Simulation of Random Variables

Prof. Americo Cunha Jr

Rio de Janeiro State University – UERJ

americo.cunha@uerj.br

www.americocunha.org

@AmericoCunhaJr   @AmericoCunhaJr

@AmericoCunhaJr   @AmericoCunhaJr

Normal samples (iid) via Box–Muller transformation

```
1   clc; clear; close all;
2
3   Ns = 256; U1 = rand(Ns,1); U2 = rand(Ns,1);
4   R = sqrt(-2*log(U1)); THETA = 2*pi*U2;
5   Z1 = R.*cos(THETA); Z2 = R.*sin(THETA);
6
7   [Xbins1,Xfreq1] = randvar_pdf(Z1,round(sqrt(Ns)));
8   [Xbins2,Xfreq2] = randvar_pdf(Z2,round(sqrt(Ns)));
9
10  figure(1)
11  plot(Z1,Z2,'*b','LineWidth',3);
12  xlim([-4 4]); ylim([-4 4]);
13
14  figure(2)
15  bar(Xbins1,Xfreq1,1.0,'FaceColor','r');
16  hold on
17  bar(Xbins2,Xfreq2,1.0,'FaceColor','c','FaceAlpha',0.5);
18  plot(-4:0.05:4,normpdf(-4:0.05:4),'b','LineWidth',3);
19  hold off
20  xlim([-4 4]);
21
22  figure(3)
23  hist3([Z1,Z2],'FaceColor','m');
24  xlim([-4 4]); ylim([-4 4]);
```

# Normal samples (iid) via Box–Muller transformation
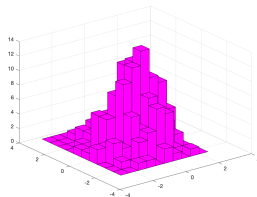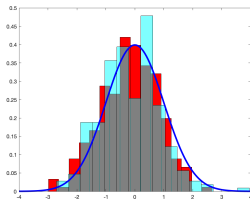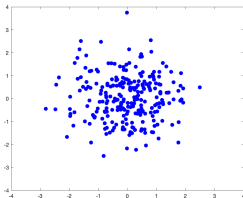
Gamma samples via inverse transform method

```
1   clc; clear; close all;
2
3   a = 7.5; b = 1.0; Nx = 500; Ns = 256; U = rand(Ns,1);
4
5   Xsupp = linspace(0.0,20.0,Nx);
6   Xcdf  = gamcdf(Xsupp,a,b);
7   Xsamp = interp1(Xcdf,Xsupp,U,'linear','extrap');
8   Xpdf  = gampdf(Xsupp,a,b);
9
10  [Xbins,Xfreq] = randvar_pdf(Xsamp,round(sqrt(Ns)));
11
12  figure(1)
13  bar(Xbins,Xfreq,1.0);
14  hold on
15  plot(Xsupp,Xpdf,'r','LineWidth',3)
16  hold off
```

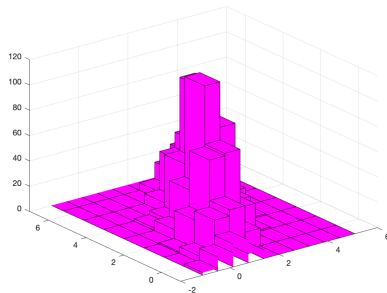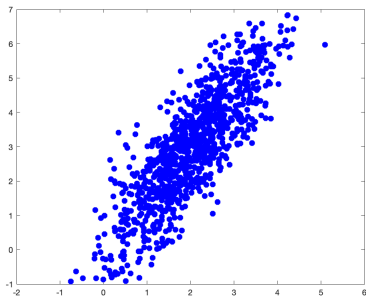# Gamma samples via inverse transform method
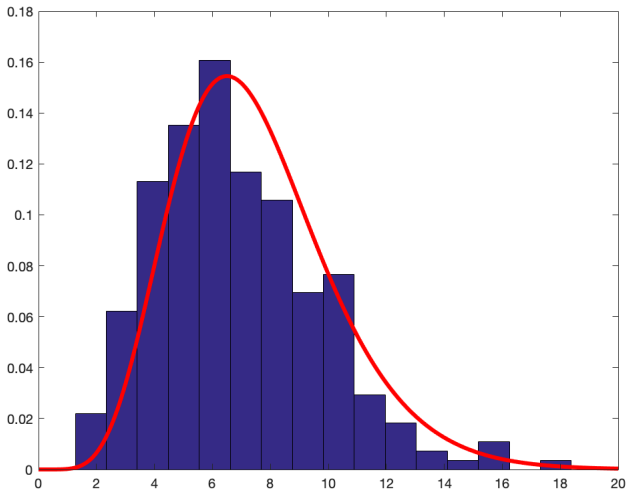
Correlated normal samples

```
1   clc; clear; close all;
2
3   Ns = 1024; mu = [2 3]; Sigma = [1.0 1.5; 1.5 3.0];
4   R = mvnrnd(mu,Sigma,Ns); Z1 = R(:,1); Z2 = R(:,2);
5
6   figure(1)
7   plot(Z1,Z2,'*b','LineWidth',3);
8   xlim([-2 6]); ylim([-1 7]);
9
10  figure(2)
11  hist3([R(:,1),R(:,2)],'FaceColor','m');
12  xlim([-2 6]); ylim([-1 7]);
```

Correlated normal samples

References

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