Bifurcation analysis and control of chaos on bistable piezoelectric energy harvesting systems

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São Paulo - SP, Brazil

Section 1

Introduction

April 4, 2005

March 13, 2013

L. Gammaitoni, **Fundamentals on energy**, NiPS Summer School 2018, University of Perugia.


IS-1
ST. JUDE MEDICAL
Sylmar, CA USA
VICTORY™ SR
5610 SSIR
S/N 1723182

Wikimedia Commons, **"File:St Jude Medical pacemaker in hand.jpg — Wikimedia Commons, the free media repository"**, 2014.

What's common in both cases?

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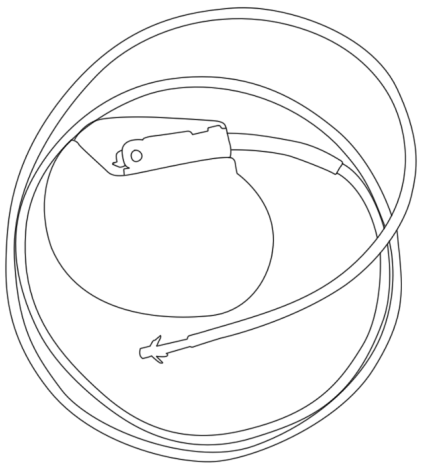
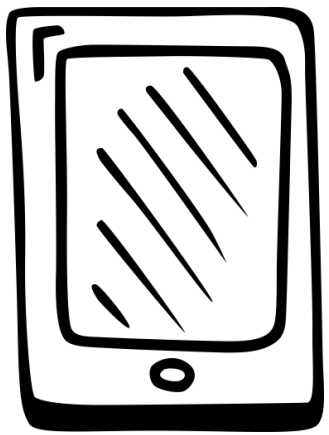
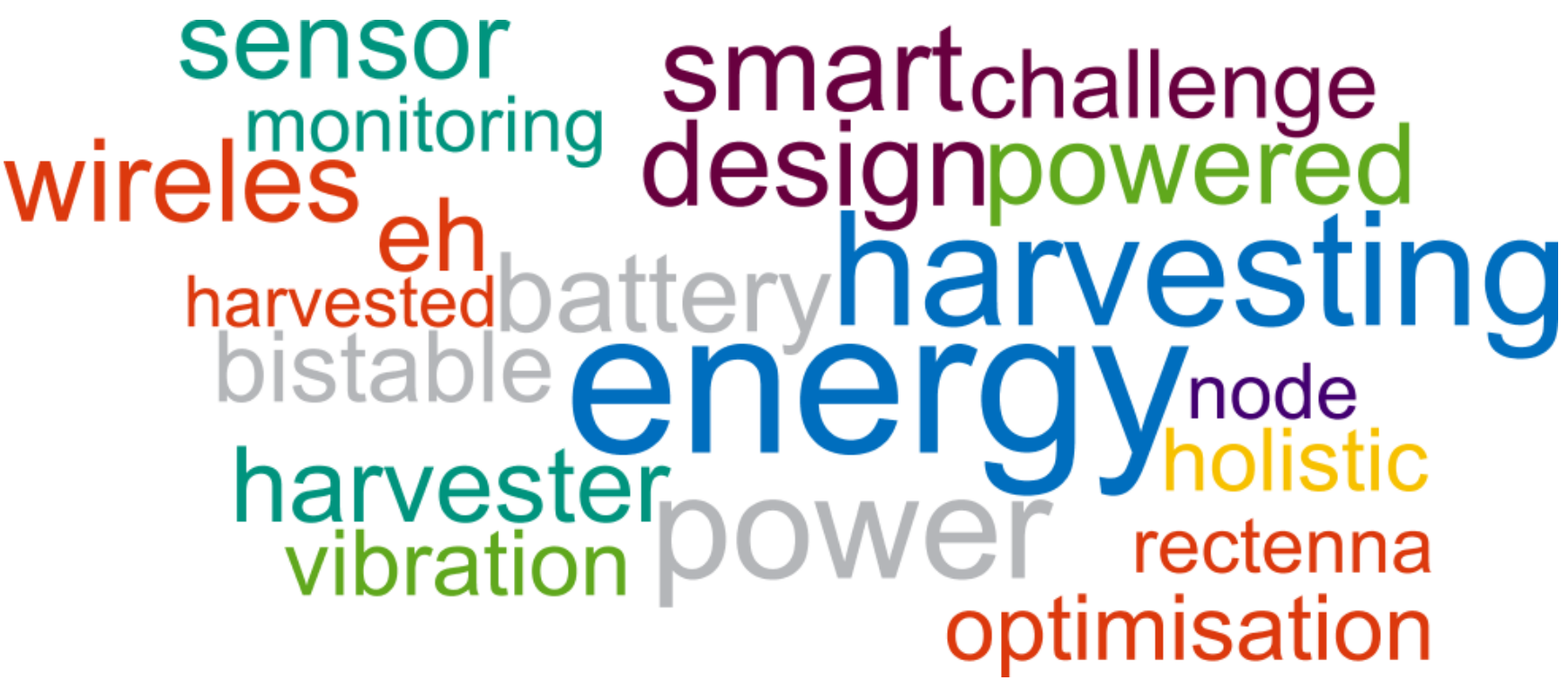
Electronic devices demanding energy!

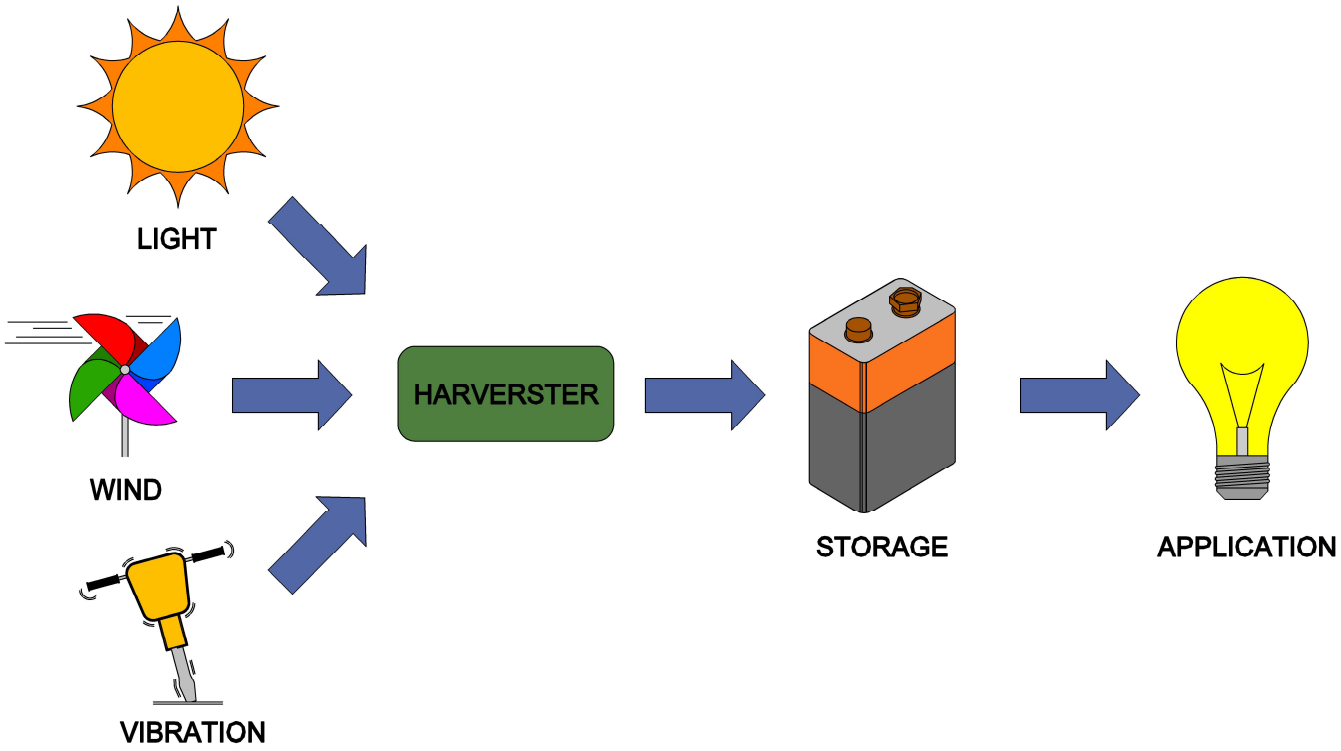






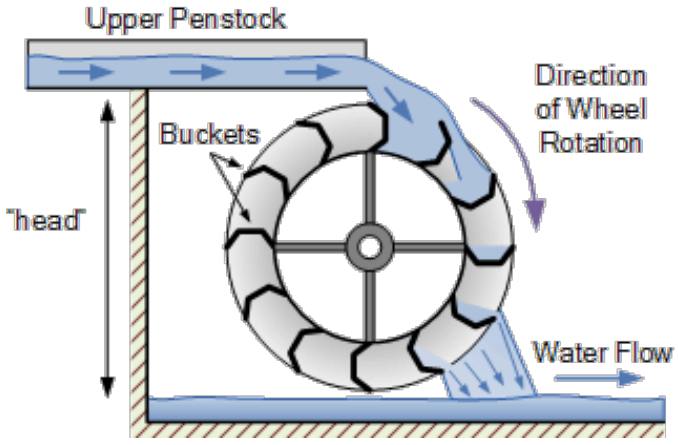
*Picture obtainded from Google Images.

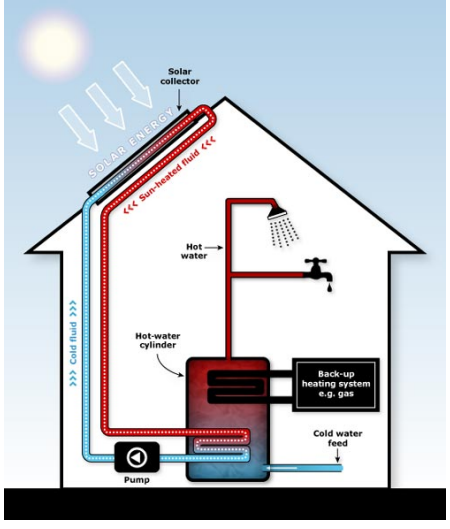
Energy Harvesting concept



- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

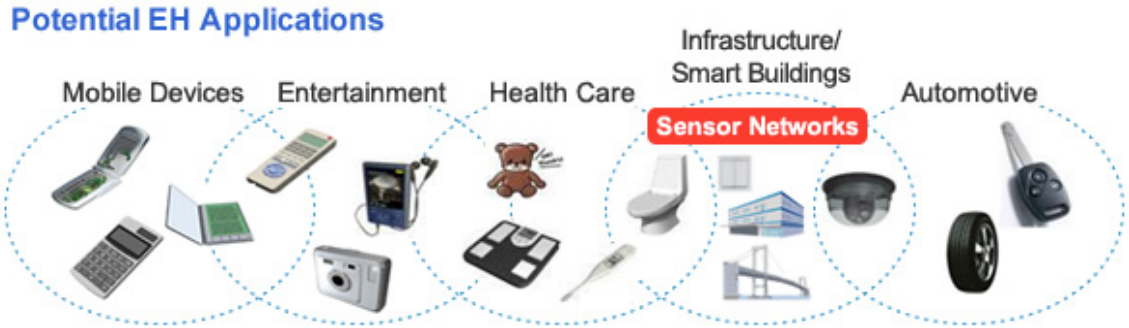
Classical technologies in Energy Harvesting

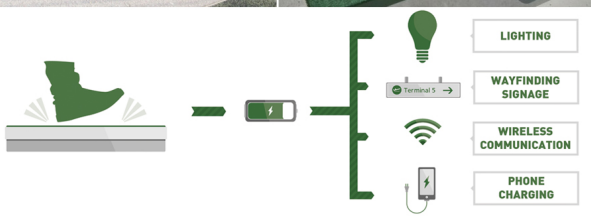


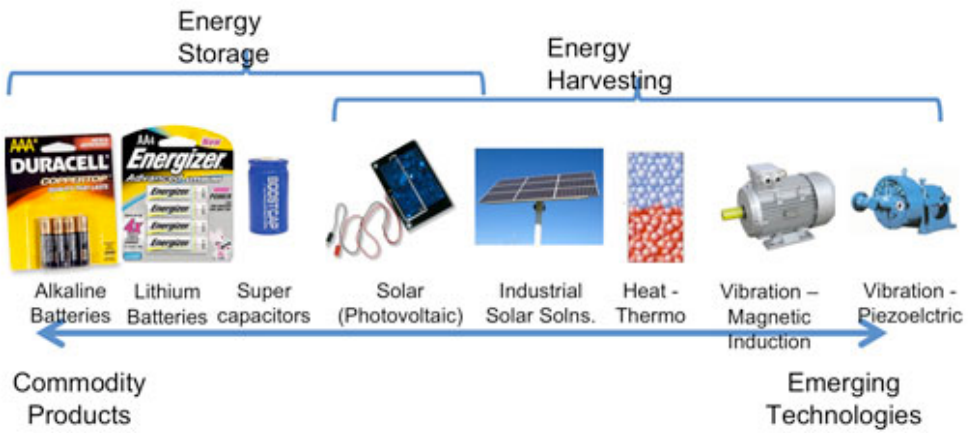


*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

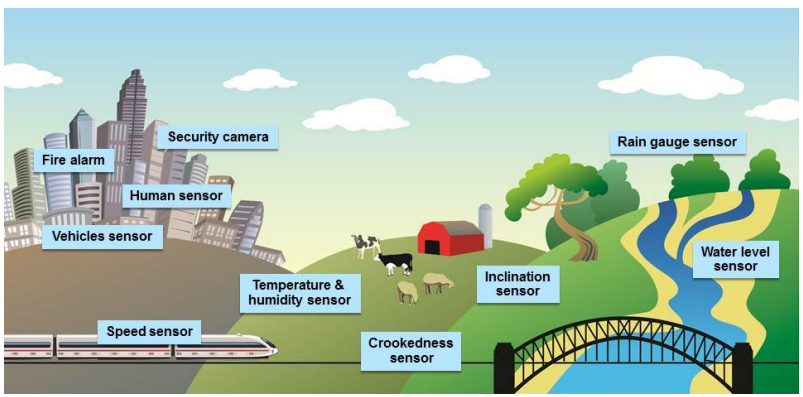
Emergent technologies in Energy Harvesting

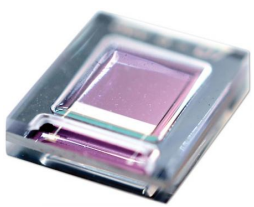


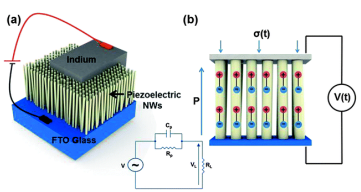




Spansion Energy Harvesting Technology Can Power the IoT







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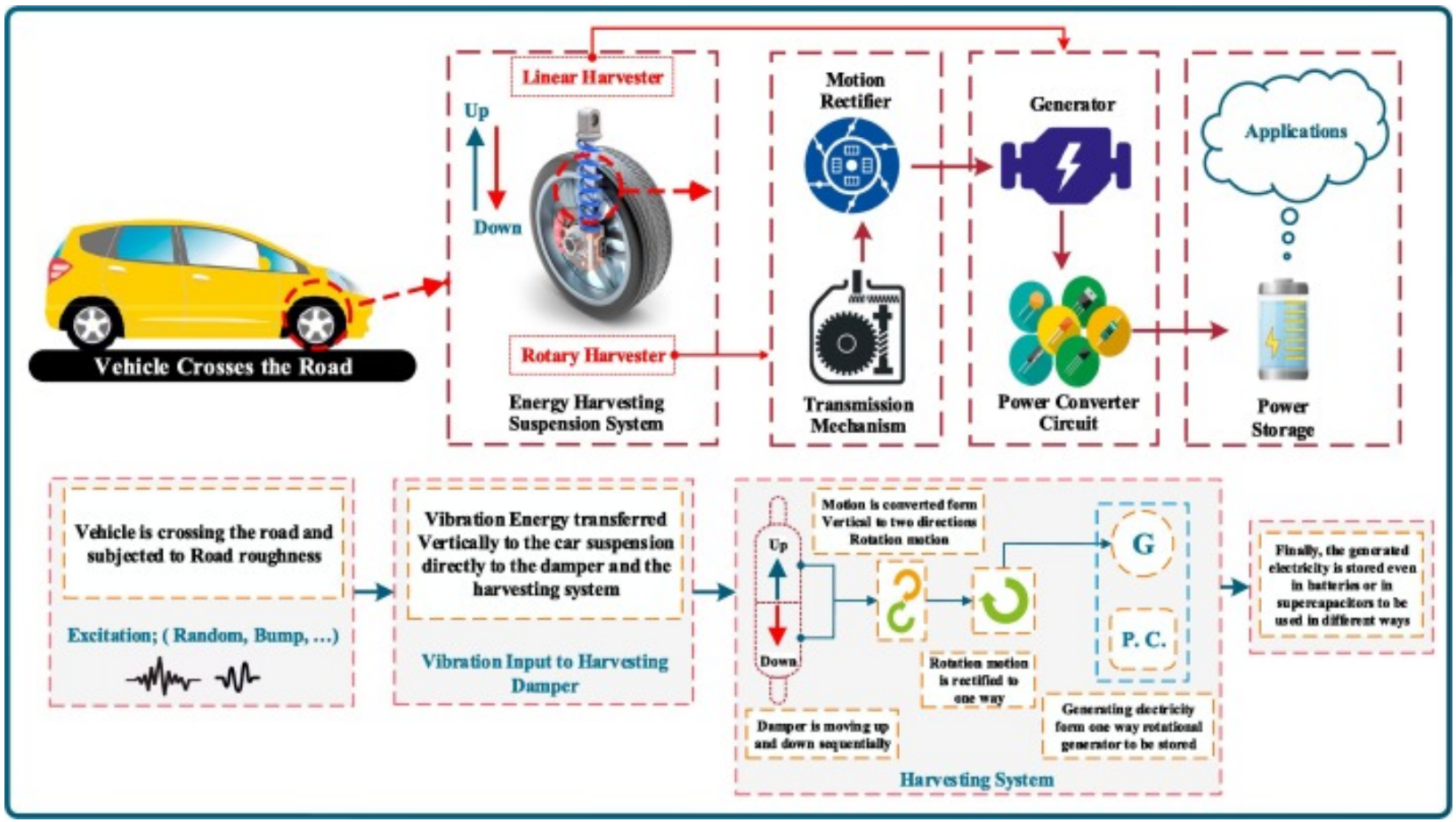
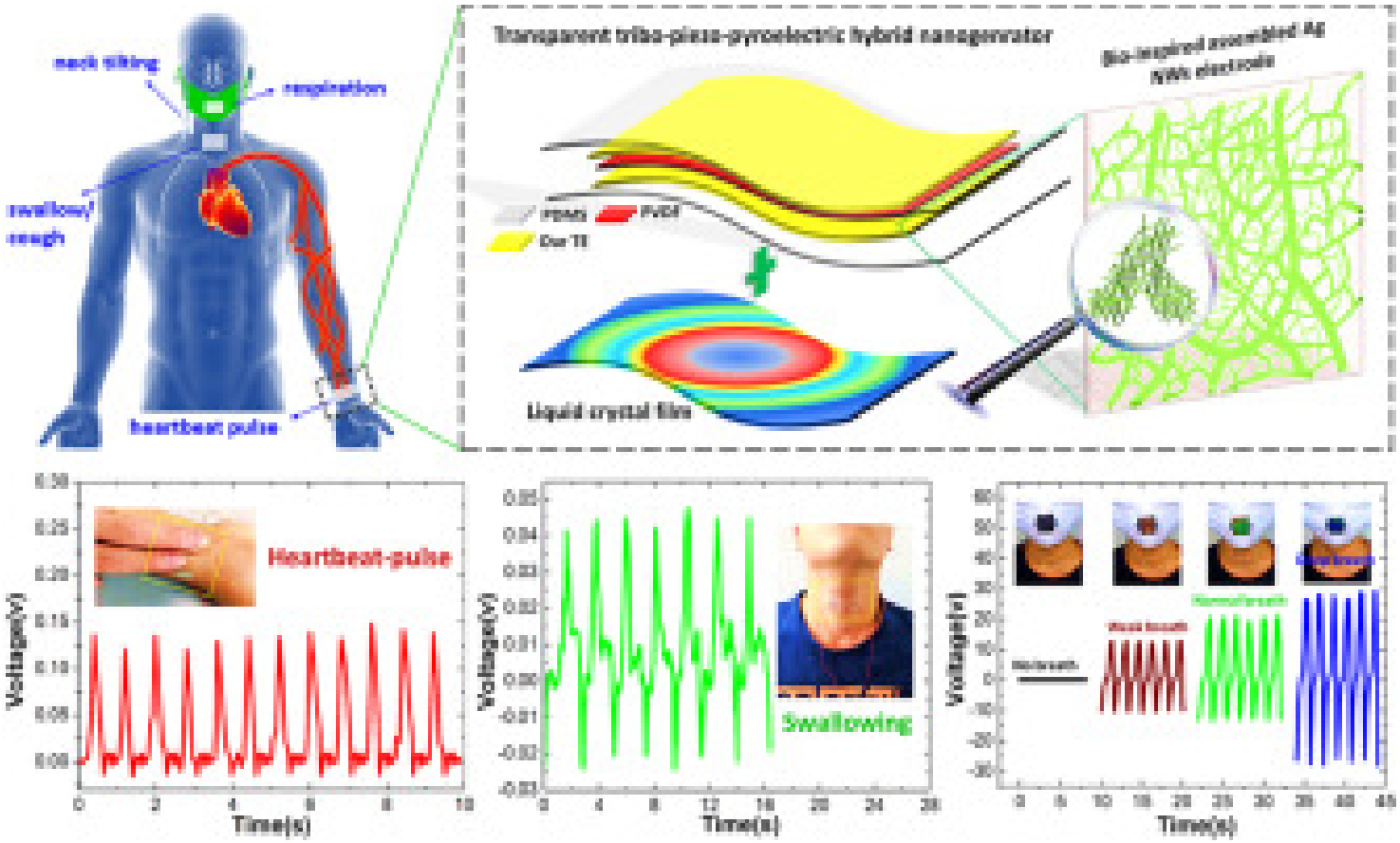
Emergent technologies in Energy Harvesting



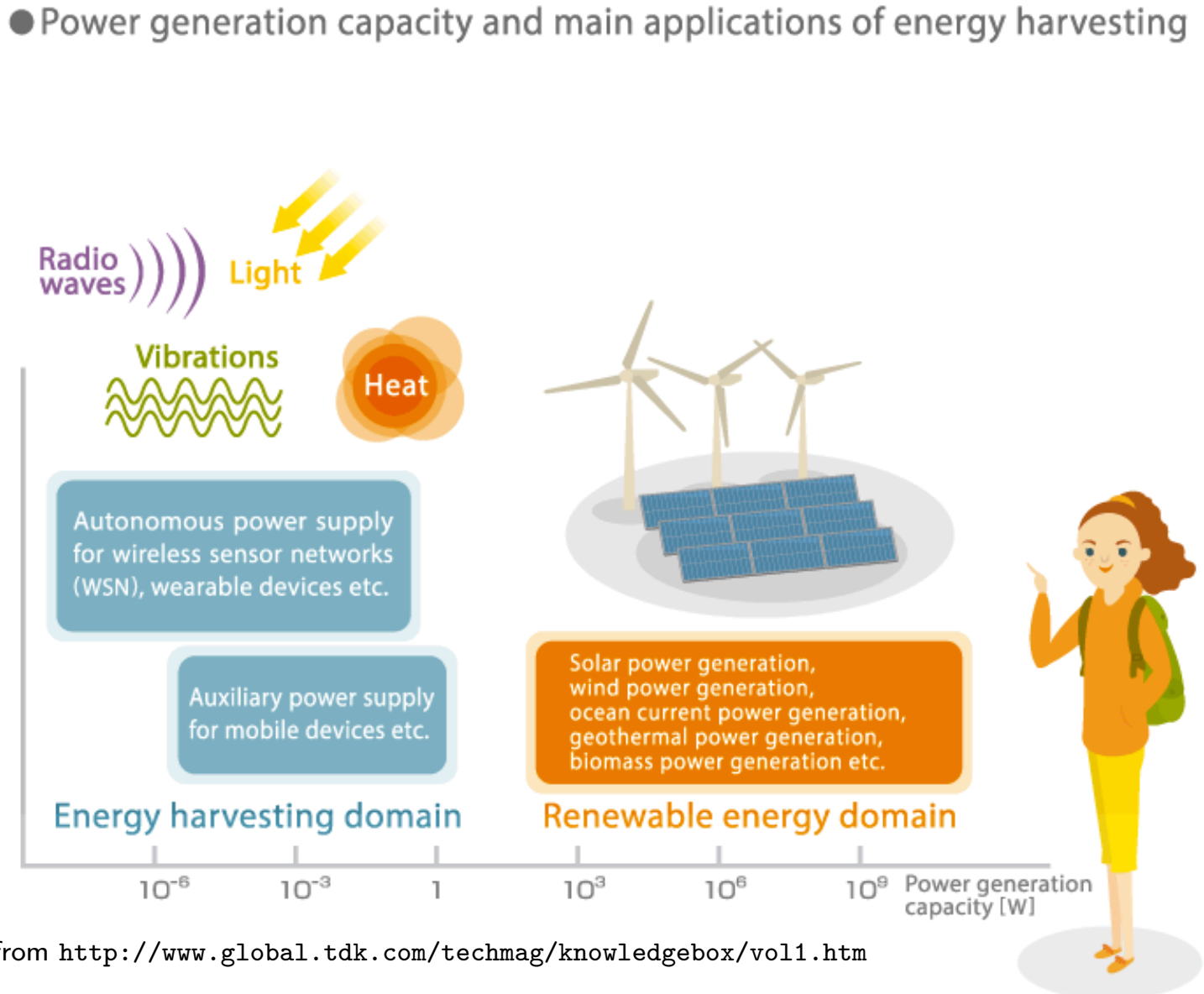
*Picture from: Mohamed A.A. Abdelkareem et al, *Vibration energy harvesting in automotive suspension system: A detailed review*, Applied Energy, Volume 229,2018 (http://www.sciencedirect.com/science/article/pii/S0306261918311851)

Emergent technologies in Energy Harvesting



*Picture from: Jian-Guo Sun et al, *A flexible transparent one-structure tribo-piezo-pyroelectric hybrid energy generator based on bio-inspired silver nanowires network for biomechanical energy harvesting and physiological monitoring*, Nano Energy, Volume 48, 2018

Energy scale for modern Energy Harvesting



*Picture from http://www.global.tdk.com/techmag/knowledgebox/vol1.htm

Energy Harvesting working principles

- Vibration based (piezoelectric effect)
- Contact based (triboelectric effect)
- Temperature based (thermoelectric effect)
- Heat based (pyroelectric effect)
- Light based (photoelectric effect)
- Fluid flow based
- Radio frequency based
- etc

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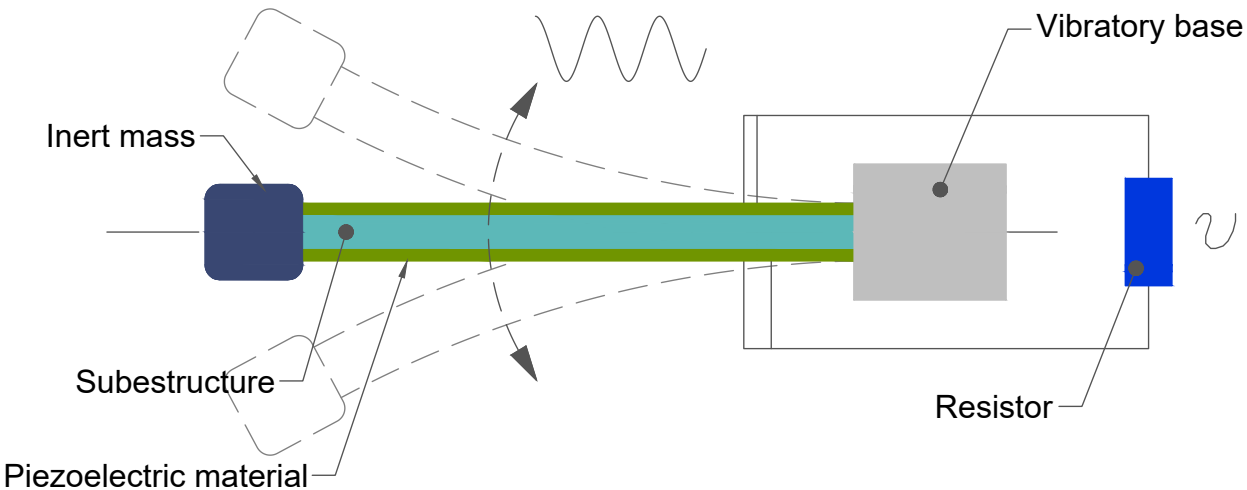
Some characteristics of Vibration Energy Harvesting

- No energy transmission systems required
- No large energy storage devices required
- Wide range of application areas
- Many physical layouts available
- System dynamics high complexity
- Low power recovering
- Recovered power quality depends on excitation
- Suitable for small eletronic powering

Linear Vibratory Harvester

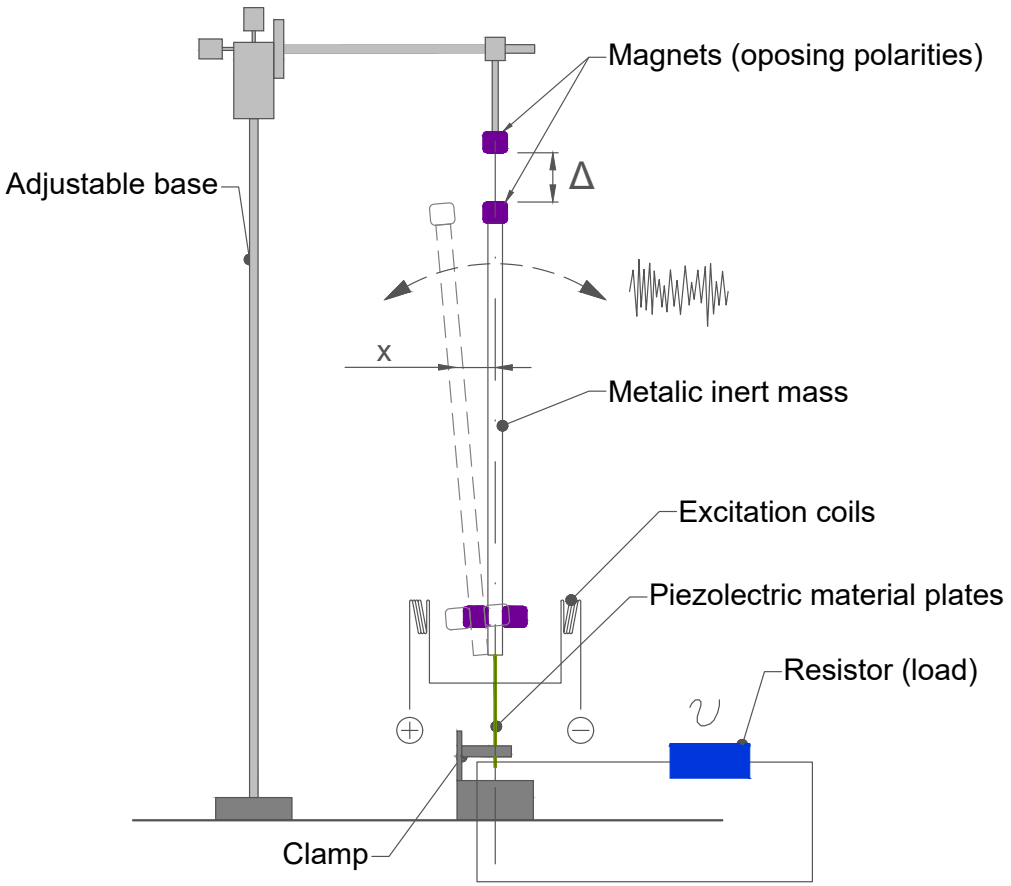
Monostable system driven by regular signal



S. Roundy, P. K. Wright and J. Rabaey, A study of low level bibrations as a power source for wireless sensor nodes. **Computer Communications**, 26: 1131-1144, 2003.

Nonlinear Vibratory Harvester

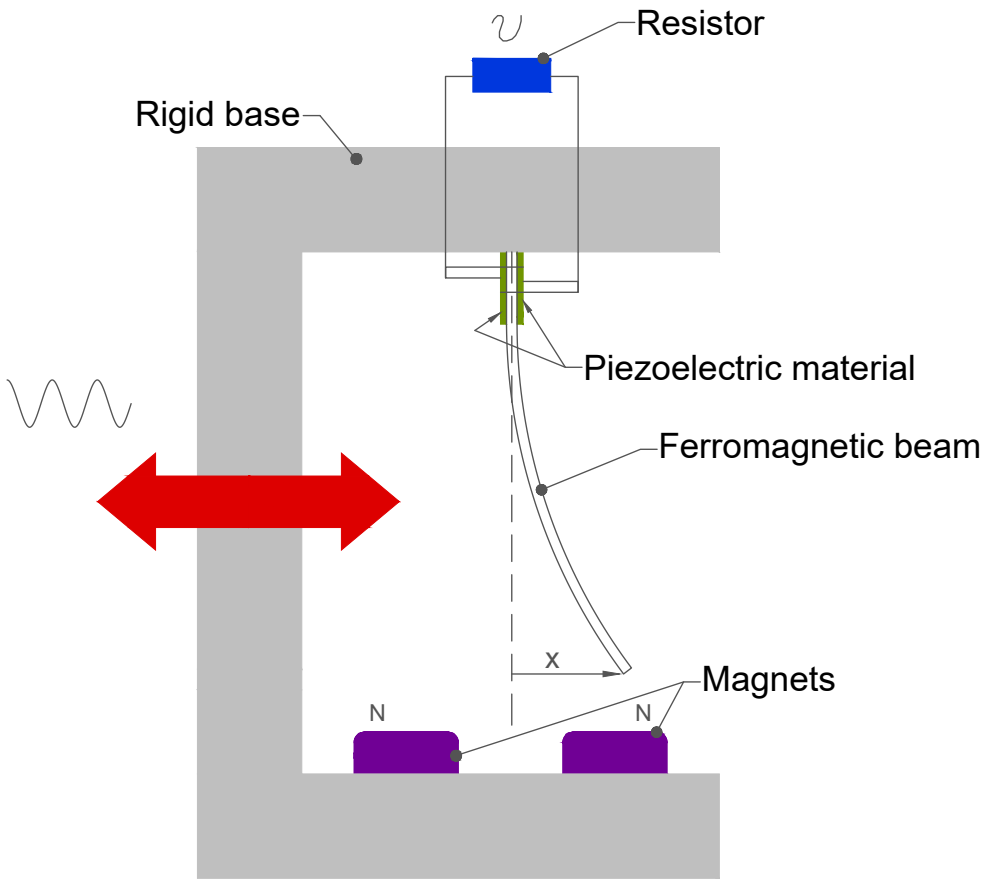
Bistable system driven by a noisy signal



F. Cottone, H. Vocca and L. Gammaitoni, Nonlinear energy harvesting. **Physical Review Letters**, 102: 080601, 2009.

Nonlinear Vibratory Harvester

Bistable system driven by regular signal



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
  - Time series
  - Poincaré sections
  - Bifurcation diagrams
  - Basis of attractions
  - Test 0-1 for chaos
- Model the underlying uncertainties and study their influence
  - System parameters variability
  - Noise in system excitation
- Propose strategies to enhance the recovered energy
  - Nonlinear optimization
  - Control of chaos

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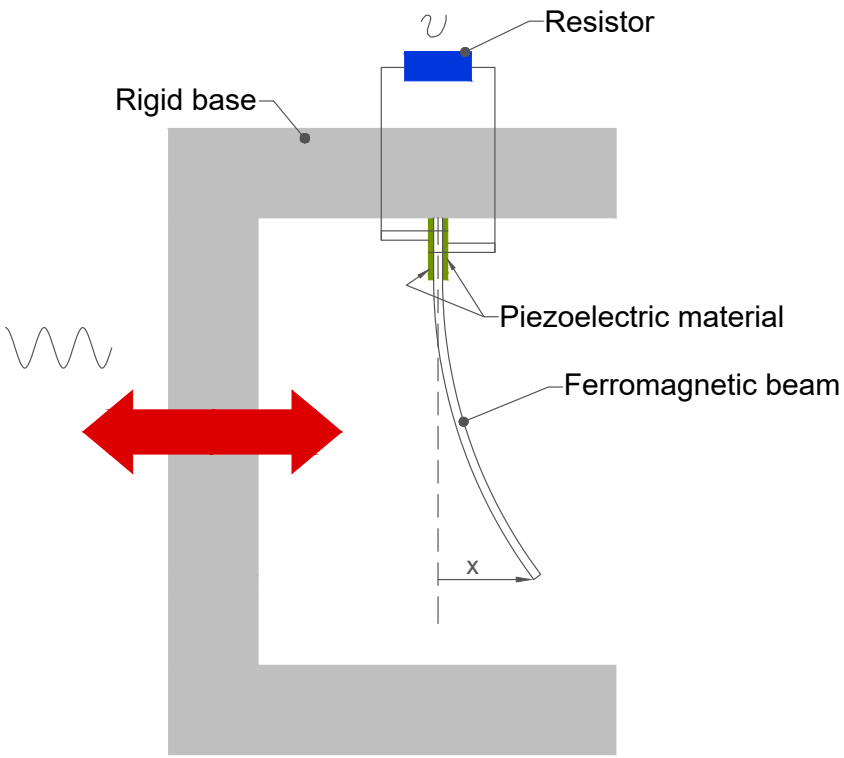
Section 2

Dynamical System

Linear Dynamics

Linear harvester driven by regular signal



$$\ddot{x} + 2\xi\dot{x} + x + \chi\upsilon = f\cos\Omega t$$

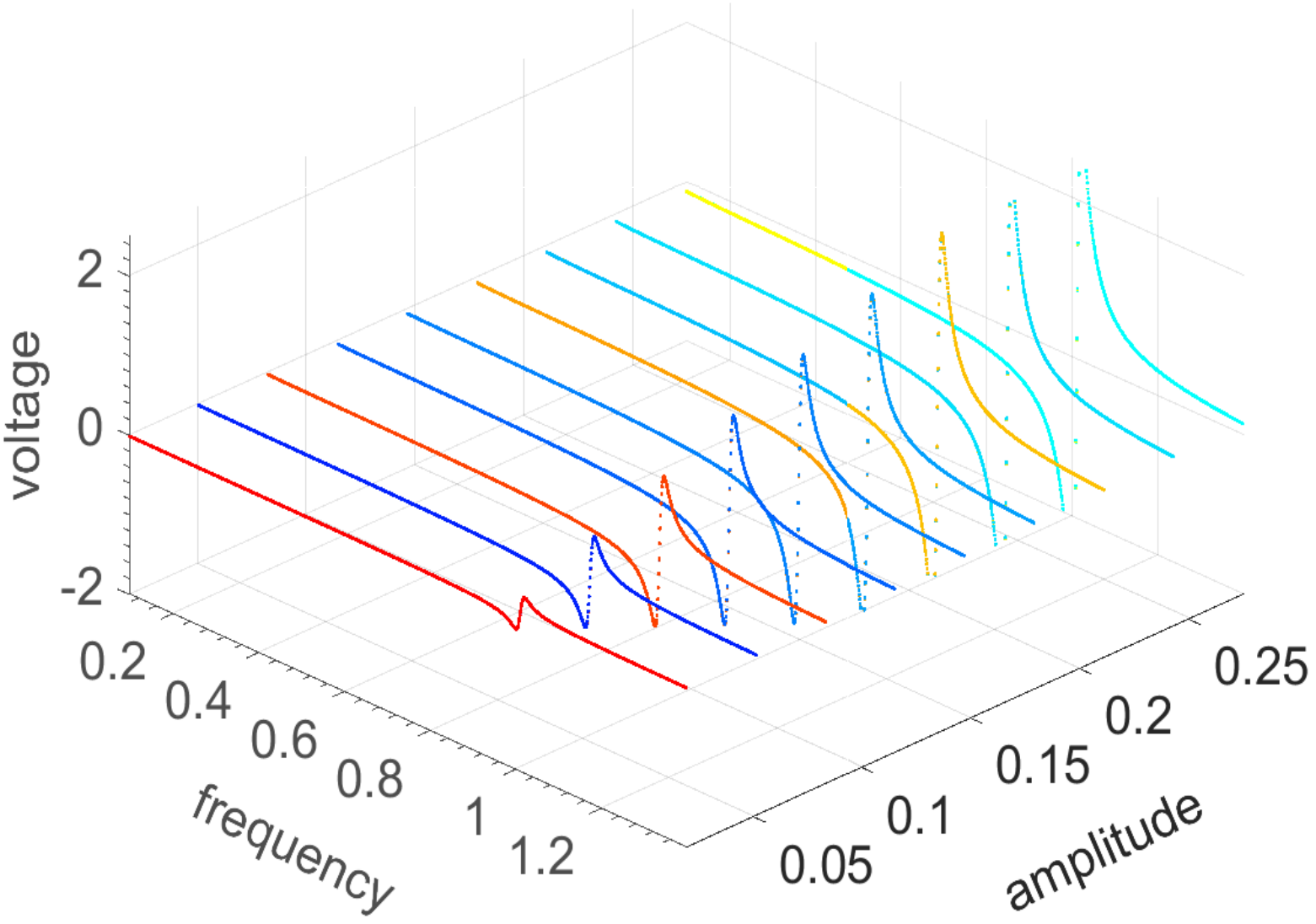
$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0,\ \dot{x}(0) = \dot{x}_0,\ \upsilon(0) = \upsilon_0$$

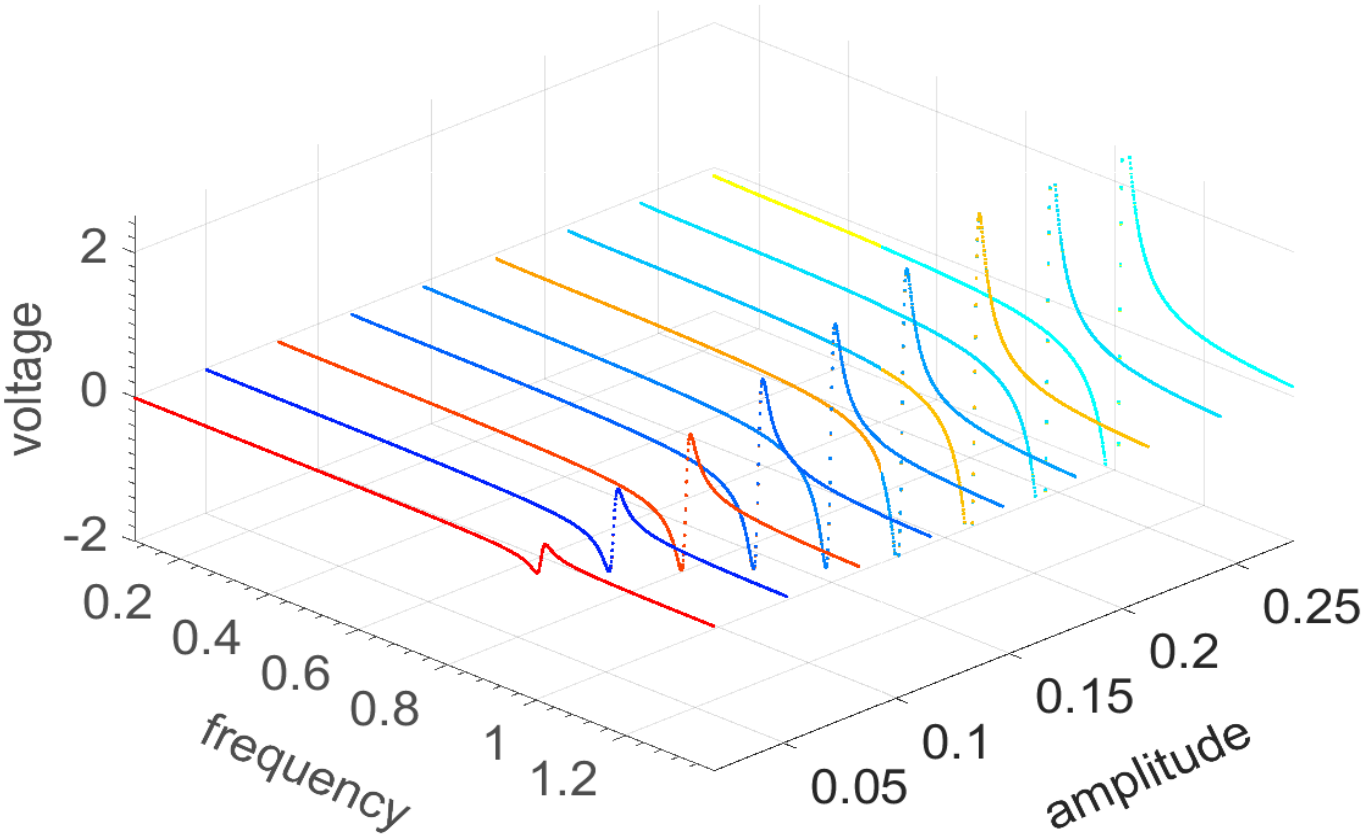
A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Linear dynamics animation

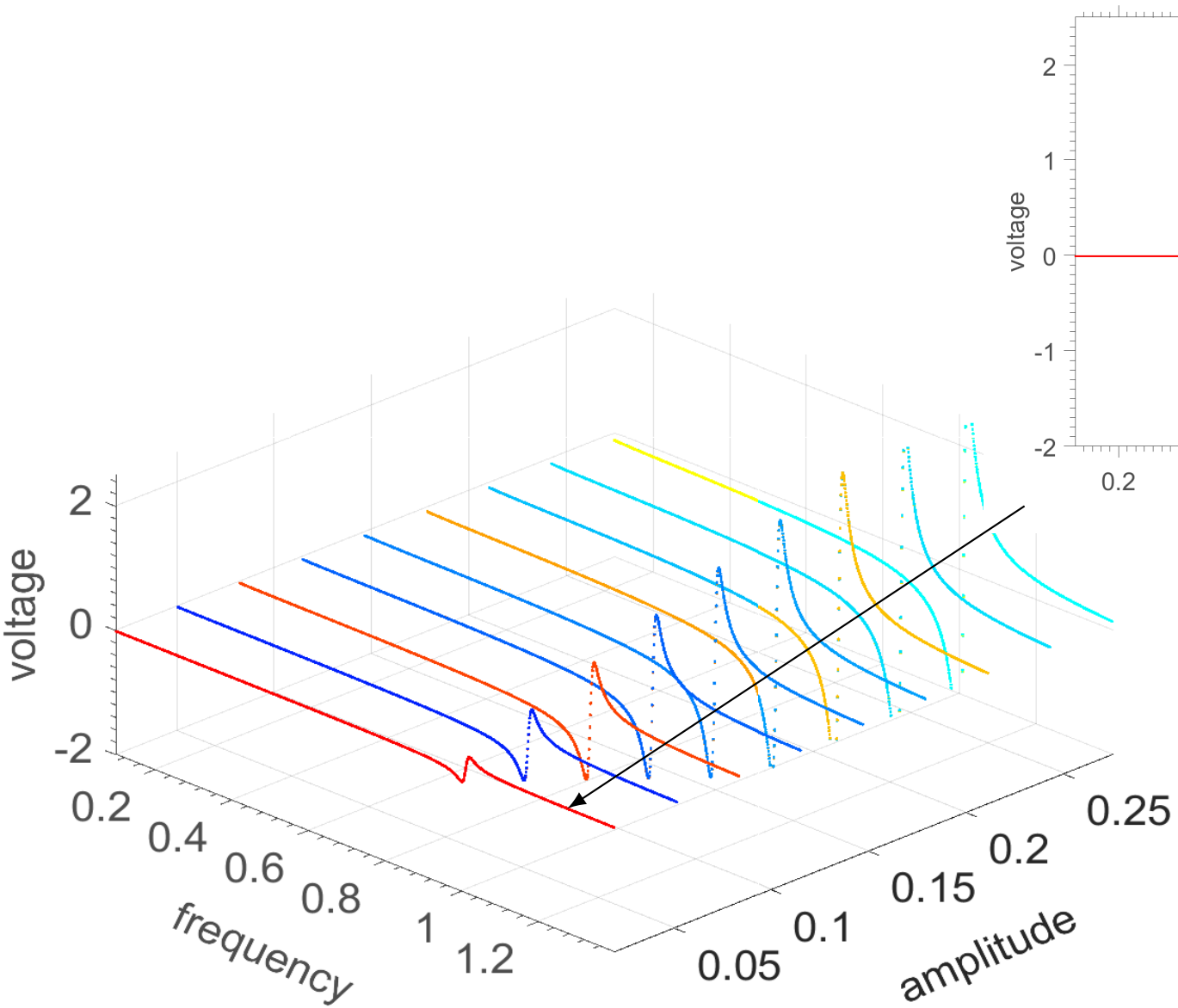
Global overview of excitation frequency effect

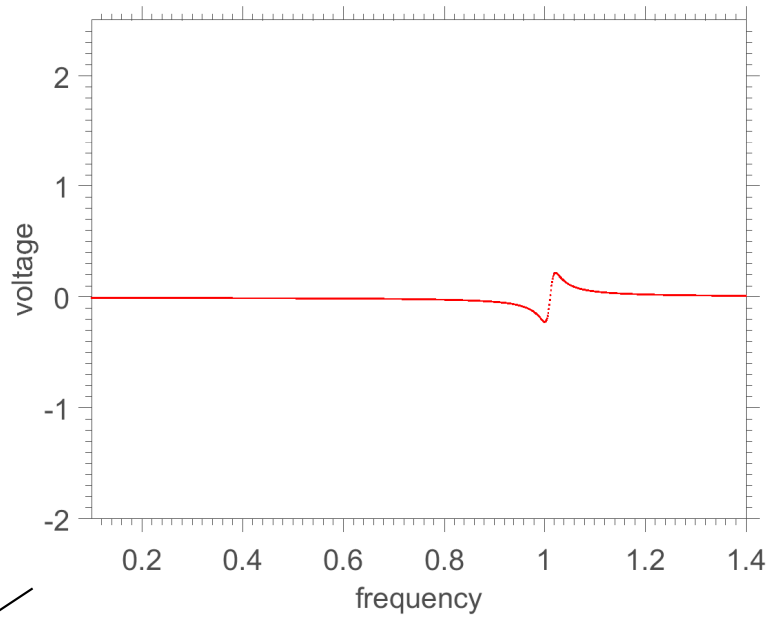


Bifurcation diagrams: voltage $\times$ frequency



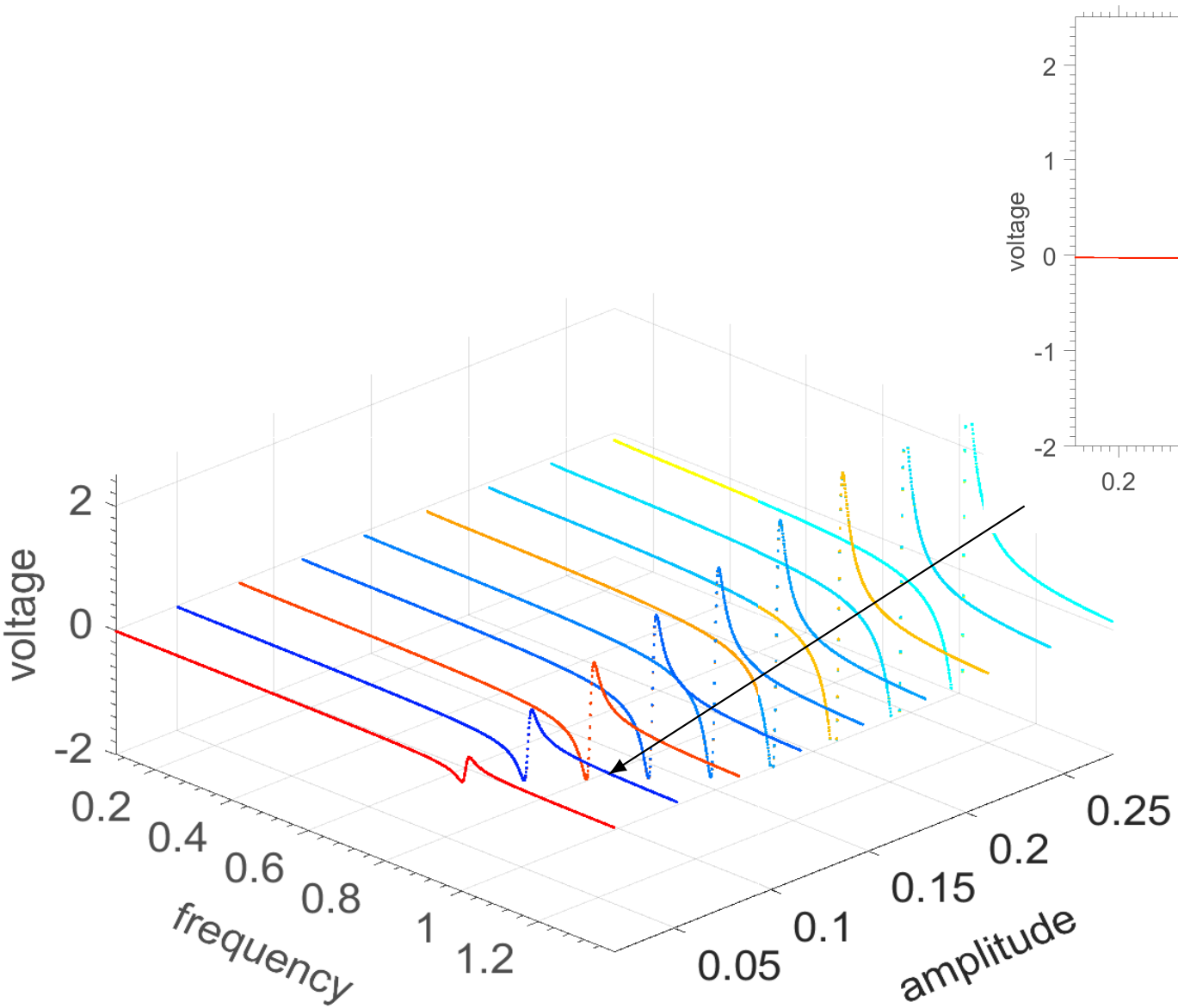
Bifurcation diagrams: voltage × frequency

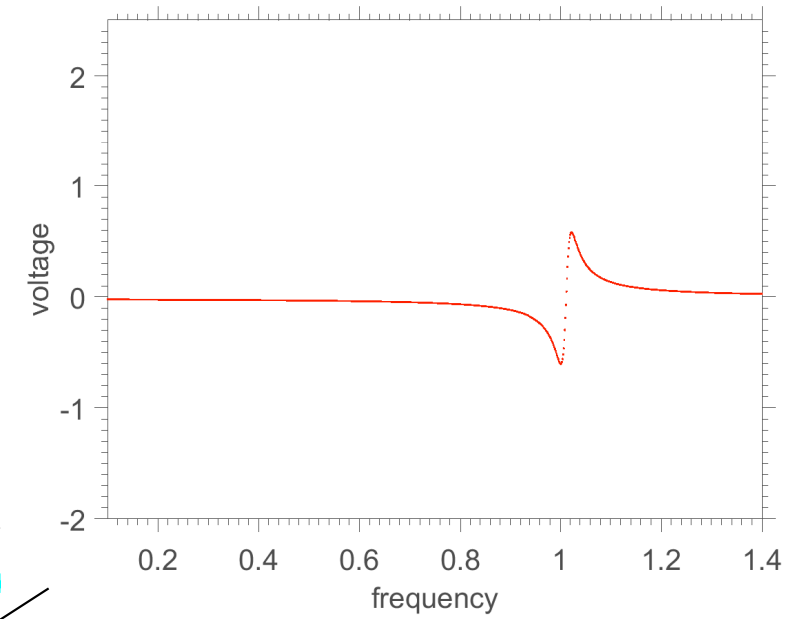




$f = 0.019$

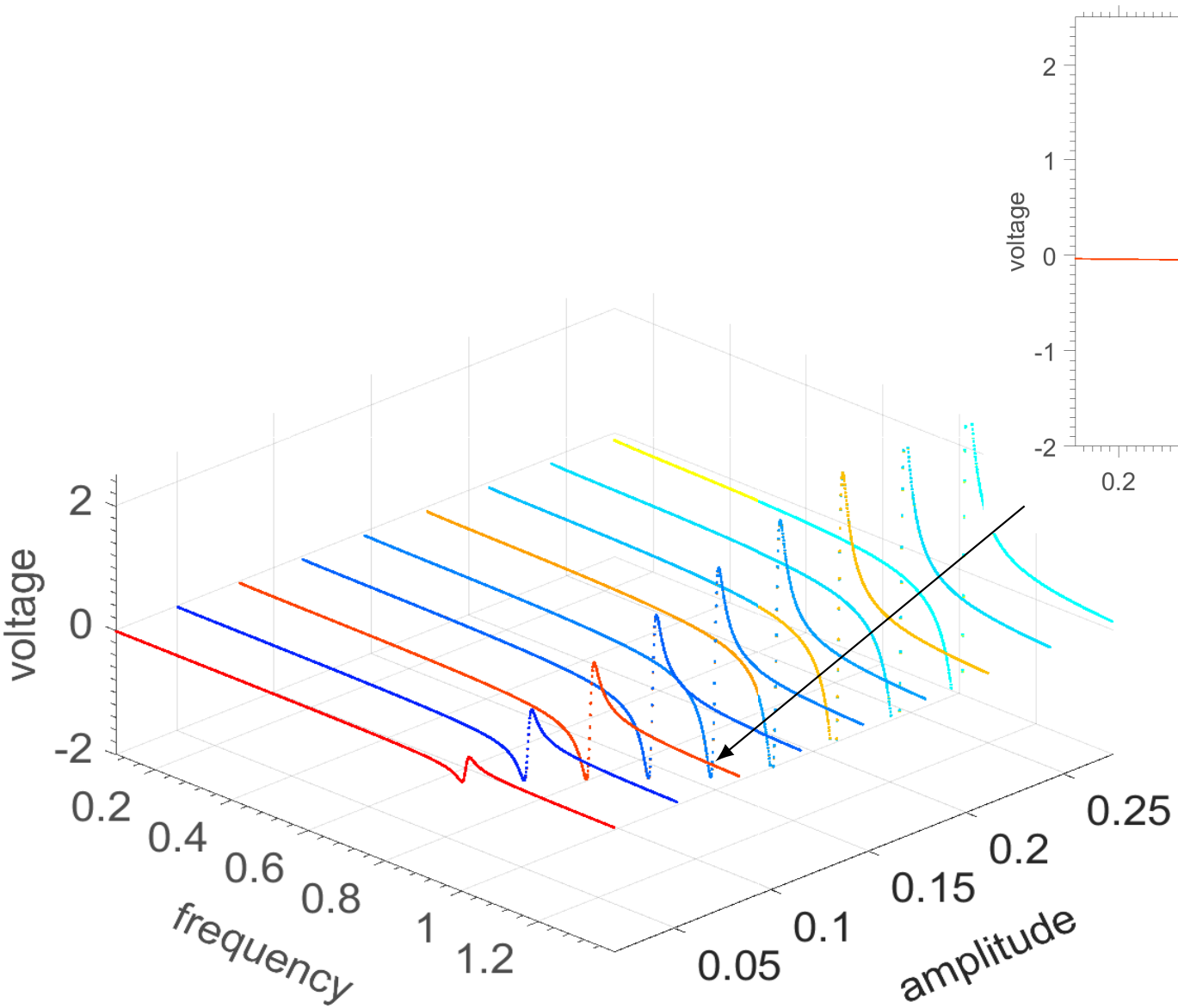
Bifurcation diagrams: voltage × frequency

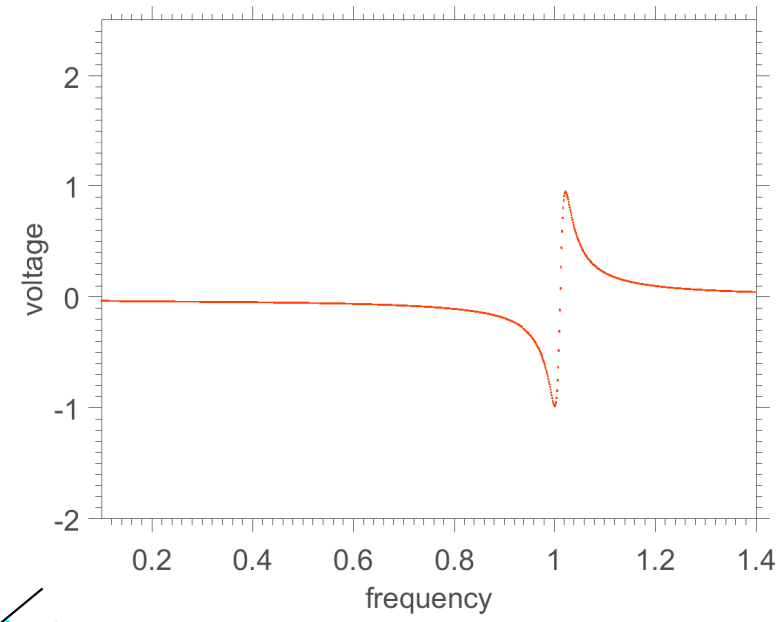




$f = 0.051$

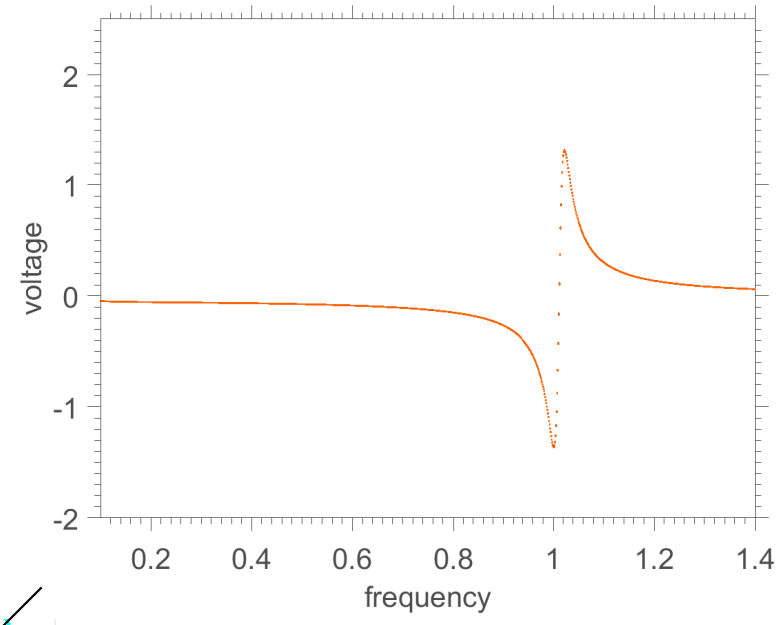
Bifurcation diagrams: voltage × frequency



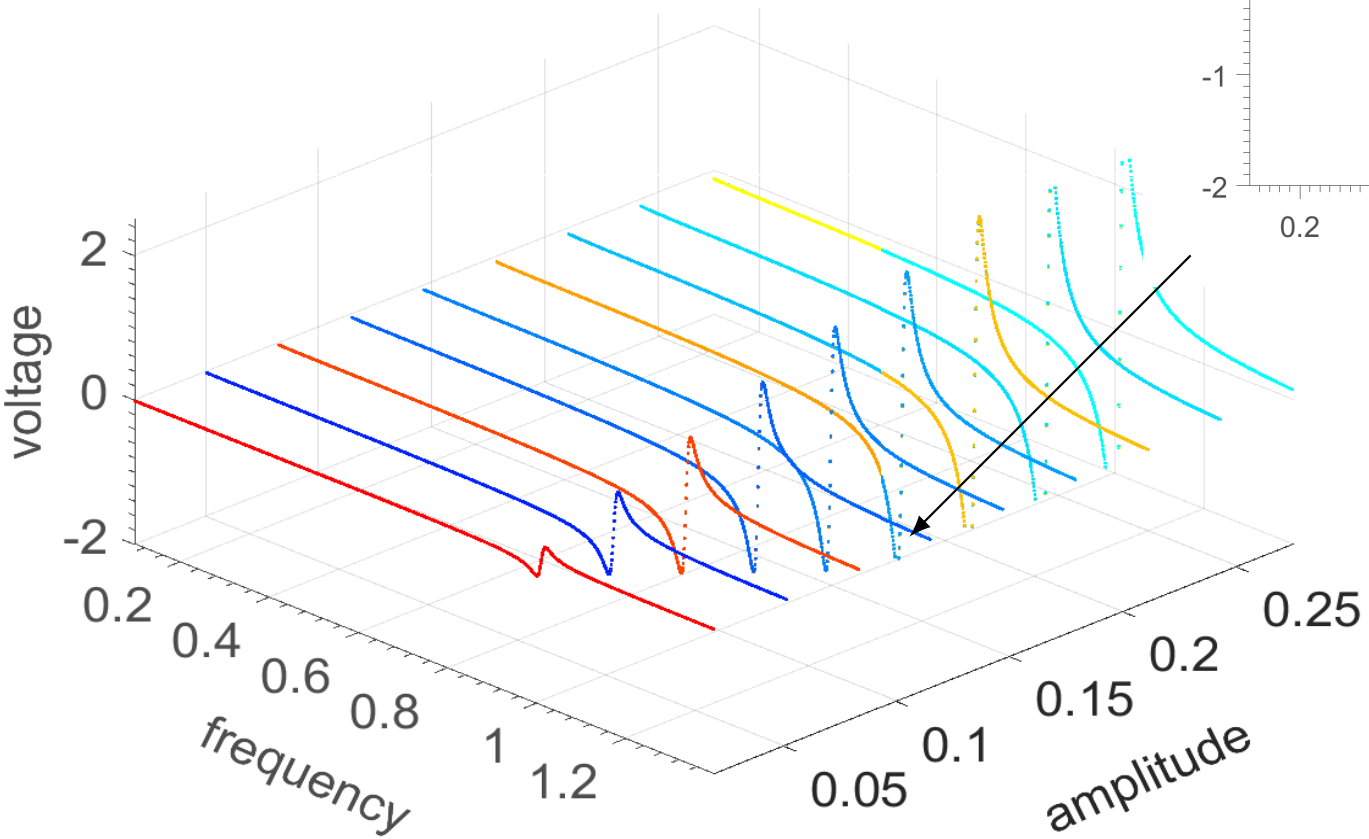


$f = 0.083$

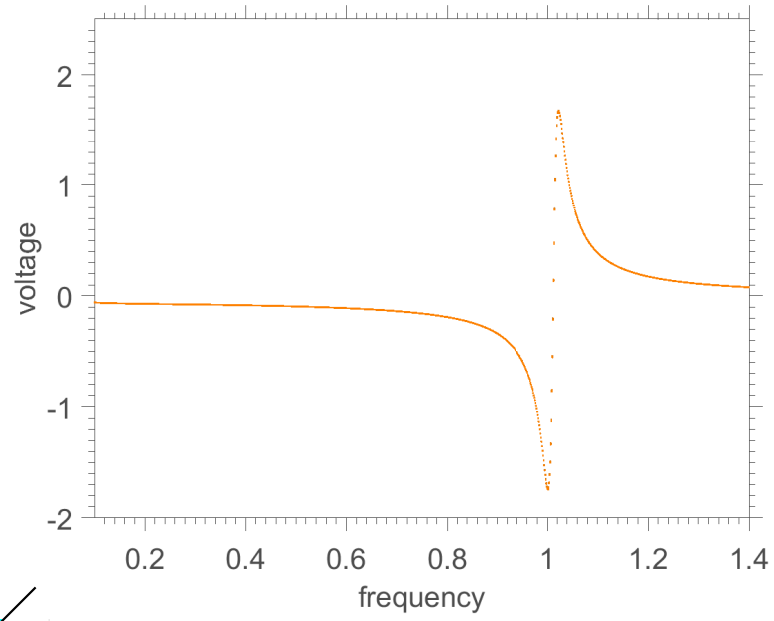
Bifurcation diagrams: voltage × frequency



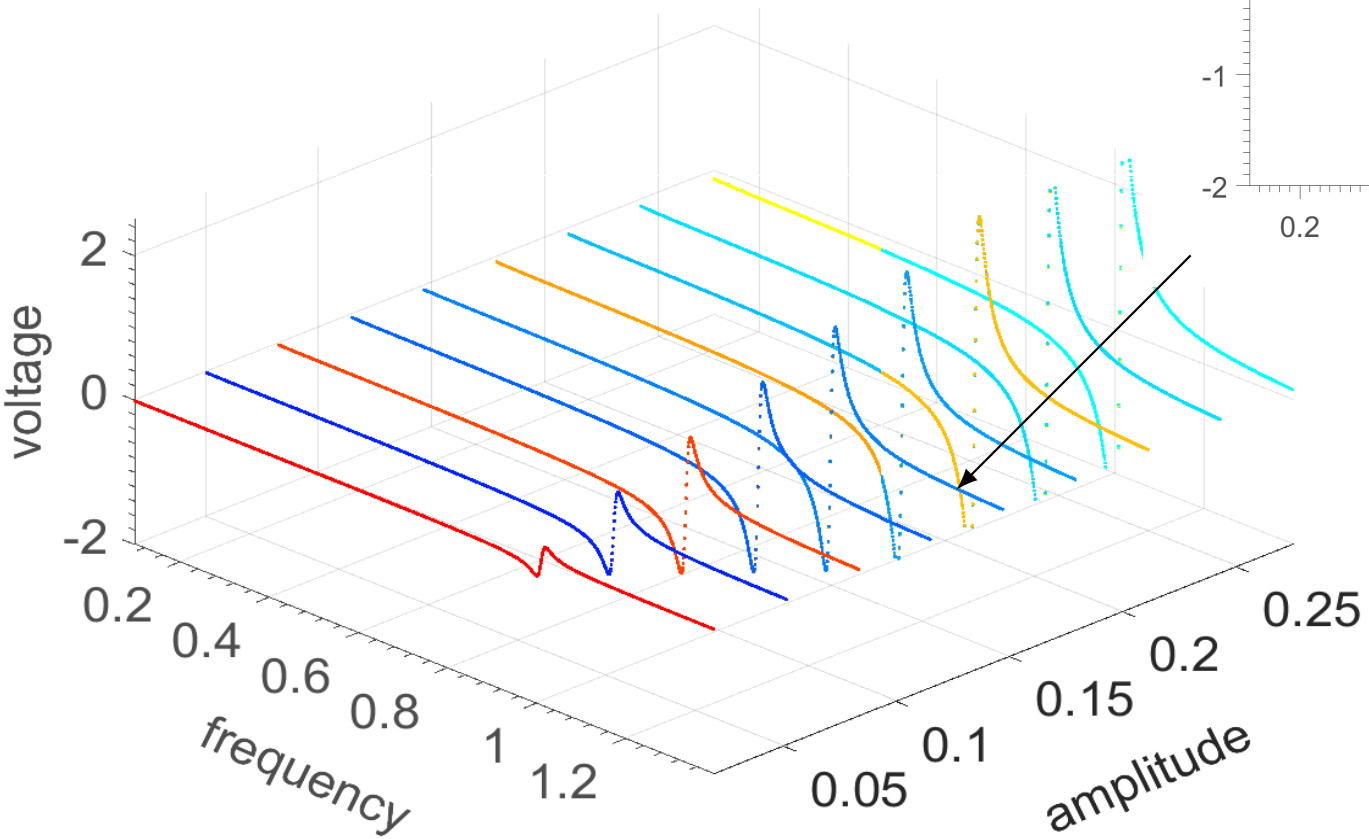
$f = 0.115$



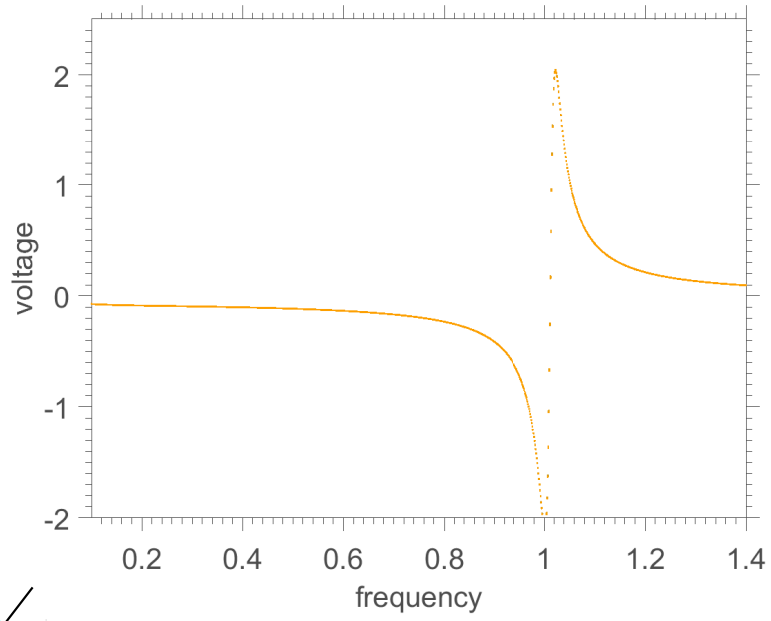
Bifurcation diagrams: voltage × frequency



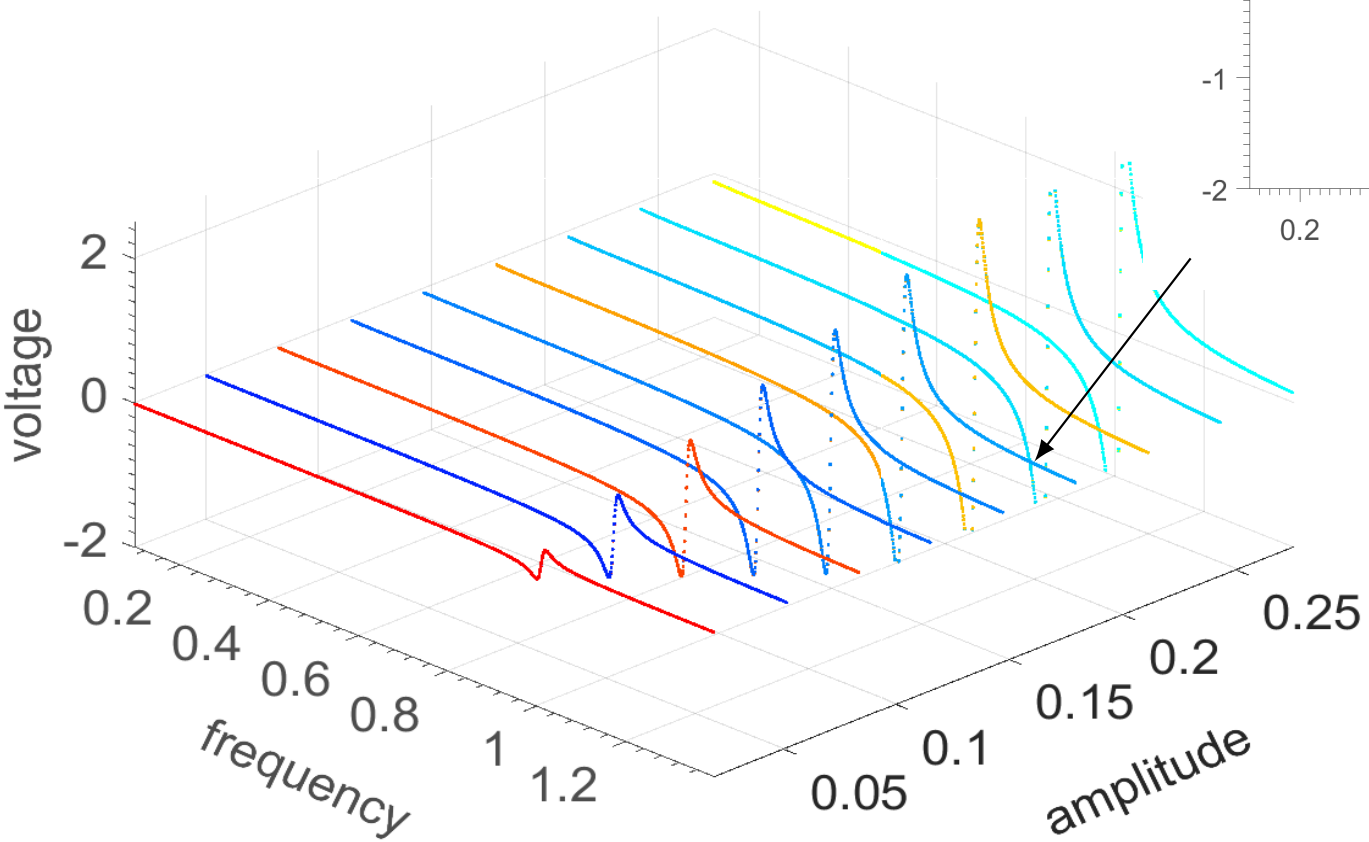
$f = 0.147$



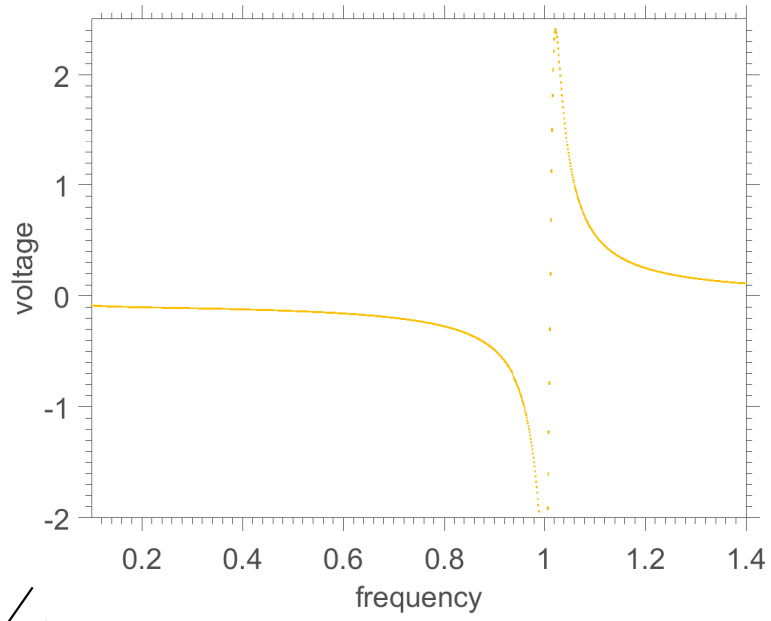
Bifurcation diagrams: voltage × frequency



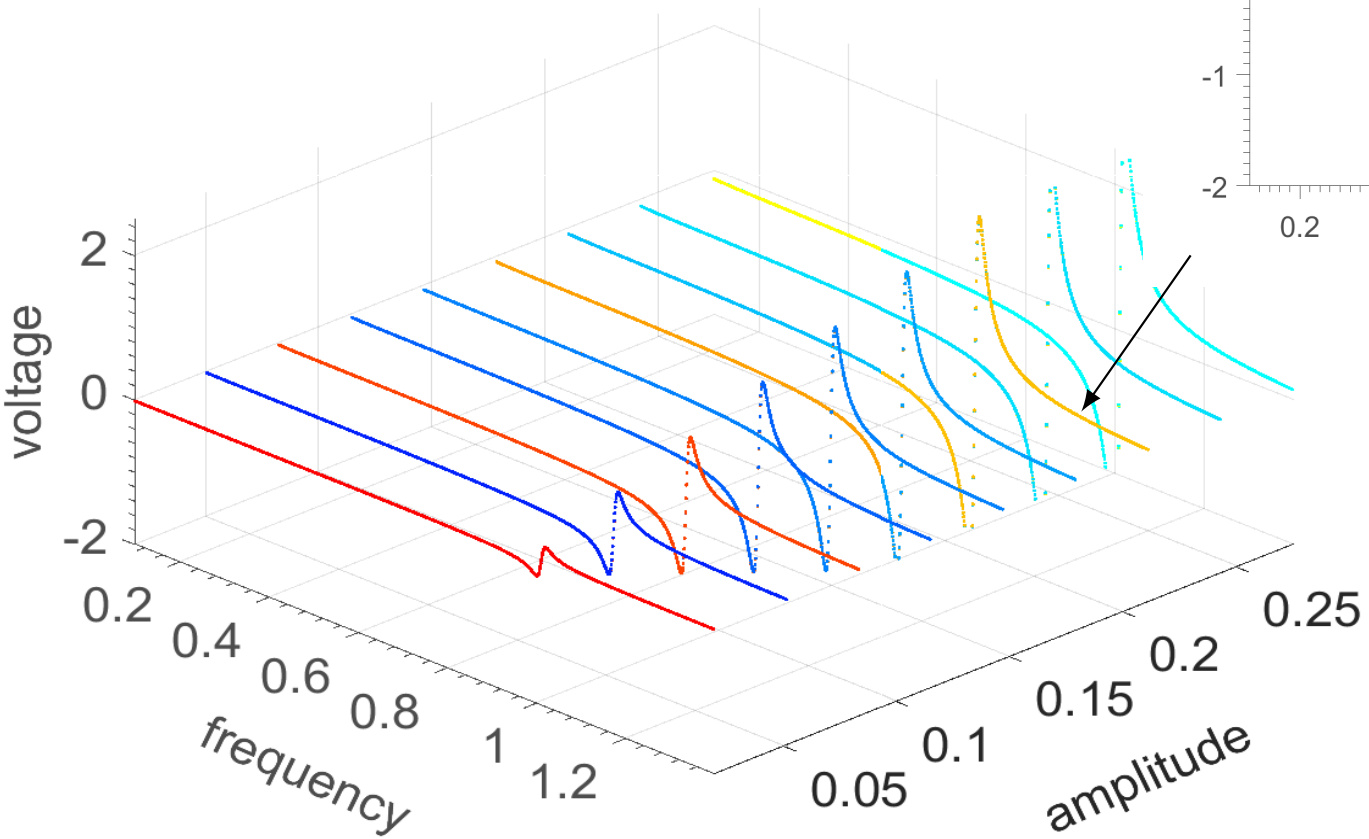
$f = 0.179$



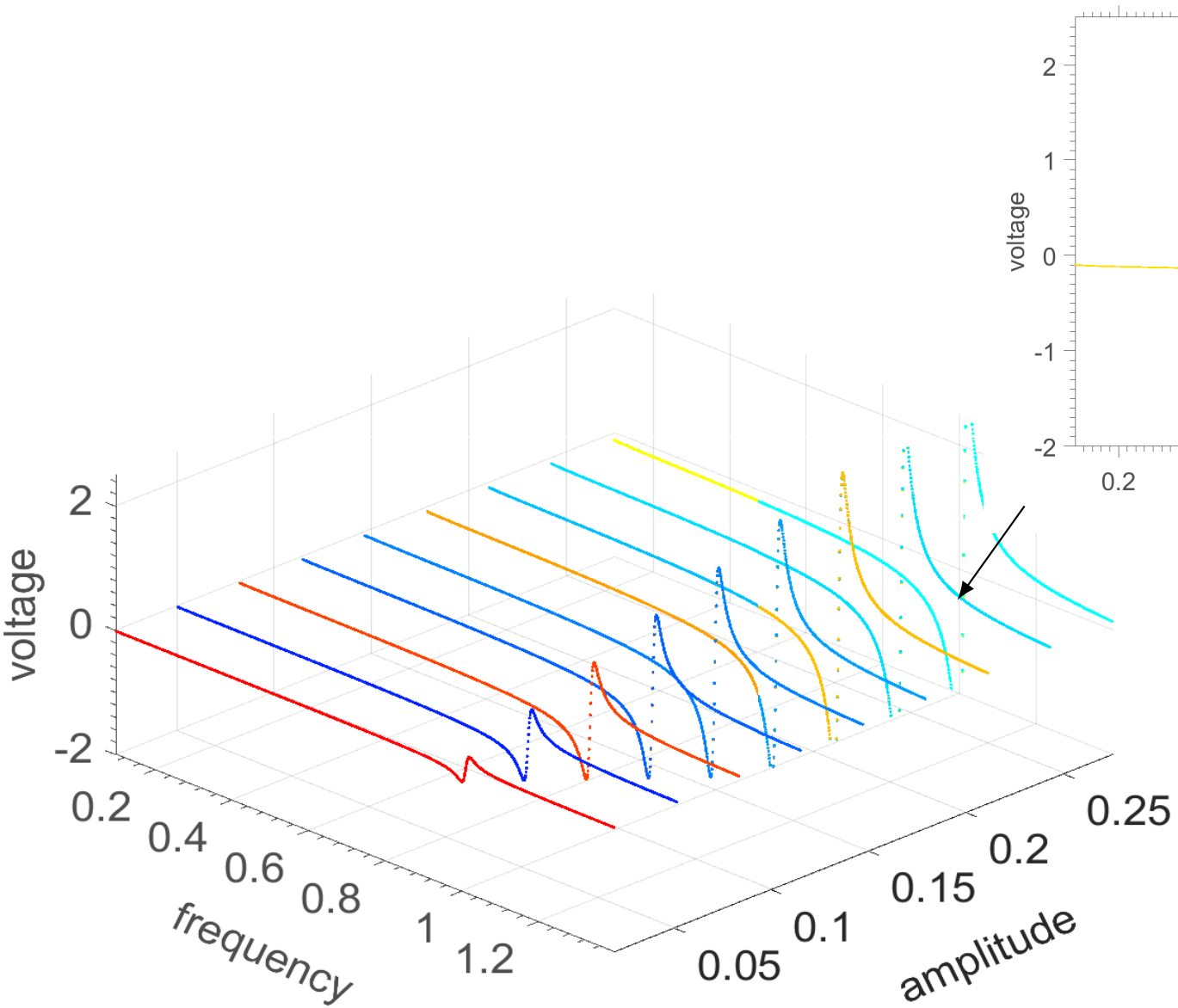
Bifurcation diagrams: voltage × frequency

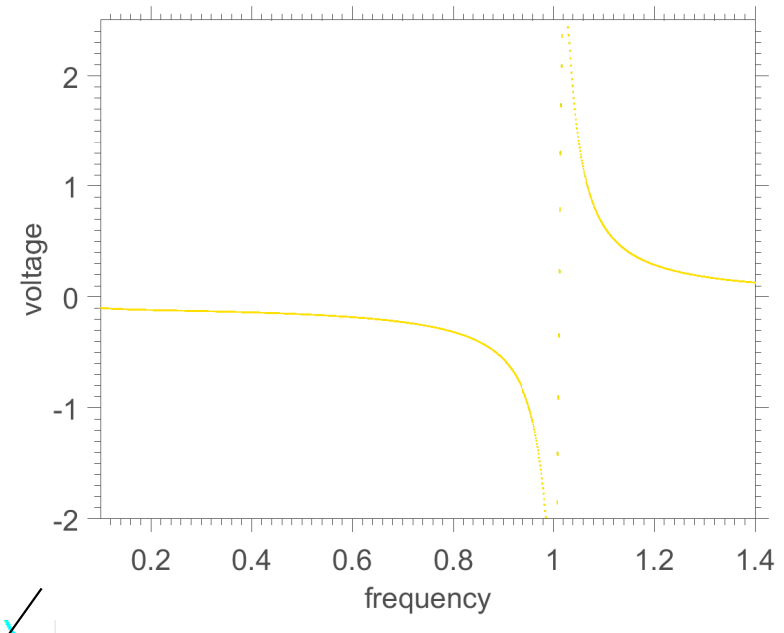


$f = 0.211$



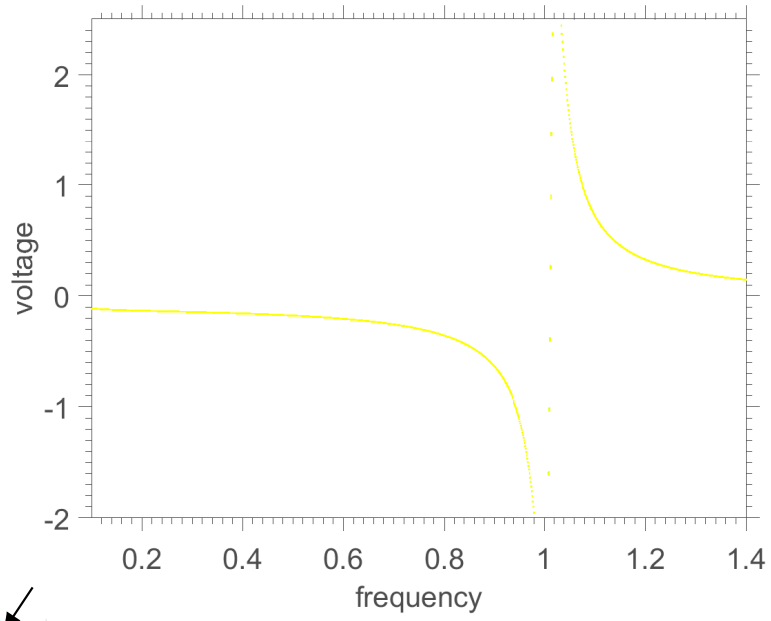
Bifurcation diagrams: voltage × frequency



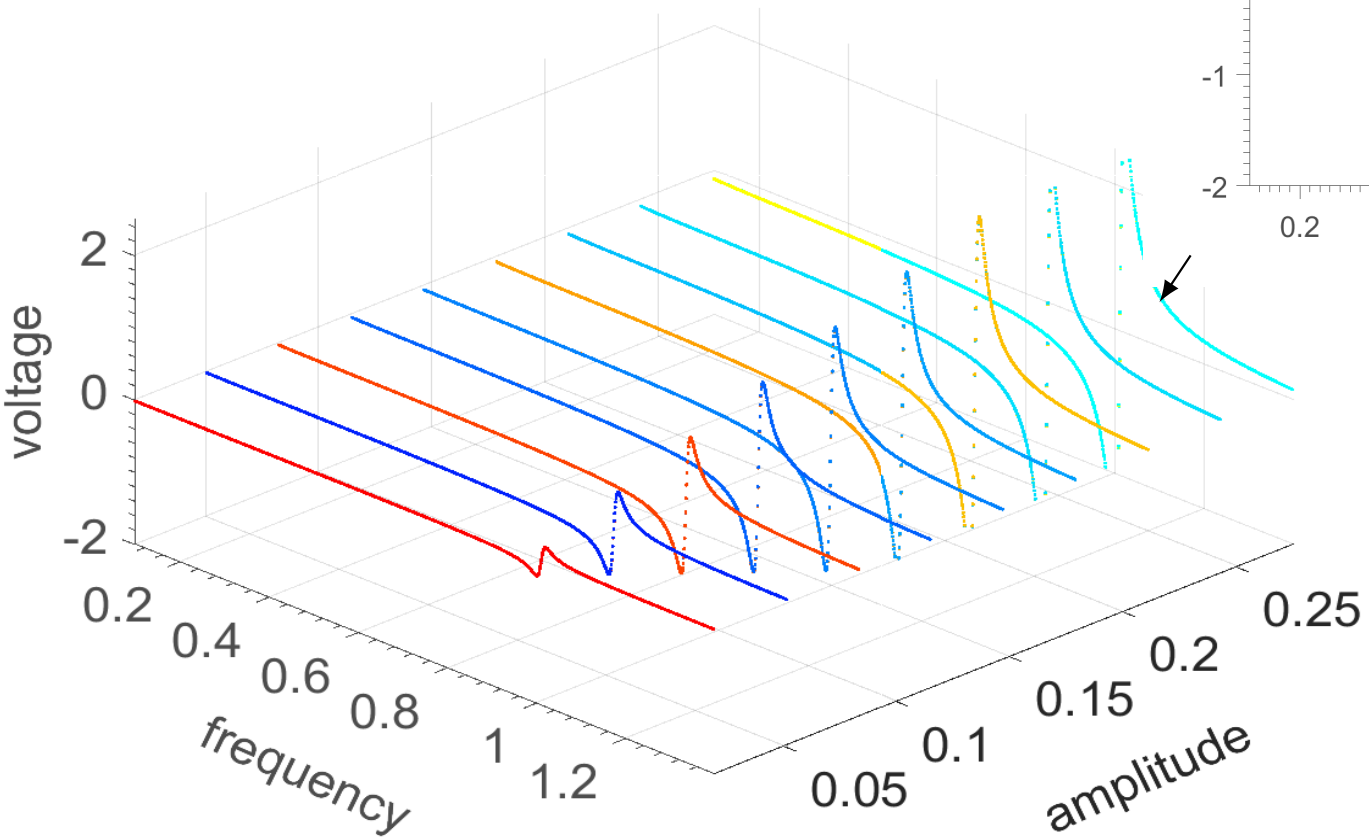


$f = 0.243$

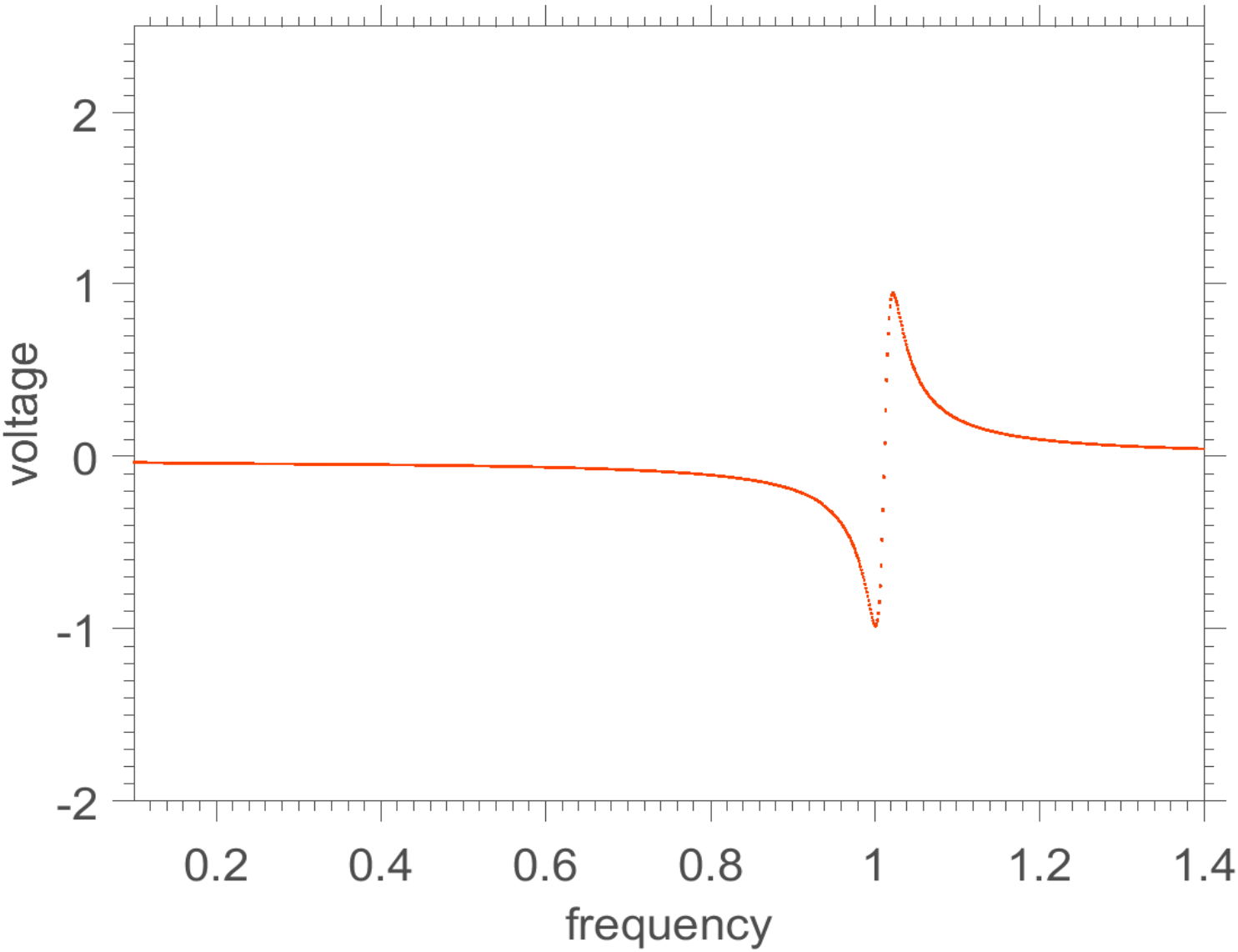
Bifurcation diagrams: voltage × frequency



$f = 0.275$



Forward and backward diagrams ($f = 0.083$)



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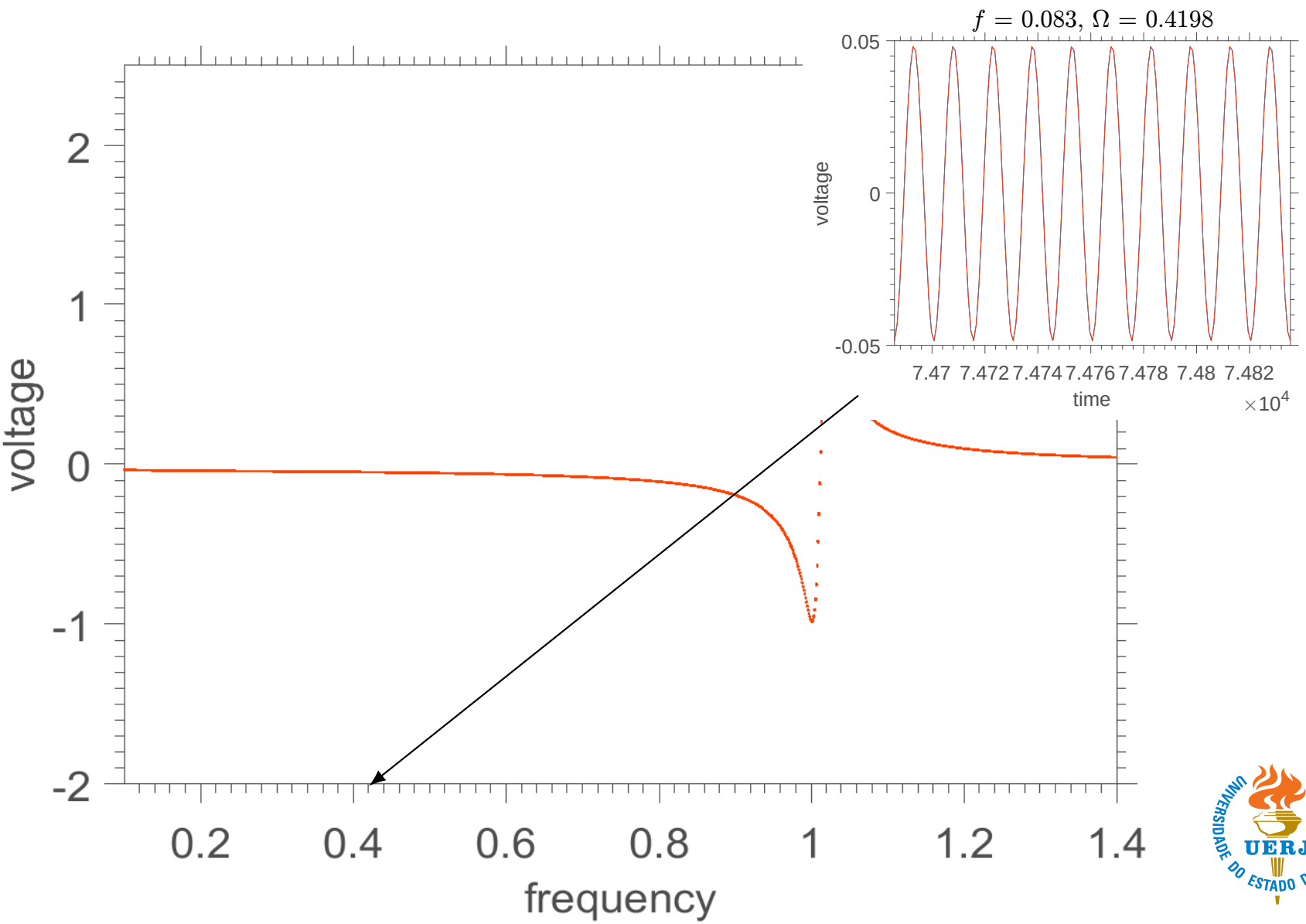
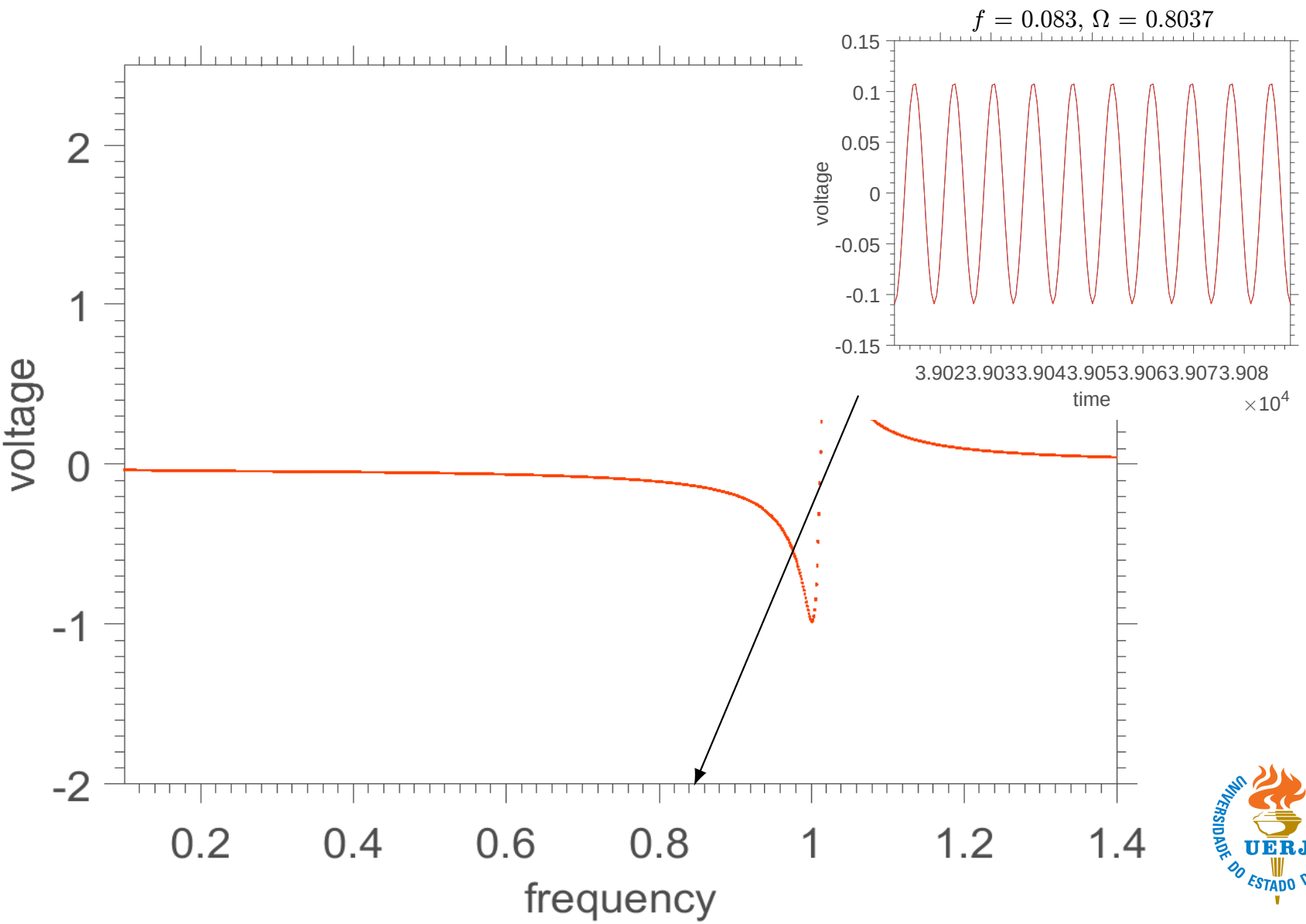
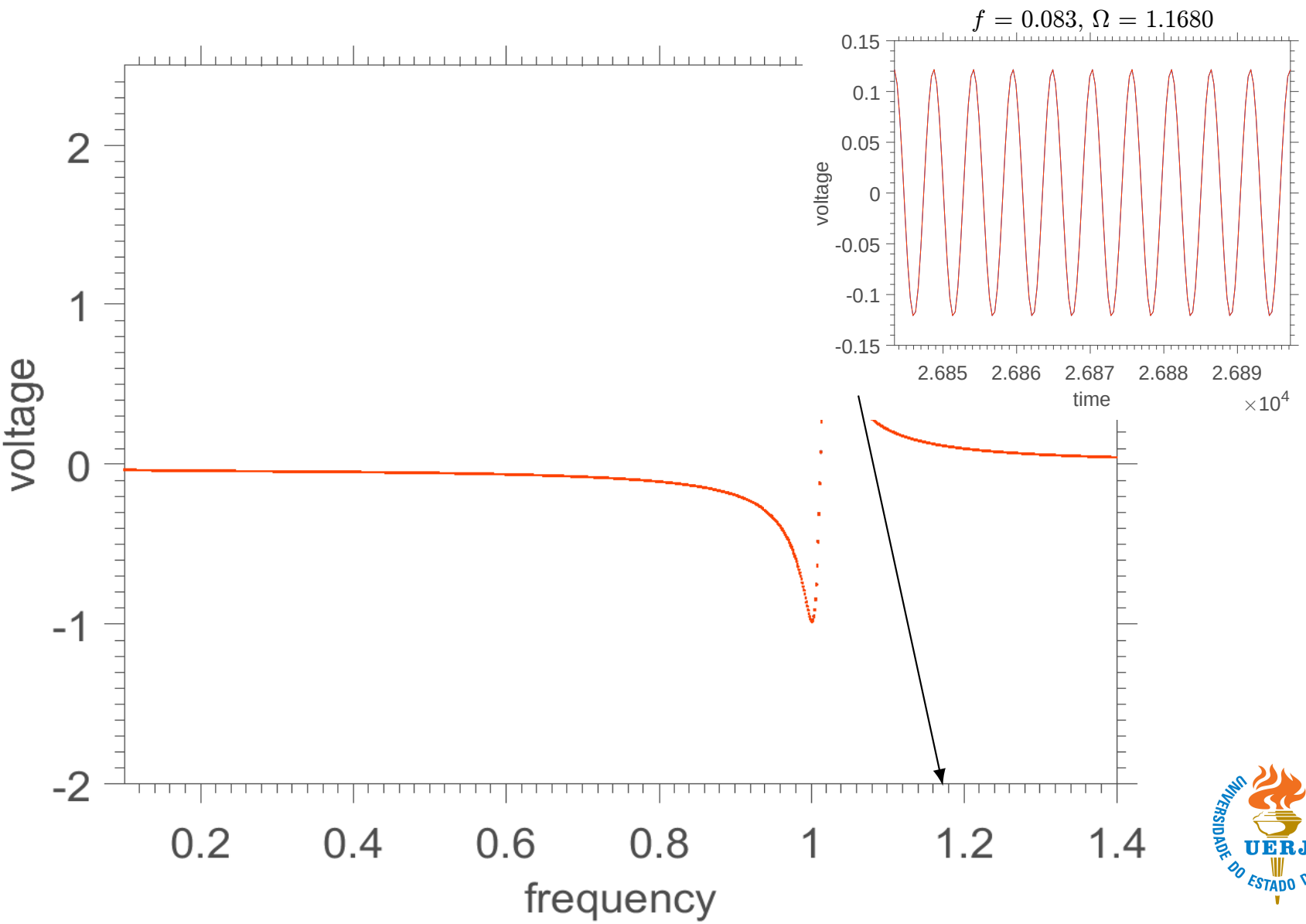


Forward and backward diagrams ($f = 0.083$)

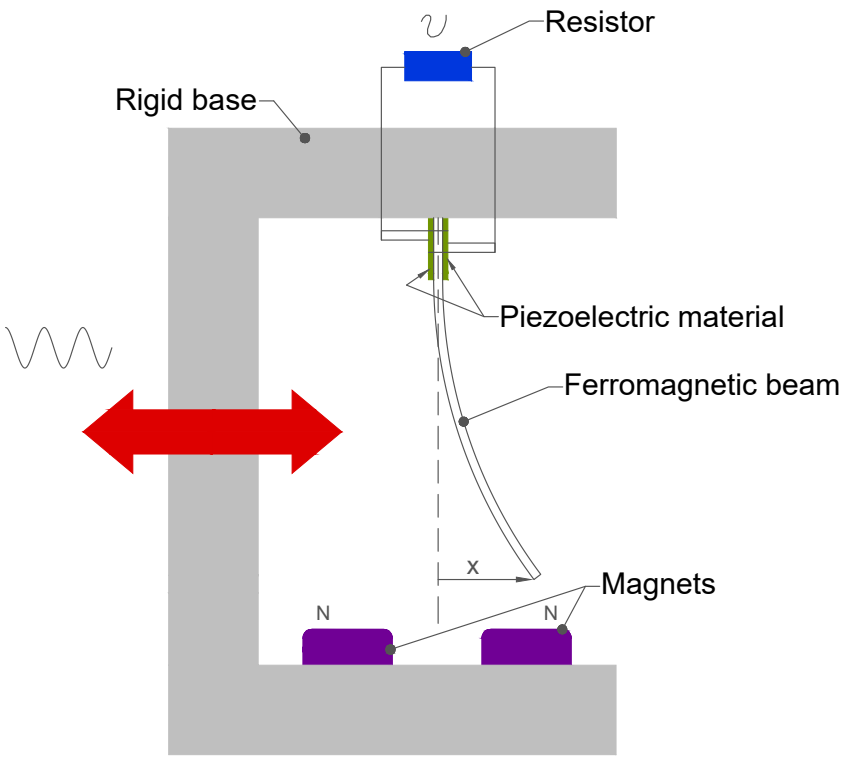


Forward and backward diagrams ($f = 0.083$)



Nonlinear Dynamics

Bistable harvester driven by regular signal



$$\ddot{x}+2\xi\dot{x}-\frac{1}{2}x(1-x^2)-\chi\upsilon = f\cos\Omega t$$

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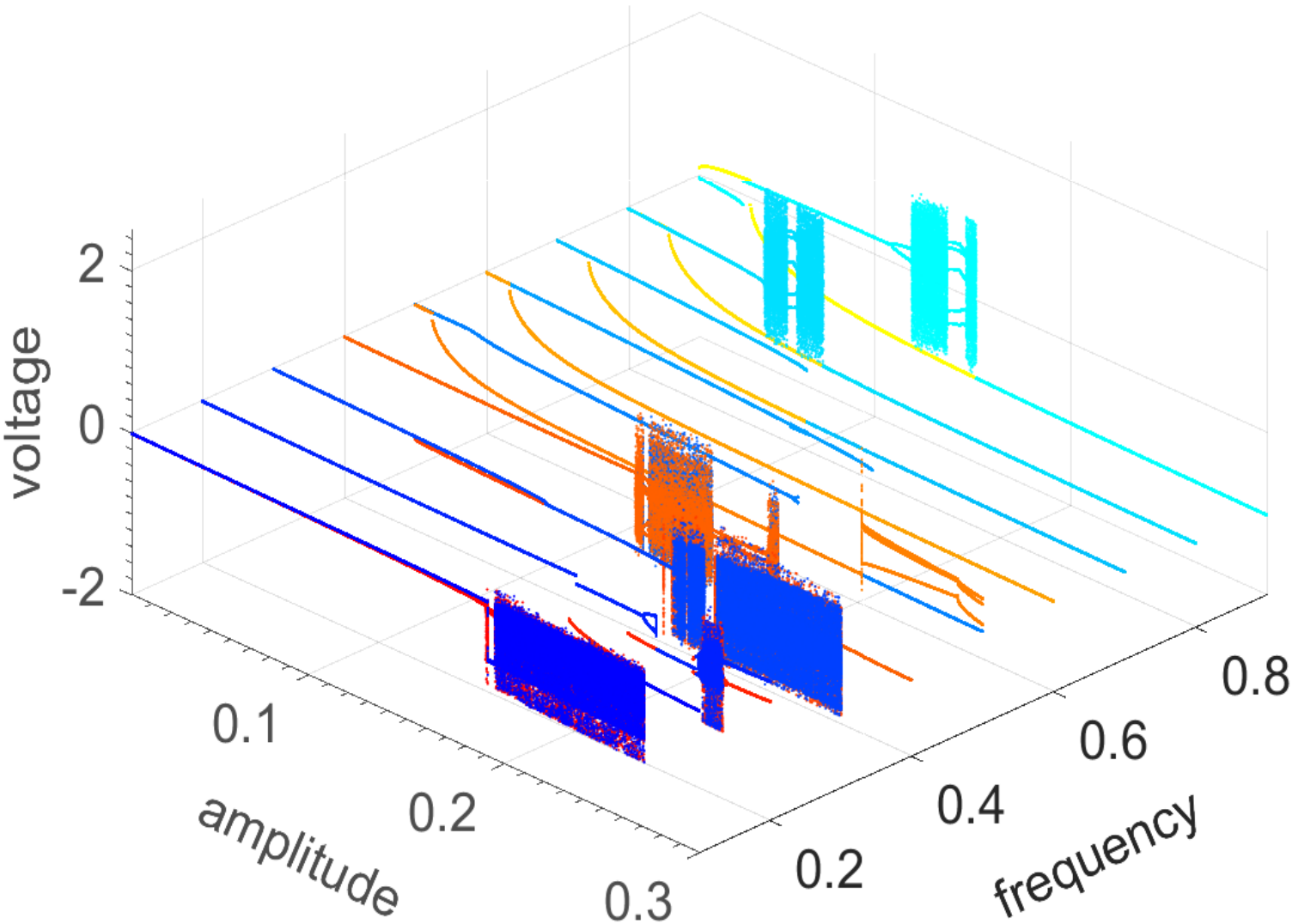
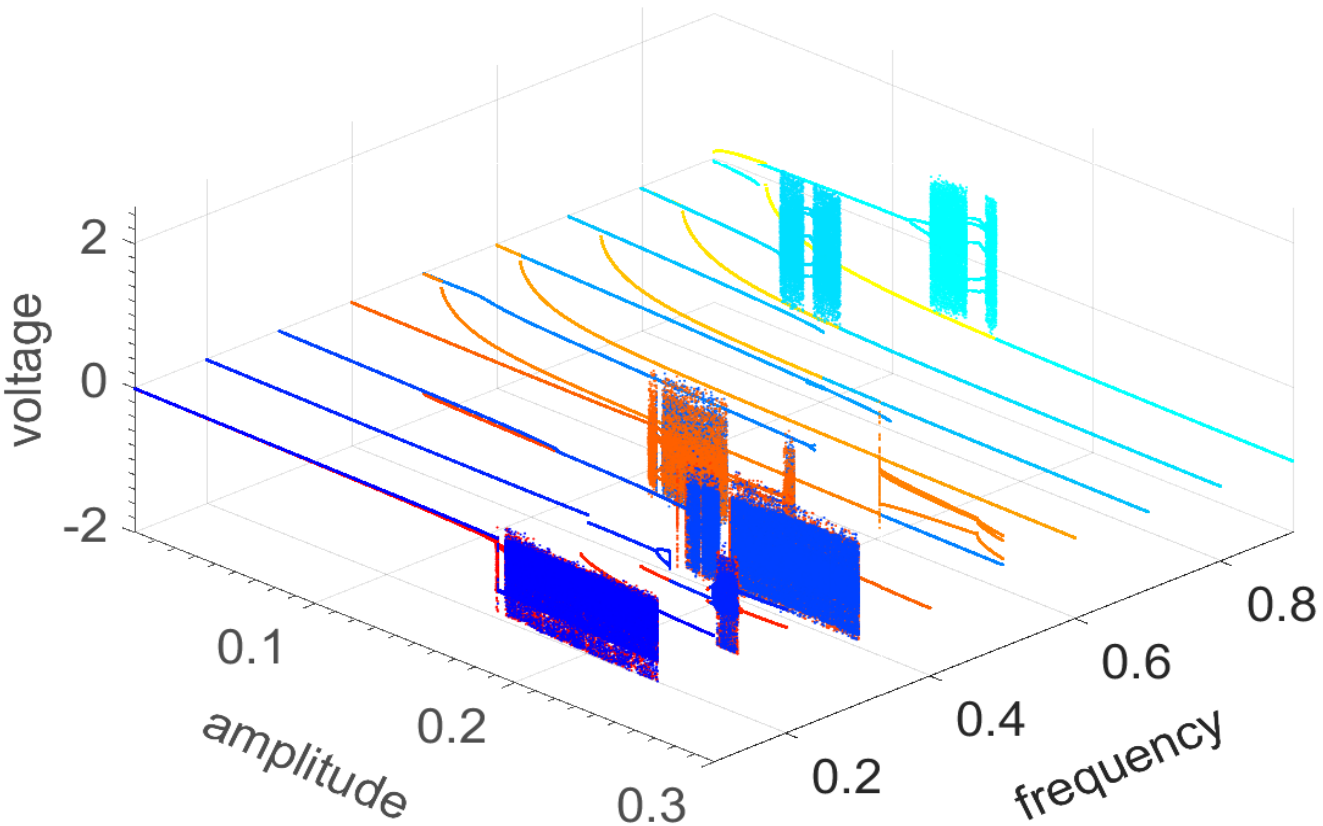
Nonlinear dynamics animation

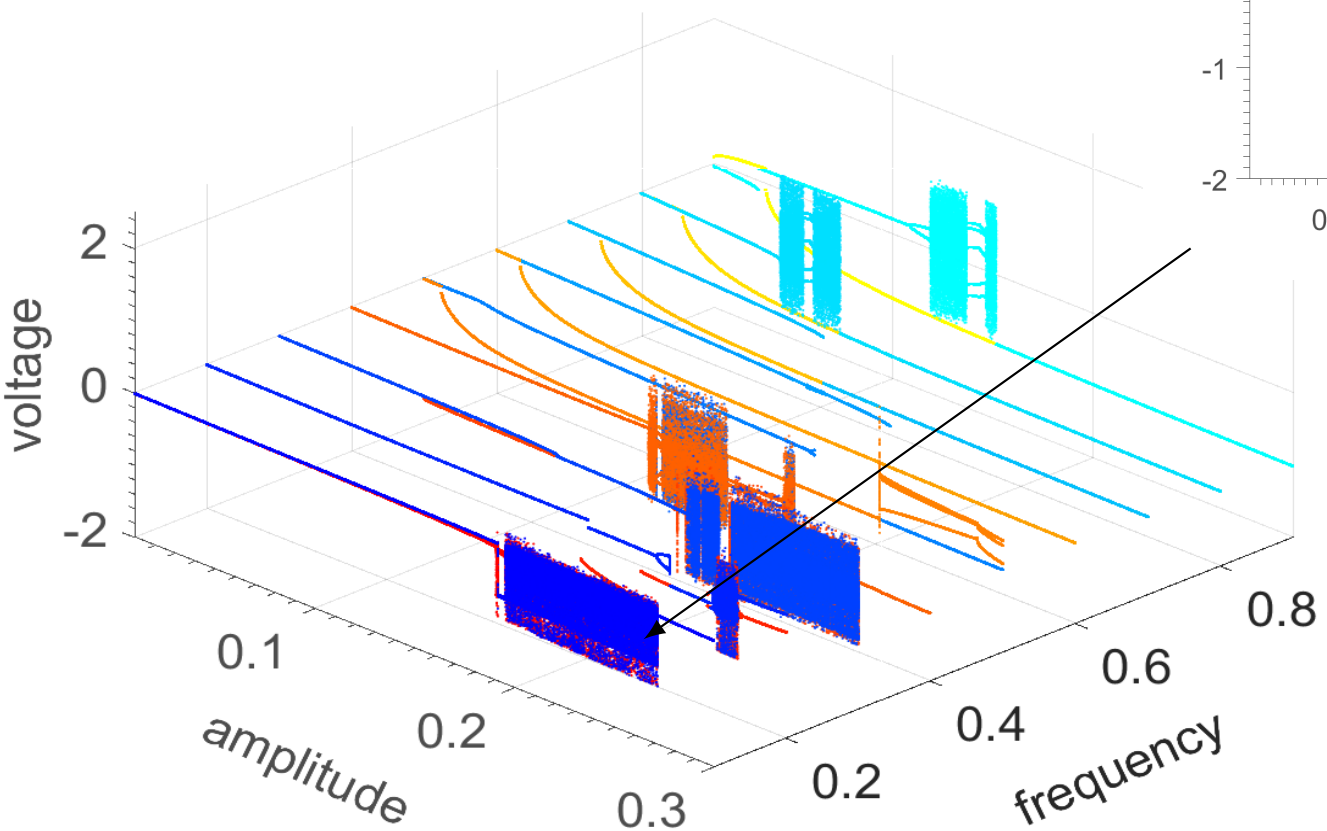
Global overview of force amplitude effect

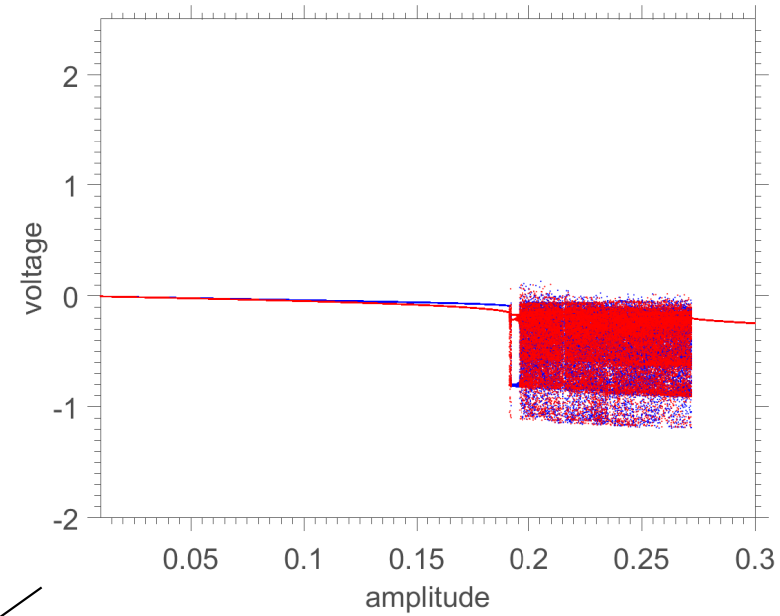


Bifurcation diagrams: voltage $\times$ amplitude



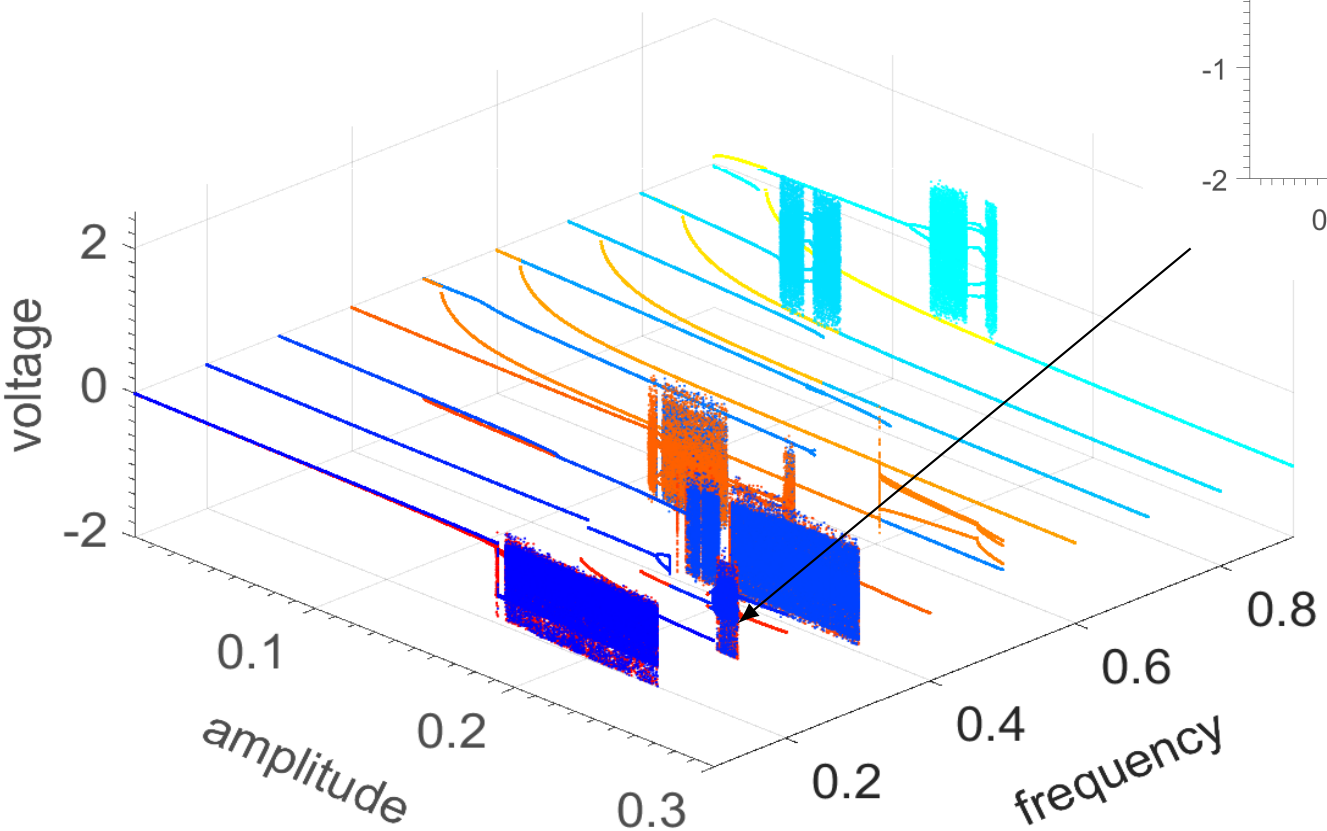
Bifurcation diagrams: voltage × amplitude

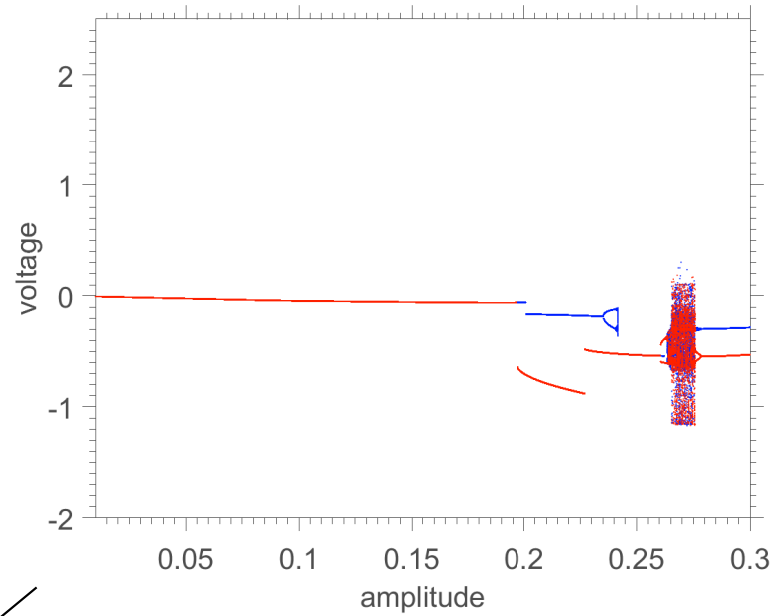




$\Omega = 0.1$

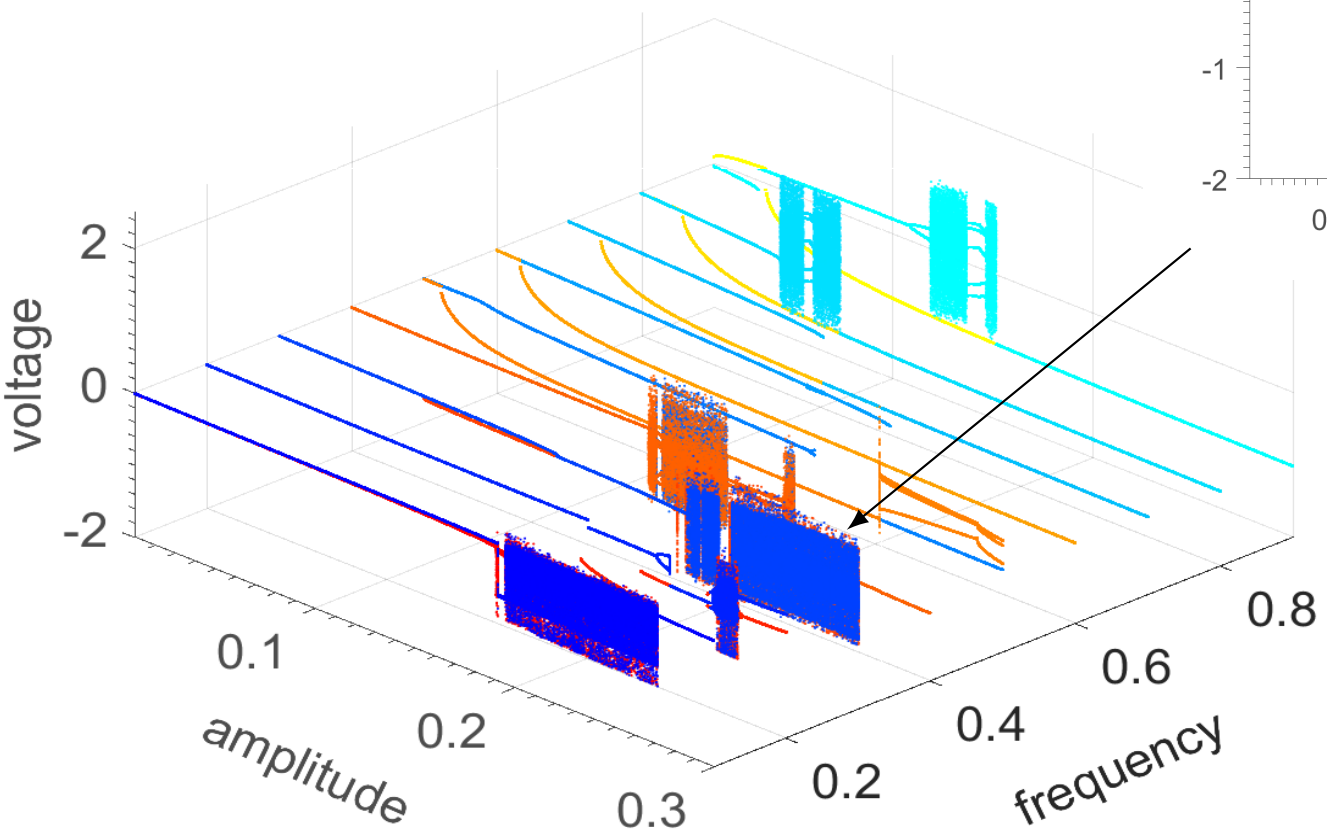
Bifurcation diagrams: voltage × amplitude

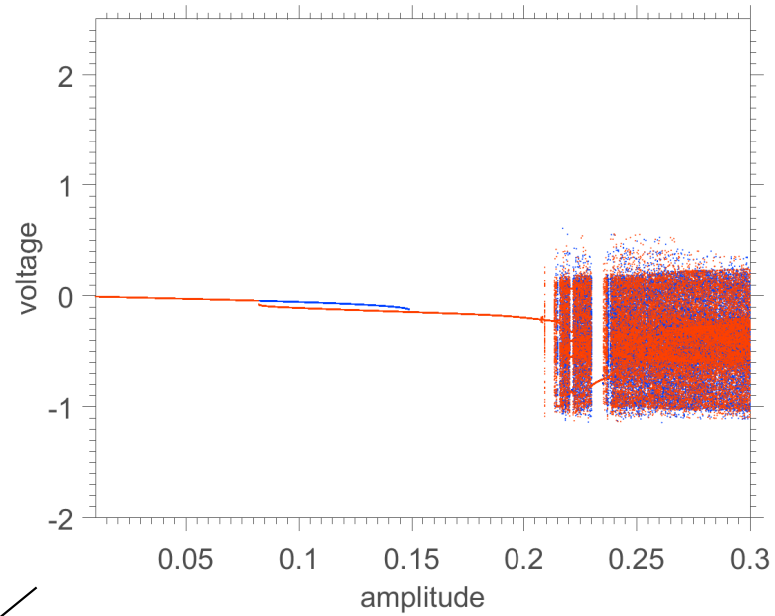




$\Omega = 0.2$

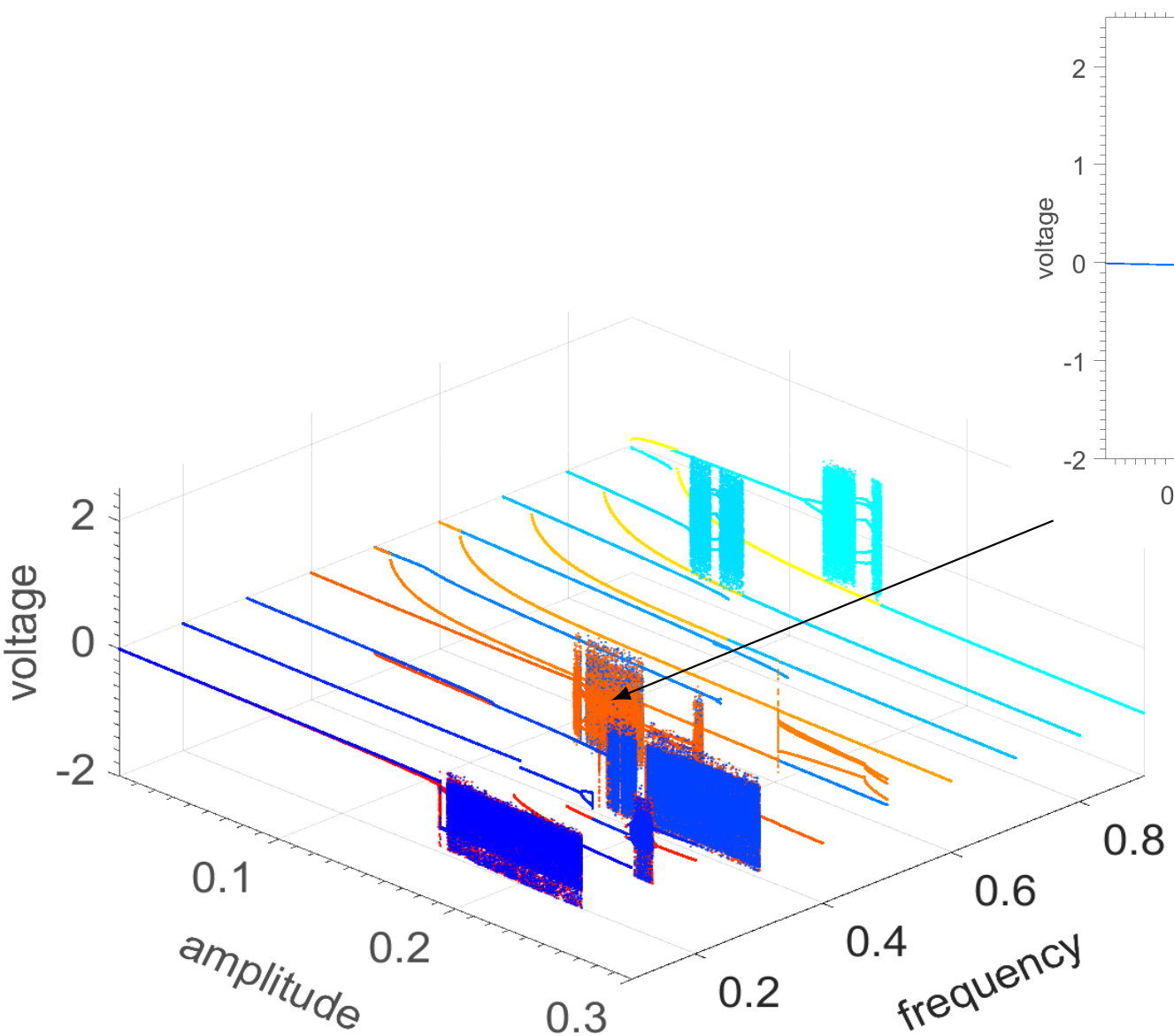
Bifurcation diagrams: voltage × amplitude

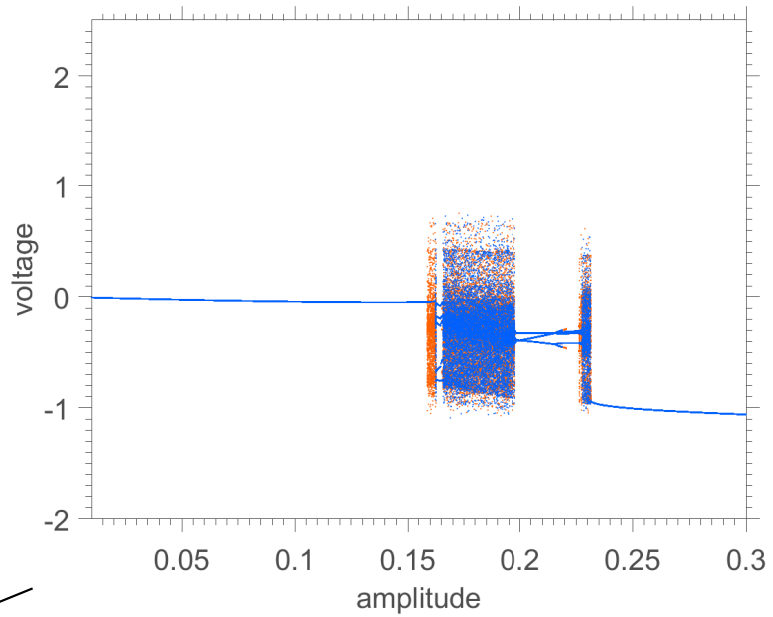




$\Omega = 0.3$

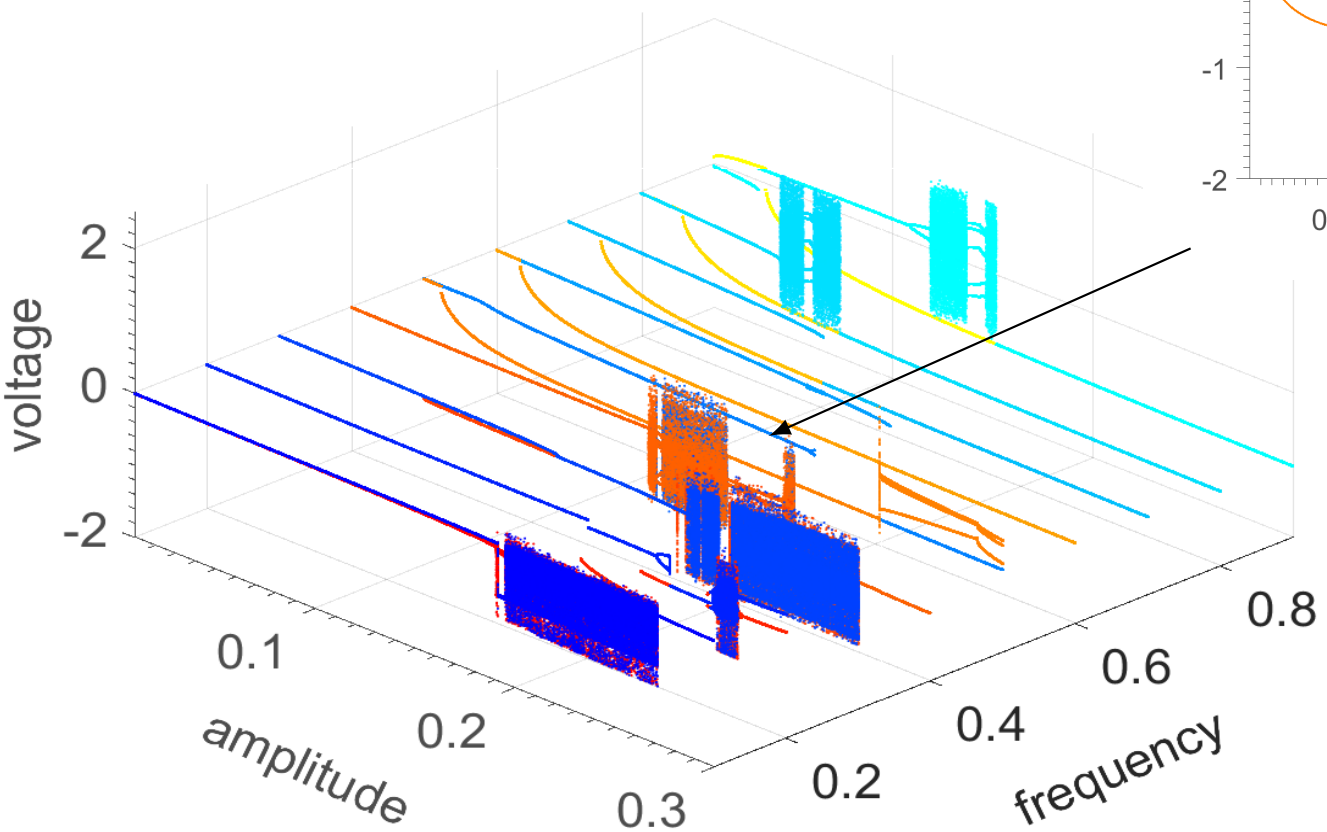
Bifurcation diagrams: voltage × amplitude

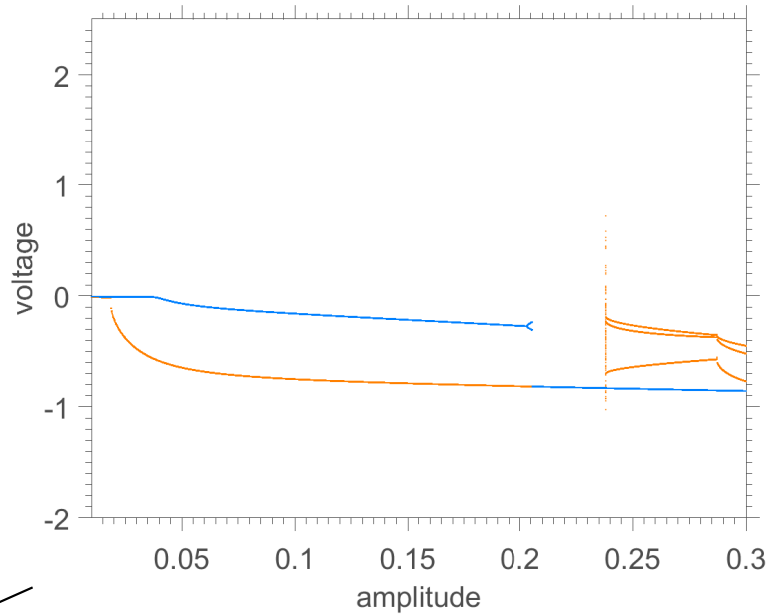




$\Omega = 0.4$

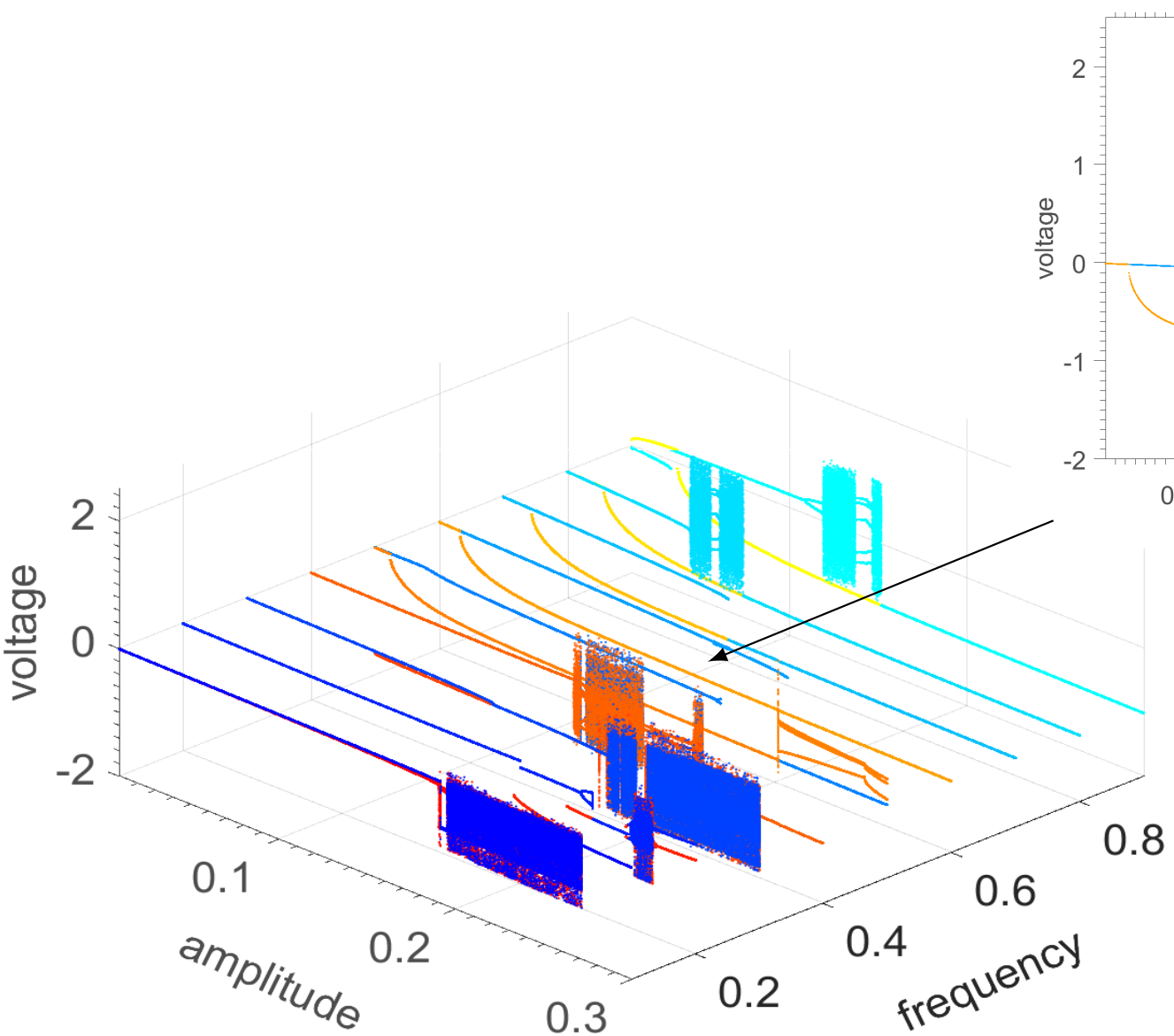
Bifurcation diagrams: voltage × amplitude

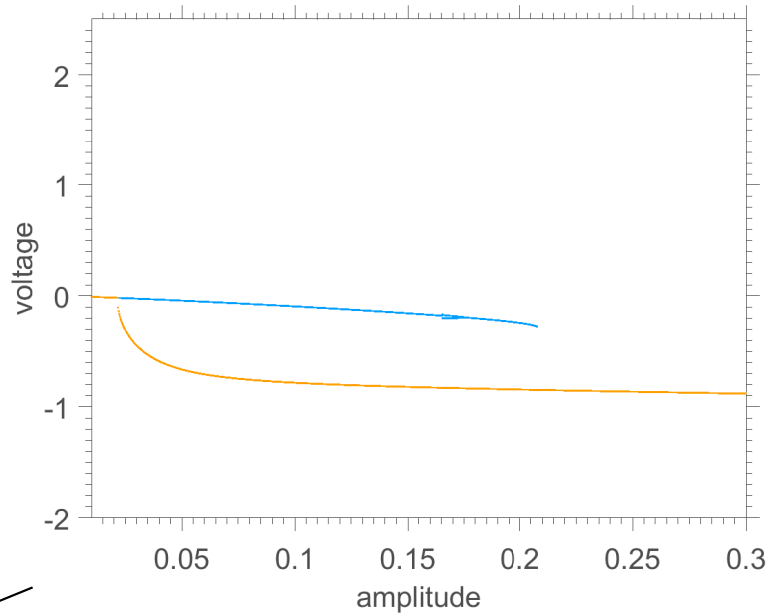




$\Omega = 0.5$

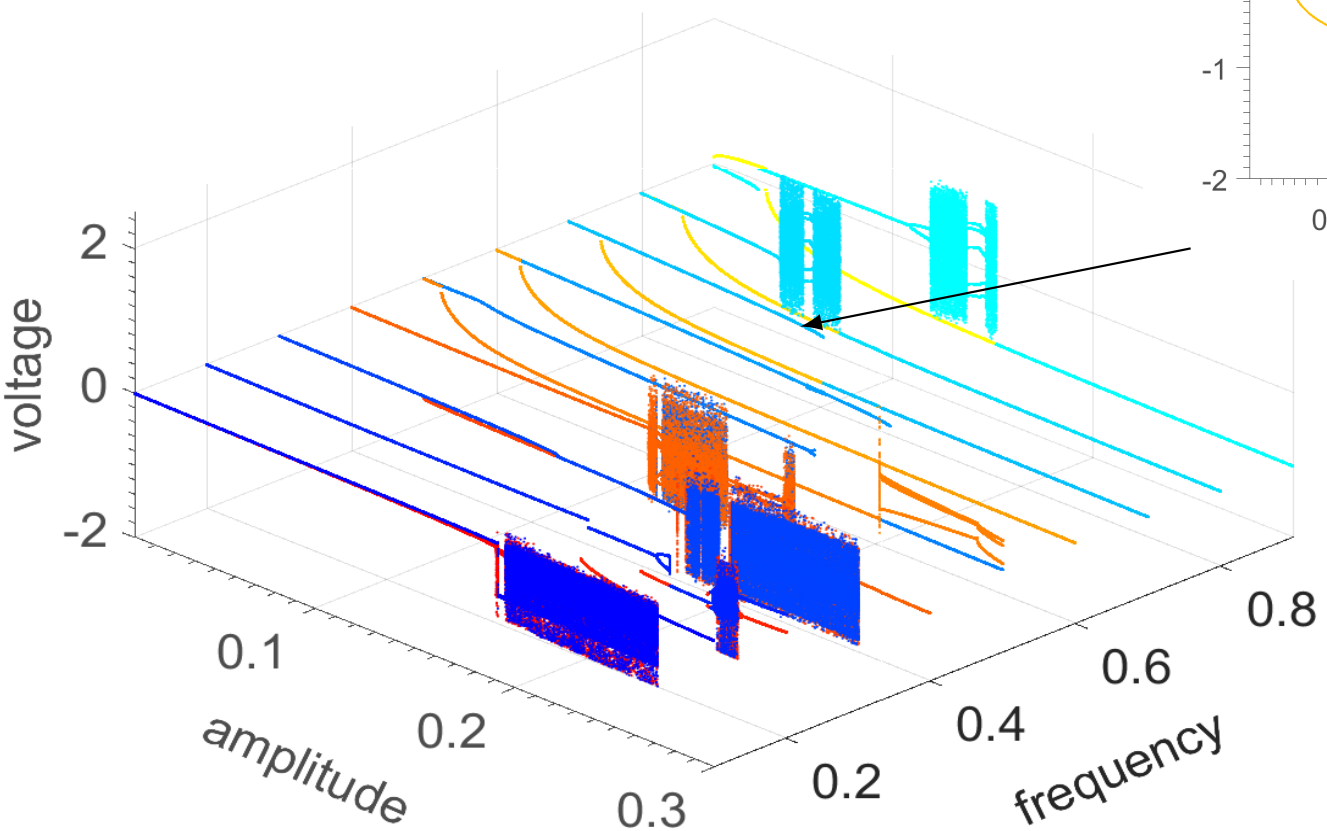
Bifurcation diagrams: voltage × amplitude

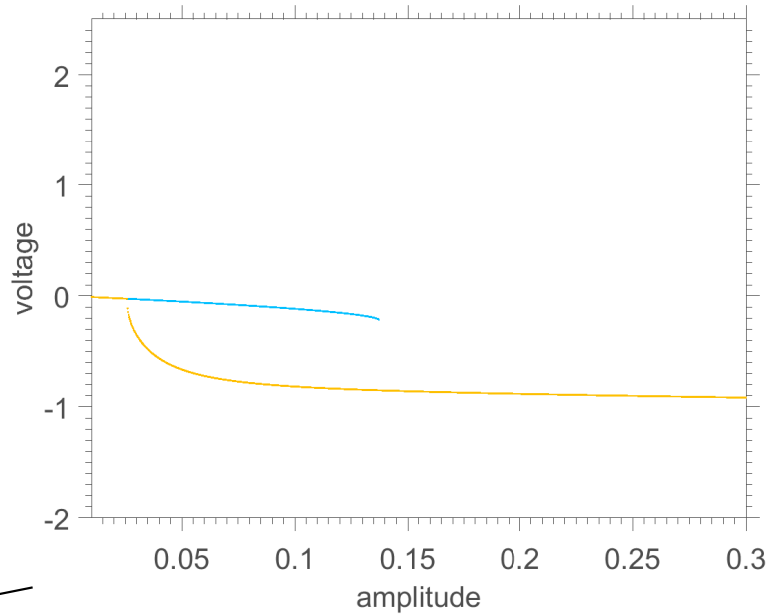




$\Omega = 0.6$

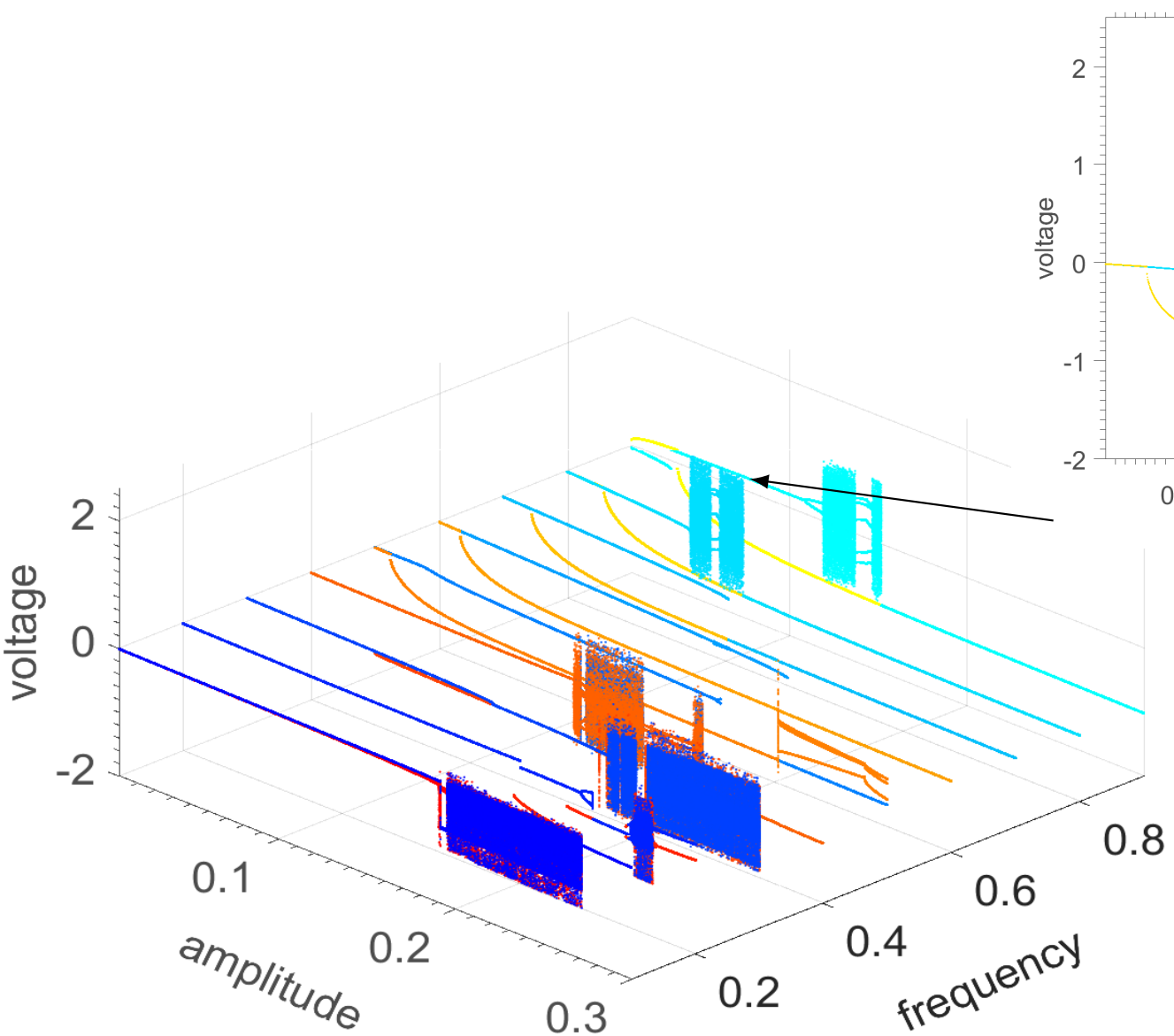
Bifurcation diagrams: voltage × amplitude

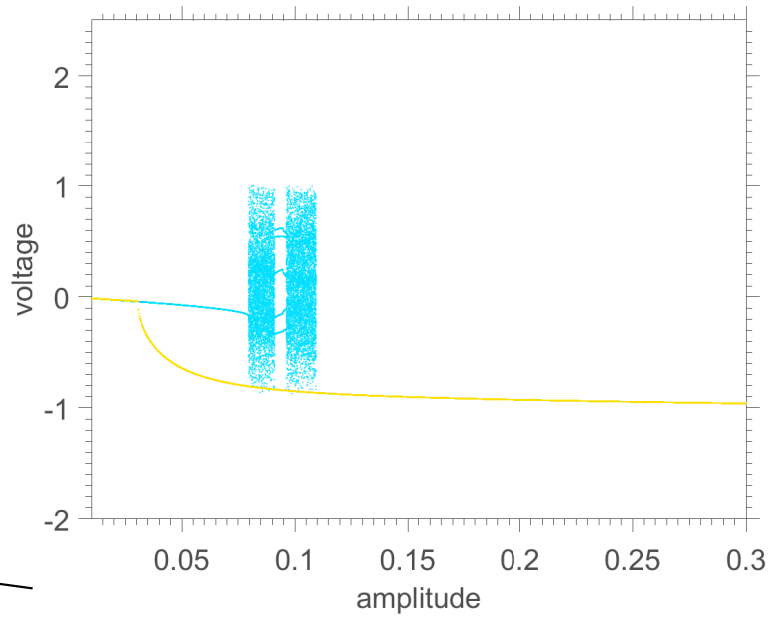




$\Omega = 0.7$

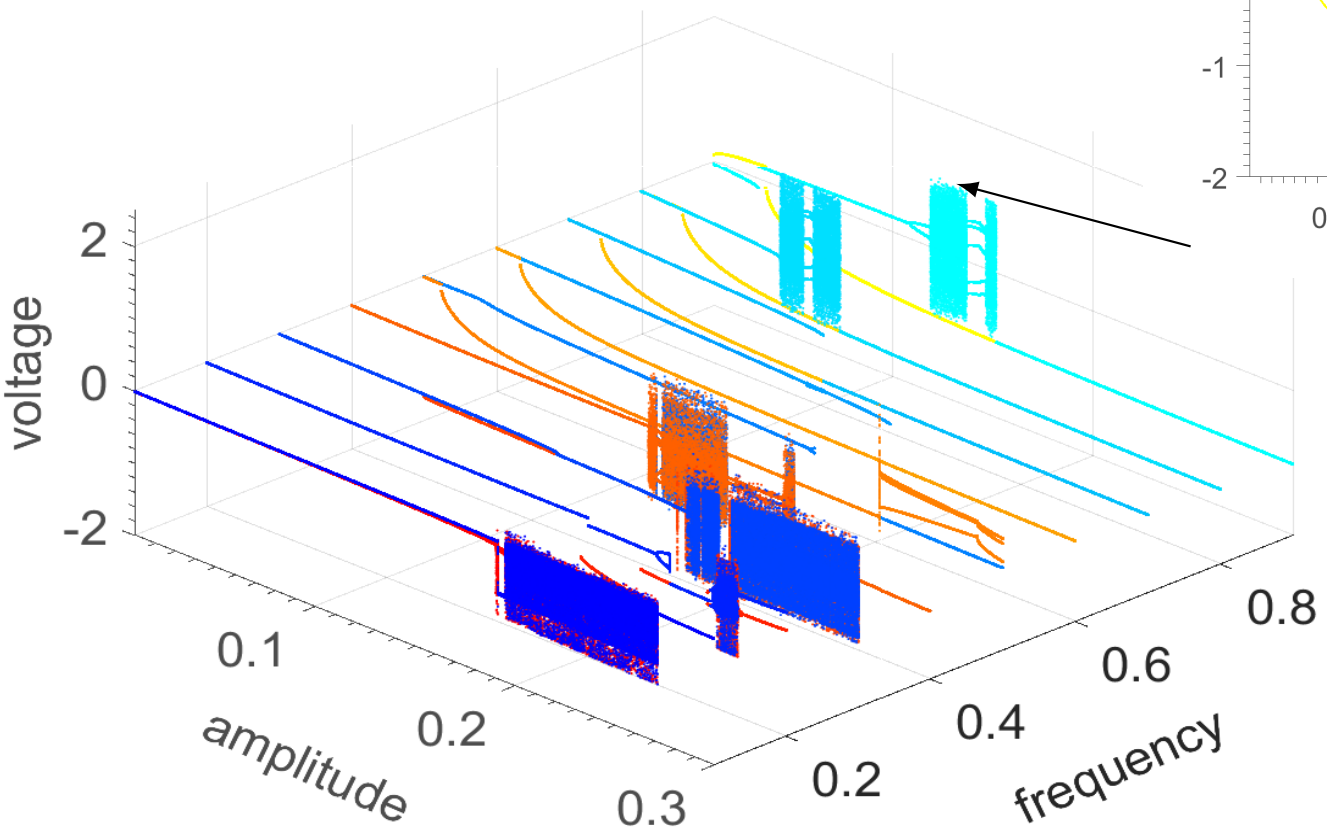
Bifurcation diagrams: voltage × amplitude

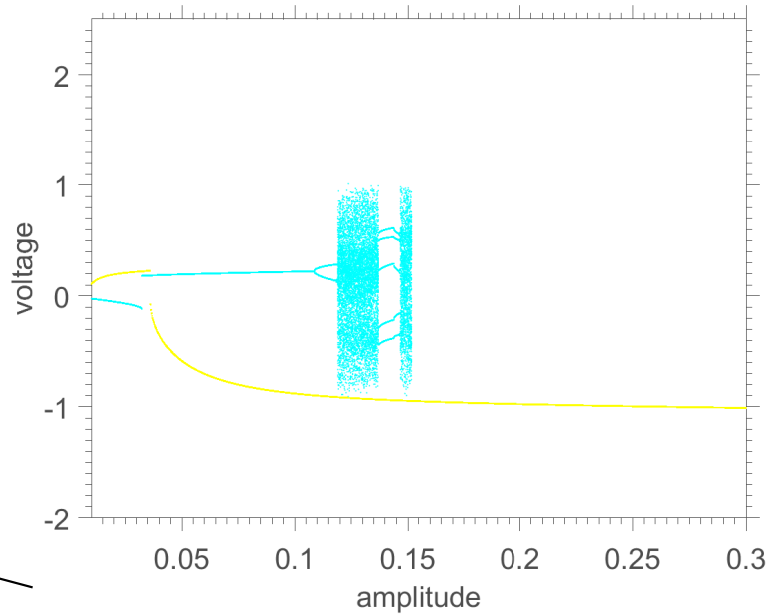




$\Omega = 0.8$

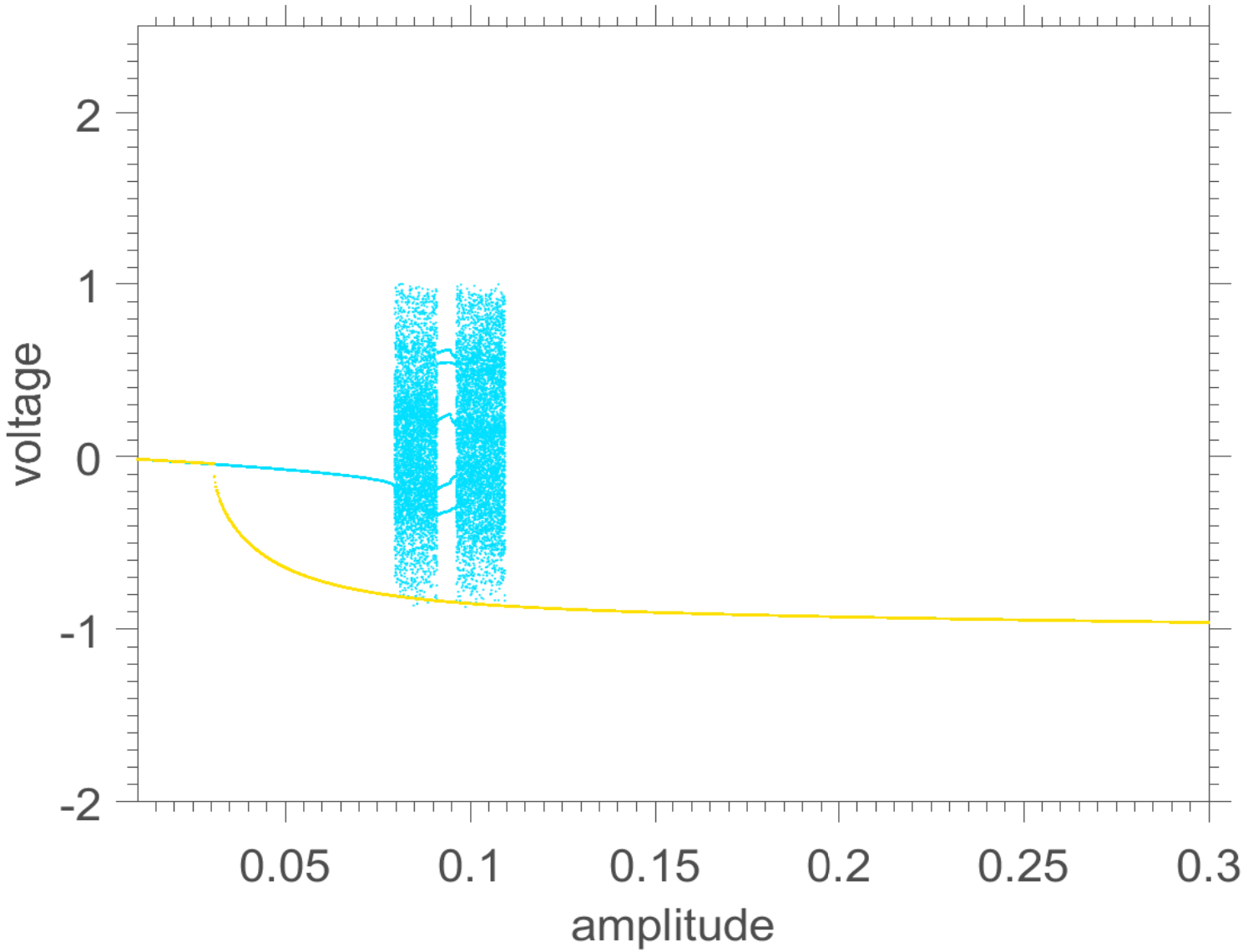
Bifurcation diagrams: voltage × amplitude



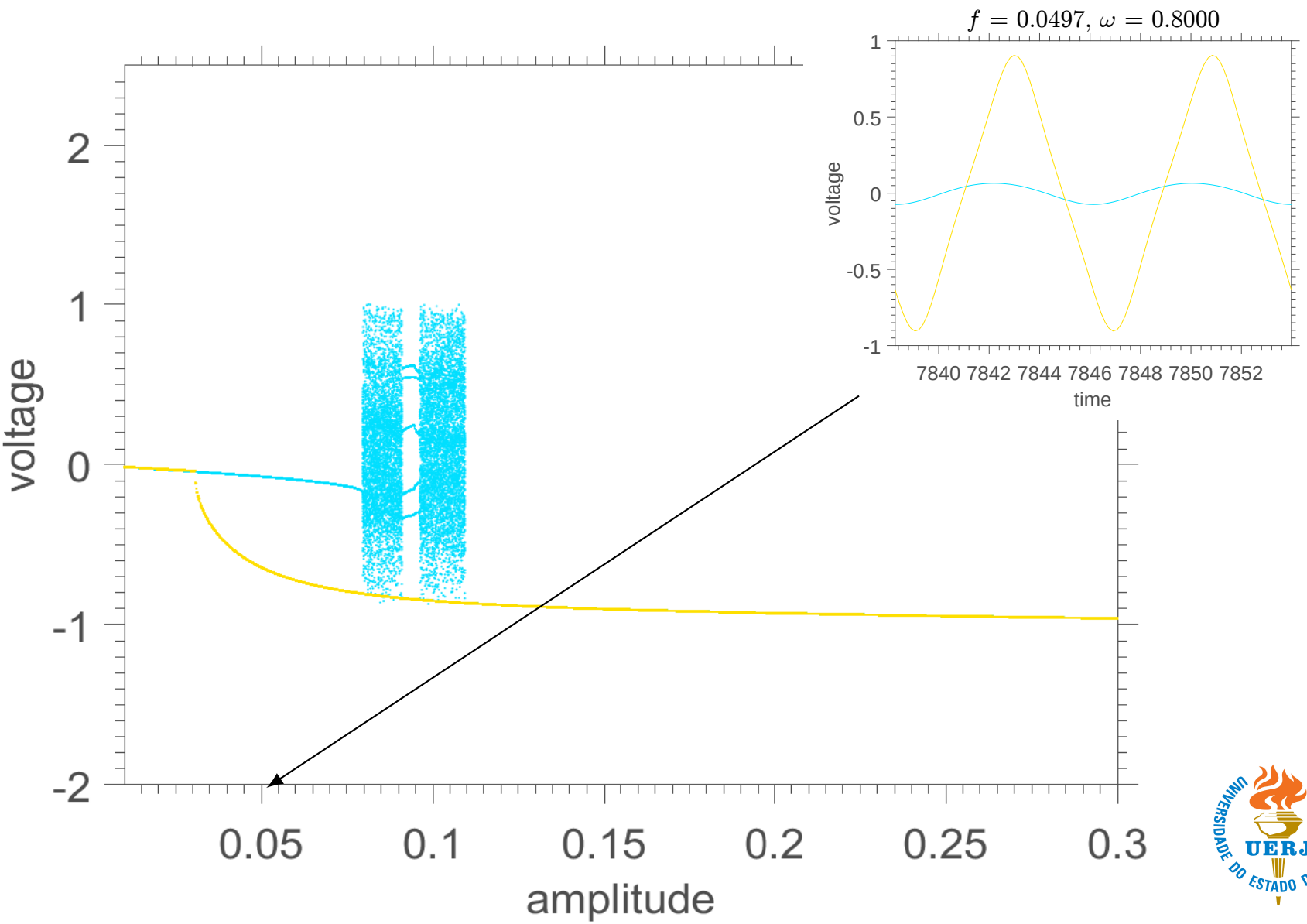


$\Omega = 0.9$

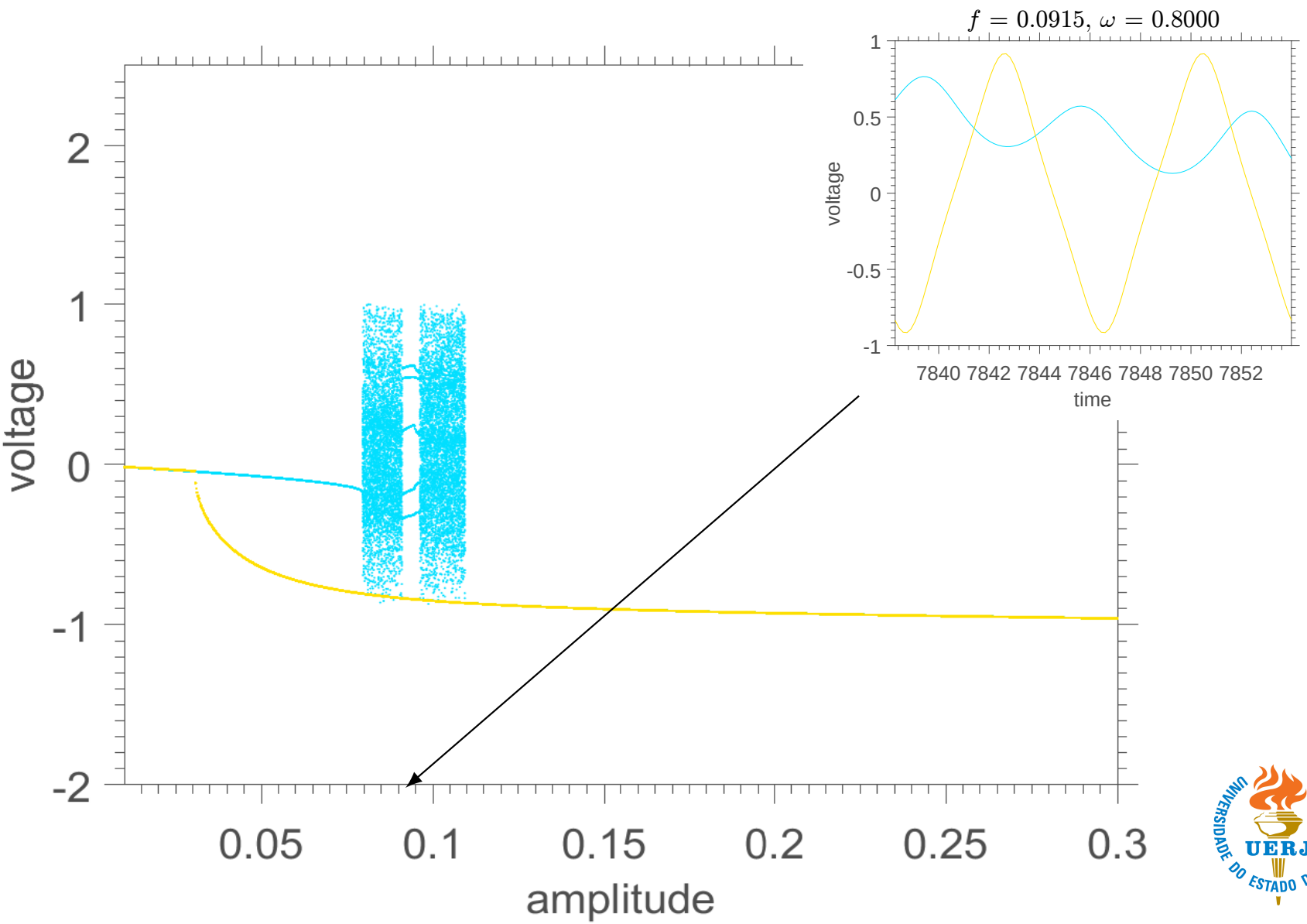
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



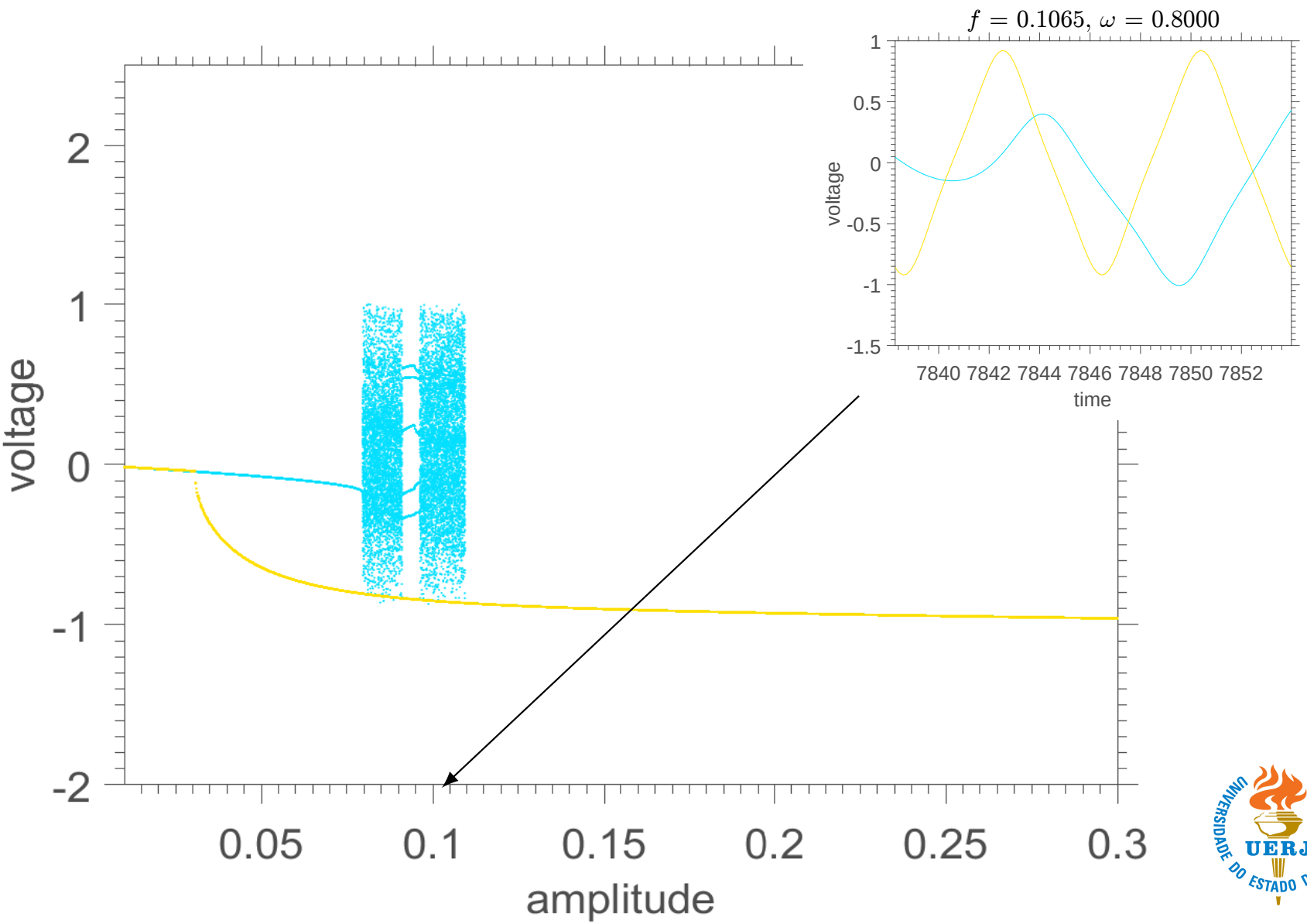
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



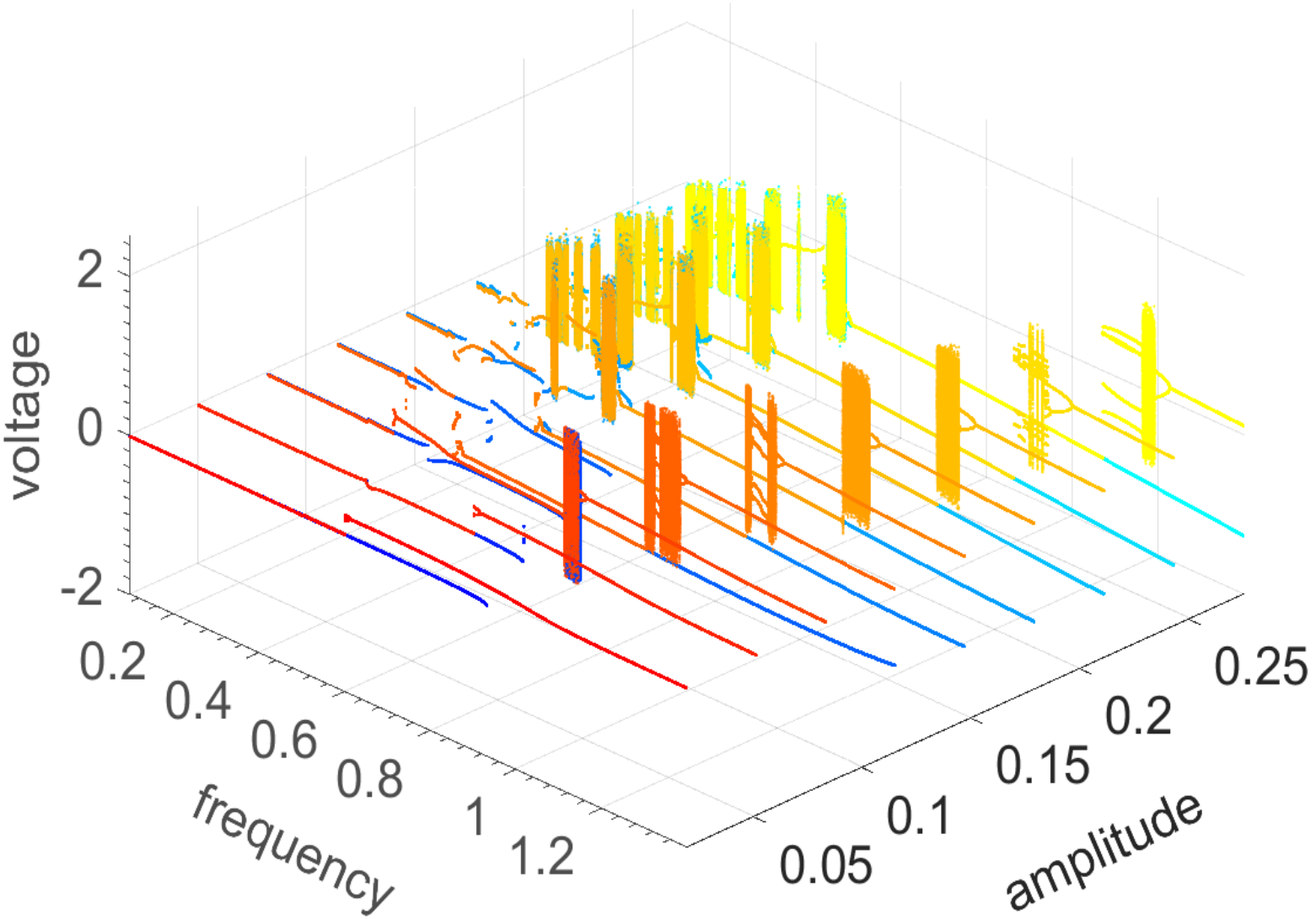
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



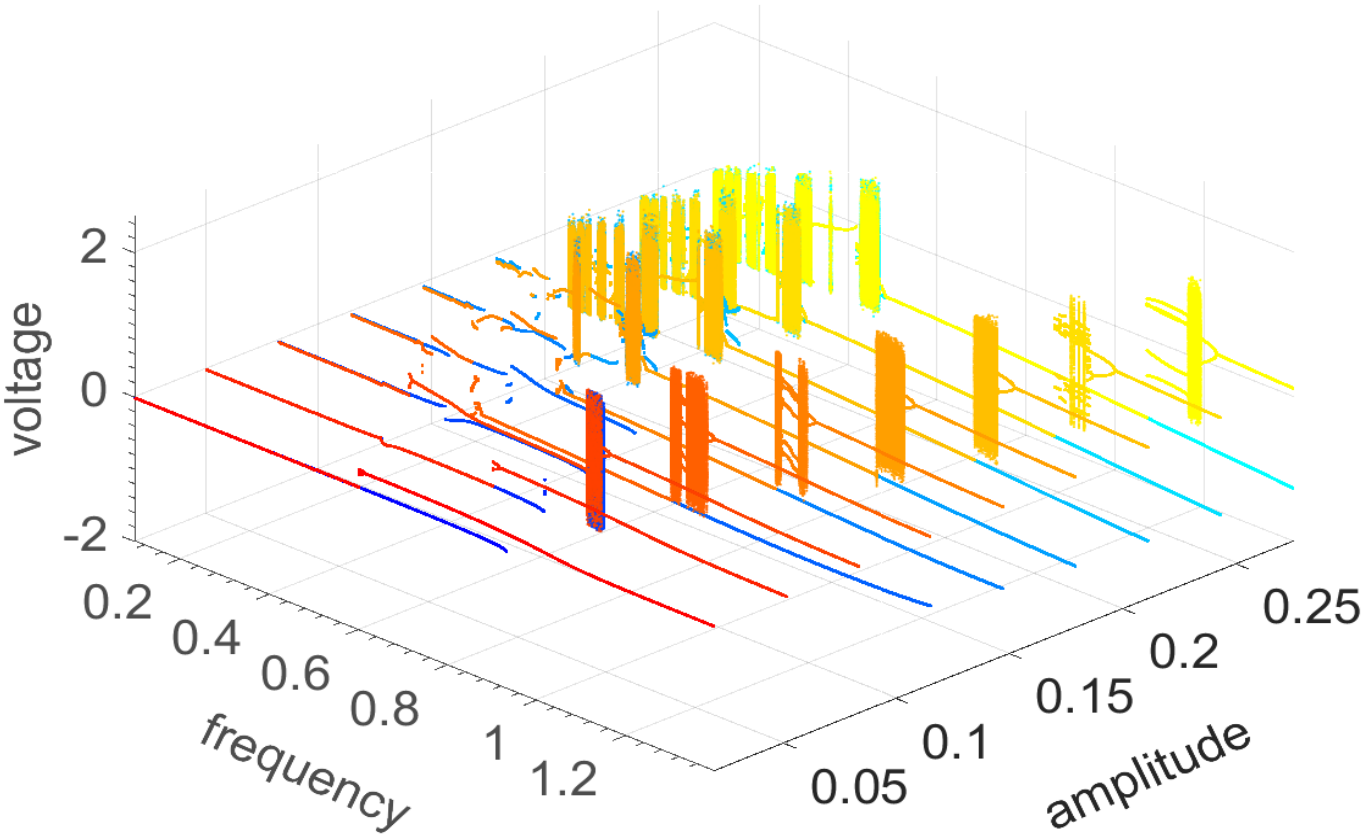
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



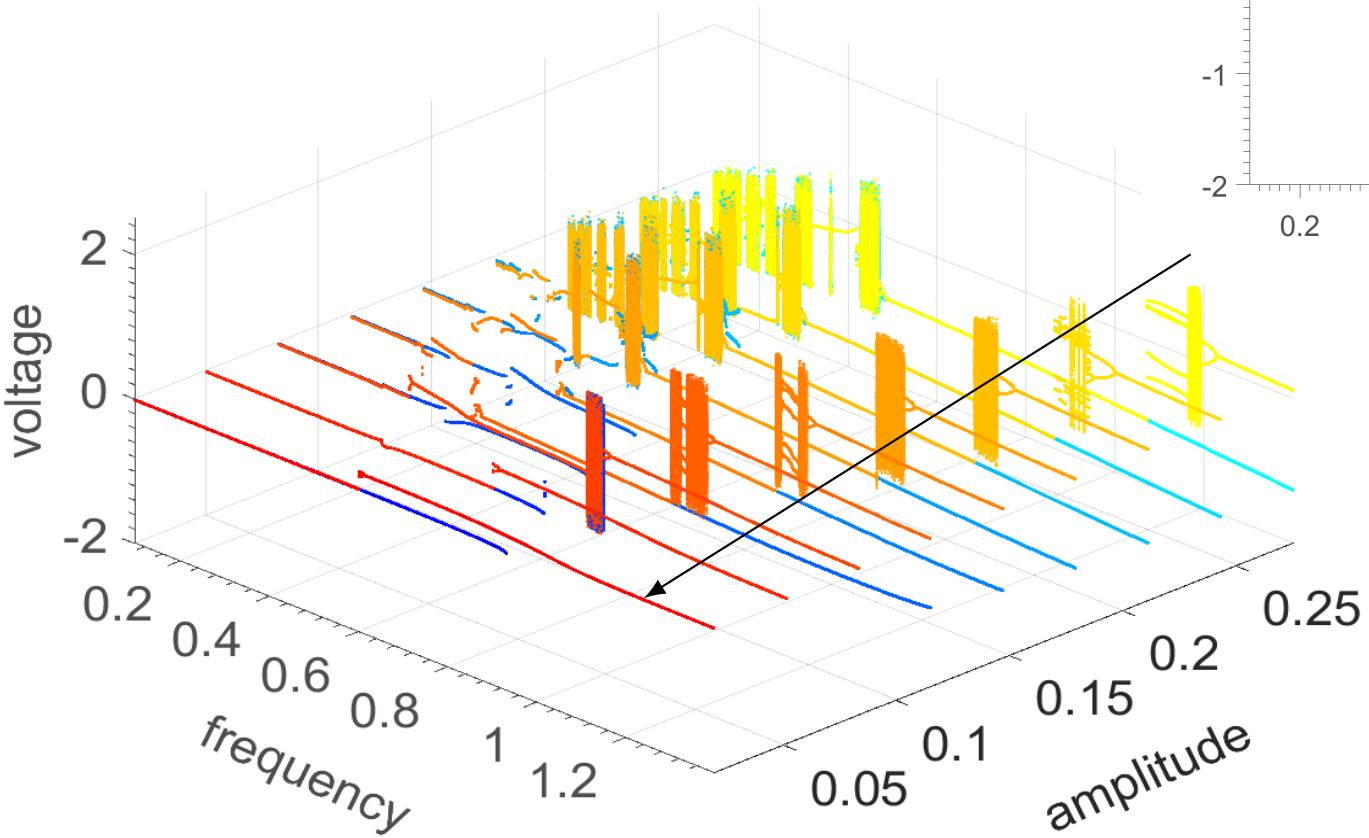
Global overview of force frequency effect

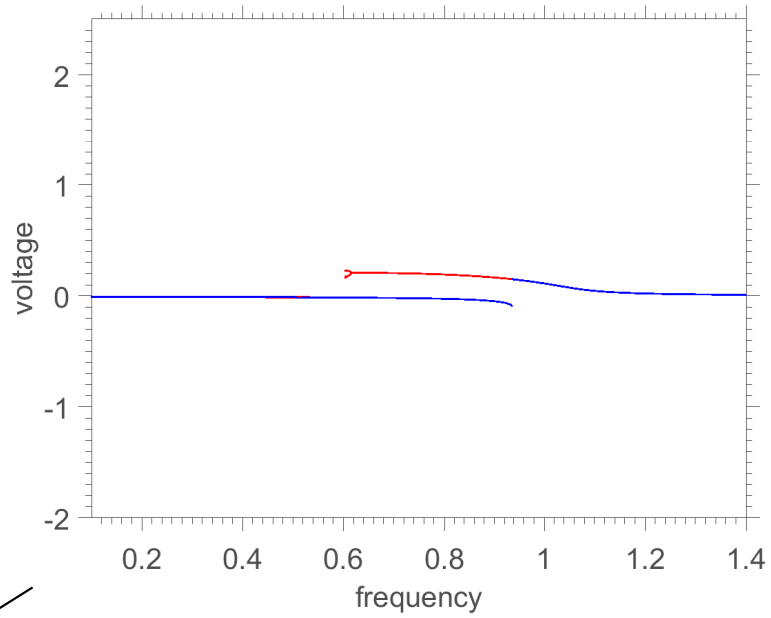


Bifurcation diagrams: voltage $\times$ frequency



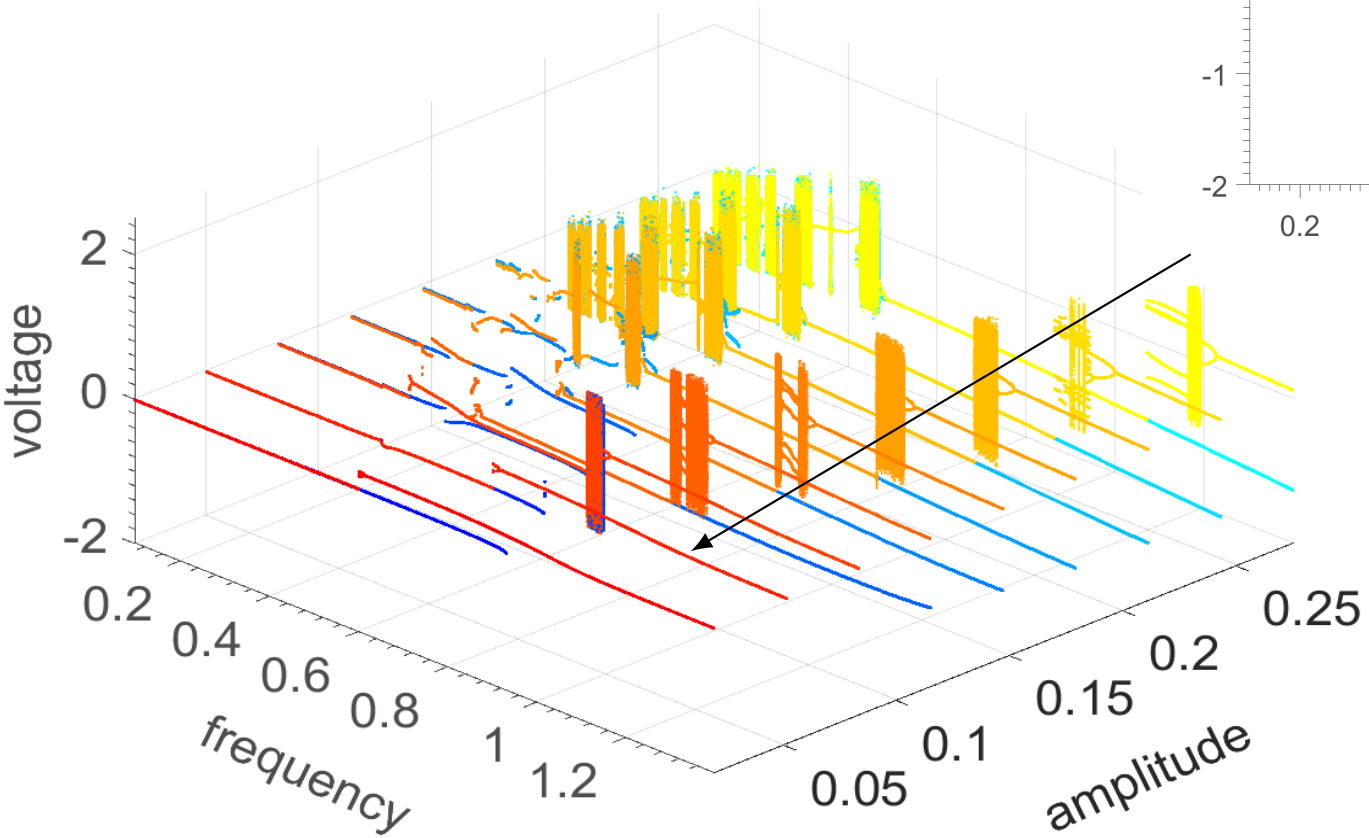
Bifurcation diagrams: voltage × frequency

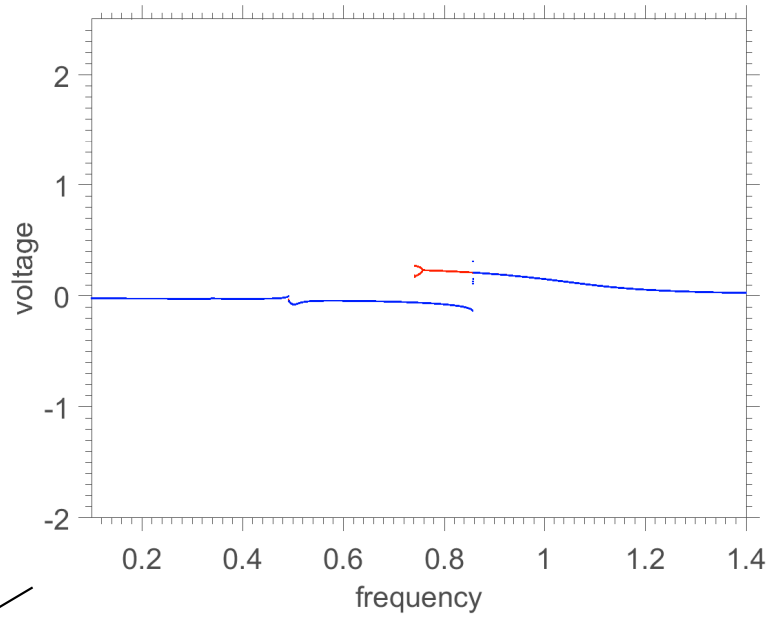




$f = 0.019$

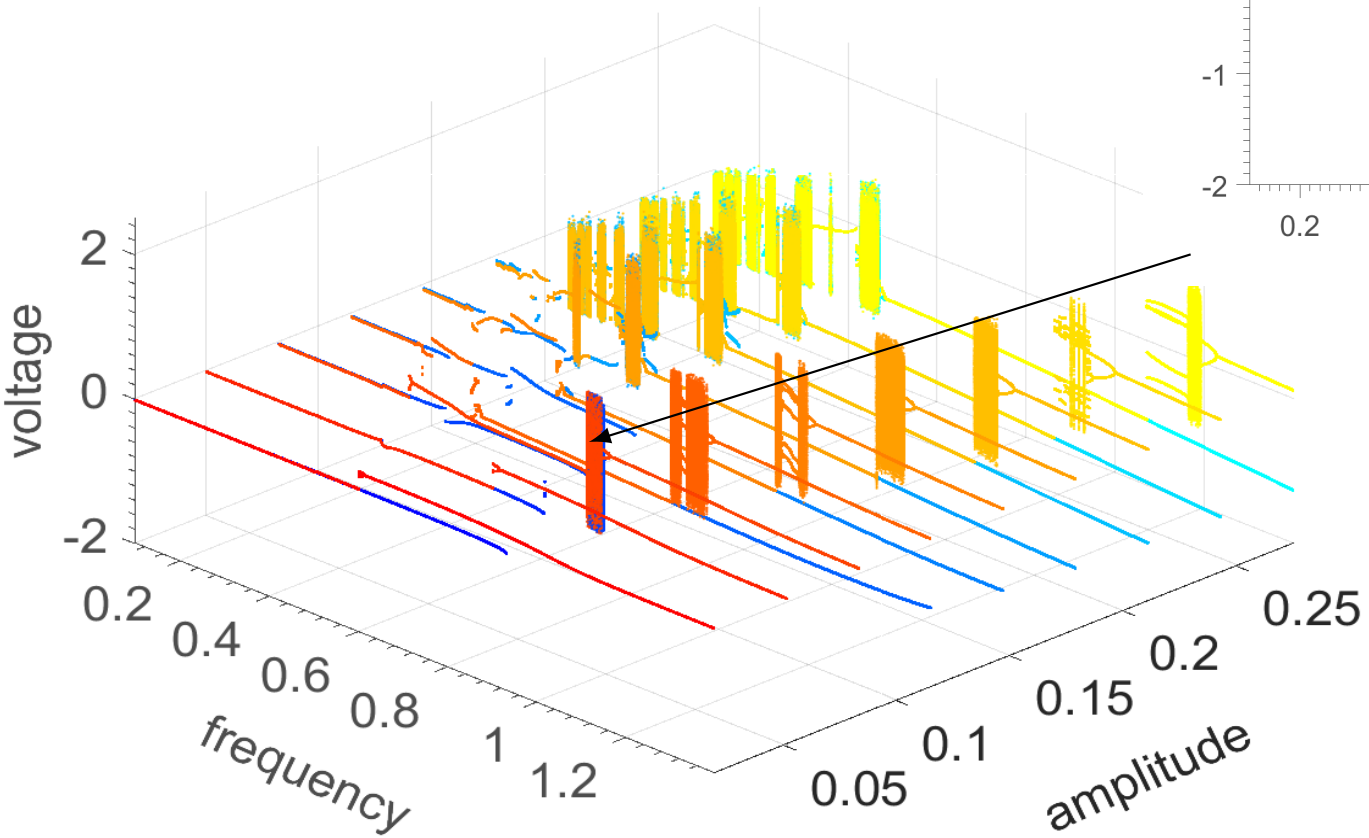
Bifurcation diagrams: voltage × frequency

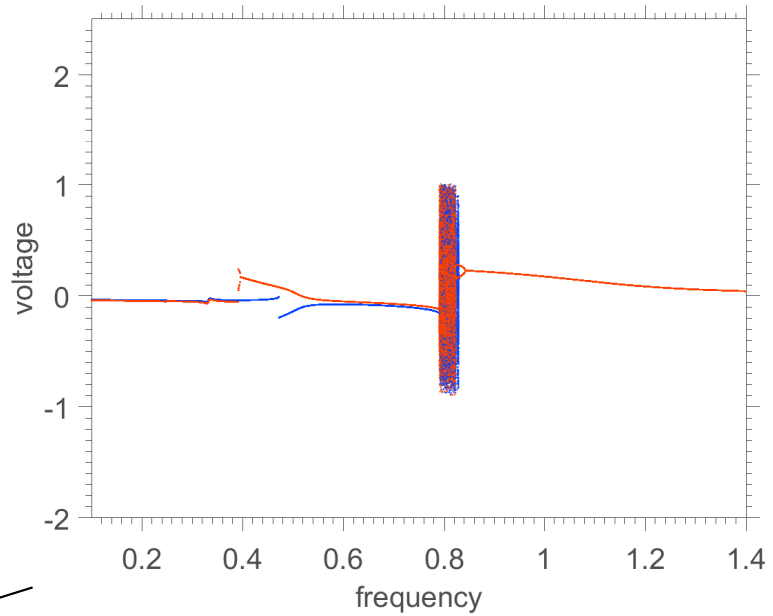




$f = 0.051$

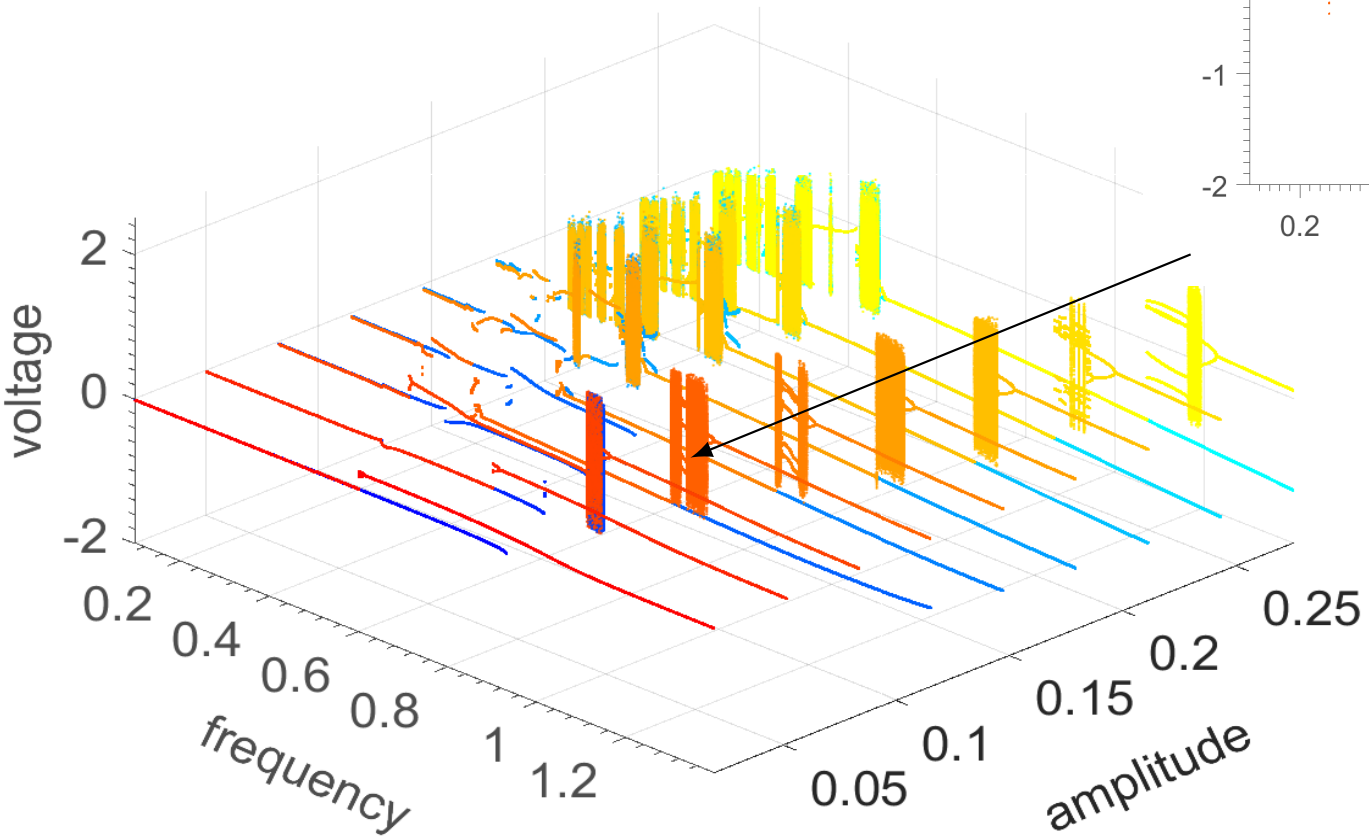
Bifurcation diagrams: voltage × frequency

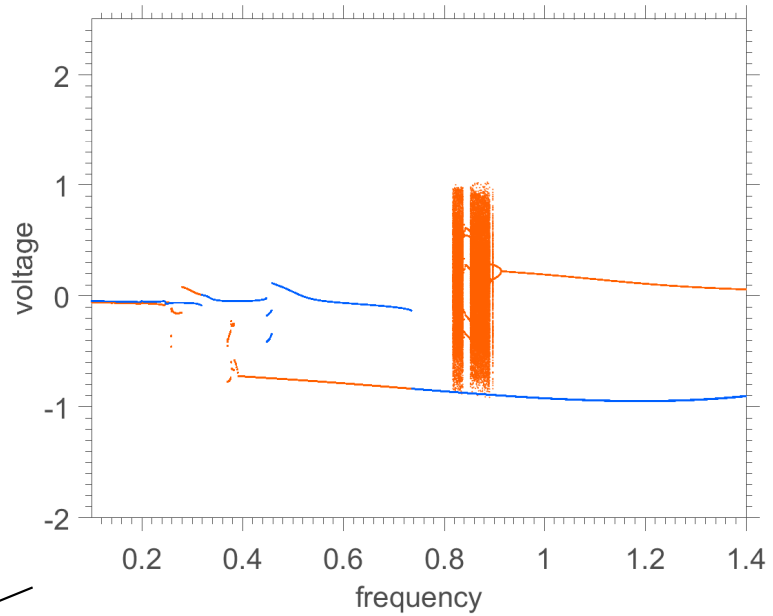




$f = 0.083$

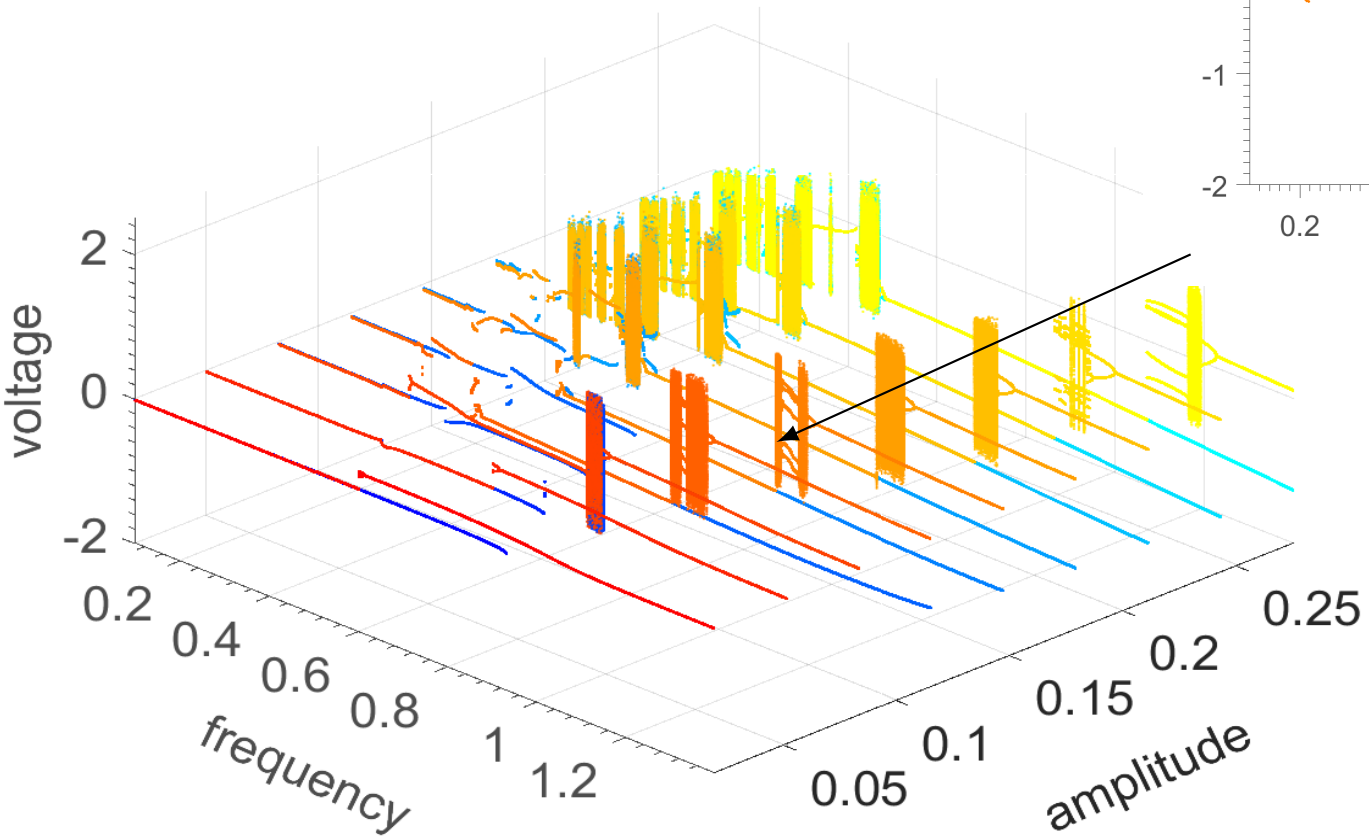
Bifurcation diagrams: voltage × frequency

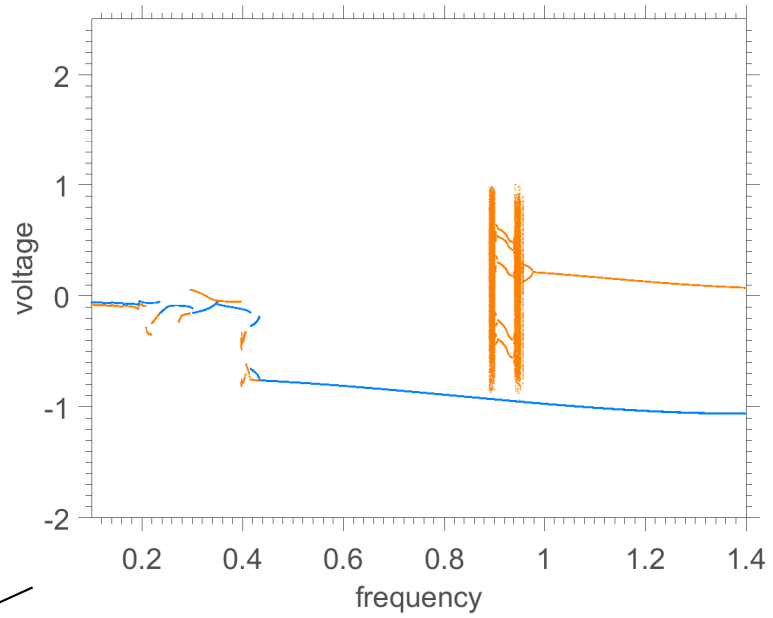




$f = 0.115$

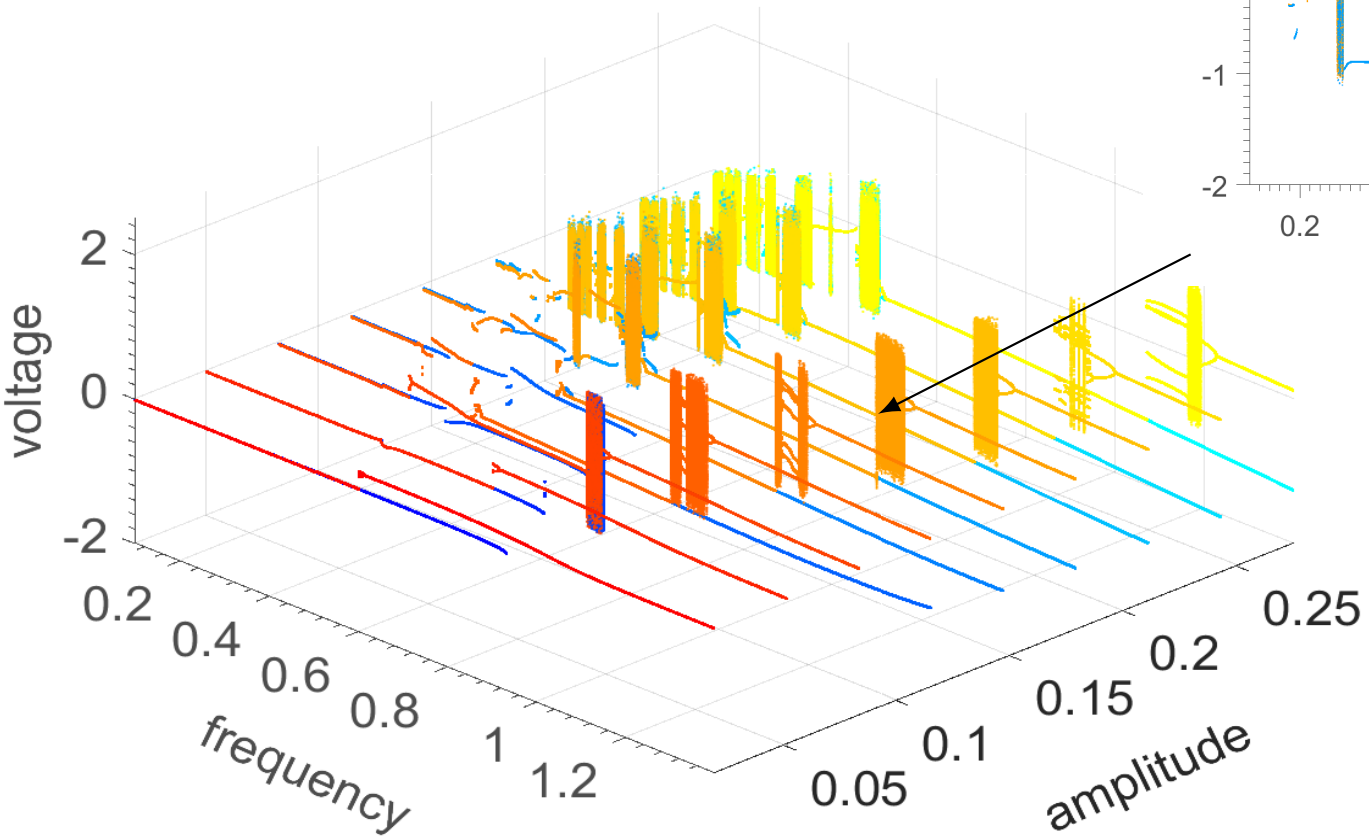
Bifurcation diagrams: voltage × frequency

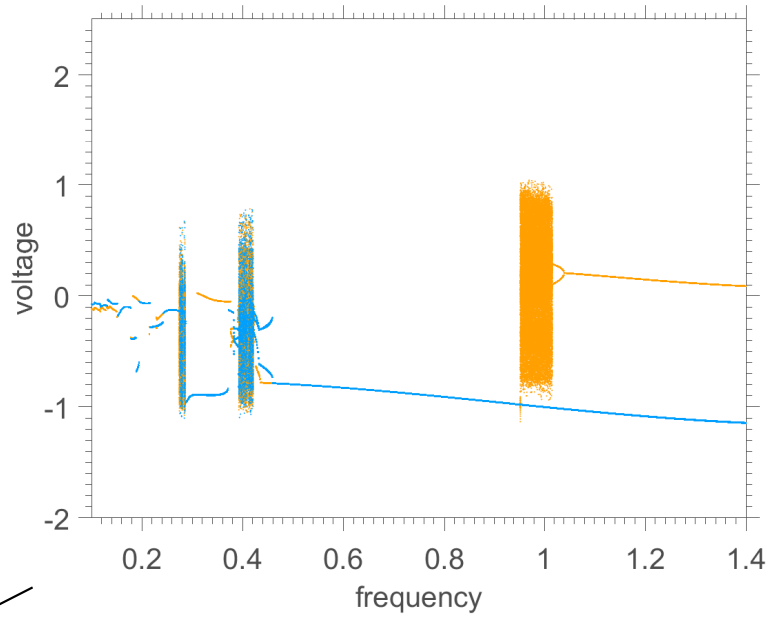




$f = 0.147$

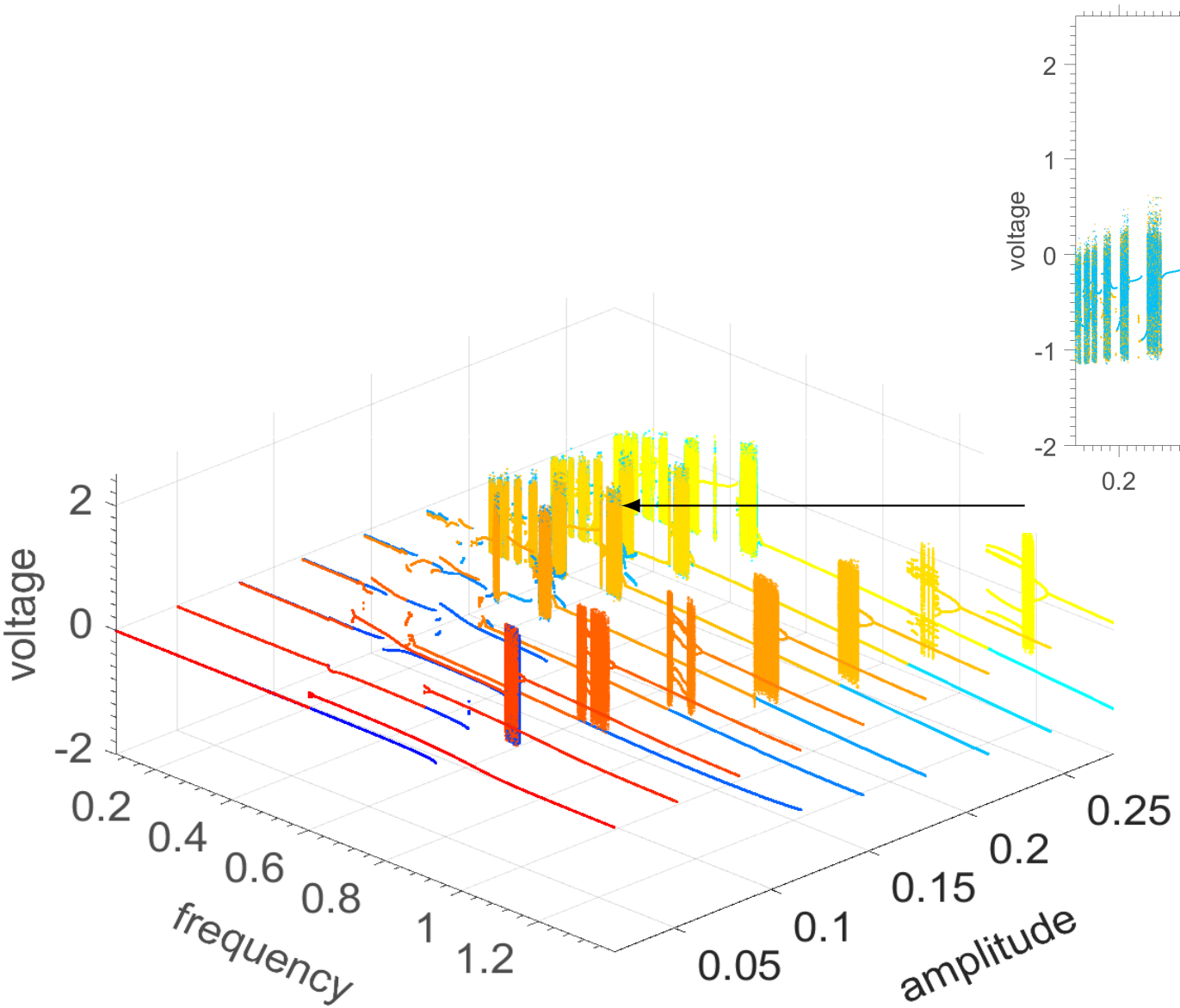
Bifurcation diagrams: voltage × frequency

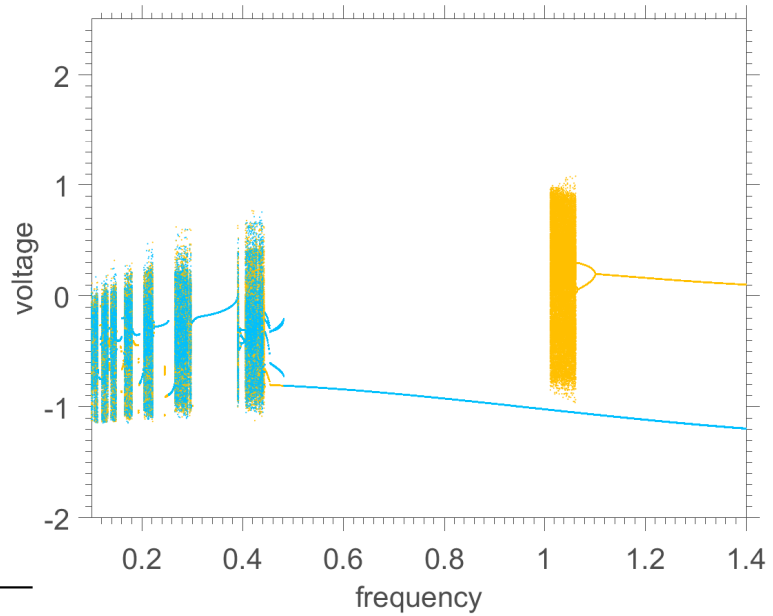




$f = 0.179$

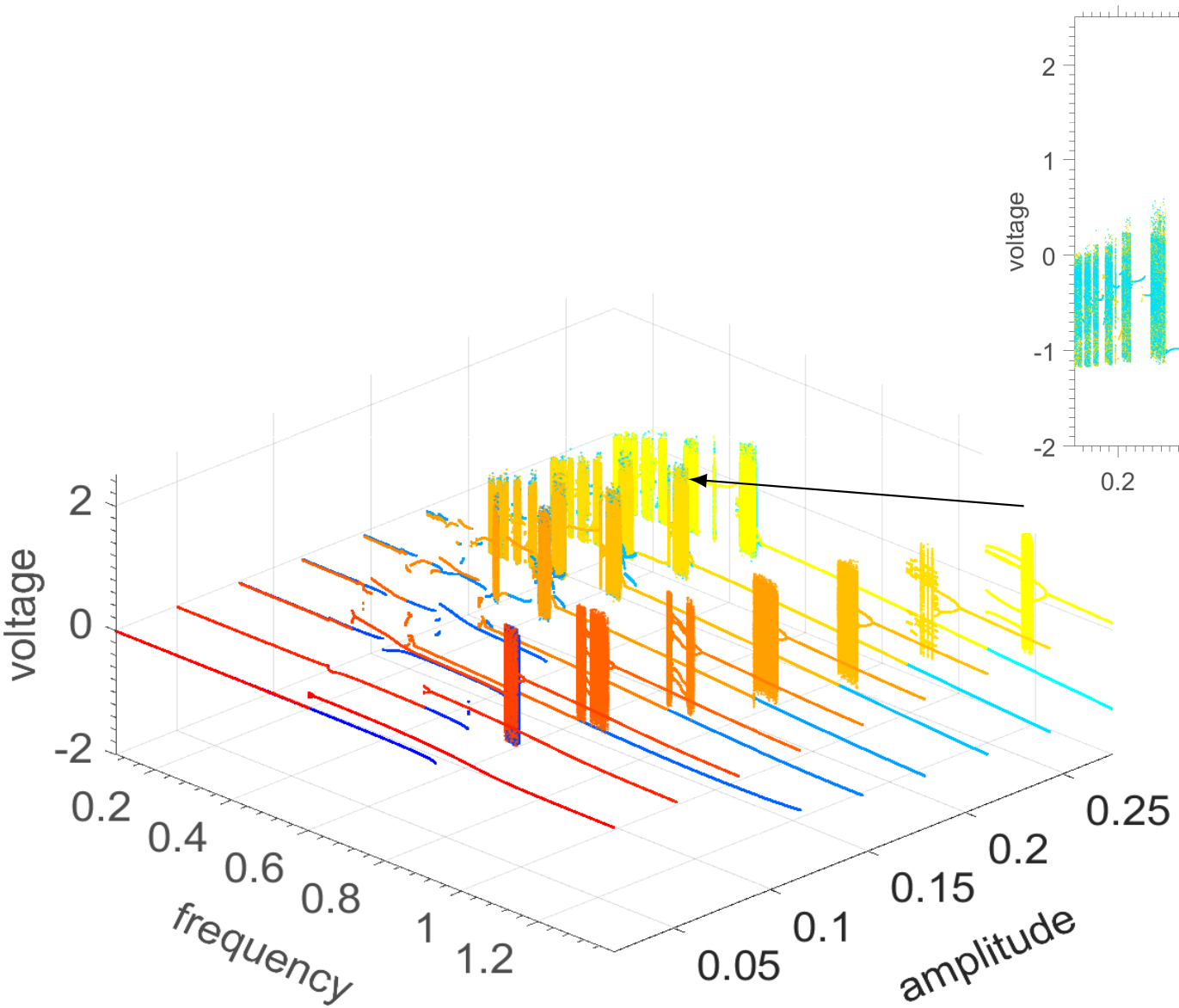
Bifurcation diagrams: voltage × frequency

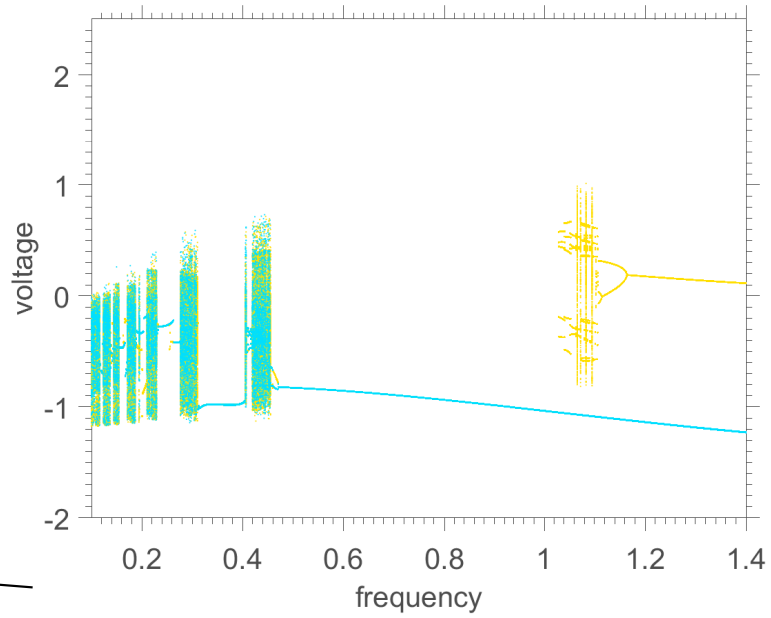




$f = 0.211$

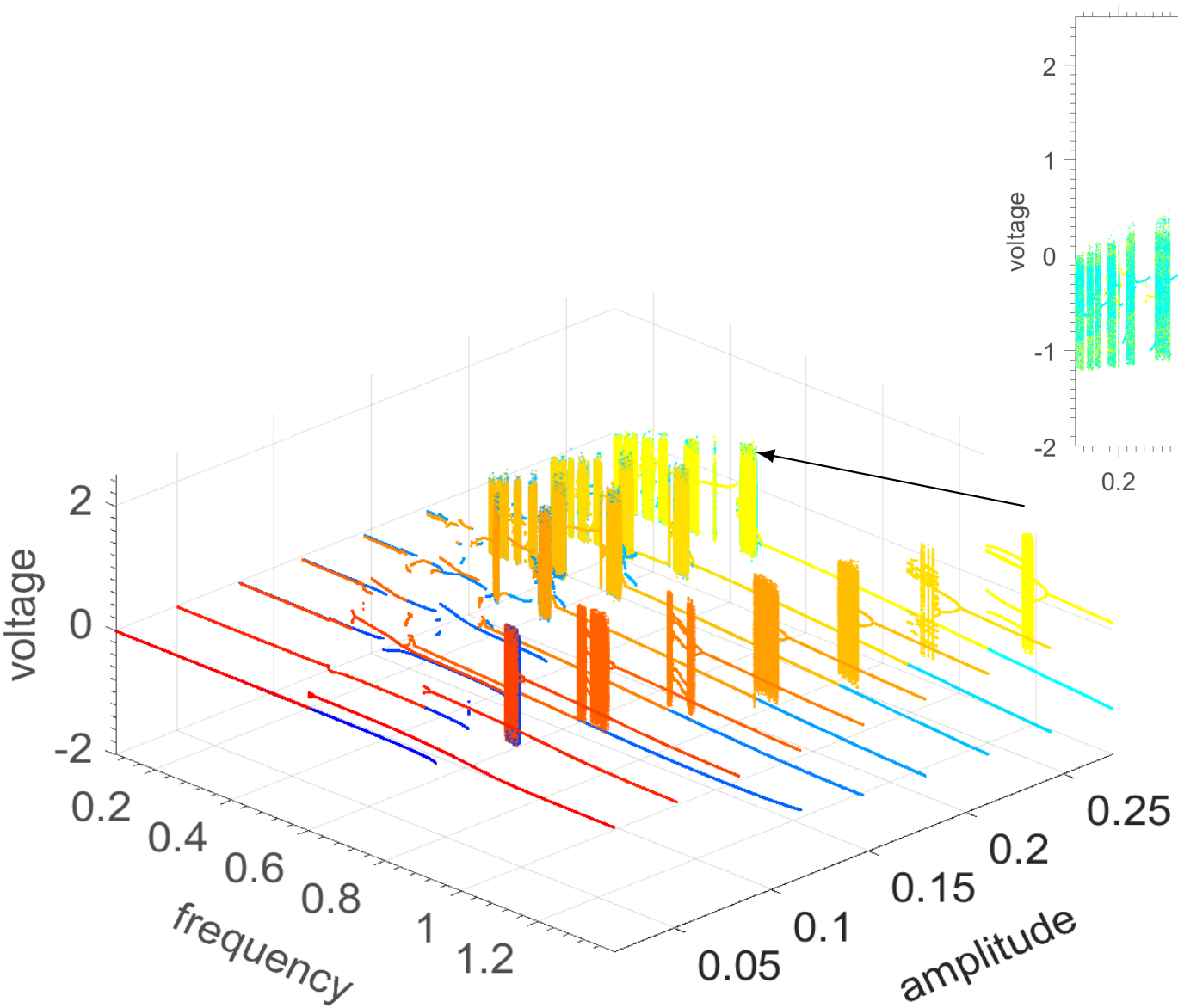
Bifurcation diagrams: voltage × frequency

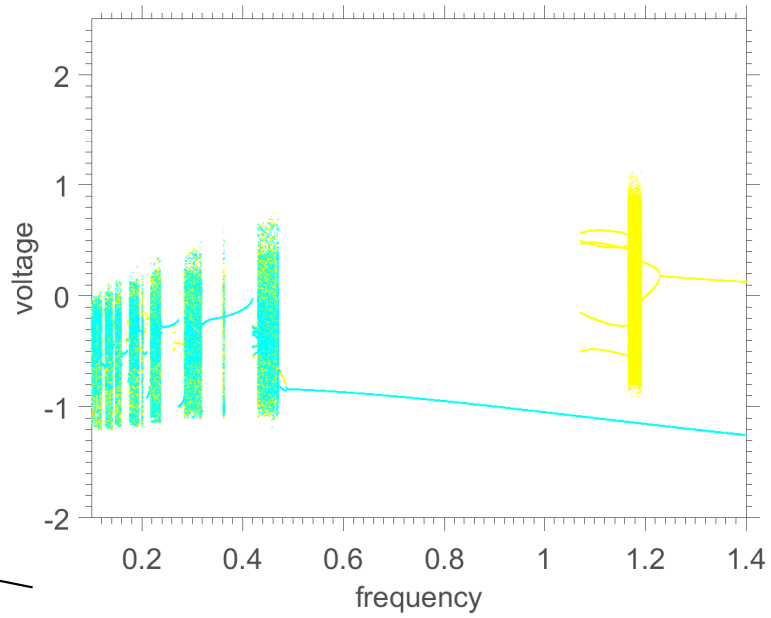




$f = 0.243$

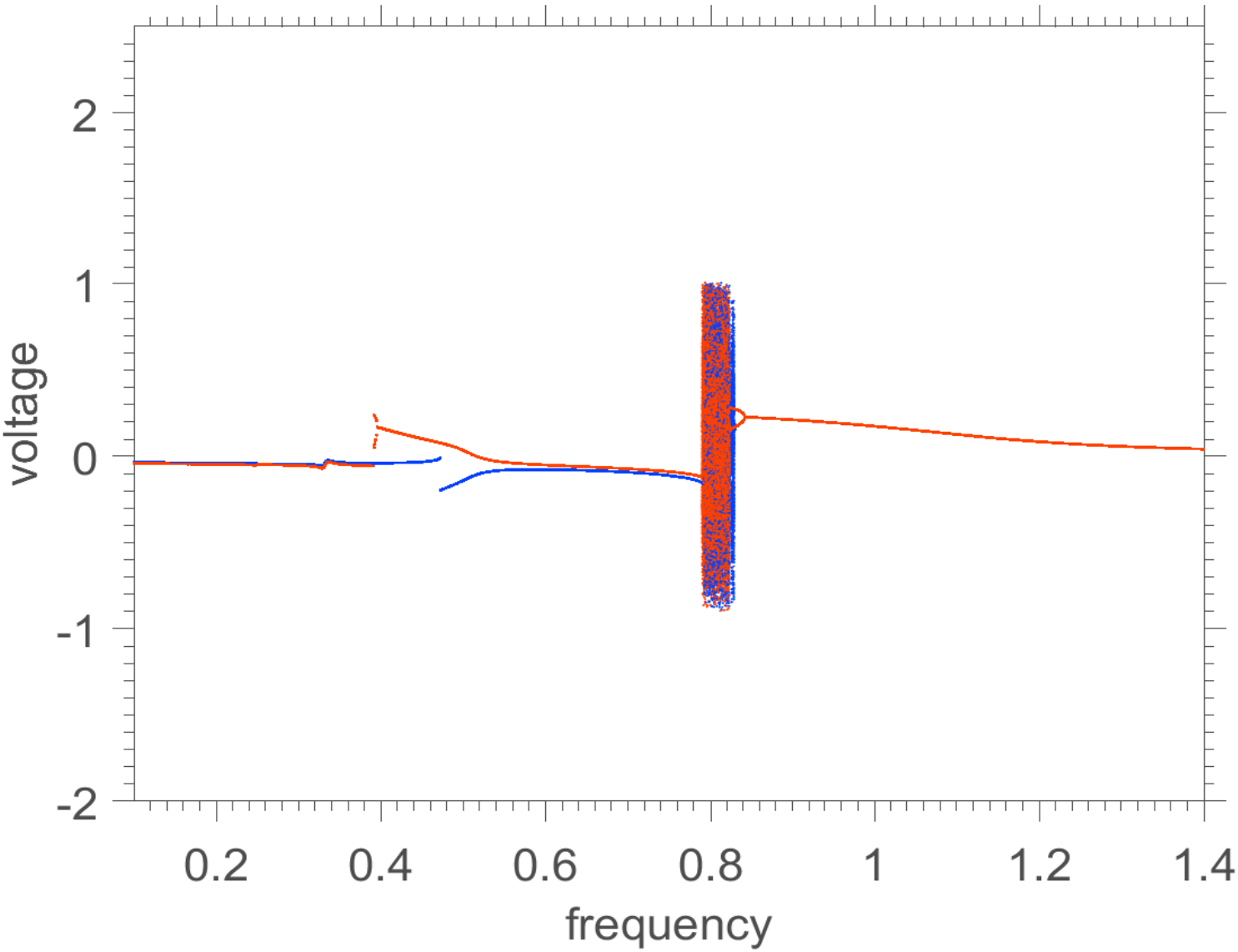
Bifurcation diagrams: voltage × frequency



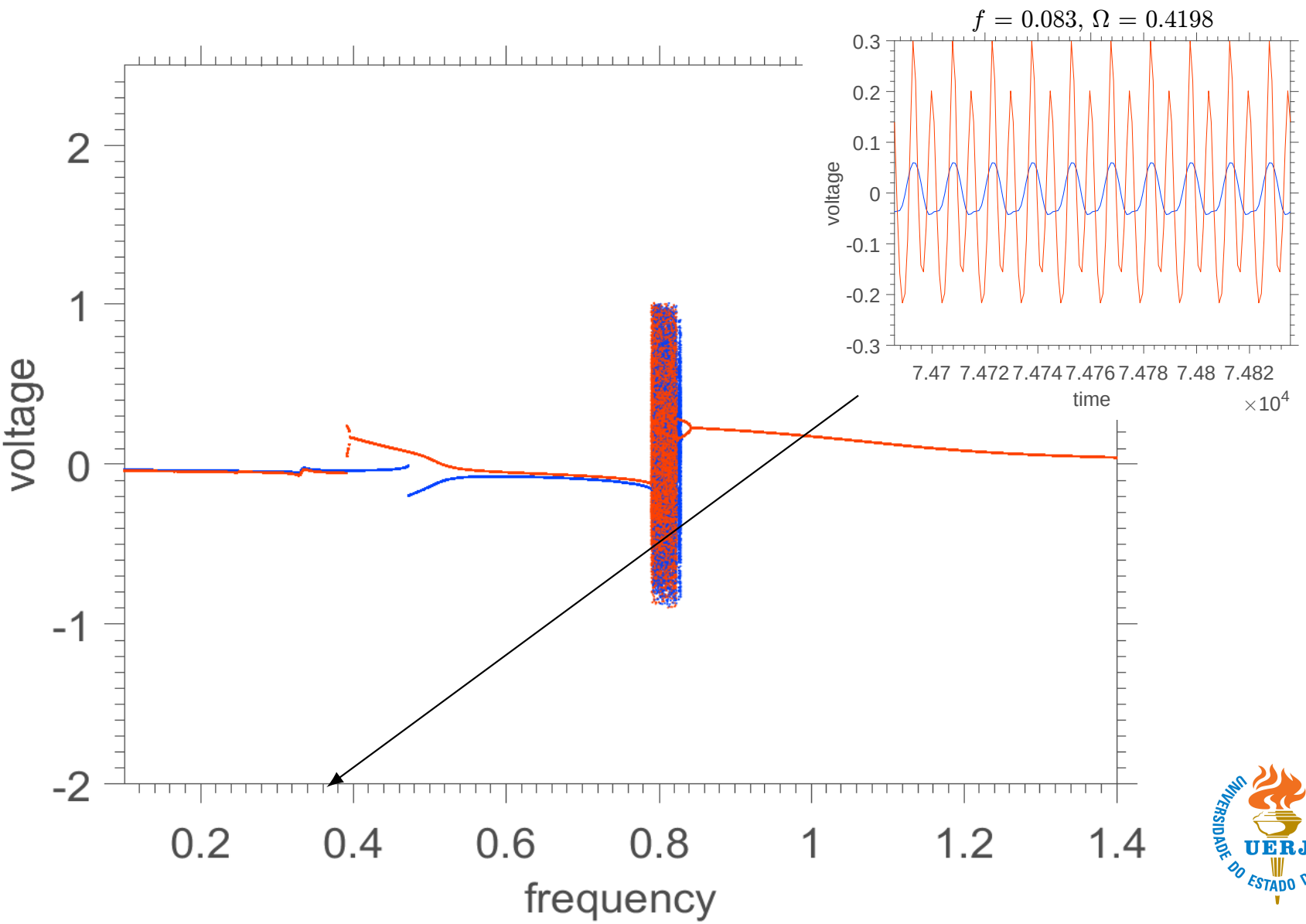


$f = 0.275$

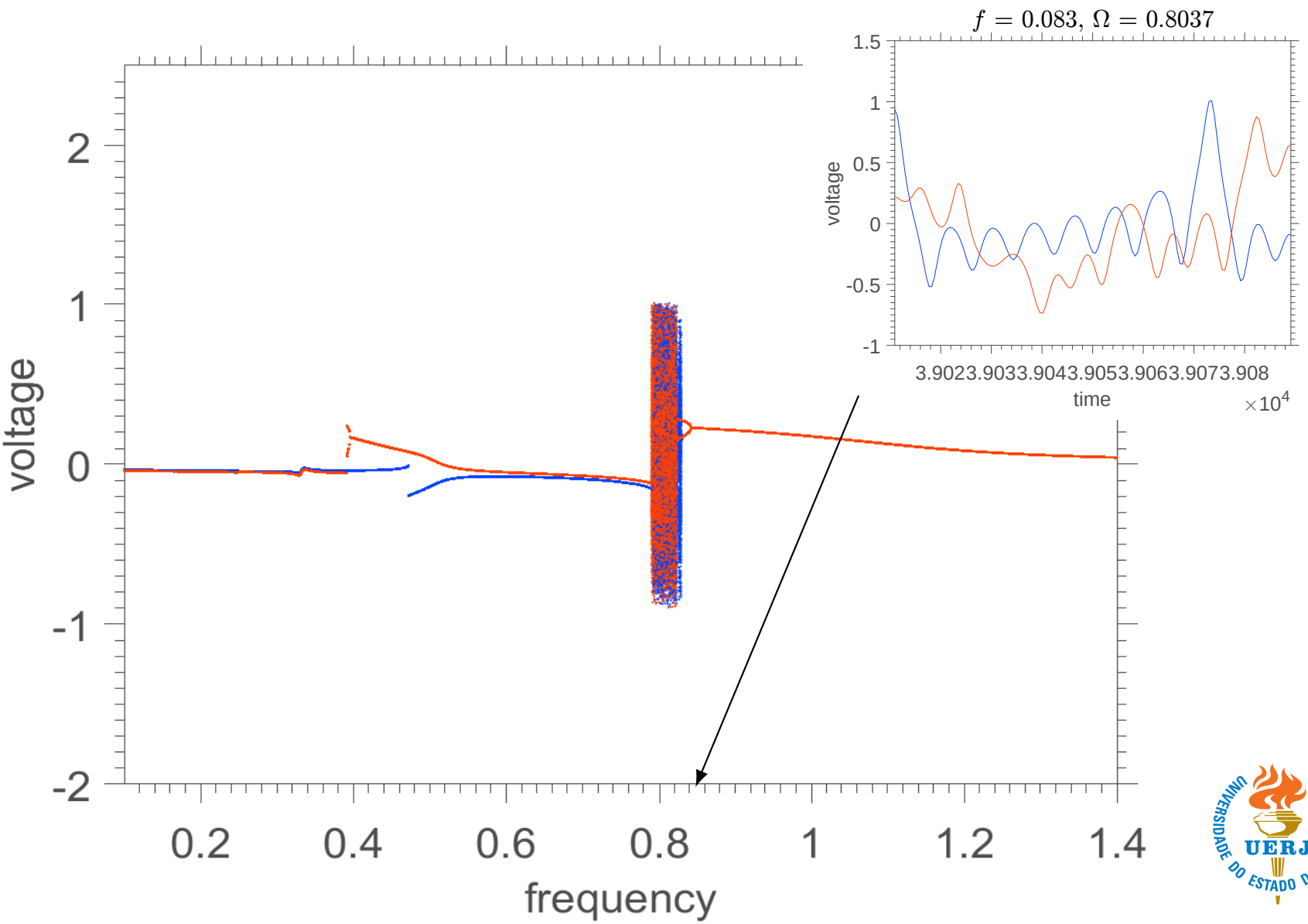
Forward and backward diagrams ($f = 0.083$)



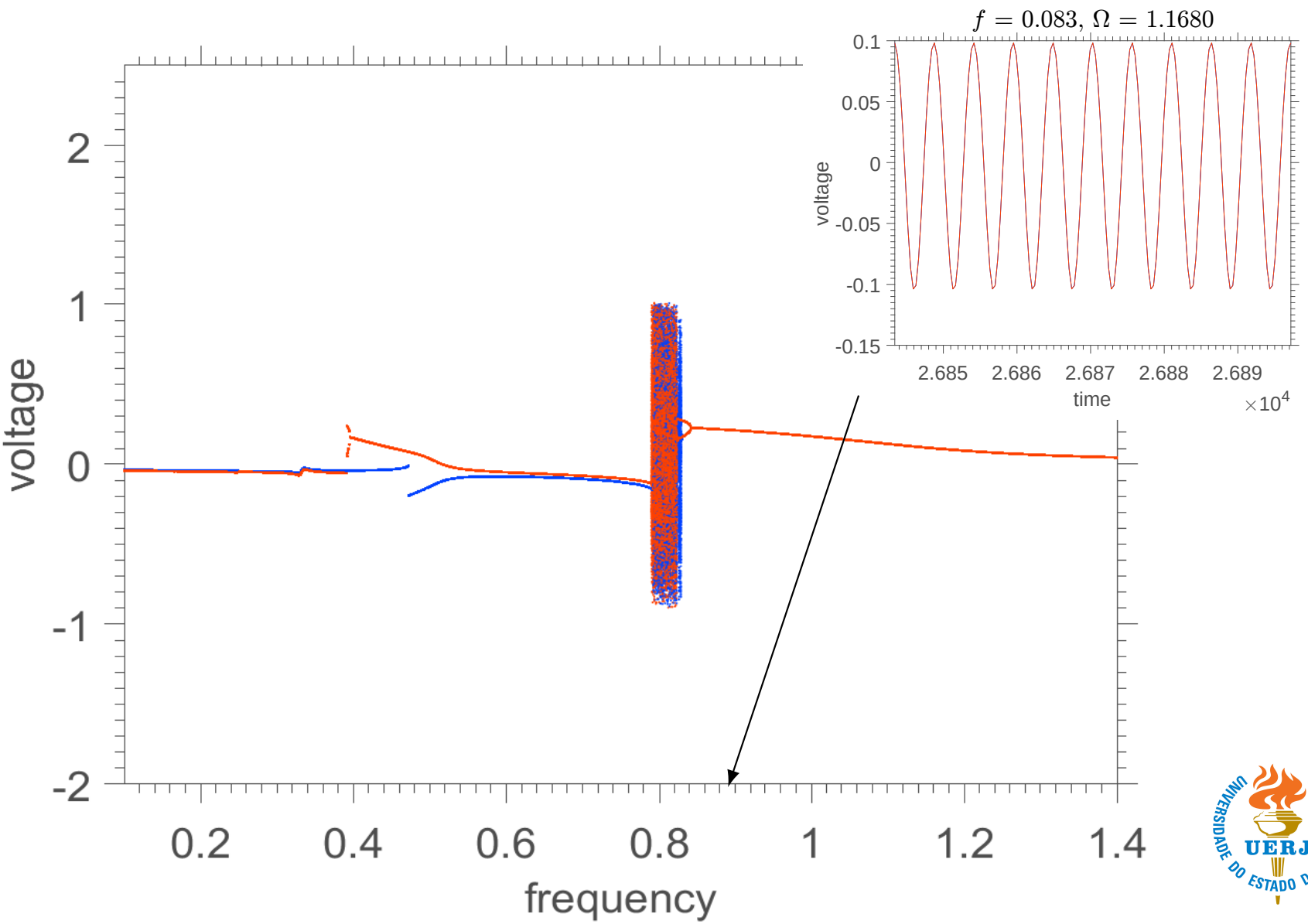
Forward and backward diagrams ($f = 0.083$)



Forward and backward diagrams ($f = 0.083$)



Forward and backward diagrams ($f = 0.083$)



Basins of attraction

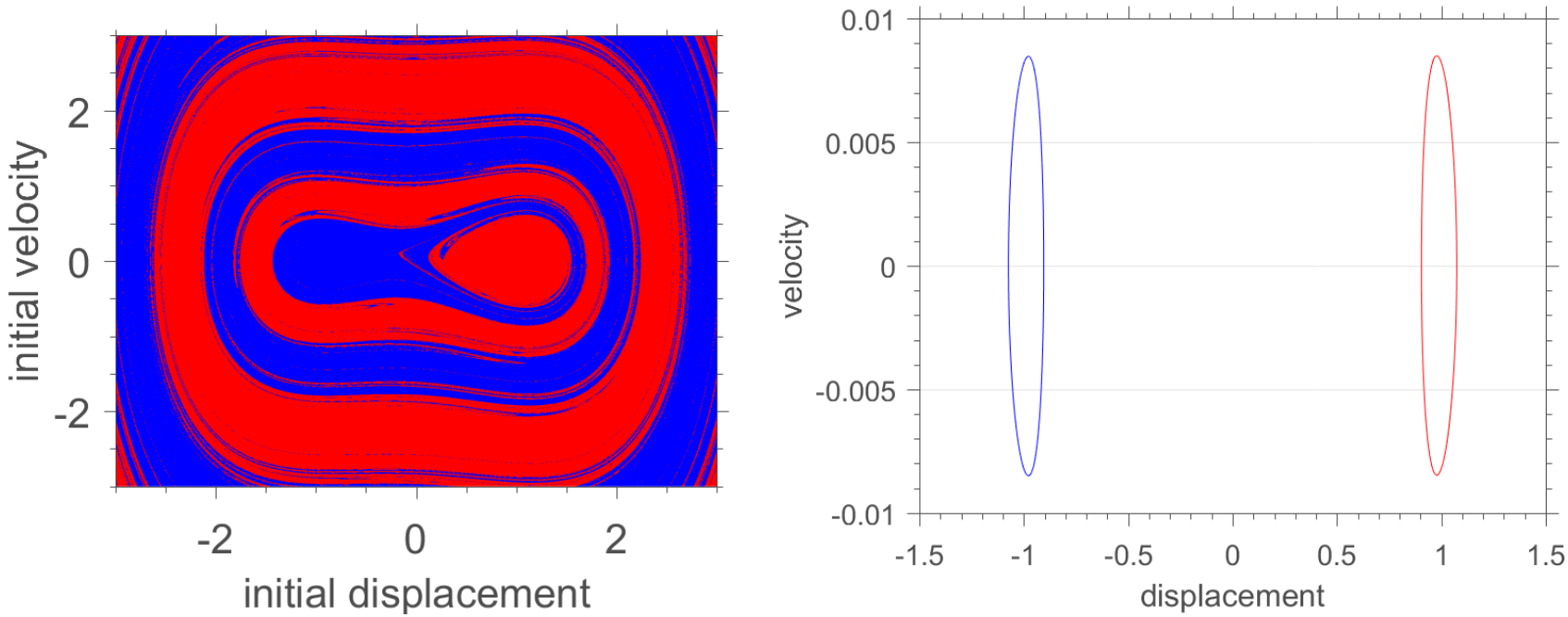


Figure: $f = 0.083$ and $\Omega = 0.1$

Basins of attraction

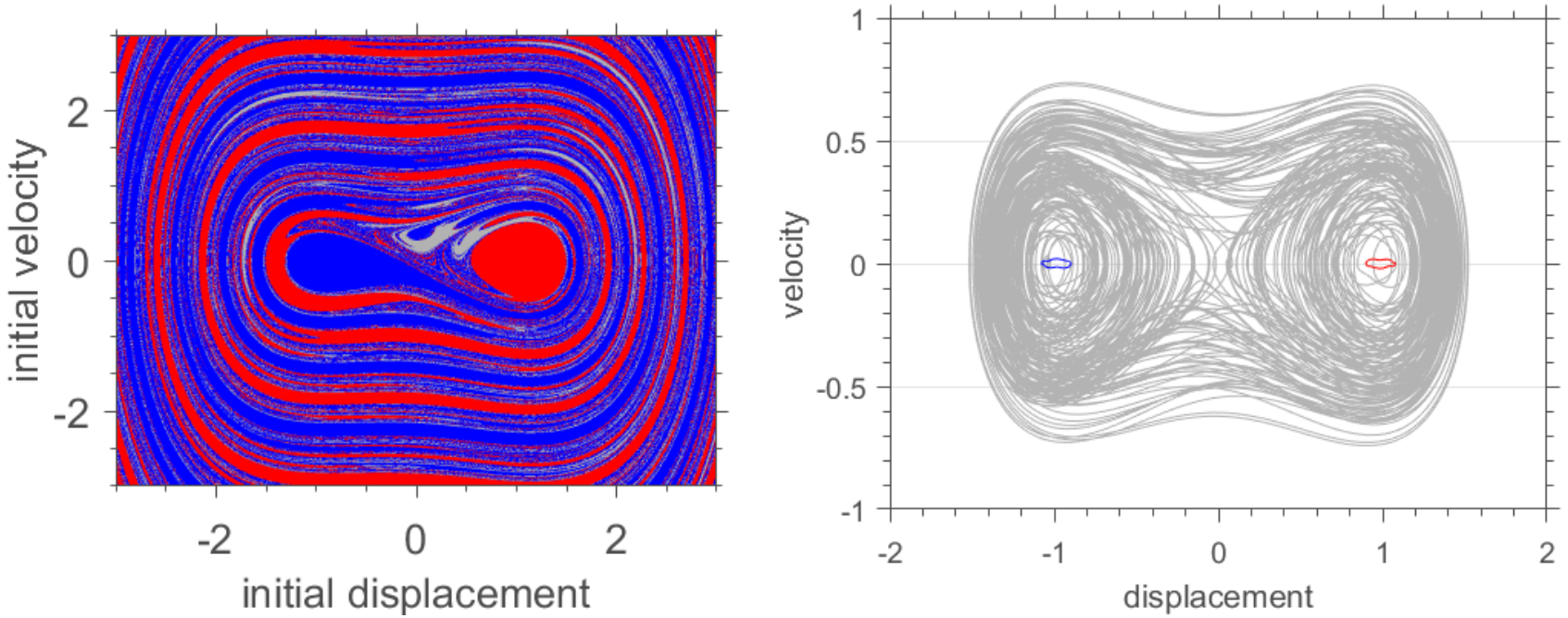


Figure: $f = 0.083$ and $\Omega = 0.2$

Basins of attraction

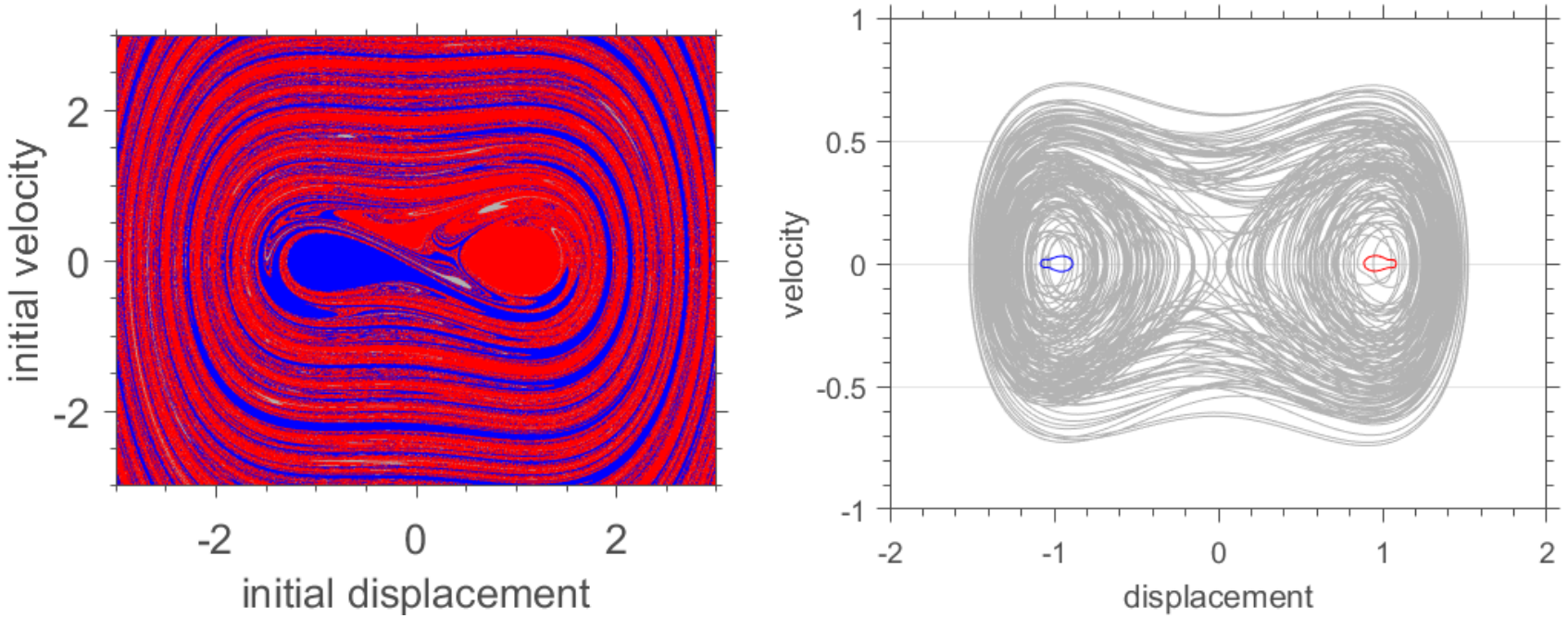


Figure: $f = 0.083$ and $\Omega = 0.3$

Basins of attraction

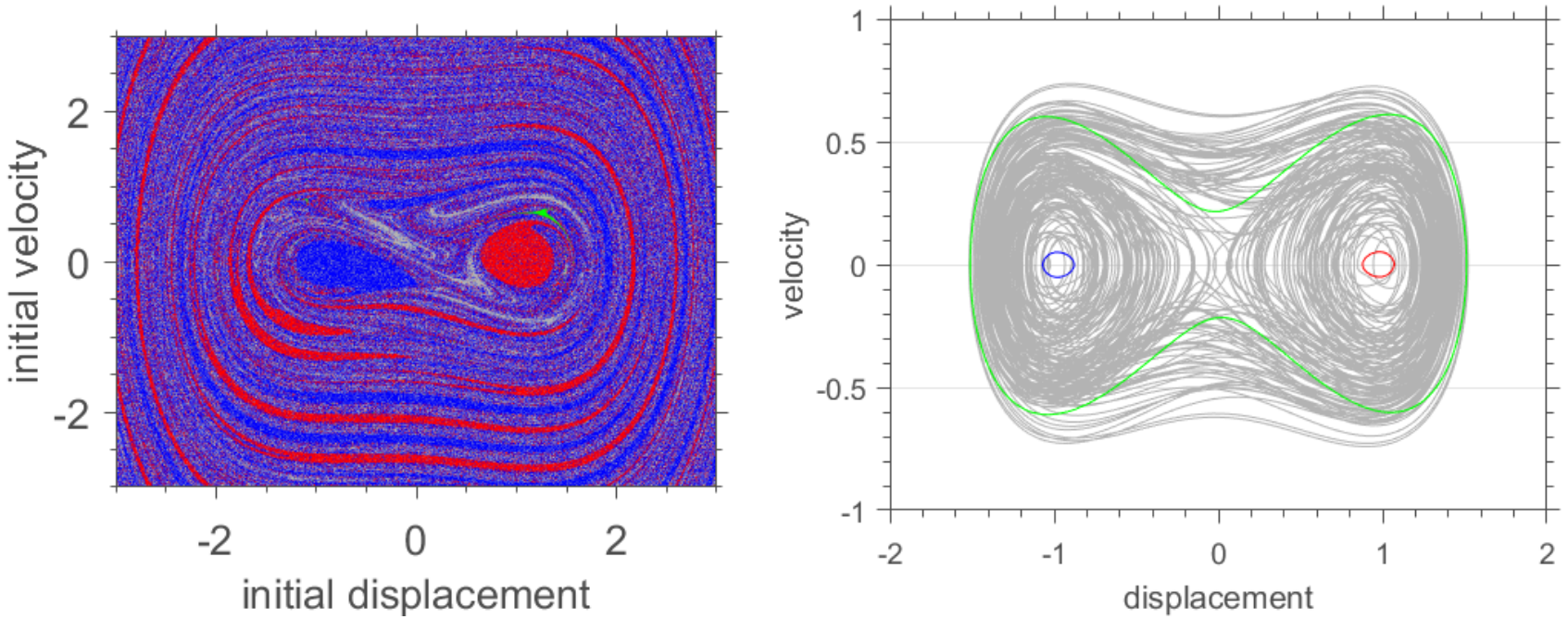


Figure: $f = 0.083$ and $\Omega = 0.4$

Basins of attraction

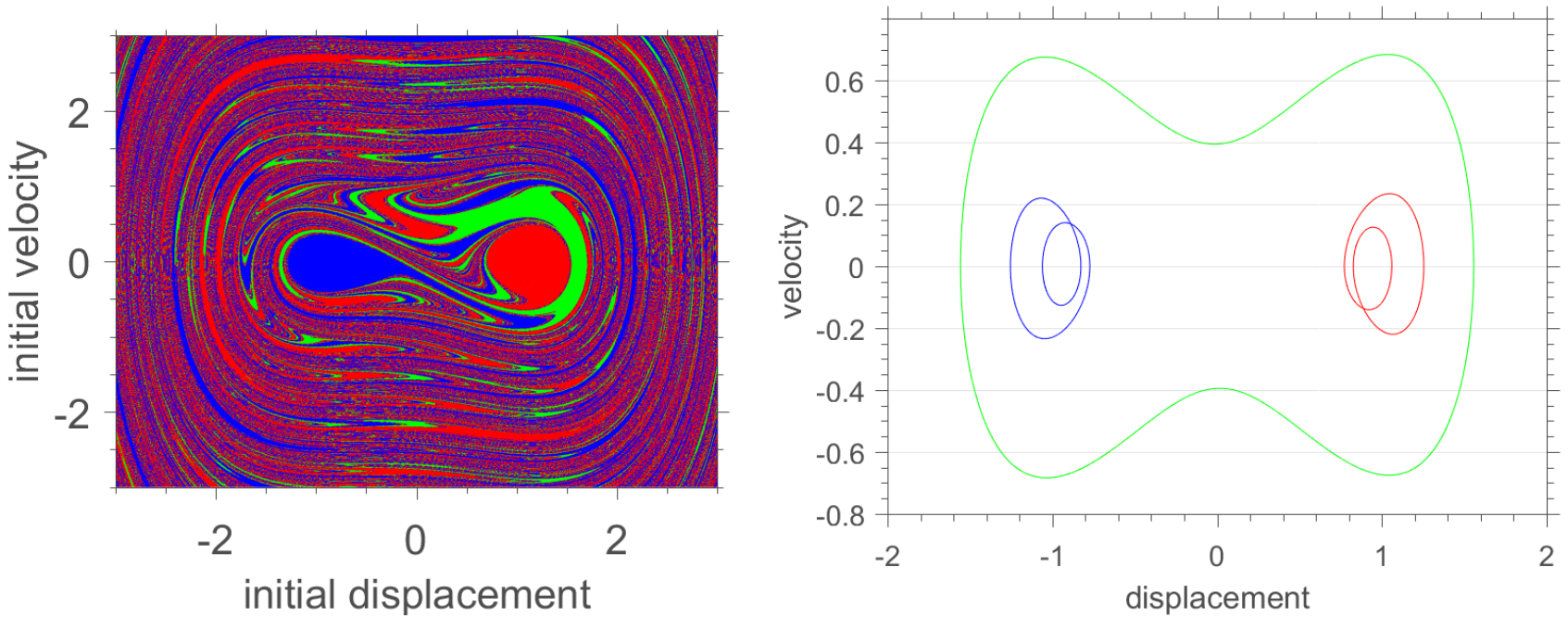


Figure: $f = 0.083$ and $\Omega = 0.5$

Basins of attraction

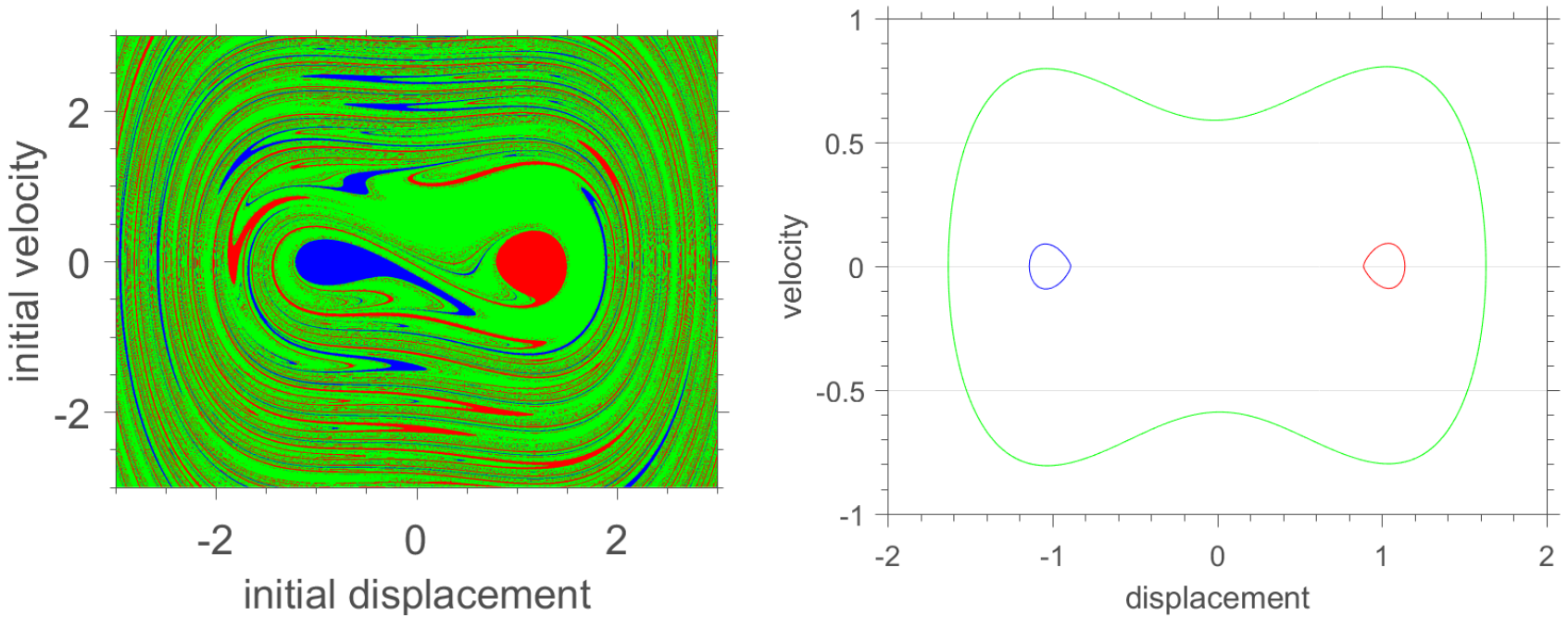


Figure: $f = 0.083$ and $\Omega = 0.6$

Basins of attraction

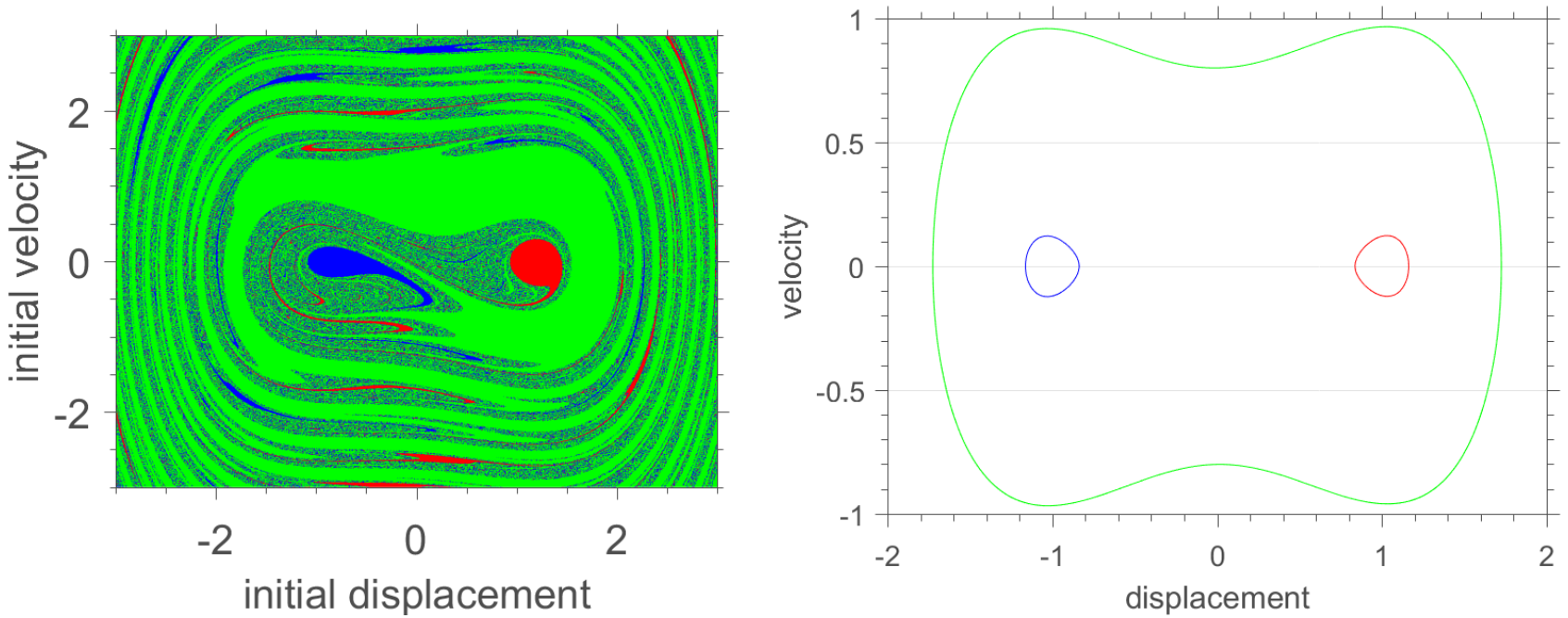


Figure: $f = 0.083$ and $\Omega = 0.7$

Basins of attraction

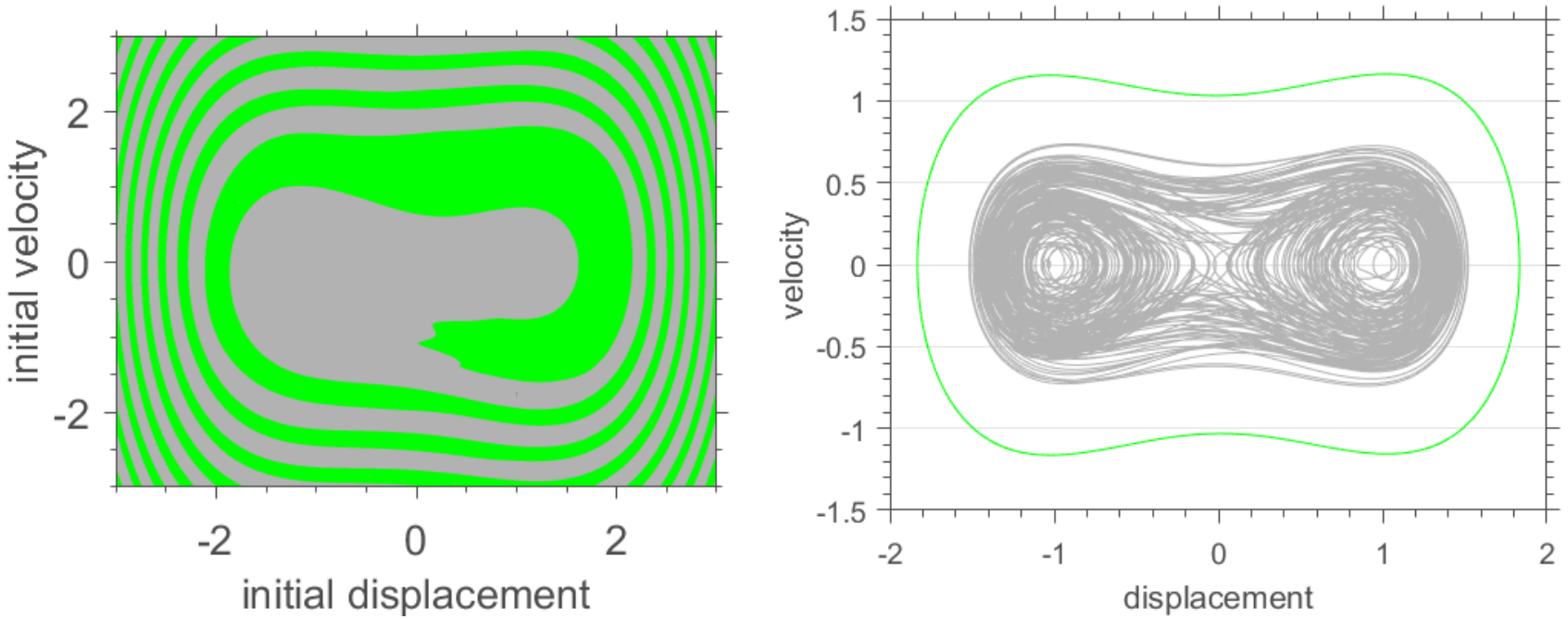


Figure: $f = 0.083$ and $\Omega = 0.8$

Basins of attraction

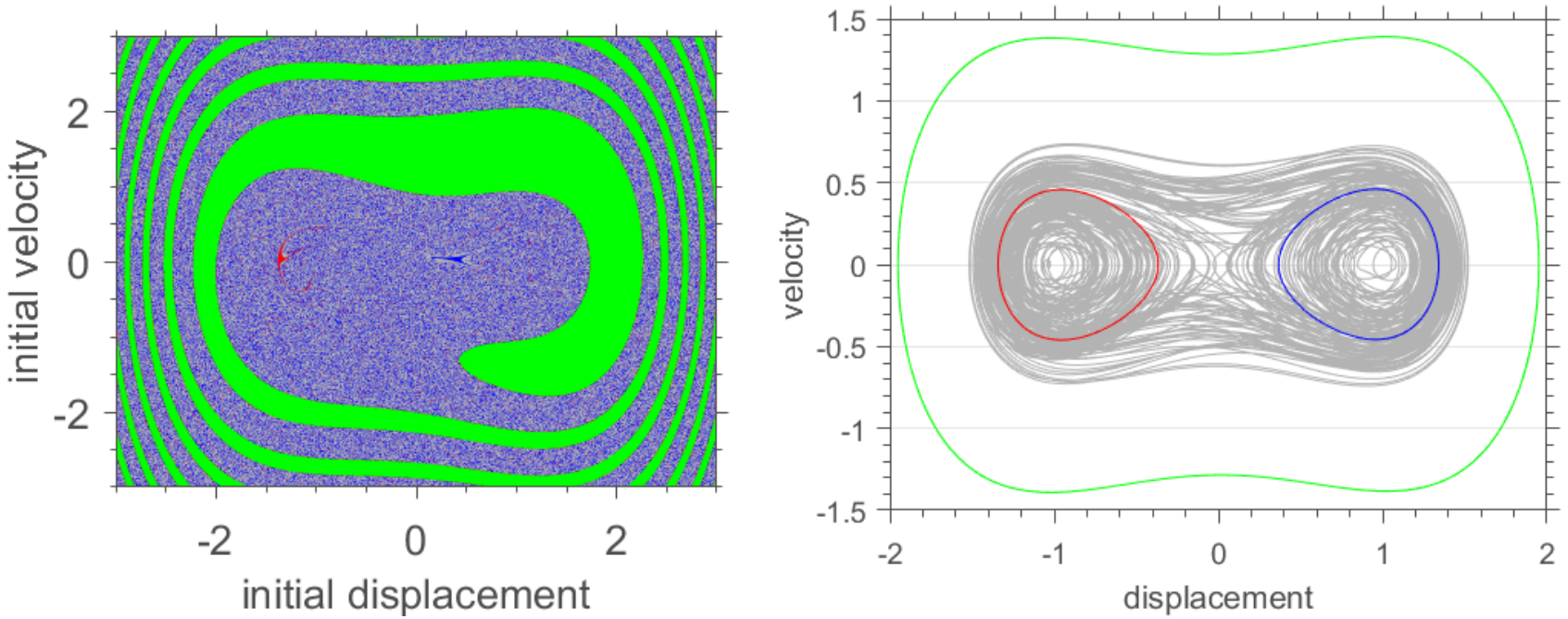
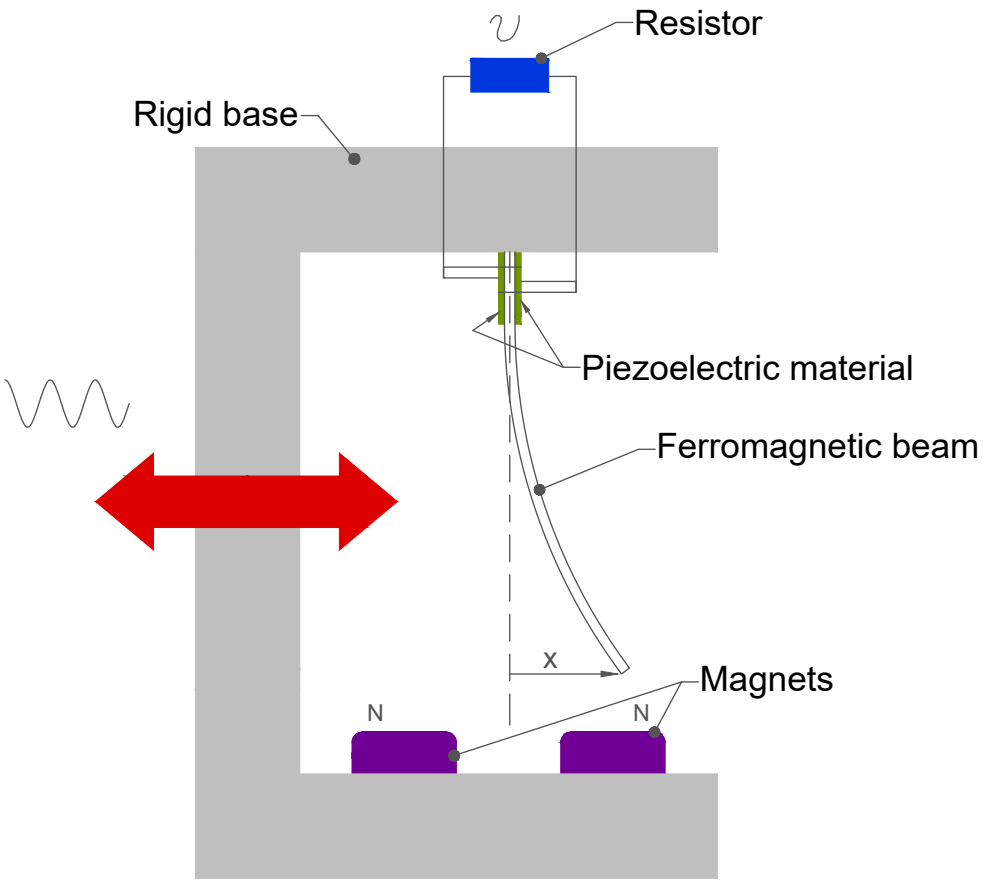


Figure: $f = 0.083$ and $\Omega = 0.9$

Stochastic Dynamics (ongoing research)

Nonlinear Vibratory Harvester

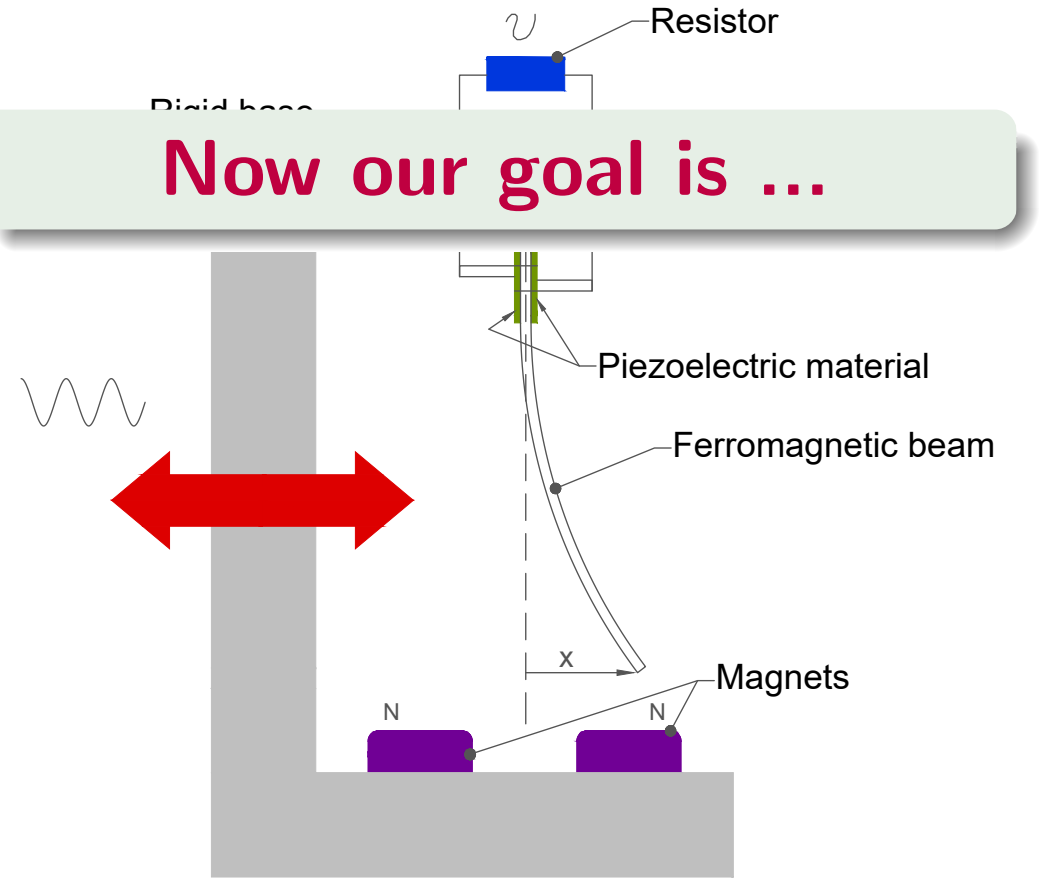
Bistable system driven by regular signal



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Nonlinear Vibratory Harvester

Bistable system driven by regular signal

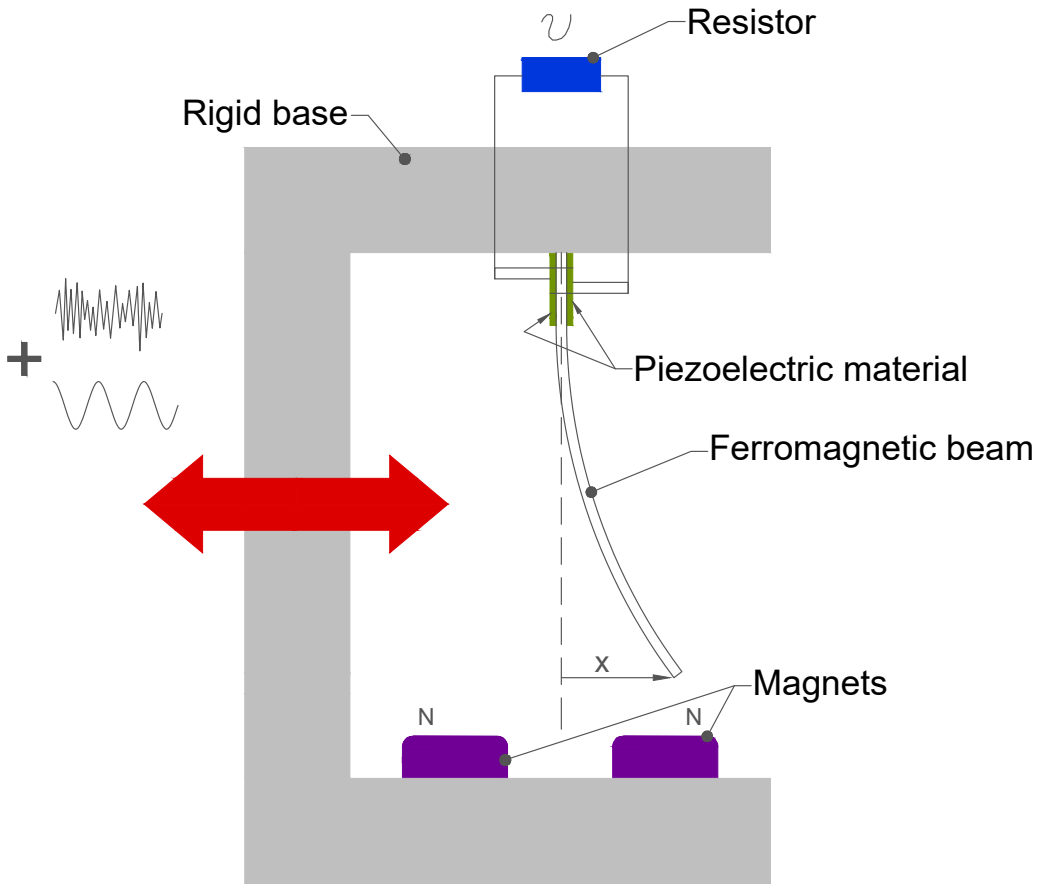


Now our goal is ...

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

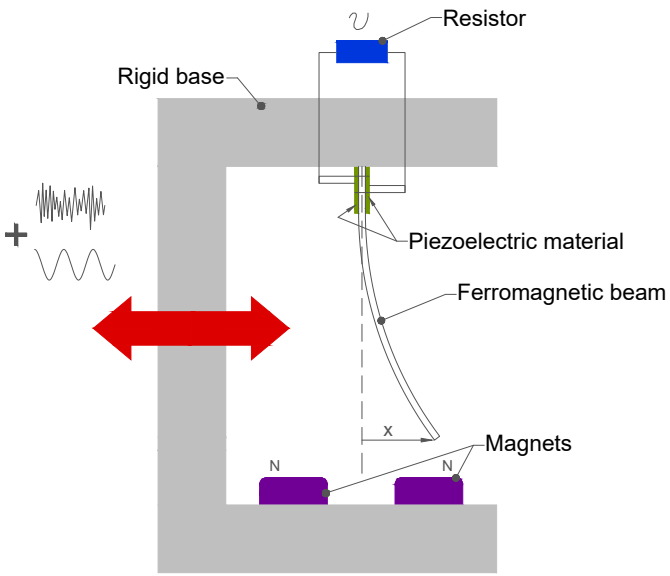
Nonlinear Vibratory Harvester

Bistable system driven by regular and noisy signals



V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr, **On the nonlinear stochastic dynamics of piezo-magneto-elastic energy harvester driven by colored noise**, (in preparation) 2019.

Bistable harvester driven by regular and noisy signals



$$\ddot{x}+2\xi\dot{x}-\frac{1}{2}x(1-x^2)-\chi\upsilon = f\cos\Omega t + \text{“noise”}$$

$$\dot{\upsilon}+\lambda\upsilon+\kappa\dot{x}=0$$

$$x(0)=x_0,\ \dot{x}(0)=\dot{x}_0,\ \upsilon(0)=\upsilon_0$$

V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr, **On the nonlinear stochastic dynamics of piezo-magneto-elastic energy harvester driven by colored noise**, (in preparation) 2019.

Nonlinear stochastic dynamical system

- external excitation N_t:
  - zero-mean stationary Gaussian process
  - covariance function:

$$\mathrm{cov}_{N_t}(t_1, t_2) = \sigma \exp\left(-\frac{|t_2 - t_1|}{\tau_{corr}}\right), \quad \frac{\sigma^2}{\tau_{corr}} = constant$$

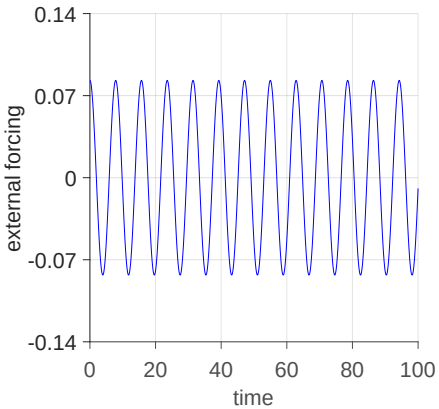
- stochastic evolution law:

$$\ddot{X} + 2\xi\dot{X} - \frac{1}{2}x(1 - X^2) - \chi V = f\cos\Omega t + N_t$$

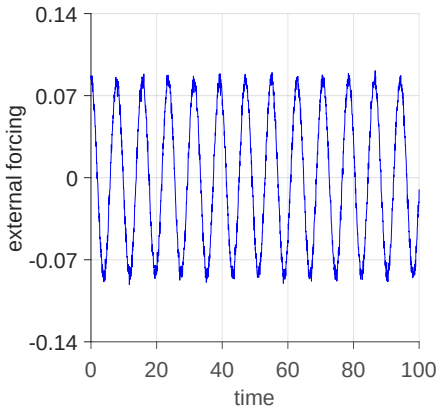
$$\dot{V} + \lambda V + \kappa\dot{X} = 0$$

$$X(0) = x_0, \ \dot{X}(0) = \dot{x}_0, \ V(0) = v_0$$

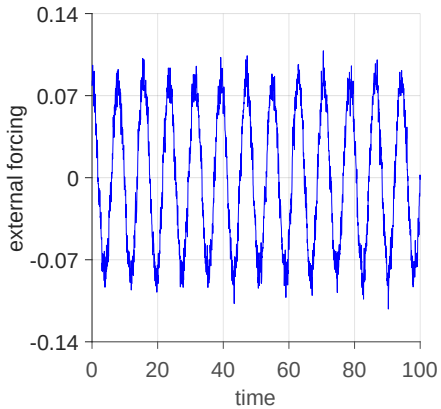
Realizations of random external force



(a) 1% of noise



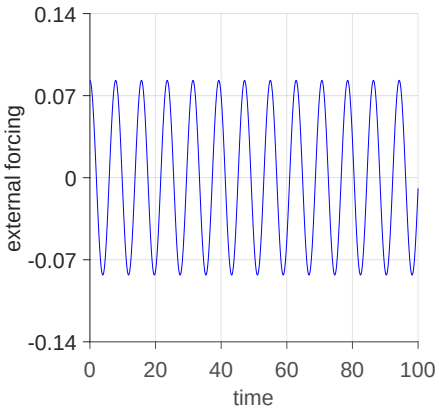
(b) 25% of noise



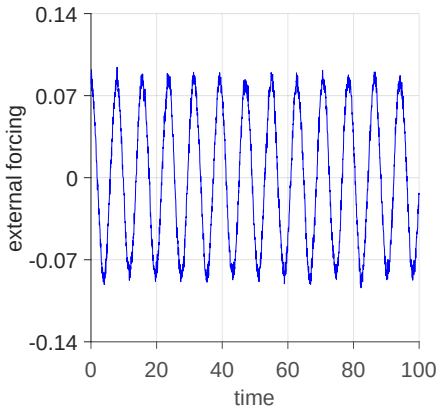
(c) 50% of noise

$$\sigma^2/\tau_{cor} = 1$$

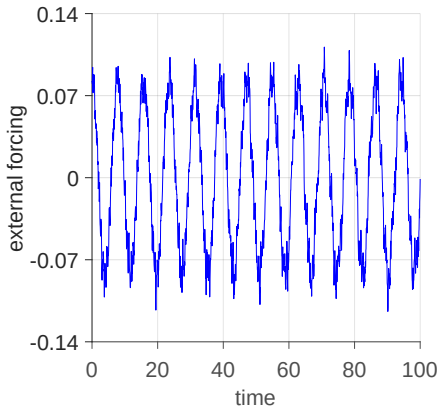
Realizations of random external force



(a) 1% of noise



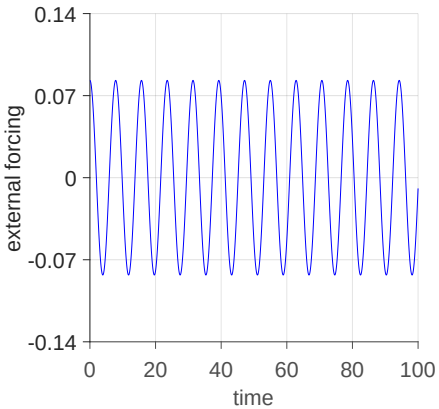
(b) 25% of noise



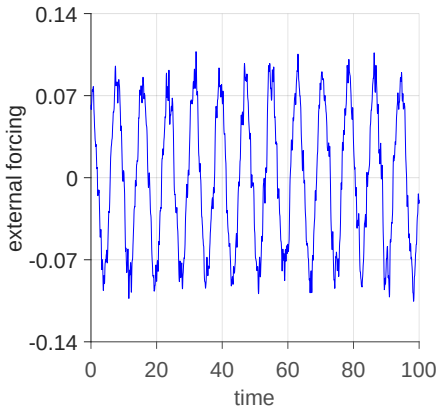
(c) 50% of noise

$$\sigma^2/\tau_{cor} = 0.5$$

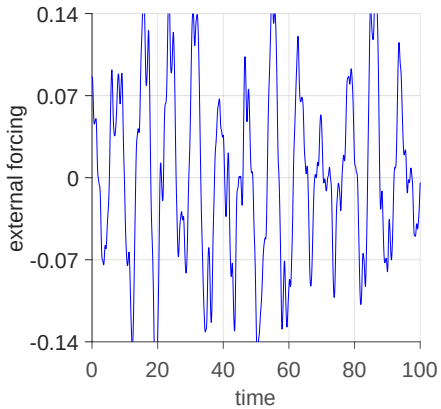
Realizations of random external force



(a) 1% of noise



(b) 25% of noise



(c) 50% of noise

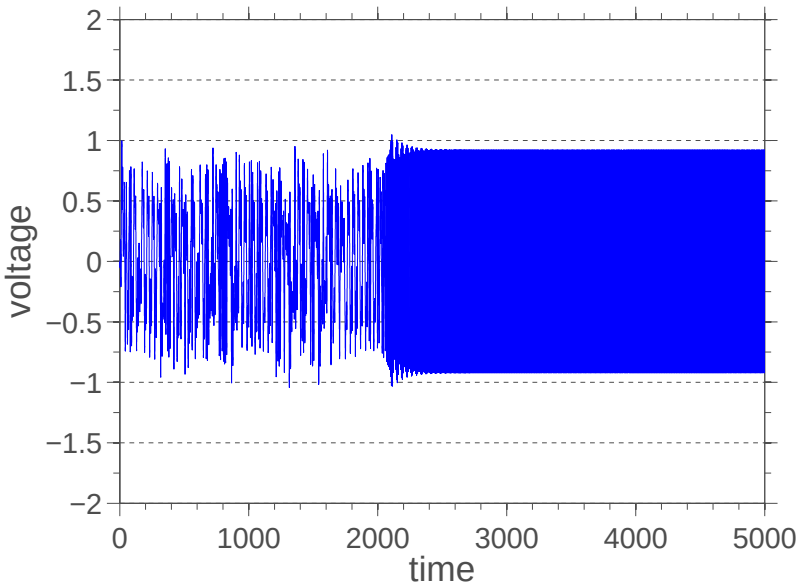
$$\sigma^2/\tau_{cor} = 0.1$$

Nonlinear stochastic dynamics animation

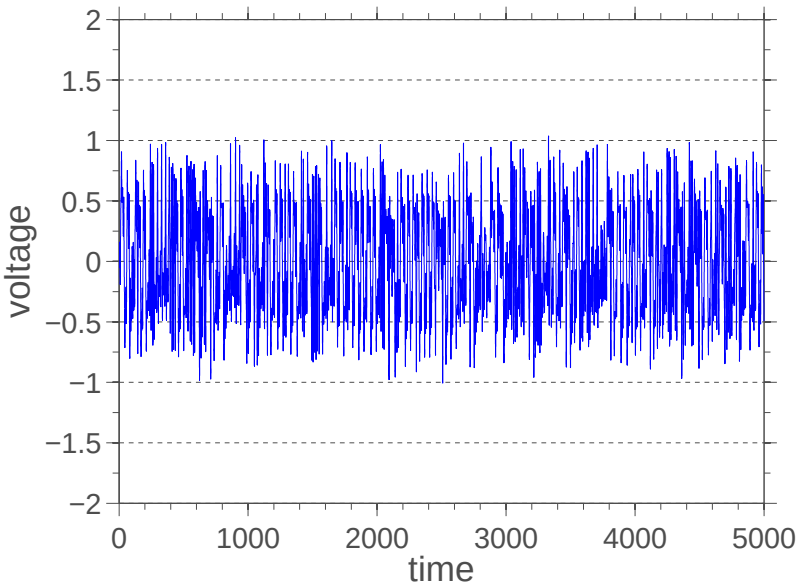
Section 3

Controlling Chaos

Typical time series for the bistable harvester

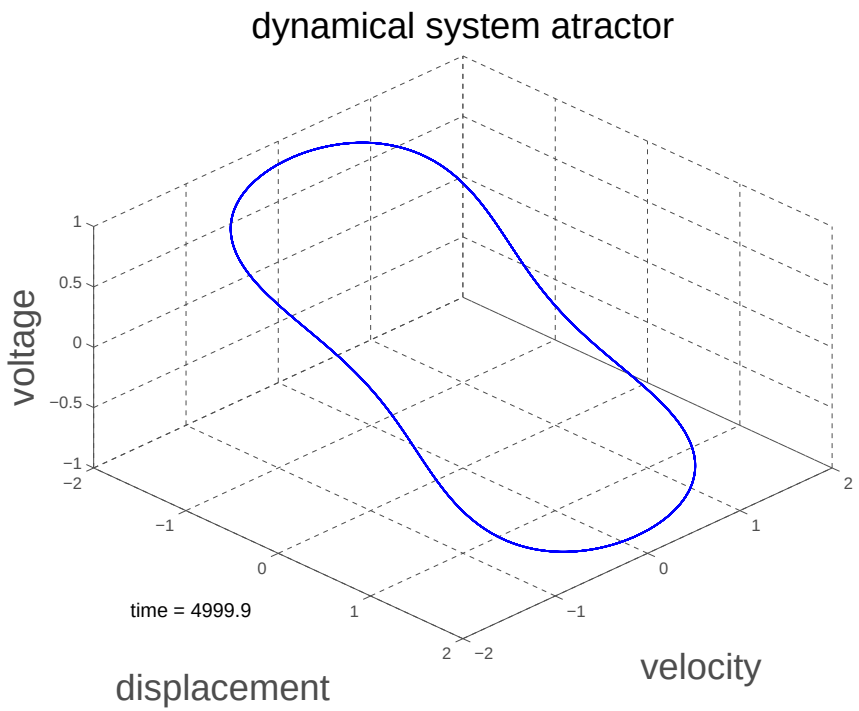


(a) regular (after chaotic transient)

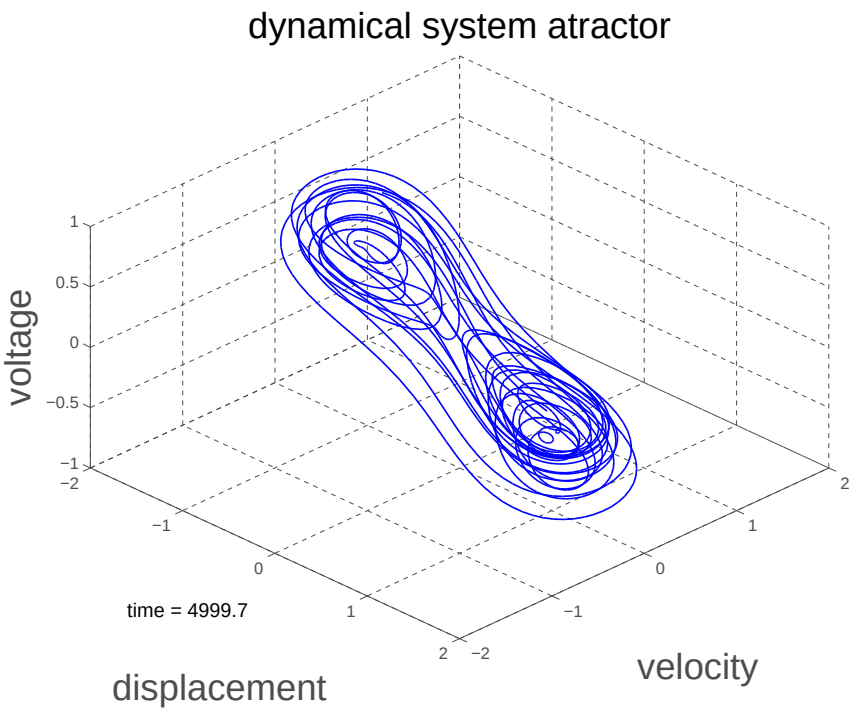


(b) strange atractor

Typical atractors for the bistable harvester

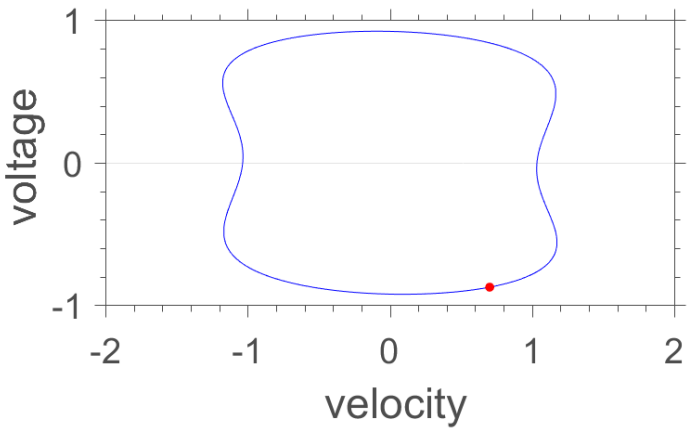


(a) regular limit cicle

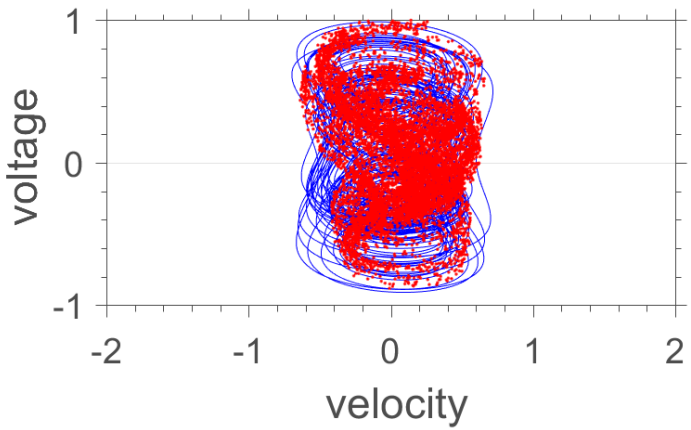


(b) strange atractor

Typical Poincaré sections for the bistable harvester



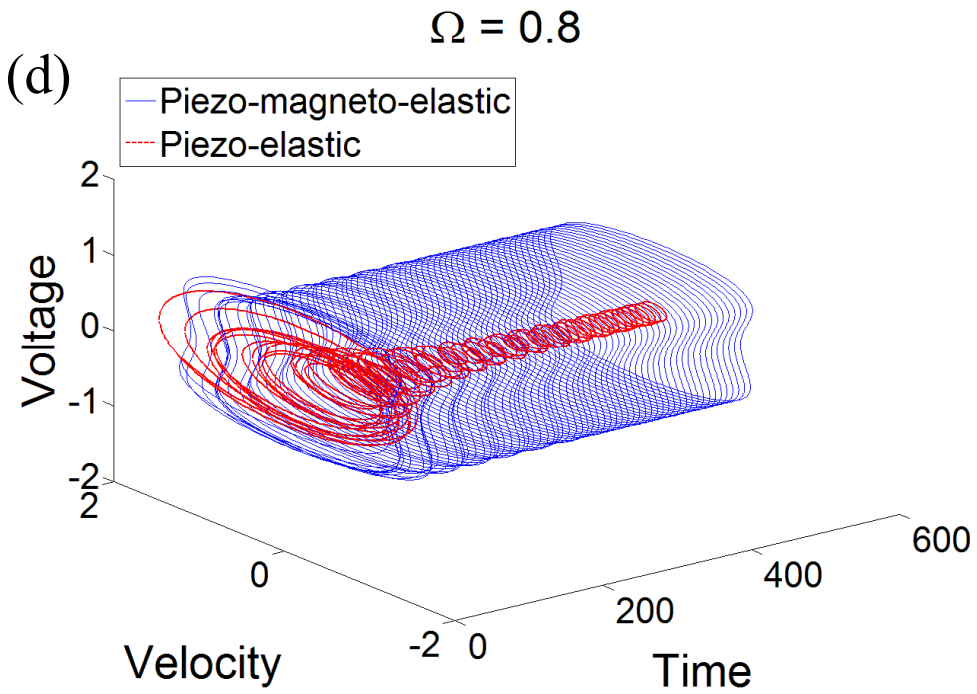
(a) regular limit cicle



(b) strange atractor

Nonlinearity and efficiency

Nonlinearity of this dynamical system may enhance output power.

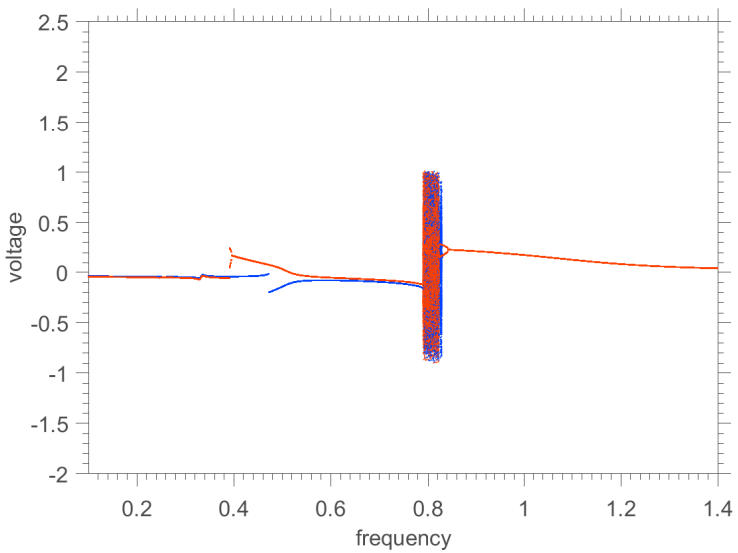


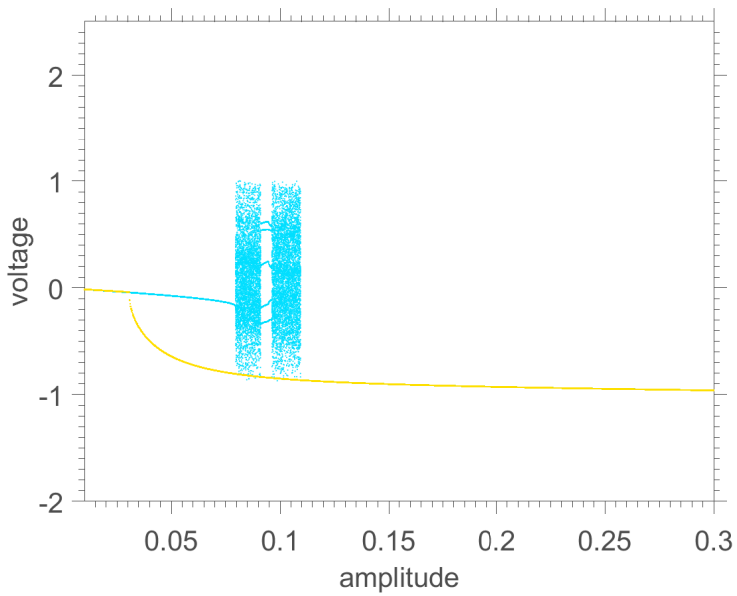
A. Erturk, *Electromechanical Modeling of Piezoelectric Energy Harvesters*, **PhD Thesis**, Virginia Tech, 2009.

*Picture from the above reference.

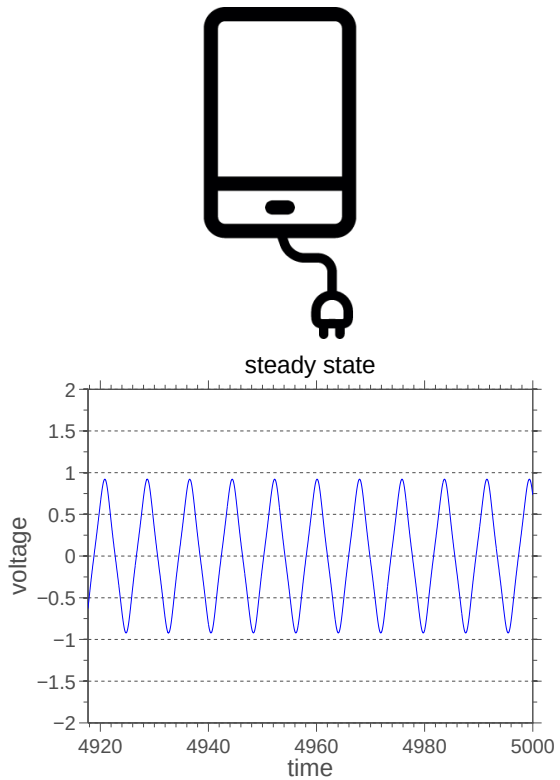
Nonlinearity and chaos

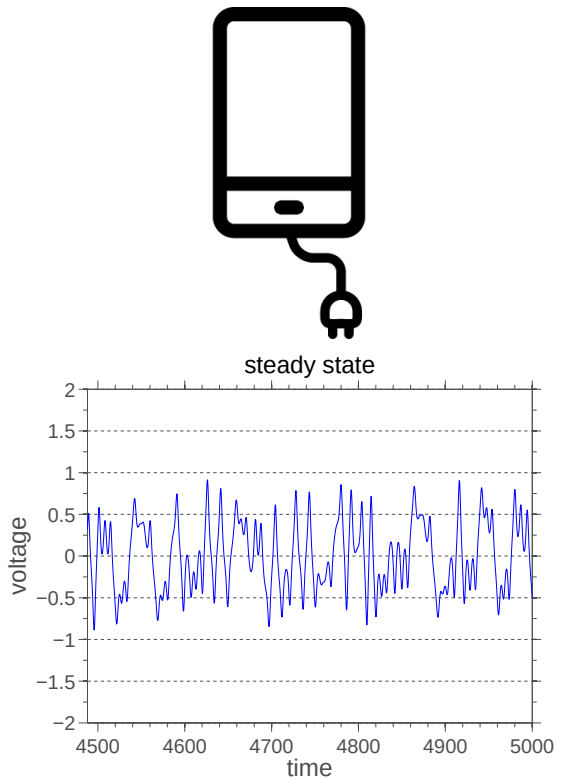
Nonlinearity of this dynamical system may also induce chaos.





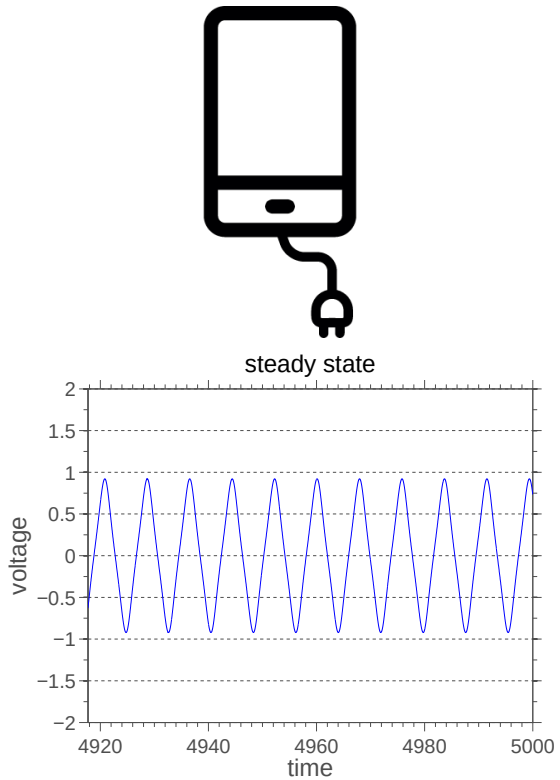
For operation of electrical devices ...

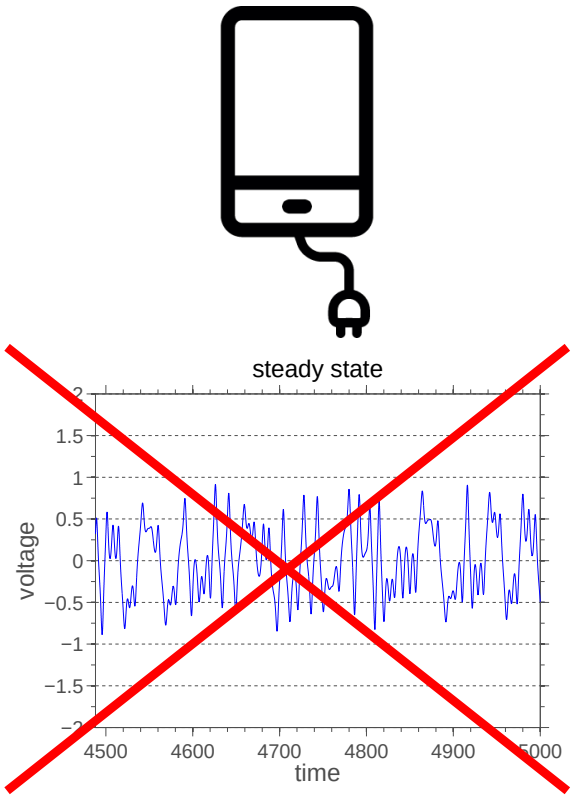




*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

For operation of electrical devices ...

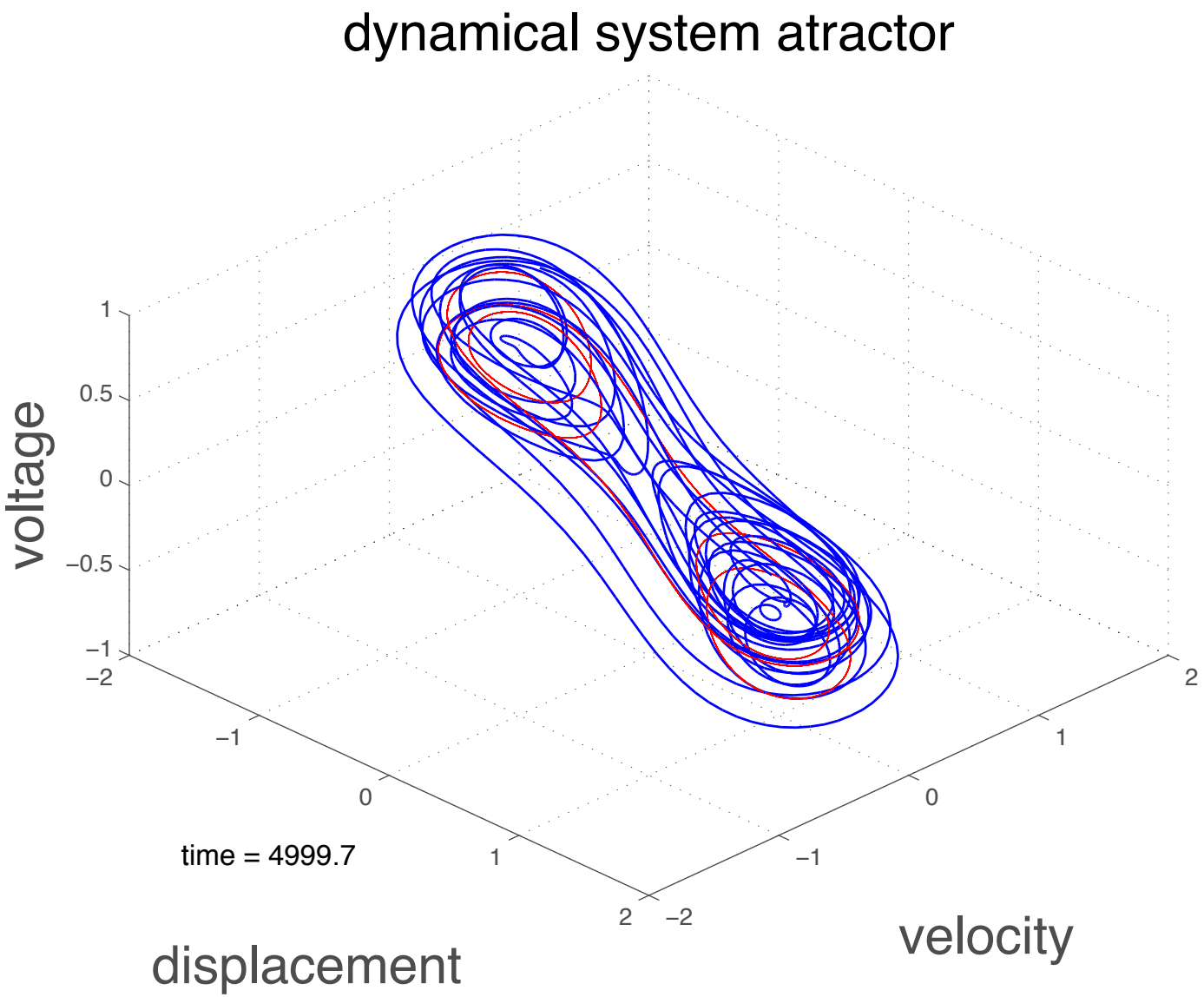




... irregular voltage is undesirable!

*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

UPO embedded into a chaotic atractor



How to explore these unstable periodic orbits?

$\Longrightarrow$ **Techniques for control of chaos**
(known UPO is required)

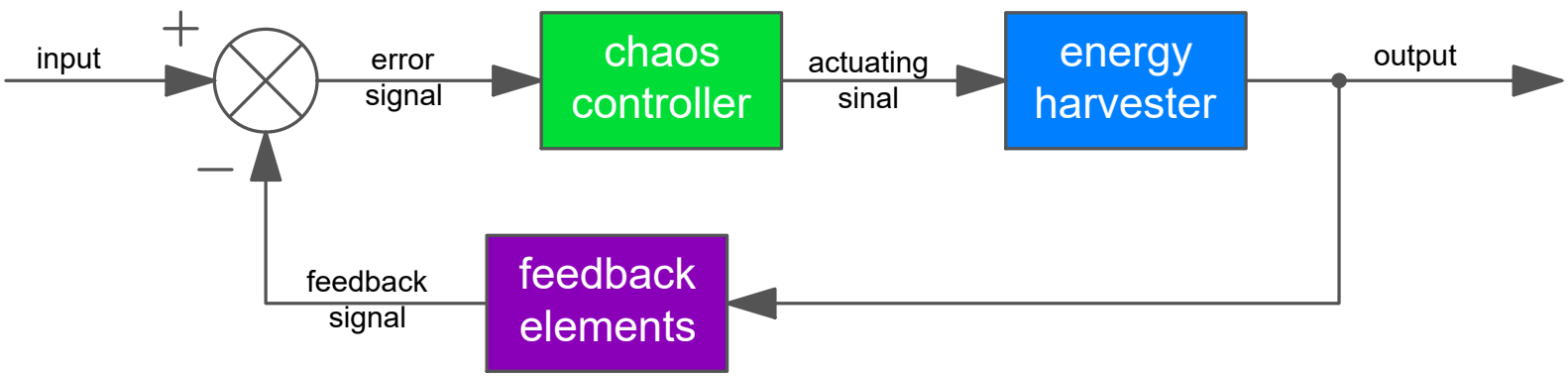
OGY:

- Control performed by a sequence of (discrete) small "kicks" that forces the system trajectory to stay in the target orbit

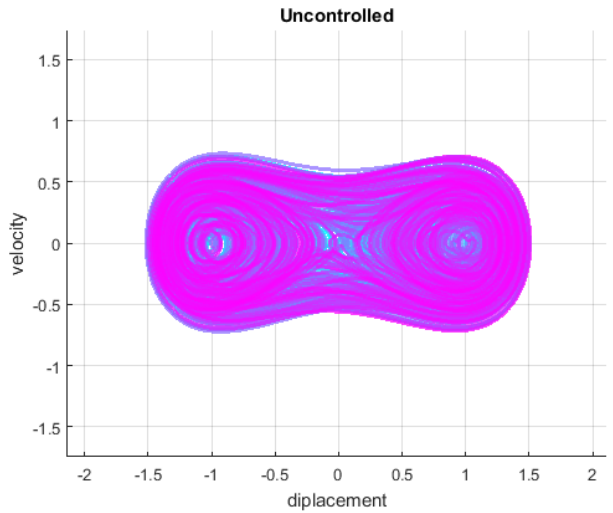
Pyragas:

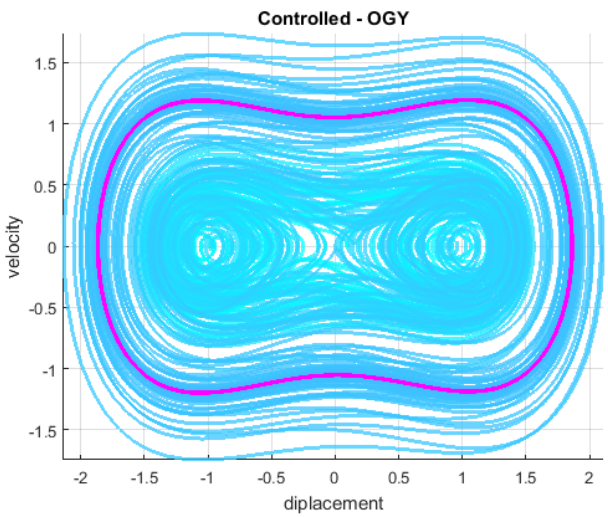
- Control performed by a (continuous) low intensity signal which is almost zero if the system evolves close to the target orbit, and increases when it starts to drift way

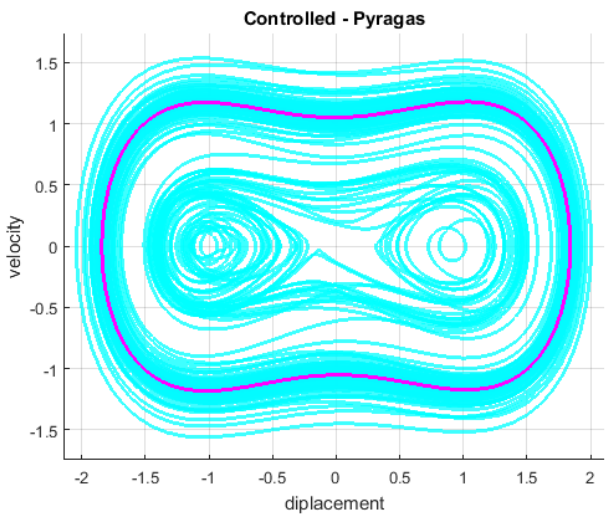
Feedback control law



Stabilization via control of chaos ($f = 0.083$, $\Omega = 0.8$)

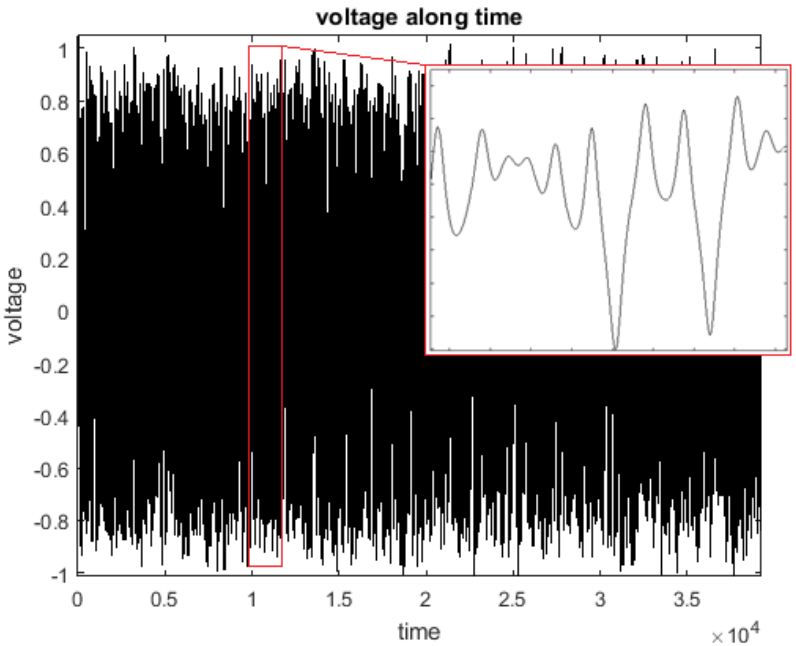




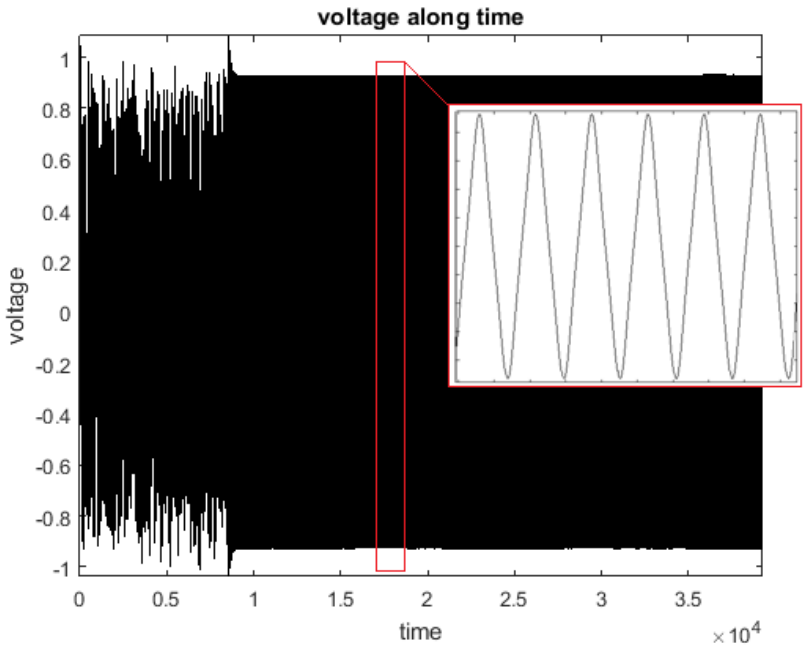


*The colors represent the evolution of the system, from blue to pink, as the time progresses.

Voltage time series ($f = 0.083$, $\Omega = 0.8$)



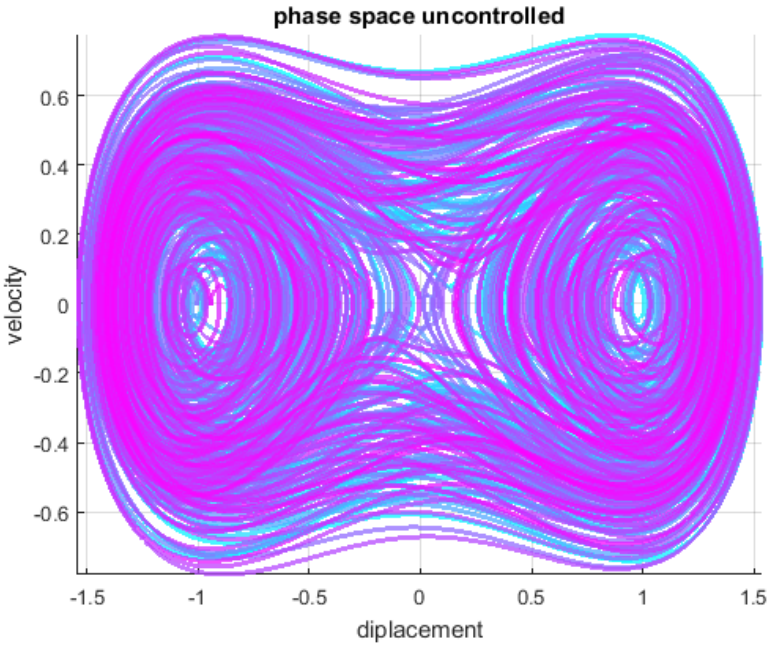
uncontrolled dynamics



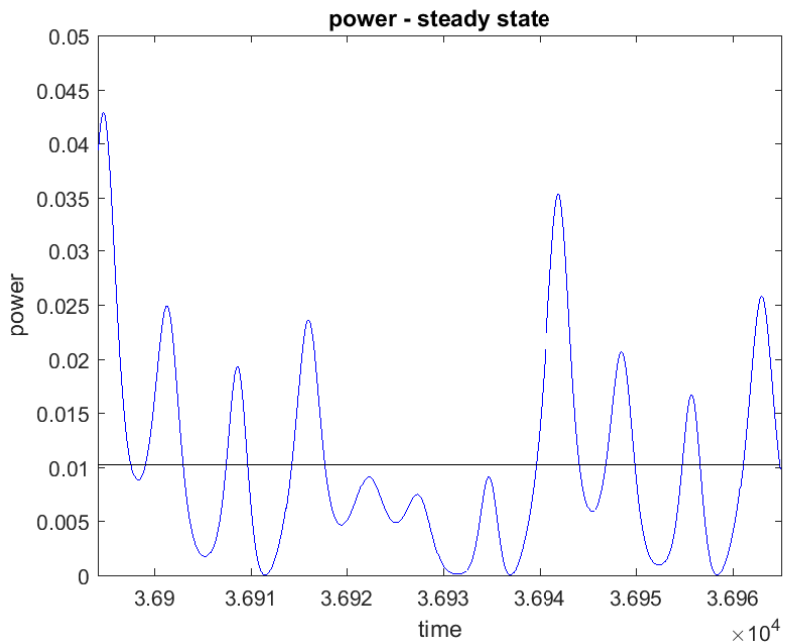
OGY controlled dynamics

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

uncontrolled dynamics



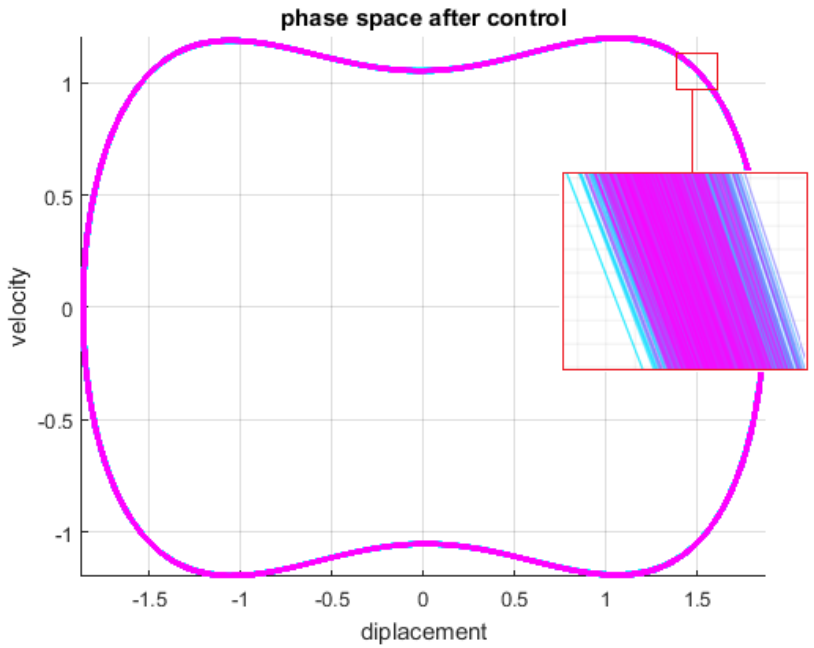
system trajectoty



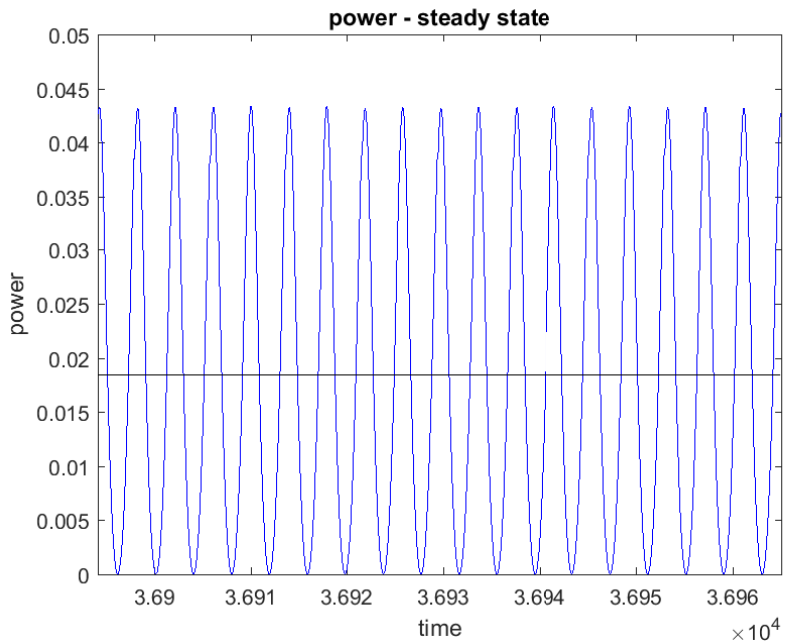
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 1 orbit



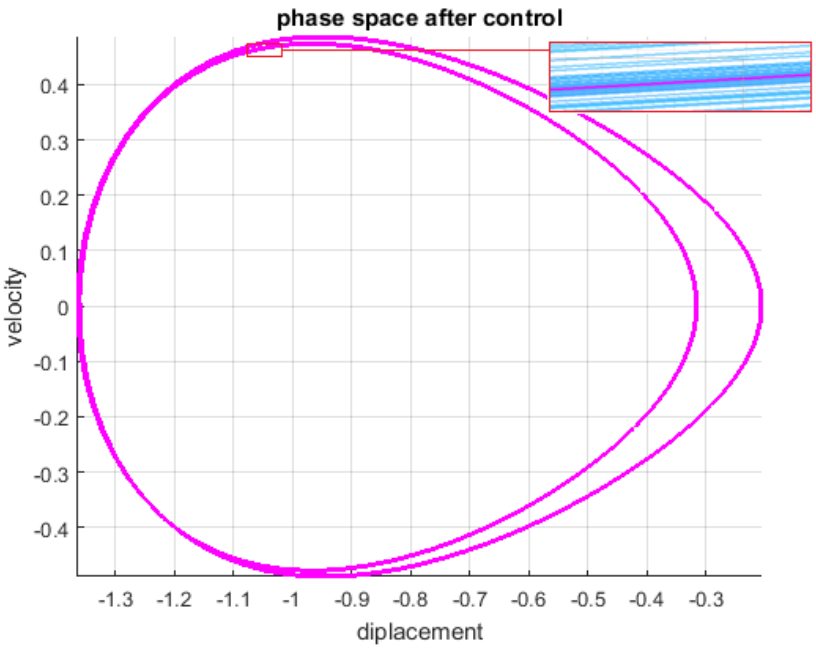
system trajectoty



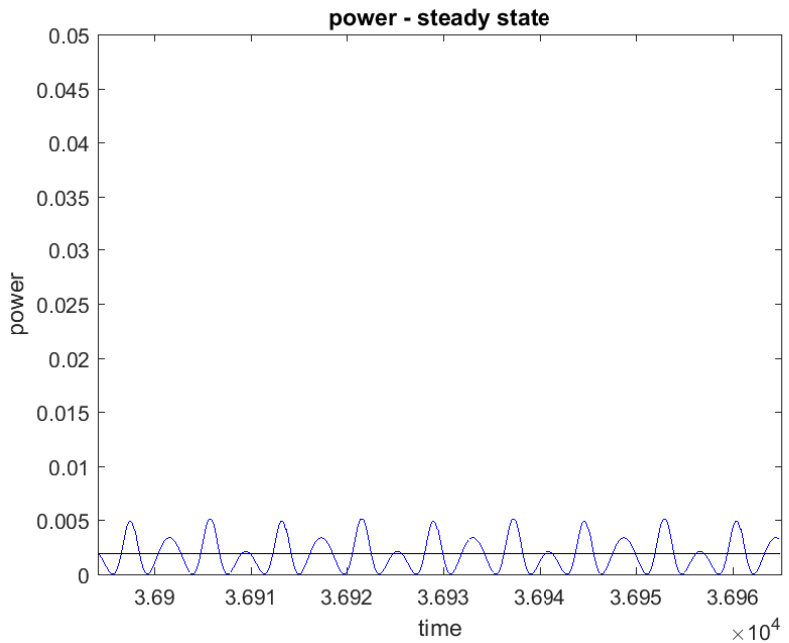
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 2 orbit



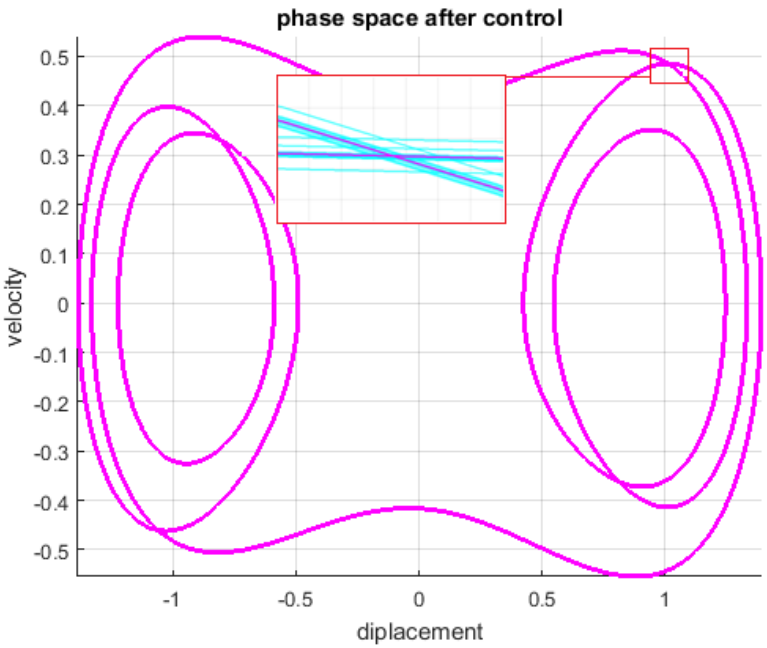
system trajectoty



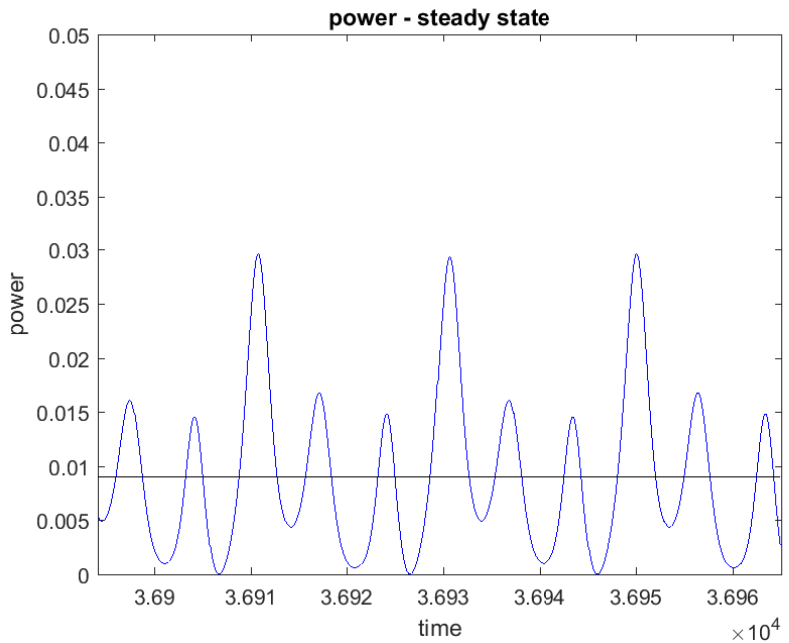
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 5 orbit



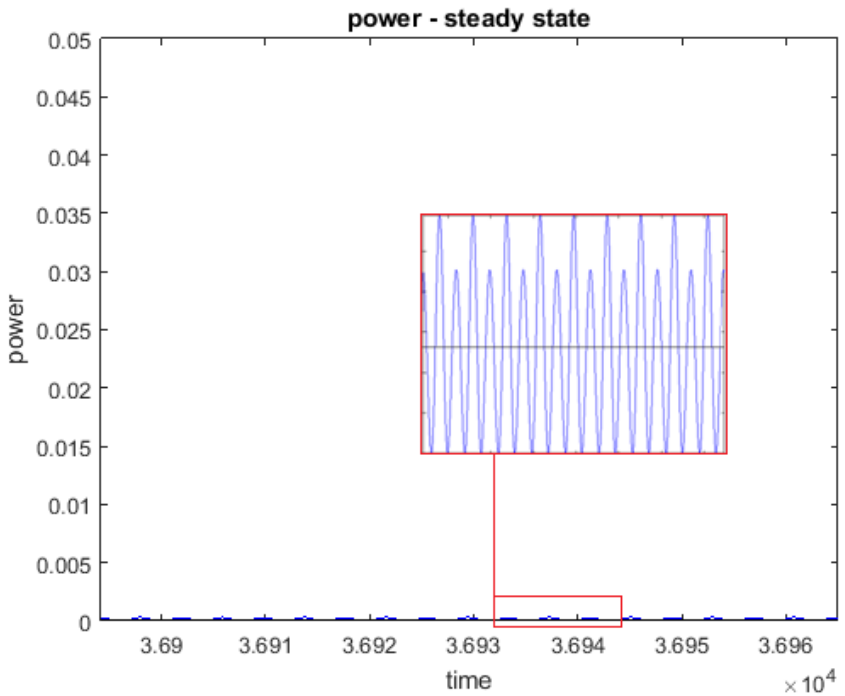
system trajectoty



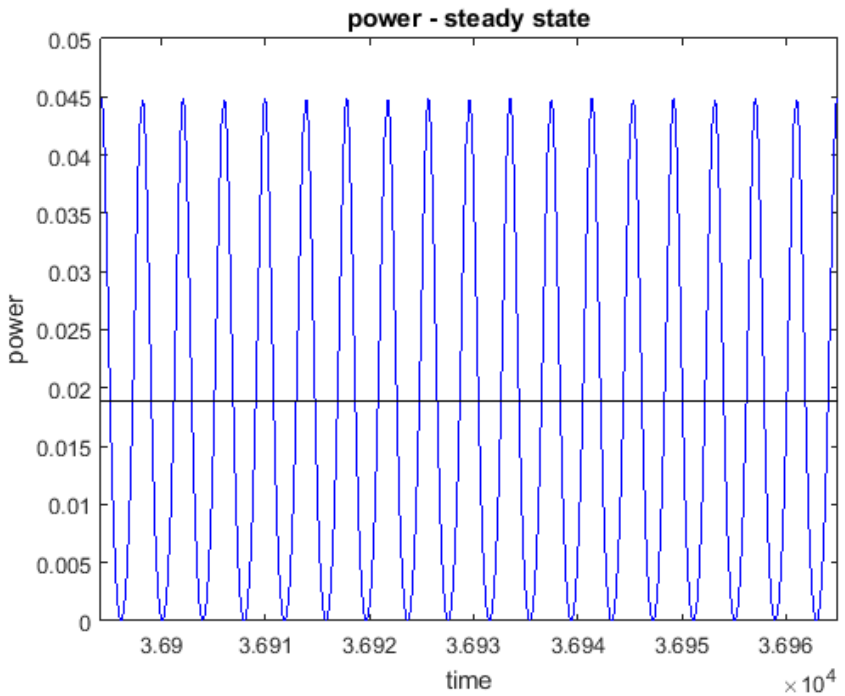
output power

Astonishing improvements are possible: Good!

$$f = 0.050,\ \Omega = 0.8$$



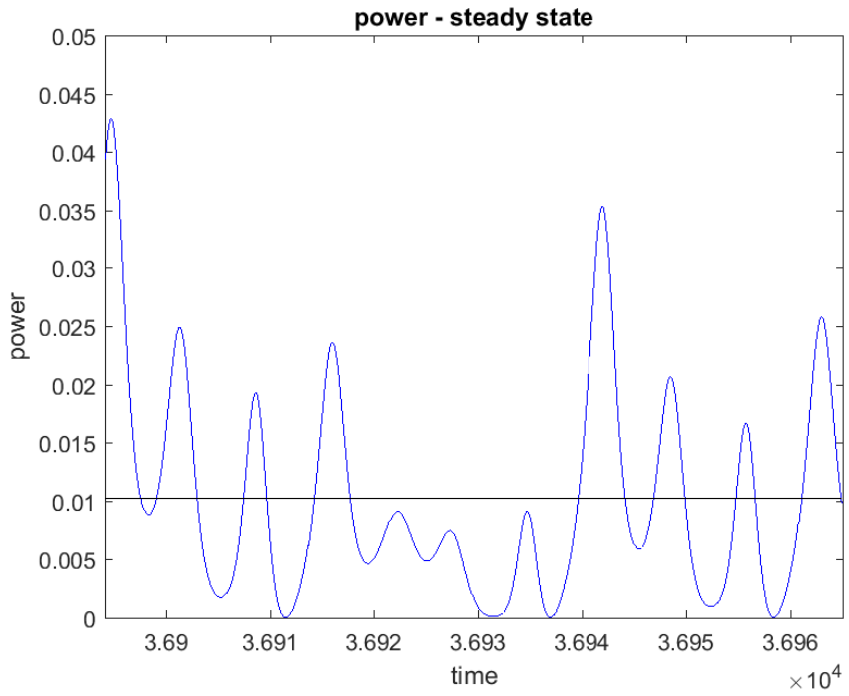
uncontrolled system



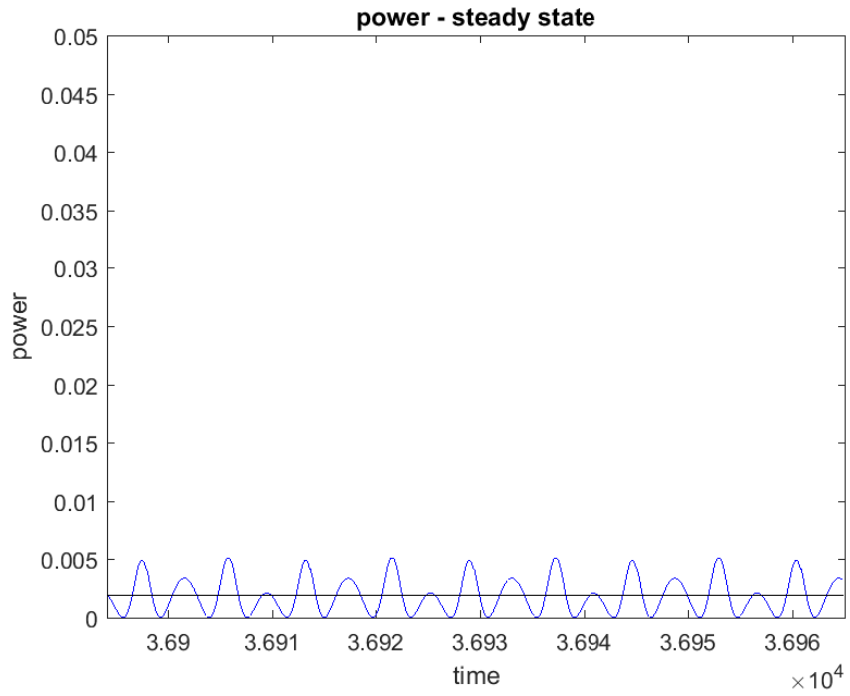
controlled system

Catastrophic effects too: Bad!

$$f = 0.090,\ \Omega = 0.8$$



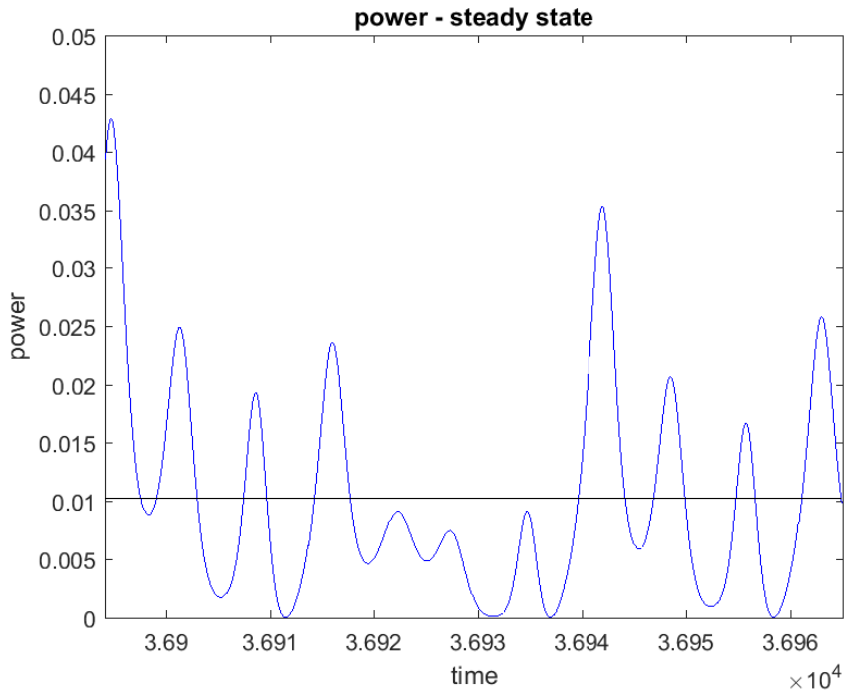
uncontrolled system



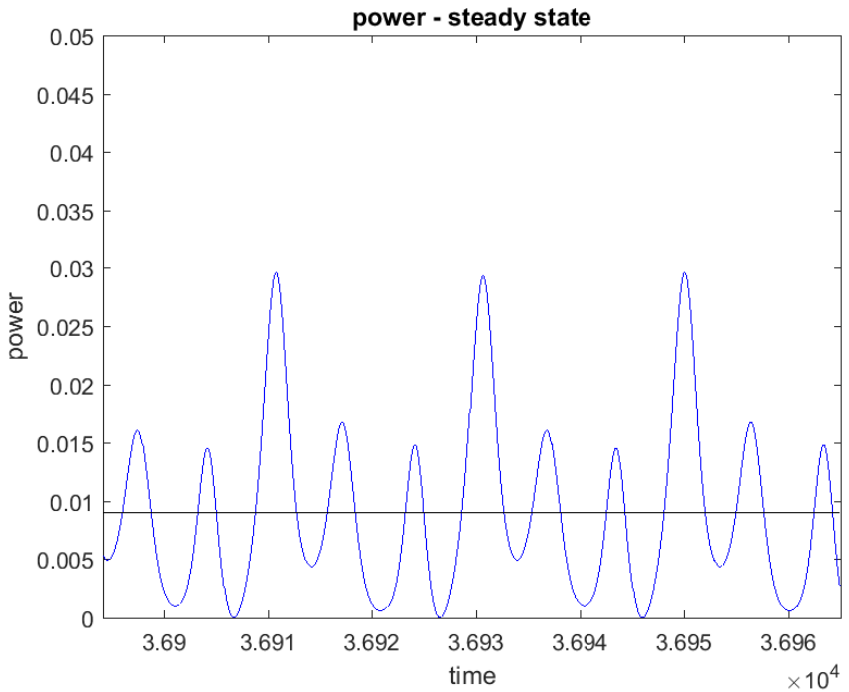
controlled system

And no significative change: Ugly!

$$f = 0.090,\ \Omega = 0.8$$



uncontrolled system



controlled system

Enhancement of power recovery

$\Omega = 0.8$ – steady state

f	uncontrolled	controlled	controller ($\times 10^{-4}$)	effective	enhancement
0.050	0.0001	0.0068	- 0.0005	0.0068	68 $\times$
0.083	0.0073	0.0131	- 0.0006	0.0131	1.8 $\times$
0.090 1-p	0.0077	0.0154	- 0.0001	0.0154	2 $\times$
0.090 2-p	0.0077	0.0037	- 0.0002	0.0037	0.5 $\times$
0.090 5-p	0.0077	0.0084	- 0.0001	0.0084	1.1 $\times$

Section 4

Final Remarks

Final remarks

Some conclusions:

- Nonlinearity is a powerful ingredient for harvesters efficiency:
  - May induce large amplitude responses
  - A plenty of energy is available in nonlinear responses
- Nonlinearity may also be an enemy:
  - May induce (very irregular) chaotic responses
  - Irregular voltage is undesirable for eletronic powering
- Control of chaos may be good, bad and ugly:
  - Astonishing improvements in efficiency are possible
  - Significative reductions of efficiency too
  - Controll of chaos may also be inert to efficiency

Ongoing research:

- Further studies in stochastic dynamics
- Further studies with Pyragas control of chaos
- Construction of an experimental apparatus

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- Profª. Aline de Paula (UnB)
- Prof. Adriano Fabro (UnB)
- Mr. Tiago Pereira (UnB)

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CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

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https://dx.doi.org/10.1007/978-981-13-9463-8_3


J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr, **Dynamic analysis and characterization of a nonlinear bi-stable piezo-magneto-elastic energy harvester**. MATEC Web of Conferences, International Conference on Structural Nonlinear Dynamics and Diagnosis (CSNDD 2018), 241, pp.01001, 2018
https://dx.doi.org/10.1007/978-981-13-9463-8_3


J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr, **Numerically exploring the nonlinear dynamics of a piezo-magneto-elastic energy harvesting device**, Pre-print, 2019
https://hal.archives-ouvertes.fr/hal-02013382


A. Cunha Jr, **Enhancing the performance of a bi-stable energy harvesting device via cross-entropy method**, Pre-print, 2017 https://hal.archives-ouvertes.fr/hal-01531845