Hyperbolic Legal Network Embedding in $\mathbb{H}^3$

Poincaré Ball Model (B^3): Hierarchical Case Law Representation | Supreme Court → High Court Precedents

Hyperbolic Geometry Formulation

Domain: $B^3 = \{x \in \mathbb{R}^3 : ||x|| < 1\}$
Metric: $ds^2 = 4 \cdot (dx^2 + dy^2 + dz^2)/(1 - r^2)^2$
Distance:
$d_B(x,y) = \text{arcosh}(1 + 2||x-y||^2/((1-||x||^2)(1-||y||^2)))$
Geodesics: Circular arcs $\perp$ to ∂B^3
Curvature: $K = -1$ (constant negative)

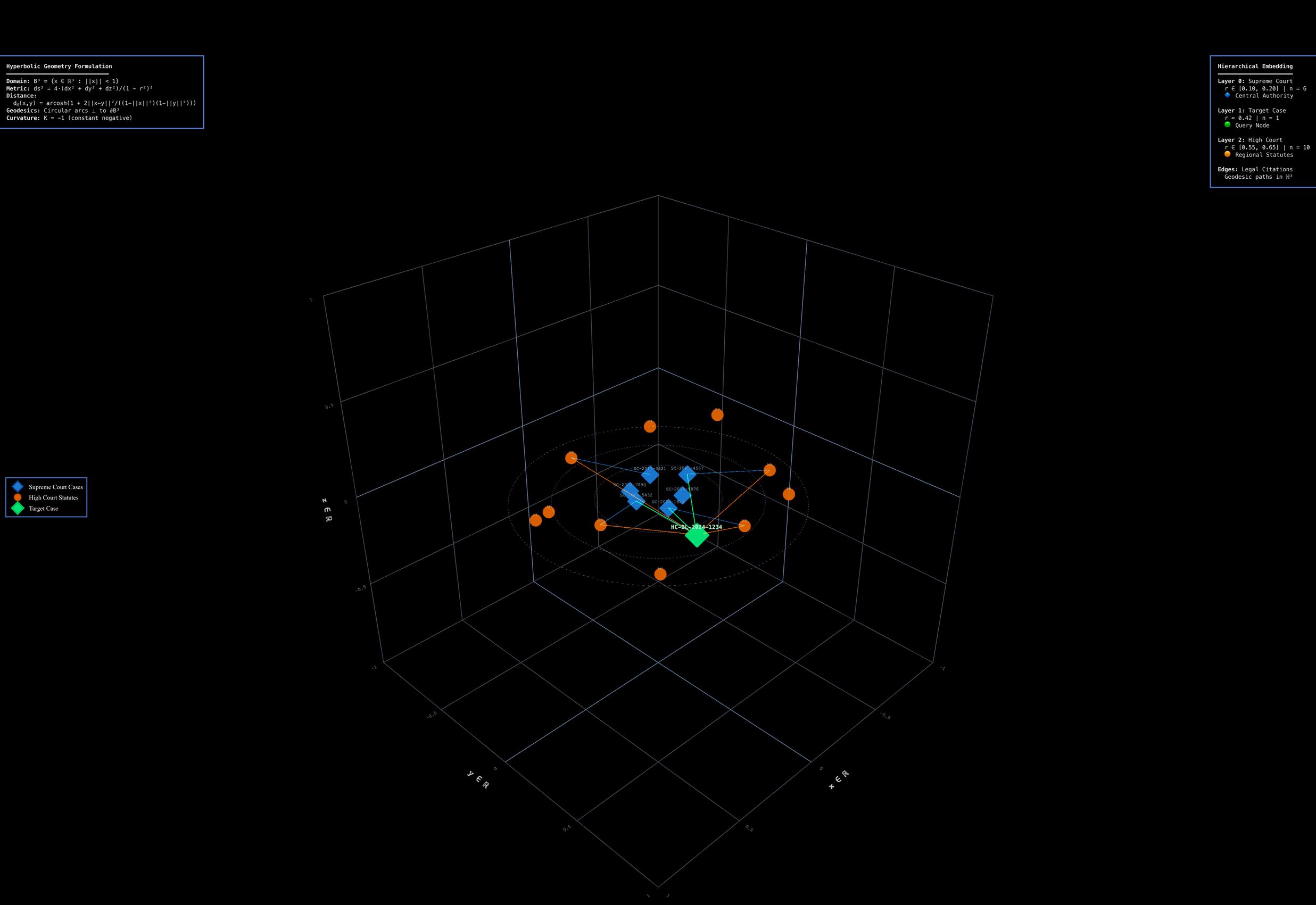
Hierarchical Embedding

Layer 0: Supreme Court
$r \in [0.10, 0.20]$ | n = 6
Central Authority

Layer 1: Target Case
$r \approx 0.42$ | n = 1
Query Node

Layer 2: High Court
$r \in [0.55, 0.65]$ | n = 10
Regional Statutes

Edges: Legal Citations
Geodesic paths in $\mathbb{H}^3$



Graph Statistics: |V| = 17 nodes, |E| = 14 geodesic edges | **Embedding:** HGCN (Hyperbolic GCN) | **Dimension:** d = 3 | **Model:** Poincaré Ball $B^3 \subset \mathbb{R}^3$