Nonlinear analysis of compensated asymmetric energy harvester

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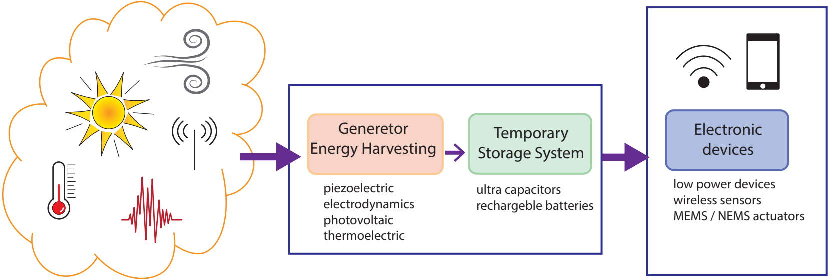
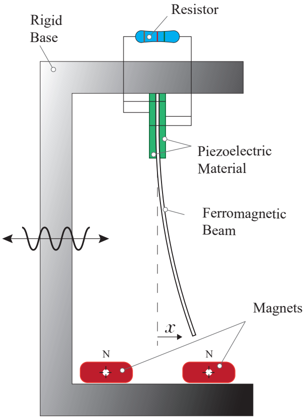
ENERGY HARVESTING AS ALTERNATIVE FOR MICROPOWERING



Adapted from: F. Cottone **Introduction to Vibration Energy Harvesting**. NiPS Energy Harvesting Summer School, August 1-5, 2011.

Classic bistable harvester



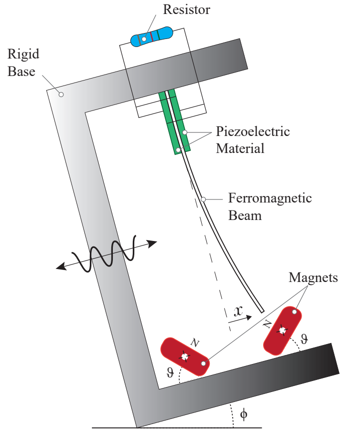
$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi v = f\cos(\Omega t)$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

$+$ initial conditions

A. Erturk, J. Hoffmann and D. Inman **A piezomagnetoelastic structure for broadband vibration energy harvesting**. Applied Physics Letters (2009).

Asymmetric bistable harvester



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 + 2\delta x - x^2) - \chi v =$$

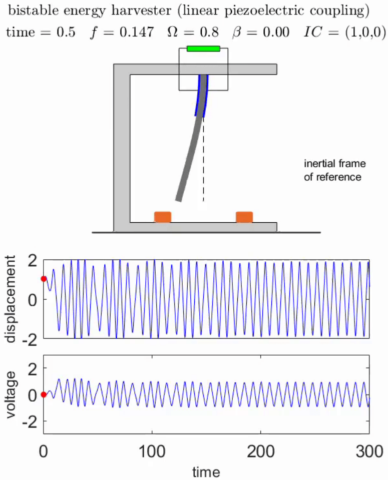
$$f\cos(\Omega t) + p\sin(\phi)$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

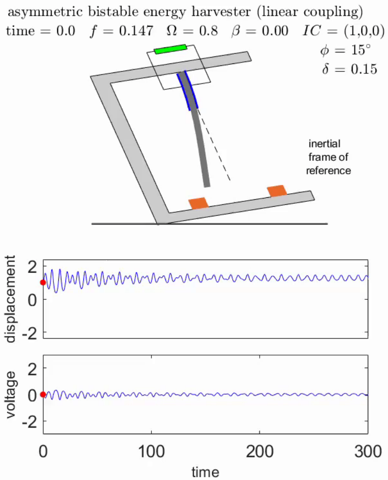
$$+ \text{ initial conditions}$$

W. Wang, C. Cao, C.R. Bowen, D.J. Inman, J. Lin **Performance enhancement of nonlinear asymmetric bistable energy harvesting from harmonic, random and human motion excitations**. Applied Physics Letters (2018)

Dynamic animation: symmetric × asymmetric



Global sensitivity analysis of (a)symmetric energy harvesters (2021)
J. P. Norenberg, A. Cunha Jr, S. da Silva, P. S. Varoto



Global sensitivity analysis of (a)symmetric energy harvesters (2021)
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Objective

Study asymmetric and compensated asymmetric bistable energy harvester:

1. Bifurcation diagram (amplitude and frequency excitation)
2. Basins of attraction based Test 0-1

Compensated asymmetric bistable harvester

Nonlinear restoring force:

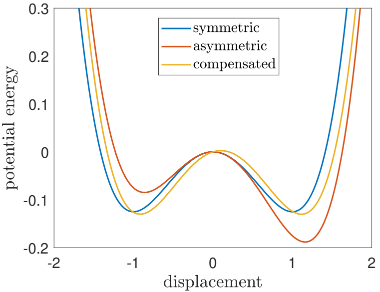
$$F_r(x) = -\frac{1}{2}\, x\,(1 + 2\delta\, x - x^2) - p\sin(\phi)$$

Equilibrium points are:

$$x_{1,2} = \frac{2\delta \pm \sqrt{4\delta^2 + 3}}{3}.$$

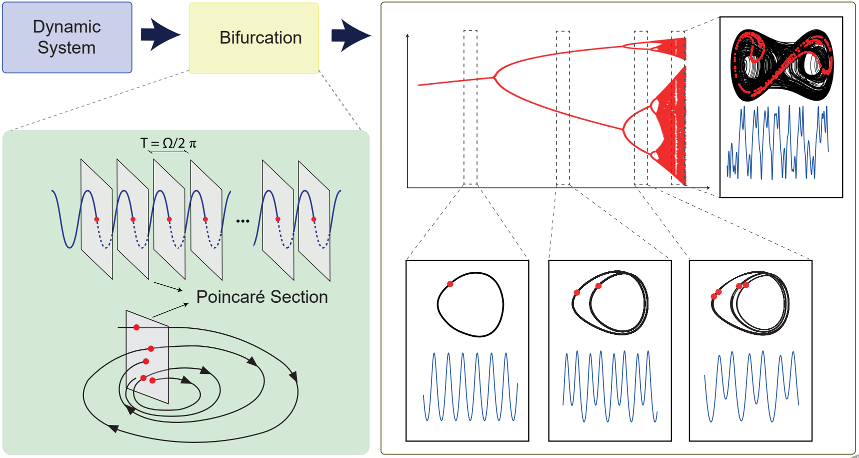
Assuming that $F_r(x_1) + F_r(x_2) = 0$

$$\phi_{opt} = \arcsin\left(\frac{8\delta^3 + 9\delta}{27p}\right)$$

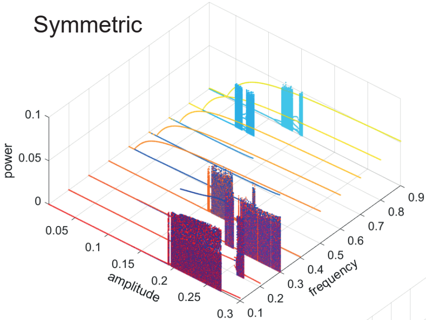


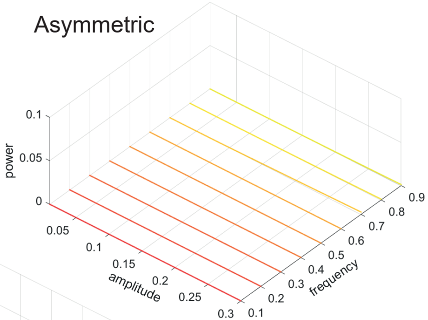
W. Wang, C. Cao, C. R. Bowen, Y. Zhang, J. Lin **Nonlinear dynamics and performance enhancement of asymmetric potential bistable energy harvesters**. Nonlinear Dynamics (2018)

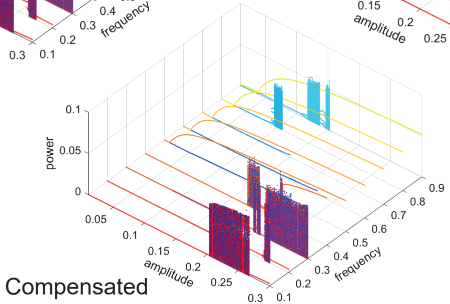
Bifurcation analysis



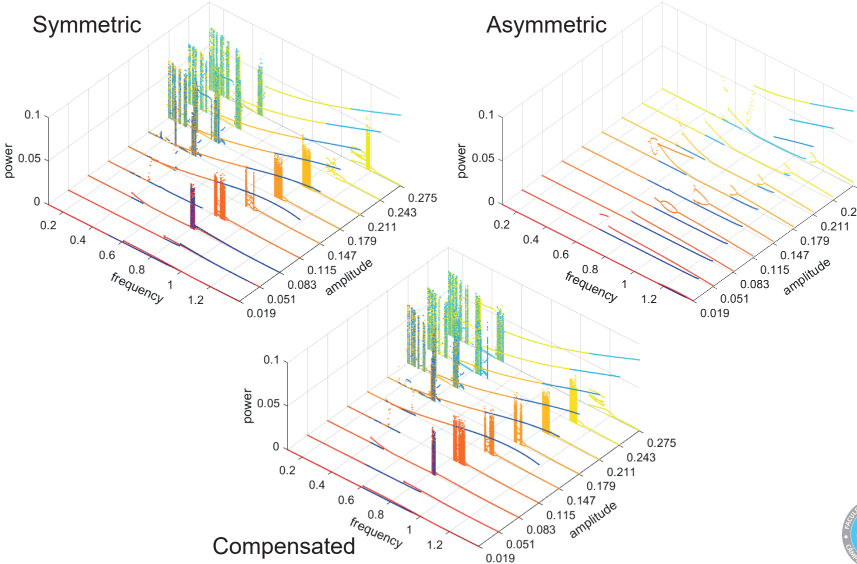
Bifurcation diagram: amplitude of excitation



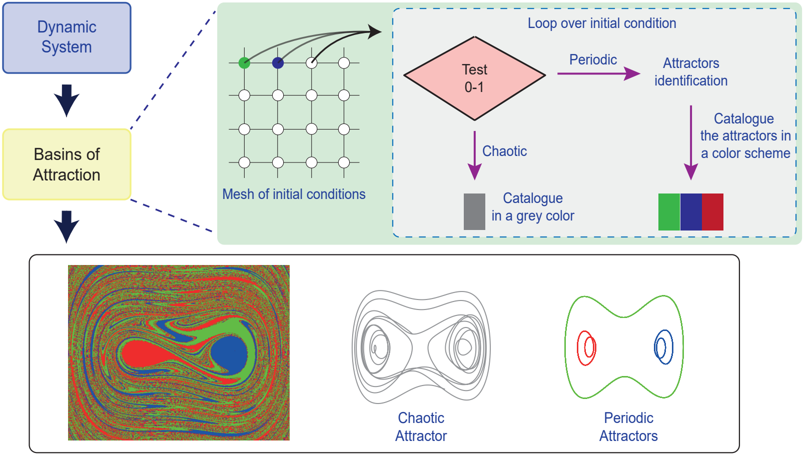




Bifurcation diagram: frequency of excitation



Basins of attraction



Test 0-1 for chaos

- The Test 0-1 for chaos is used to characterize the dynamic numerically
- This classification is obtained through time-series response
- It is cheaper than Lyapunov exponents
- It is a statistical method
- Depends on sampling process

G. Gottwald, I. Melbourne **The 0-1 test for chaos: A review.**. Chaos Detection and Predictability (2016)

Test 0-1 for chaos: Euclidean extension

The time-serial $x(t)$ is divided into two coordinates:

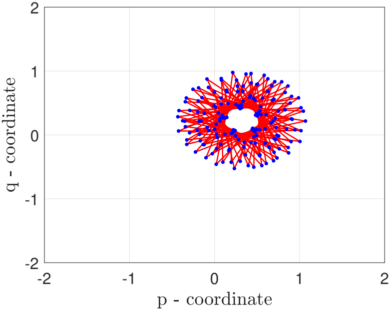
$$p_n(c) = \sum_{j=1}^{n} x(t_j) \cos jc$$

$$q_n(c) = \sum_{j=1}^{n} x(t_j) \sin jc$$

which c is a random value with support $(0, 2\pi)$ and $n = 1, 2, \ldots, N$

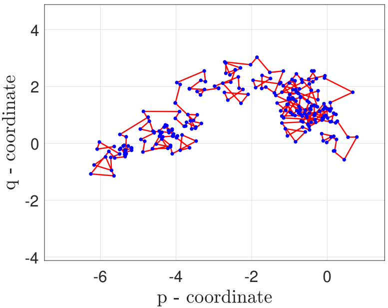
Euclidean extension: regular × chaotic

Regular behavior



non-diffuse projection

Chaotic behavior



diffuse projection

Test 0-1 for chaos: statistical classifier

The mean square deviation is calculated by:

$$M_n(c) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{n} ([p_{j+n}(c) - p_j(c)]^2 + [q_{j+n}(c) - q_j(c)]^2)$$

Finally, the classifier is defined by

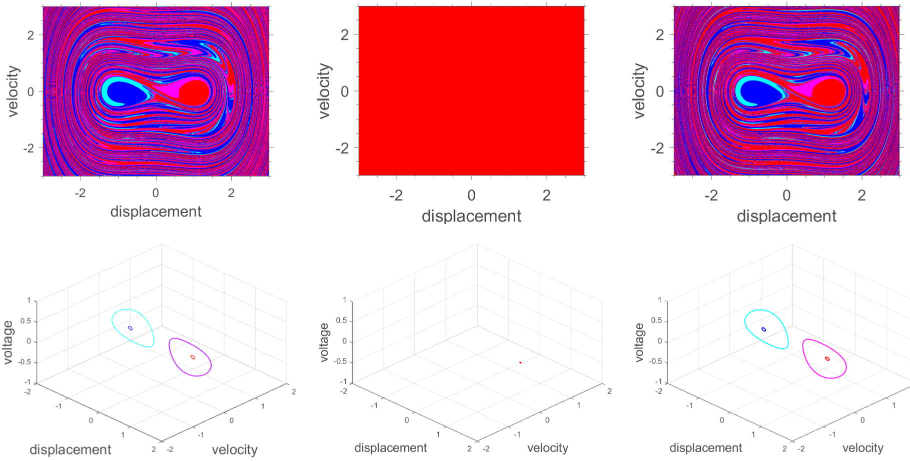
$$K_c = \lim_{N \to \infty} \frac{cov(t_n, M_n)}{\sqrt{var(t_n) var(M_n)}}$$

where $M_n = (M_1, M_2, \ldots, M_n)$, $t_n = (t_1, t_2, \ldots, t_n)$, *cov* and *var* are covariance and variance operator.

$$\begin{cases} K_c < 0.2 & \Rightarrow \text{regular} \\ 0.2 \leq K_c \leq 0.8 & \Rightarrow \text{inconclusive} \\ K_c > 0.8 & \Rightarrow \text{chaotic} \end{cases}$$

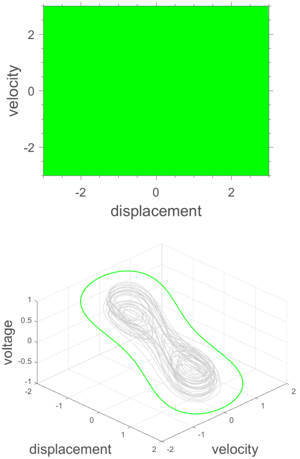
$f = 0.019$ and $\Omega = 0.8$

Symmetric | Asymmetric | Compensated asymmetric

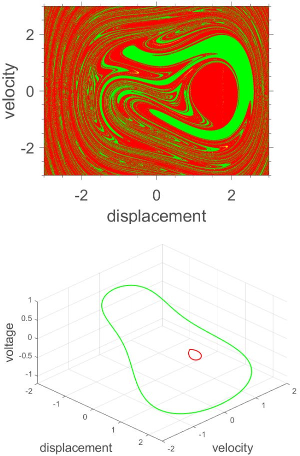


$f = 0.211$ and $\Omega = 0.8$

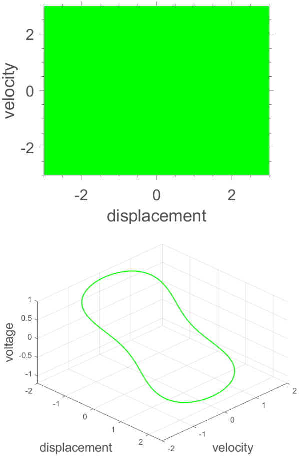
Symmetric



Asymmetric



Compensated asymmetric



Final remarks

- Dynamic investigations of symmetric and asymmetric models were performed
- An optimal asymmetric model was obtained
- Bifurcation diagrams of amplitude and frequency of excitation were plotted
- Basins of attraction based on Test 0-1 for chaos were proposed
- Asymmetries demonstrated harmful for harvesting energy
- The optimal value of the sloping angle compensated the asymmetric potential
- The dynamic behavior of the optimal asymmetric model is similar to the symmetric

Code availability

Contents lists available at ScienceDirect

Software Impacts

journal homepage: www.journals.elsevier.com/software-impacts

Original software publication

STONEHENGE — Suite for nonlinear analysis of energy harvesting systems

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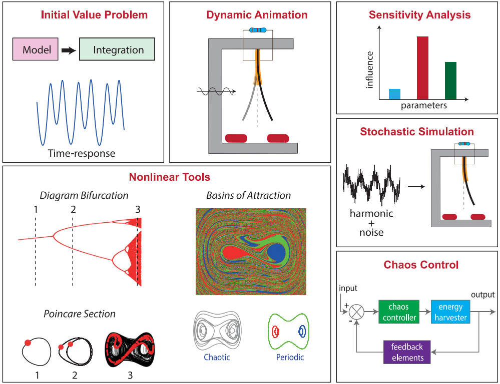
[b] Rio de Janeiro State University – UERJ, Rio de Janeiro - RJ, Brazil



STONEHENGE

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Thank you for your attention

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