Control of chaos via OGY method on a bistable energy harvester

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José Geraldo Telles
Elbert Macau

COBEM 2019
October 20 – 25, 2019
Uberlândia - MG, Brazil

Section 1

Introduction

April 4, 2005

March 13, 2013

L. Gammaitoni, **Fundamentals on energy**, NiPS Summer School 2018, University of Perugia.


IS-1
ST. JUDE MEDICAL
Sylmar, CA USA
VICTORY™ SR
5610 SSIR
S/N 1723182

Wikimedia Commons, **"File:St Jude Medical pacemaker in hand.jpg — Wikimedia Commons, the free media repository"**, 2014.

What's common in both cases?

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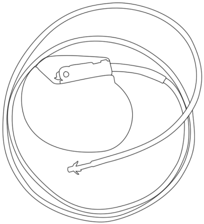
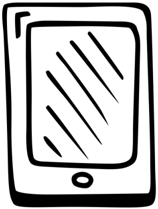
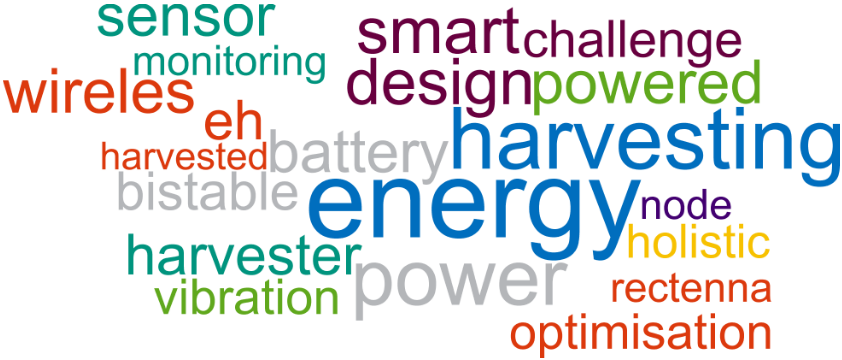
Electronic devices demanding energy!

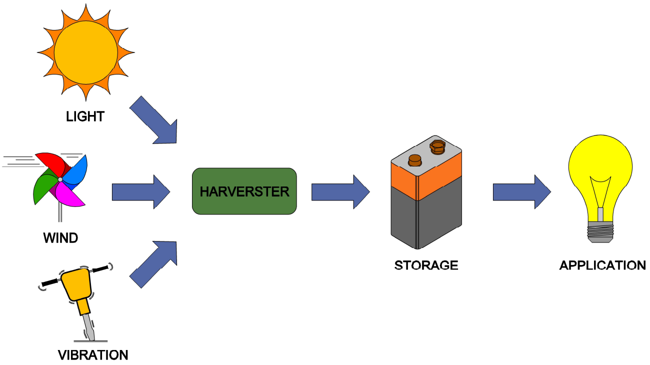






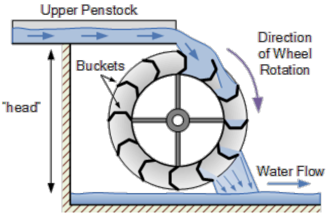
*Picture obtainded from Google Images.

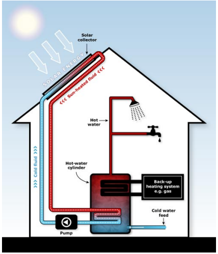
Energy Harvesting concept



- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

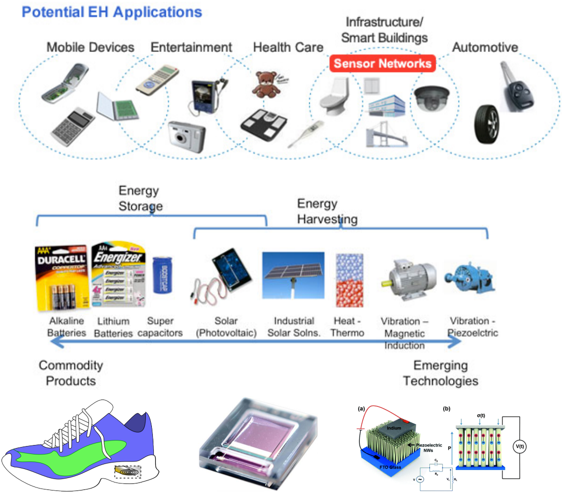
Classical technologies in Energy Harvesting

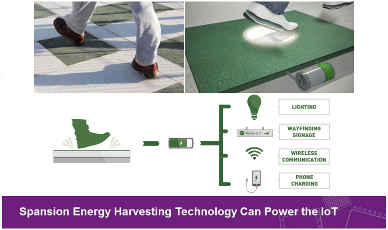


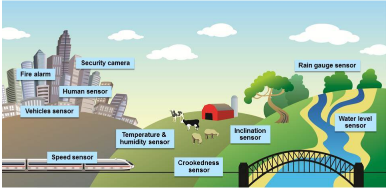


*Pictures obtainded from Google Images, several sources. If you are the owner of any one of these images, consider its use a compliment.

Emergent technologies in Energy Harvesting







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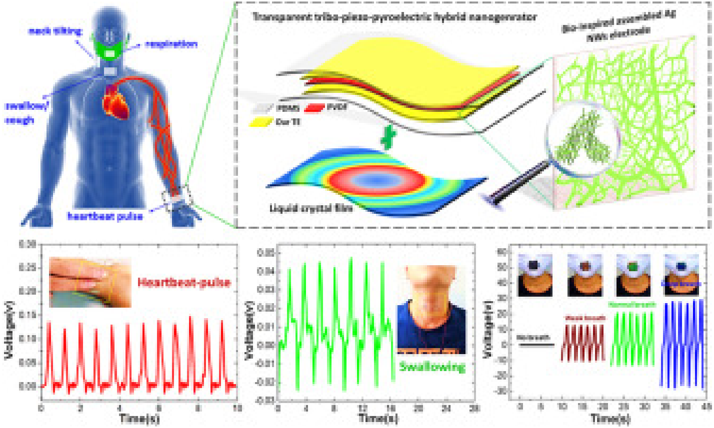
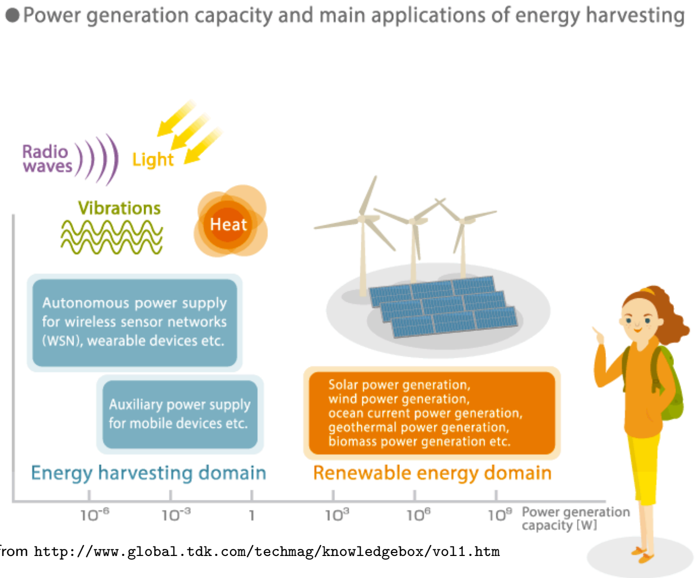
Emergent technologies in Energy Harvesting



*Picture from the reference.

J-G Sun et al. A flexible transparent one-structure tribo-piezo-pyroelectric hybrid energy generator based on bio-inspired silver nanowires network for biomechanical energy harvesting and physiological monitoring. **Nano Energy**, 48, 2018.

Energy scale for modern Energy Harvesting



*Picture from http://www.global.tdk.com/techmag/knowledgebox/vol1.htm

Energy Harvesting working principles

- Vibration based (piezoelectric effect)
- Contact based (triboelectric effect)
- Temperature based (thermoelectric effect)
- Heat based (pyroelectric effect)
- Light based (photoelectric effect)
- Fluid flow based
- Radio frequency based
- etc

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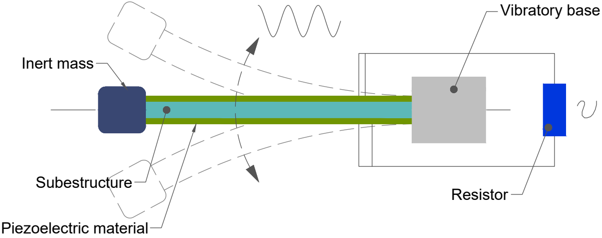
Linear Vibratory Harvester

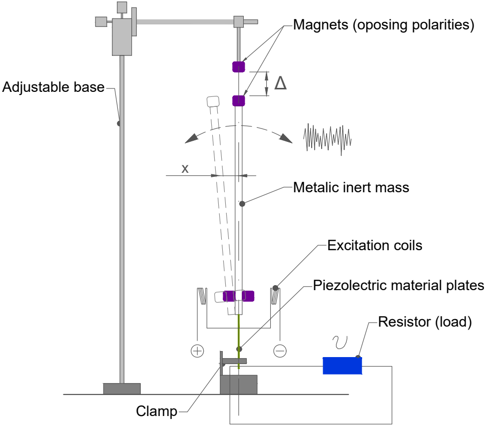
Monostable system driven by regular signal



S. Roundy, P. K. Wright and J. Rabaey, A study of low level bibrations as a power source for wireless sensor nodes. **Computer Communications**, 26: 1131-1144, 2003.

Nonlinear Vibratory Harvester

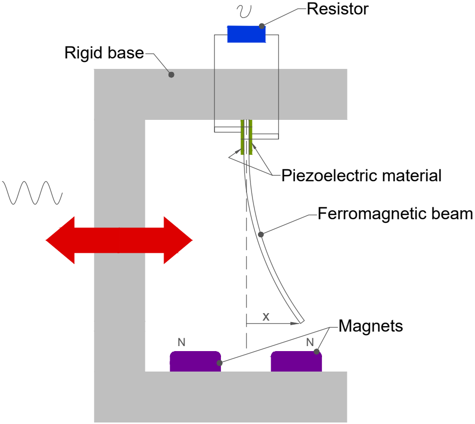
Bistable system driven by a noisy signal



F. Cottone, H. Vocca and L. Gammaitoni, Nonlinear energy harvesting. **Physical Review Letters**, 102: 080601, 2009.

Nonlinear Vibratory Harvester

Bistable system driven by regular signal



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Research objectives

This research has several objectives:

- Investigate in detail the underlying nonlinear dynamics
  - Time series
  - Poincaré sections
  - Bifurcation diagrams
  - Basis of attractions
  - Test 0-1 for chaos
- Model the underlying uncertainties and study their influence
  - System parameters variability
  - Noise in system excitation
- Propose strategies to enhance the recovered energy
  - Nonlinear optimization
  - Control of chaos

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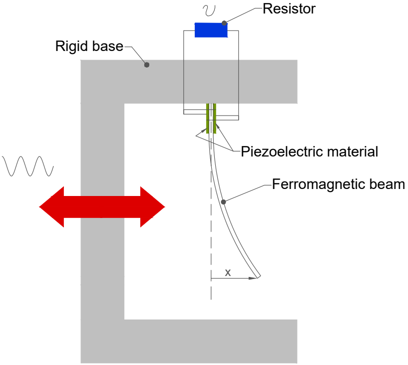
Section 2

Dynamical System

Linear Dynamics

Linear harvester driven by regular signal



$$\ddot{x} + 2\xi\dot{x} + x + \chi\upsilon = f\cos\Omega t$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

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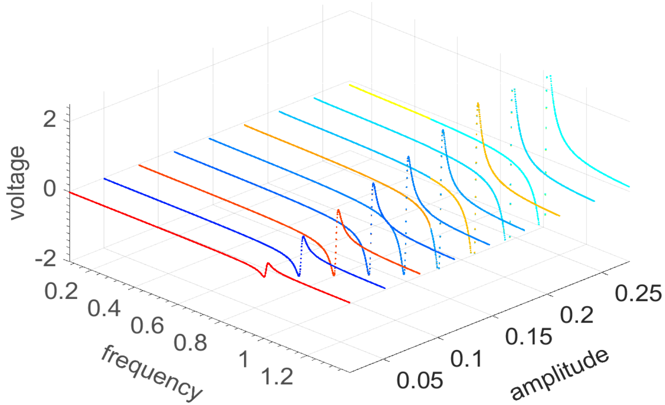
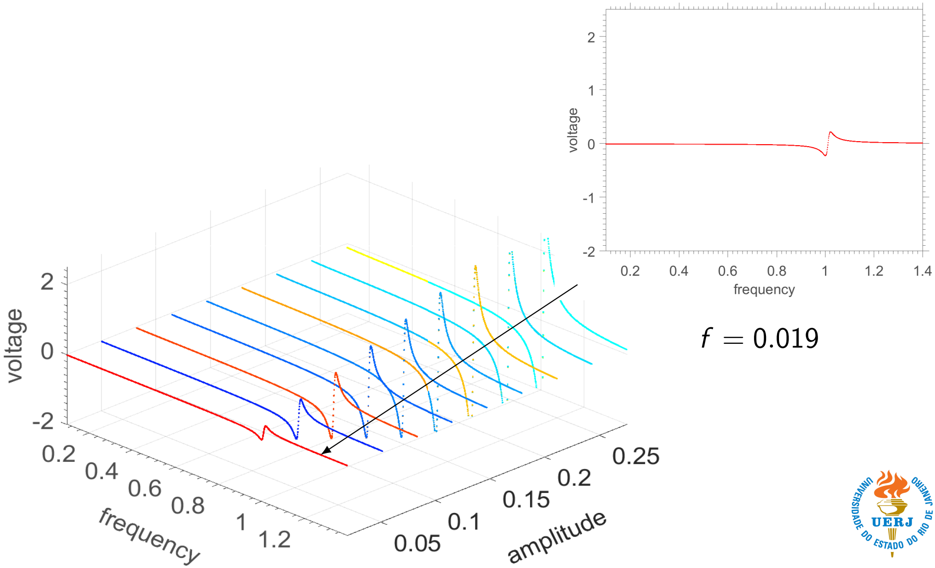
A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Linear dynamics animation

Bifurcation diagrams: voltage $\times$ frequency

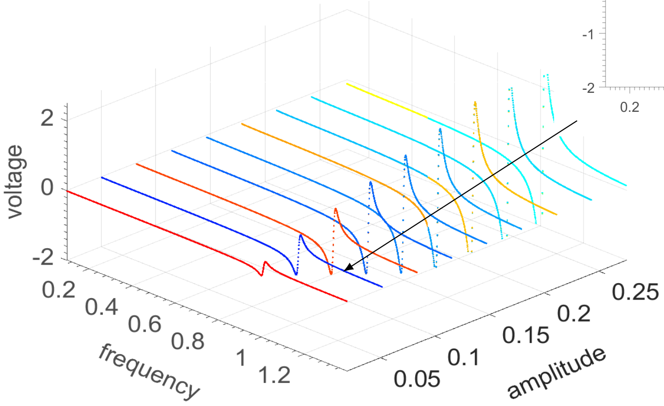


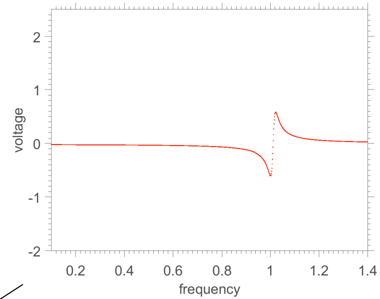
Bifurcation diagrams: voltage × frequency



$f = 0.019$

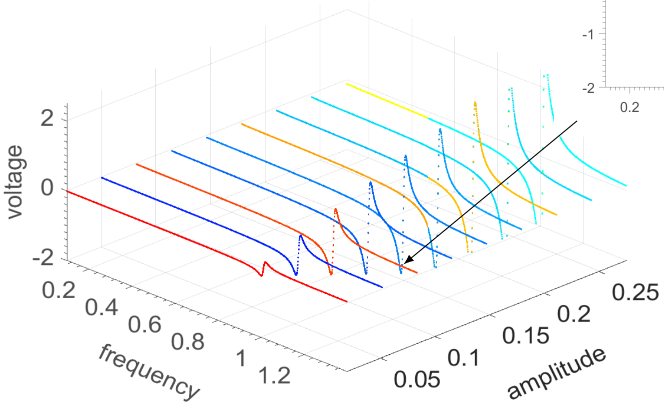
Bifurcation diagrams: voltage $\times$ frequency

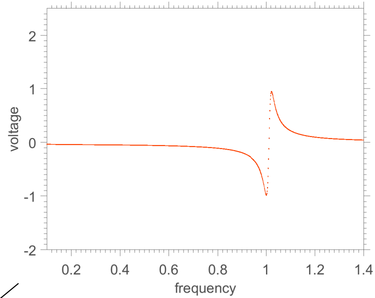




$f = 0.051$

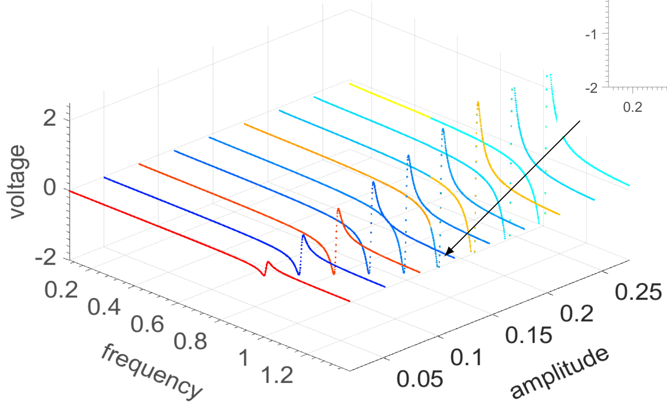
Bifurcation diagrams: voltage $\times$ frequency

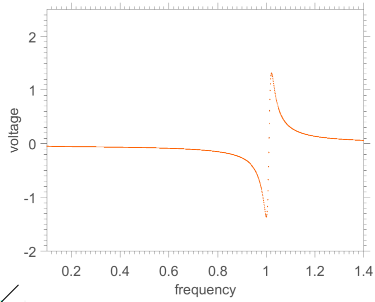




$$f = 0.083$$

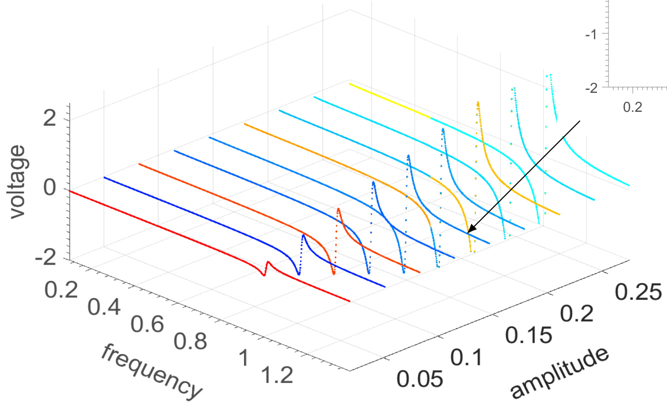
Bifurcation diagrams: voltage $\times$ frequency

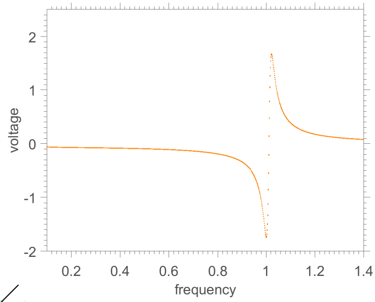




$f = 0.115$

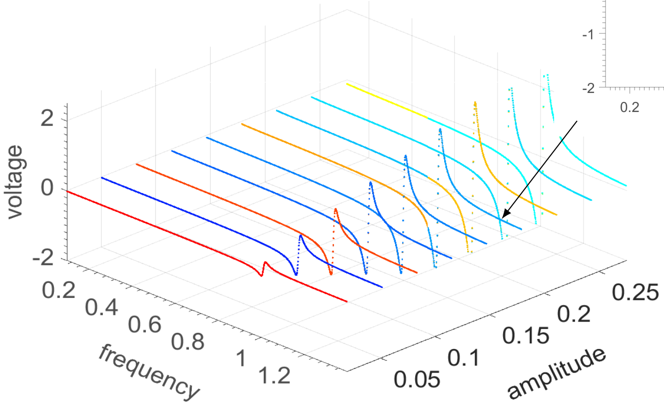
Bifurcation diagrams: voltage × frequency

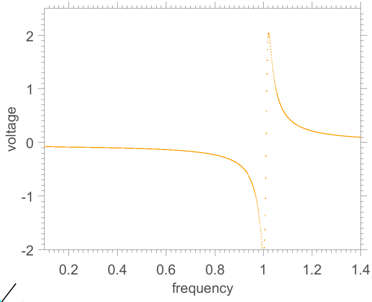




$f = 0.147$

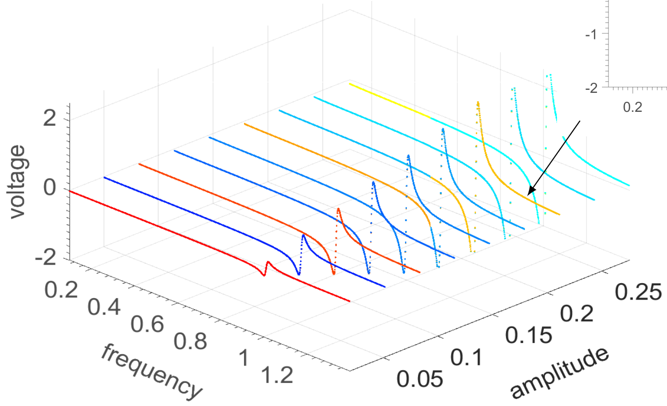
Bifurcation diagrams: voltage × frequency

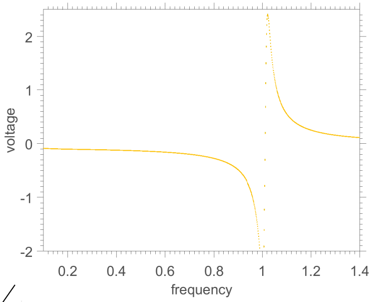




$f = 0.179$

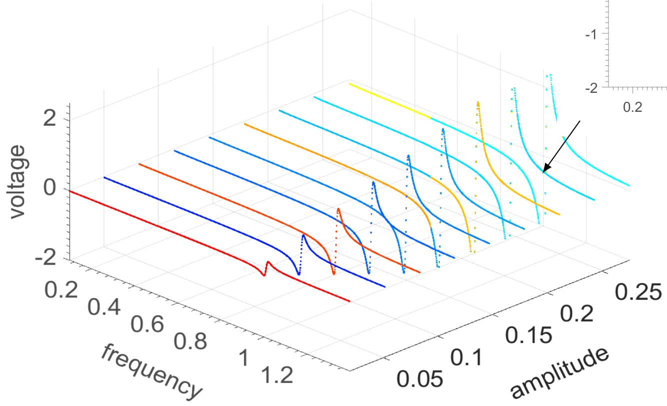
Bifurcation diagrams: voltage $\times$ frequency

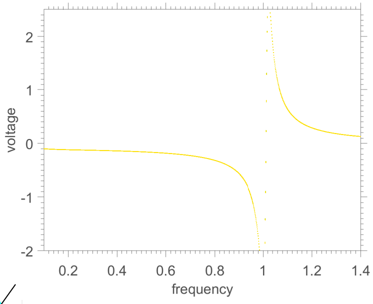




$f = 0.211$

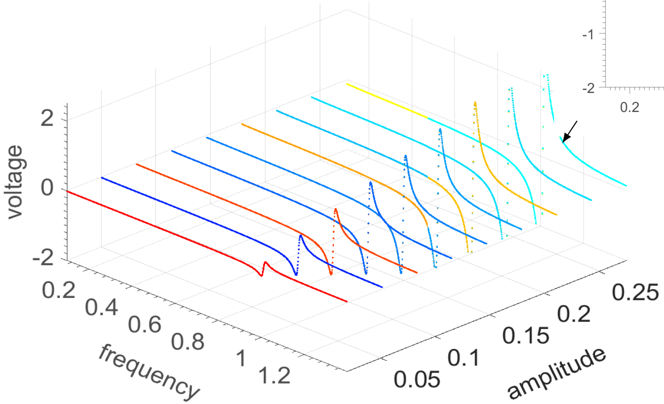
Bifurcation diagrams: voltage × frequency

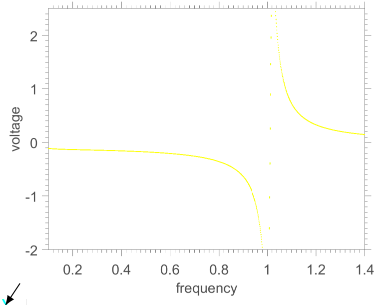




$f = 0.243$

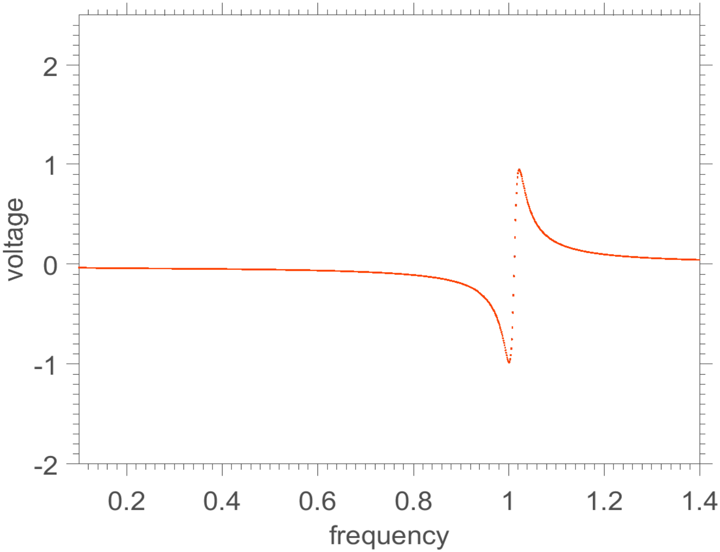
Bifurcation diagrams: voltage × frequency



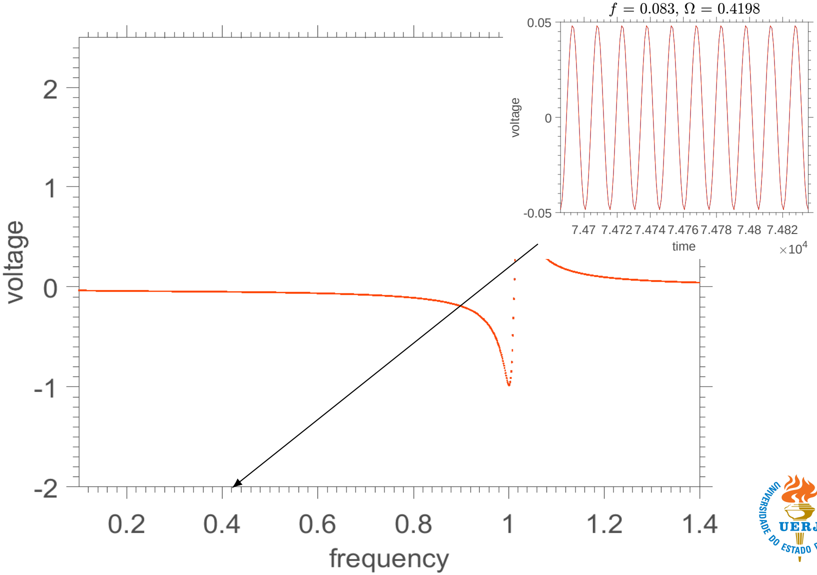


$f = 0.275$

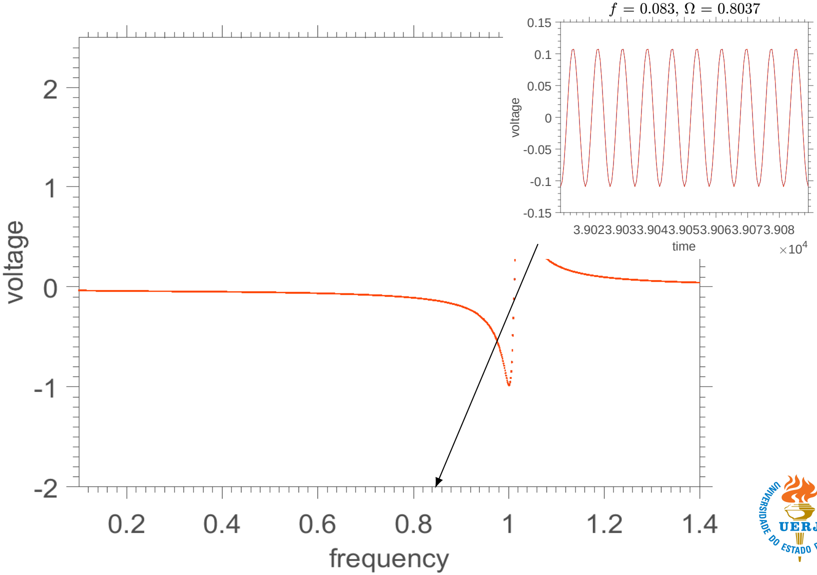
Forward and backward diagrams ($f = 0.083$)



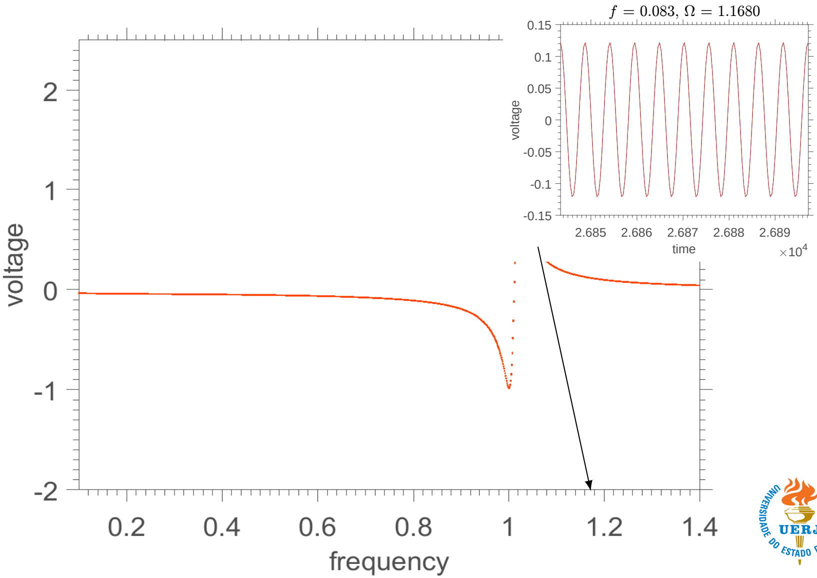
Forward and backward diagrams ($f = 0.083$)



Forward and backward diagrams ($f = 0.083$)

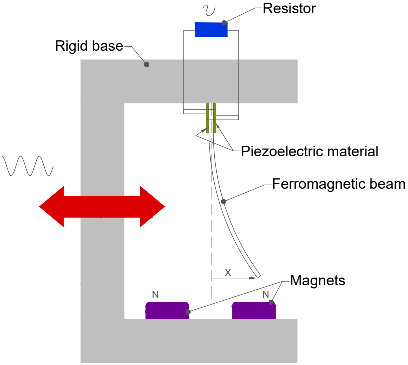


Forward and backward diagrams ($f = 0.083$)



Nonlinear Dynamics

Bistable harvester driven by regular signal



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t$$

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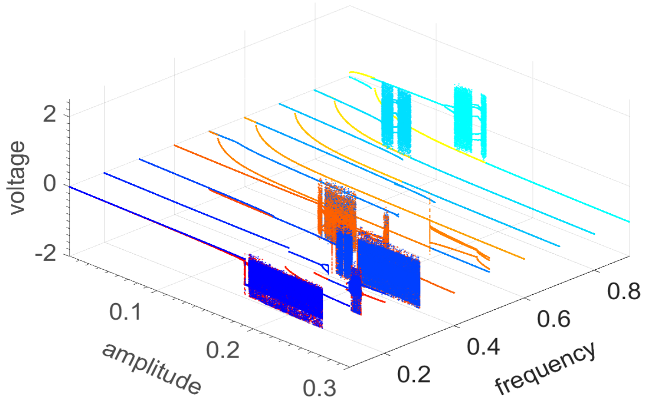
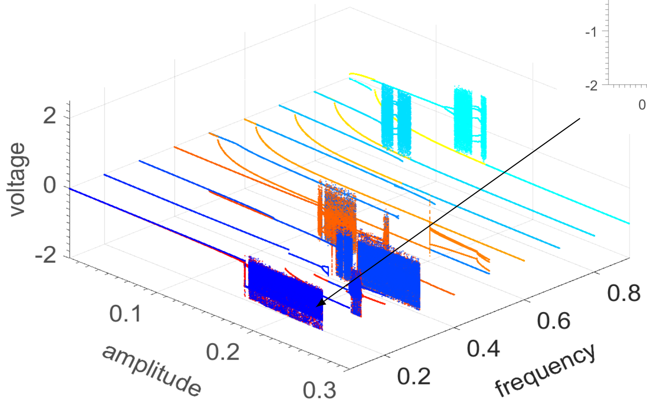
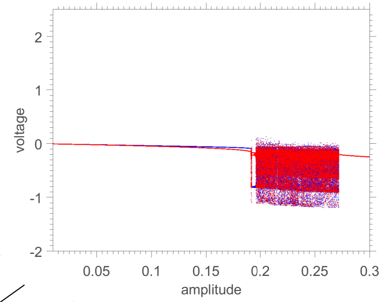
Nonlinear dynamics animation

Bifurcation diagrams: voltage × amplitude



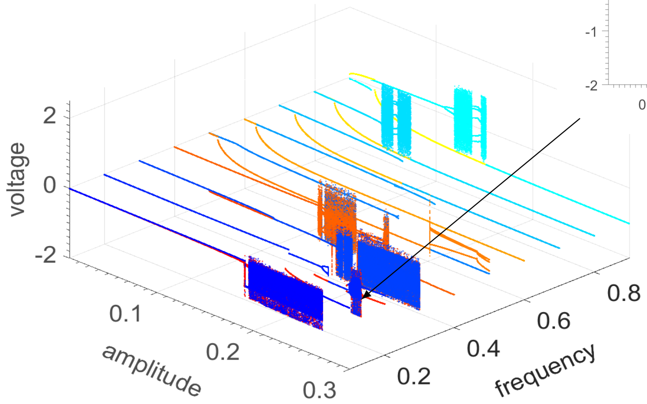
Bifurcation diagrams: voltage $\times$ amplitude

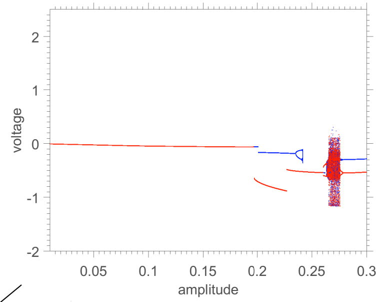




$\Omega = 0.1$

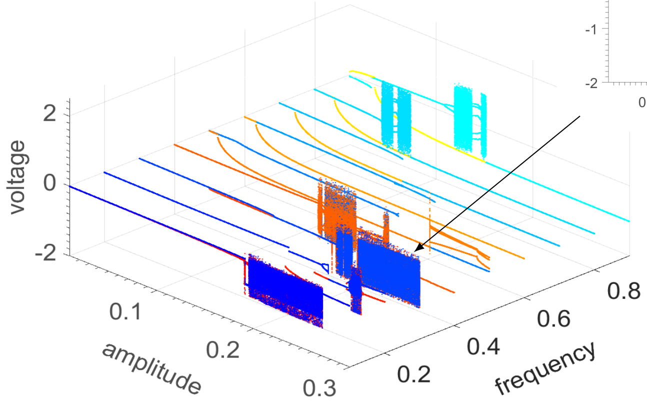
Bifurcation diagrams: voltage $\times$ amplitude

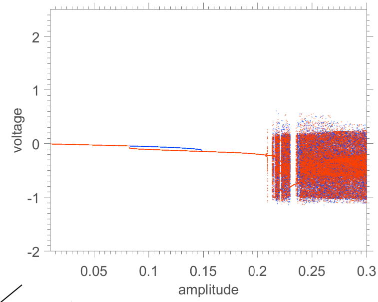




$\Omega = 0.2$

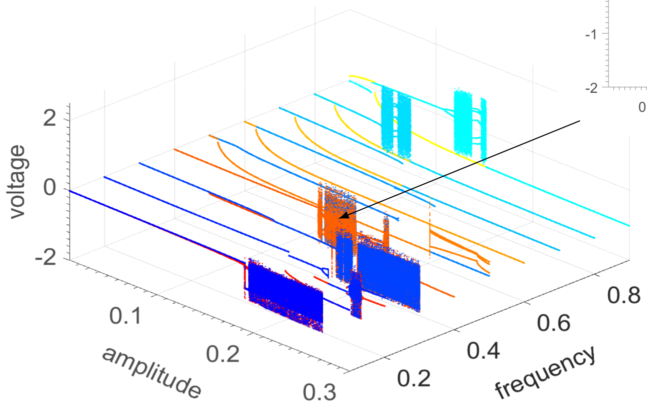
Bifurcation diagrams: voltage × amplitude

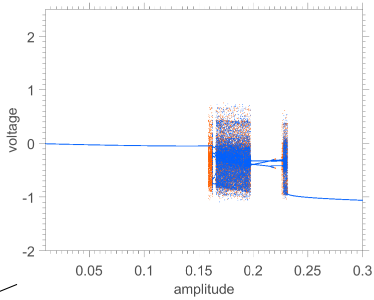




$\Omega = 0.3$

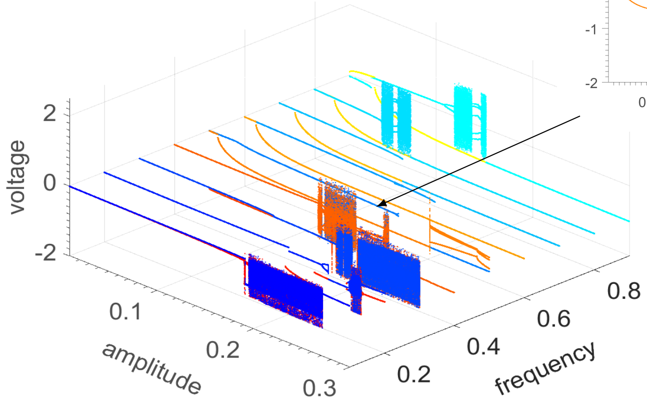
Bifurcation diagrams: voltage $\times$ amplitude

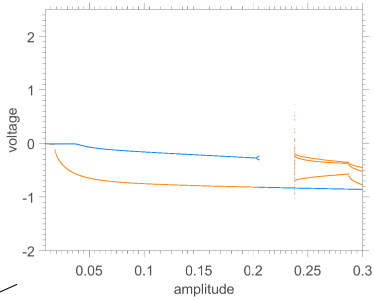




$\Omega = 0.4$

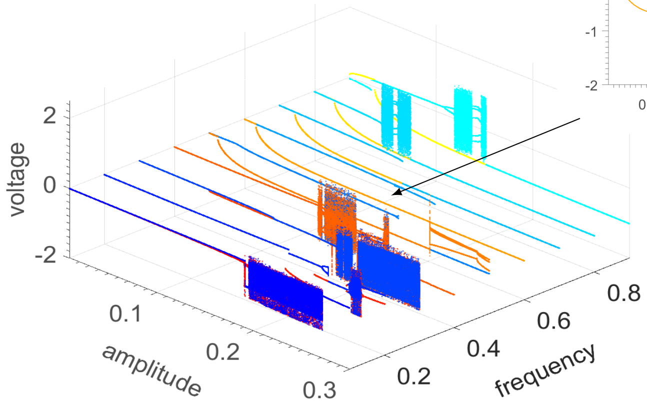
Bifurcation diagrams: voltage $\times$ amplitude

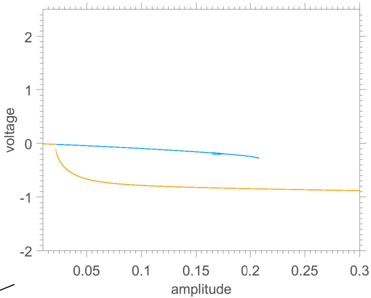




$\Omega = 0.5$

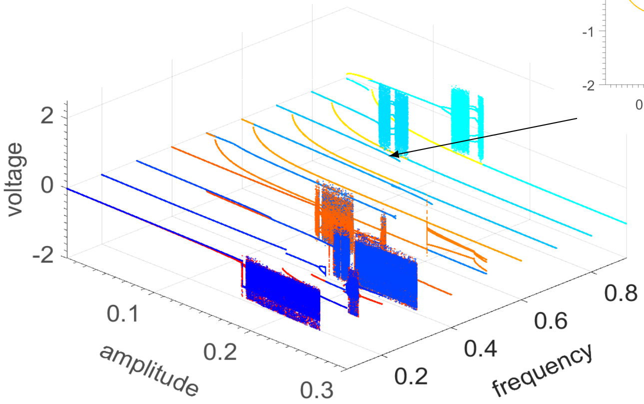
Bifurcation diagrams: voltage × amplitude

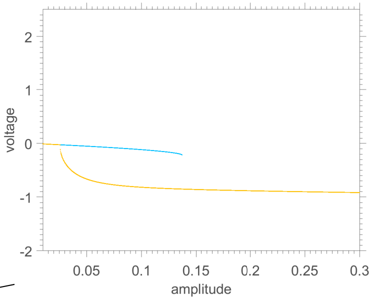




$\Omega = 0.6$

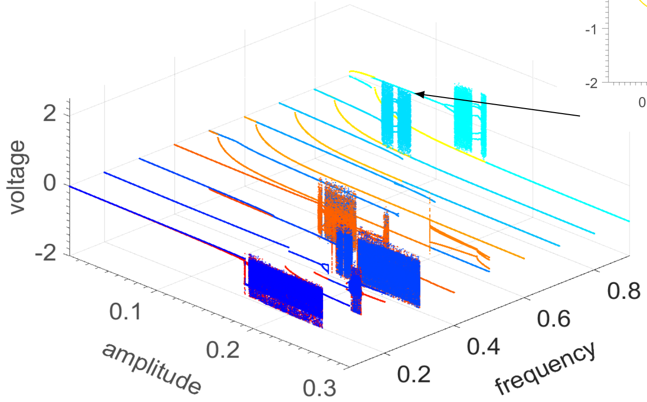
Bifurcation diagrams: voltage $\times$ amplitude

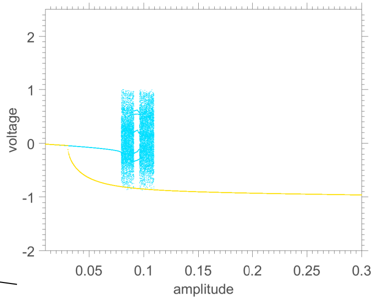




$\Omega = 0.7$

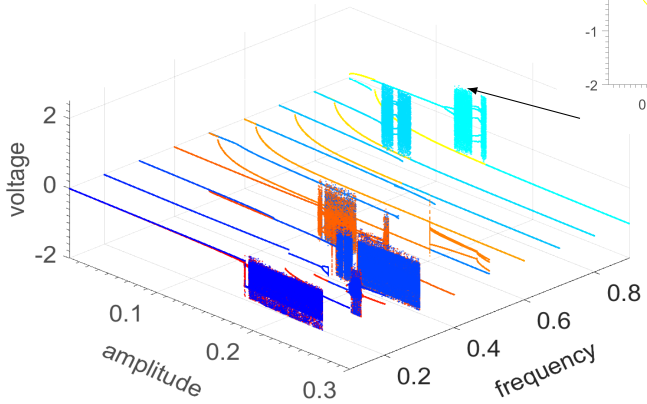
Bifurcation diagrams: voltage × amplitude

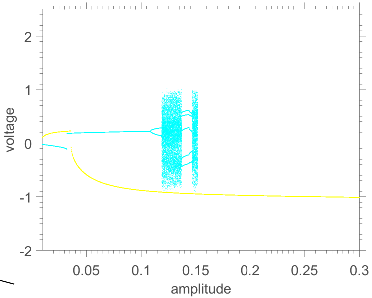




$\Omega = 0.8$

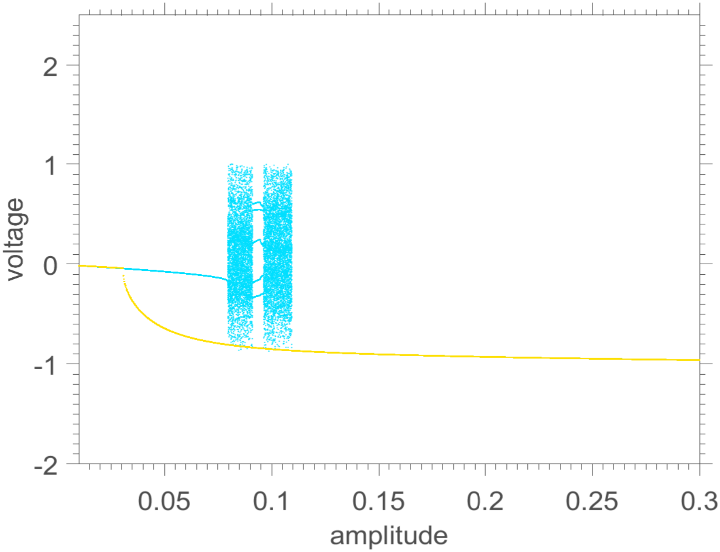
Bifurcation diagrams: voltage $\times$ amplitude



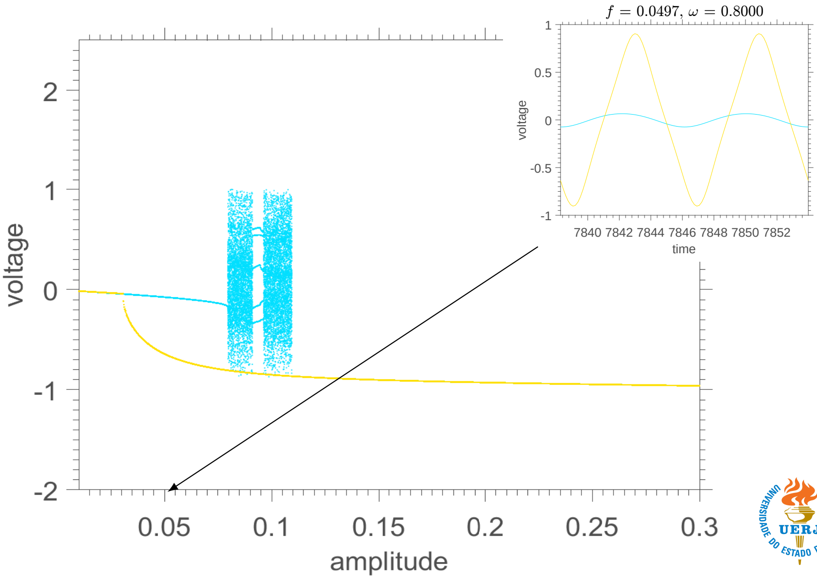


$\Omega = 0.9$

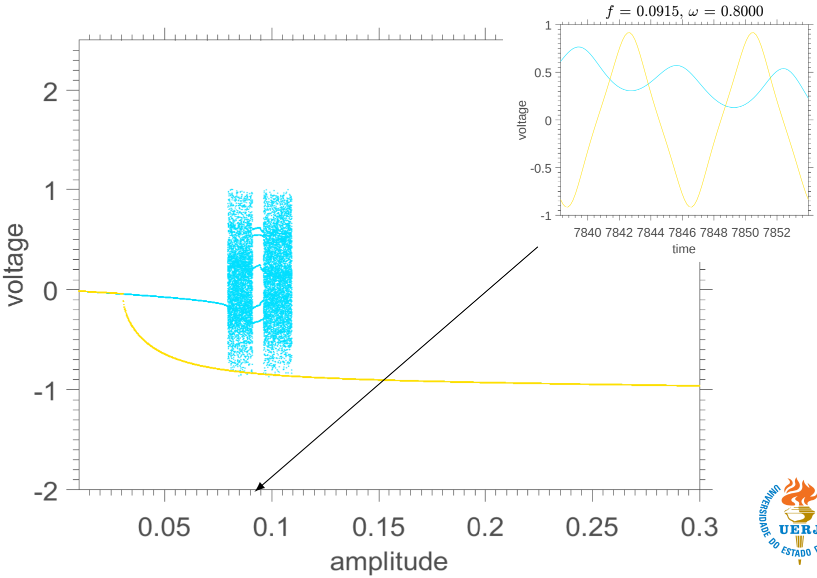
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



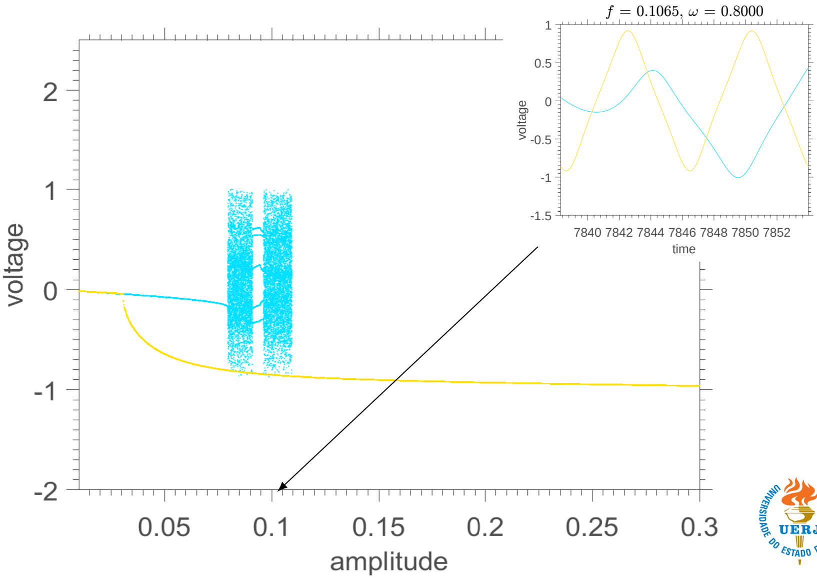
Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Forward and backward bifurcation diagrams ($\Omega = 0.8$)



Basins of attraction

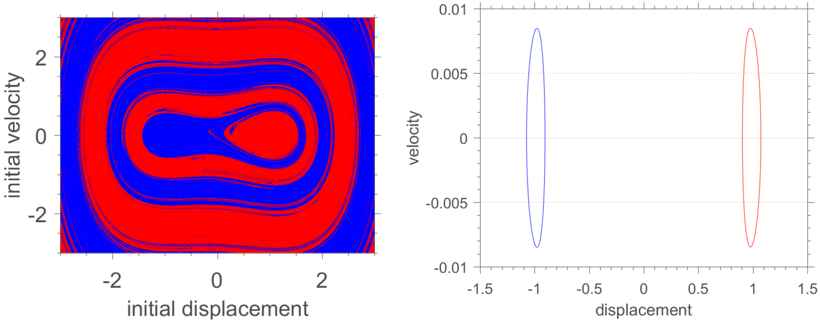


Figure: $f = 0.083$ and $\Omega = 0.1$

Basins of attraction

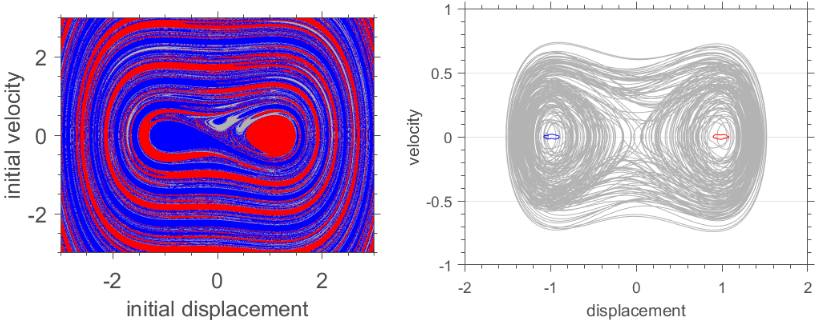


Figure: $f = 0.083$ and $\Omega = 0.2$

Basins of attraction

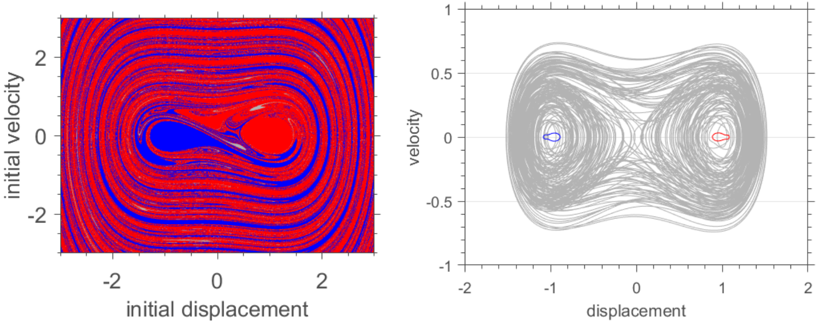


Figure: $f = 0.083$ and $\Omega = 0.3$

Basins of attraction

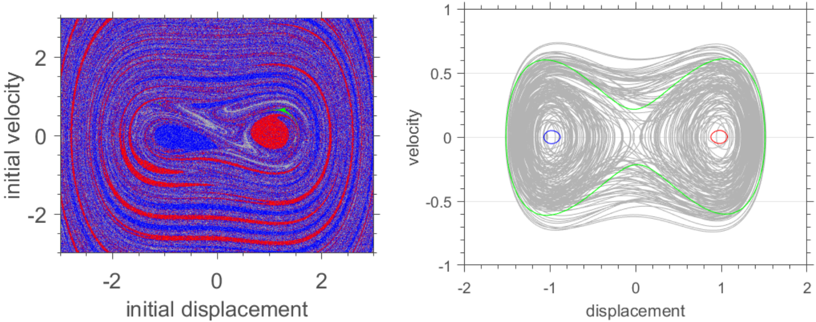
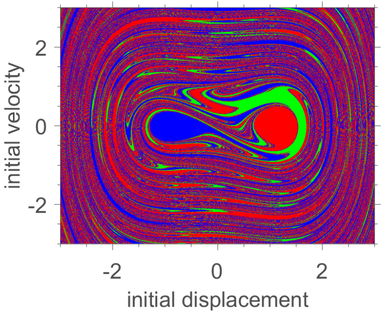


Figure: $f = 0.083$ and $\Omega = 0.4$

Basins of attraction



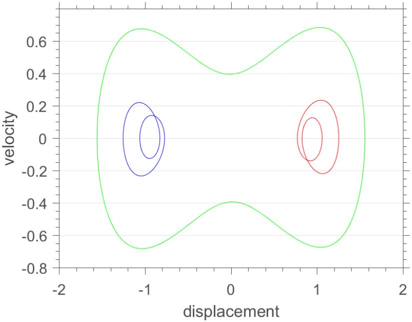


Figure: $f = 0.083$ and $\Omega = 0.5$

Basins of attraction

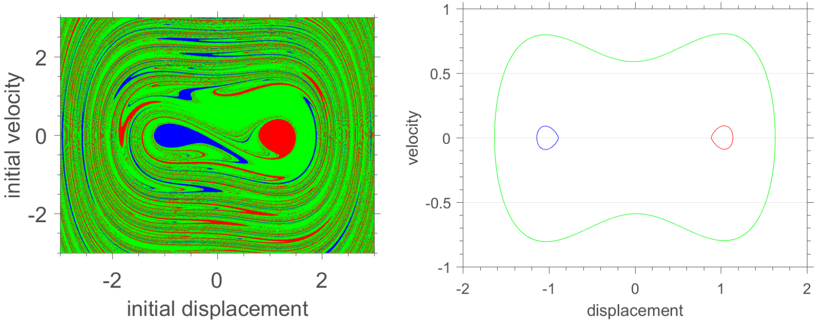
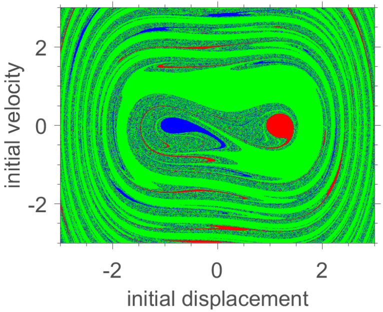


Figure: $f = 0.083$ and $\Omega = 0.6$

Basins of attraction



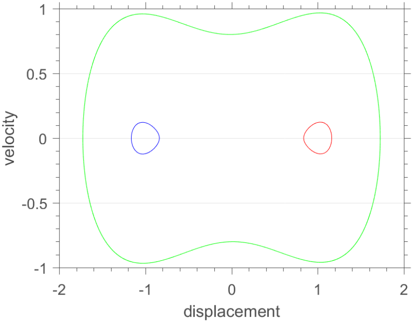


Figure: $f = 0.083$ and $\Omega = 0.7$

Basins of attraction

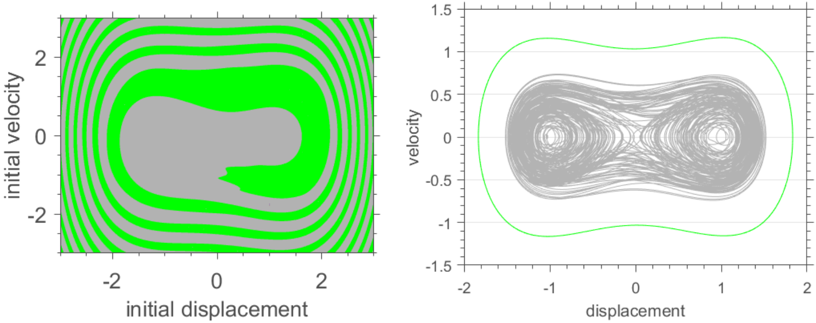


Figure: $f = 0.083$ and $\Omega = 0.8$

Basins of attraction

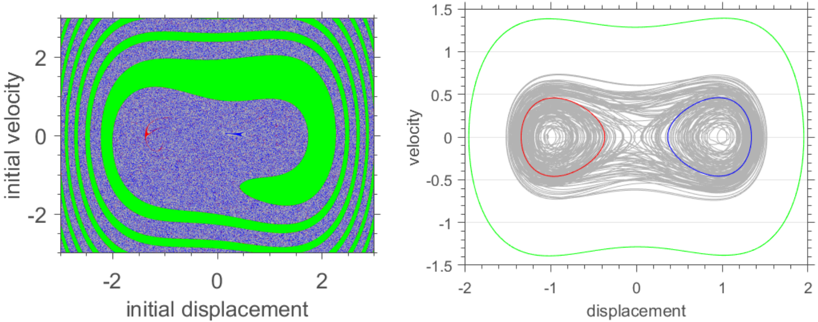
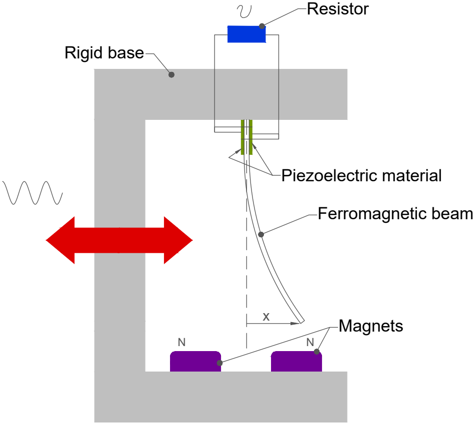


Figure: $f = 0.083$ and $\Omega = 0.9$

Stochastic Dynamics (ongoing research)

Nonlinear Vibratory Harvester

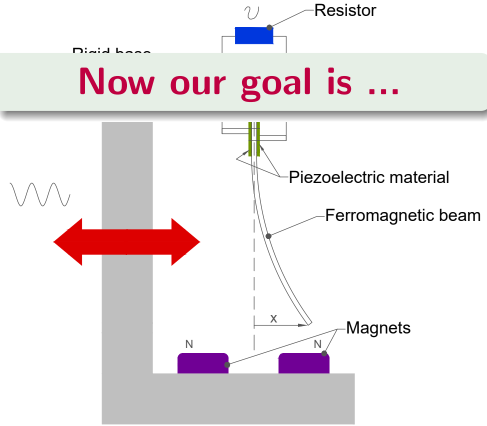
Bistable system driven by regular signal



A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

Nonlinear Vibratory Harvester

Bistable system driven by regular signal

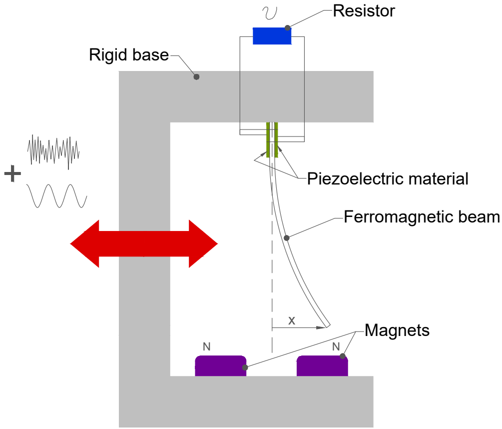


Now our goal is ...

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting*. **Applied Physics Letters**, 94: 254102, 2009.

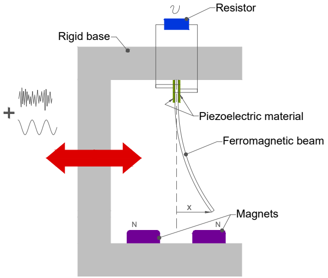
Nonlinear Vibratory Harvester

Bistable system driven by regular and noisy signals



V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr, **On the nonlinear stochastic dynamics of piezo-magneto-elastic energy harvester driven by colored noise**, (in preparation) 2019.

Bistable harvester driven by regular and noisy signals



$$\ddot{x}+2\xi\dot{x}-\frac{1}{2}x(1-x^2)-\chi\upsilon = f\cos\Omega t + \text{"noise"}$$

$$\dot{\upsilon}+\lambda\upsilon+\kappa\dot{x}=0$$

$$x(0)=x_0,\ \dot{x}(0)=\dot{x}_0,\ \upsilon(0)=\upsilon_0$$

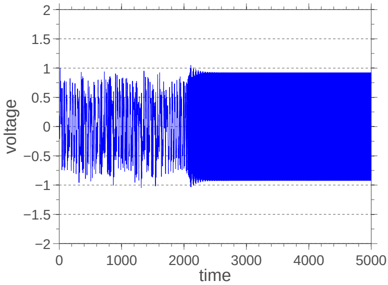
V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr, **On the nonlinear stochastic dynamics of piezo-magneto-elastic energy harvester driven by colored noise**, (in preparation) 2019.

Nonlinear stochastic dynamics animation

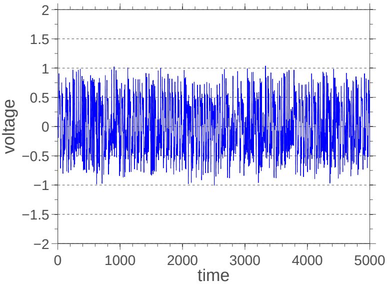
Section 3

Controlling Chaos

Typical time series for the bistable harvester

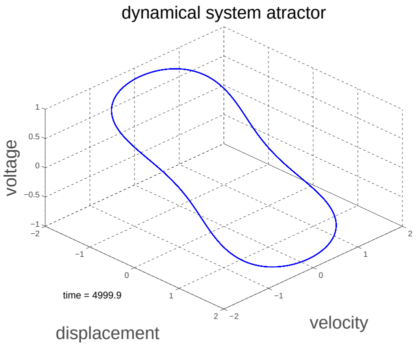


(a) regular (after chaotic transient)

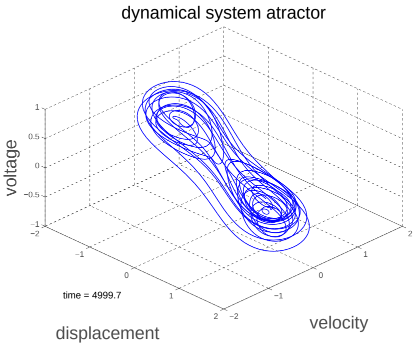


(b) strange atractor

Typical atractors for the bistable harvester

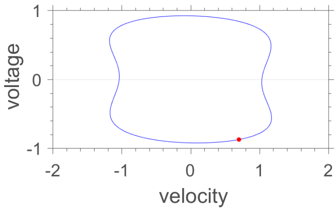


(a) regular limit cicle

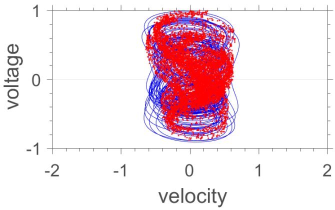


(b) strange atractor

Typical Poincaré sections for the bistable harvester



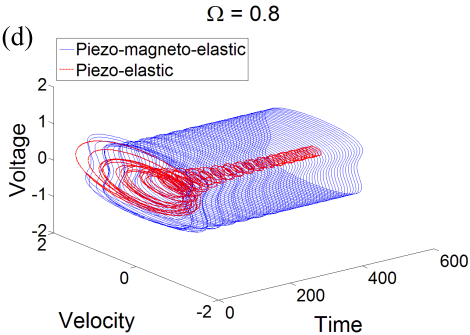
(a) regular limit cicle



(b) strange atractor

Nonlinearity and efficiency

Nonlinearity of this dynamical system may enhance output power.

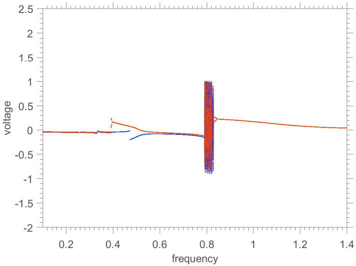


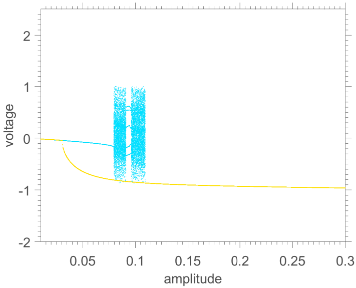
A. Erturk, *Electromechanical Modeling of Piezoelectric Energy Harvesters*, **PhD Thesis**, Virginia Tech, 2009.

*Picture from the above reference.

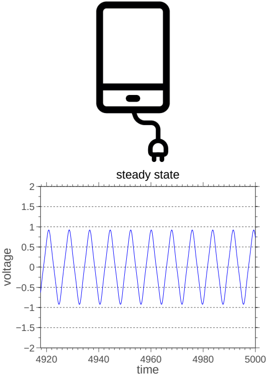
Nonlinearity and chaos

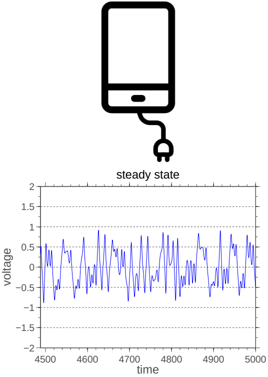
Nonlinearity of this dynamical system may also induce chaos.





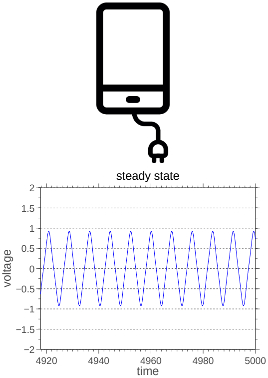
For operation of electrical devices ...

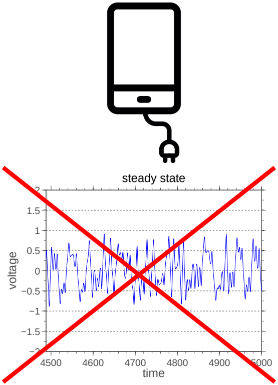




*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

For operation of electrical devices ...

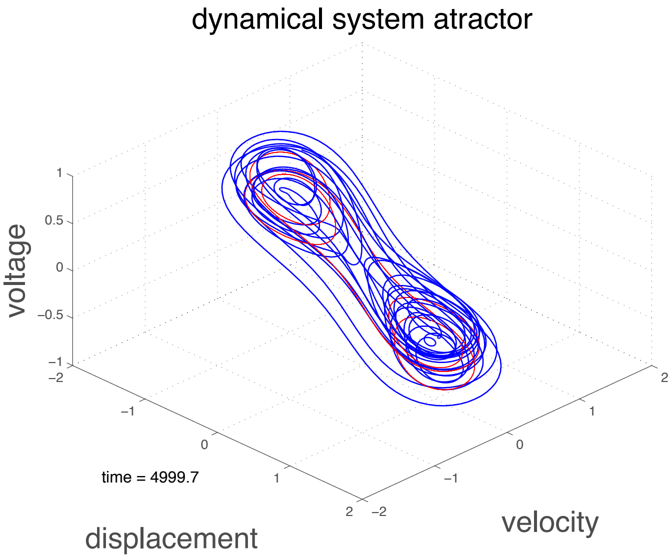




... irregular voltage is undesirable!

*Phone picture from http://freevector.co/vector-icons/technology/charging-phone.html

UPO embedded into a chaotic atractor



How to explore these unstable periodic orbits?

$\Longrightarrow$ **Techniques for control of chaos (known UPO is required)**

OGY:

- Control performed by a sequence of (discrete) small "kicks" that forces the system trajectory to stay in the target orbit

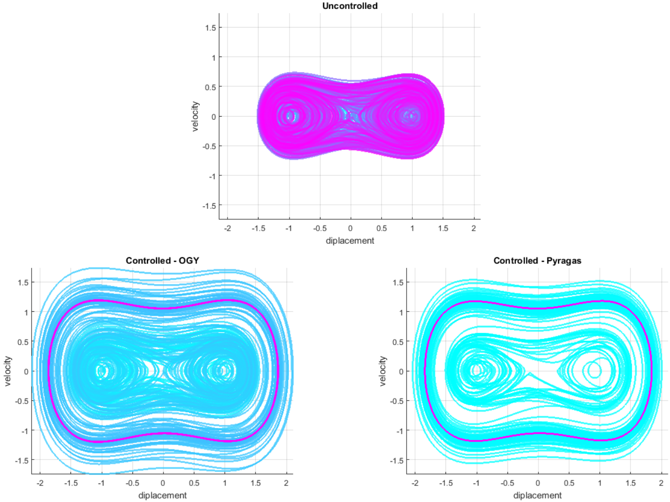
E. Ott, C. Grebogi, J. Yorke, Controlling chaos, **Physical Review Letters**, 64:1196, 1990.

Pyragas:

- Control performed by a (continuous) low intensity signal which is almost zero if the system evolves close to the target orbit, and increases when it starts to drift way

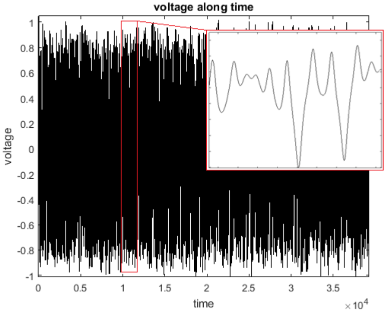
K. Pyragas, Continuous control of chaos by self-controlling feedback, **Physics Letters A**, 170:421, 1992.

Stabilization via control of chaos ($f = 0.083$, $\Omega = 0.8$)

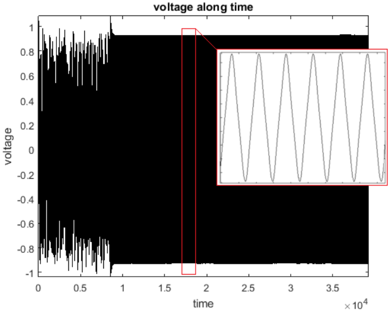


*The colors represent the evolution of the system, from blue to pink, as the time progresses.

Stabilization via control of chaos ($f = 0.083$, $\Omega = 0.8$)

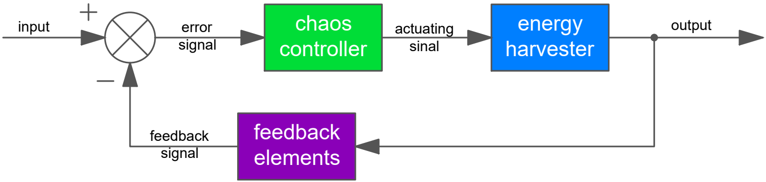


uncontrolled dynamics



OGY controlled dynamics

Feedback control law



E. Ott, C. Grebogi, J. Yorke, Controlling chaos, **Physical Review Letters**, 64:1196, 1990.

OGY control of chaos

- Poincaré map:

$$y_{n+1} = g(y_n,\, p)$$

- Linearization around a deviation from the target orbit:

$$y_{n+1} - y^* = \partial_y\, g(y^*,\, p^*)\,(y_n - y^*) + \partial_p\, g(y^*,\, p^*)\,(p - p^*)$$

- Goal:

$$\left\| y_{n+1} - y^* \right\| \to 0$$

- Controller project:

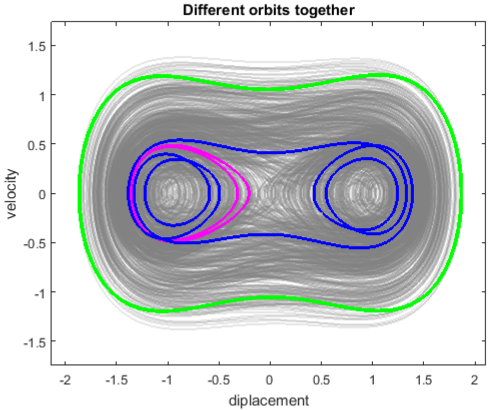
$$p - p^* = -K\,(y_n - y^*), \qquad \|K\| \leq \frac{1 - \left\|\partial_y\, g(y^*,\, p^*)\right\|}{\left\|\partial_p\, g(y^*,\, p^*)\right\|}$$

E. Ott, C. Grebogi, J. Yorke, `Controlling chaos`, **Physical Review Letters**, 64:1196, 1990.

T. Kapitaniak, **Chaos for Engineers: theory, applications and control**, Springer, 2nd Ed, 2000.

Orbits - $f = 0.090$, $\Omega = 0.8$

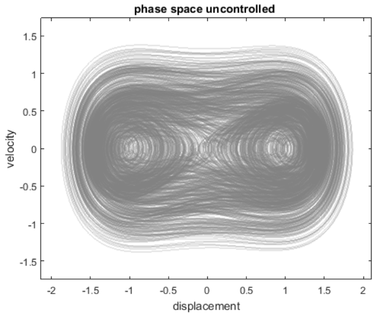
Phase space with controlled and uncontrolled orbits.



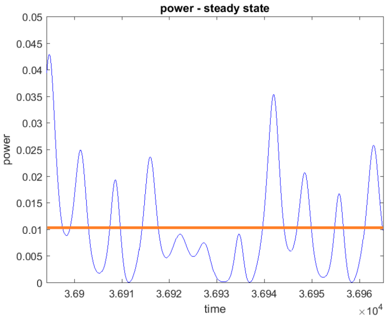
Periodic orbits within the chaotic attractor

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

uncontrolled dynamics



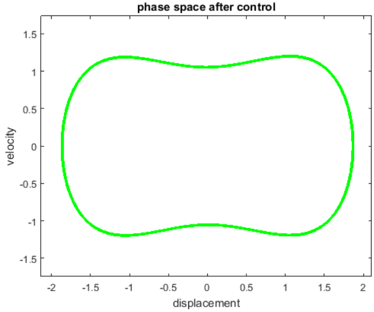
system trajectoty



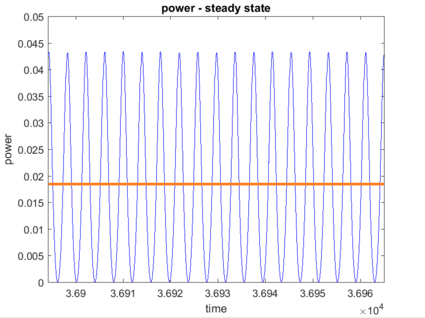
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 1 orbit



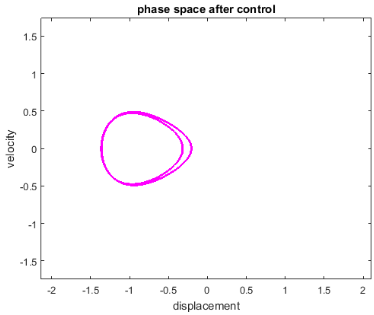
system trajectoty



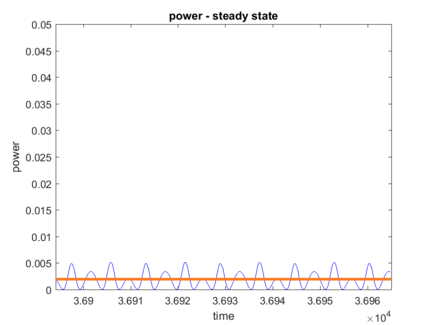
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 2 orbit



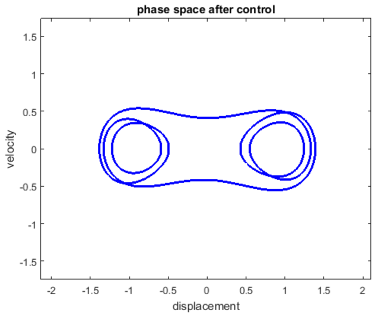
system trajectoty



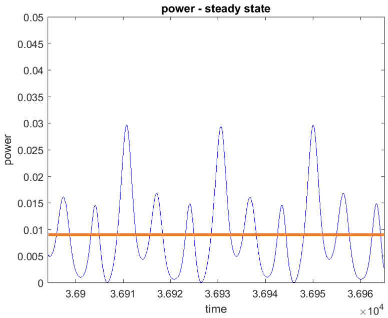
output power

OGY control with different orbits ($f = 0.090$, $\Omega = 0.8$)

controlled dynamics in a period 5 orbit



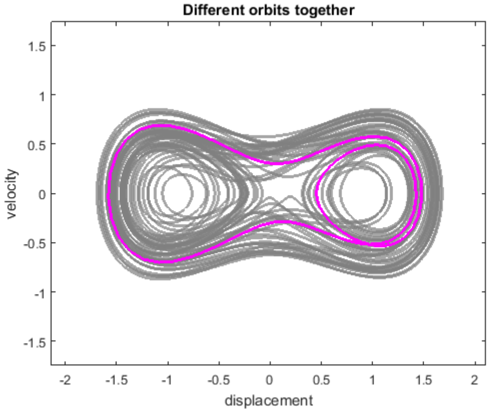
system trajectoty



output power

Orbits - $f = 0.115$, $\Omega = 0.3$

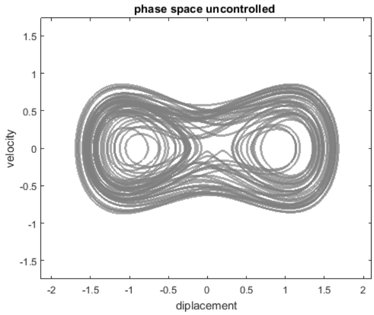
Phase space with controlled and uncontrolled orbits.



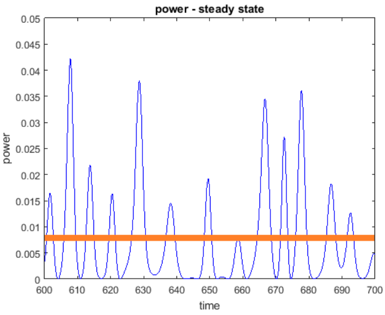
Periodic orbit within the chaotic attractor

OGY control with different orbits ($f = 0.115$, $\Omega = 0.3$)

uncontrolled dynamics



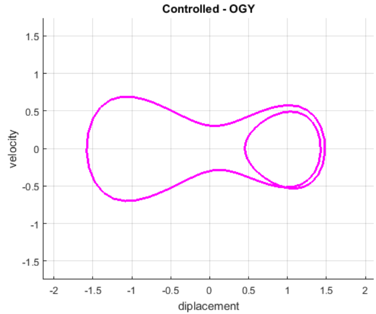
system trajectoty



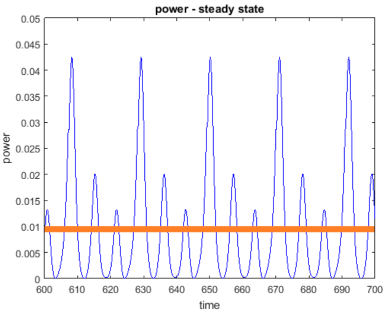
output power

OGY control with different orbits ($f = 0.115$, $\Omega = 0.3$)

controlled dynamics in a period 2 orbit



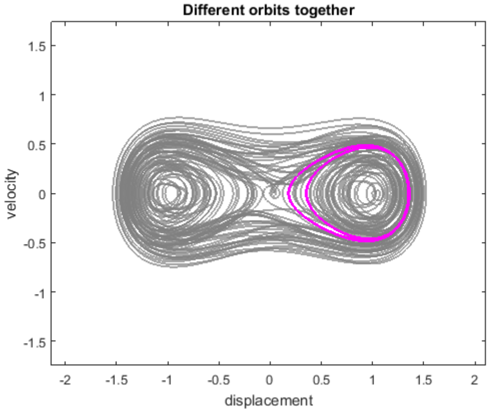
system trajectoty



output power

Orbits - $f = 0.088$, $\Omega = 0.8$

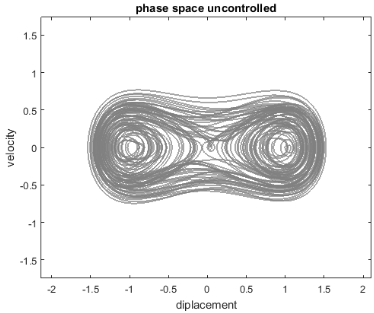
Phase space with controlled and uncontrolled orbits.



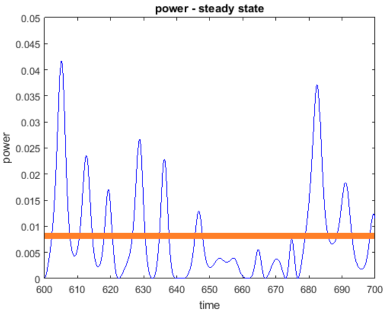
Periodic orbit within the chaotic attractor

OGY control with different orbits ($f = 0.088$, $\Omega = 0.8$)

uncontrolled orbit



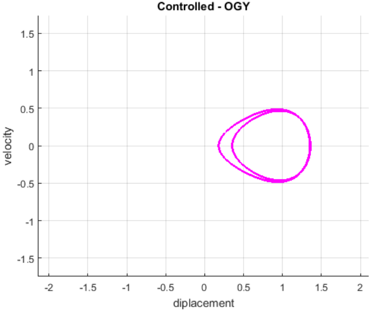
system trajectoty



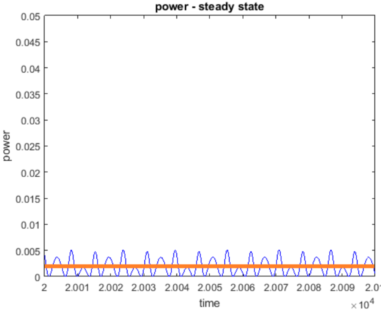
output power

OGY control with different orbits ($f = 0.088$, $\Omega = 0.8$)

controlled dynamics in a period 2 orbit



system trajectoty



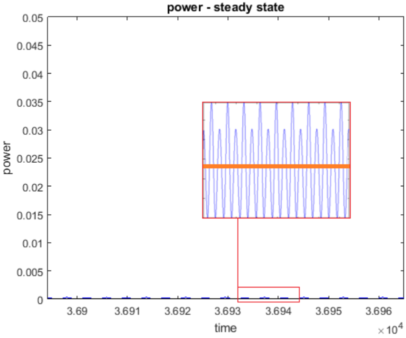
output power

Performance of OGY controller

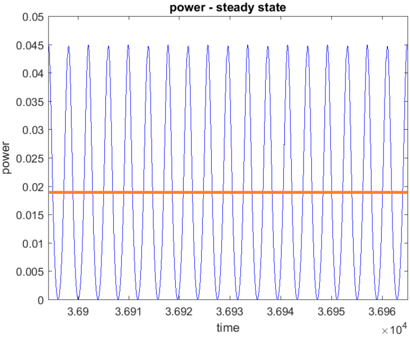
*Picture from https://www.amazon.com

Astonishing improvements are possible: Good!

$$f = 0.050, \ \Omega = 0.8$$



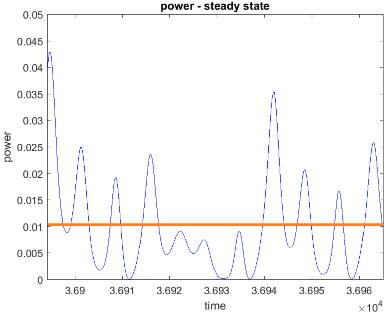
uncontrolled system



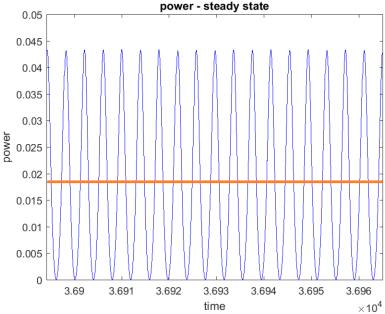
controlled system

Astonishing improvements are possible: Good!

$$f = 0.090,\ \Omega = 0.8$$



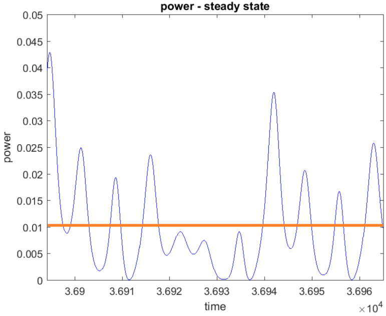
uncontrolled system



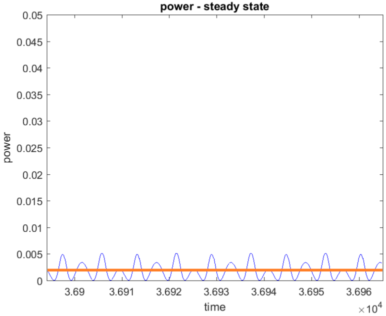
controlled system

Catastrophic effects too: Bad!

$f = 0.090,\ \Omega = 0.8$



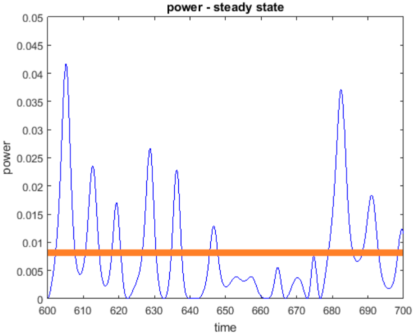
uncontrolled system



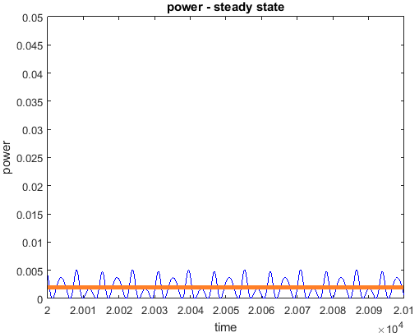
controlled system

Catastrophic effects too: Bad!

$$f = 0.088,\ \Omega = 0.8$$



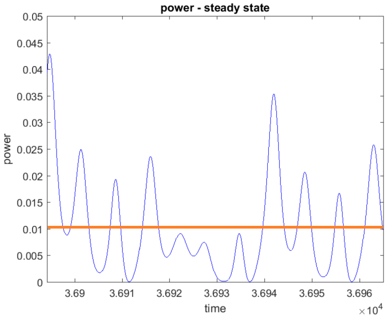
uncontrolled system



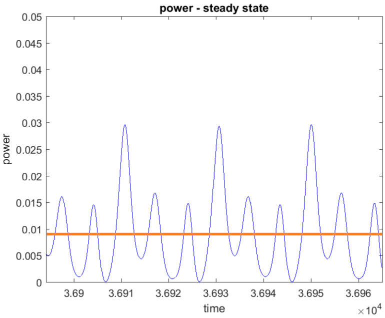
controlled system

And no significative change: Ugly!

$$f = 0.090,\ \Omega = 0.8$$



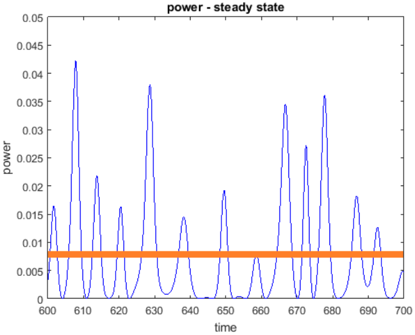
uncontrolled system



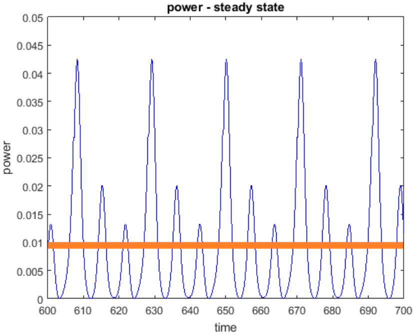
controlled system

And no significative change: Ugly!

$$f = 0.115,\ \Omega = 0.3$$



uncontrolled system



controlled system

Enhancement of power recovery (steady state dynamics)

excitation frequency: $\Omega = 0.8$

excitation amplitude	output power: uncontrolled system	output power: controlled system	output power: controller ($\times 10^{-4}$)	output power: effective value	power enhancement
0.050	0.0001	0.0068	- 0.0005	0.0068	$\times$ **68**
0.083	0.0073	0.0131	- 0.0006	0.0131	$\times$ **1.8**
0.088 (2-p)	0.0077	0.0019	- 0.0007	0.0019	$\times$ **0.2**
0.090 (1-p)	0.0077	0.0154	- 0.0001	0.0154	$\times$ **2.0**
0.090 (2-p)	0.0077	0.0037	- 0.0002	0.0037	$\times$ **0.5**
0.090 (5-p)	0.0077	0.0084	- 0.0001	0.0084	$\times$ **1.1**

excitation frequency: $\Omega = 0.3$

excitation amplitude	output power: uncontrolled system	output power: controlled system	output power: controller ($\times 10^{-4}$)	output power: effective value	average power enhancement
0.115 (2-p)	0.0078	0.0096	- 0.0004	0.0096	$\times$ **1.2**

Section 4

Final Remarks

Final remarks

Some conclusions:

- Nonlinearity is a powerful ingredient for harvesters efficiency:
  - May induce large amplitude responses
  - A plenty of energy is available in nonlinear responses
- Nonlinearity may also be an enemy:
  - May induce (very irregular) chaotic responses
  - Irregular voltage is undesirable for eletronic powering
- Control of chaos may be good, bad and ugly:
  - Astonishing improvements in efficiency are possible
  - Significative reductions of efficiency too
  - Controll of chaos may also be inert to efficiency

Ongoing research:

- Further studies in stochastic dynamics
- Further studies with Pyragas control of chaos
- Construction of an experimental apparatus

Acknowledgments

Academic discussion:

- Prof. Lucca Gammaitoni (NiPS Lab / Univ. Perugia)
- Profª. Aline de Paula (UnB)
- Prof. Adriano Fabro (UnB)
- Mr. Tiago Pereira (UnB)

Financial support:

FAPERJ Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

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Thank you for your attention!

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