Honours Degree of Bachelor of Science in Information Technology
Batch 17 - Level 4 (Semester II)
CM 4110: Advanced Topics in Mathematics

Assignment 2 - Group Theory

<u>**NO LATE ASSIGNMENTS**</u> **will be accepted under any circumstances.**

<u>**INSTRUCTIONS**</u>

- **Submit on or before 16$^{\text{th}}$ January, 2022**.

- Answer **ALL** questions.

- All the necessary steps for the answers should be clearly indicated.

- **Only PDF (.pdf)** format files can be submitted through the MOODLE, and there is no page limit.

- Suppose your index number is 174106T.
  Then you can name your files as ***CM*4110_*174106T.pdf***.

- Copy the figures and paste them in to your document to answer the questions.

- Submissions by anyone other than corresponding student will not be accepted.

- **Additional time will not be given** to upload the file. It is the students' responsibility to upload the answer file before the expiration of the link.

- **Strictly no plagiarism. Any evidence of plagiarism will set the mark to zero.**

Continued...

(i) Show that a necessary and sufficient condition that a non-empty subset H of a group G to be a subgroup is $a, b \in H \Rightarrow ab^{-1} \in H$.

(ii) Let a be any element of a group G. The subset

$$C(a) = \{x \in G \mid xa = ax\}$$

is called *centralizer* of a in G.
Prove that $C(a)$ is a subgroup of G.

(iii) Show that

(a) $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{R}, \ ad - bc \neq 0 \right\}$ is a group under matrix multiplication.

(b) $H = \left\{ \begin{pmatrix} \cos(\theta) & -2\sin(\theta) \\ \frac{1}{2}\sin(\theta) & \cos(\theta) \end{pmatrix} \mid \theta \in \mathbb{R} \right\}$ is a subgroup of G.

Compute the centralizer in G of the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$.