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ASSIGNMENT

[Parameter Estimation]

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Q1 - Let $(X_1, X_2, \ldots)$ be a random sample of size n taken from a Normal Population with Parameters:

mean = θ_1
Variance = θ_2

Find the maximum likelihood estimates of these 2 parameters.

Ans 1-

$$L(\theta_1, \theta_2 | X_1, X_2, X_3 \ldots X_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Taking Log Likelihood

$$\ln L(\theta_1, \theta_2 | X_1, X_2, X_3 \ldots X_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Maximum likelihood estimation for θ_1

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2 | X_1 \ldots X_n) \Rightarrow \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (x_i) - n\theta_1 = 0$$

$$\Rightarrow \theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\Rightarrow \bar{x}$$

Maximum likelihood estimation for θ_2:

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 \mid x_1 \ldots x_n) \Rightarrow \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\Rightarrow -n + \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\Rightarrow \theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\Rightarrow s^2$$

Ans:

Maximum likelihood estimation of θ_1 = $\bar{x}$ (mean)
Maximum likelihood estimation of θ_2 = s^2 (variance)

Q2 → Let $X_1, X_2 \ldots X_n$ be a random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown and 'm' is also known positive integer. Compute value of θ using the MLE.

Ans 2 →

$$L(\theta \mid x_1, x_2 \ldots x_n) = \sum_{i=1}^{n} \left[\ln \binom{m}{x_i} + x_i \ln(\theta) + (m - x_i) \ln(1-\theta) \right]$$

Maximum likelihood estimation for θ

$$\frac{\partial}{\partial \theta} \ln L(\theta \mid x_1, x_2 \ldots x_n) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right] \Rightarrow 0$$

$$\Rightarrow \sum_{i=1}^{n} \left[\frac{x(1-\theta) - \theta(m - x_i)}{\theta(1-\theta)} \right] = 0$$

$$\Rightarrow \hat{\theta} = \frac{\left(\sum_{i=1}^{n} x_i \right)}{(mn + \sum_{i=1}^{n} x_i)}$$

$$\left[\text{Let } \sum_{i=1}^{n} x_i = s \right]$$

↓ Success cases

Ans:

$$\therefore \hat{\theta} = \frac{s}{nm + s}$$