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Effect of Stochastic Excitation on Sub-harmonic Solutions in a Bistable Energy Harvester

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Abstract. Broadband frequency energy harvesting systems are based on nonlinear resonators. Ambient kinetic energy is transformed to electric energy by a piezoelectric patch installed on the elastic beam. In this paper we study a bistable energy harvester consisted of the resonator with some nonlinearities induced by magnets. The system is excited with the combined effect of the frame harmonic and stochastic motions. Starting from initial conditions with minimum energy we observe considerable improvement of performance of the system in the presence of stochastic excitation component. Our initial calculations clearly indicate the existence of stochastic bifurcations in that system.

Keywords: nonlinear vibration, energy harvesting, piezoelectric

INTRODUCTION

The piezoelectric Energy Harvesting (EH) from ambient vibration sources are considered as an important application for distributed energy supply in case of structural health monitoring systems and/or small electric energy devices. A device for EH is usually consisting of the cantilever beam as a mechanical resonator with a piezo-ceramic patch as a energy transducer. During the beam excitation strains occur by the deflected beam resulting by voltage output on the electrodes. The nonlinear potential of the beam induced by magnets is dedicated to the frequency broadband effect to optimize the energy harvesting procedure [1, 2, 3].

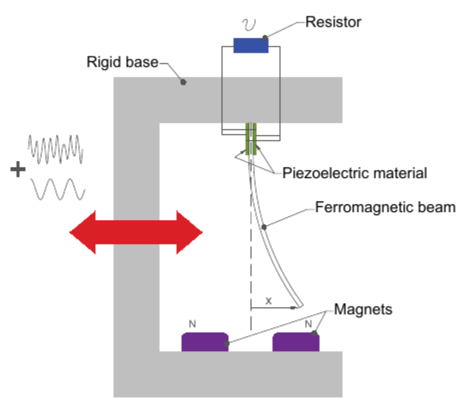

FIGURE 1. Schematic of the energy harvester with kinematic excitation through the frame [6].

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The EH system under investigation was proposed by Erturk et al. [1], is the piezo-magneto-elastic beam, driven by a noise-disturbed rigid base (frame) movement, as is illustrated in Figure 1.

MATHEMATICAL MODEL

The nonlinear stochastic dynamics evolves according to the following system of stochastic differential equations (dimensionless)

$$\ddot{X} + 2\,\xi\,\dot{X} - \frac{1}{2}\,X\,(1 - X^2) - \chi\,V = f\,\Omega^2\,\cos(\Omega t) + N_t,$$
$$\dot{V} + \lambda\,V + \kappa\,\dot{X} = 0, \tag{1}$$

where the beam free edge displacement, its velocity and the system voltage output are represented by the random processes X, $\dot{X}$, V respectively, ξ denotes the mechanical damping ratio, χ and κ are the piezoelectric coupling terms, and λ is a reciprocal time constant. Such dimensionless parameters are fixed as $\xi = 0.01$, $\chi = 0.05$, $\kappa = 0.5$, $\lambda = 0.05$, and the initial conditions are adopted as being $X_0 = 1$, $\dot{X}_0 = 0$ and $V_0 = 0$ [5].

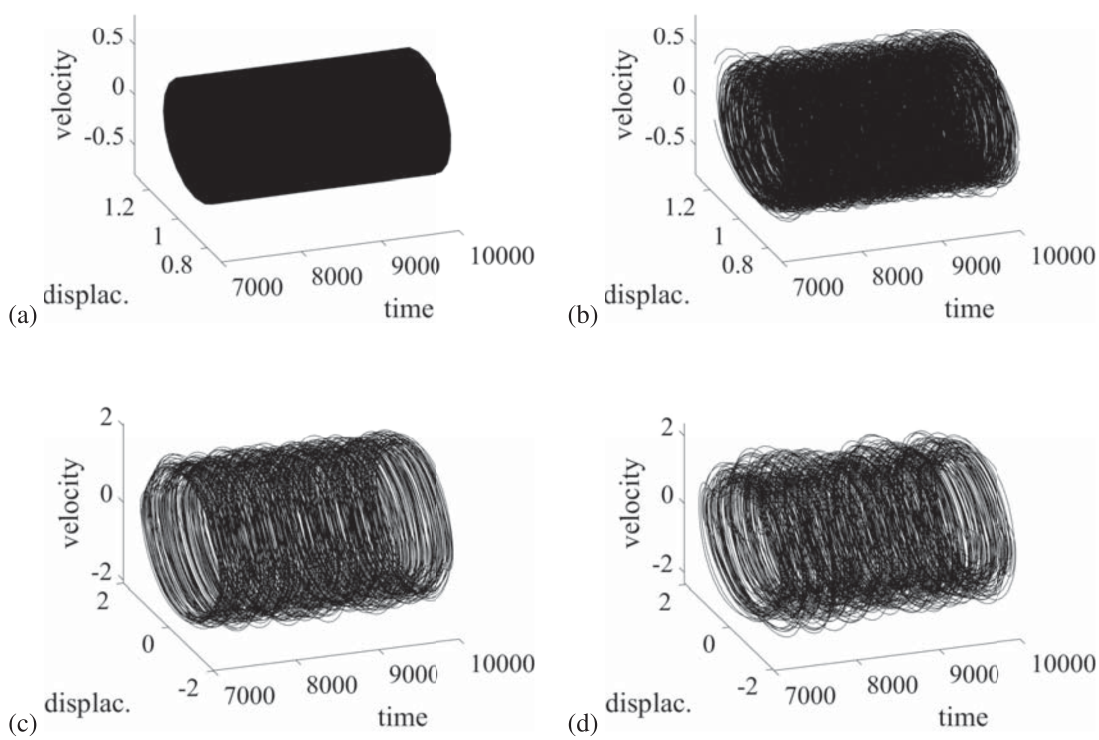

FIGURE 2. Time histories of velocity-displacement for various standard deviation values σ, (a) deterministic case $\sigma / f\,\Omega^2 = 0$, (b)-(d) $\sigma / f\,\Omega^2 = 0.08, 0.016, 0.032$, respectively.

The external excitation consists of a harmonic force with amplitude and frequency respectively given by $f = \omega^2 A$, $A = 0.183$ and $\Omega = 2.595$ [5], and disturbance component N_t , assumed as a zero-mean Gaussian colored noise with covariance function [6]

$$\langle N_t, N_{t'} \rangle = \frac{D}{\tau}\,\exp\left(-\frac{|t - t'|}{\tau}\right), \tag{2}$$

where τ denotes the correlation time, $D = \sigma^2 \tau$ is the noise intensity, defined in terms of the colored noise standard deviation σ. The last parameter values are assumed as percentages of harmonic forcing amplitude with $\tau = \pi/\Omega$.

NUMERICAL RESULTS

Numerical simulations were done with the help of the Monte Carlo algorithm in Matlab. The realizations of noise are generated by deterministic approximations, obtained by the Karhunen-Loeve expansion. This method allows the dynamics to be integrated with the standard 4th-order Runge-Kutta method with automatic time step. Figures 2-4 present results of displacement, velocity and voltage output sampled from Monte Carlo simulations, for $\sigma/f\Omega^2 = 0$, 0.08, 0.16, 0.32.

Phase portraits time evolution in the stationary interval show the evolution of mechanical resonator responses with increasing noise (Fig. 2). Note that for the deterministic case the solution is of small amplitude located around the minimum of potential $X = 1$ (see Fig. 2a). The increase of noise leads to the enlargement of the attractor (Fig. 2b) and finally to escape from the potential well for $\sigma/f\Omega^2 = 0.16$ and 0.32 (Figs. 2c and d). Note that the increase of noise level leads to noisy fluctuations along the time flow. The same investigated cases were compared in terms of the corresponding Fourier spectra (Fig. 3). Interestingly, the higher amplitude solutions with inter-well switching dynamics is characterized by relatively small frequency. Namely, it is subharmonic 3 solution (with dominating 1/3 frequency component). This is in agreement of recent studies on subharmonics [4, 5] where various initial conditions were used to get the high orbit solutions. Here, instead of that we destabilized the deterministic small orbit solution by the stochastic effect. Note that in our approach the fixed initial conditions are not so important.

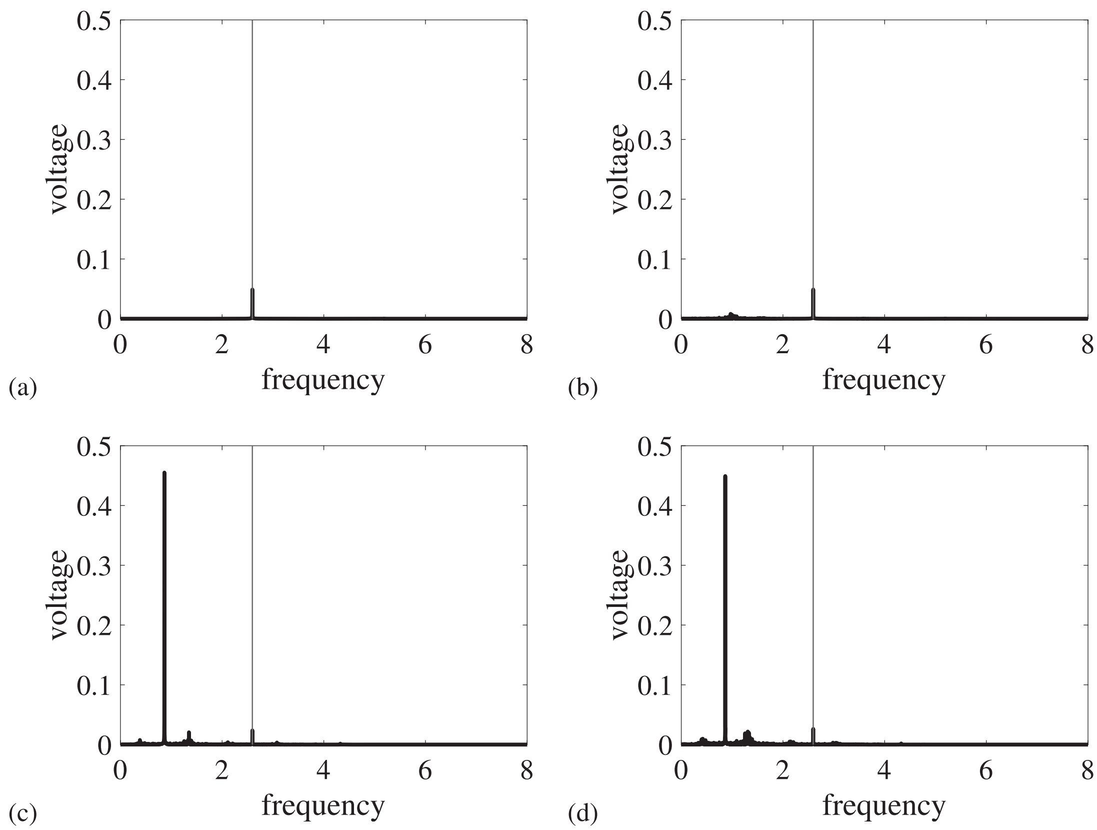

FIGURE 3. Fourier spectra of corresponding voltage output (the casses as in Fig. 2). The gray thin line denotes the excitation frequency. The power output of corresponding cases are estimated in terms of voltage variance $var(V) = 0.0057$ (a), 0.0062 (b), 0.4262 (c), 0.4221 (d).

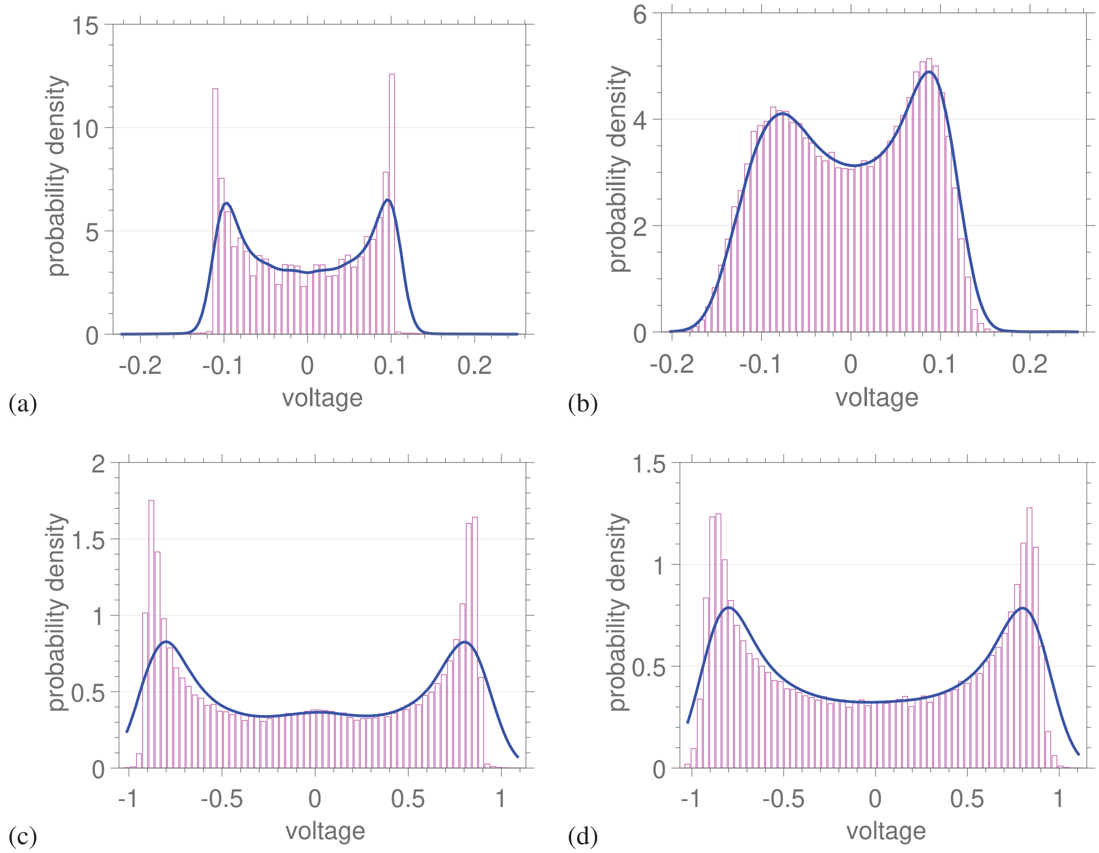

FIGURE 4. Distribution of voltage output for four cases (a)-(d) with corresponding noise levels $\sigma / f\,\Omega^2 = 0$, 0.08, 0.16, 0.32, respectively.

Finally, we show the histograms of voltage output (Fig. 4) for four noise levels. Our results indicate that there are at least two qualitative changes in the distribution of the voltage output.

CONCLUSION

The paper presented the responses of the bistable EH system excited simultaneously by harmonic and stochastic components. Our initial calculations show that the system can bifurcate with the change of noise level. Fig. 4 reflects changes from symmetric to non-symmetric distribution observed in intra-well oscillations (Figs. 4a and b). While for higher noise inter-well oscillations appear with a large distribution interval (Figs. 4c and d). More conclusions about these bifurcations could be drown after systematic study of multiple stochastic realizations.

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