Sobol global sensitivity analysis on a bistable energy harvester

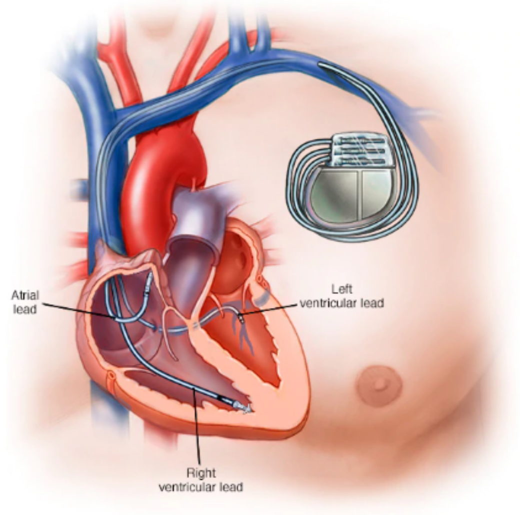
João Pedro C. V. Norenberg[1] **Americo Cunha Jr**[2]
Samuel da Silva[1] Paulo Sérgio Varoto[3]

[1]São Paulo State University – UNESP
[2]Rio de Janeiro State University – UERJ
[3]University of São Paulo – USP

ICTAM 2020+1

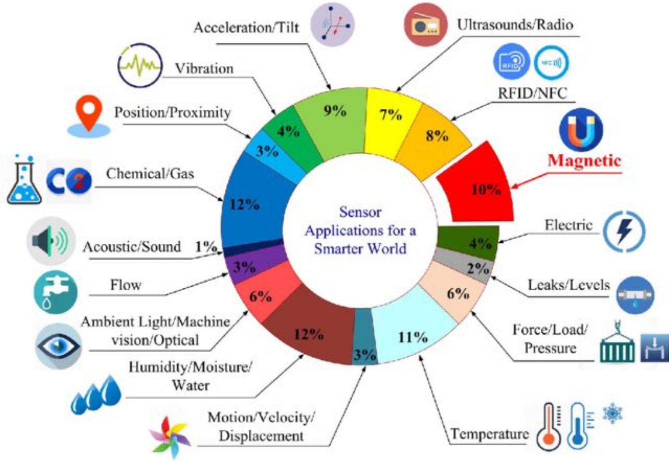
Virtual Congress, August 22-27, 2021

On average a pacemaker must be changed every 7 years



*Picture from `http://muonray.blogspot.com/2017/11/thermal-energy-harvesting-projects.html`

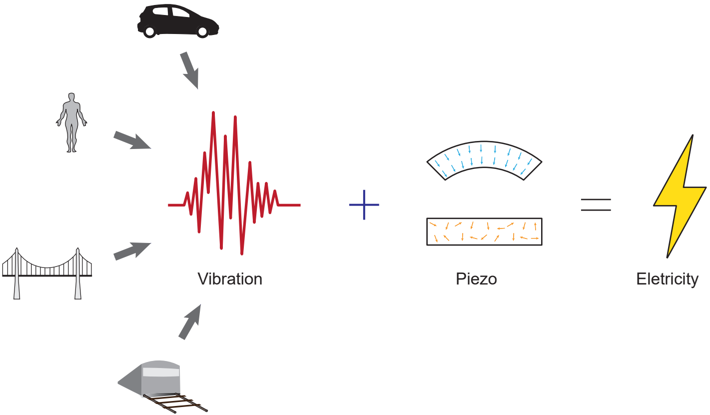
IoT will demand more than a trillion sensors by 2025

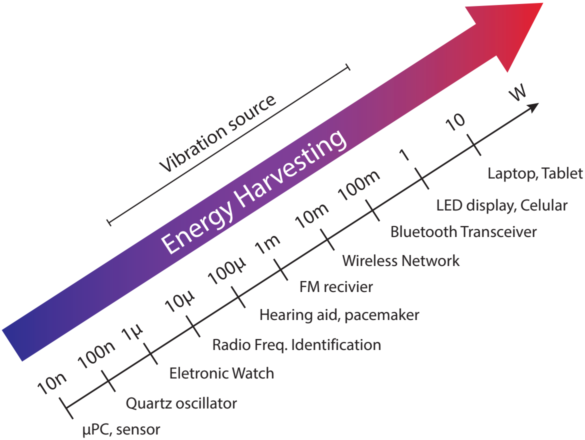


*Picture from this reference.

L. Xuyang et al., *Overview of Spintronic Sensors With Internet of Things for Smart Living.* **IEEE Transactions on Magnetics**, 0018-9464, 2019.

Vibration Energy Harvesting Concept

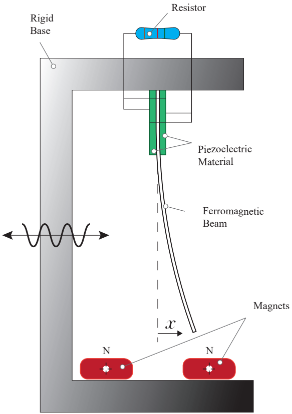




Adapted from: https://passive-components.eu/energy-harvesting-is-not-fiction-anymore-2/

Research objectives

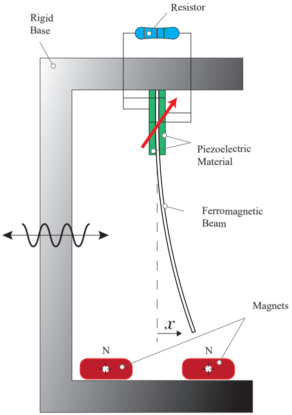
- Study variability effects on bistable energy harvesters
- Apply global sensitivity analysis to:
    1. extract features of the dynamic behavior;
    2. identify the most sensitivity parameters.



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi v = f\cos(\Omega t)$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

$+$ initial conditions

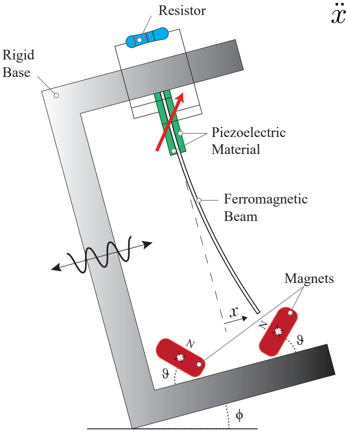


$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t)$$

$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta\,|x|)$$

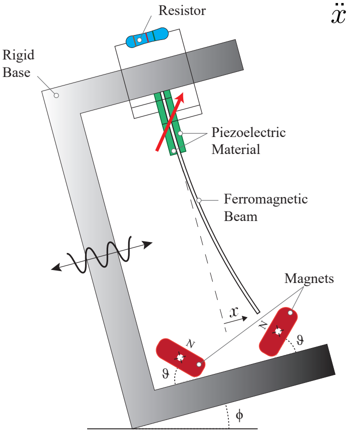


$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 + 2\delta x - x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t) + p\sin(\phi)$$

$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta\,|x|)$$



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 + 2\delta x - x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t) + p\sin(\phi)$$

$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta\,|x|)$$

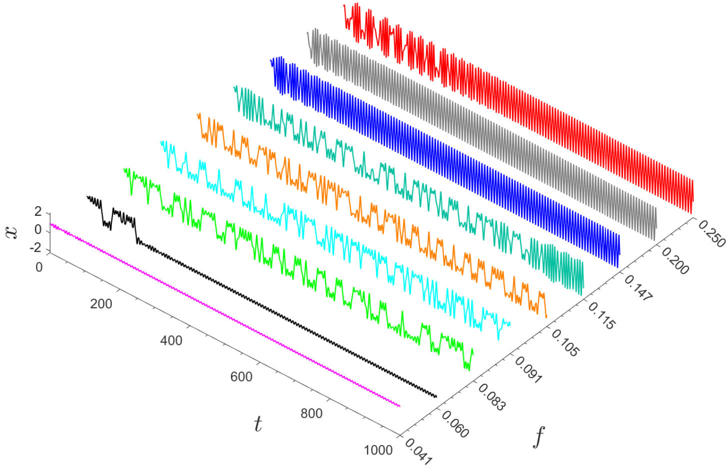
$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} \lambda\, v\,(t)^2 \; dt$$

Dynamic animation (symmetric + linear coupling)

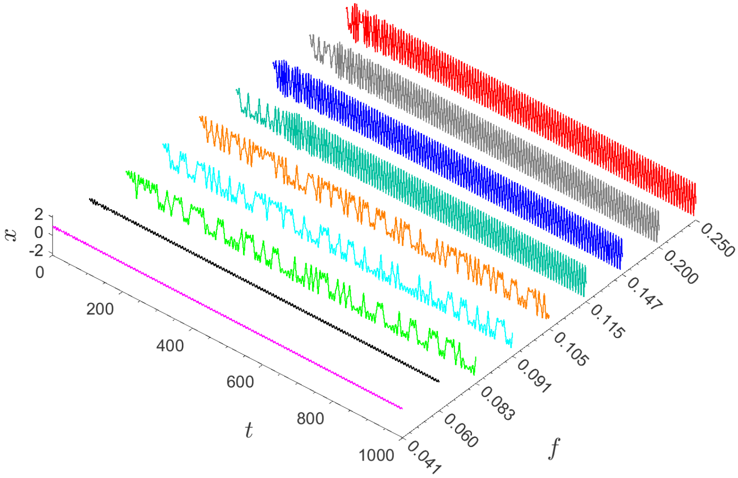
Dynamic animation (symmetric + nonlinear coupling)

Dynamic animation (asymmetric + nonlinear coupling)

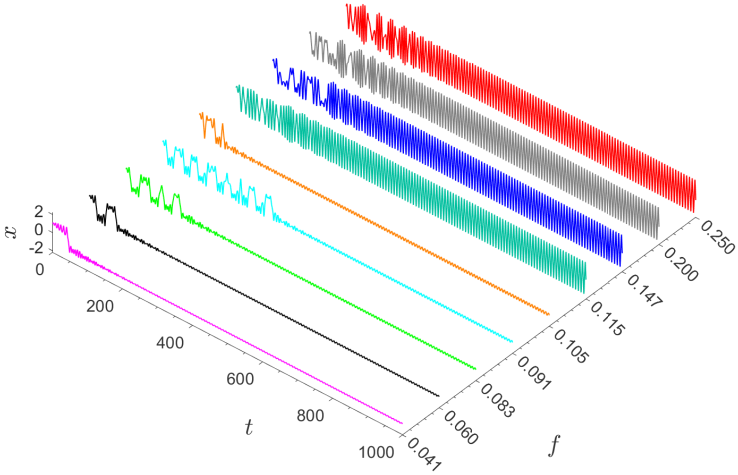
Time series (symmetric + linear coupling)



Time series (symmetric + nonlinear coupling)



Time series (asymmetric + nonlinear coupling)



Global sensitivity analysis

Main contributions:

- Simpler probabilistic model constructions
- Nontrivial insight into the behavior
- Important step for robustness and optimization problems

Mathematical Model:

$$\text{Quantity of Interest} = \mathcal{M}(\text{system parameters})$$

$$Y = \mathcal{M}(\mathbf{X}) \quad , \quad X_i \sim \mathcal{U}(0,1)$$

Hoeffding-Sobol decomposition:

$$Y = \mathcal{M}_0 + \sum_{i=1}^{k} \mathcal{M}_i\,(X_i) + \sum_{i<j}^{k} \mathcal{M}_{ij}\,(X_i, X_j) + \cdots + \mathcal{M}_{1\ldots k}\,(X_1 \ldots X_k)$$

An orthogonal decomposition in terms of conditional expectations:

- $\mathcal{M}_0 = \mathbb{E}\{Y\}$
- $\mathcal{M}_i(X_i) = \mathbb{E}\{Y\,|X_i\} - \mathcal{M}_0$
- $\mathcal{M}_{ij}(X_i, X_j) = \mathbb{E}\{Y\,|X_i, X_j\} - \mathcal{M}_i - \mathcal{M}_j - \mathcal{M}_0$
- $\ldots$

First-order Sobol' indices:

$$S_i = Var[\mathcal{M}_i(X_i)] \ / \ Var[\mathcal{M}(\mathbf{X})]$$

(quantify the additive effect of each input separately)

Second-order Sobol' indices:

$$S_{ij} = Var[\mathcal{M}_{ij}(X_i, X_j)] \ / \ Var[\mathcal{M}(\mathbf{X})]$$

(quantify interaction effect of inputs X_i and X_j)

I.M. Sobol *Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates*. **Mathematics and Computers in Simulation**, 55(1-3): 271-280, 2001.

Polynomial Chaos Expansion:

$$Y = \mathcal{M}(\mathbf{X}) \approx \sum_{\alpha \in \mathcal{A}} y_\alpha \, \psi_\alpha(\mathbf{X})$$

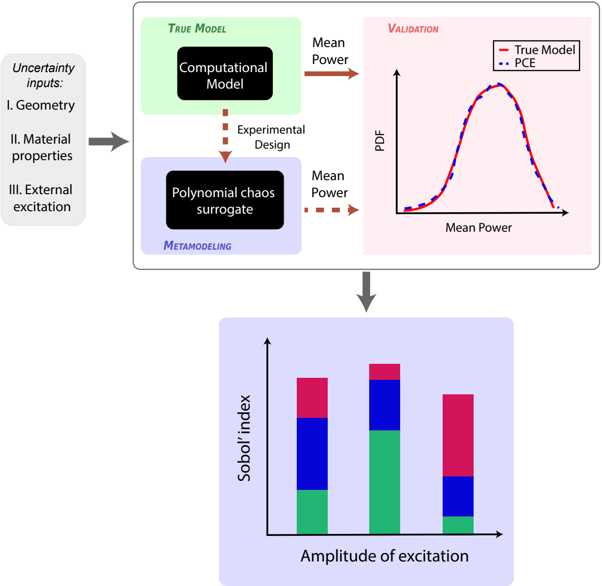
Analytical calculation of Sobol' indices:

$$S_{\boldsymbol{u}} = \sum_{\boldsymbol{\alpha} \in \mathcal{A}_{\boldsymbol{u}}} y_{\boldsymbol{\alpha}}^2 \Bigg/ \sum_{\boldsymbol{\alpha} \in \mathcal{A} \setminus 0} y_{\boldsymbol{\alpha}}^2$$

$$\mathcal{A}_{\boldsymbol{u}} = \{\boldsymbol{\alpha} \in \mathcal{A} : i \in \boldsymbol{u} \Longleftrightarrow \alpha_i \neq 0\}$$

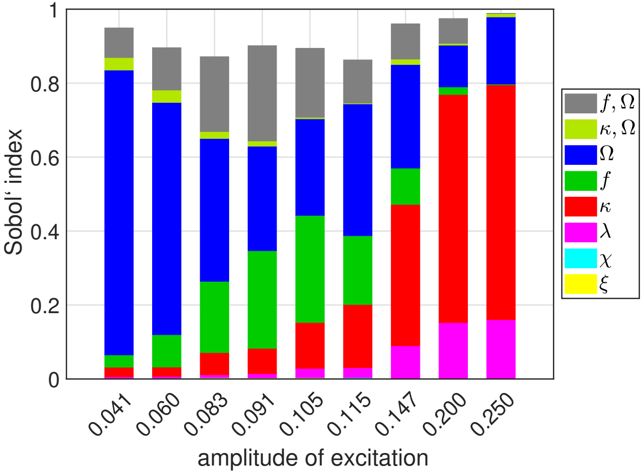
B. Sudret, *Global sensitivity analysis using polynomial chaos expansions.* **Reliability Engineering & System Safety, 2016**, 93(7): 964–979, 2008.

Global sensitivity analysis framework



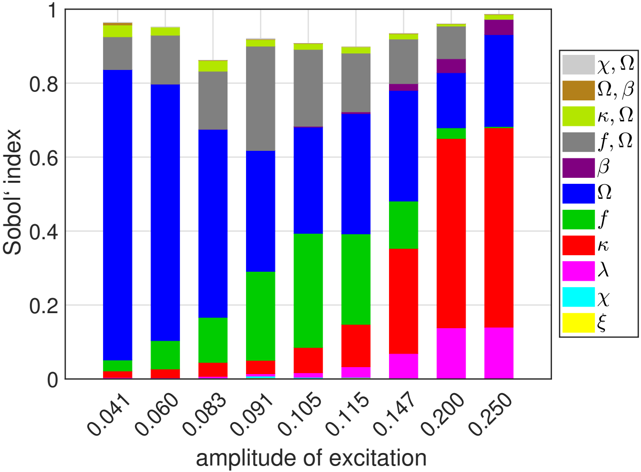
Mean power sensitivity (symmetric + linear piezo)

$\beta = 0 \;\; \phi = 0^o \;\; f = 0.147 \;\; \Omega = 0.8$



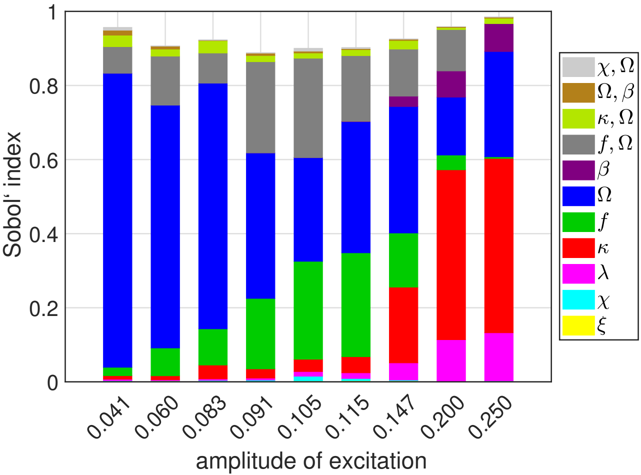
Mean power sensitivity (symmetric + nonlinear piezo)

$$\beta = 0.5 \;\; \phi = 0^{o} \;\; f = 0.147 \;\; \Omega = 0.8$$



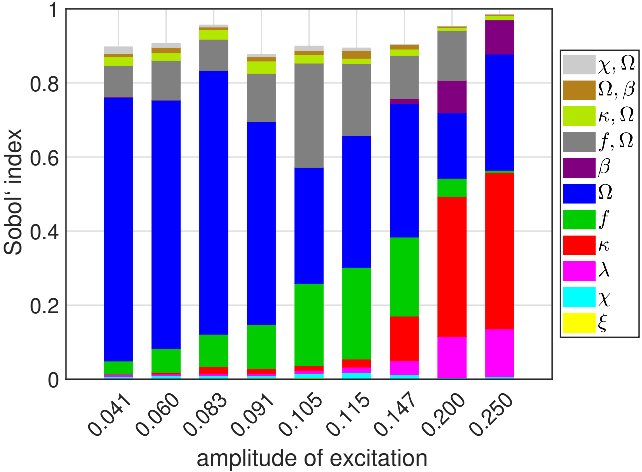
Mean power sensitivity (symmetric + nonlinear piezo)

$\beta = 1.0 \;\; \phi = 0^{o} \;\; f = 0.147 \;\; \Omega = 0.8$

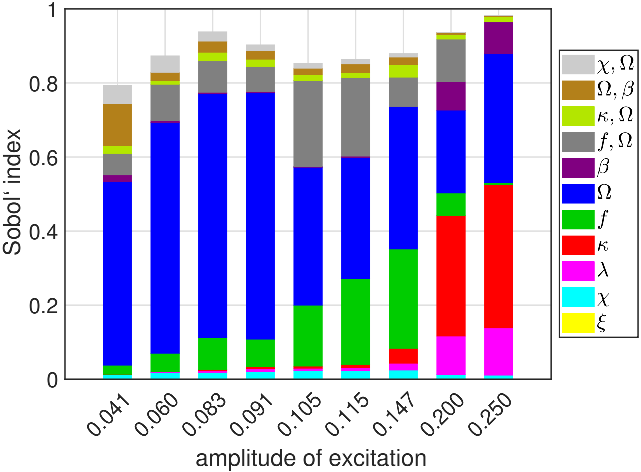


Mean power sensitivity (symmetric + nonlinear piezo)

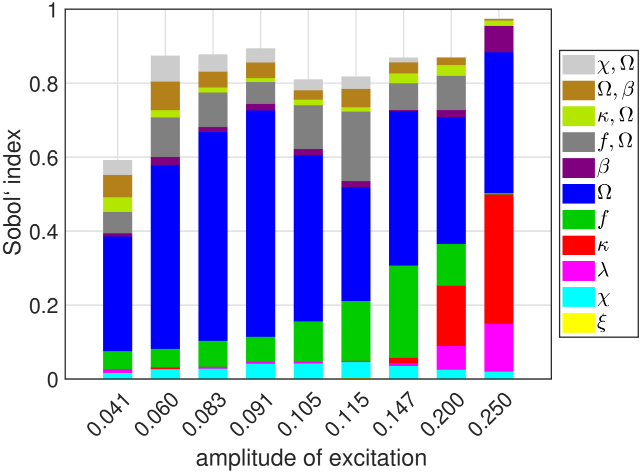
$\beta = 1.5 \;\; \phi = 0^{\circ} \;\; f = 0.147 \;\; \Omega = 0.8$



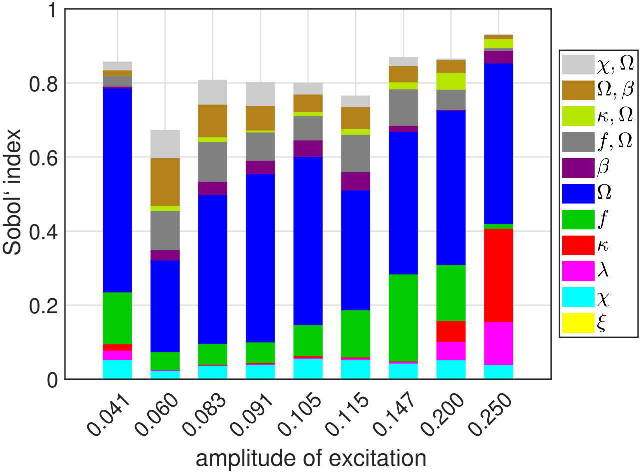
$\beta = 2.0 \ \ \phi = 0^{o} \ \ f = 0.147 \ \ \Omega = 0.8$



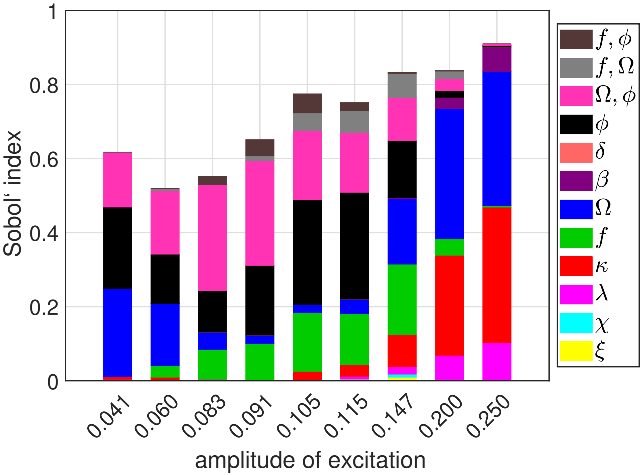
$\beta = 2.5 \;\; \phi = 0^{\circ} \;\; f = 0.147 \;\; \Omega = 0.8$



$\beta = 3.0 \;\; \phi = 0^{o} \;\; f = 0.147 \;\; \Omega = 0.8$



$$\beta = 1.0 \;\; \phi \in [-15^o, 15^o] \;\; f = 0.147 \;\; \Omega = 0.8$$



Sensitivities in a nutshell

parameter	sensitivity	optimal condition
ξ	no	-
χ	no	-
λ	low	high energy
κ	high	high energy
f	high	low and middle energy
Ω	high	all spectrum
β	low	high values
δ	no	-
ϕ	high	low and middle energy

Findings:

- The order of importance of the parameters changes a lot with vibration regime
- Frequency and amplitude of excitation, asymmetric bias angle, and piezoelectric coupling at the electrical domain are the most influential parameters that affect the mean power harvested
- Bias angle is a better parameter to optimize the harvested power for middle and low forcing amplitudes
- Piezoelectric electrical properties are better parameters to optimize the harvested power for high forcing amplitude

Ongoing research:

- Uncertainty quantification for parameters disturbances

Acknowledgments

Institutional support:

Universidade de São Paulo

Financial support:

Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

Conselho Nacional de Desenvolvimento Científico e Tecnológico

Thank you for your attention!

americo.cunha@uerj.br

www.americocunha.org

WEBSITE

@AmericoCunhaJr @AmericoCunhaJr

J. P. Norenberg, A. Cunha Jr, S. da Silva, P. S. Varoto, *Global sensitivity analysis of (a)symmetric energy harvesters*, **arXiv:2107.04647**, 2021. https://arxiv.org/abs/2107.04647

PAPER