A Zoo of Computational Models

Tutorial 01

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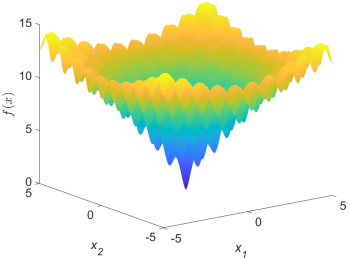
The codes for the tutorials are available on GitHub

https://github.com/americocunhajr/UQ-CSE

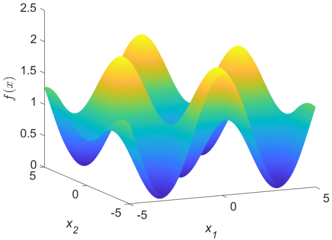
Ackley function



$$f(x) = -a \exp\left(-b\sqrt{\frac{1}{2}\sum_{i=1}^{2} x_i^2}\right) - \exp\left(\frac{1}{2}\sum_{i=1}^{2} cos(2\pi x_i)\right) + a + \exp(1)$$

* Tutorial elaborated by Marcos Vinícius Issa.

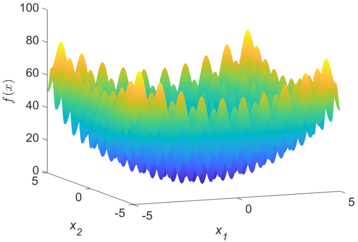
Griewank function



$$f(x) = \sum_{i=1}^{2} \frac{x_i^2}{4000} - \prod_{i=1}^{2} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

* Tutorial elaborated by Marcos Vinícius Issa.

Rastrigin function



$$f(x) = 20 + \sum_{i=1}^{2} \left(x_i^2 - 10cos(2\pi x_i) \right)$$

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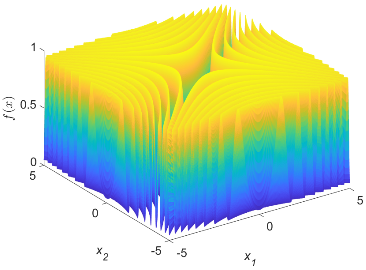
Schaffer function

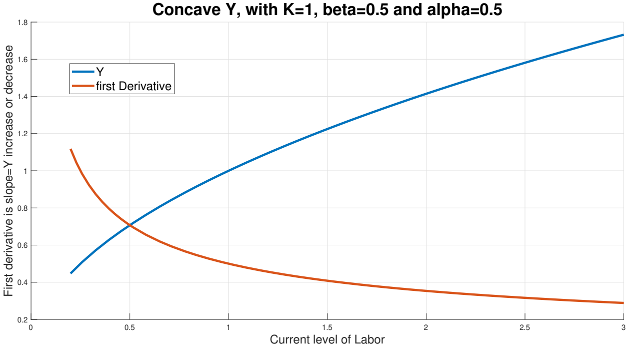


$$f(x) = 0.5 + \frac{sin^2(x_1^2 - x_2^2) - 0.5}{(1 + 0.001(x_1^2 - x_2^2))^2}$$

* Tutorial elaborated by Marcos Vinícius Issa.

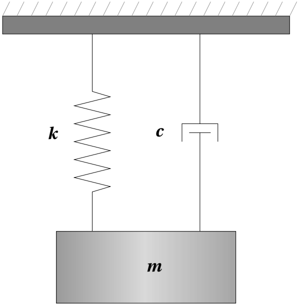
Cobb–Douglas production function

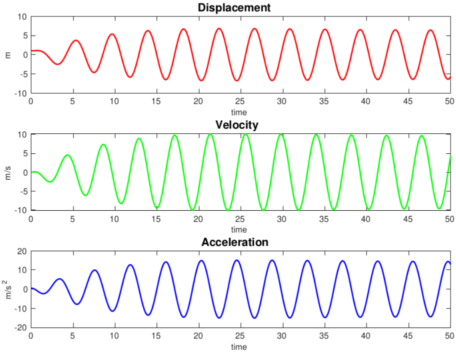
$$Y = L^{\beta} K^{\alpha}$$



* Tutorial elaborated by Bruna Pavlack.

Harmonic oscillator



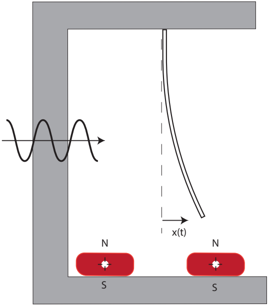


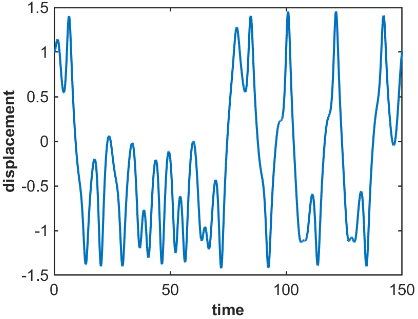
$$m\ddot{x} + c\dot{x} + kx = f\cos(\omega t)$$

+ initial conditions

* Tutorial elaborated by Diego Matos.

Duffing Oscillator



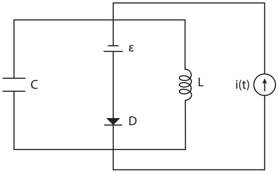


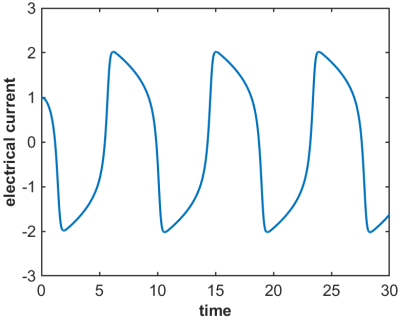
$$\ddot{x} + \delta\,\dot{x} + \alpha\,x + \beta\,x^3 = \gamma\,\cos(\Omega t)$$

$+$ initial conditions

* Tutorial elaborated by Joao Pedro Norenberg.

Van der Pol Oscillator





$$\ddot{x} - \mu(1 - x^2)\,\dot{x} + x = 0$$

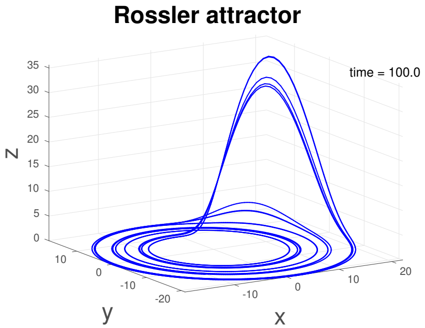
$+$ initial conditions

* Tutorial elaborated by Joao Pedro Norenberg.

Rössler system

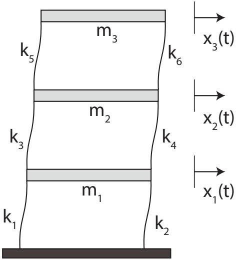
$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + \alpha y \\ \dot{z} &= \beta + z(x - \gamma) \end{aligned}$$

$+$ initial conditions



* Tutorial elaborated by Diego Matos.

Shear Building



Generalized eigenvalue problem:

$$[K]\,\mathbf{u}_n = \omega_n^2\,[M]\,\mathbf{u}_n$$

$$\omega_n = 2\,\pi\,f_n \qquad n \in \{1, 2, 3\}$$

Natural frequencies:

- $f_1 = 7.01$ Hz
- $f_2 = 55.00$ Hz
- $f_3 = 114.84$ Hz

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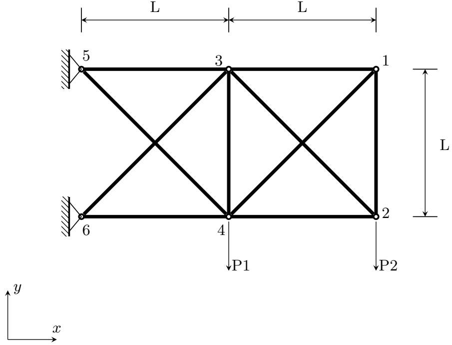
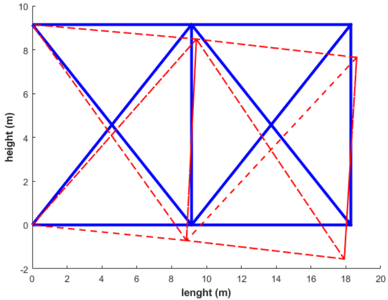
2D Truss





$$[K]\,\mathbf{u} = \mathbf{f}$$

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SIR-type temporal model

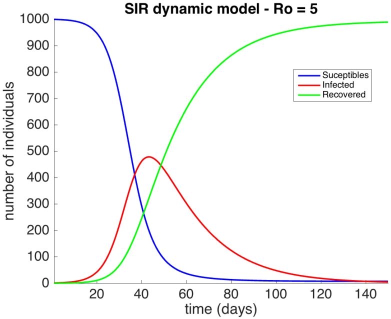
$$S \xrightarrow{\beta} I \xrightarrow{\gamma} R$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\beta \, \frac{I}{N} \, S$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \beta \, \frac{I}{N} \, S - \gamma \, I$$

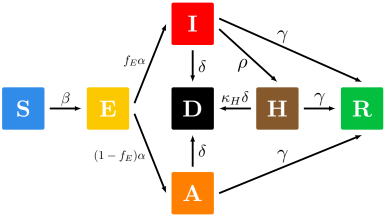
$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma \, I$$

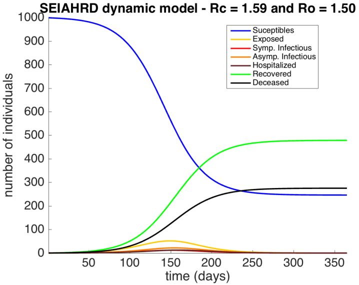
$+$ initial conditions



* Tutorial elaborated by Rachel Lucena.

SEIAHRD-type temporal model





$$\frac{dS}{dt} = -\beta\, S\, \frac{(I + A + H)}{N}$$

$$\frac{dE}{dt} = \beta\, S\, \frac{(I + A + H)}{N} - \alpha\, E$$

$$\frac{dI}{dt} = f_E\, \alpha\, E - (\gamma + \rho + \delta)\, I$$

$$\frac{dA}{dt} = (1 - f_E)\, \alpha\, E - (\gamma + \delta)\, A$$

$$\frac{dH}{dt} = \rho\, I - (\gamma + \kappa_H\, \delta)\, H$$

$$\frac{dR}{dt} = \gamma\, (I + A + H)$$

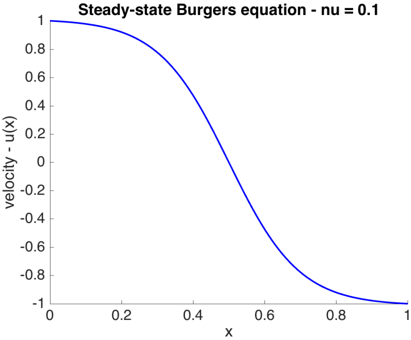
$$\frac{dD}{dt} = \delta\, (I + A + \kappa_H\, H)$$

$+$ initial conditions

* Tutorial elaborated by Rachel Lucena.

Steady-state Burgers equation

$$\nu \frac{\mathrm{d}^2 u}{\mathrm{d}x^2} - u \frac{\mathrm{d}u}{\mathrm{d}x} = 0 \, , \qquad 0 < x < 1$$
$$u = \;\; 1 \, , \qquad x = 0$$
$$u = -1 \, , \qquad x = 1$$



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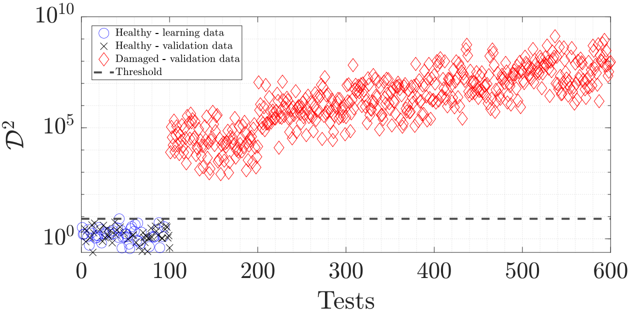
Data-driven damage detection

$$\mathcal{D} = \sqrt{(\mathbf{T}^i - \boldsymbol{\mu}_B)^t \boldsymbol{\Sigma}^{-1} (\mathbf{T}^i - \boldsymbol{\mu}_B)}$$



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