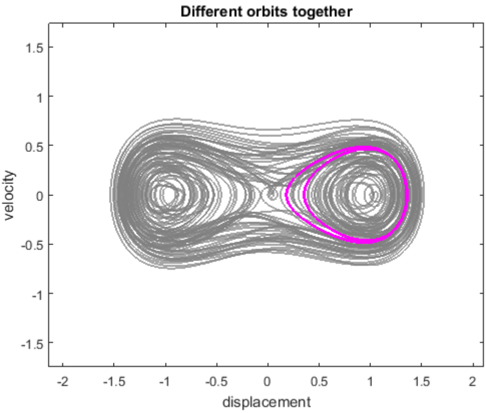
system trajectoty



output power

Orbits - $f = 0.088$, $\Omega = 0.8$

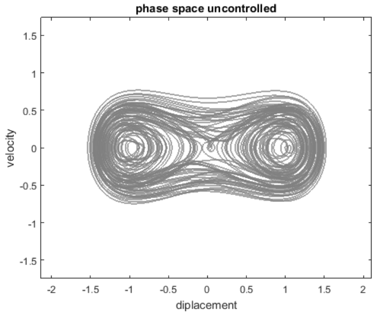
Phase space with controlled and uncontrolled orbits.



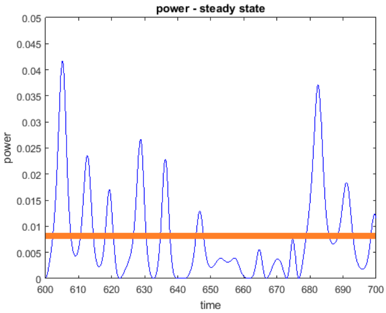
Periodic orbit within the chaotic attractor

OGY control with different orbits ($f = 0.088$, $\Omega = 0.8$)

uncontrolled orbit



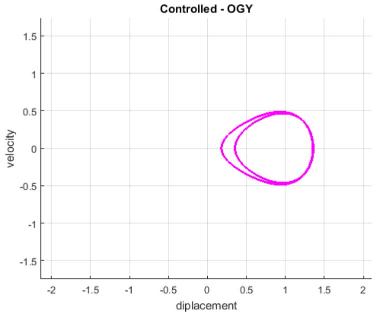
system trajectoty



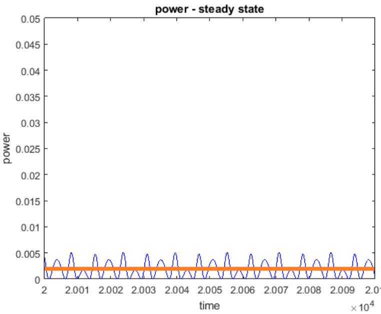
output power

OGY control with different orbits ($f = 0.088$, $\Omega = 0.8$)

controlled dynamics in a period 2 orbit



system trajectoty



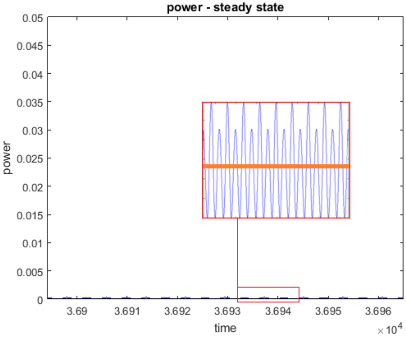
output power

Performance of OGY controller

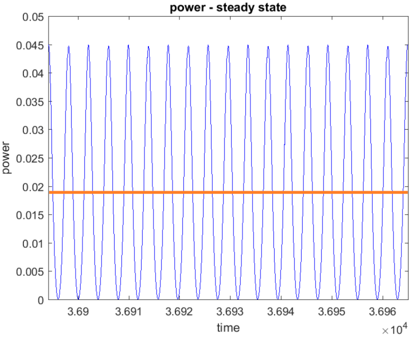
*Picture from https://www.amazon.com

Astonishing improvements are possible: Good!

$$f = 0.050,\ \Omega = 0.8$$



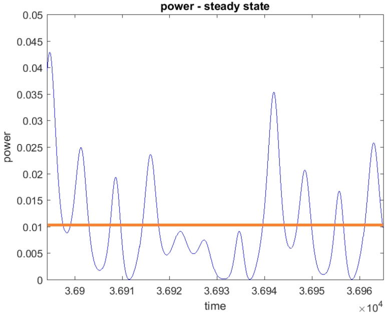
uncontrolled system



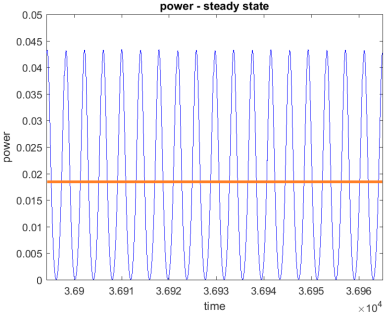
controlled system

Astonishing improvements are possible: Good!

$f = 0.090, \ \Omega = 0.8$



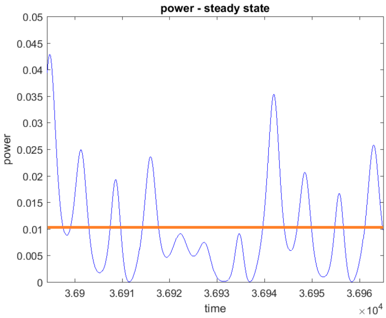
uncontrolled system



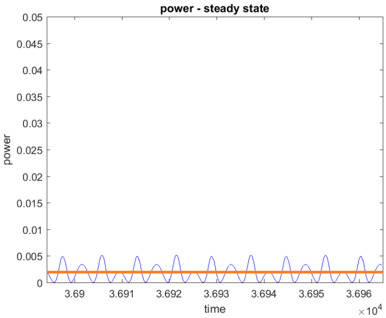
controlled system

Catastrophic effects too: Bad!

$f = 0.090,\ \Omega = 0.8$



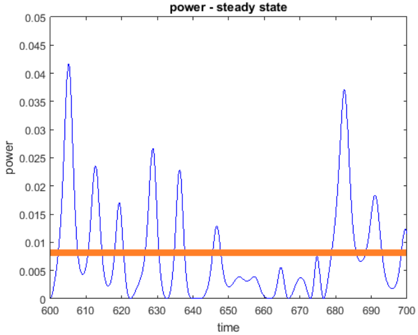
uncontrolled system



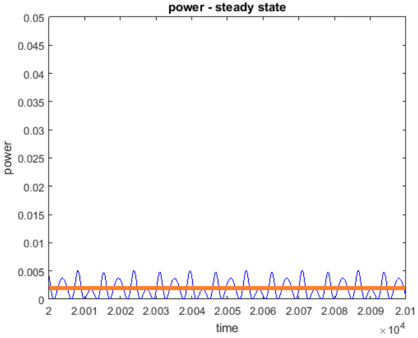
controlled system

Catastrophic effects too: Bad!

$$f = 0.088, \ \Omega = 0.8$$



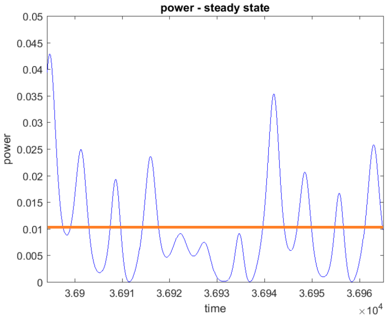
uncontrolled system



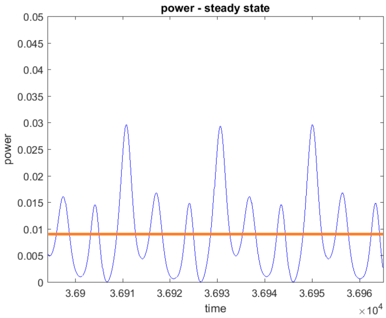
controlled system

And no significative change: Ugly!

$f = 0.090$, $\Omega = 0.8$



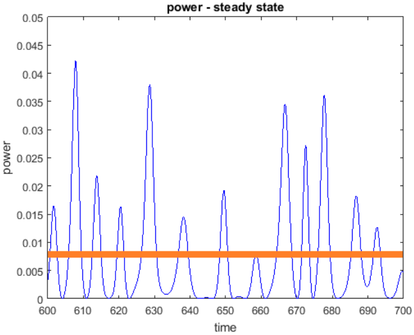
uncontrolled system



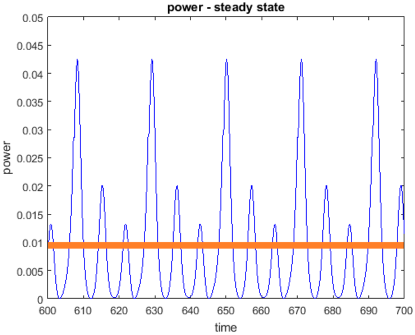
controlled system

And no significative change: Ugly!

$$f = 0.115, \; \Omega = 0.3$$



uncontrolled system



controlled system

Enhancement of power recovery (steady state dynamics)

excitation frequency: $\Omega = 0.8$

excitation amplitude	output power				power enhancement
	uncontrolled system	controlled system	controller ($\times 10^{-4}$)	effective value	
0.050	0.0001	0.0068	- 0.0005	0.0068	× **68**
0.083	0.0073	0.0131	- 0.0006	0.0131	× **1.8**
0.088 (2-p)	0.0077	0.0019	- 0.0007	0.0019	× **0.2**
0.090 (1-p)	0.0077	0.0154	- 0.0001	0.0154	× **2.0**
0.090 (2-p)	0.0077	0.0037	- 0.0002	0.0037	× **0.5**
0.090 (5-p)	0.0077	0.0084	- 0.0001	0.0084	× **1.1**

excitation frequency: $\Omega = 0.3$

excitation amplitude	output power				average power enhancement
	uncontrolled system	controlled system	controller ($\times 10^{-4}$)	effective value	
0.115 (2-p)	0.0078	0.0096	- 0.0004	0.0096	× **1.2**

Section 4

Final Remarks

Final remarks

Some conclusions:

- Nonlinearity is a powerful ingredient for harvesters efficiency:
  - May induce large amplitude responses
  - A plenty of energy is available in nonlinear responses
- Nonlinearity may also be an enemy:
  - May induce (very irregular) chaotic responses
  - Irregular voltage is undesirable for eletronic powering
- Control of chaos may be good, bad and ugly:
  - Astonishing improvements in efficiency are possible
  - Significative reductions of efficiency too
  - Controll of chaos may also be inert to efficiency

Ongoing research:

- Further studies with Pyragas control of chaos
- Studies in the stochastic dynamics

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Thank you for your attention!

americo@ime.uerj.br

www.americocunha.org

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