On the nonlinear dynamics of a bi-stable piezoelectric energy harvesting device

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Universidade do Estado do Rio de Janeiro (UERJ)

NUMERICO – **Nu**cleus of **M**odeling and **E**xpe**ri**mentation with **Co**mputers

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In collaboration with: Vinicius Lopes (UERJ)
João Peterson (UERJ)

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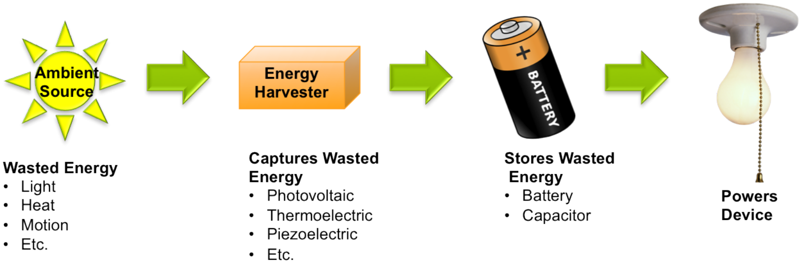
Section 1

Introduction

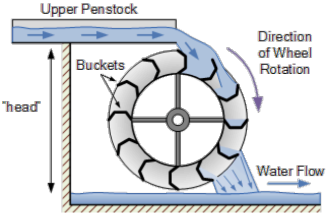
Energy Harvesting

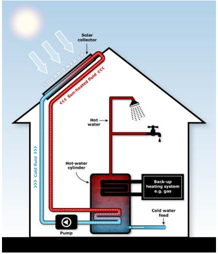


- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

*Picture from http://hiddenjoules.com/intro/what/

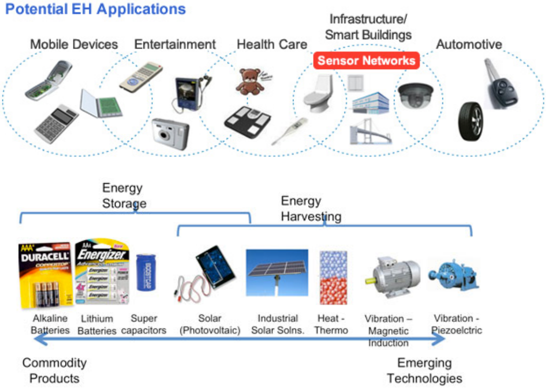
Classical Technologies in Energy Harvesting

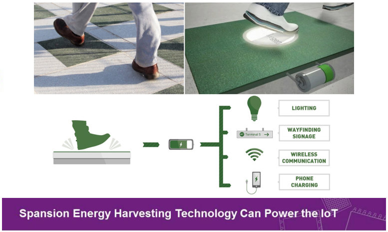


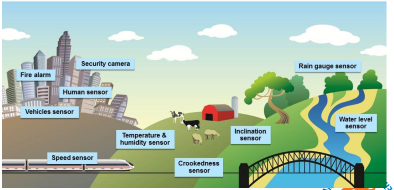


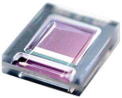
*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

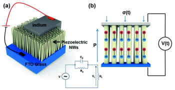
Emergent Technologies in Energy Harvesting





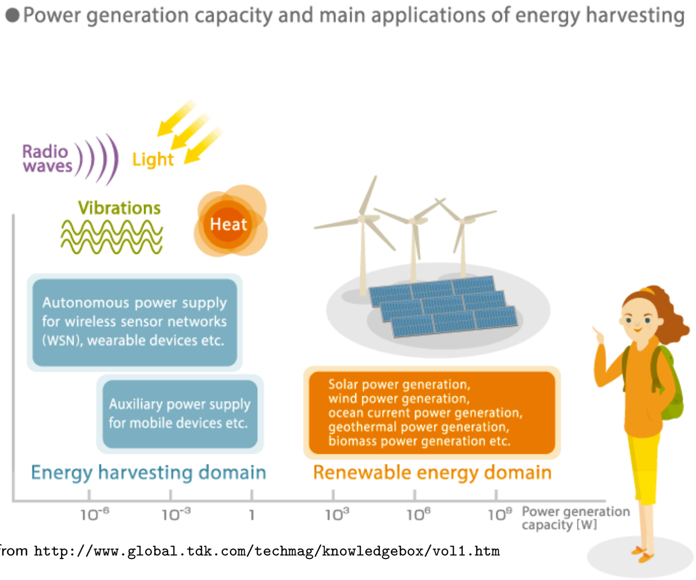






*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

Energy Scale for Modern Harvesting Devices



*Picture from `http://www.global.tdk.com/techmag/knowledgebox/vol1.htm`

Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
  - Time series
  - Bifurcation diagrams
  - Basis of attractions
- Propose strategies to enhance the recovered energy
  - Nonlinear optimization
  - Control of chaos
- Model the underlying uncertainties and study their influence
  - System parameters variability
  - Noise in system excitation

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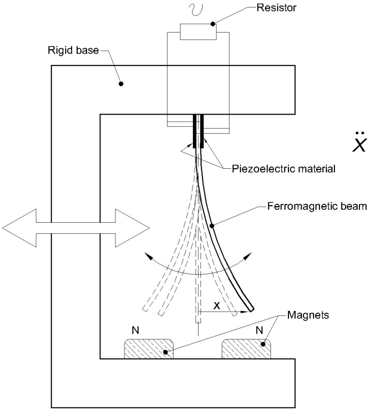
Section 2

Nonlinear Dynamics

Bi-stable energy harvesting device



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0, \;\; \dot{x}(0) = \dot{x}_0, \;\; \upsilon(0) = \upsilon_0$$

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Section 3

Numerical Experiments

Nonlinear dynamics animation

Displacement vs excitation frequency ($f = 0.050$)

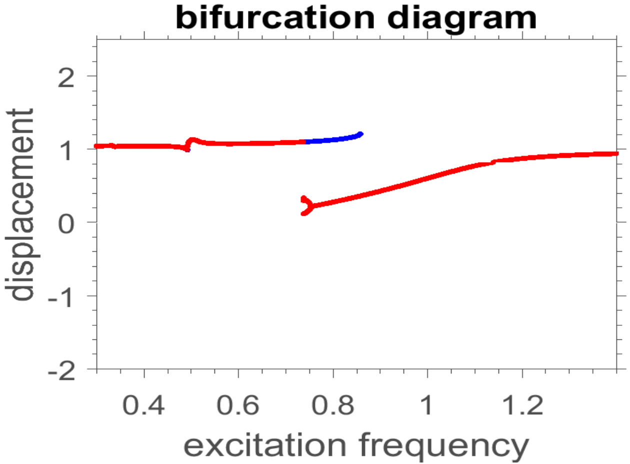


Figure: $(x_0, \dot{x}_0, v_0) = (1\ 0\ 0)$, forward (blue) and backward (red).

Displacement vs excitation frequency ($f = 0.050$)

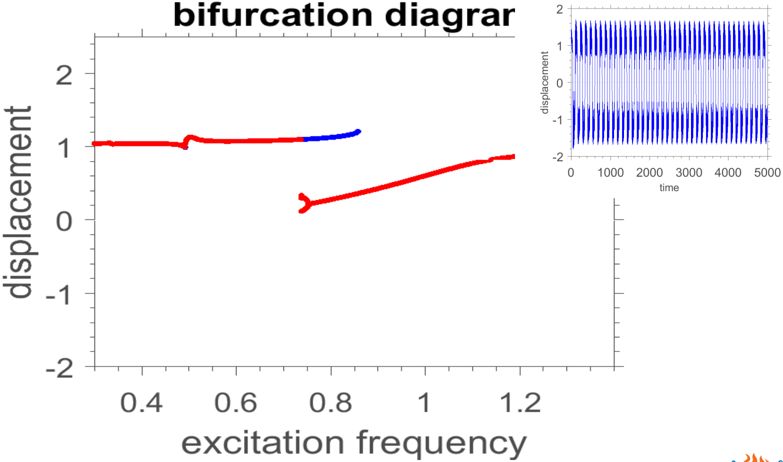


Figure: Time series for $f = 0.4$.

Displacement vs excitation frequency ($f = 0.050$)

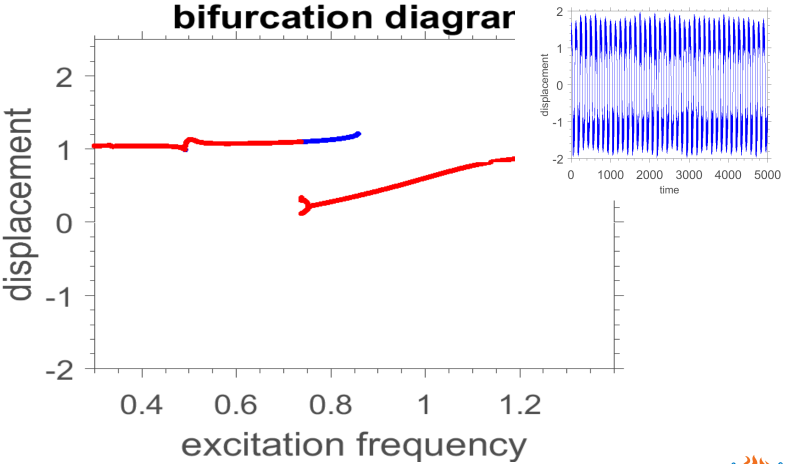


Figure: Time series for $f = 0.73$.

Displacement vs excitation frequency ($f = 0.050$)

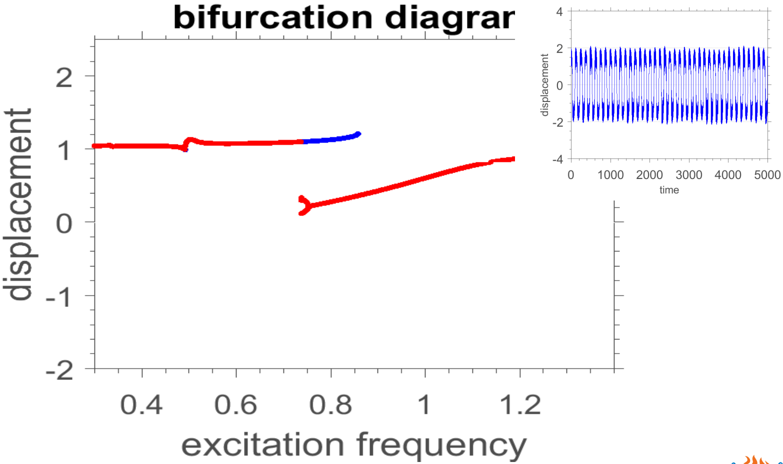


Figure: Time series for $f = 1.2$.

Displacement vs excitation frequency ($f = 0.083$)

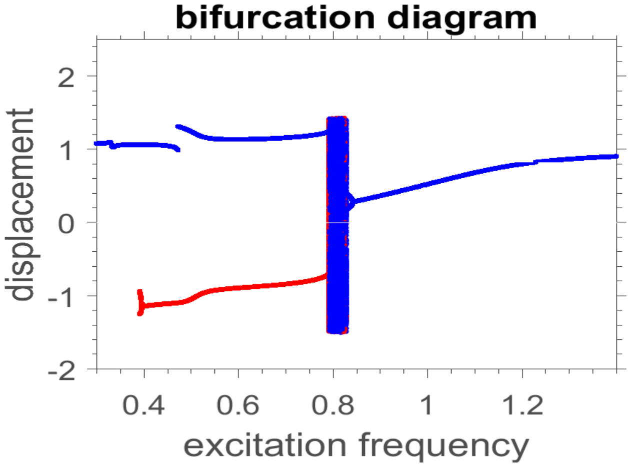


Figure: $(x_0, \dot{x}_0, v_0) = (1\ 0\ 0)$, forward (blue) and backward (red).

Displacement vs excitation frequency ($f = 0.083$)

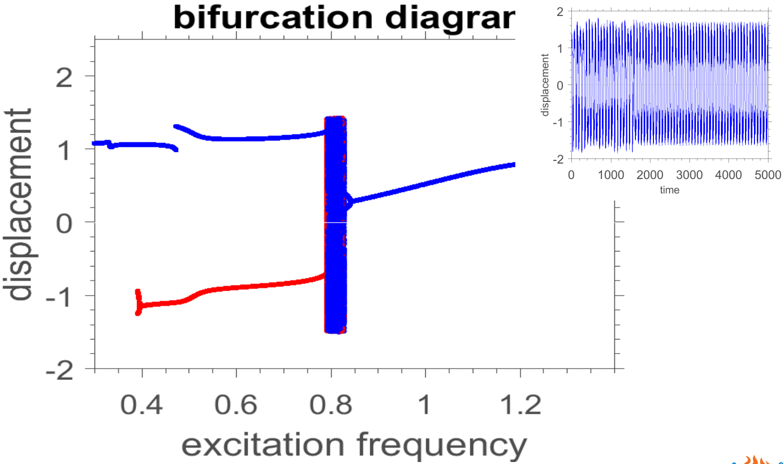


Figure: Time series for $f = 0.38$.

Displacement vs excitation frequency ($f = 0.083$)

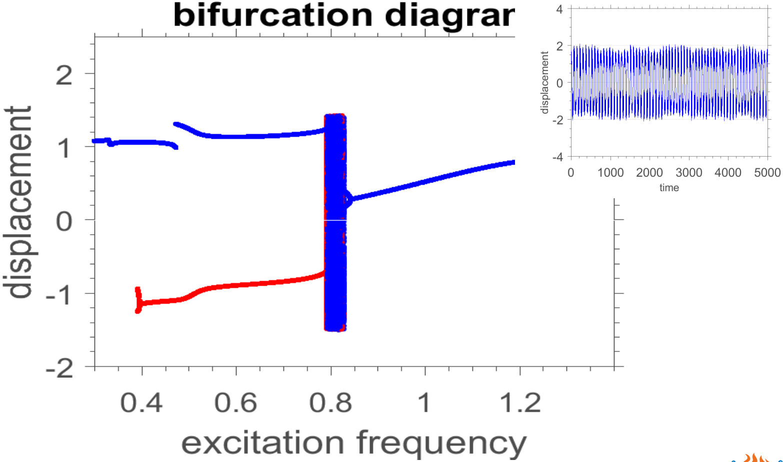


Figure: Time series for $f = 0.8$.

Displacement vs excitation frequency ($f = 0.083$)

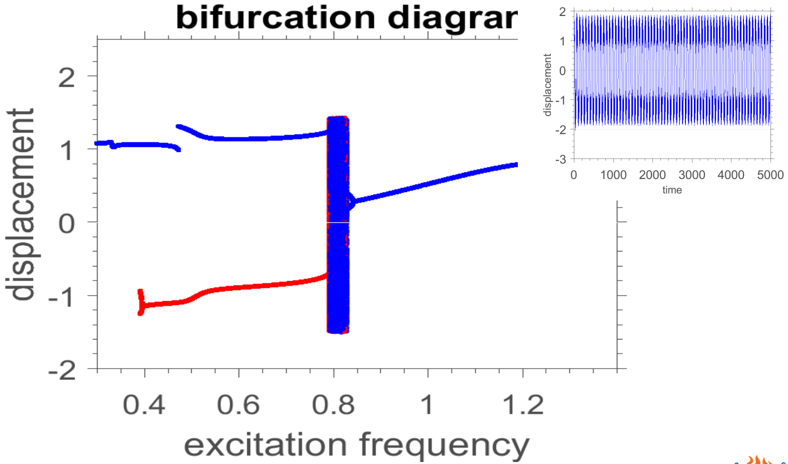


Figure: Time series for $f = 0.83$.

Displacement vs excitation frequency ($f = 0.083$)

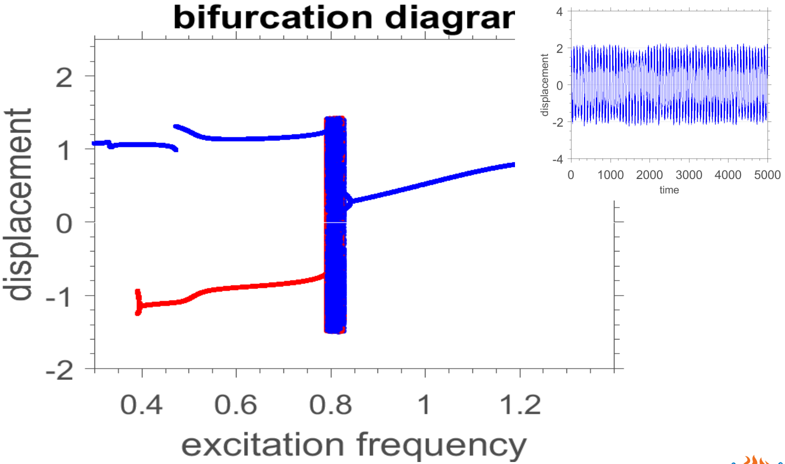


Figure: Time series for $f = 1.2$.

Displacement vs excitation frequency ($f = 0.100$)

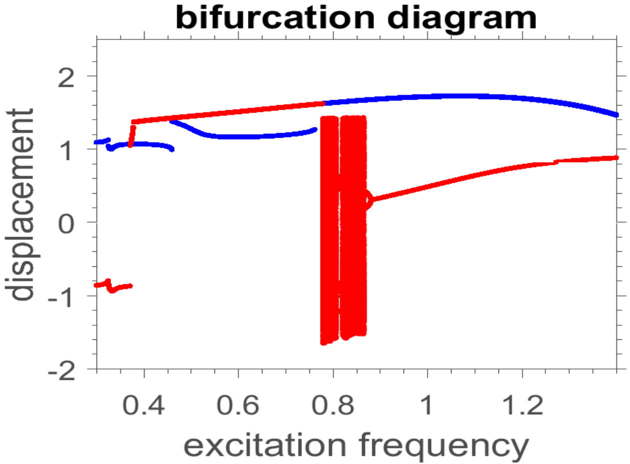
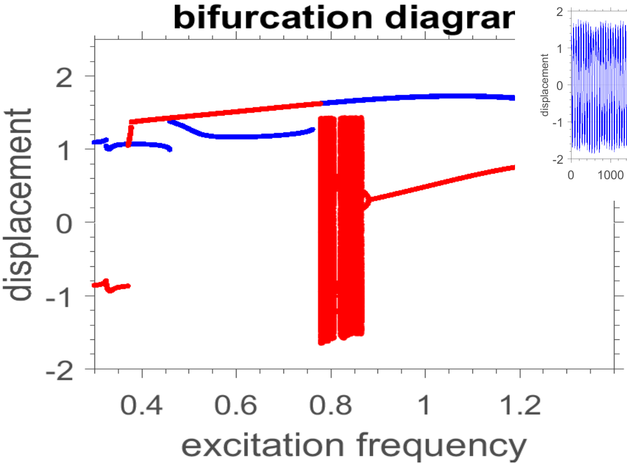


Figure: $(x_0, \dot{x}_0, v_0) = (1\ 0\ 0)$, forward (blue) and backward (red).

Displacement vs excitation frequency ($f = 0.100$)



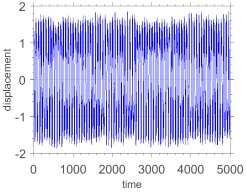


Figure: Time series for $f = 0.38$.

Displacement vs excitation frequency ($f = 0.100$)

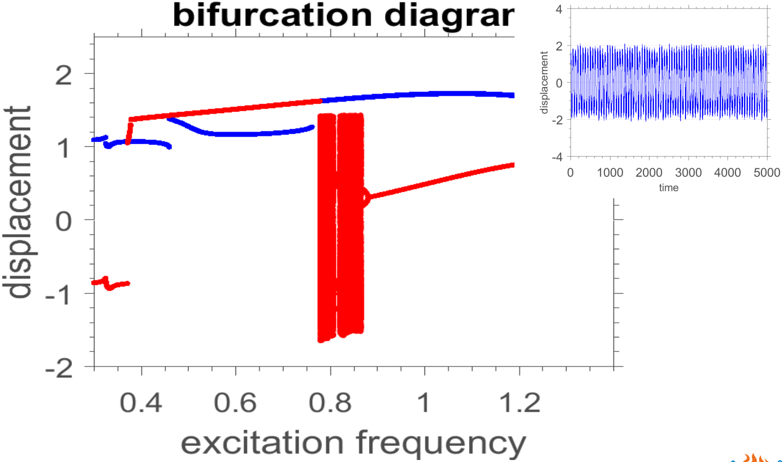


Figure: Time series for $f = 0.8$.

Displacement vs excitation frequency ($f = 0.100$)

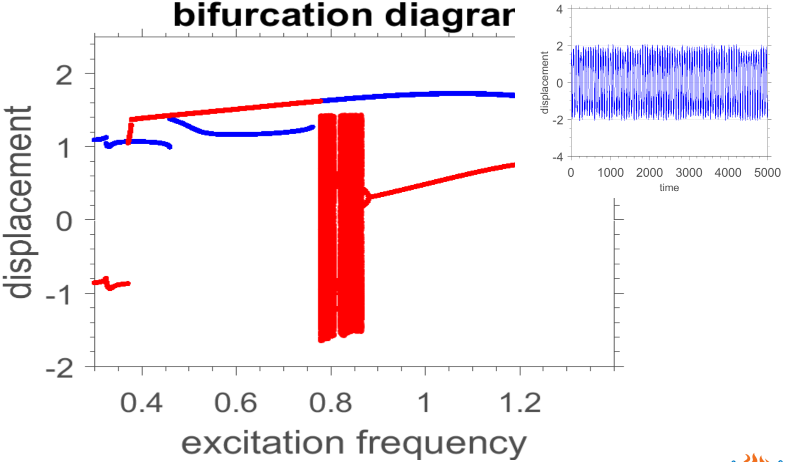


Figure: Time series for $f = 0.82$.

Displacement vs excitation frequency ($f = 0.100$)

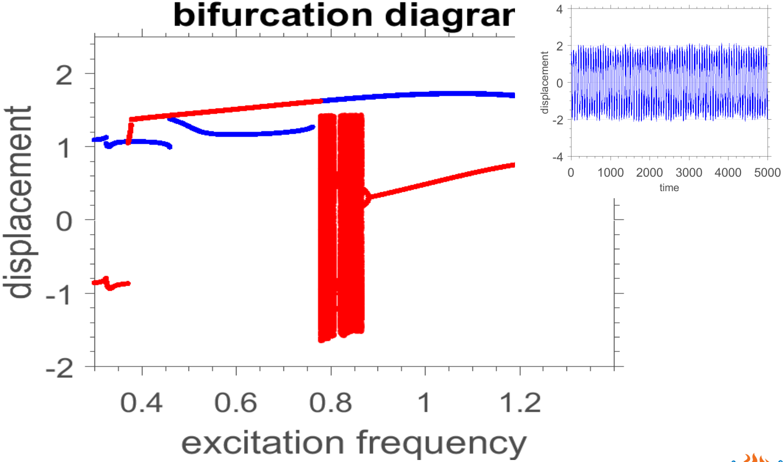


Figure: Time series for $f = 0.87$.

Displacement vs excitation frequency ($f = 0.100$)

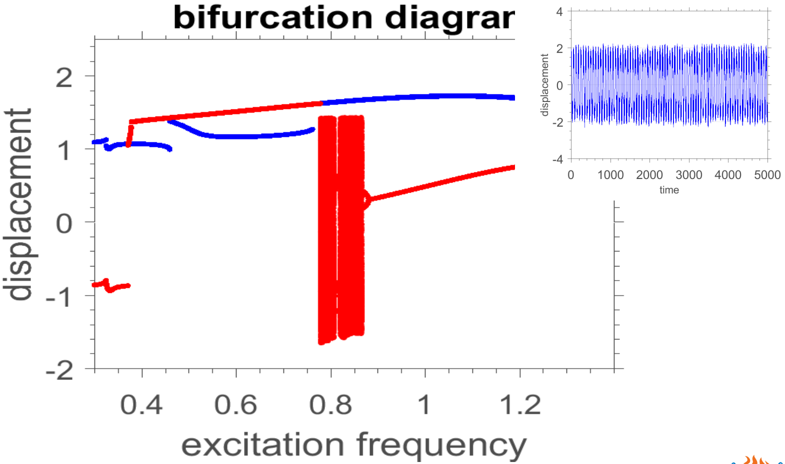


Figure: Time series for $f = 1.2$.

Displacement vs excitation frequency ($f = 0.115$)

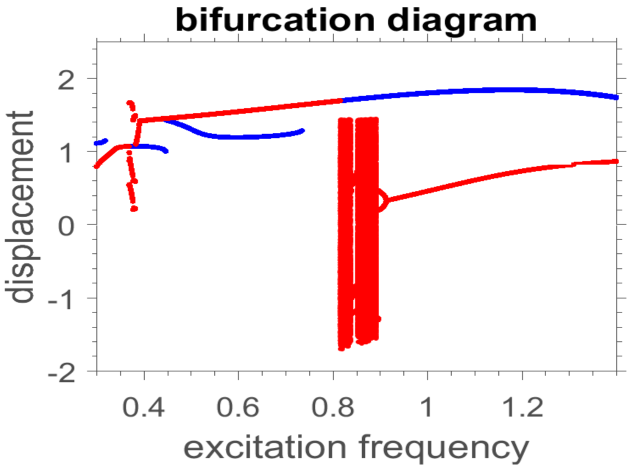


Figure: $(x_0, \dot{x}_0, \upsilon_0) = (1\ 0\ 0)$, forward (blue) and backward (red).

Displacement vs excitation frequency ($f = 0.115$)

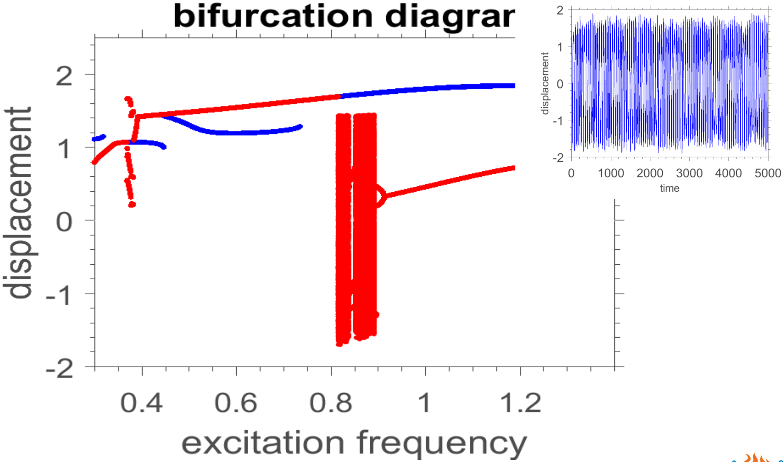
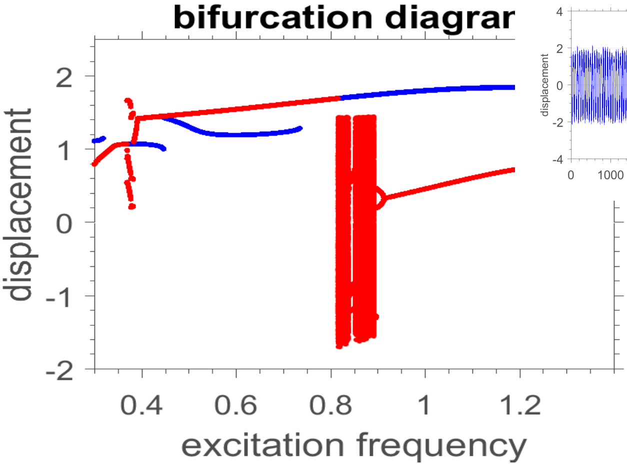


Figure: Time series for $f = 0.40$.

Displacement vs excitation frequency ($f = 0.115$)



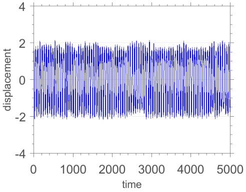


Figure: Time series for $f = 0.85$.

Displacement vs excitation frequency ($f = 0.115$)

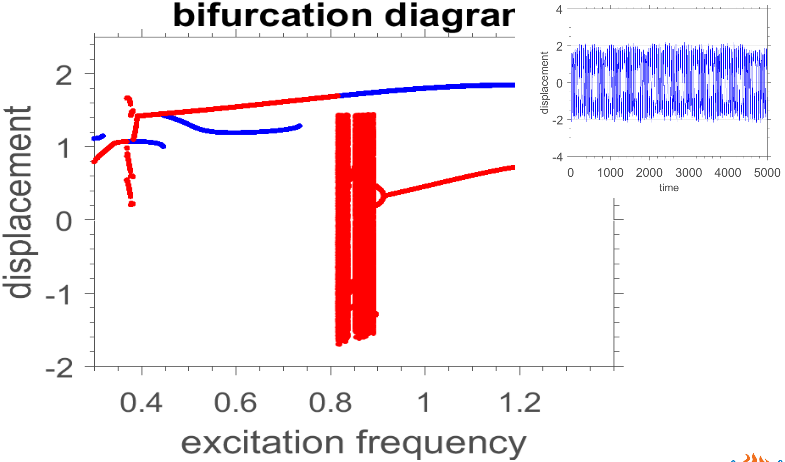
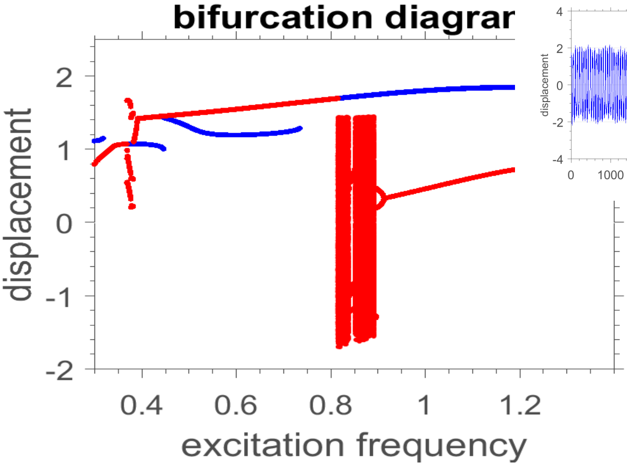


Figure: Time series for $f = 0.88$.

Displacement vs excitation frequency ($f = 0.115$)



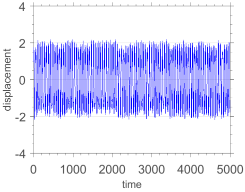


Figure: Time series for $f = 0.90$.

Displacement vs excitation frequency ($f = 0.115$)

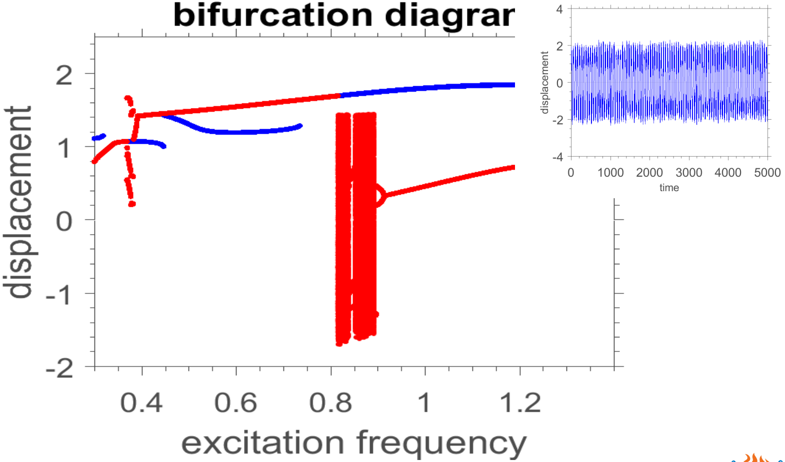
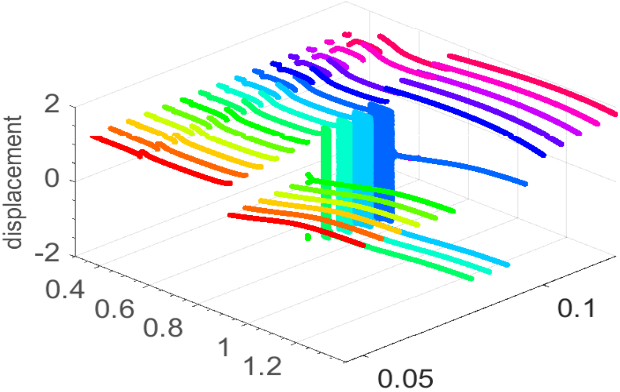


Figure: Time series for $f = 1.2$.

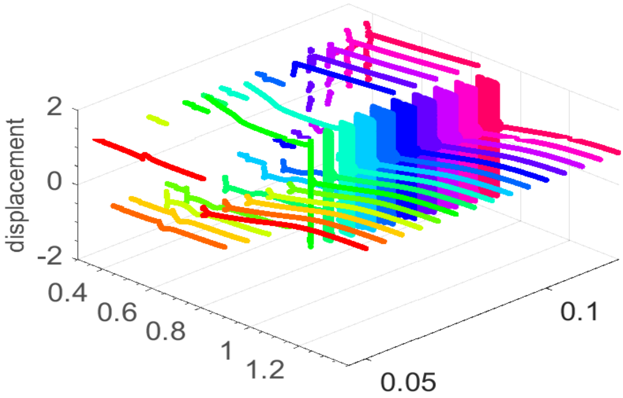
Displacement vs excitation frequency (forward)



$$0.3 \leq \Omega \leq 1.4$$

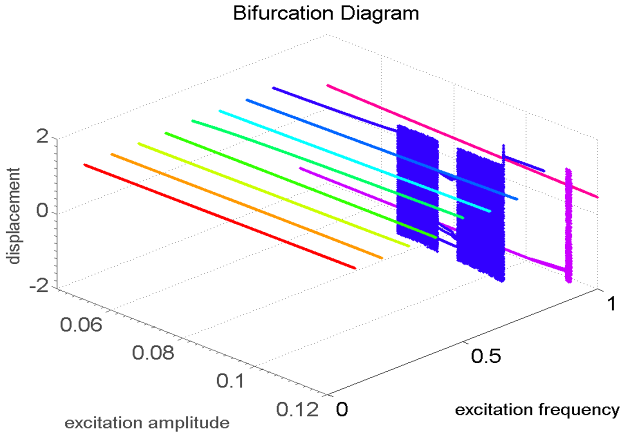
$$f \in \{0.05, 0.06, 0.07, 0.08, 0.09\}$$

Displacement vs excitation frequency (backward)



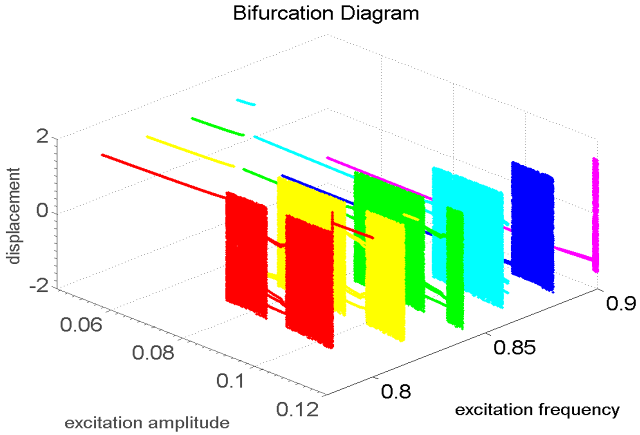
$$0.3 \leq \Omega \leq 1.4$$
$$f \in \{0.05, 0.06, 0.07, 0.08, 0.09\}$$

Displacement vs excitation amplitude (forward)



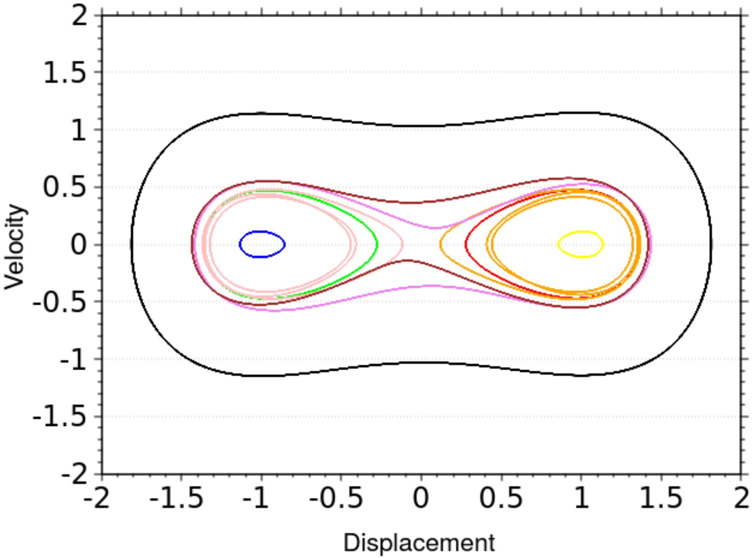
$$0.04 \leq f \leq 0.12$$
$$\Omega \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$$

Displacement vs excitation amplitude (forward)

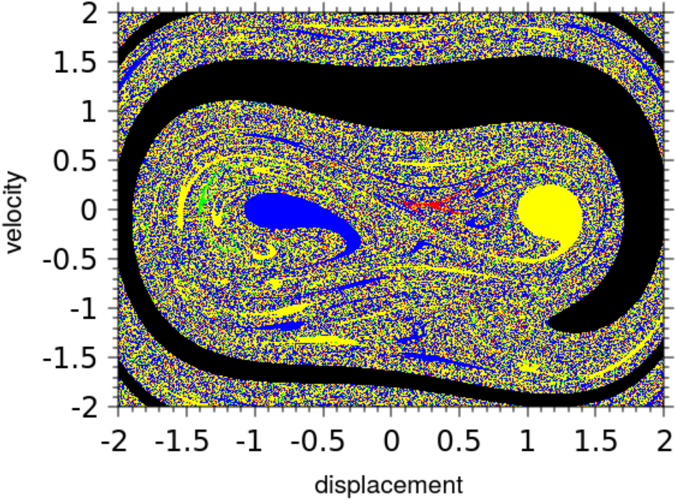


$$0.04 \leq f \leq 0.12$$
$$\Omega \in \{0.8, 0.82, 0.84, 0.86, 0.88, 0.9\}$$

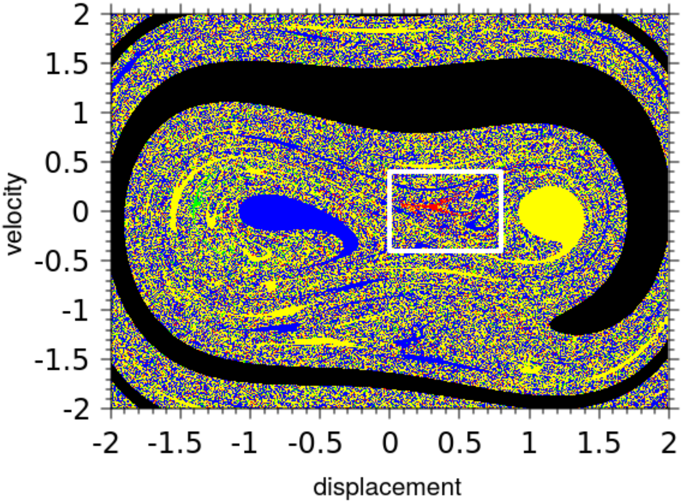
Basins of Attraction ($f = 0.05$ and $\Omega = 0.8$)



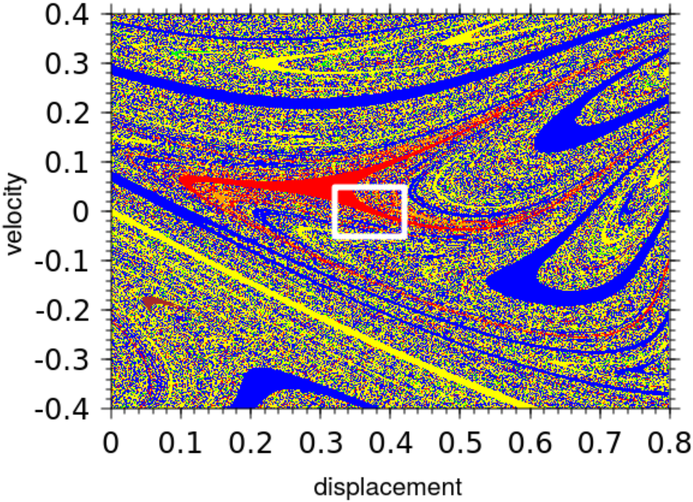
Basins of Attraction ($f = 0.05$ and $\Omega = 0.8$)



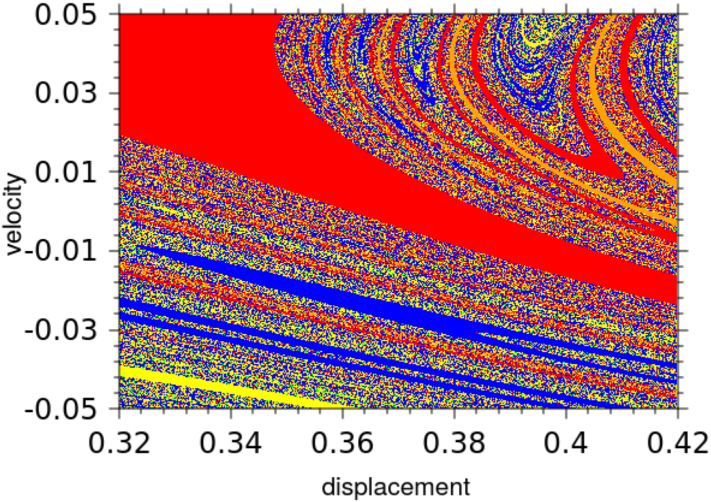
Basins of Attraction ($f = 0.05$ and $\Omega = 0.8$)



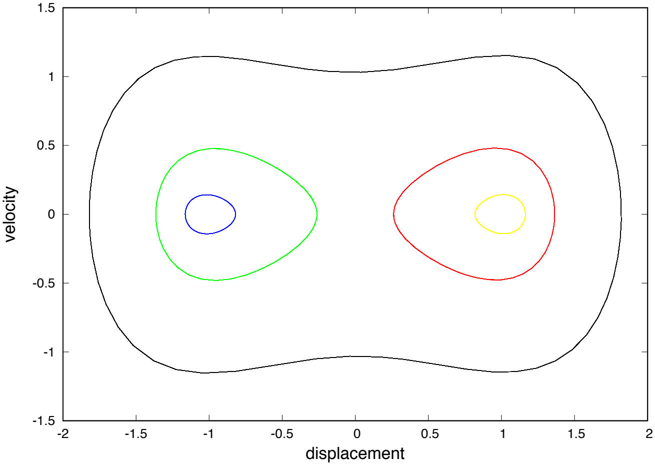
Basins of Attraction ($f = 0.05$ and $\Omega = 0.8$)



Basins of Attraction ($f = 0.05$ and $\Omega = 0.8$)



Basins of Attraction ($f = 0.12$ and $\Omega = 0.8$)



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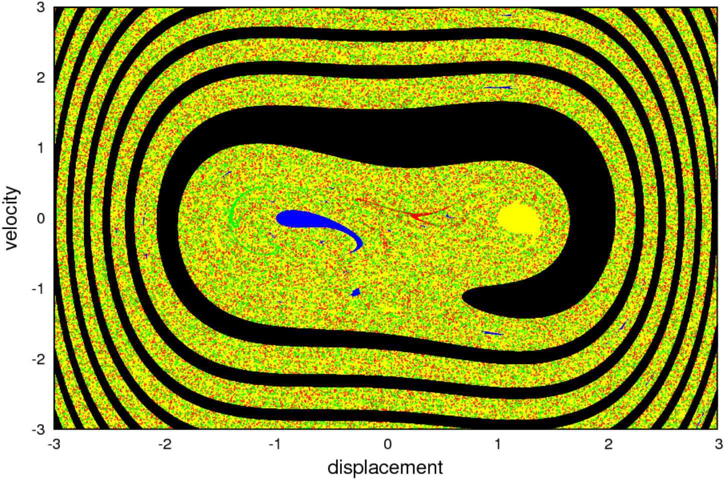
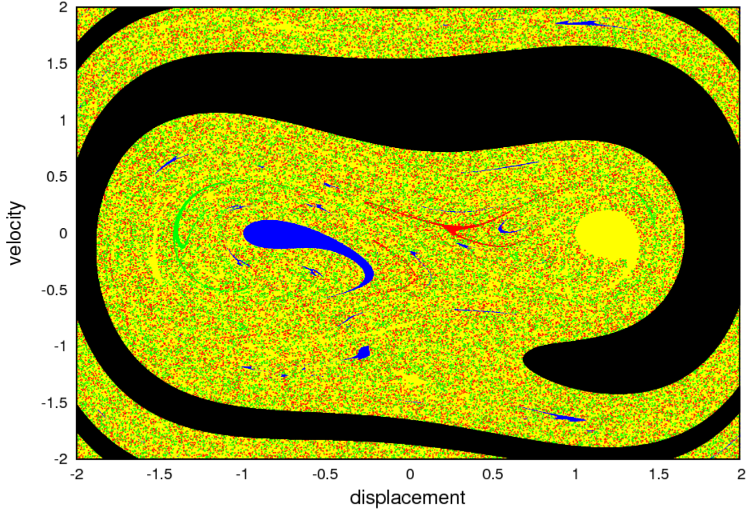
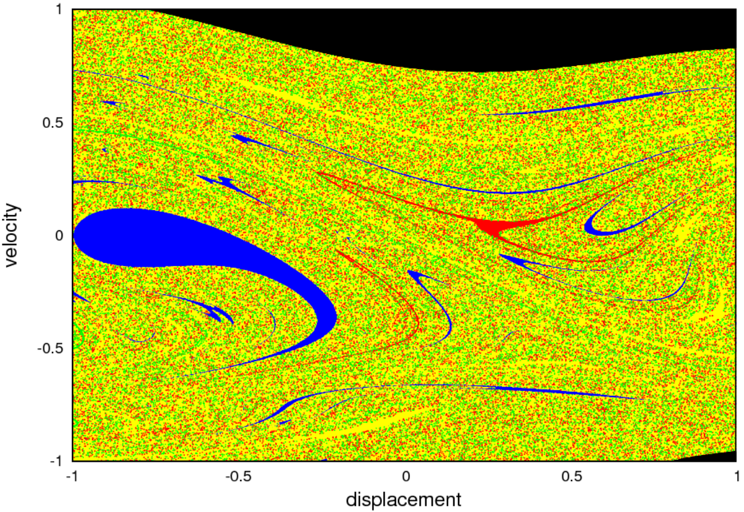


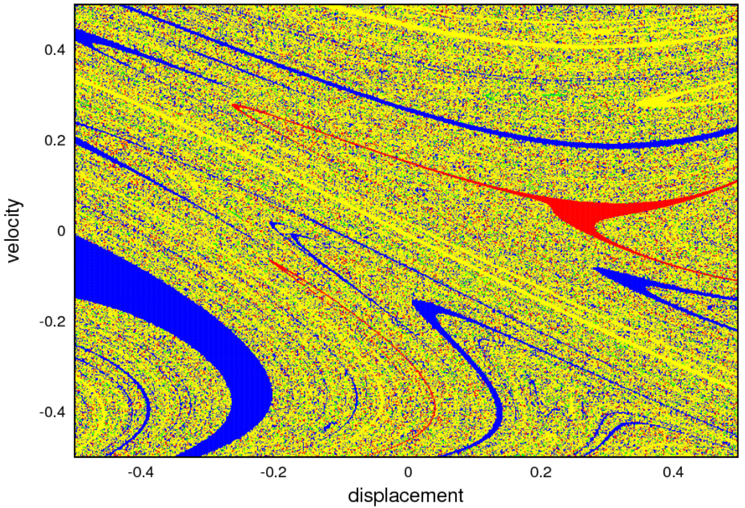
Basins of Attraction ($f = 0.12$ and $\Omega = 0.8$)



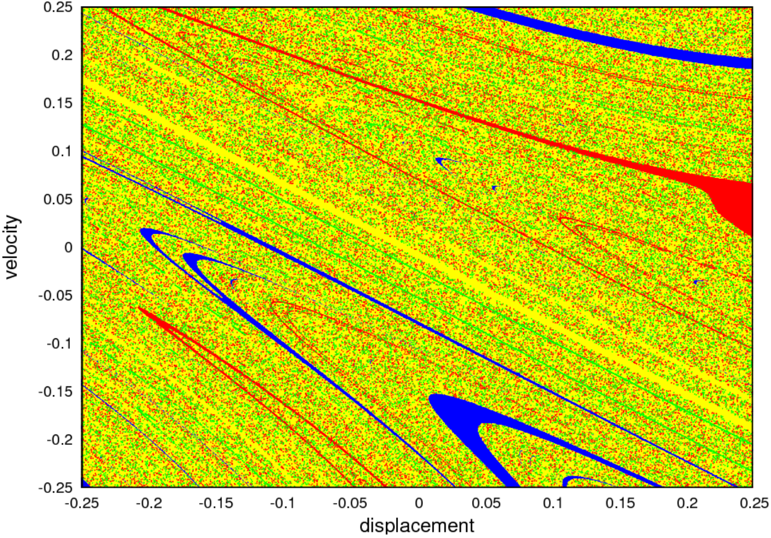
Basins of Attraction ($f = 0.12$ and $\Omega = 0.8$)



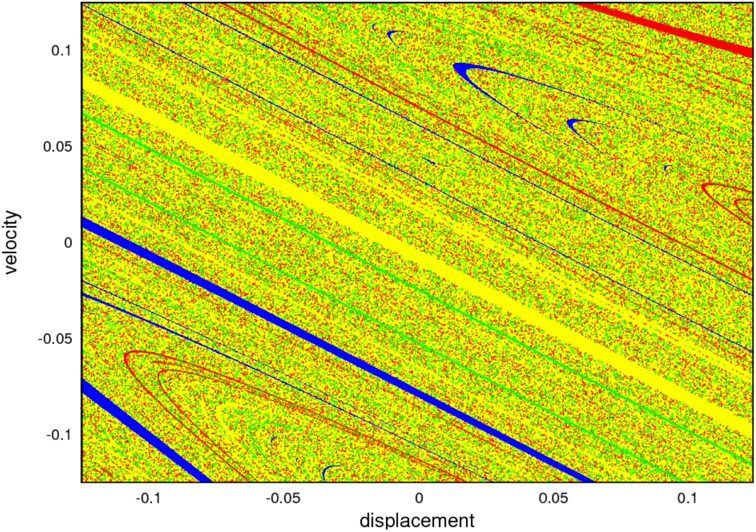
Basins of Attraction ($f = 0.12$ and $\Omega = 0.8$)



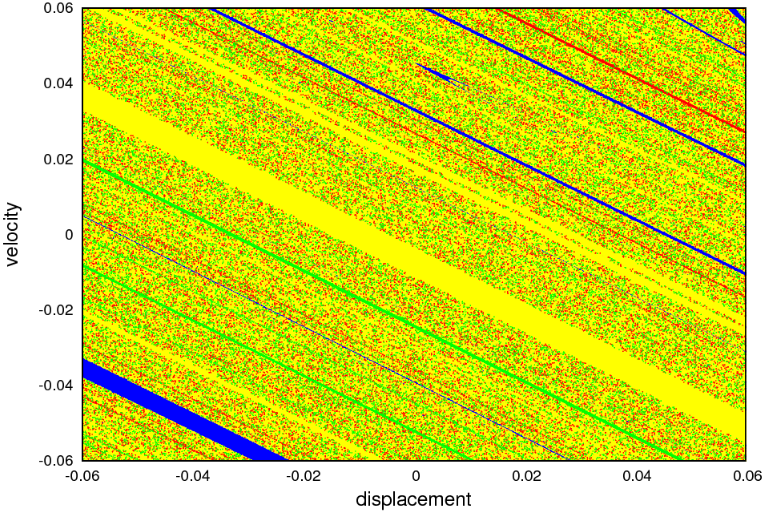
Basins of Attraction ($f = 0.12$ and $\Omega = 0.8$)



Basins of Attraction ($f = 0.12$ and $\Omega = 0.8$)



Basins of Attraction ($f = 0.12$ and $\Omega = 0.8$)



Section 4

Final Remarks

Ongoing research

Stochastic nonlinear dynamical system:

$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t + \text{“noise”}$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

$$x(0) = x_0,\ \dot{x}(0) = \dot{x}_0,\ \upsilon(0) = \upsilon_0$$

Ongoing research

Nonlinear dynamics animation (with noise):

Final remarks

Contributions:

- Detailed investigation of the system nonlinear dynamics
  - Bifurcation in system response due to external excitation
  - Sensitivity to initial conditions

Ongoing research:

- Modeling of system parameters uncertainties
- Control of chaos to improve the system efficiency

Acknowledgments

Academic discussion:

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- Prof. Adriano Fabro (UnB)
- Mr. Tiago Pereira (UnB)

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CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

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Thank you for your attention!

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www.americocunha.org

A. Cunha Jr,
Enhancing the performance of a bi-stable energy harvesting device via cross-entropy method.
(under review) https://hal.archives-ouvertes.fr/hal-01531845

J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr,
Numerically exploring the nonlinear dynamics of a piezo-magneto-elastic energy harvesting device.
(in preparation)

Physical system parameters

parameter	value
ξ	0.01
χ	0.05
f	0.083
Ω	0.8
λ	0.05
κ	0.5