Probabilistic Warm-up

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Deck of cards

Deck of 52 playing cards

spades ♠, clubs ♣, hearts ♡, diamonds ♢

2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

A card is drawn from this deck. Find the probability of:

1. '2' of spades
2. a jack
3. a king of red colour
4. a card of diamond
5. a king or a queen
6. a non-face card
7. a black face card
8. a black card
9. a non-ace
10. a non-face black card
11. neither a spade nor a jack
12. neither a heart nor a red king

http://www.math-only-math.com/playing-cards-probability.html

Deck of cards

Deck of 52 playing cards

spades ♠, clubs ♣, hearts ♡, diamonds ♢

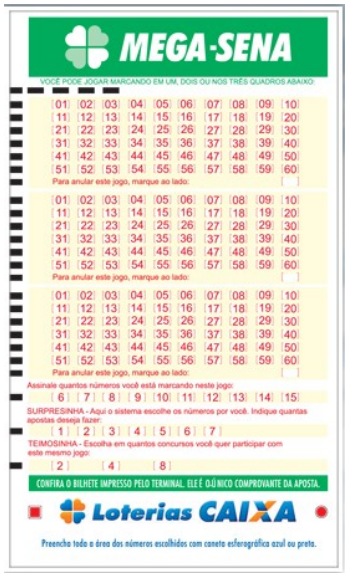
2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

A card is drawn from this deck. Find the probability of:

1. $1/52$
2. $4/52 = 1/13$
3. $2/52 = 1/26$
4. $13/52 = 1/4$
5. $8/52 = 2/13$
6. $40/52 = 10/13$
7. $6/52 = 3/26$
8. $26/52 = 1/2$
9. $48/52 = 12/13$
10. $20/52 = 5/13$
11. $36/52 = 9/13$
12. $38/52 = 19/26$

http://www.math-only-math.com/playing-cards-probability.html

Mega Sena



Biggest Brazilian lottery

Choose 6 integer numbers (between 01 and 60)

1st prize: hit 6 numbers

2nd prize: hit 5 numbers

3rd prize: hit 4 numbers

A player buys n tickets in a single extraction. Other player buys a single ticket in n extractions. Both bet the same value.

Which strategy is best?

Mega Sena

This lotery has $N = 50.063.860$ possible games.

Player 1:
(n tickets in a single extraction)

Player 2:
(a single ticket in n extractions)

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Probability of winning:

$$\underbrace{1/N + 1/N + \cdots + 1/N}_{n \text{ times}} = n/N$$

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Number of possible bets: N^n

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Number of possible bets: N^n
Number of losing bets: $(N-1)^n$

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(a single ticket in n extractions)

Number of possible bets: N^n
Number of losing bets: $(N-1)^n$
Probability of winning:

$$1 - \frac{(N-1)^n}{N^n} = 1 - \left(1 - \frac{1}{N}\right)^n$$

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This lotery has $N = 50.063.860$ possible games.

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Probability of winning:

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(a single ticket in n extractions)

Number of possible bets: N^n
Number of losing bets: $(N-1)^n$
Probability of winning:

$$1 - \frac{(N-1)^n}{N^n} = 1 - \left(1 - \frac{1}{N}\right)^n$$

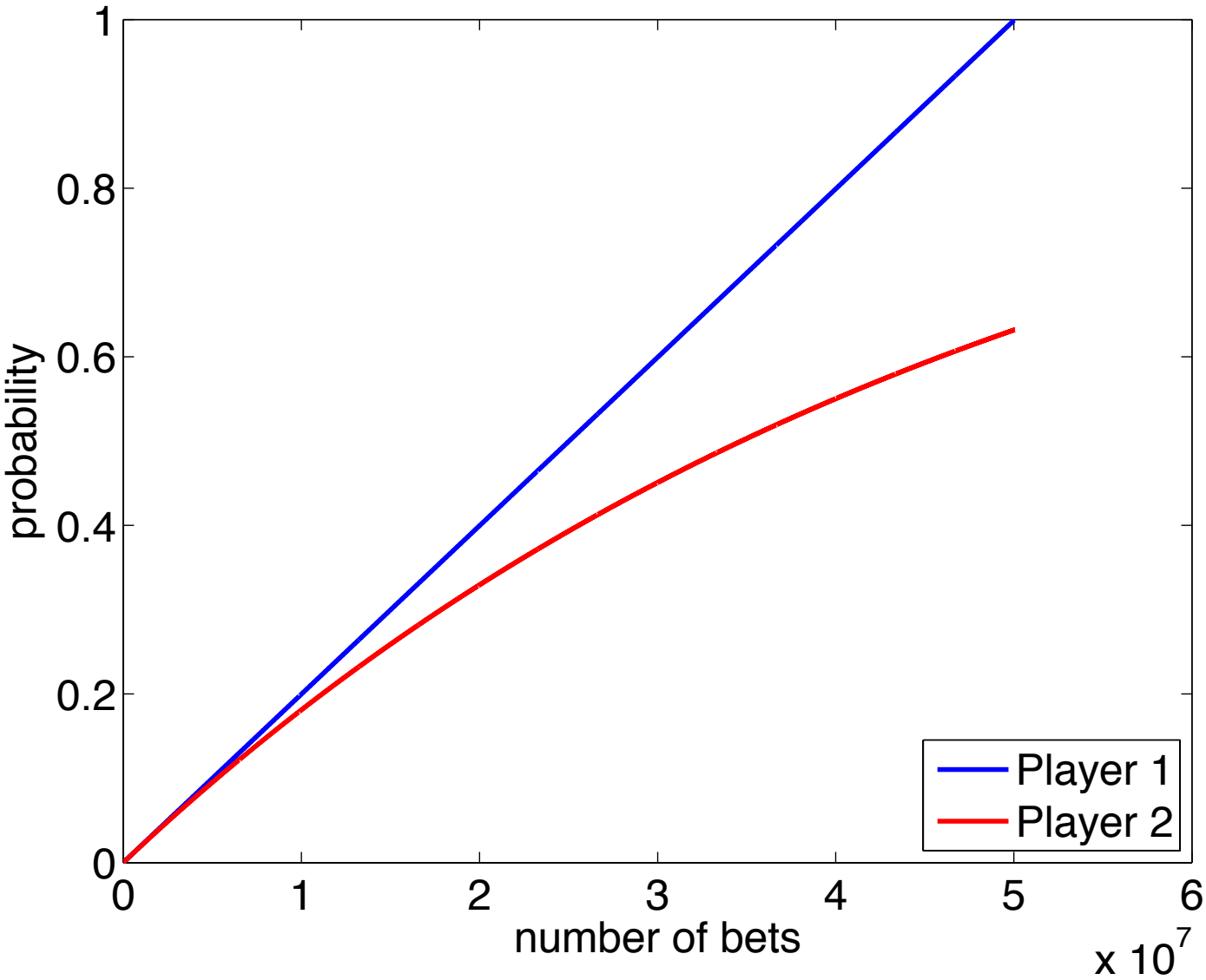
The first strategy is better!

$$\frac{n}{N} \geq 1 - \left(1 - \frac{1}{N}\right)^n$$

MegaSena.m

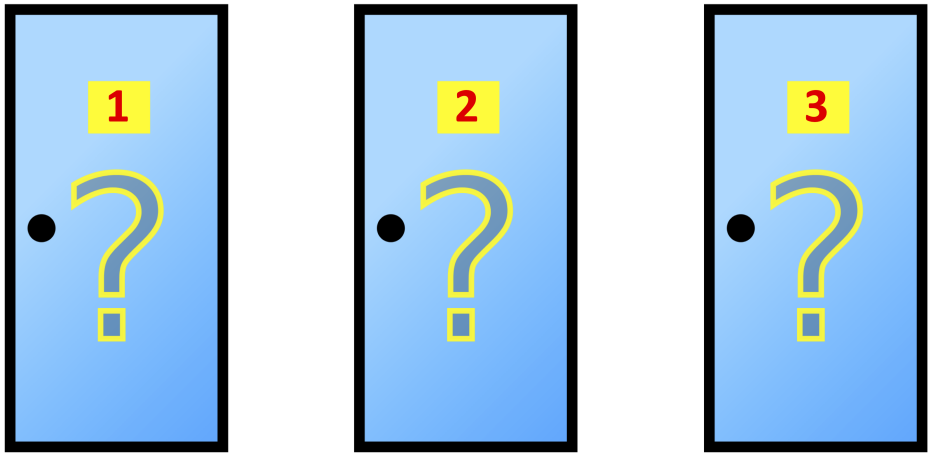
```
clc; clear; close all

% number of possible games
N = 50063860;

% number of bets time-series
n = 1:1:N;

% Player 1 probability
prob1 = n/N;

% Player 2 probability
prob2 = 1 - (1 - 1/N).^n;

% plot graphs
plot(n,prob1,'b',n,prob2,'r','LineWidth',2);
set(gca,'fontsize',18)
legend('Player 1','Player 2')
xlabel('number of bets')
ylabel('probability')
```

MegaSena.m

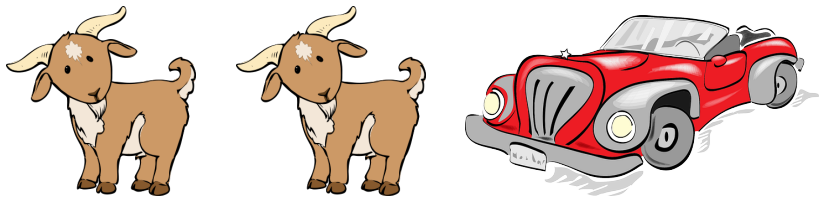

probability
number of bets
x 10^7
Player 1
Player 2
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
6

Monty Hall problem

Choose a port:



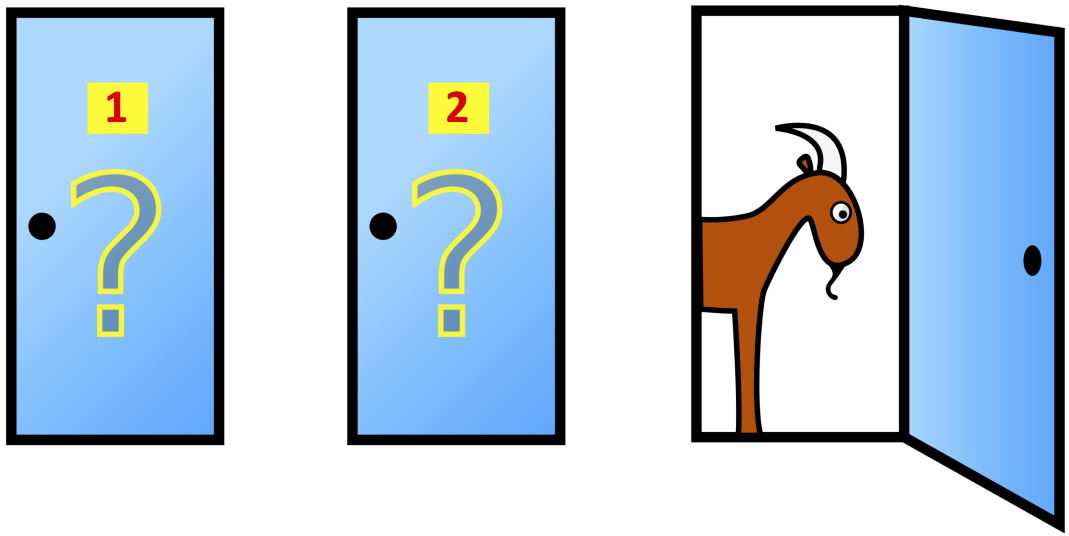
Behind the doors (2 goats and 1 car):



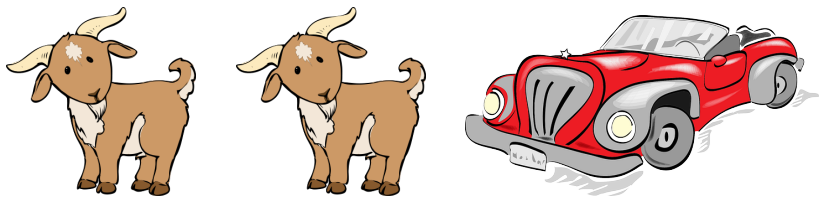
Monty Hall problem — Wikipedia, The Free Encyclopedia, 2021.

Monty Hall problem

Choose a port:



Behind the doors (2 goats and 1 car):



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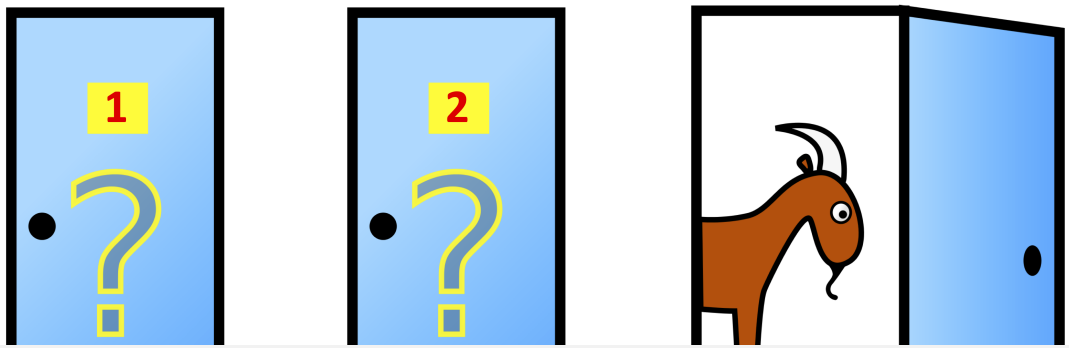
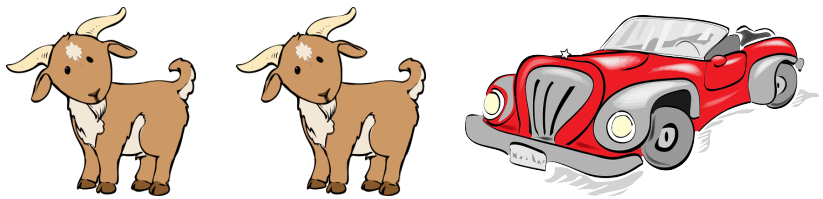
Monty Hall problem

Choose a port:



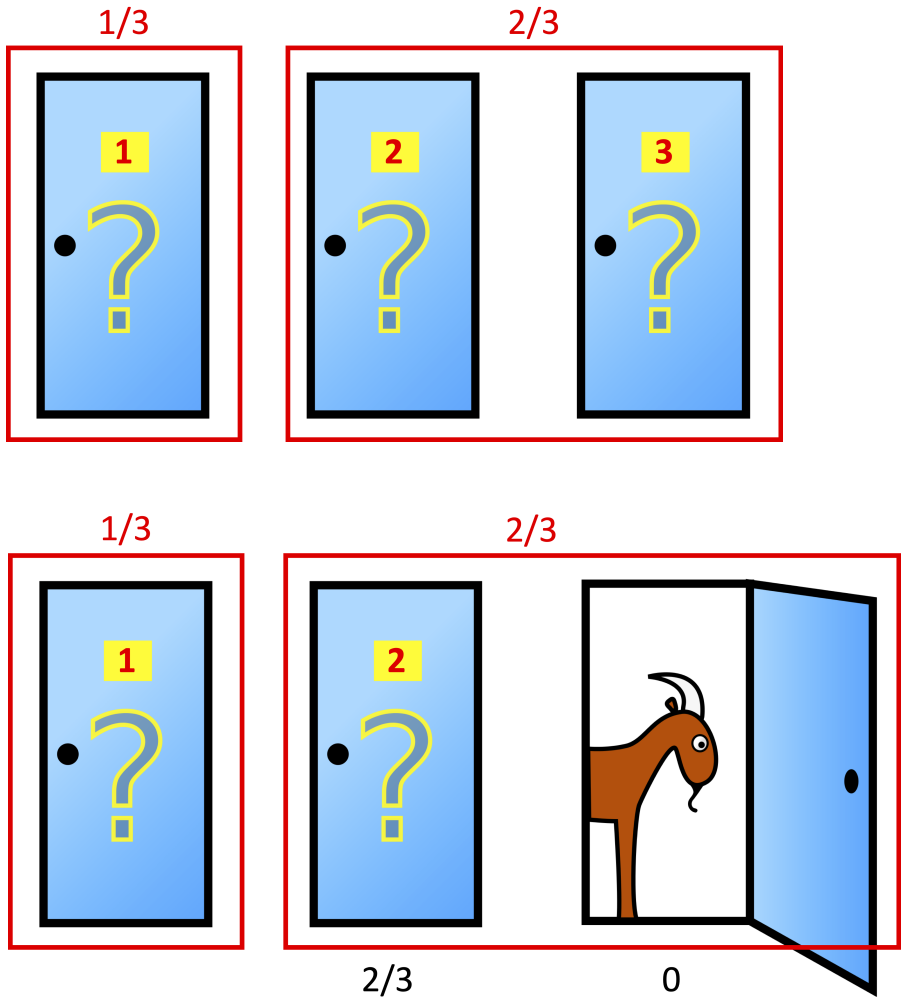
One goat is revealed!
Would you switch doors if allowed?

Behind the doors (2 goats and 1 car):



Monty Hall problem — Wikipedia, The Free Encyclopedia, 2021.

Switching ports is a smart strategy!



Monty Hall problem — Wikipedia, The Free Encyclopedia, 2021.

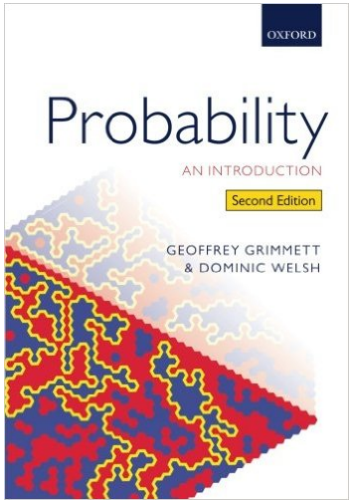
MontyHall.m

```
clc; clear; close all;
N = 100; d = 3; win=0; lose=0; Monty=0;
for i = 1:N
  % define the doors content
  Car = randi(d)-1; Choose = randi(d)-1;
  while (Monty == Car || Monty == Choose), Monty = randi(d)-1; end
  % check the result
  if Choose == Car, lose = lose + 1; else win = win + 1; end
  % display round results
  disp(['Round ',num2str(i)]); disp('Chosen door:');
  disp([zeros(1,Choose),1,zeros(1,(d-1)-Choose)]);
  if Car < Choose, disp('Monty opens:');
    disp([ones(1,Car),0,ones(1,Choose-Car-1),0,ones(1,(d-1)-Choose)]);
  end
  if Car > Choose, disp('Monty opens:');
    disp([ones(1,Choose),0,ones(1,Car-Choose-1),0,ones(1,(d-1)-Car)]);
  end
  if Car == Choose
    if Car < Monty, disp('Monty opens:');
        disp([ones(1,Car),0,ones(1,Monty-Car-1),0,ones(1,(d-1)-Monty)]);
    else disp('Monty opens:');
        disp([ones(1,Monty),0,ones(1,Car-Monty-1),0,ones(1,(d-1)-Car)]);
    end
  end
  disp('Car is behind:'); disp([zeros(1,Car),1,zeros(1,(d-1)-Car)]);
  if Choose==Car, disp('Switching loses'); else disp('Switching wins');end
  disp(' '); disp('   Win | Lose'); disp([win,lose]);
  disp('---------------------');
end
disp(['Won  ',num2str(win)]); disp(['Lost ',num2str(lose)]);
disp(['Estimated chance of winning after switch: ',num2str(win/N)]);
```

* Code adapted from https://www.mathworks.com/matlabcentral/fileexchange/26398-monty-hall

References

G. Grimmett and D. Welsh, **Probability: An Introduction**. Oxford University Press, 2 edition, 2014.



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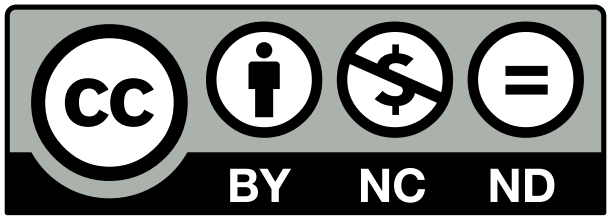

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