Optimization of an energy harvesting device via cross-entropy method

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NUMERICO – **Nu**cleus of **M**odeling and **E**xperi**m**entation with **Co**mputers

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31º Colóquio Brasileiro de Matemática
Simpósio de Matemática Aplicada na Indústria
31 Julho - 04 Agosto, 2017
Rio de Janeiro, Brasil

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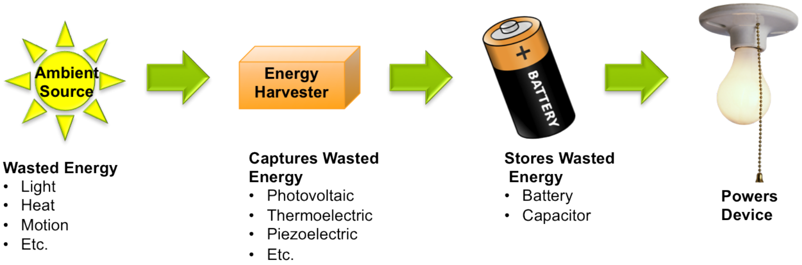
2. Nonlinear Dynamics
3. Optimization Problem
4. Numerical Experiments
5. Final Remarks

Section 1

Introduction

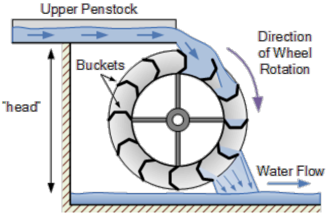
Energy Harvesting

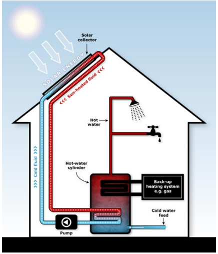


- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

*Picture from http://hiddenjoules.com/intro/what/

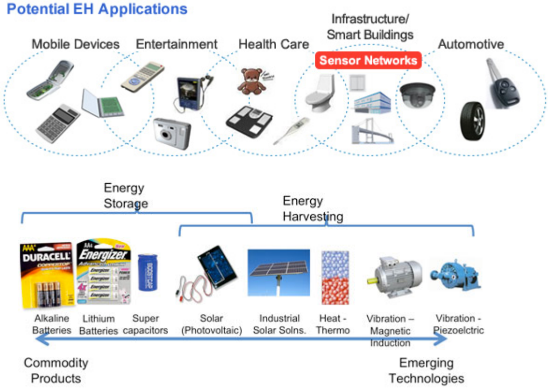
Classical Technologies in Energy Harvesting

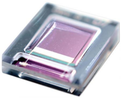


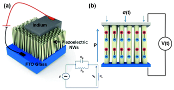


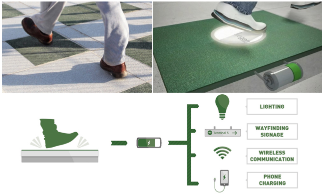
*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

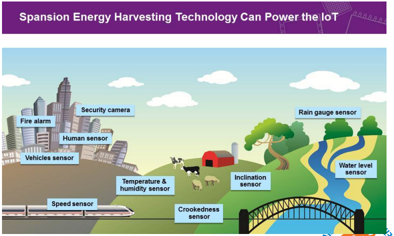
Emergent Technologies in Energy Harvesting











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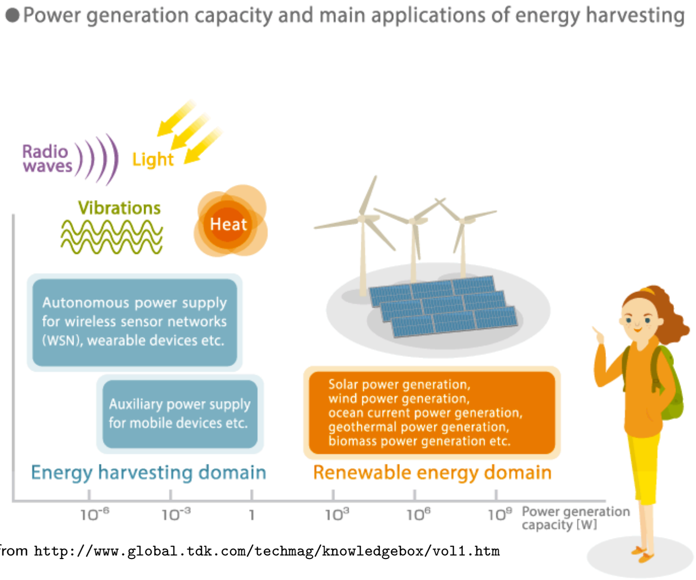
Energy Scale for Modern Harvesting Devices



*Picture from `http://www.global.tdk.com/techmag/knowledgebox/vol1.htm`

Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
- Propose strategies to enhance the recovered energy
- Model the underlying uncertainties and study their influence

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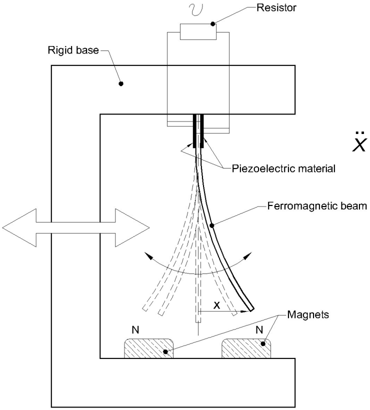
Section 2

Nonlinear Dynamics

Bi-stable energy harvesting device



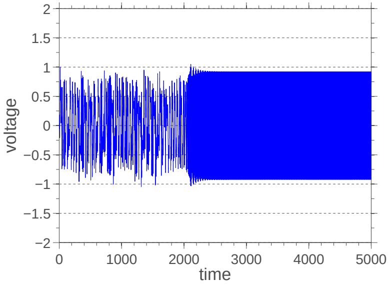
$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0,\ \dot{x}(0) = \dot{x}_0,\ \upsilon(0) = \upsilon_0$$

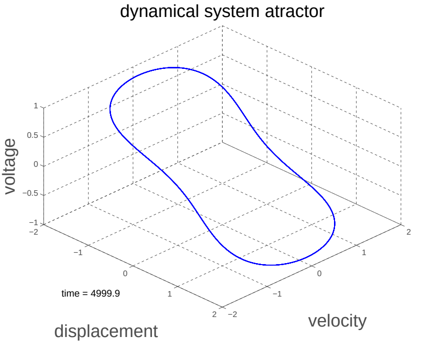
A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Nonlinear dynamics: time series

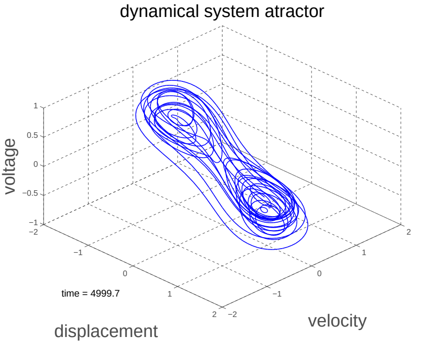


(a) f = 0.11 (b) f = 0.10

Nonlinear dynamics: system atractor

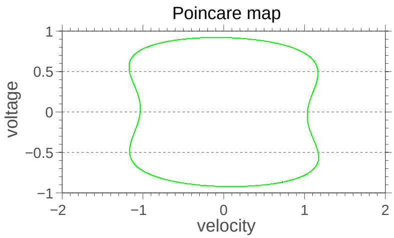


(a) $f = 0.11$



(b) $f = 0.10$

Nonlinear dynamics: Poincaré maps



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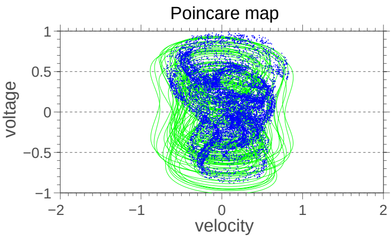
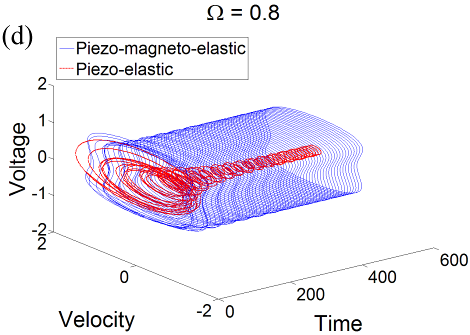


(b) f = 0.10

Nonlinearity and efficiency

Nonlinearity of this dynamical system may enhance output power.

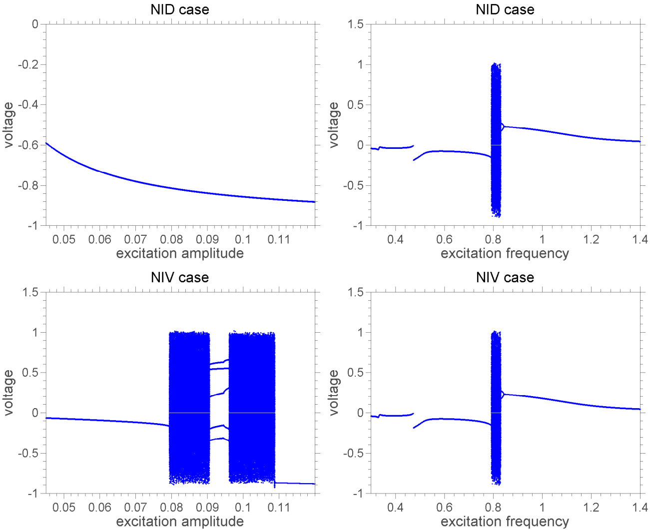


A. Erturk, *Electromechanical Modeling of Piezoelectric Energy Harvesters*, **PhD Thesis**, Virginia Tech, 2009.

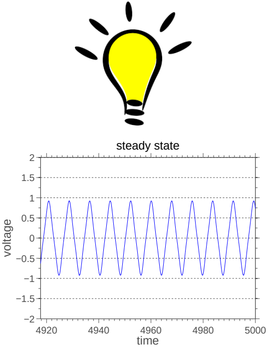
*Picture from the above reference.

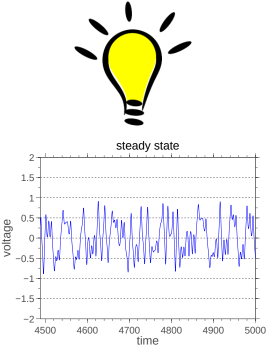
Nonlinearity and chaos

Nonlinearity of this dynamical system may also induce chaos.



For practical use of the electrical energy ...





*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

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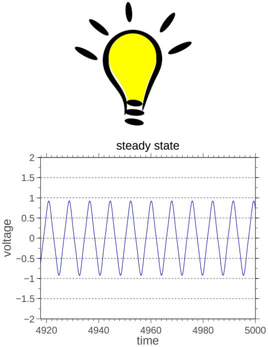
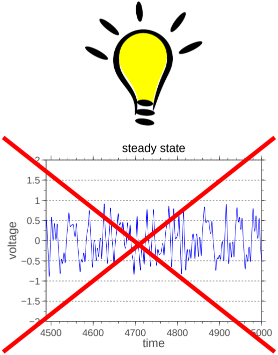




... irregular voltage is undesirable!

*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

Detection of chaos: 0-1 test

Given a time series $(x_1, \cdots, x_N)$, for several $c \in (0, 2\pi)$:

1. change from coordinates $(x, \dot{x})$ to (p, q):

   $p(n) = \sum_{j=1}^{n} x_j \cos(j\,c), \quad q(n) = \sum_{j=1}^{n} x_j \sin(j\,c)$

2. compute mean square displacement (for $0 \ll n \ll N$):

   $M(n) = \frac{1}{N} \sum_{j=1}^{N} \left(\left[p(j+n) - p(j)\right]^2 + \left[q(j+n) - q(j)\right]^2 \right)$

3. compute correlation:

   $K_c = \mathtt{corr} \left\{ (1, 2, \cdots, n), (M(1), M(2), \cdots, M(n)) \right\}$

4. compute median:

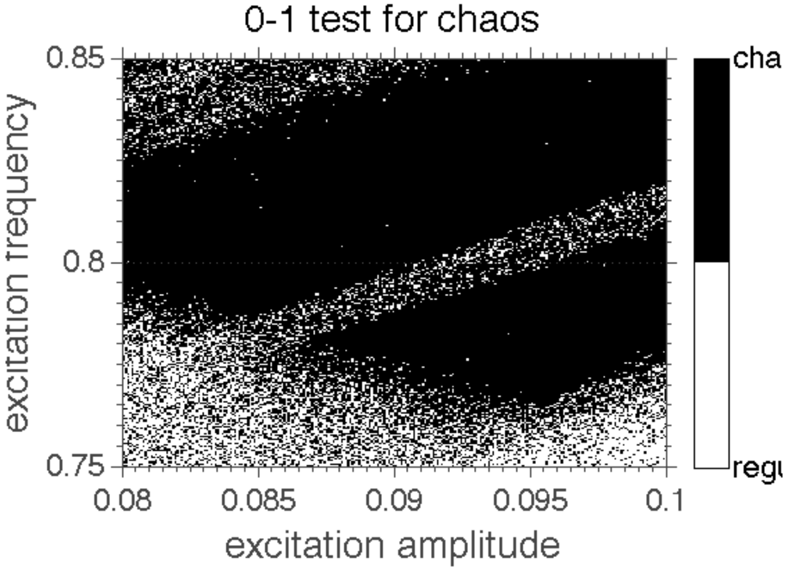
   $K = \mathtt{median} \left\{ K_c \right\}$

Numerical indicator:

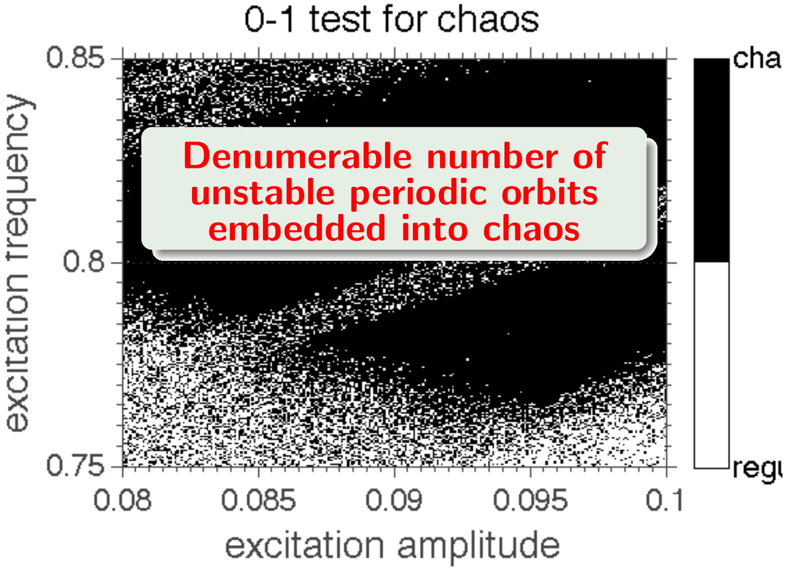
$K \approx 0$: regular dynamics

$K \approx 1$: chaotic dynamics

Characterization of chaos

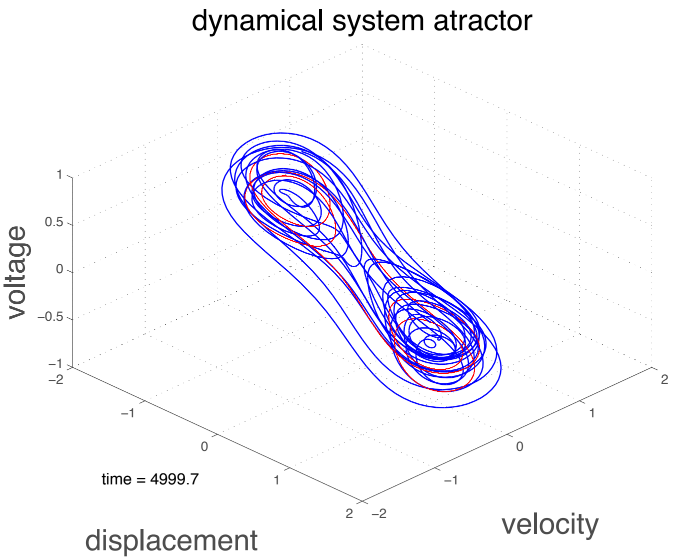


Characterization of chaos



Denumerable number of unstable periodic orbits embedded into chaos

UPO embedded into a chaotic atractor



How to explore these unstable periodic orbits?

- Control of chaos
  - OGY method
  - Pyragas method
- Nonlinear optimization
  - Direct search on a fine grid
  - Genetic algorithm
  - Particle swarm optimization
  - Cross-Entropy method
  - Many other ...

How to explore these unstable periodic orbits?

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Section 3

Optimization Problem

Optimization of recovered power

Find a pair (f, Ω) which maximize

$$P = \frac{1}{T} \int_{\tau=0}^{T} \lambda \, v^2(\tau) \, d\tau$$

(mean power)

such that

$$f_{min} \leq f \leq f_{max}$$

$$\Omega_{min} \leq \Omega \leq \Omega_{max}$$

(control parameters are acceptable)

$$K \leq \texttt{tolerance}$$

(system dynamics is regular, i.e., not chaotic)

Optimization framework

- constrained problem

  Find $\mathbf{x}^\star = \arg\max \mathcal{S}(\mathbf{x})$ s.t. $\mathcal{G}_m(\mathbf{x}) \leq 0, \;\; m = 1, \cdots, M$

- penalized problem

  Find $\mathbf{x}^\star = \arg\max \left\{ \mathcal{S}(\mathbf{x}) + \sum_{m=1}^{M} H_m \max\left[0, \mathcal{G}_m(\mathbf{x})\right] \right\}$

Characteristics:

- Test 0-1 for chaos constraint is a discontinuous function of $\mathbf{x}$
- Gradient-based methods are not applicable
- Evolutionary algorithms/metaheuristics can be used (but we prefer not!)

An Idea

Solve this optimization problem through the calculation of a rare-event probability (Cross-Entropy Method)

Cross-entropy method

Hypothesis: There is a single maximum

$$\mathcal{S}(\mathbf{x}^\star) = \gamma^\star = \max_{\mathbf{x}\in\mathcal{X}} \mathcal{S}(\mathbf{x})$$

The optimization problem is randomized

$$\mathcal{P}\left\{\mathcal{S}(\mathbf{X}) \geq \gamma\right\} = \mathbb{E}\left\{\mathcal{I}_{\{\mathcal{S}(\mathbf{X})\geq\gamma\}}\right\}$$

where $\mathbf{X}$ is a random vector with probability density $f(\mathbf{x};\mathbf{v})$

$\mathcal{S}(\mathbf{X}) \geq \gamma$ is a rare-event when $\gamma \approx \gamma^\star$

R. Y. Rubinstein, and Dirk P. Kroese, **Simulation and the Monte Carlo Method**, Wiley, 3rd Edition, 2017.

Cross-entropy method

CE method generates an "optimal sequence" of estimators $(\widehat{\gamma}_t, \widehat{\mathbf{v}}_t)$ such that $\widehat{\gamma}_t \to \gamma^\star$ and $f(\mathbf{x}, \widehat{\mathbf{v}}_t) \to \delta(\mathbf{x} - \mathbf{x}^\star)$

"minimize KL divergence between $\mathcal{I}_{\{\mathcal{S}(\mathbf{X}) \geq \gamma\}}$ and $f(\cdot\,, \mathbf{v})$"

This method has two main steps:

1. **Generate** an iid sample of objects in the search space $\mathcal{X}$ according to a specified probability distribution $f(\cdot; \mathbf{v})$
2. **Update** distribution parameters, based on the best performing samples (elite samples), using cross-entropy minimization

R. Y. Rubinstein, and Dirk P. Kroese, **Simulation and the Monte Carlo Method**, Wiley, 3rd Edition, 2017.

Cross-entropy algorithm

1. Define N, N^e, t_{max}, $t = 0$, $f(\cdot, \mathbf{v})$ and $\widehat{\mathbf{v}}_0$
2. Update level $t = t + 1$
3. Generate $\mathbf{X}_1, \cdots, \mathbf{X}_N$ (iid) samples from $f(\cdot, \widehat{\mathbf{v}}_{t-1})$
4. Evaluate performance function $\mathcal{S}(\mathbf{X}_n)$ at samples $\mathbf{X}_1, \cdots, \mathbf{X}_N$ and sort the results $\mathcal{S}_{(1)} \leq \cdots \leq \mathcal{S}_{(N)}$
5. Update estimators $\widehat{\gamma}_t$ and $\widehat{\mathbf{v}}_t$
6. Repeat ② — ⑤ while stopping criterion is not met

R. Y. Rubinstein, and Dirk P. Kroese, **Simulation and the Monte Carlo Method**, Wiley, 3rd Edition, 2017.

Cross-Entropy Estimators

- Level estimator:

$$\widehat{\gamma}_t = \mathcal{S}_{(N-N^e+1)}$$

- Elite samples:

$$\mathcal{E}_t = \left\{ \mathbf{X}_k : \mathcal{S}(\mathbf{X}_k) \geq \widehat{\gamma}_t \right\}$$

- Parameter estimator:

$$\widehat{\mathbf{v}}_t = \arg\max_{\mathbf{v}} \sum_{\mathbf{X}_k \in \mathcal{E}_t} \ln f(\mathbf{X}_k; \mathbf{v})$$

(maximum likelihood estimator)

- For $f(\cdot; \mathbf{v}) \sim \mathcal{N}(\mu, \sigma)$ one has

$$\widehat{\mu}_t = \frac{\sum_{\mathbf{X}_k \in \mathcal{E}_t} X_k}{|\mathcal{E}_t|} \quad \text{and} \quad \widehat{\sigma}_t = \sqrt{\frac{\sum_{\mathbf{X}_k \in \mathcal{E}_t} (X_k - \widehat{\mu}_t)^2}{|\mathcal{E}_t|}}$$

Section 4

Numerical Experiments

Reference solution

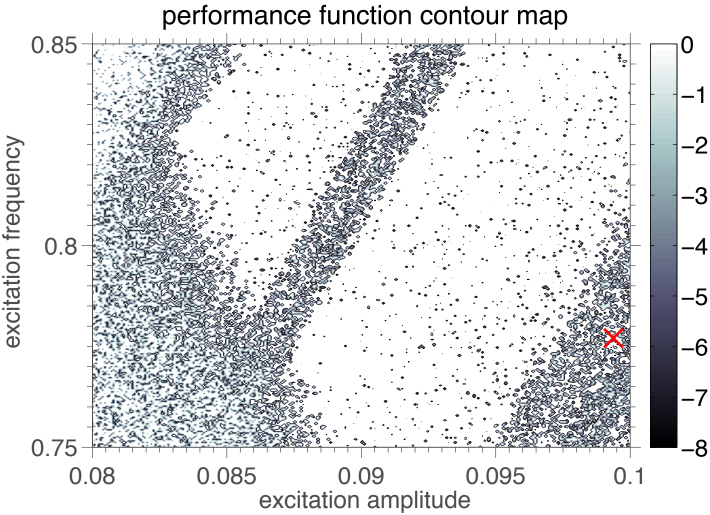
- Domain:

$$0.08 \leq f \leq 0.1$$
$$0.75 \leq \Omega \leq 0.85$$

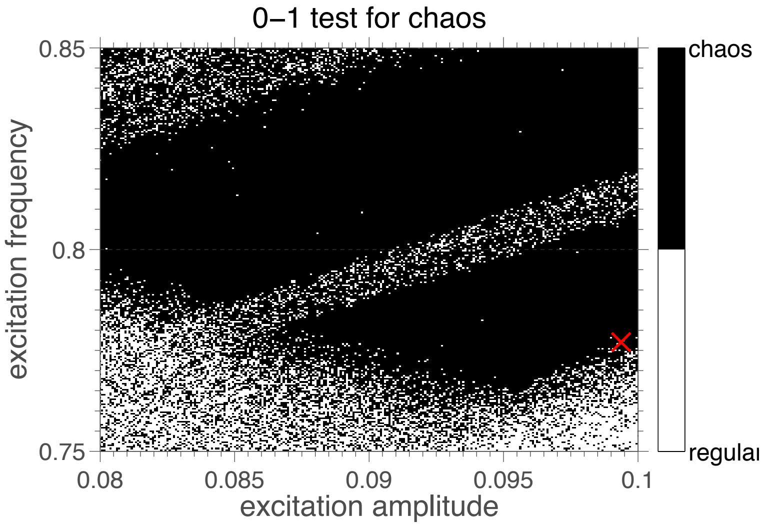
- Penalty parameter: 10
- Direct search on a uniform grid: 256×256 points
- CPU time:[1] $\approx$ 14 hours

[1] MacBook Pro “Core i7” 2.2 GHz 16GB 1333 MHz DDR3

Reference solution: performance function



Reference solution: chaos indicator



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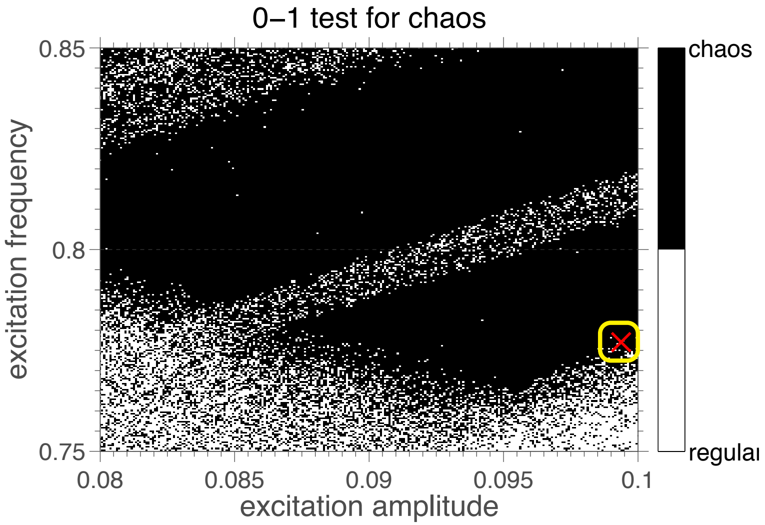

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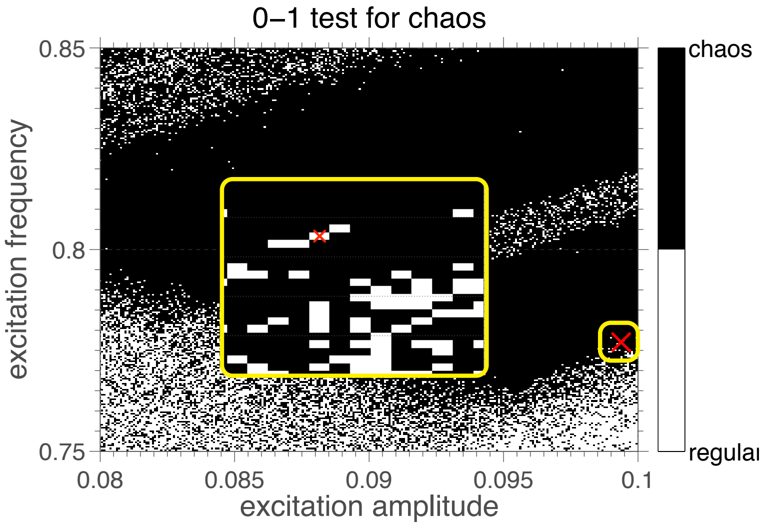

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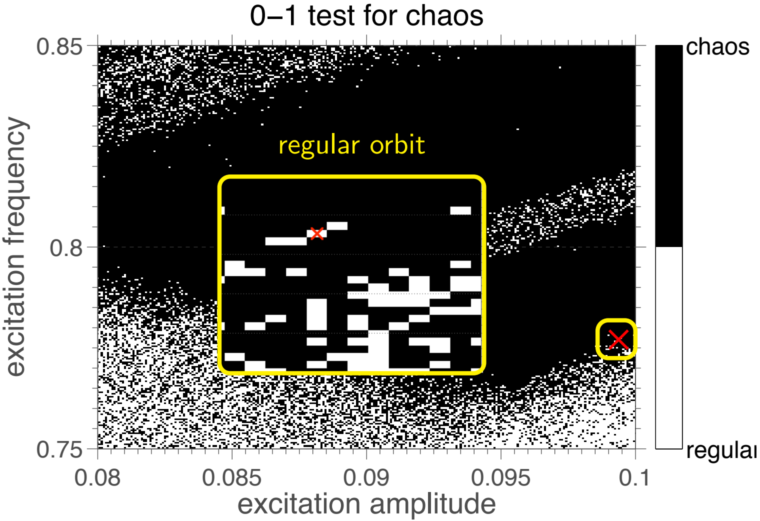
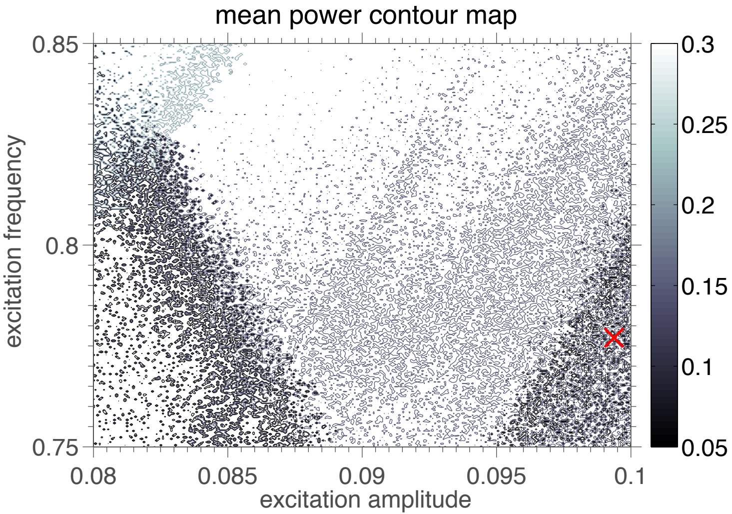


Reference solution: mean power



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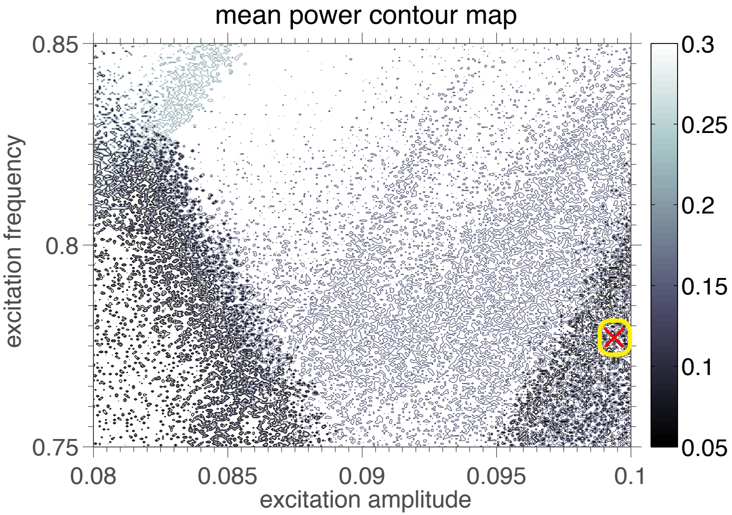

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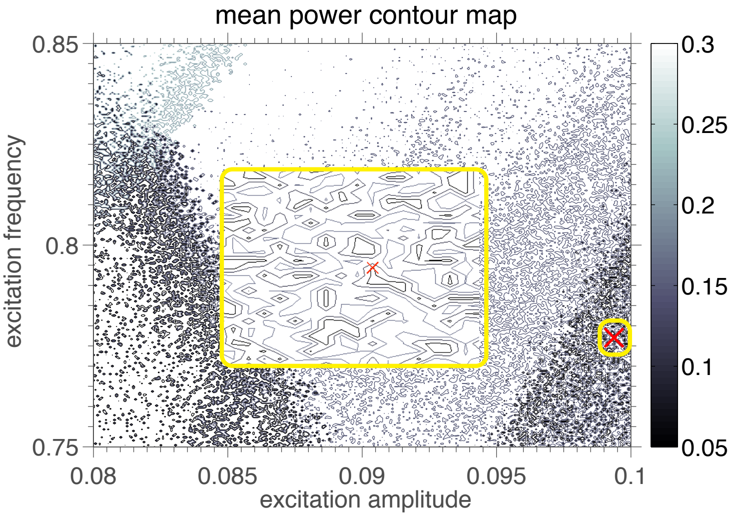

Cross-entropy solution

- Domain:

$$0.08 \leq f \leq 0.1$$
$$0.75 \leq \Omega \leq 0.85$$

- Penalty parameter: 10
- Number of CE samples: 50
- Percentage of elite samples: 10%
- CE samples distribution: Truncated Gaussian
- Convergence criterium: $||\boldsymbol{\sigma}||_\infty < 1 \times 10^{-3}$
- Levels until convergence: 17
- CPU time:[2] $\approx$ 12 minutes

[2] MacBook Pro "Core i7" 2.2 GHz 16GB 1333 MHz DDR3

Cross-entropy animation (50 samples)

Cross-entropy animation (75 samples)

Cross-entropy animation (25 samples)

Cross-entropy performance

- Percentage of elite samples: 10%
- Convergence criterium: $||\boldsymbol{\sigma}||_{\infty} < 1 \times 10^{-3}$

CE samples	CE levels	CPU time[3] (seconds)	speed-up
reference	—	50 632	—
25	15	304	167
50	17	696	73
75	29	1736	29
100	42	3286	15

[3] MacBook Pro "Core i7" 2.2 GHz 16GB 1333 MHz DDR3

Section 5

Final Remarks

Concluding remarks

Contributions:

- Formulation of a nonlinear optimization problem to enhance power recovered by a bi-stable energy harvesting device
- Efficient solution of this optimization problem by means of cross-entropy method

Ongoing research:

- Study the effect of sampling from non-Gaussian distributions on the proposed cross-entropy approach
- Combine this cross-entropy approach with techniques of chaos control to enhance energy harvesting performance

Acknowledgments

Academic discussion:

- Prof. Welington Oliveira (MINES ParisTech)

Invitation for the symposium:

- Prof. Antônio José da Silva Neto (UERJ)
- IMPA

Financial support:

- CNPq
- CAPES
- FAPERJ

Thank you for your attention!

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www.americocunha.org

A. Cunha Jr **Enhancing the performance of a bi-stable energy harvesting device via cross-entropy method** (under review).

https://hal.archives-ouvertes.fr/hal-01531845

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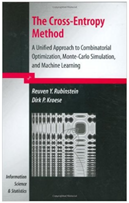

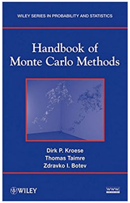

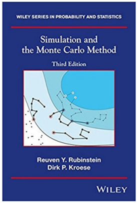

Physical system parameters

parameter	value
ξ	0.01
χ	0.05
f	0.083
Ω	0.8
λ	0.05
κ	0.5
x_0	1.0
$\dot{x}_0$	0.0
v_0	0.0