Probabilistic analysis of an asymmetric bistable energy harvester

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Samuel da Silva[1] Paulo Sérgio Varoto[3]

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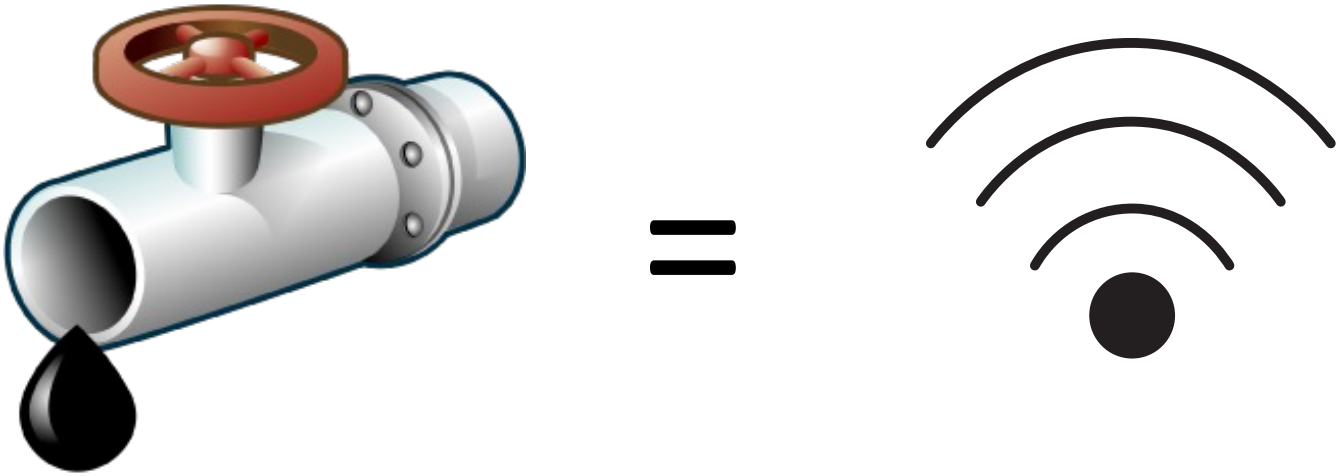
Third International Nonlinear Dynamics Conference (NODYCON 2023)

Rome, Italy, June 19-22, 2023

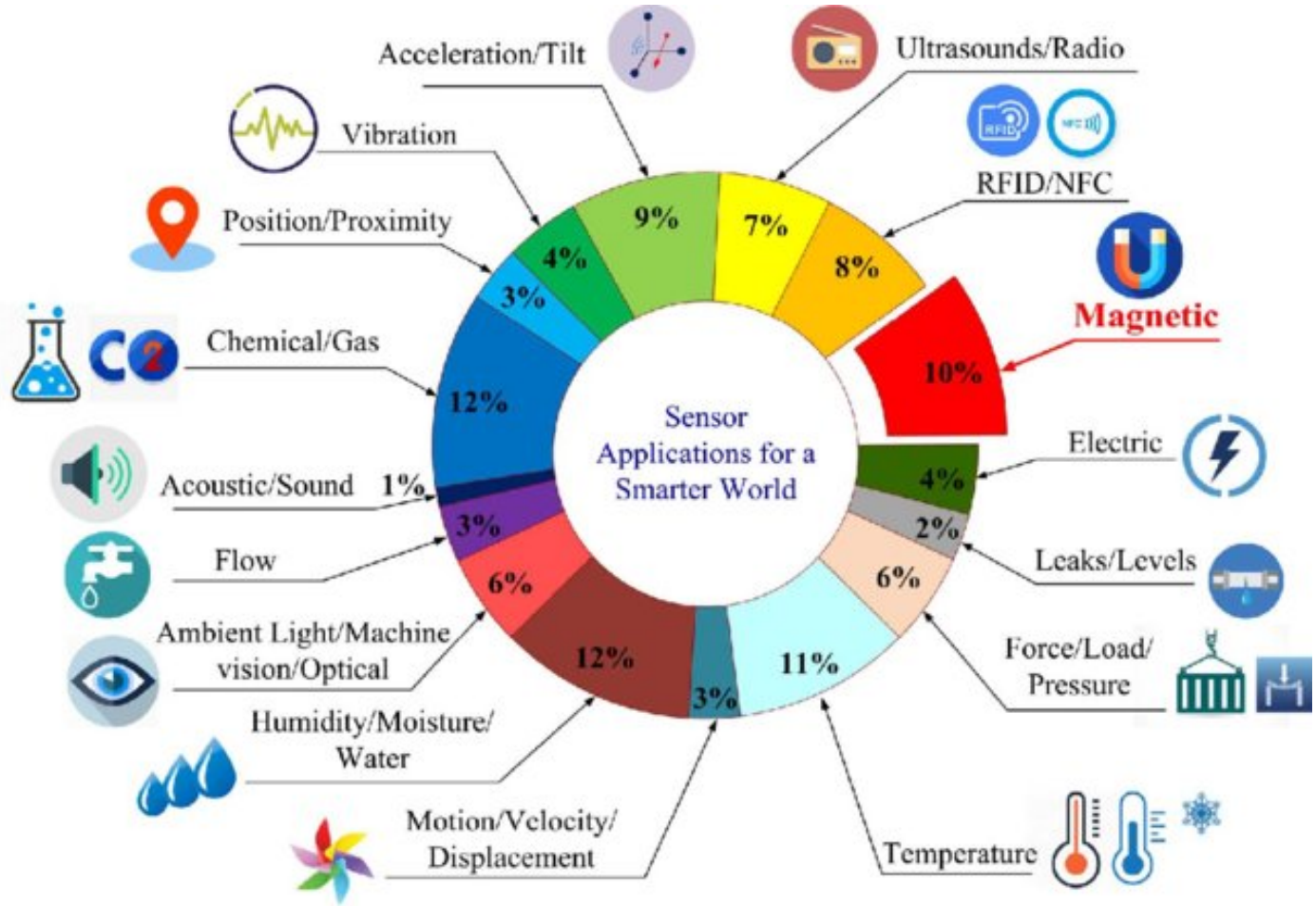
Is data (in fact) the new oil?

https://www.economist.com/leaders/2017/05/06/the-worlds-most-valuable-resource-is-no-longer-oil-but-data

If so, sensors are the new pipeline!



IoT will demand more than a trillion sensors by 2025



L. Xuyang et al., *Overview of Spintronic Sensors With Internet of Things for Smart Living,*

IEEE Transactions on Magnetics, 0018-9464, 2019.

Energy and environmental challenges are huge!

Up to 78 million batteries will be discarded daily by 2025, researchers warn

An EU-funded project makes it plain: batteries need to live longer than the Internet of things (IoT) devices they power. A position paper published by the project outlines the key actions to power IoT devices reliably and sustainably.

CLIMATE CHANGE AND ENVIRONMENT

ENERGY

SCIENTIFIC ADVANCES

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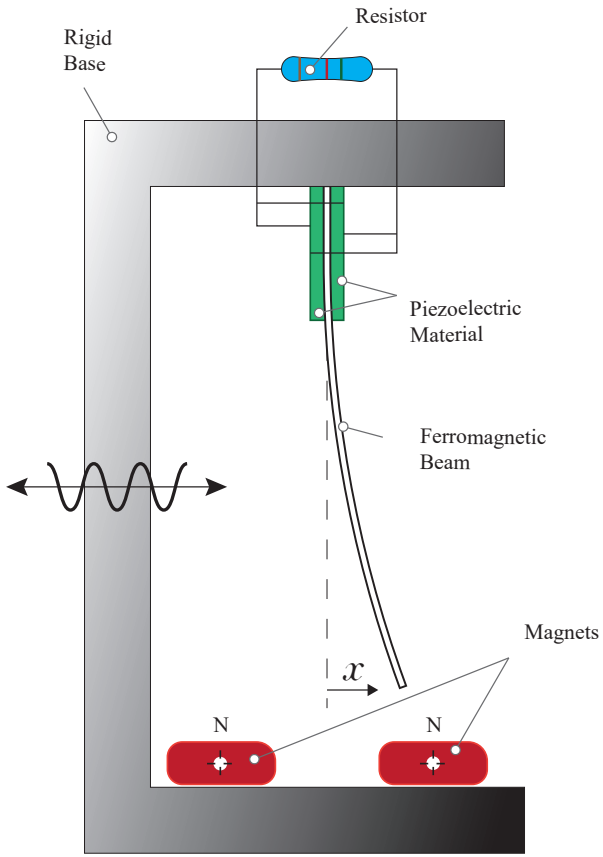
CORDIS — European Commission, *Up to 78 million batteries will be discarded daily by 2025, researchers warn (July 23, 2021)*

Research objectives

Study variability effects on an asymmetric bistable energy harvester with nonlinear transduction, seeking to find paths to optimize the energy-recovering process

- Global sensitivity analysis:
  1. Extract features of the dynamic behavior
  2. Identifies the most sensitivity parameters
- Uncertanty quantification:
  1. Parametric variability modeling
  2. Low-dimensional probabilistic models
  3. Probabilistic maps addressing uncertainty

Bistable harvester: symmetric + linear coupling



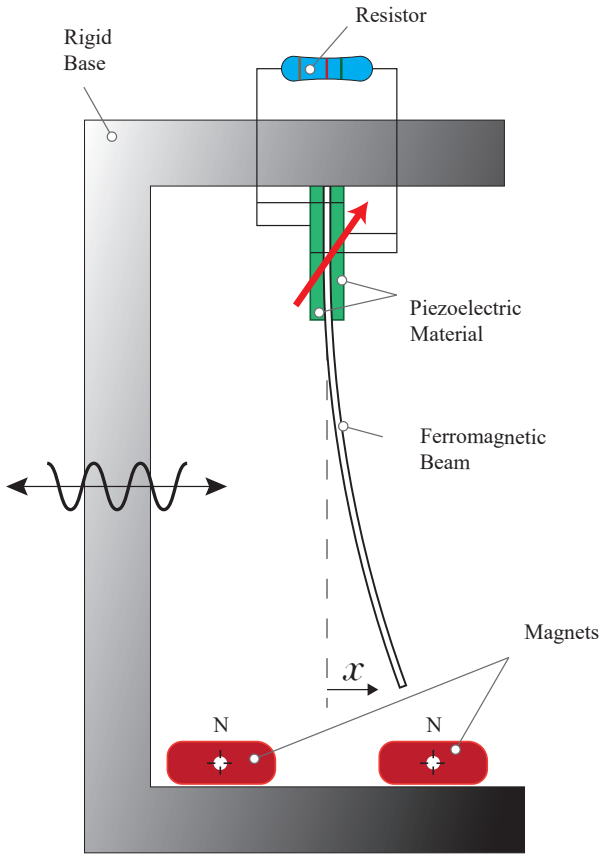
$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi v = f\cos(\Omega t)$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

$+$ initial conditions

A. Erturk, J. Hoffmann and D. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*, **Applied Physics Letters**, 2009

Bistable harvester: symmetric + nonlinear coupling



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1-x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t)$$

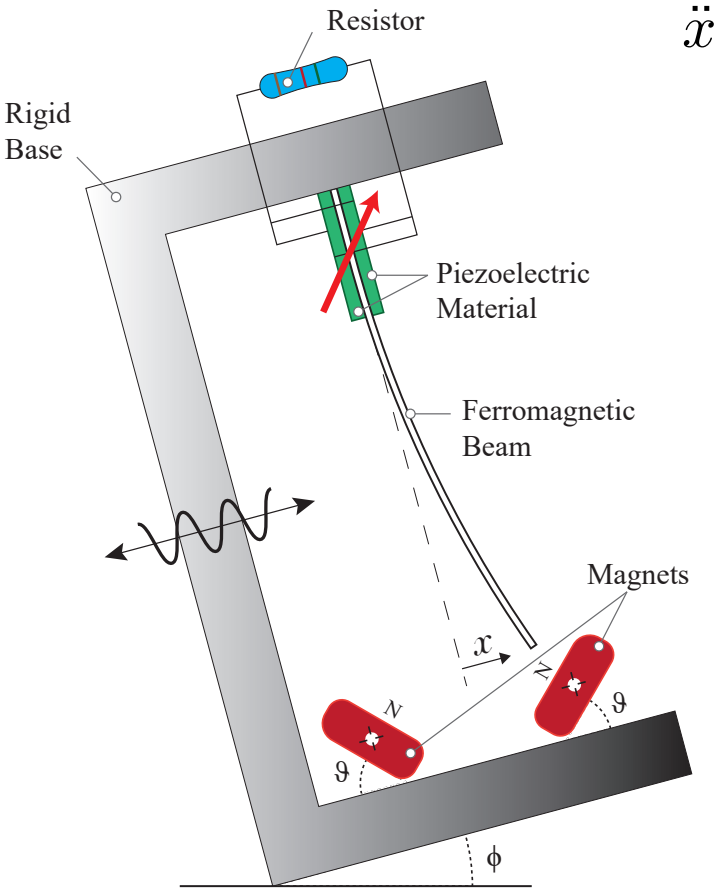
$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

$$+ \quad \text{initial conditions}$$

$$\widehat{\Theta}(x) = (1 + \beta|x|)$$

A Triplett, DD Quinn, *The effect of non-linear piezoelectric coupling on vibration-based energy harvesting*, **Journal of Intelligent Material Systems and Structures**, 2009.

Bistable harvester: asymmetric + nonlinear coupling



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 + 2\delta x - x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t) + p\sin(\phi)$$

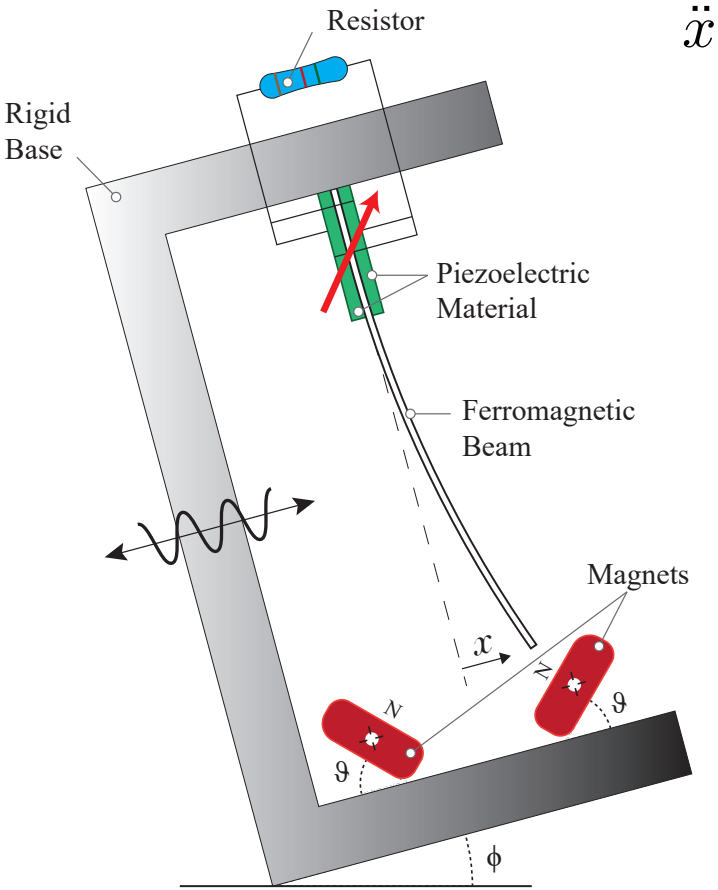
$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta|x|)$$

W. Wang et al., *Performance enhancement of nonlinear asymmetric bistable energy harvesting from harmonic, random and human motion excitations.* **Applied Physics Letters**, 2018

Bistable harvester: asymmetric + nonlinear coupling



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 + 2\delta x - x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t) + p\sin(\phi)$$

$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

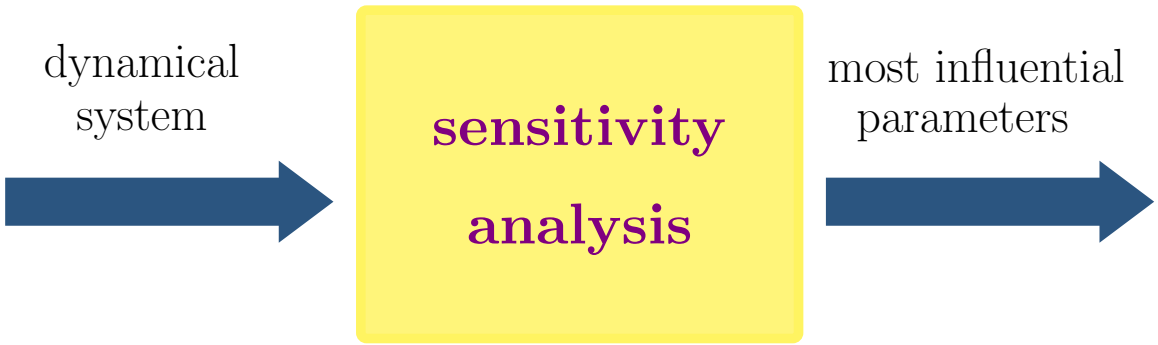
$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta|x|)$$

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} \lambda\, v(t)^2\, dt$$

W. Wang et al., *Performance enhancement of nonlinear asymmetric bistable energy harvesting from harmonic, random and human motion excitations.* **Applied Physics Letters**, 2018

Global sensitivity analysis



- Insight about nontrivial parameters interactions
- Construction of simpler probabilistic models
- Important step for design and optimization

Sensitivity analysis based on variance decomposition

Mathematical Model:

$$Y = \mathcal{M}(\mathbf{X}) \quad , \quad X_i \sim \mathcal{U}(0,1)$$

Hoeffding-Sobol decomposition:

$$Y = \mathcal{M}_0 + \sum_{i=1}^{k} \mathcal{M}_i\left(X_i\right) + \sum_{i<j}^{k} \mathcal{M}_{ij}\left(X_i, X_j\right) + \cdots + \mathcal{M}_{1\ldots k}\left(X_1 \ldots X_k\right)$$

An orthogonal decomposition in terms of conditional expectations:

- $\mathcal{M}_0 = \mathbb{E}\{Y\}$
- $\mathcal{M}_i(X_i) = \mathbb{E}\left\{Y \,|X_i\right\} - \mathcal{M}_0$
- $\mathcal{M}_{ij}(X_i, X_j) = \mathbb{E}\left\{Y \,\middle|X_i, X_j\right\} - \mathcal{M}_i - \mathcal{M}_j - \mathcal{M}_0$
- $\ldots$

Sobol' indices

First-order Sobol' indices:

$$S_i = Var[\mathcal{M}_i(X_i)] \ / \ Var[\mathcal{M}(\mathbf{X})]$$

(quantify the additive effect of each input separately)

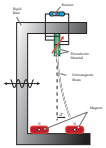
Second-order Sobol' indices:

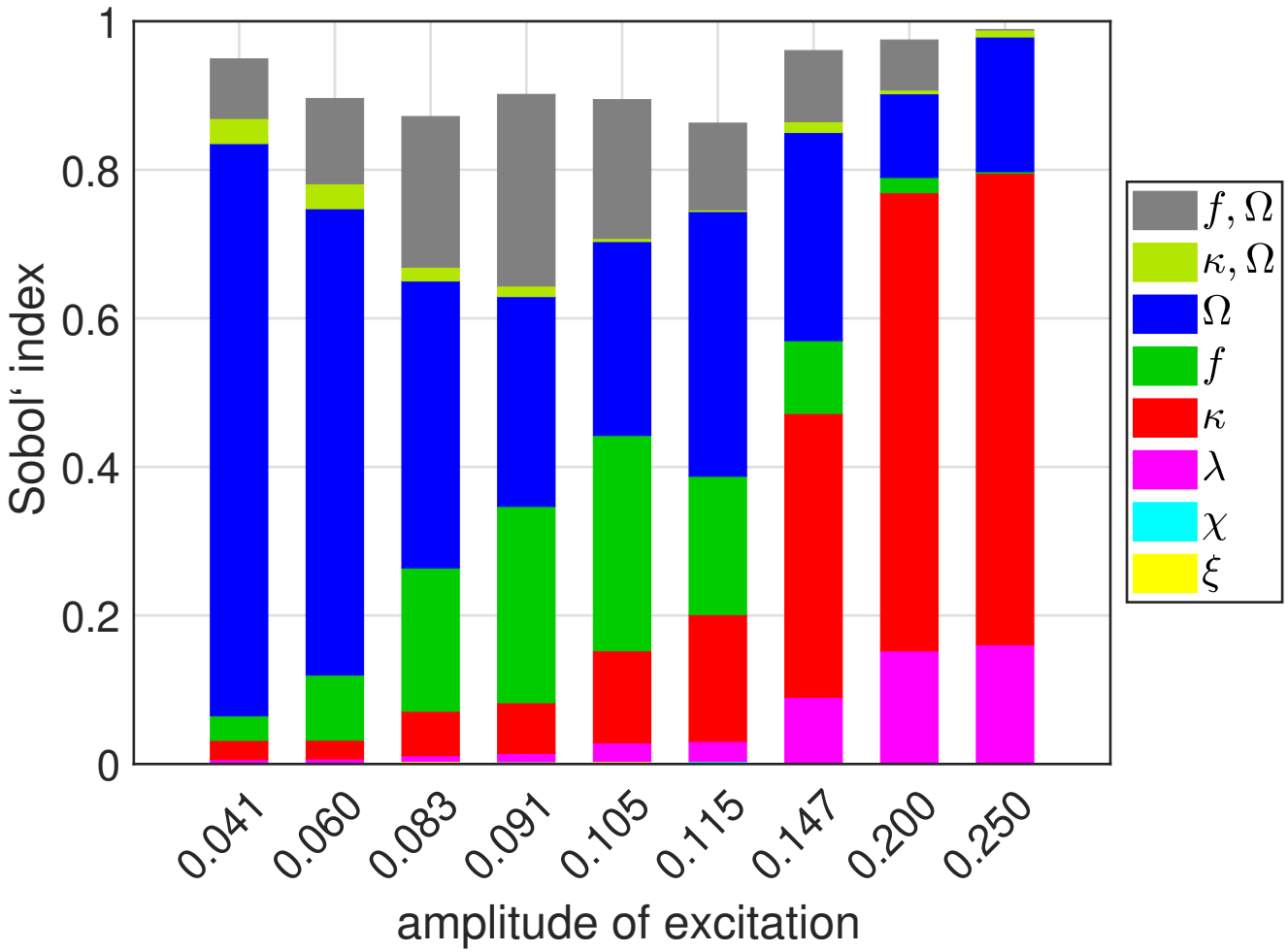
$$S_{ij} = Var[\mathcal{M}_{ij}(X_i, X_j)] \ / \ Var[\mathcal{M}(\mathbf{X})]$$

(quantify interaction effect of inputs X_i and X_j)

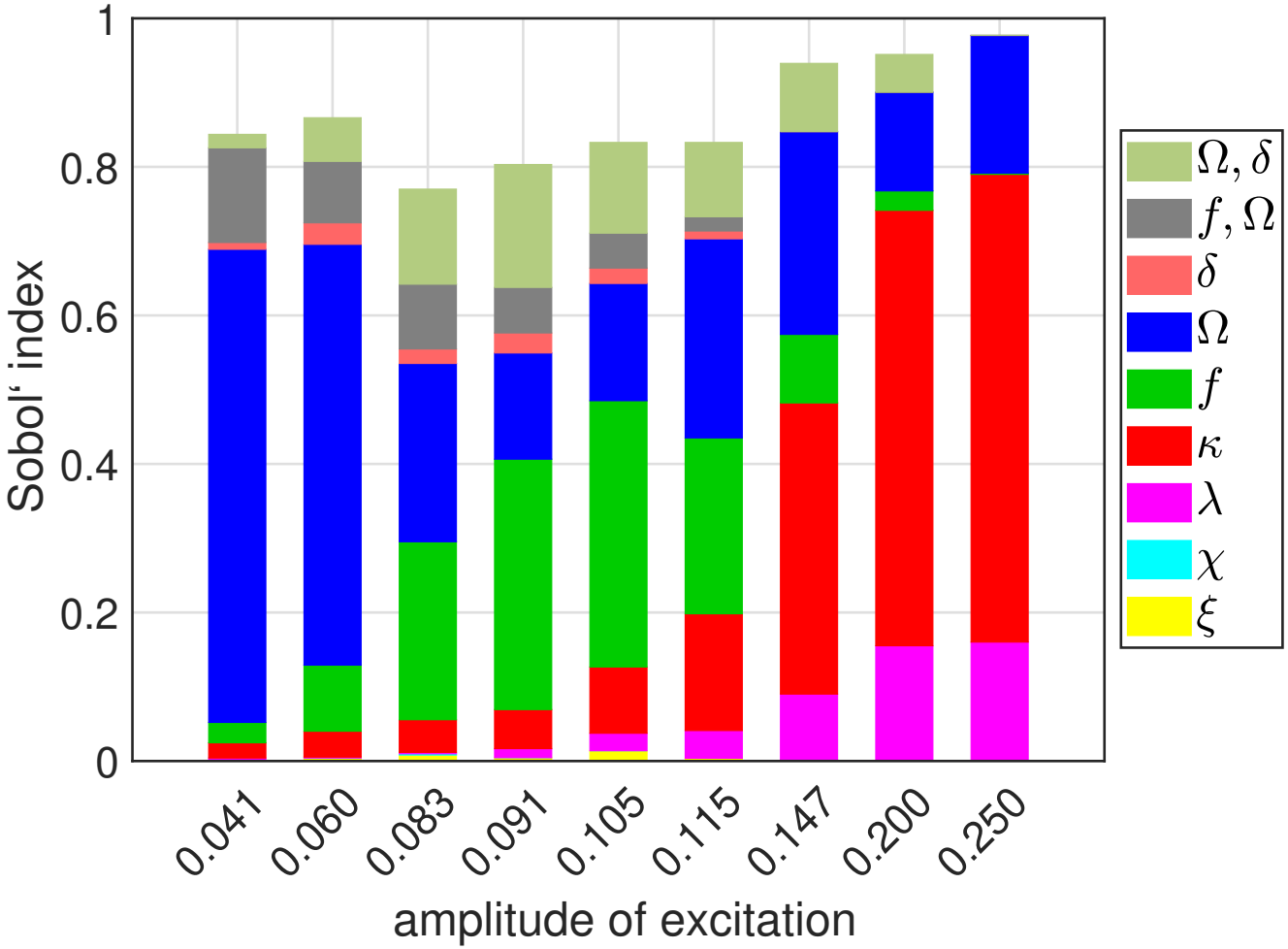
I.M. Sobol *Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates.* **Mathematics and Computers in Simulation**, 55(1-3): 271-280, 2001.

Mean power sensitivity: symmetric system

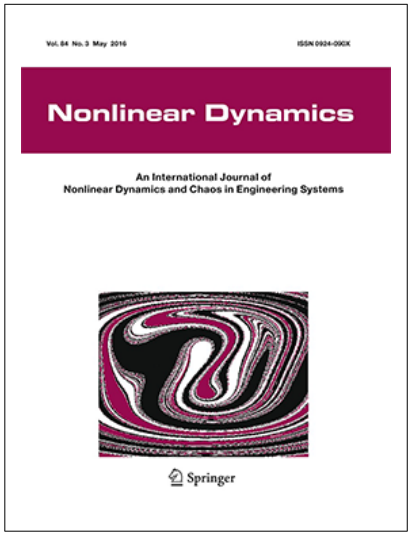




Mean power sensitivity: asymmetric system



Download the paper for details!



Nonlinear Dyn (2022) 109:443–458
https://doi.org/10.1007/s11071-022-07563-8

ORIGINAL PAPER

Global sensitivity analysis of asymmetric energy harvesters

João Pedro Norenberg · Americo Cunha Jr ·
Samuel da Silva · Paulo Sergio Varoto

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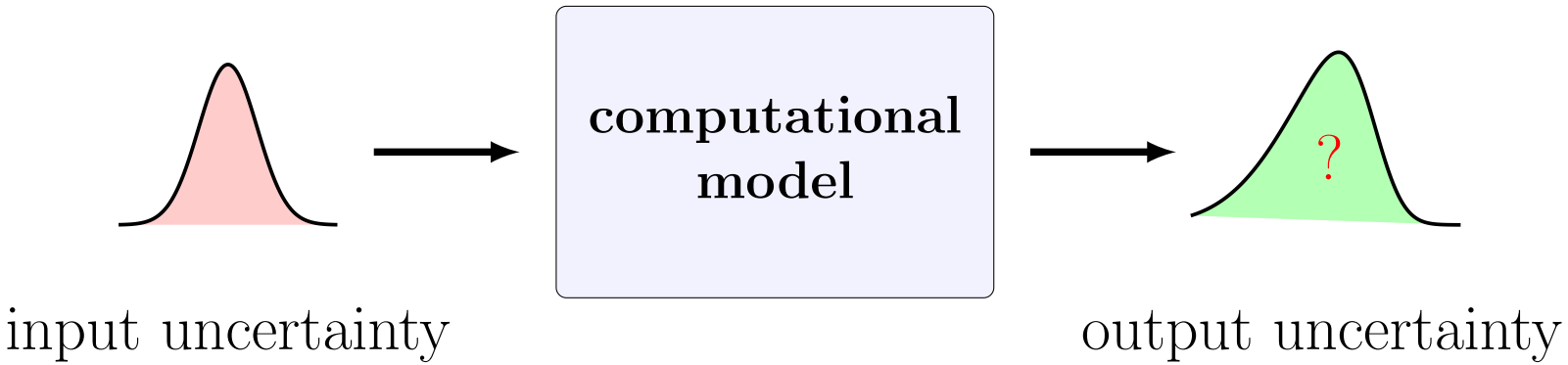
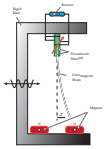
Uncertainty quantification

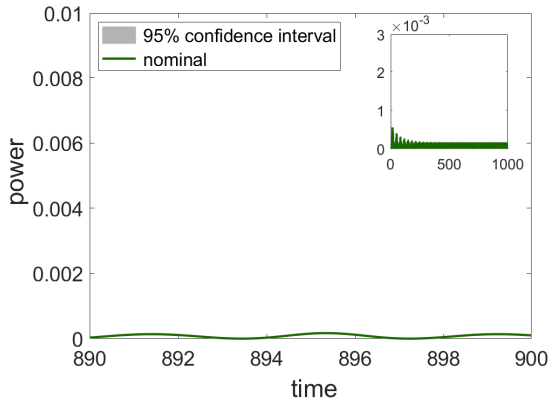
Seek to quantify how parameters variability influences the recovered power uncertainty

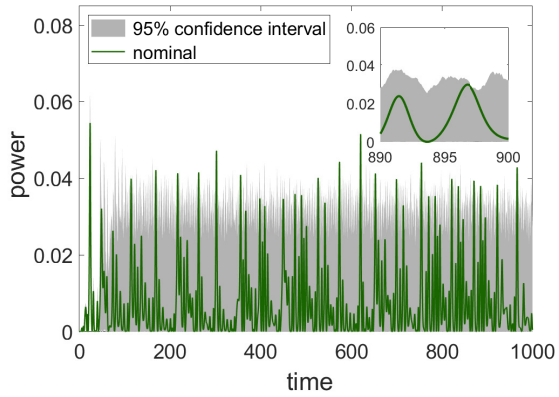


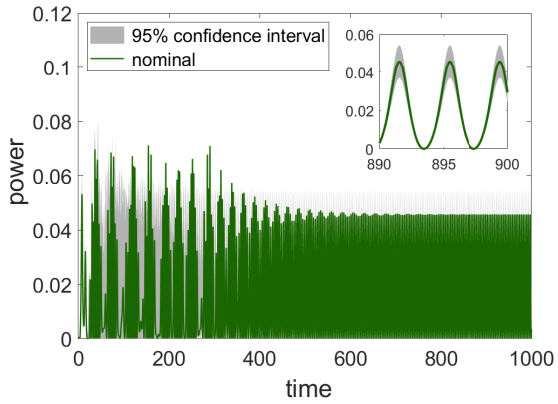
Recovered power evolution: symmetric system



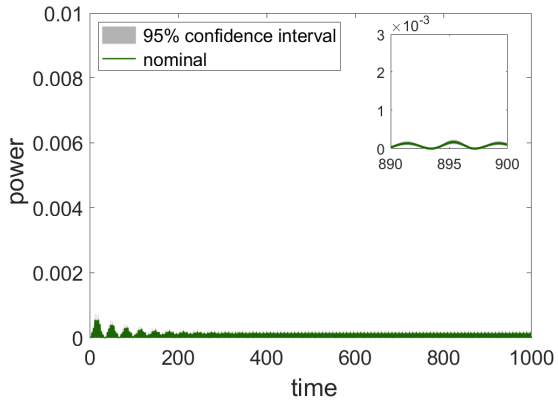
$\lambda \sim \text{Uniform}(c.v. = 20\%)$

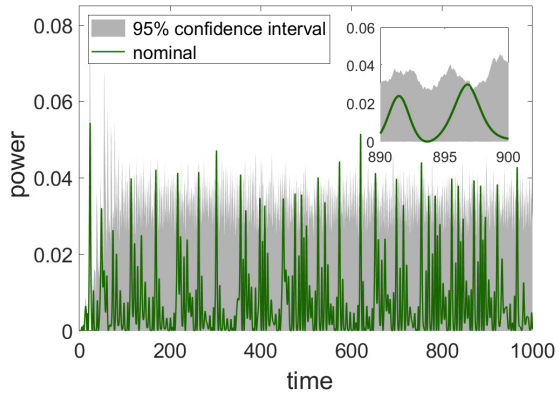


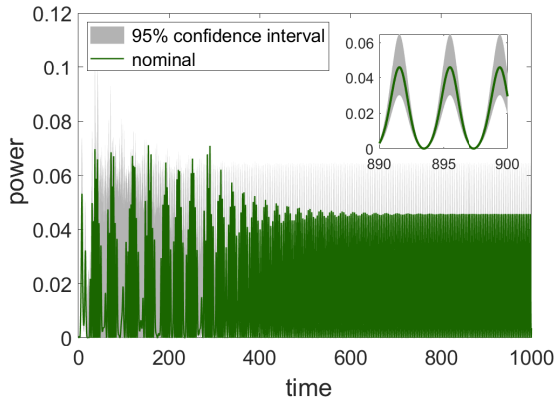




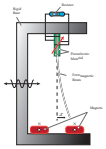
$\kappa \sim \text{Uniform}(c.v. = 20\%)$



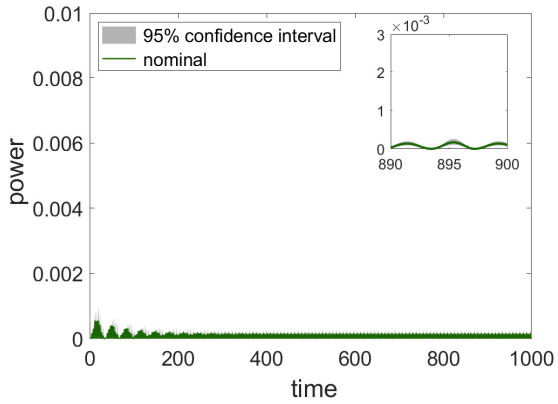


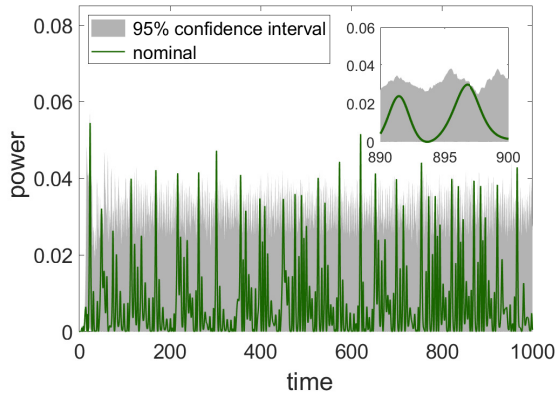


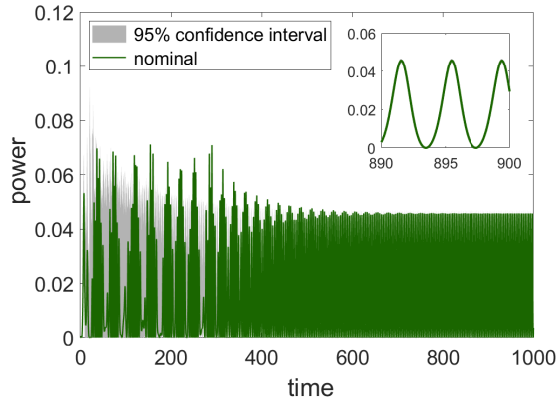
Recovered power evolution: symmetric system



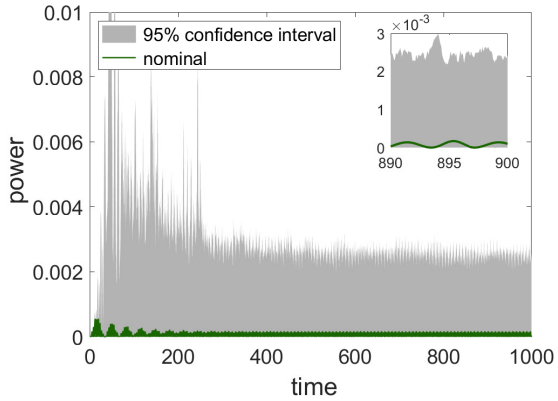
$$f \sim \text{Uniform}(c.v. = 20\%)$$

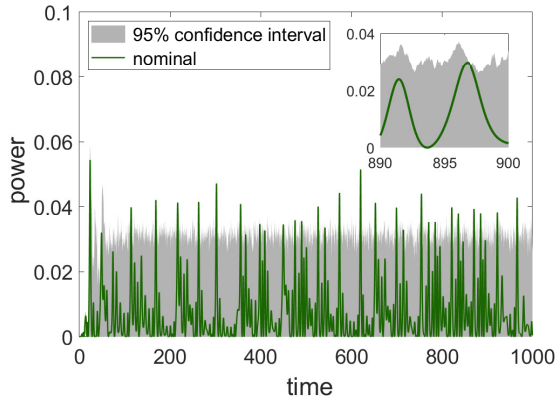


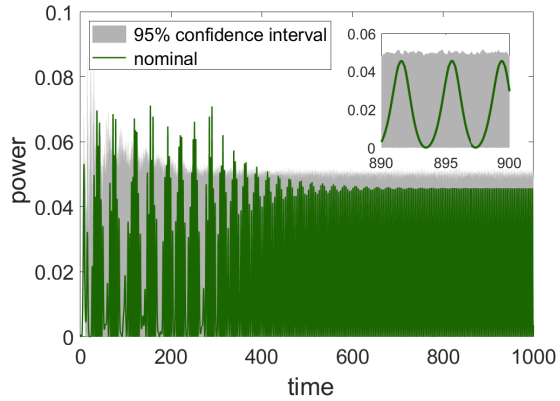




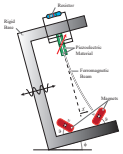
$$\Omega \sim \text{Uniform}(c.v. = 20\%)$$



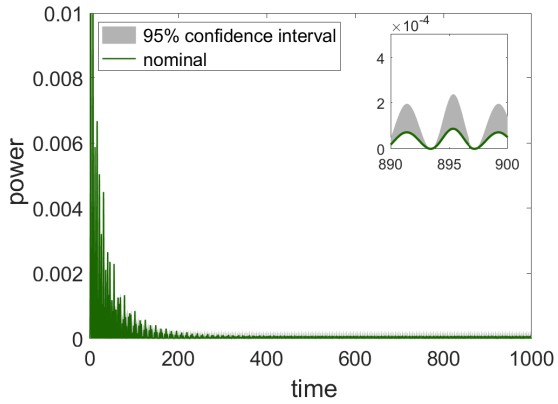


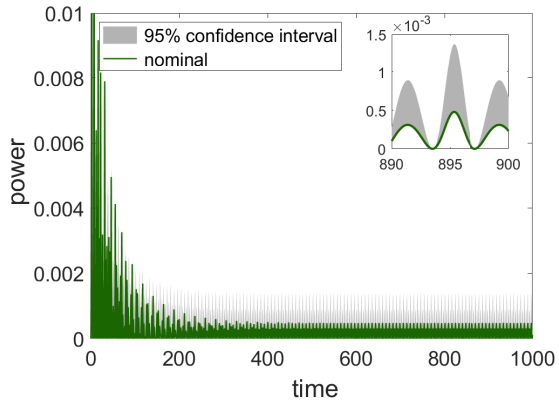


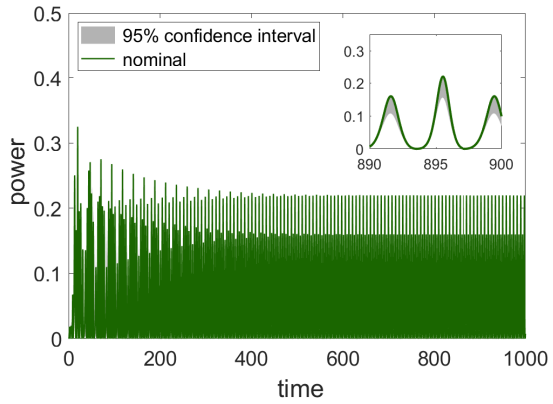
Recovered power evolution: asymmetric system



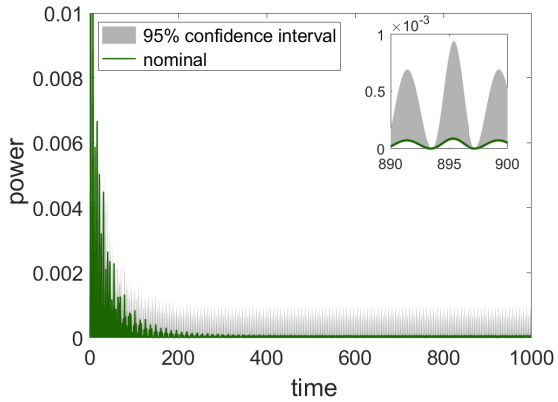
$\delta \sim \text{Uniform}\ [-0.15\ 0.15]$

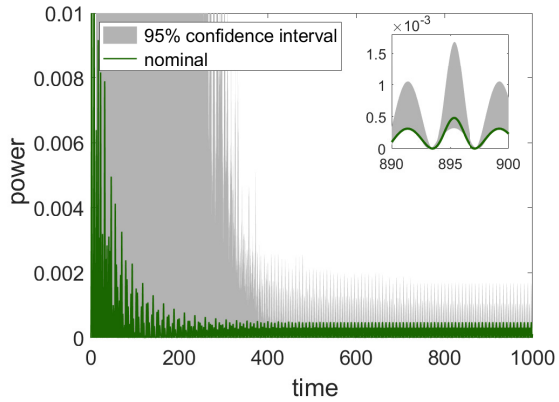


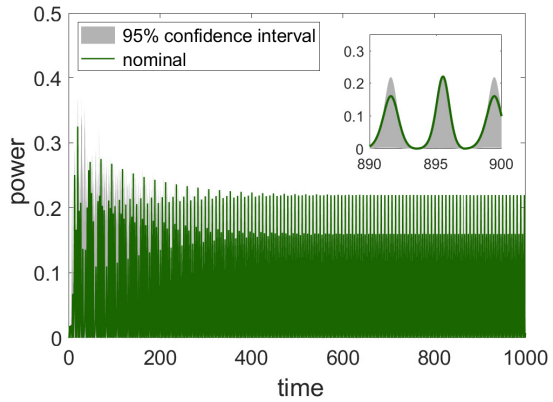




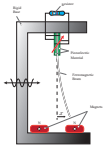
$\phi \sim \text{Uniform}\ [-15^\circ\ 15^\circ]$

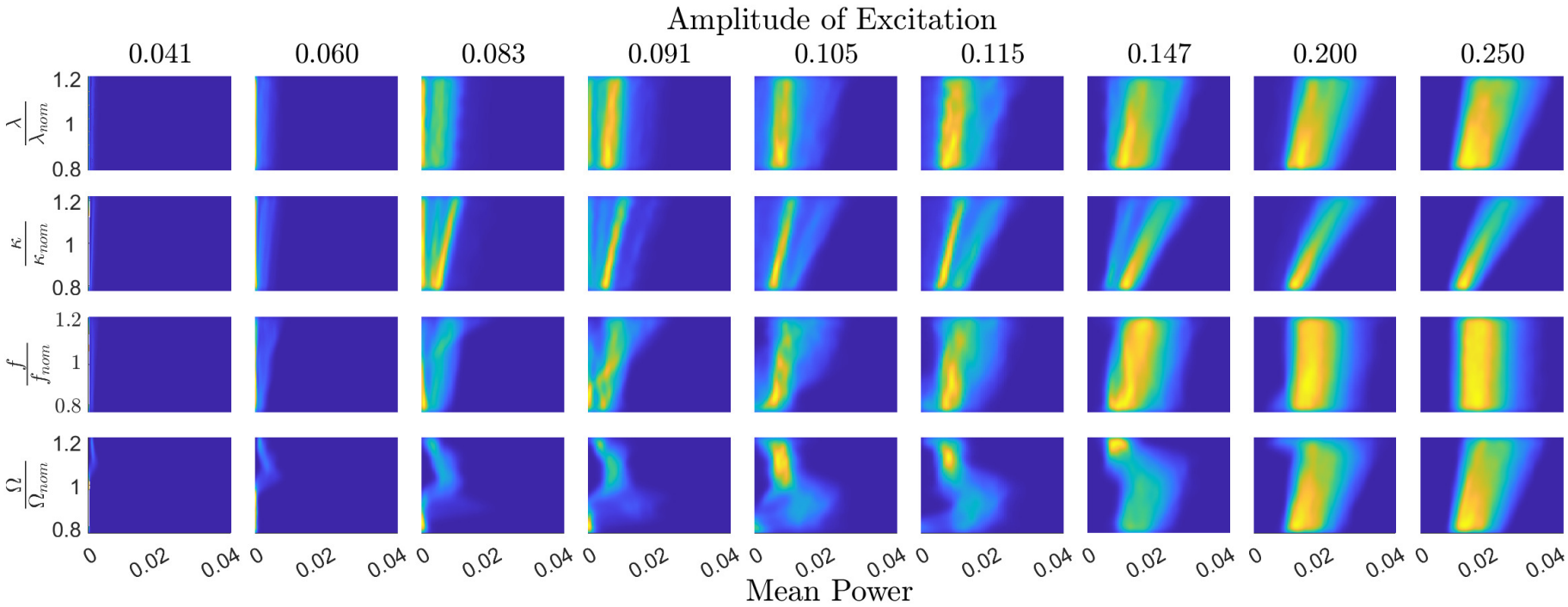




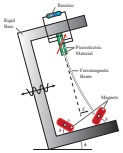


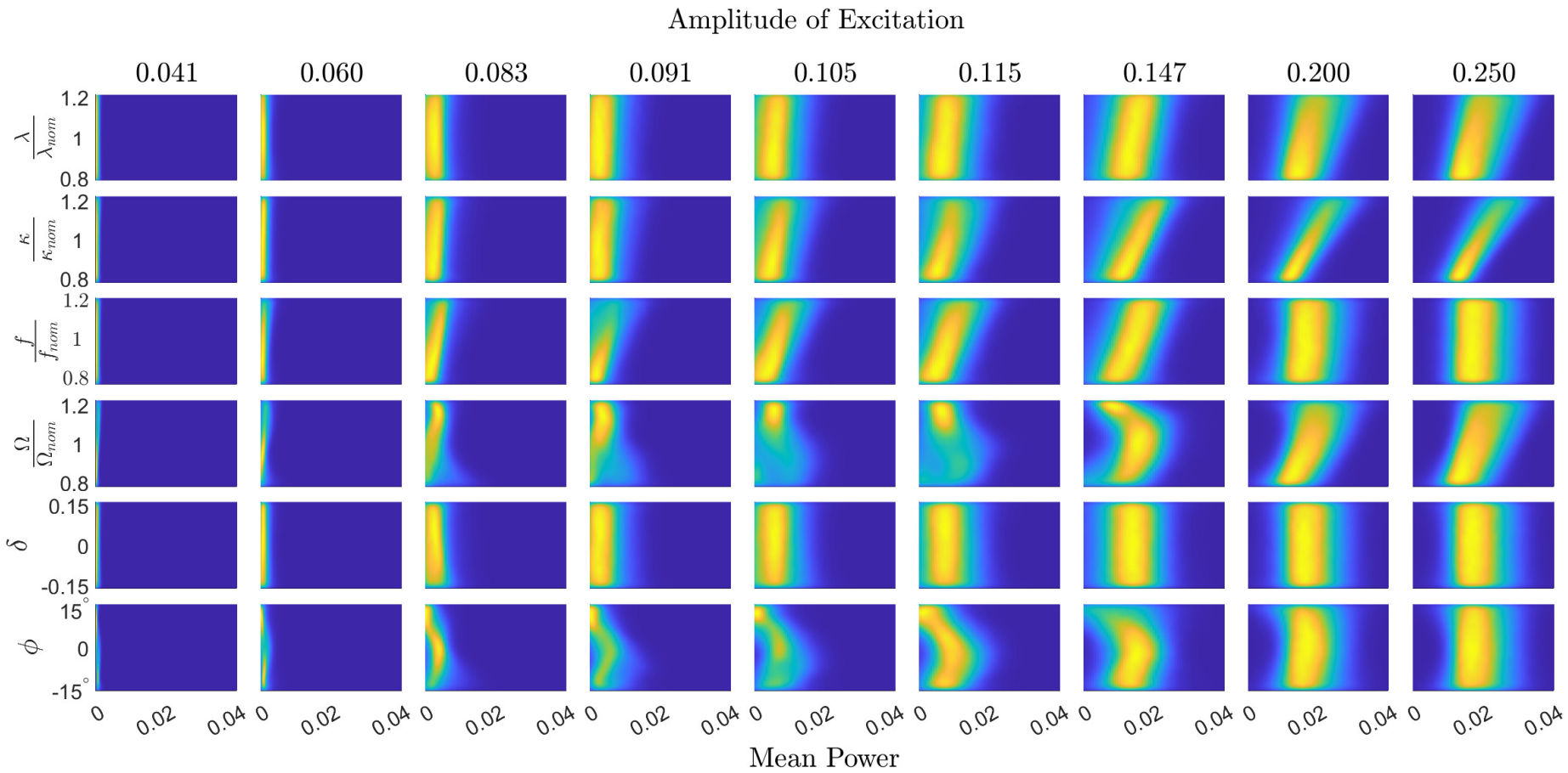
Mean power joint-CDF: symmetric system



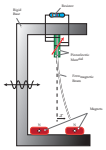


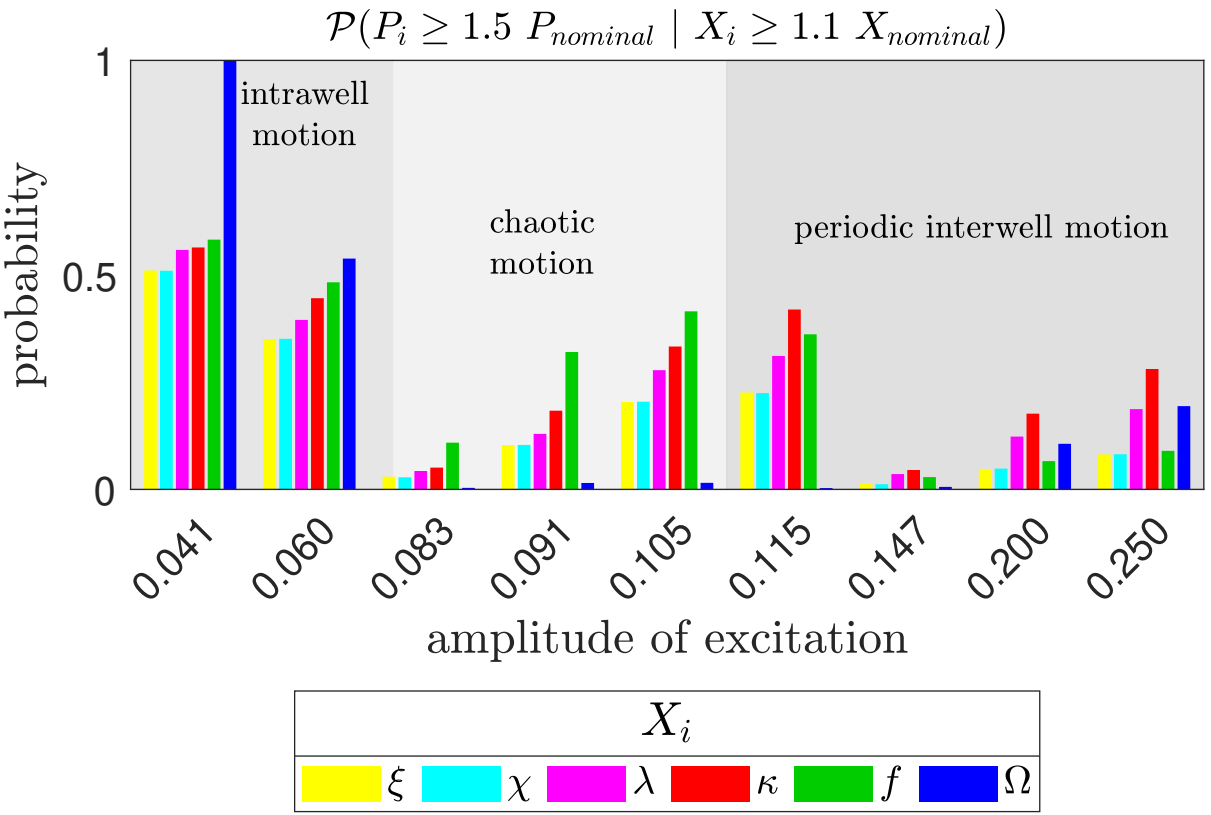
Mean power joint-CDF: asymmetric system





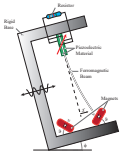
Conditional probability: symmetric system

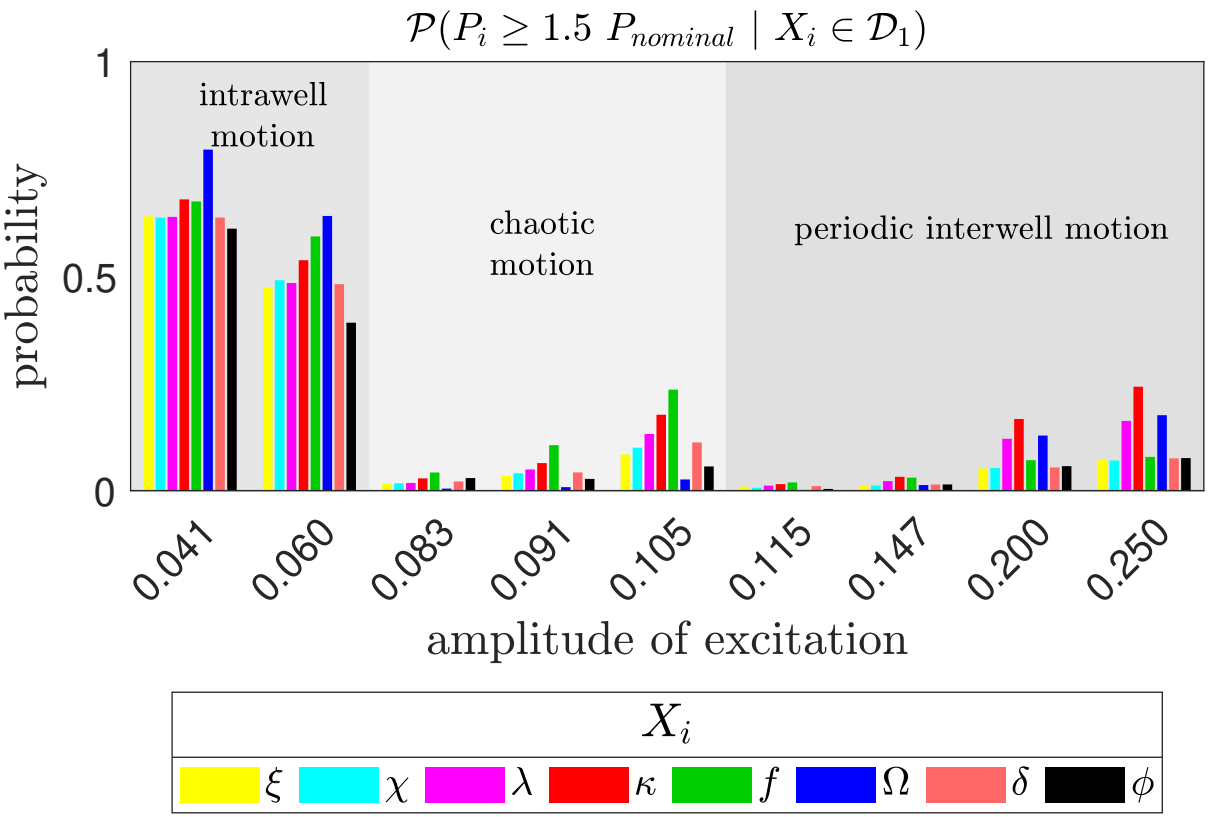




"Probability of nominal power increases by 50% given X_i increased by 10%"

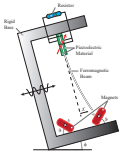
Conditional probability: strongly asymmetry system

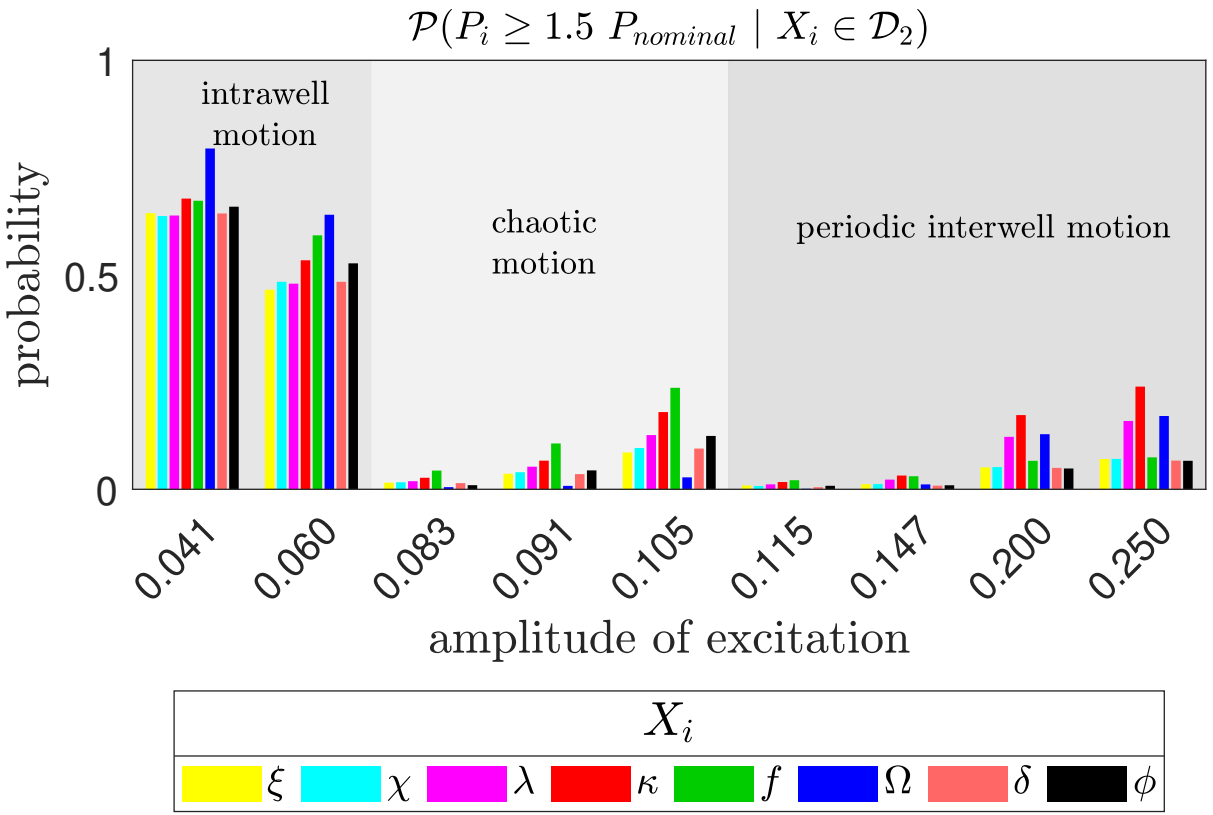




"Probability of nominal power increases by 50% given the system has strong asymmetry"

Conditional probability: weak asymmetry





"Probability of nominal power increases by 50% given the system has weak asymmetry"

Final remarks

Findings:

- Increasing the frequency of excitation leads to a greater probability of increasing the power harvested at intrawell motion regions
- Increasing the amplitude of excitation is the best alternative to improve energy harvesting at chaotic regime
- Increasing the piezoelectric coupling showed a greater chance of increasing the power at the interwell region.
- Asymmetry configurations and the excitation condition are crucial to determine power performance

Ongoing research:

- Effect of environmental noise
- Experimental analysis

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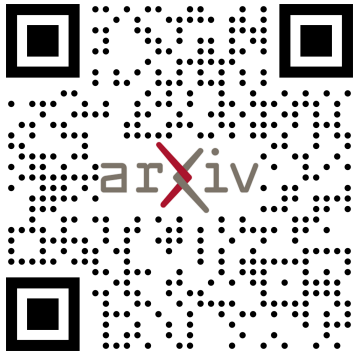
Probabilistic maps on bistable vibration energy harvesters

João Pedro Norenberg · Americo Cunha Jr. · Samuel da Silva · Paulo Sergio Varoto



J. P. Norenberg, A. Cunha Jr, S. da Silva, P. S. Varoto, *Probabilistic maps on bistable vibration energy harvesters* , **arXiv**, 2023

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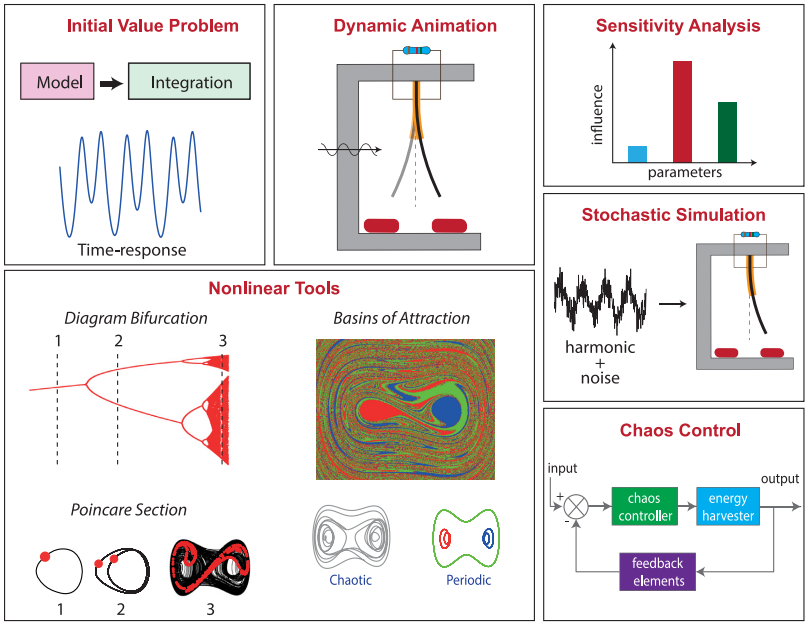
Original software publication

STONEHENGE — Suite for nonlinear analysis of energy harvesting systems

João Pedro Norenberg [a], João Victor Peterson [b], Vinicius Gonçalves Lopes [b], Roberto Luo [b], Leonardo de la Roca [b], Marcelo Pereira [b], José Geraldo Telles Ribeiro [b], Americo Cunha Jr [b,*]

[a] São Paulo State University – UNESP, Ilha Solteira - SP, Brazil
[b] Rio de Janeiro State University – UERJ, Rio de Janeiro - RJ, Brazil

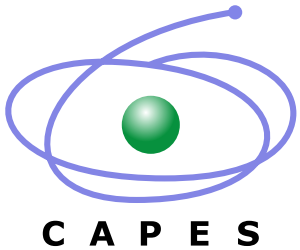


STONEHENGE

J. P. Norenberg et al. *STONEHENGE — Suite for nonlinear analysis of energy harvesting systems*, **Software Impacts**, 10:100161, 2021

Acknowledgments

Financial support:



Institutional support:

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