Application of PWYW Pricing in Film Industry

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Abstract

Film industry has become one of the most valuable entertainment industries and is still expanding rapidly under the fast development of mobile internet network and time of popular culture. However, the current fixed price pricing strategy has great limitation that block the growth of movies' both profit and social effect. We bring out a new pricing model, considering the complex realistic features of film industry, introducing the PWYW strategy into film pricing. Though both theoretical analysis and random simulation, our model is proved to have better performance than common model in current market.

1 Introduction

1.1 Film industry

Film industry has become one of the most valuable entertainment industries and is still expanding rapidly under the fast development of mobile internet network and time of popular culture. However, currently, the price of a movie at the most time is fixed for everyone and every screening. Moreover, the price is usually expensive according to audience's respond. High price results in low occupancy, making cinemas raise price again to guarantee the profit. That becomes a vicious cycle. Thus, a better model of pricing is needed to solve the problem.

1.2 Pay What You Want

In this paper, we introduce Pay-What-You-Want (PWYW) pricing strategy, bringing out a new film pricing model in order to improve profit as well as attract more audience.

Pay-What-You-Want (PWYW) pricing is a pricing strategy where buyers pay their desired amount for a given commodity instead of some fixed prices or prices set by others. [1]Sometimes the payment can be zero, while sometimes a minimum price may be set. A successful application of PWYW is that in 2007, the rock band Radiohead sell their new album In Rainbows with this strategy. Consumers can buy the album with any price they want, from 1 pound to 100 pounds or even more. The result is that both sales volume and the sales multiplied several times.[2]

This result seems counter-intuitive, for the sales also increase as well as sales volume. The explanation is that PWYW subdivide customers and apply price discrimination on precisely every consumer. The infinite continuous pricing holds almost all target customers, which greatly expands its customer groups and raises the influence of the product. Since the margin cost is low enough, there will not be large deficit even if most customers pay the lowest price. In fact, setting an appropriate minimum price, with large consumer base, those majors paying low prices still contribute considerable profit, while sales contributed by those minors who are willing to pay a high price is also sizable.

In reality, PWYW is currently getting more and more popular in all kinds of applications, since the speed and impact of network's information transmission has been quicken. Right now, almost all streaming platforms are using this pricing strategy. In particular, PWYW strategy fits those goods with low marginal cost compared with high marginal profit in particular, since low minimum price can be low enough with low marginal cost. It also fits goods that have fans economy or value varies from one to another consumer, for that raises possibility that some consumers pay very high price. Thus, PWYW pricing strategy is fairly suitable for film industry.

1.3 Our work

Our research explores the potential application of PWYW pricing strategy in film industry. Based on actual situations of the film market, we set some fundamental assumptions and build a mathematical model of film pricing. We do strict mathematical derivation to get a general insight on this problem. After that, through random data simulations using computer program, we verified out model and mathematical derivation, get results of our model, compare with currently widely used model and finally draw conclusions. Our model is proved to have better performance than the fixed price model commonly used in current market.

2 Fundamental Assumptions and Main Model

To make out model close to the reality, we have to consider some features of movies. For example, former audience's evaluation influence later audience numbers and valuations. The cost is related to the number of screening rooms but not audience number. The audience number of the early screenings are influenced by the hype and are usually higher.

In our setting, there is one movie that will be on show twice, representing two main stages of a movie's schedule. Each time the screening has infinite capacity. The number of audience in the first showing will have a positive effect on the total number of potential customers for the second showing.

The cost function of the movie will be dependent on the number of audiences, and should be some kind of step function:

$$c(x) = [\frac{x}{c_0}] \times C$$

here x is the number of audiences, C is some constant representing how much each screening room would cost. c_0 is an integer representing the number of audiences one screening room can hold.

Every potential customer i will have a valuation v_i, which is obedient to some kind of random distribution. In practice, it should be some heavy-tailed distribution. [3][4] v_i will also be affected by the kind of the movie. It could also be different from screening to screening.

The movie theater have two pricing strategy:

Fixed price:

This is the traditional pricing strategy. The theater sets a fixed price p_f, which is only dependent on the number of screenings of the movie. (For example, in the first showing, the payment will be 25$ and in the second showing it will be 15$) The theater will charge everyone that comes to the movie with p_{f}.

Pay What You Want(PWYW):

First, the theater sets a baseline price p_b. The audiences can decide their payments, as long as the payment is greater than p_b.

For any customer, if the movie's price is below their valuation, i.e. p_f or $p_b < v_i$, there will be a certain probability for the customer to go to the movie. This probability can be affected by the number of audiences from the previous showing.

If a customer goes to a movie, and the movie's pricing is PWYW, then the customer's payment will be different from traditional model. In a traditional setting, since the customers can decide their payment, they will pay p_b in order to maximize their utility. But in our setting, there will be a certain probability for the customer to decide to pay their actual valuation v_i.

There are two goals for us. The first is to maximize the movie theater's utility, generally:

$$u = \sum_i p_i - c$$

where p_i represents the payment of each audience, and c represents the cost. The second goal is to maximize social welfare without reducing the cinema's utility if the cinema is going to switch from fixed pricing to PWYW pricing.

3 Detailed Model and Technical Approach

Based on the main model, some details need to be confirmed to use technical approach to simulate the model and get the results. Simulations are essential because of the limitation of mathematical expressions, as we have to use complex measuring valuations to make our model closer to the reality.

3.1 Way of measuring valuation

We use lognormal distribution to measure the valuation of customers.

Why lognormal distribution?

In real life, most people are only willing to pay a small amount of money for the movie, but very few people are willing to pay a very high price. There are a lot of anecdotes where fans of a movie will go to the theater again and again to contribute more box office to the movie. So the distribution of the customers' valuations should be some sort of distribution with a long 'tail', where customers with very high valuations lie. [3][4]

So we decide to use lognormal distribution to measure the valuation of people for the movie.

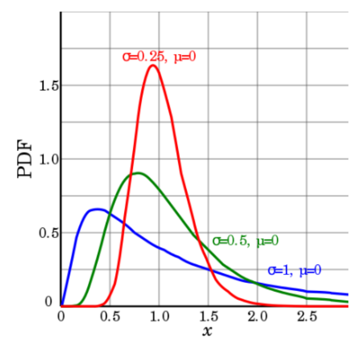

Figure 1: PDF of Lognormal Distribution

As shown in Figure 1, PDF of lognormal distribution has a peak when x is small. Also, it has a long tail which is very close to our description mentioned in the first paragraph.

What is lognormal distribution?

In probability theory, a log-normal (or lognormal) distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable X is log-normally distributed, then Y = ln(X) has a normal distribution. Likewise, if Y has a normal distribution, then the exponential function of Y, X = exp(Y), has a log-normal distribution. A random variable which is log-normally distributed takes only positive real values. The distribution is occasionally referred to as the Galton distribution or Galton's distribution, named after Francis Galton. The log-normal distribution also has been associated with other names, such as McAlister, Gibrat and Cobb– Douglas.[5]

The attributions of lognormal distribution

Notation	$Lognormal(\mu,\, \sigma^2)$
Parameters	$\mu \in (-\infty, +\infty)$
Support	$x \in (0, +\infty)$
PDF	$\frac{1}{x\sigma\sqrt{2\pi}}\ \exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right)$
CDF	$\frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left[\frac{\ln x-\mu}{\sqrt{2}\sigma}\right]$
Mean	$\exp\left(\mu+\frac{\sigma^2}{2}\right)$
Median	$\exp\left(\mu+\frac{\sigma^2}{2}\right)$
Variance	$[\exp(\sigma^2)-1]\exp(2\mu+\sigma^2)$
Fisher Information	$\begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 1/2\sigma^4 \end{pmatrix}$

3.2 How do we program?

Input: pricing strategy.
Output: utility of theater.
Processing:

1. Simulate the lognormal distribution.
2. Generate the initial valuation for potential customers for the first showing.
3. Calculate the total utility of the theater in the first showing.
4. Generate the valuations for potential customers for the second showing according to the results from first showing.
5. Simulate the second showing.

The detailed description will be shown in the section Simulation Results.

4 Theoretical Analysis

With clear model, we can get the mathematical expression of the model. Though because of the limitation of mathematical expression, it is impossible to get certain answer in expression of the real model, we can still derive general expressions, and use simple distributions to verify the model.

4.1 Fixed Price

First let's consider the fixed price scenario. In this case, the movie theater uses fixed price payment and charge everyone with price p_1 for the first showing and p_2 for the second showing.

Let the total number of people in the first stage be N_0, their probability of going to the movie if the price is lower than their valuation is p_{g1}, and the probability density function of the distribution of valuations be $f(x)$, the CDF of the distribution be $F(x) = \int_{-\infty}^{\infty} f(x)\mathrm{d}x$. Let the price of the movie be p_1. Then, the expected number of people that will go the first showing of the movie will be:

$$N_1 = (1 - F(p_1))p_{g1}N_0$$

Then, since the number of audience of the first showing will have some impact on the second showing, we can describe this effect with some function $g : N_1 \to N_2'$, and the total number of people in the second showing can be expressed as

$$N_2 = N_2' + N_1 = g(n_1) + N_1$$

So, the expected number of audience of the second showing is:

$$N_2 = (1 - F(p_2))p_{g2}N_2$$

where p_{g2} is one's probability of going to the movie if the price is lower than their valuation.

The cost function, if we were to use our original definition in the model section, will be a step function, which is not good for our analysis where everything is arbitrary. So instead we use a linear function to represent the cost:

$$c = \frac{(n_1 + n_2)}{c_0}$$

As for the payment, the expected payment for an audience is simply p_1 and p_2 for these two showings. So the total utility of the theater is:

$$\begin{aligned} u &= n_1p_1 + n_2p_2 - c \\ &= p_{g1}(1 - F(p_1))N_0p_1 + [g(n_1) + N_1]p_{g2}[1 - F(p_2)]p_2 \\ &- \frac{(n_1 + n_2)}{c_0} \end{aligned}$$

Where $n_1 = (1 - F(p_1))p_{g1}N_0$. We didn't consider the cost function.

A good method of analysis in game theory is to start from a simple case were we set all distribution to a uniform distribution. By analyzing the problem in its simplest form, we can get an overview of the problem. So here we set the distribution of valuations to be a uniform distribution from 0 to 1. So $v \sim U(0, 1)$. Also we set $g(x) = kx$, so each audience in the first showing will bring k additional potential customer to the second showing. After this, the utility will be:

$$\begin{aligned} u &= p_{g1}(1 - p_1)N_0p_1 + (k(1 - p_1)p_{g1}N_0 + N_1)p_{g2}(1 - p_2)p_2 \\ &- \frac{(1 - p_1)p_{g1}N_0 + (1 - p_2)p_{g2}N_2}{c_0} \end{aligned}$$

To find the optimal price to maximize the theater's utility, first we can take the derivative of this with respect to p_1 and p_2 separately and find all the stationary points.

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}p_1} &= p_{g1}N_0(1 - 2p_1) - kp_{g1}N_0p_{g2}(1 - p_2)p_2 \\ &+ \frac{p_{g1}N_0 + (1 - p_2)p_{g2}kp_{g1}N_0}{c_0} \end{aligned}$$

Solving $\frac{\mathrm{d}u}{\mathrm{d}p_1} = 0$ gives us:

$$p_1 = \frac{1}{2} + \frac{(1 - c_0p_2)(1 - p_2)p_{g2}k + 1}{2c_0}$$

We can do the same thing with p_2. We have:

$$\frac{\mathrm{d}u}{\mathrm{d}p_2} = [N_2]p_{g_2}(1-2p_2) + \frac{p_{g2}N_2}{c_0}$$

And $\frac{\mathrm{d}u}{\mathrm{d}p_2} = 0$ gives us

$$p_2 = \frac{1}{2} + \frac{1}{2c_0}$$

Now if we back substitute p_2 into the expression of p_1, we can get the full expression of p_1.

4.2 PWYW

Now we can move on to analyze the case where PWYW pricing strategy is used.

If PWYW is used, then there are two possible ways to optimize this situation.

1. Find a better pricing so that the utility of the theater is optimized.
2. On condition that the utility not falling, maximize the number of audience a.k.a the social effect.

But either way, we can derive the expression of the utility first. The expected number of audience is still the same as before, with:

$$\begin{cases} n_1 = (1-F(p_1))p_{g1}N_0 \\ n_2 = (1-F(p_2))p_{g2}[g(n_1)+N_1] \end{cases}$$

But the expected payment for a audience is different. In our model, we defined p_{pay} as the probability of a audience deciding to pay his actual valuation instead of the baseline price. In more advanced model, this probability could be different from audience to audience, or could be related to the audience's actual valuation. We may look further into the issue in our simulation section, but for now let's view this as a constant. The expected payment for a audience is given by:

$$\overline{p_1} = (1-p_{pay})p_1 + p_{pay}\int_{p_1}^{\infty} f(x)\cdot x\mathrm{d}x$$

For the second showing, we have:

$$\overline{p_2} = (1-p_{pay})p_2 + p_{pay}\int_{p_2}^{\infty} f(x)\cdot x\mathrm{d}x$$

So the utility of the theater is given by:

$$\begin{aligned} u &= n_1\overline{p_1} + n_2\overline{p_2} - c \\ &= (1-F(p_1))p_{g1}N_0[(1-p_{pay})p_1 + p_{pay}\int_{p_1}^{\infty} f(x)\cdot x\mathrm{d}x] \\ &+ (1-F(p_2))p_{g2}[g(n_1)+N_1] \\ &\cdot [(1-p_{pay})p_2 + p_{pay}\int_{p_2}^{\infty} f(x)\cdot x\mathrm{d}x] \\ &- \frac{(n_1+n_2)}{c_0} \end{aligned}$$

Then like before, if we need to go further, we can assume the distribution of the valuations is uniform between 0 and 1, and set $g(x) = kx$ so each audience in the first showing will bring k additional potential customer to the second showing. In this case, the utility is given by:

$$\begin{aligned} u &= (1-p_1)p_{g1}N_0[p_1 - p_1p_{pay} + \frac{p_{pay}}{2} - \frac{p_{pay}}{2}p_1^2] \\ &+ (1-p_2)p_{g2}[k(1-p_1)p_{g1}N_0 + N_1] \\ &\cdot [p_2 - p_2p_{pay} + \frac{p_{pay}}{2} - \frac{p_{pay}}{2}p_2^2] \\ &- \frac{(1-p_1)p_{g1}N_0 + (1-p_2)p_{g2}[k(1-p_1)p_{g1}N_0 + N_1]}{c_0} \end{aligned}$$

First we can find the optimal price to maximize the theater's utility. Like before, we need to take partial derivative with respect to p_1 and p_2.

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}p_1} &= p_{g1}N_0[\frac{3}{2}p_{pay}p_1^2 + (p_{pay}-2)p_1 + 1 - \frac{3}{2}p_{pay}] \\ &- \overline{p_2}(1-p_2)p_{g2}kp_{g1}N_0 + \frac{p_{g1}N_0}{c_0}[1-k(1-p_2)p_{g2}] \end{aligned}$$

Then the stationary points is given by the quadratic equation:

$$\frac{3}{2}p_{pay}p_1^2 + (p_{pay}-2)p_1 + 1 - \frac{3}{2}p_pay + \frac{1}{c_0} - (\frac{1}{c_0} + \overline{p_2})(1-p_2)p_{g2}k = 0$$

Here $\overline{p_2}$ is the expected payment for an audience, as defined before. We can do the same thing to p_2.

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}p_2} &= p_{g2}N_2[\frac{3}{2}p_{pay}p_2^2 + (p_{pay}-2)p_2 + 1 - \frac{3}{2}p_{pay}] \\ &+ \frac{p_{g2}N_2}{c_0} \end{aligned}$$

Then the stationary points is given by the quadratic equation:

$$\frac{3}{2}p_{pay}p_2^2 + (p_{pay}-2)p_2 + 1 - \frac{3}{2}p_{pay} + \frac{1}{c_0} = 0$$

Solving for p_2 and back substitute it into p_1 will give us desired result.

5 Simulation and Results

In our simulation, after parameters' adjustment, we set the probability k of customers who pay their true valuation to $k = 0.5$. Note that this is based on long tailed distribution of the valuation, which means most people's true valuation are already low.

The potential customer of the first screening is 500 and that of the second is 2000. The public rating, which means the rating given by the people watching movie in the first stage, how many additional people will go to see the movie in the second stage is 0.7.

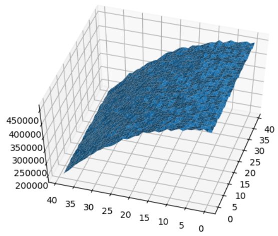

Figure 2: Using PWYW with uniform distribution

5.1 Uniform Distribution

To verify our model, we first simulated with the model using PWYW strategy with uniform distribution.

As shown in Figure 2, when the lowest price paid by the people in the first stage is 9 and the lowest price paid by the people in the second stage is 2, you can get the max net profit which is 46484. The result graph seems to be the increasing half of a parabola. The result data fits our theoretical analysis very well.

5.2 Long-tail Distribution

Now we simulate the long tail distribution valuation.

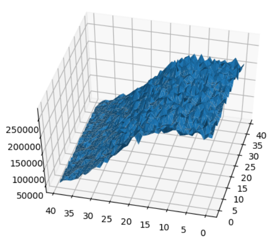

Figure 3: Using PWYW with long tail distribution

We first simulate the situation with linear cost function, as stated in the Theoretical Analysis part. As shown in Figure 3, the result is very similar to that of stage cost function, which is shown below, except that the graph is more smoothness.

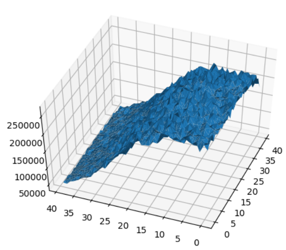

Figure 4: Using PWYW with long tail distribution

Figure 4 shows the simulation result of PWYW and long tail distribution, setting the cost function to more realistic stage function. From the graph and data, when we set the base price in the first screening to 19 and the base price in the second screening to 10, we can get the maximum net profit which is 272754.

The general trend is that the lower the baseline price, the higher net profit you will get. But if your baseline prices are too low, your net profit will go down. That also verify out general mathematical induction.

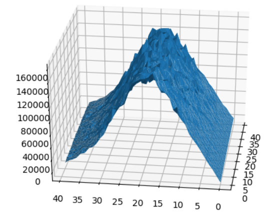

Figure 5: Using PWYW with long tail distribution

To compare, Figure 5 shows the simulation result of Fixed Price and long tail distribution, which represents the current market situation. We can see that as baseline prices rise, the net revenue will rise and drop thereafter like a parabola. When the base price in the first screening is 5 and the base price in the second screening is 19, we get the max net profit 175622 ($175622 < 272754$). Thus, with PWYW, the profit increased by 55.3%.

5.3 Maximum Social Effect

As stated above, another way using PWYW strategy is that on condition that the utility not falling, maximize the number of audience, a.k.a the social effect.

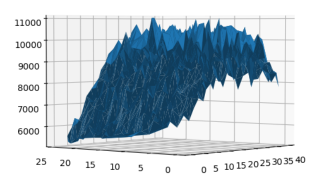

Figure 6: Using PWYW with long tail distribution

Figure 6 shows this situation. Z axis depicts audience number. The result shows that if we set the first screening baseline to 1 and second screening baseline to 6, we get the most people (11024), with the profit also higher than with fixed price strategy.

6 Conclusion

Our simulations verified our model and theory, showing that the PWYW film pricing model is better than currently widely used fixed price model in both utility and social effect. We also find that even in fixed price model, blindly raising the price of the ticket may not be a wise choice - on the contrary, reducing the price properly brings more profit and social effect.

In summary, we did a lot of market investigation and desktop research, considering all kinds of aspects, those full preparation laid our research a solid foundation. We abstract out two stages of film screening, and take the influence from the previous audience and audience customer psychology into consideration to make out model realistic. We introduce PWYW into film pricing, using both theoretical analysis and random simulation and prove our model gain better result than common model in current market.

7 Acknowledgement

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