Exploration of the nonlinear stochastic dynamics of a bi-stable energy harvester

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NUMERICO – **Nu**cleus of **M**odeling and **E**xpe**ri**mentation with **Co**mputers

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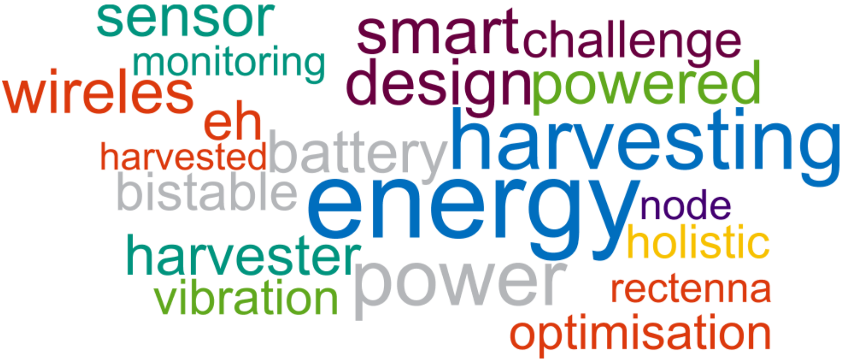
In collaboration with: Vinicius Lopes (UERJ)
João Peterson (UERJ)

DDays LAC 2018
November 26 – 30, 2018
Punta del Este, Uruguay

1. Introduction
2. Nonlinear Dynamics
3. Stochastic Dynamics
4. Final Remarks

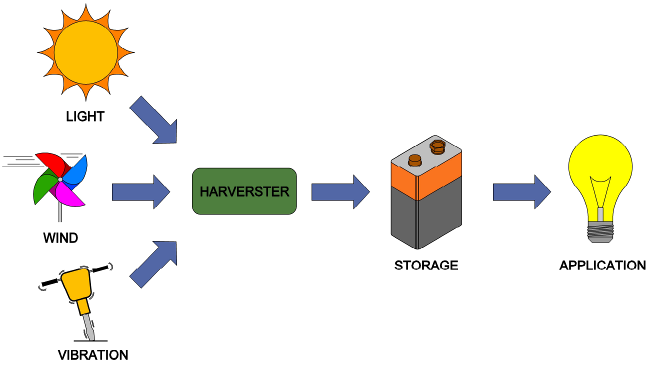
Section 1

Introduction



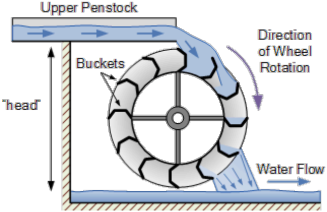
*Picture obtainded from Google Images.

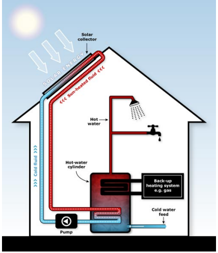
Energy Harvesting concept



- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

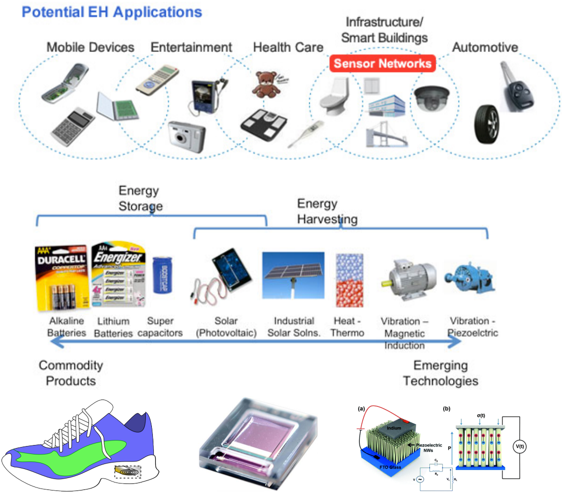
Classical technologies in Energy Harvesting

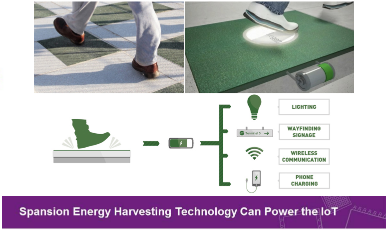


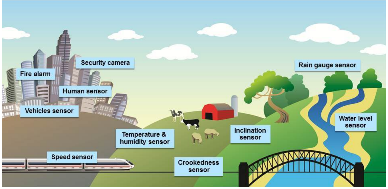


*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

Emergent technologies in Energy Harvesting







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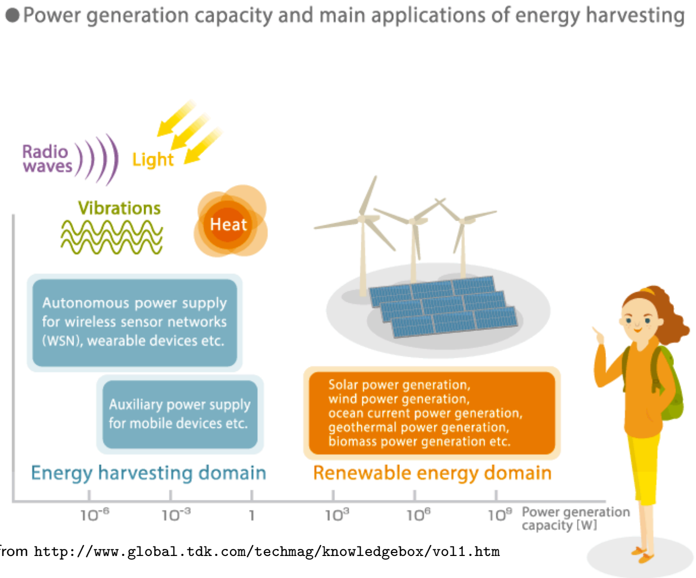
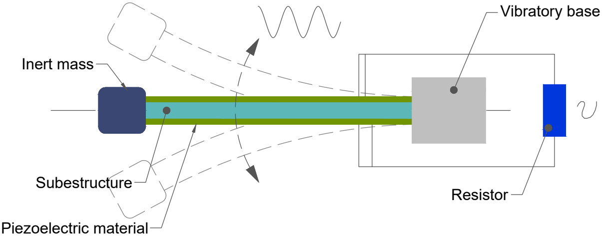
Energy scale for modern Energy Harvesting technologies



*Picture from `http://www.global.tdk.com/techmag/knowledgebox/vol1.htm`

Vibration based Energy Harvesting

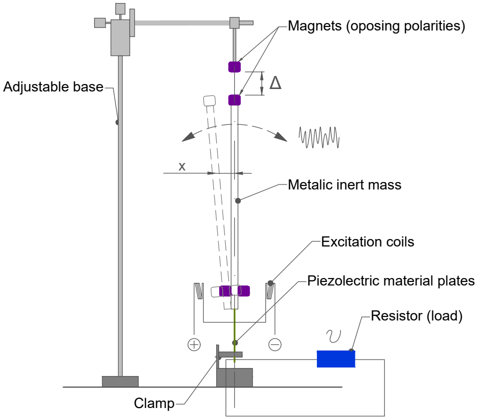
Monostable system driven by regular signal



S. Roundy, P. K. Wright and J. Rabaey, A study of low level bibrations as a power source for wireless sensor nodes. **Computer Communications**, 26: 1131-1144, 2003.

Vibration based Energy Harvesting

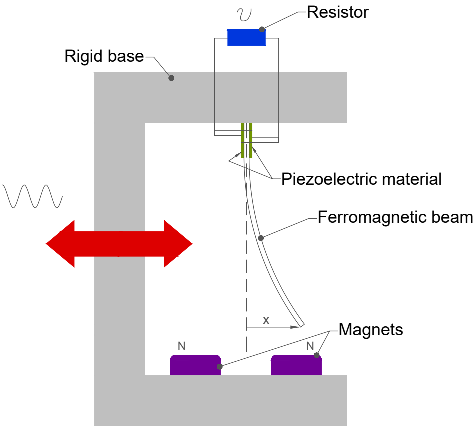
Bistable system driven by a noisy signal



F. Cottone, H. Vocca and L. Gammaitoni, Nonlinear energy harvesting. **Physical Review Letters**, 102: 080601, 2009.

Vibration based Energy Harvesting

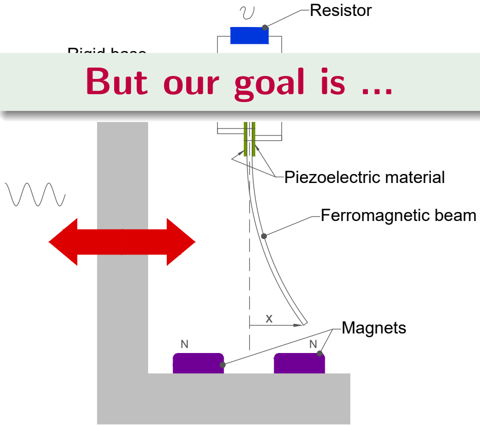
Bistable system driven by regular signal



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Vibration based Energy Harvesting

Bistable system driven by regular signal

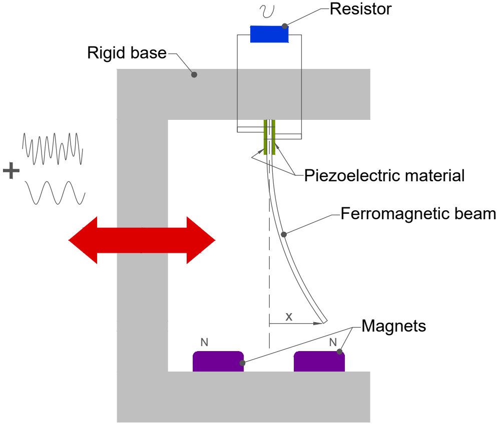


But our goal is ...

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Vibration based Energy Harvesting

Bistable system driven by regular and noisy signals



J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr, **On the nonlinear stochastic dynamics of piezo-magneto-elastic energy harvester driven by colored noise**, (in preparation) 2018.

Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
  - Time series
  - Poincaré sections
  - Bifurcation diagrams
  - Basis of attractions
  - Test 0-1 for chaos
- Propose strategies to enhance the recovered energy
  - Nonlinear optimization
  - Control of chaos
- Model the underlying uncertainties and study their influence
  - System parameters variability
  - Noise in system excitation

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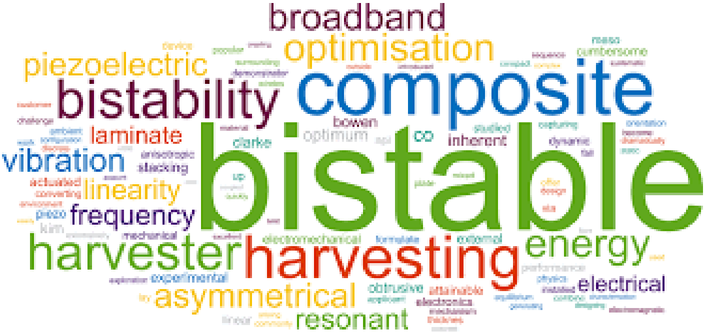
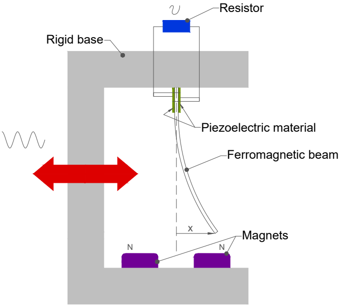
Section 2

Nonlinear Dynamics



*Picture obtainded from Google Images.

Bistable harvester driven by regular signal



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t$$

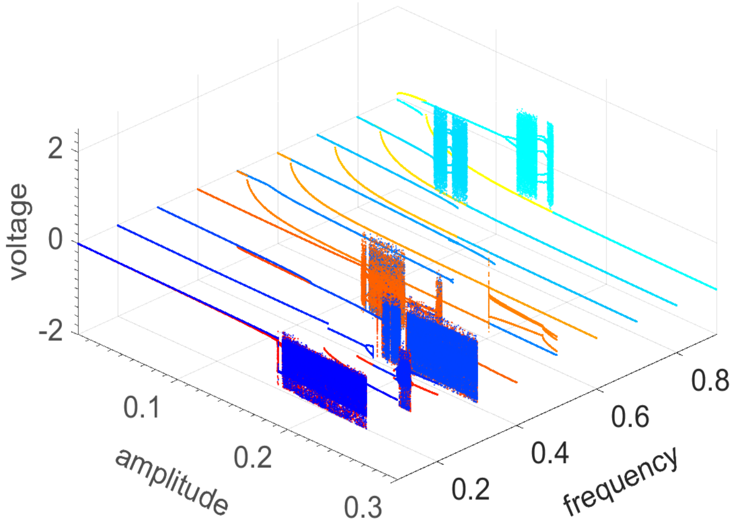
$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0,\ \dot{x}(0) = \dot{x}_0,\ \upsilon(0) = \upsilon_0$$

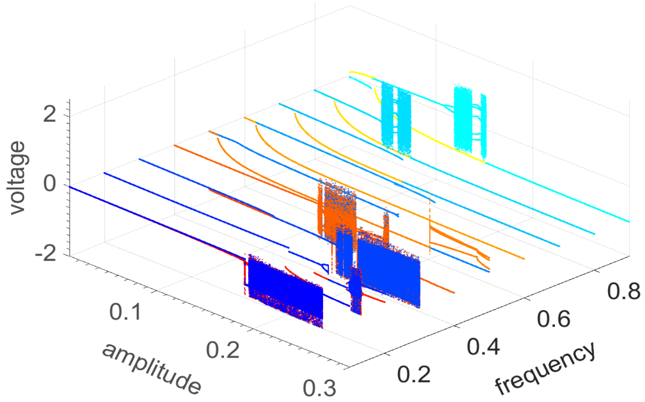
A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Nonlinear dynamics animation

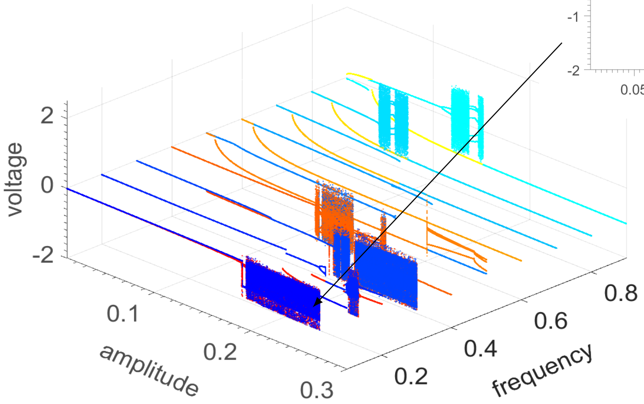
Global overview of force amplitude effect

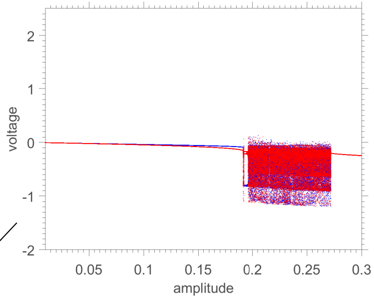


Bifurcation diagrams: voltage vs amplitude



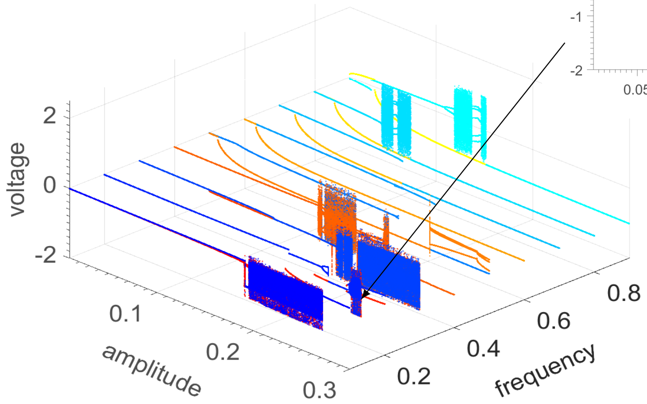
Bifurcation diagrams: voltage vs amplitude

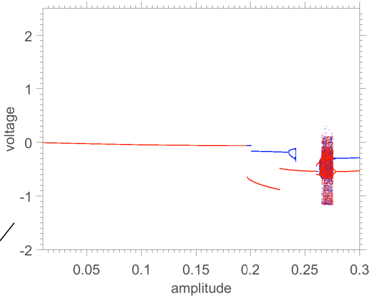




$\Omega = 0.1$

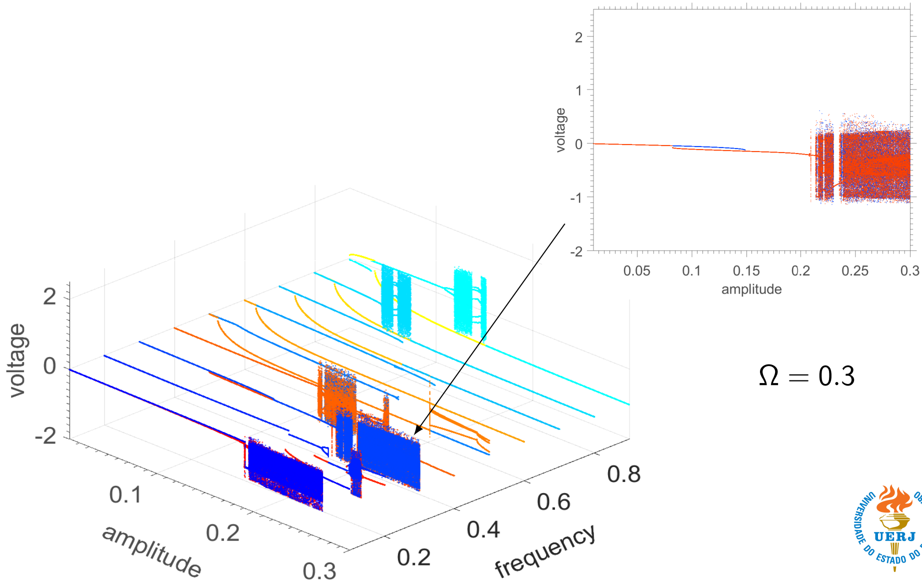
Bifurcation diagrams: voltage vs amplitude





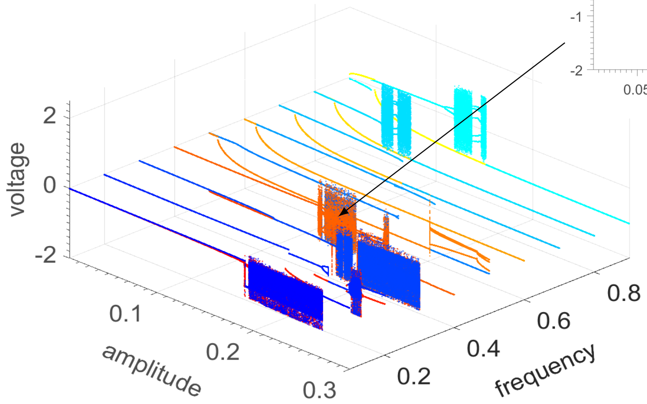
$\Omega = 0.2$

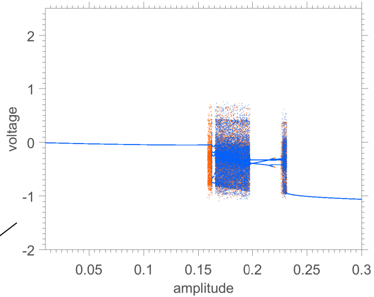
Bifurcation diagrams: voltage vs amplitude



$\Omega = 0.3$

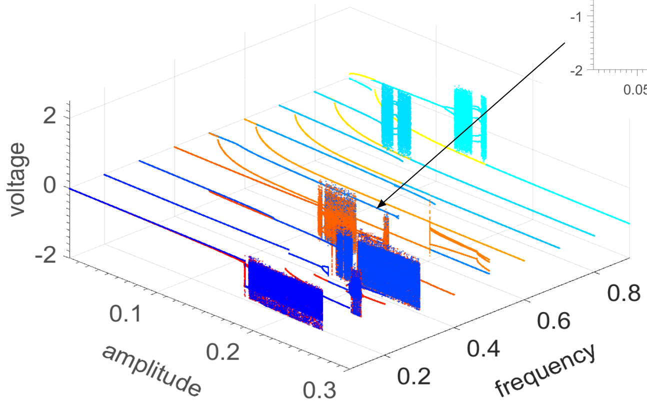
Bifurcation diagrams: voltage vs amplitude

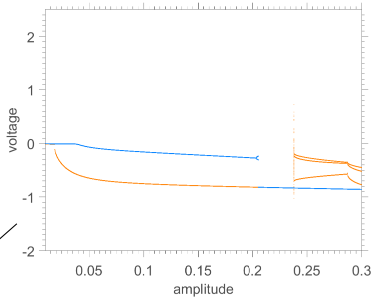




$\Omega = 0.4$

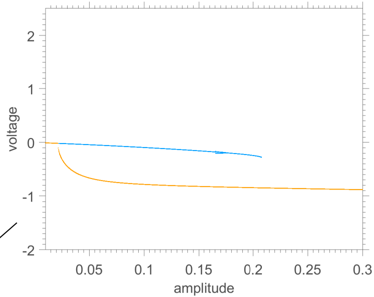
Bifurcation diagrams: voltage vs amplitude

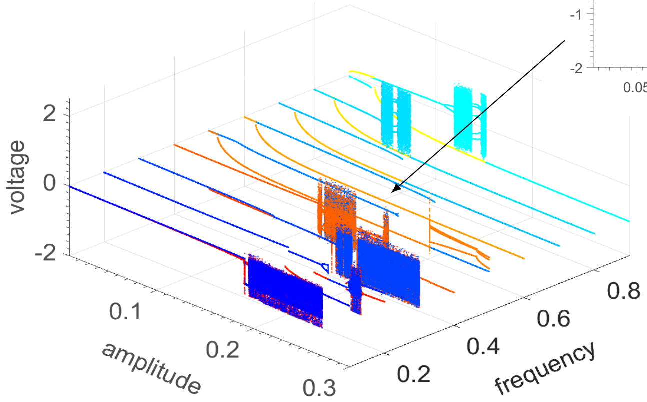




$\Omega = 0.5$

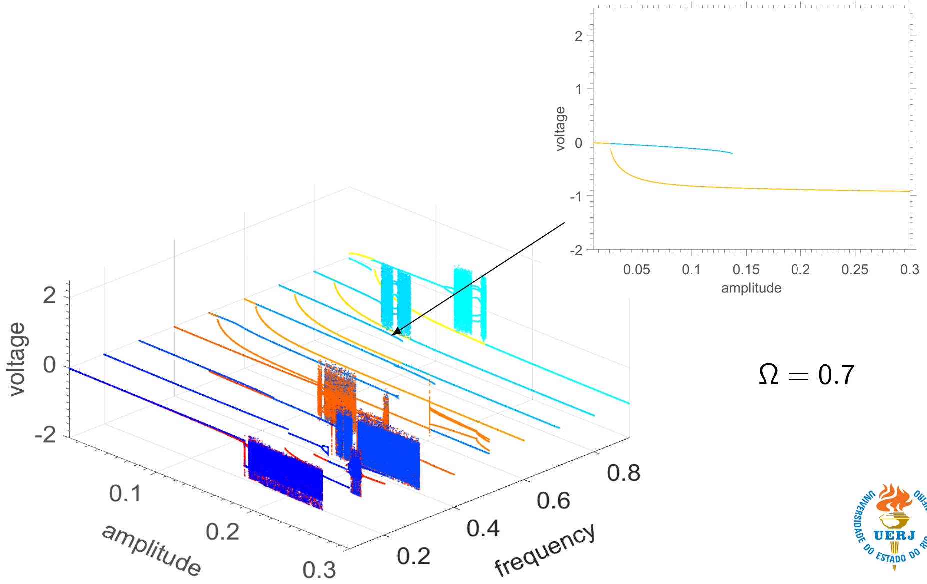
Bifurcation diagrams: voltage vs amplitude





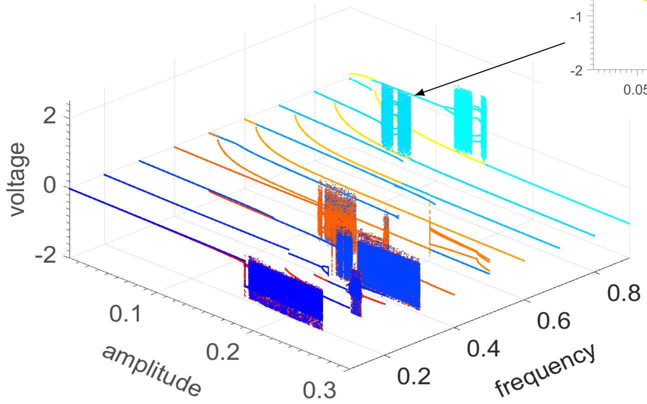
$\Omega = 0.6$

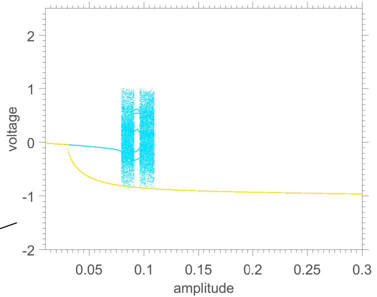
Bifurcation diagrams: voltage vs amplitude



$\Omega = 0.7$

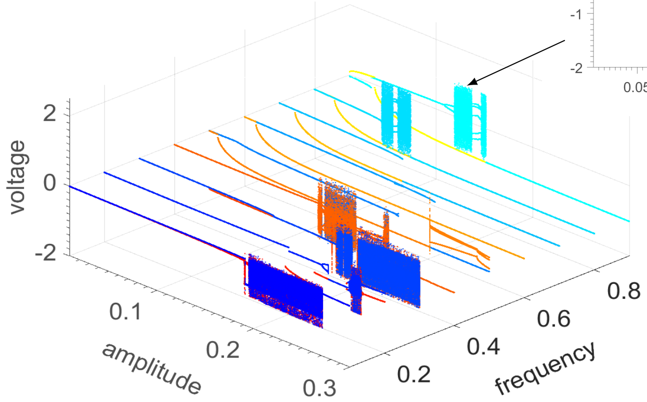
Bifurcation diagrams: voltage vs amplitude

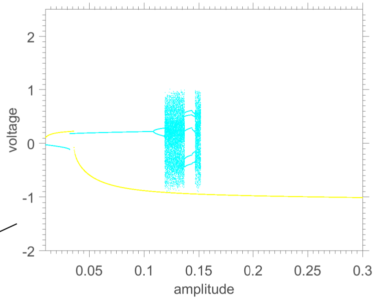




$\Omega = 0.8$

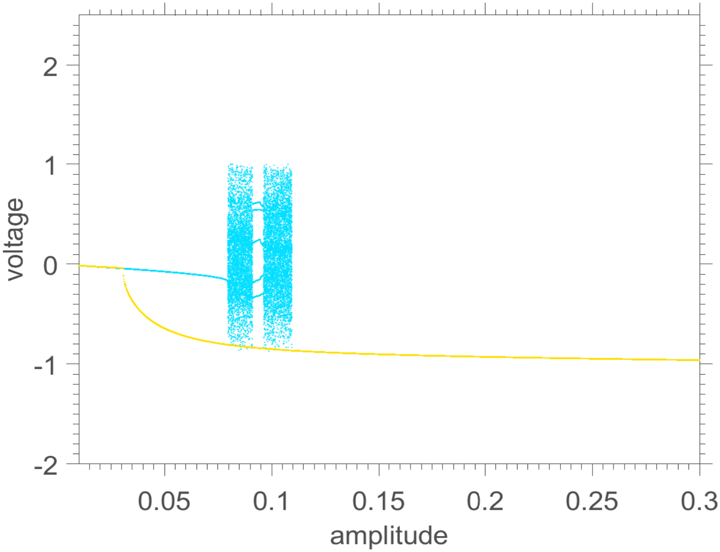
Bifurcation diagrams: voltage vs amplitude



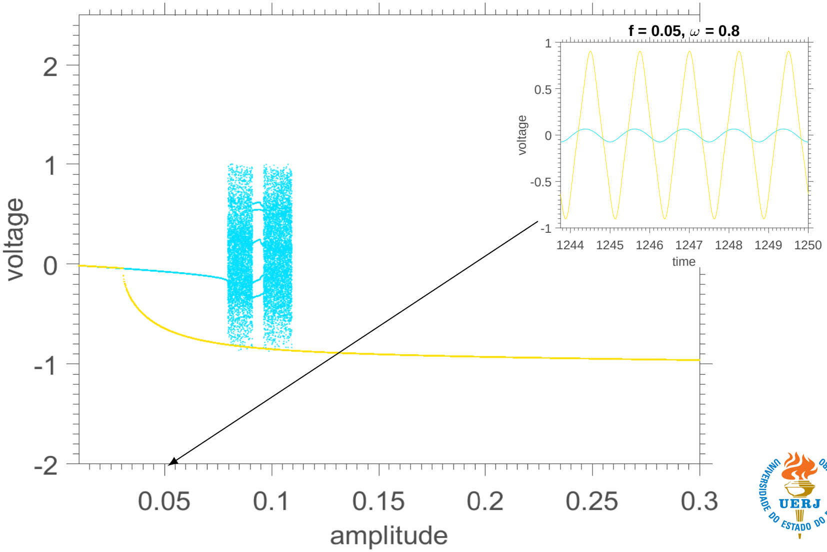


$\Omega = 0.9$

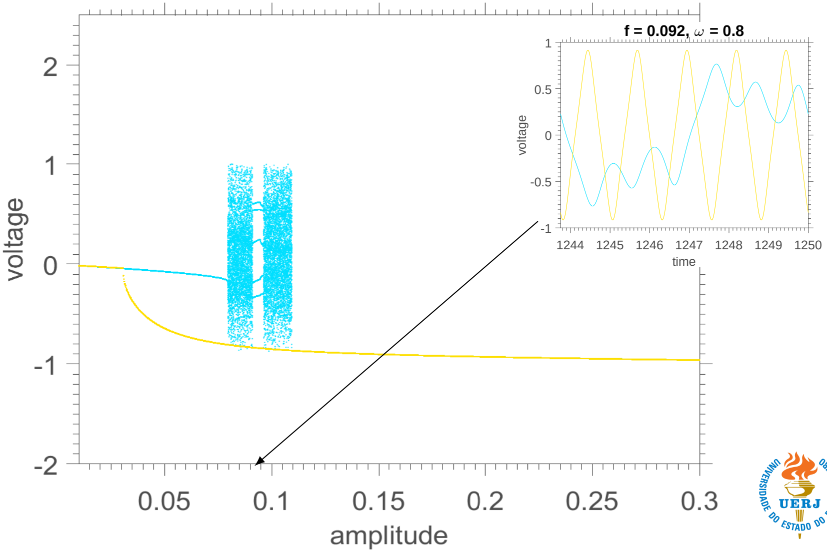
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



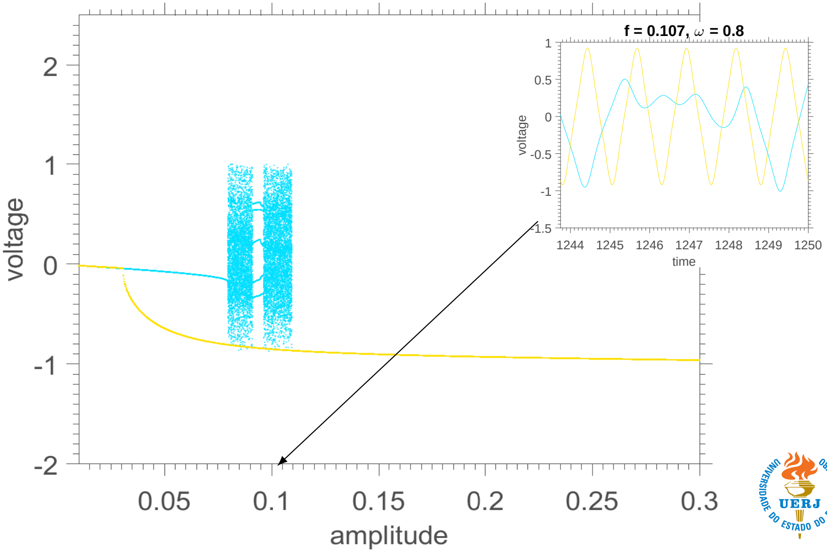
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



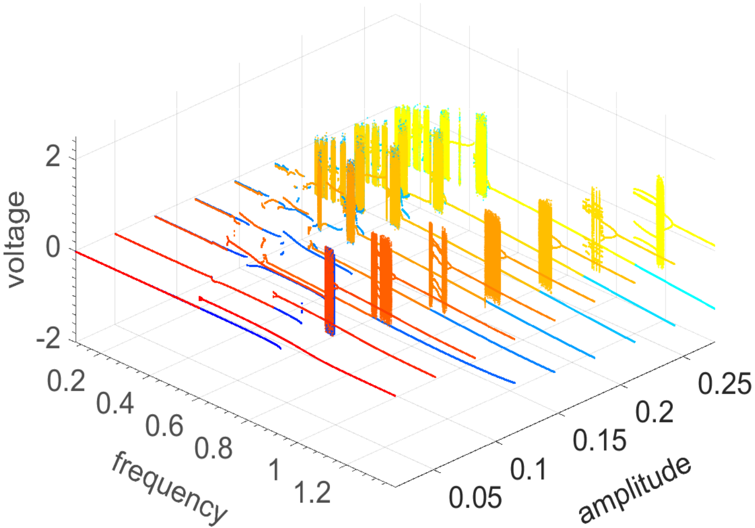
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



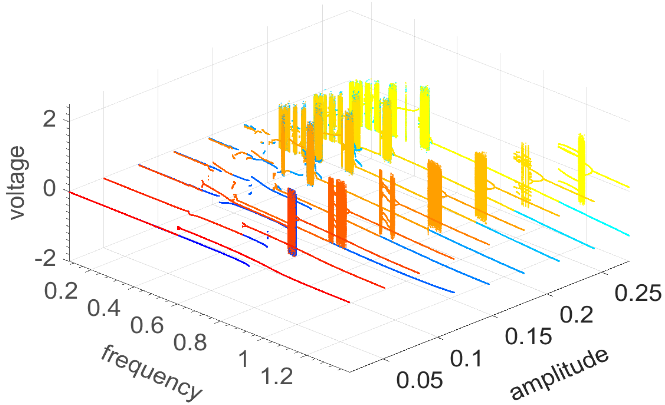
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



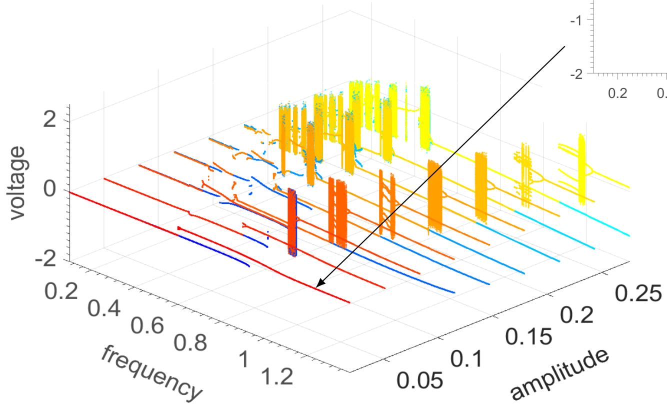
Global overview of force frequency effect

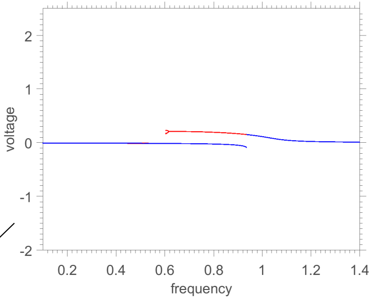


Bifurcation diagrams: voltage vs frequency



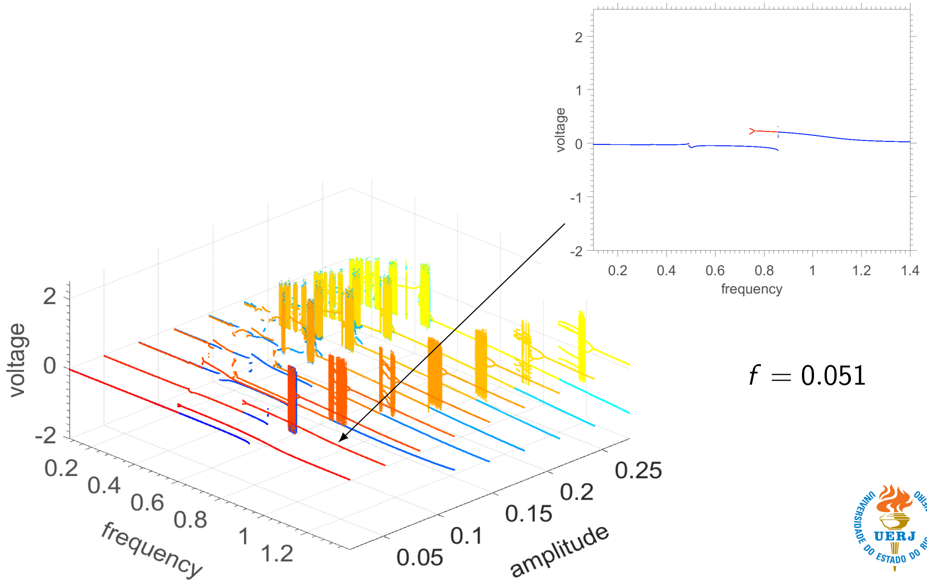
Bifurcation diagrams: voltage vs frequency





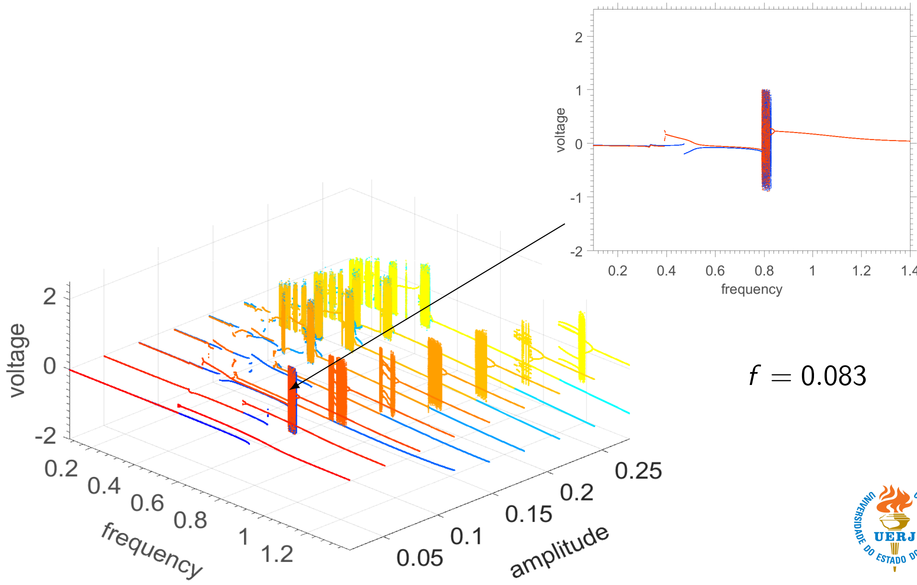
$f = 0.019$

Bifurcation diagrams: voltage vs frequency



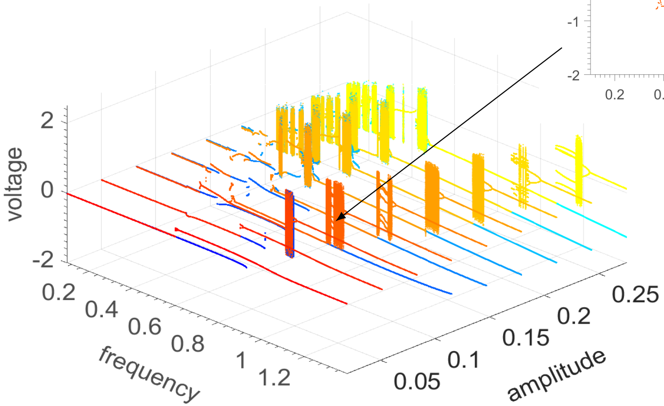
$f = 0.051$

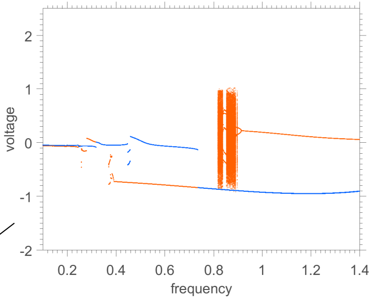
Bifurcation diagrams: voltage vs frequency



$f = 0.083$

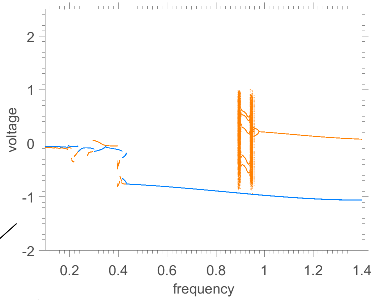
Bifurcation diagrams: voltage vs frequency

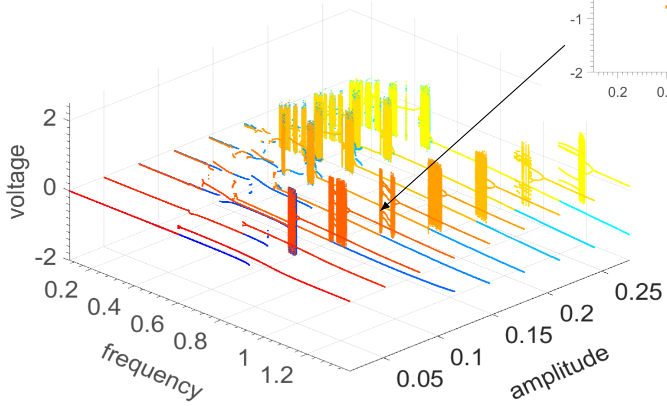




$f = 0.115$

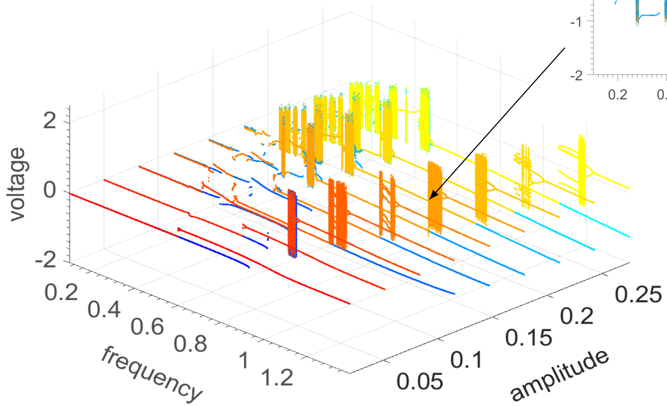
Bifurcation diagrams: voltage vs frequency

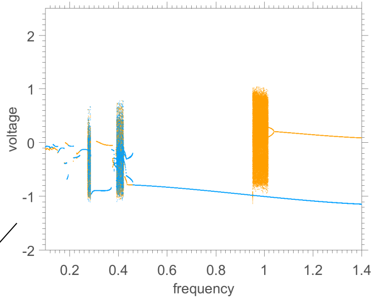




$f = 0.147$

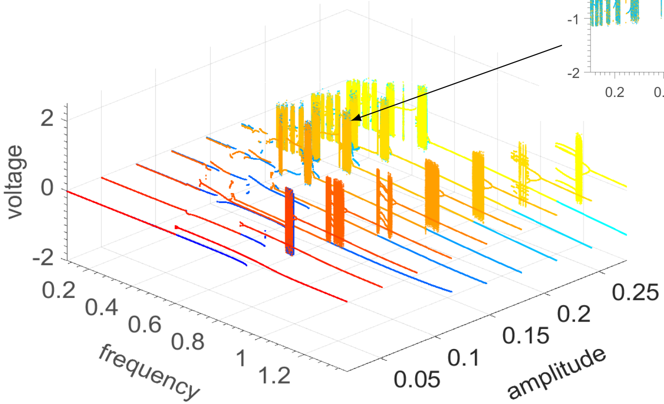
Bifurcation diagrams: voltage vs frequency

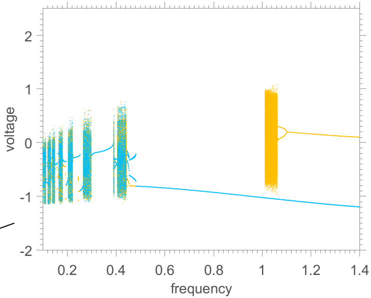




$f = 0.179$

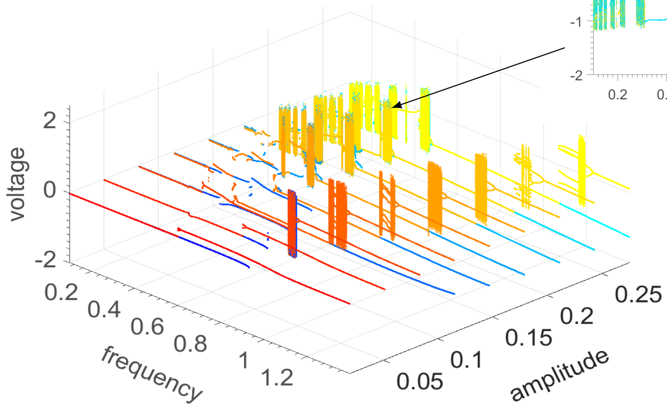
Bifurcation diagrams: voltage vs frequency

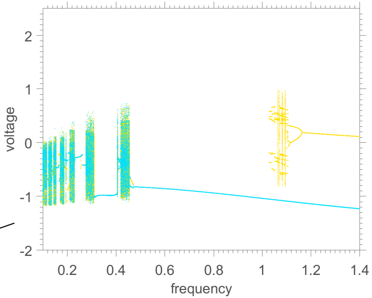




$f = 0.211$

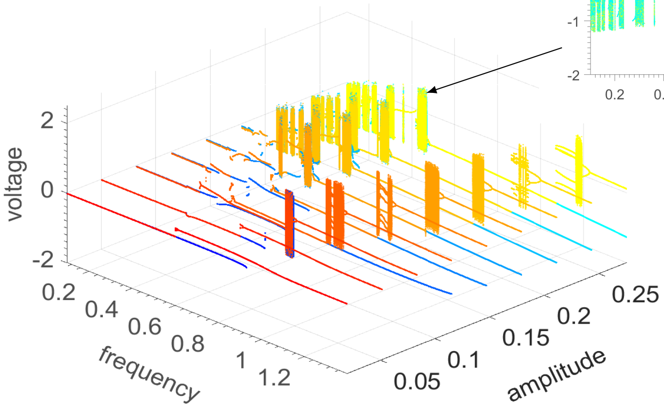
Bifurcation diagrams: voltage vs frequency

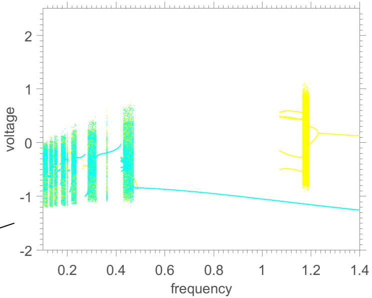




$f = 0.243$

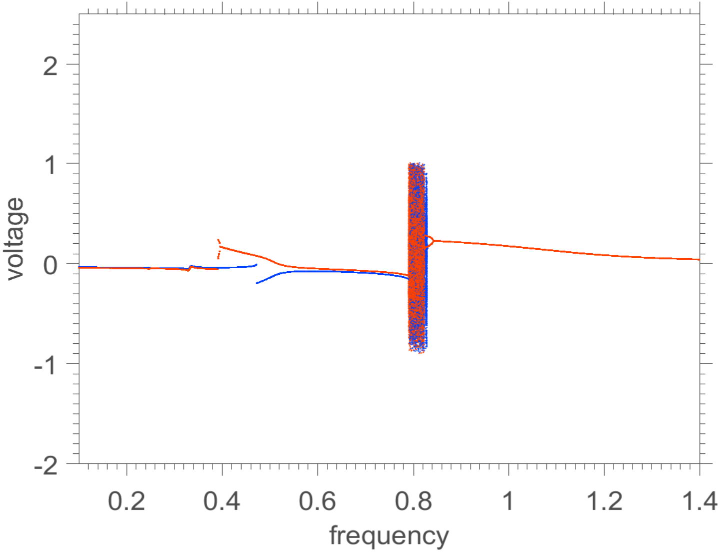
Bifurcation diagrams: voltage vs frequency



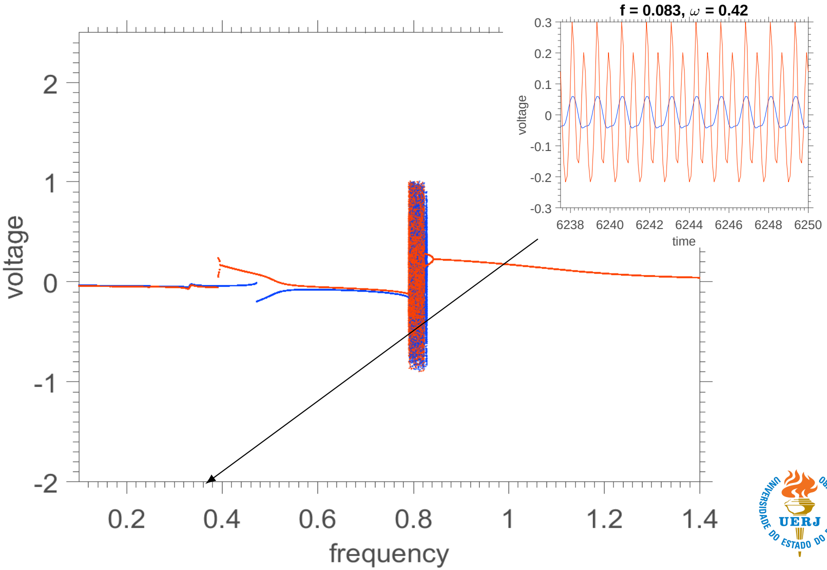


$f = 0.275$

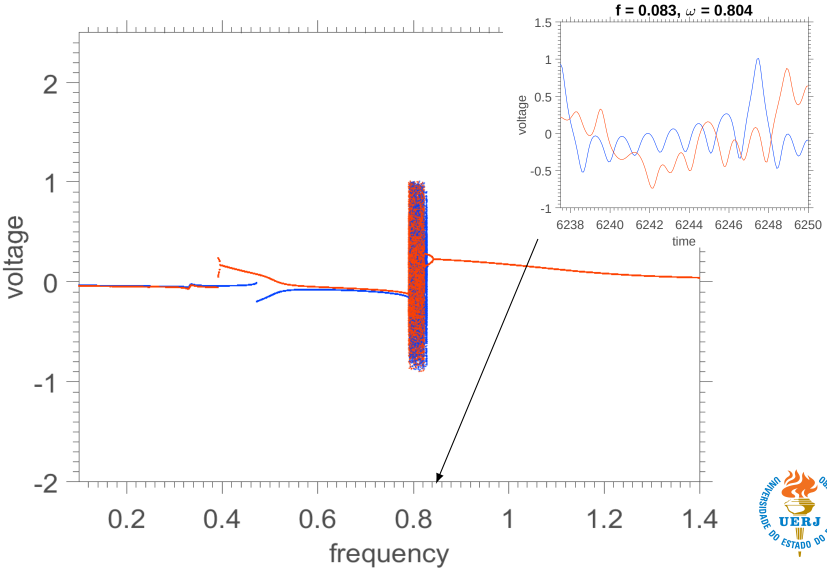
Forward and backward diagrams ($f = 0.083$)



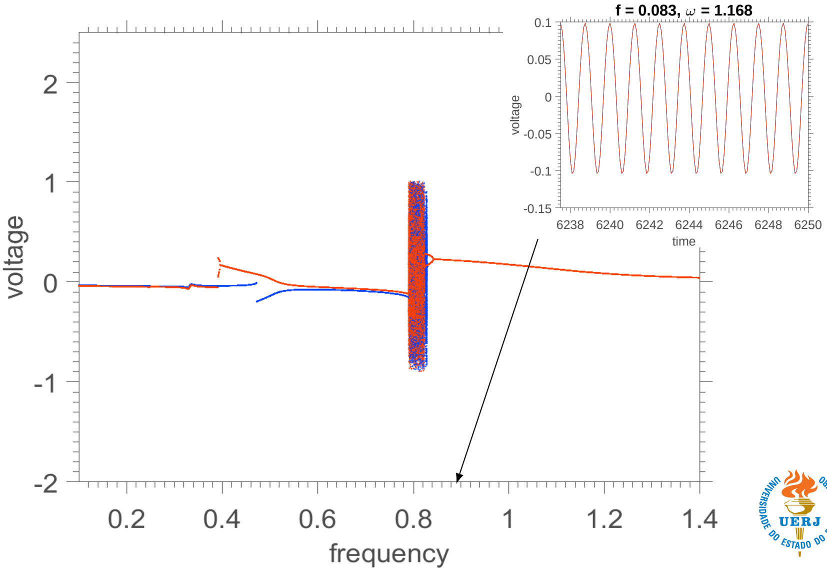
Forward and backward diagrams ($f = 0.083$)



Forward and backward diagrams ($f = 0.083$)



Forward and backward diagrams ($f = 0.083$)



Basins of attraction

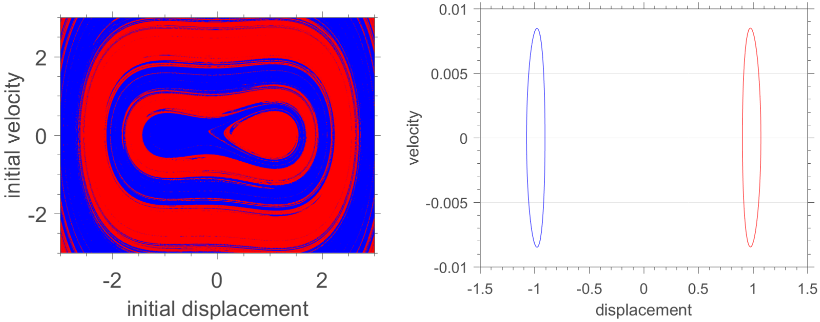


Figure: $f = 0.083$ and $\Omega = 0.1$

Basins of attraction

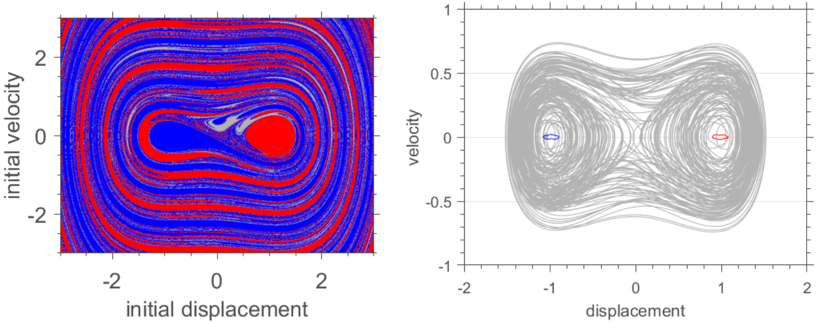


Figure: $f = 0.083$ and $\Omega = 0.2$

Basins of attraction

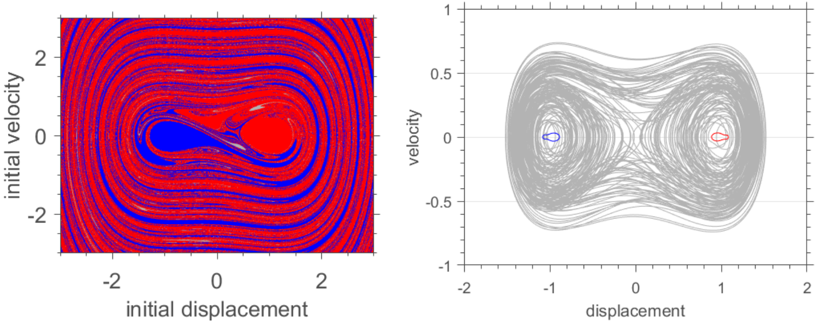


Figure: $f = 0.083$ and $\Omega = 0.3$

Basins of attraction

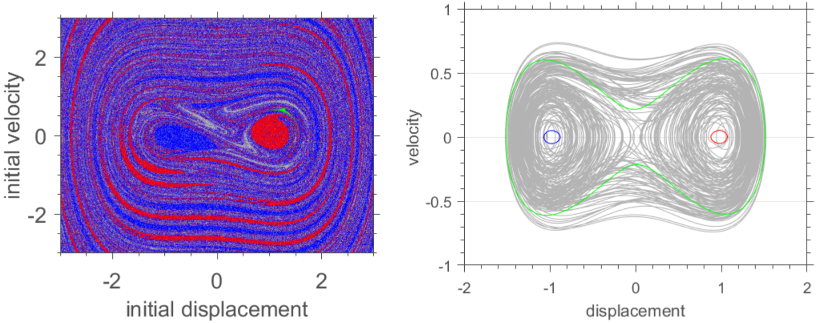


Figure: $f = 0.083$ and $\Omega = 0.4$

Basins of attraction

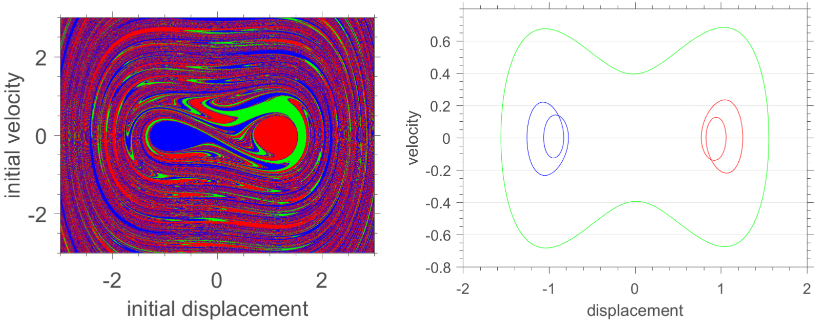


Figure: $f = 0.083$ and $\Omega = 0.5$

Basins of attraction

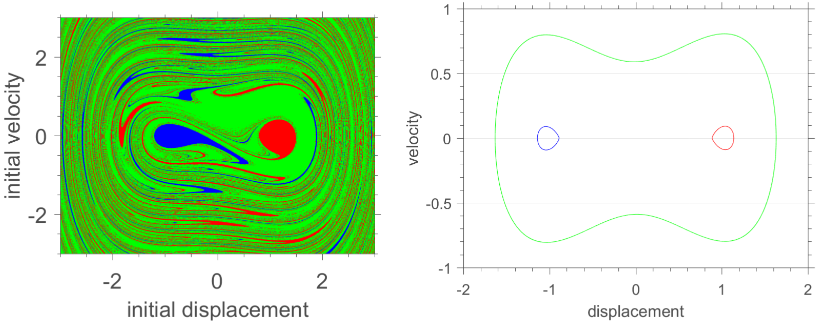


Figure: $f = 0.083$ and $\Omega = 0.6$

Basins of attraction

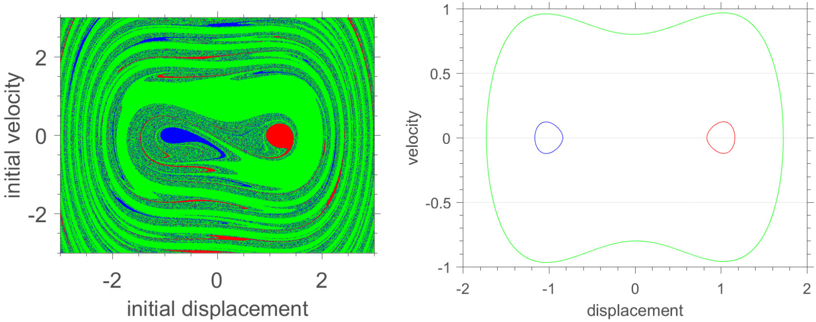


Figure: $f = 0.083$ and $\Omega = 0.7$

Basins of attraction

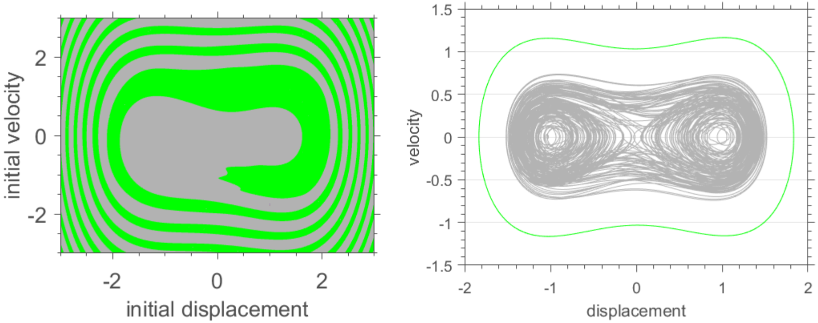


Figure: $f = 0.083$ and $\Omega = 0.8$

Basins of attraction

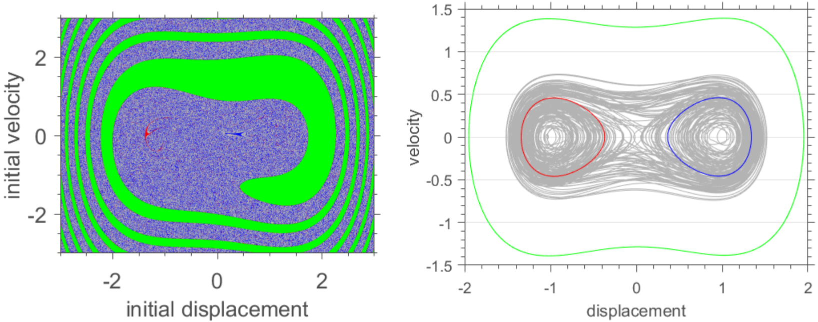
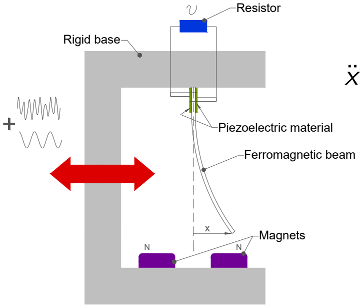


Figure: $f = 0.083$ and $\Omega = 0.9$

Section 3

Stochastic Dynamics

Bistable harvester driven by regular and noisy signals



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1-x^2) - \chi\upsilon = f\cos\Omega t + \text{"noise"}$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0, \;\; \dot{x}(0) = \dot{x}_0, \;\; \upsilon(0) = \upsilon_0$$

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Probabilistic model for system dynamics

- nonlinear stochastic dynamical system:

$$\ddot{X} + 2\xi\dot{X} - \frac{1}{2}x(1 - X^2) - \chi V = f\cos\Omega t + N_t$$

$$\dot{V} + \lambda V + \kappa\dot{X} = 0$$

$$X(0) = x_0,\ \dot{X}(0) = \dot{x}_0,\ V(0) = \upsilon_0$$

- external excitation (zero-mean stationary Gaussian process):

$$N_t = \left\{N(t), t \geq 0\right\}, \quad \mathbb{E}\left\{N_t\right\} = 0$$

- covariance function (colored noise):

$$\mathrm{cov}_N(t_1, t_2) = \sigma_N \exp\left(-\frac{|t_2 - t_1|}{\tau_{corr}}\right)$$

Computational representation of the noise

Karhunen-Loève decomposition:

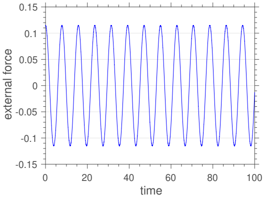
$$N_t = \mathbb{E}\left\{N_t\right\} + \sum_{n=1}^{\infty} \sigma_N \sqrt{\lambda_n}\, \varphi_n(t)\, Y_n$$

$$\int_{\mathbb{R}} \mathrm{cov}_N(t_1, t_2)\, \varphi_n(t_2)\, dt_1 = \lambda_n\, \varphi_n(t_1), \quad t_1 \in \mathbb{R}$$

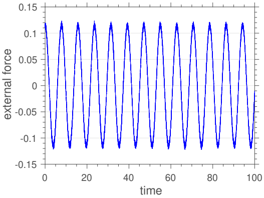
$$\mathbb{E}\left\{Y_n\right\} = 0 \qquad \text{and} \qquad \mathbb{E}\left\{Y_n Y_m\right\} = \delta_{mn}$$

D. Xiu, **Numerical Methods for Stochastic Computations: A Spectral Method Approach**, Princeton University Press, 2010.

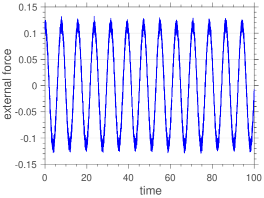
Realizations of random external force



(a) 1% of noise



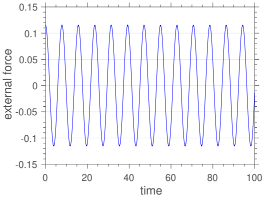
(b) 25% of noise



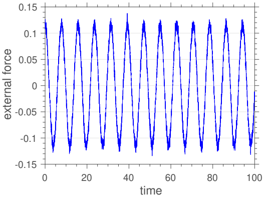
(c) 50% of noise

$$\tau_{cor}/\tau_{\Omega} = 5\%$$

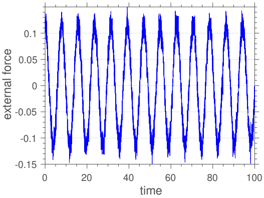
Realizations of random external force



(a) 1% of noise



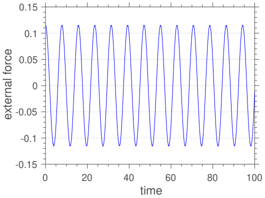
(b) 25% of noise



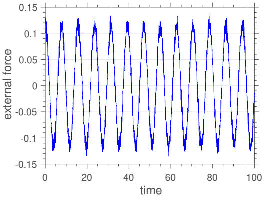
(c) 50% of noise

$$\tau_{cor}/\tau_{\Omega} = 25\%$$

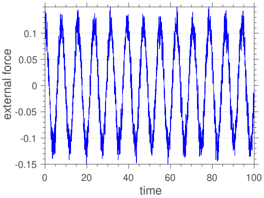
Realizations of random external force



(a) 1% of noise



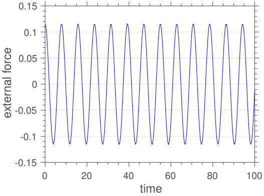
(b) 25% of noise



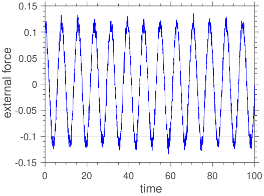
(c) 50% of noise

$$\tau_{cor}/\tau_{\Omega} = 50\%$$

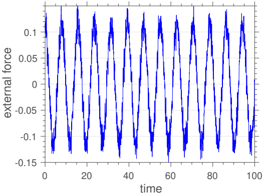
Realizations of random external force



(a) 1% of noise



(b) 25% of noise



(c) 50% of noise

$$\tau_{cor}/\tau_{\Omega} = 75\%$$

Propagation of uncertainties: Monte Carlo method

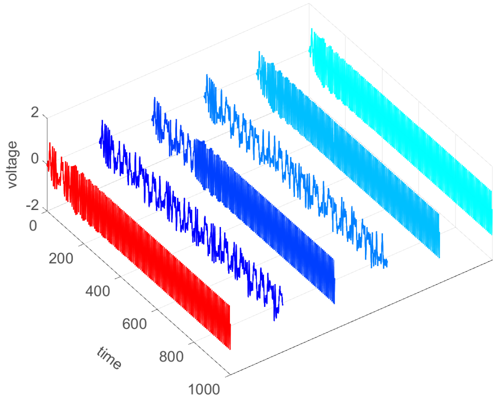
Monte Carlo Method

pre-processing	**processing**	**post-processing**
generation of scenarios	solution of model equations	computation of statistics
$\boldsymbol{X}_1$ $\vdots$ $\boldsymbol{X}_M$	$\boldsymbol{U} = h(\boldsymbol{X})$ **computational model**	$\boldsymbol{U}_1 = h(\boldsymbol{X}_1)$ $\vdots$ $\boldsymbol{U}_M = h(\boldsymbol{X}_M)$
known $F_{\boldsymbol{X}}$		estimated $F_{\boldsymbol{U}}$
generator of random vector $\boldsymbol{X}$	deterministic solver of $\boldsymbol{u} = h(\boldsymbol{x})$	statistical inference to estimate convergence and distribution of $\boldsymbol{U}$

A. Cunha Jr, R. Nasser, R. Sampaio, H. Lopes, and K. Breitman, *Uncertainty quantification through Monte Carlo method in a cloud computing setting.* **Computer Physics Communications**, 185: 1355–1363, 2014.

Nonlinear stochastic dynamics animation

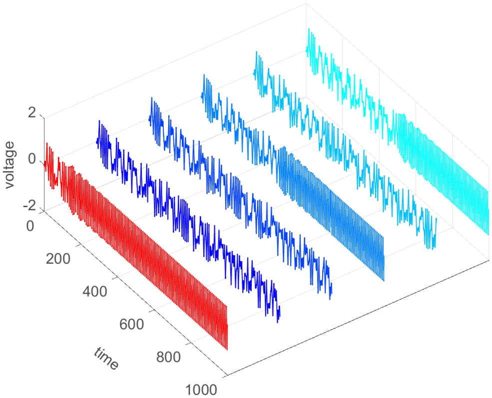
Typical voltage time series (1% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$f = 0.115,\ \Omega = 0.8$

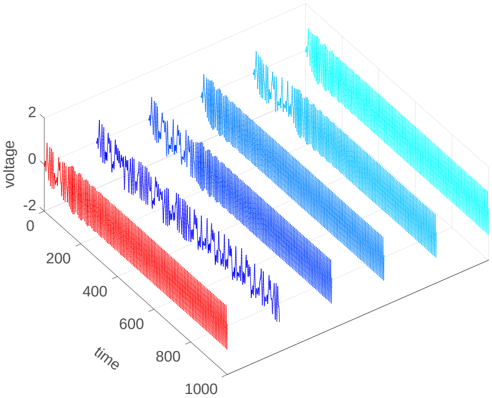
Typical voltage time series (1% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$f = 0.115,\ \Omega = 0.8$

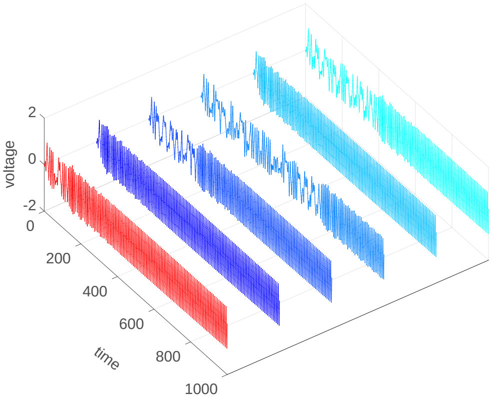
Typical voltage time series (1% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$f = 0.115$, $\Omega = 0.8$

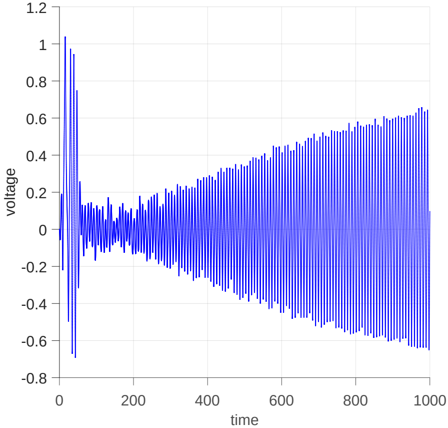
Typical voltage time series (1% of noise)



(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

$$f = 0.115,\ \Omega = 0.8$$

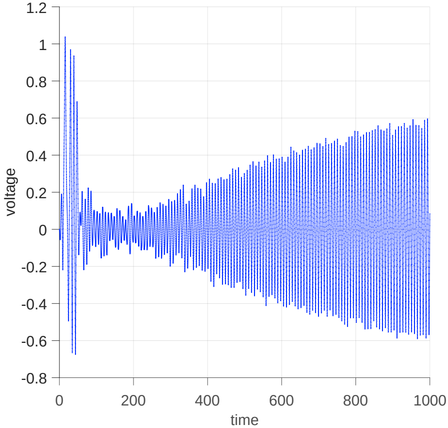
Mean of voltage time series (1% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$$f = 0.115,\ \Omega = 0.8$$

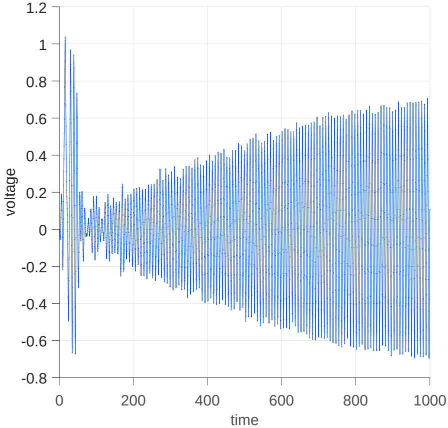
Mean of voltage time series (1% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$f = 0.115,\ \Omega = 0.8$

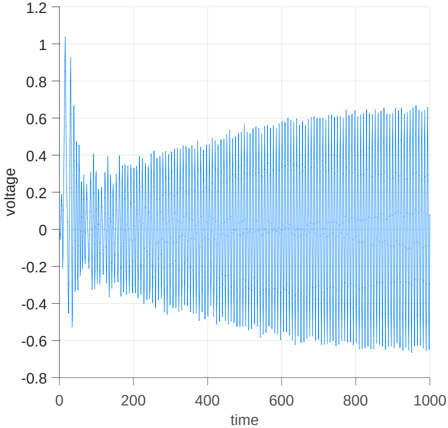
Mean of voltage time series (1% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$f = 0.115,\ \Omega = 0.8$

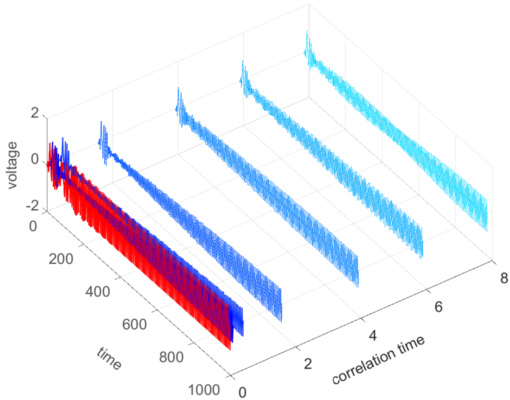
Mean of voltage time series (1% of noise)



(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

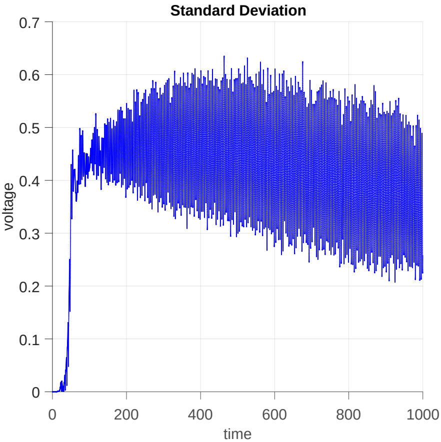
$f = 0.115,\ \Omega = 0.8$

Mean of voltage time series (1% of noise)



$f = 0.115$, $\Omega = 0.8$

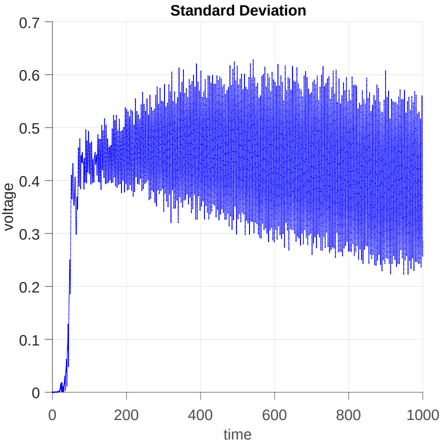
Standard deviation of voltage time series (1% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$f = 0.115,\ \Omega = 0.8$

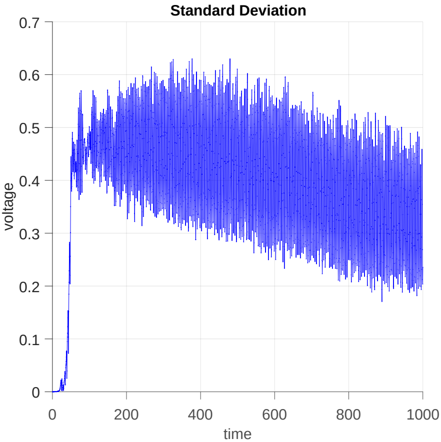
Standard deviation of voltage time series (1% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$f = 0.115$, $\Omega = 0.8$

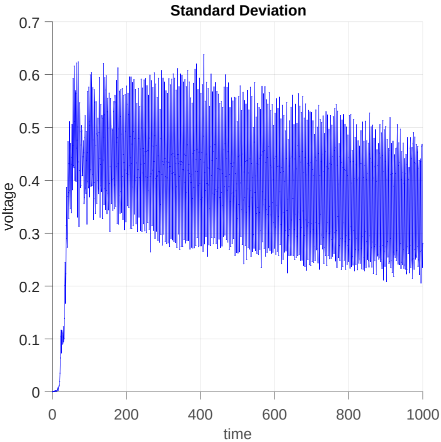
Standard deviation of voltage time series (1% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$f = 0.115,\ \Omega = 0.8$

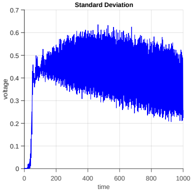
Standard deviation of voltage time series (1% of noise)



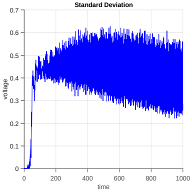
(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

$f = 0.115,\ \Omega = 0.8$

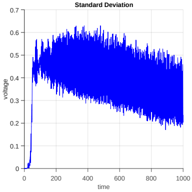
Standard deviation of voltage time series (1% of noise)



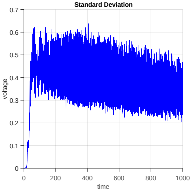
(e) $\tau_{cor}/\tau_{\Omega} = 5\%$



(f) $\tau_{cor}/\tau_{\Omega} = 25\%$



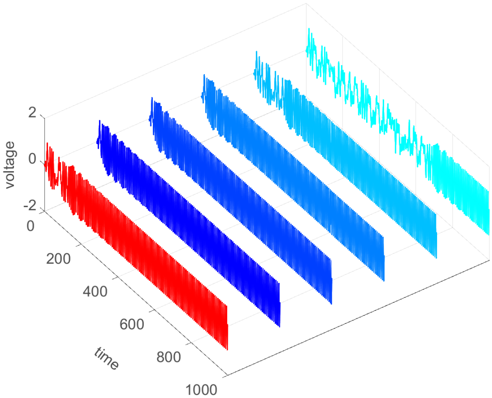
(g) $\tau_{cor}/\tau_{\Omega} = 50\%$



(h) $\tau_{cor}/\tau_{\Omega} = 75\%$

$$f = 0.115,\ \Omega = 0.8$$

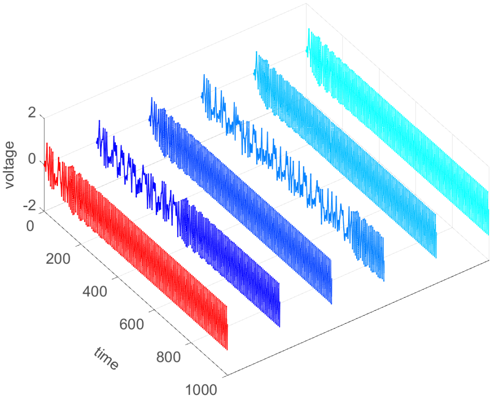
Typical voltage time series (25% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$f = 0.115,\ \Omega = 0.8$

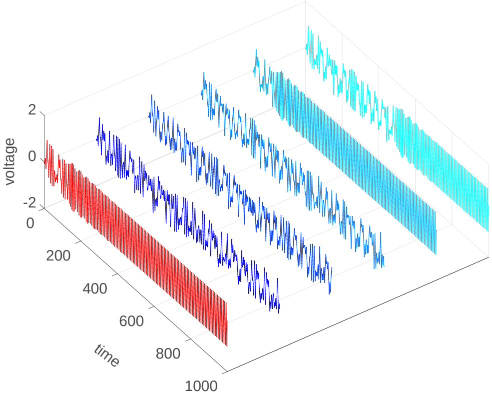
Typical voltage time series (25% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$$f = 0.115,\ \Omega = 0.8$$

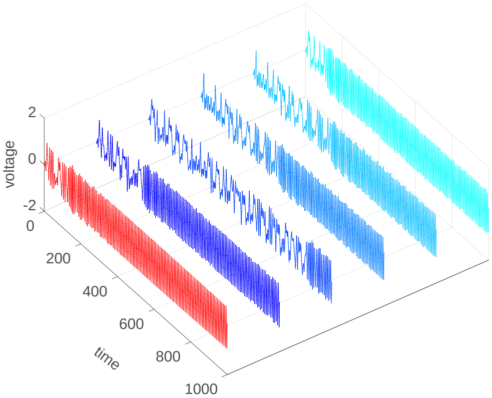
Typical voltage time series (25% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$$f = 0.115,\ \Omega = 0.8$$

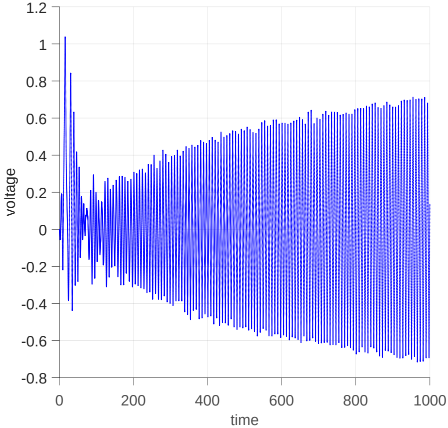
Typical voltage time series (25% of noise)



(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

$f = 0.115,\ \Omega = 0.8$

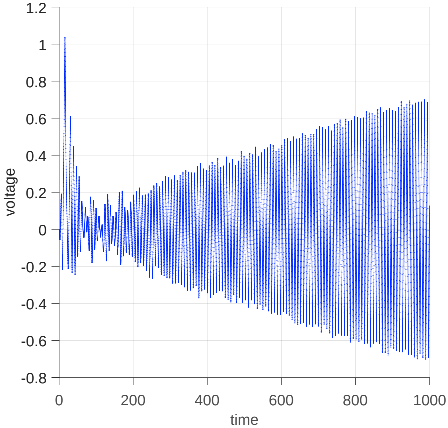
Mean of voltage time series (25% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$$f = 0.115,\ \Omega = 0.8$$

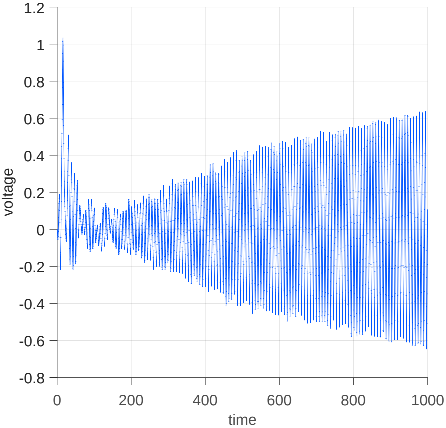
Mean of voltage time series (25% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$f = 0.115$, $\Omega = 0.8$

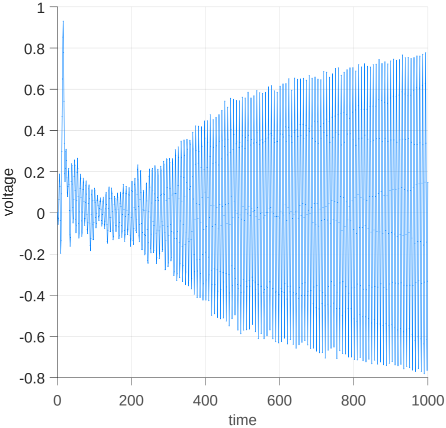
Mean of voltage time series (25% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$f = 0.115,\ \Omega = 0.8$

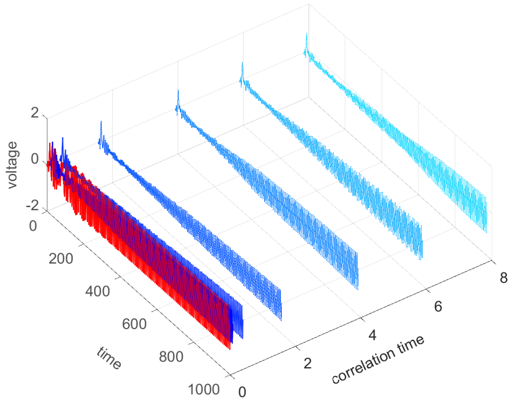
Mean of voltage time series (25% of noise)



(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

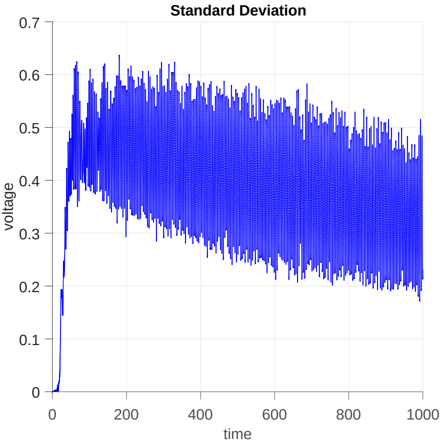
$f = 0.115$, $\Omega = 0.8$

Mean of voltage time series (25% of noise)



$f = 0.115, \ \Omega = 0.8$

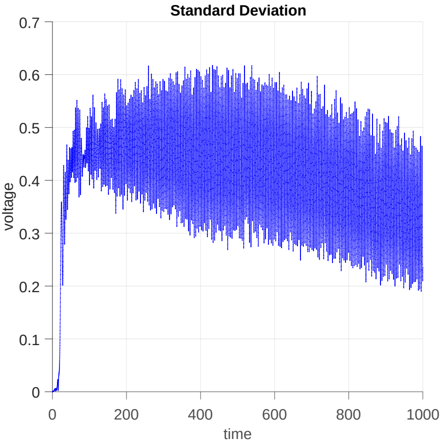
Standard deviation of voltage time series (25% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$$f = 0.115,\ \Omega = 0.8$$

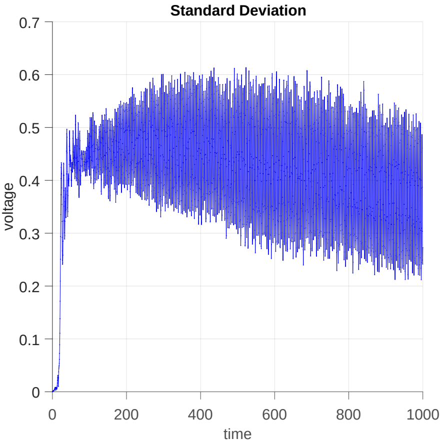
Standard deviation of voltage time series (25% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$f = 0.115$, $\Omega = 0.8$

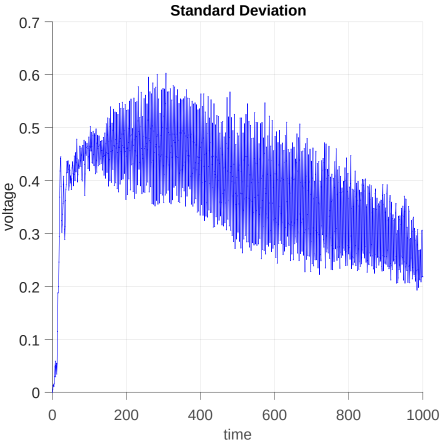
Standard deviation of voltage time series (25% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$f = 0.115,\ \Omega = 0.8$

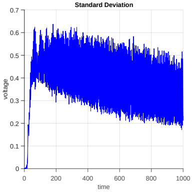
Standard deviation of voltage time series (25% of noise)



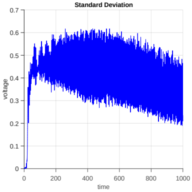
(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

$f = 0.115,\ \Omega = 0.8$

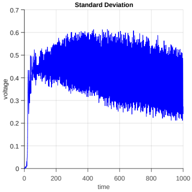
Standard deviation of voltage time series (25% of noise)



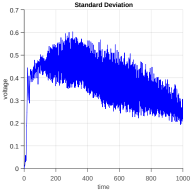
(e) $\tau_{cor}/\tau_{\Omega} = 5\%$



(f) $\tau_{cor}/\tau_{\Omega} = 25\%$



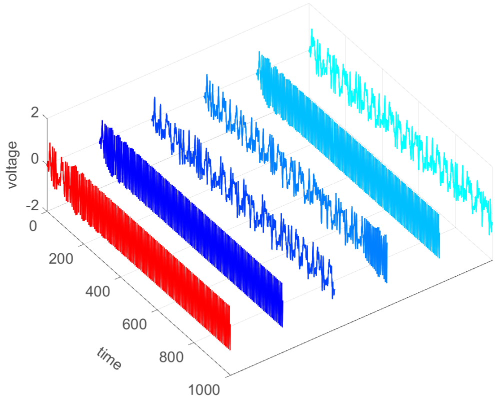
(g) $\tau_{cor}/\tau_{\Omega} = 50\%$



(h) $\tau_{cor}/\tau_{\Omega} = 75\%$

$$f = 0.115, \ \Omega = 0.8$$

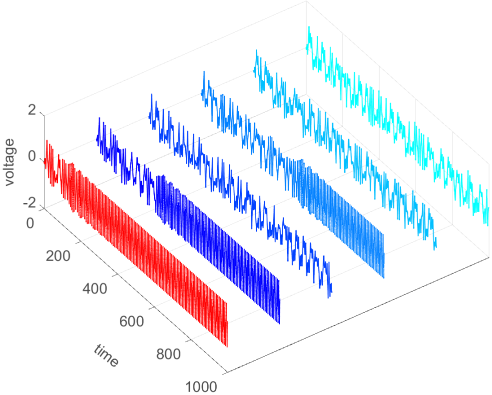
Typical voltage time series (50% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$f = 0.115,\ \Omega = 0.8$

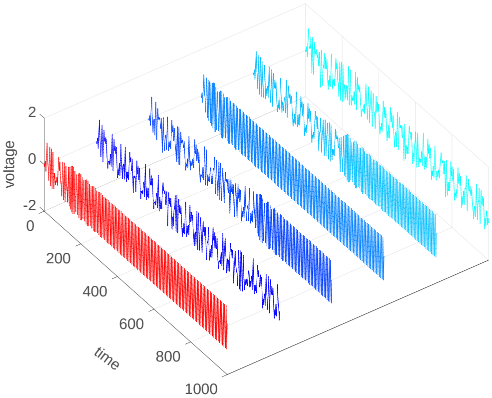
Typical voltage time series (50% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$f = 0.115$, $\Omega = 0.8$

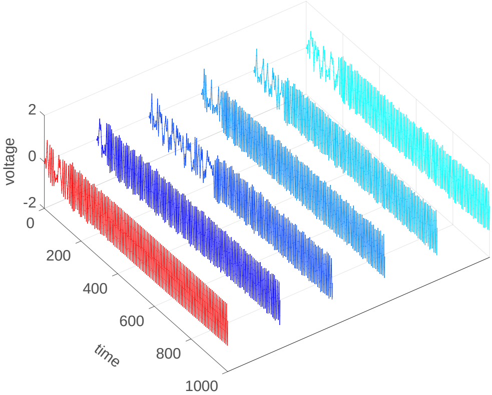
Typical voltage time series (50% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$$f = 0.115, \ \Omega = 0.8$$

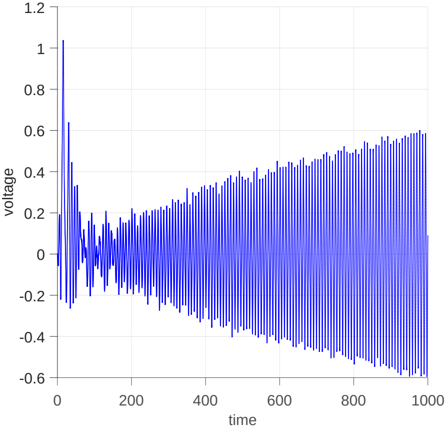
Typical voltage time series (50% of noise)



(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

$f = 0.115$, $\Omega = 0.8$

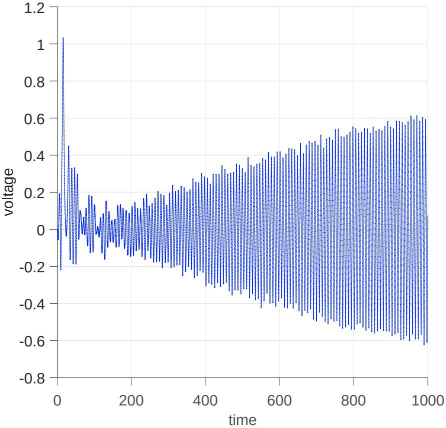
Mean of voltage time series (50% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$f = 0.115,\ \Omega = 0.8$

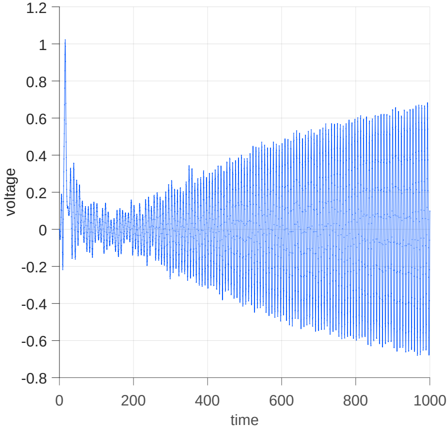
Mean of voltage time series (50% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$$f = 0.115,\ \Omega = 0.8$$

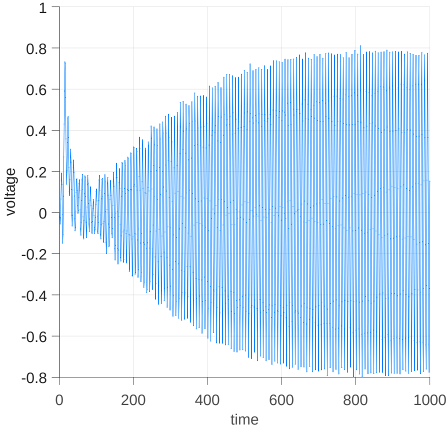
Mean of voltage time series (50% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$f = 0.115$, $\Omega = 0.8$

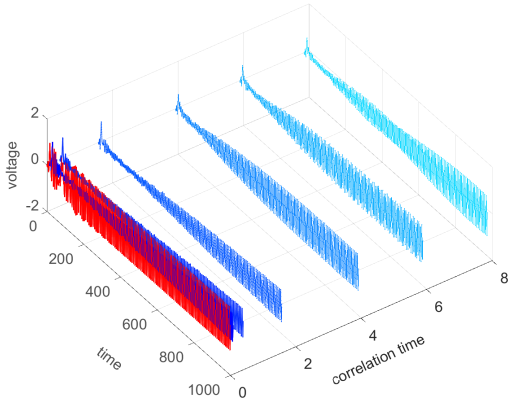
Mean of voltage time series (50% of noise)



(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

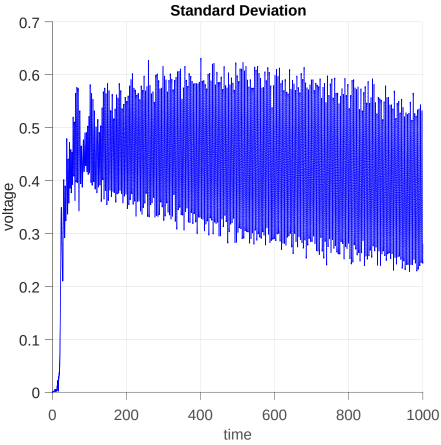
$f = 0.115,\ \Omega = 0.8$

Mean of voltage time series (50% of noise)



$f = 0.115$, $\Omega = 0.8$

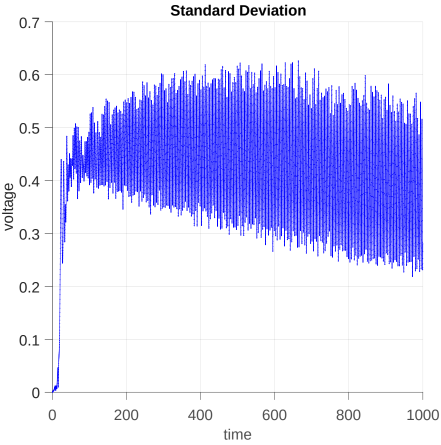
Standard deviation of voltage time series (50% of noise)



(a) $\tau_{cor}/\tau_{\Omega} = 5\%$

$f = 0.115,\ \Omega = 0.8$

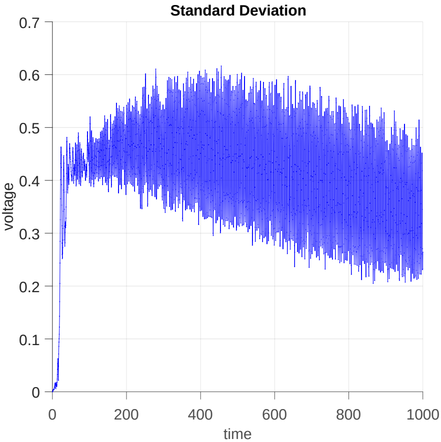
Standard deviation of voltage time series (50% of noise)



(b) $\tau_{cor}/\tau_{\Omega} = 25\%$

$f = 0.115,\ \Omega = 0.8$

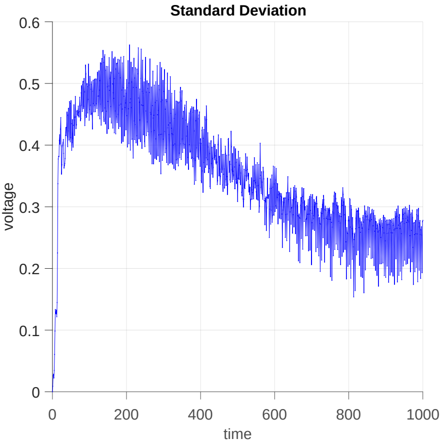
Standard deviation of voltage time series (50% of noise)



(c) $\tau_{cor}/\tau_{\Omega} = 50\%$

$f = 0.115,\ \Omega = 0.8$

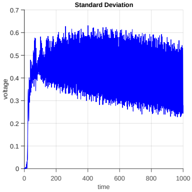
Standard deviation of voltage time series (50% of noise)



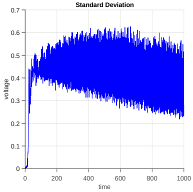
(d) $\tau_{cor}/\tau_{\Omega} = 75\%$

$f = 0.115,\ \Omega = 0.8$

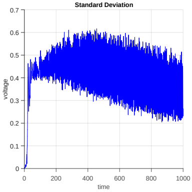
Standard deviation of voltage time series (50% of noise)



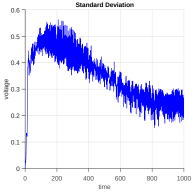
(e) $\tau_{cor}/\tau_{\Omega} = 5\%$



(f) $\tau_{cor}/\tau_{\Omega} = 25\%$



(g) $\tau_{cor}/\tau_{\Omega} = 50\%$



(h) $\tau_{cor}/\tau_{\Omega} = 75\%$

$$f = 0.115,\ \Omega = 0.8$$

Section 4

Final Remarks

Final remarks

Contributions:

- Investigation of the system deterministic dynamics
  - bifurcation analsysis
  - basis of attractions exploration
- Investigation of the system stochastic dynamics
  - colored noise disturbance
  - intensity and correlation time effects

Ongoing research:

- Improve system efficiency via control of chaos
- Construction of an experimental apparatus

Acknowledgments

Financial support:

FAPERJ
Fundação Carlos Chagas Filho de Amparo
à Pesquisa do Estado do Rio de Janeiro

CNPq
Conselho Nacional de Desenvolvimento
Científico e Tecnológico

CAPES

UERJ
UNIVERSIDADE DO ESTADO DO RIO DE JANEIRO

Thank you for your attention!

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www.americocunha.org

A. Cunha Jr,
Enhancing the performance of a bi-stable energy harvesting device via cross-entropy method.
(under review) https://hal.archives-ouvertes.fr/hal-01531845

J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr,
Numerically exploring the nonlinear dynamics of a piezo-magneto-elastic energy harvesting device.
(under review)

Physical system parameters

parameter	value
ξ	0.01
χ	0.05
f	0.083
Ω	0.8
λ	0.05
κ	0.5