

LegalNexus: A Multi-Agent Hyperbolic Framework for Legal Reasoning and Retrieval

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Abstract—Legal information retrieval faces unique challenges due to the strict hierarchical nature of judicial systems and the complex, often contradictory, web of citations. Traditional Euclidean embeddings fail to capture the tree-like authority structure of courts, while standard retrieval systems lack the reasoning capabilities to resolve conflicting precedents. In this paper, we present *LegalNexus*, a comprehensive platform integrating six novel contributions: (1) Hyperbolic Legal Networks (HGCN), which embed 49,633 cases into a Poincaré ball to explicitly encode court authority in the radial dimension, justified by Gromov δ -hyperbolicity analysis; (2) A Game-Theoretic Multi-Agent Swarm, utilizing Nash Equilibrium to coordinate Linker, Interpreter, and Conflict agents in constructing a consistent knowledge graph; (3) An Adversarial Hybrid Retrieval System, which combines semantic, structural, and citation-based search with a Prosecutor-Defense-Judge agent simulation; (4) A Toulmin Argumentation Framework for extracting structured legal arguments and enabling argument chain traversal; (5) Temporal Scoring with precedent decay and resurrection mechanisms; and (6) A Counterfactual “What-If” Engine for identifying legal pivot points. Our experiments demonstrate that HGCN embeddings naturally cluster Supreme Court cases near the origin (radius < 0.10), achieving 0.92 precision@5, while the multi-agent swarm resolves 94% of citation conflicts through iterative debate.

Index Terms—Hyperbolic Embeddings, Multi-Agent Systems, Nash Equilibrium, Legal Tech, Graph Neural Networks, Toulmin Argumentation, Counterfactual Analysis

I. INTRODUCTION

The legal domain is characterized by two fundamental structures: *hierarchy* and *argumentation*. Judicial authority flows downwards from Supreme Courts to lower tribunals, creating a tree-like citation network best represented in hyperbolic space. Simultaneously, legal reasoning involves adversarial argumentation, where contradictions (e.g., overruling judgments) must be resolved to establish the current state of law.

Existing solutions often rely on flat Euclidean embeddings (e.g., BERT) which distort hierarchical relationships, or static knowledge graphs that cannot dynamically resolve conflicts. We propose *LegalNexus*, a unified frame-

work that addresses these limitations through geometric deep learning and game-theoretic agent coordination.

The contributions of this work are sixfold: (1) We introduce Hyperbolic Graph Convolutional Networks (HGCN) for legal case embeddings, demonstrating that hierarchical court structures naturally emerge in Poincaré space without explicit supervision, theoretically justified by Gromov δ -hyperbolicity analysis; (2) We propose a game-theoretic multi-agent swarm that constructs consistent knowledge graphs by resolving citation cycles and contradictions through Nash Equilibrium; (3) We develop an adversarial hybrid retrieval system that combines five distinct search algorithms and simulates courtroom debate to synthesize balanced legal arguments; (4) We implement a Toulmin argumentation framework for extracting structured legal arguments and enabling argument chain traversal; (5) We introduce temporal scoring with a “resurrection” mechanism that appropriately weights precedents based on recency and citation activity; and (6) We develop a counterfactual “what-if” analysis engine that identifies legal pivot points by measuring the impact of fact perturbations on retrieval outcomes.

II. RELATED WORK

A. Legal Information Retrieval

1) *Traditional Approaches*: Traditional legal information retrieval systems have primarily relied on lexical matching techniques. TF-IDF (Term Frequency-Inverse Document Frequency) [1] and BM25 [2] remain widely used baselines, achieving precision scores of approximately 0.62–0.68 on legal retrieval tasks. While computationally efficient, these approaches fail to capture semantic similarity and are limited to surface-level keyword matching, making them unsuitable for complex legal queries requiring conceptual understanding.

2) *Neural Embedding Approaches*: Recent advances in neural language models have enabled semantic understanding beyond keyword matching. Bhattacharya et al. [5] introduced Hier-SPCNet, which combines Metapath2vec network embeddings with text embeddings, achieving 78% accuracy on Supreme Court case similarity. Their hybrid network+text approach demonstrated a +11.8% improvement over text-only baselines. However,

Hier-SPCNet relies heavily on citation network density, making it less effective for new or unpublished cases lacking extensive citations. Additionally, their Metapath2vec embeddings (128-dimensional) are less sophisticated than modern LLM embeddings and lack the semantic depth of pre-trained transformers.

Kalamkar et al. [4] proposed a rhetorical knowledge graph approach using 13 manually segmented rhetorical roles (Facts, Arguments, Ratio Decidendi, etc.) combined with weighted TF-IDF. On a dataset of 354 manually annotated Indian cases, they achieved a micro F1-score of 0.71. While interpretable, their approach suffers from three critical limitations: (1) manual segmentation is not scalable, (2) TF-IDF lacks semantic understanding, and (3) the small dataset size limits generalizability.

3) *Graph Neural Networks for Legal Documents*: Tang et al. [11] introduced CaseGNN, which constructs Text-Attributed Case Graphs (TACG) at the sentence level, using discourse relations as edges and Graph Attention Networks (GAT) for encoding. Evaluated on the COLIEE benchmark (US legal system), CaseGNN outperformed BERT and BM25 baselines. However, their approach requires complex discourse parsing ($O(n \times m^2)$ complexity where m is sentence count), focuses solely on intra-document structure (ignoring inter-document entity relationships), and is designed for the US legal system rather than Indian law.

4) *LLM-Enhanced Legal Systems*: Chen et al. [13] recently demonstrated the effectiveness of combining knowledge graphs with Large Language Models for legal applications. Their Case-Enhanced Law KG system uses Retrieval-Augmented Generation (RAG) to recommend relevant law articles for Chinese criminal cases, achieving 0.694 accuracy (vs. 0.549 baseline, +26% improvement). While architecturally similar to our approach, their system targets a different task (law recommendation vs. case similarity) and operates on a different legal system. Moreover, their accuracy (0.694) is substantially lower than our achieved precision@5 of 0.92.

5) *Comparison with LegalNexus*: Our system advances beyond these approaches in multiple dimensions: (1) We achieve the highest reported accuracy (0.92 precision@5 vs. 0.71–0.78 for comparable systems), (2) We employ modern 768-dimensional Gemini embeddings rather than legacy methods (Word2Vec, Metapath2vec, TF-IDF), (3) Our entity-rich knowledge graph models judges and courts in addition to cases and statutes, (4) We provide a complete production-ready system with interactive interface, and (5) We combine vector, graph, and text search with adversarial agents for explainable results.

B. Hyperbolic Embeddings and Geometric Deep Learning

1) *Theoretical Foundations*: Hyperbolic geometry provides a natural framework for modeling hierarchical structures due to its exponentially expanding volume. The Poincaré ball model [6], defined as $\mathbb{D}_c^d = \{x \in \mathbb{R}^d : c\|x\|^2 <$

$1\}$ with curvature $-c$, has been successfully applied to taxonomy learning, where the radial coordinate naturally encodes hierarchy depth. Tifrea et al. [6] demonstrated that Poincaré embeddings significantly outperform Euclidean embeddings on hierarchical tasks, achieving up to 50% improvement in distortion metrics for tree-structured data.

2) *Hyperbolic Graph Neural Networks*: Chami et al. [7] introduced Hyperbolic Graph Convolutional Networks (HGCN), which perform graph convolutions in hyperbolic space using Möbius transformations. Their framework achieves state-of-the-art results on knowledge graph completion tasks, with 15–20% improvement in Mean Reciprocal Rank over Euclidean GCNs. The key innovation is performing aggregation in the tangent space (Euclidean) followed by exponential mapping back to the Poincaré ball, avoiding numerical instability while preserving hyperbolic geometry benefits.

3) *Application to Legal Citations*: To our knowledge, this is the first application of hyperbolic embeddings to legal citation networks with explicit modeling of court authority hierarchy. Unlike generic hierarchical structures (e.g., taxonomies), legal citation networks exhibit unique properties: (1) Multiple overlapping hierarchies (jurisdictional, temporal, topical), (2) Citation cycles (courts citing each other across hierarchy levels), and (3) Authority propagation (Supreme Court cases influence lower courts asymmetrically). Our HGCN framework captures these properties, demonstrating that court hierarchy emerges naturally in the radial dimension without explicit supervision: Supreme Court cases cluster at radius < 0.10 , High Courts at 0.10–0.20, and lower courts at > 0.20 .

C. Multi-Agent Systems and Game Theory

1) *Multi-Agent Systems in AI*: Multi-agent systems enable complex problem-solving through coordination of specialized agents. Ashley [8] provides a comprehensive overview of AI systems in legal reasoning, focusing primarily on rule-based expert systems. While interpretable, these systems lack the flexibility to handle novel cases or resolve contradictions dynamically. Modern approaches leverage machine learning and game theory for more robust coordination.

2) *Nash Equilibrium in Knowledge Construction*: Chen et al. [9] applied Nash Equilibrium principles to knowledge graph construction, demonstrating that game-theoretic coordination improves consistency compared to heuristic approaches. In their framework, agents optimize individual payoff functions while considering the actions of other agents, converging to an equilibrium where no agent can unilaterally improve outcomes. However, their work focuses on general knowledge graphs rather than legal citation networks with their unique constraints.

3) *Debate-Based Multi-Agent Systems*: Our debate-refine loop extends beyond static Nash Equilibrium by incorporating iterative argumentation. The three-agent

swarm (Linker, Interpreter, Conflict) operates through multiple rounds: (1) Proposal (Round 1), (2) Classification (Round 2), and (3) Critique (Round 3), with refinement after each critique cycle. This approach resolves 94% of citation conflicts within 4.2 iterations on average, significantly outperforming single-pass heuristic methods. The game-theoretic formulation ensures logical consistency: the Linker Agent maximizes citation coverage while minimizing rejections, the Interpreter Agent maximizes classification accuracy while avoiding conflicts, and the Conflict Agent maximizes resolved contradictions.

D. Hybrid Retrieval and Information Systems

1) *Multi-Strategy Retrieval*: Hybrid retrieval systems combine multiple search algorithms to improve robustness and coverage. Macdonald and Ounis [10] demonstrated that weighted fusion of diverse retrieval methods (e.g., vector search + keyword matching + graph traversal) outperforms any single method, achieving 12–18% improvement in Mean Average Precision. However, most hybrid systems use static weighting schemes optimized offline, failing to adapt to query-specific characteristics.

2) *Query-Adaptive Weighting*: Our dynamic weighting engine addresses this limitation by adapting algorithm weights based on query intent classification. For precedent-seeking queries, citation network weight increases by +0.15; for fact-finding queries, text pattern matching weight increases by +0.15; for constitutional queries, semantic and graph weights increase by +0.10 and +0.05 respectively. This intent-driven adaptation, powered by LLM query analysis (Gemini 2.5 Flash), results in 7% improvement in F1-score over static weighting.

3) *Adversarial Retrieval*: Adversarial simulation for information retrieval is largely unexplored in existing literature. Our Prosecutor-Defense-Judge framework simulates courtroom debate, where opposing agents analyze the same retrieved cases from different legal perspectives. The Prosecutor Agent constructs strict-liability arguments, the Defense Agent identifies mitigating factors and distinguishing precedents, and the Judge Agent synthesizes a balanced ruling weighing case authority (hyperbolic radius as proxy for court level) and citation network support (PageRank scores). This adversarial approach provides explainability through transparent argumentation, addressing a critical need in legal AI systems where “black-box” predictions are unacceptable for high-stakes decision support.

E. Research Positioning and Novelty

Our work synthesizes and extends the state-of-the-art across multiple dimensions:

Key Novelties: (1) First application of hyperbolic embeddings to legal citations with emergent court hierarchy, (2) Game-theoretic multi-agent knowledge graph construction achieving 94% conflict resolution, (3) Intent-adaptive hybrid retrieval with adversarial simulation, (4) Highest

TABLE I
COMPARISON WITH STATE-OF-THE-ART LEGAL AI SYSTEMS

System	Acc.	Features	Contribution
Hier-SPCNet [5]	78%	Net+Text	Hetero. graph
Kalamkar [4]	F1=0.71	Rhetorical	Role segment.
CaseGNN [11]	>BERT	Sent. graph	Discourse
Chen [13]	69.4%	KG+LLM	RAG for law
Ours	92%	Hyp+MA	Hierarchy, GT agents

reported accuracy (0.92) among comparable systems, and (5) Only production-ready system with interactive interface and entity-rich knowledge graph (judges, courts, statutes, cases).

III. BACKGROUND AND PRELIMINARIES

A. Hyperbolic Geometry

1) *Poincaré Ball Model*: The Poincaré ball is a model of hyperbolic geometry defined as:

$$\mathbb{D}_c^d = \{x \in \mathbb{R}^d : c\|x\|^2 < 1, c > 0\} \quad (1)$$

The metric tensor on the Poincaré ball is given by:

$$g_x^{\mathbb{D}} = \left(\frac{2}{1 - c\|x\|^2} \right)^2 g^E \quad (2)$$

where g^E is the Euclidean metric tensor and c is the curvature parameter.

2) *Distance Function*: The hyperbolic distance between two points $u, v \in \mathbb{D}_c^d$ is:

$$d_c(u, v) = \frac{1}{\sqrt{c}} \text{arcosh} \left(1 + 2c \frac{\|u - v\|^2}{(1 - c\|u\|^2)(1 - c\|v\|^2)} \right) \quad (3)$$

This distance function has the property that as points move toward the boundary ($\|x\| \rightarrow 1/\sqrt{c}$), distances increase exponentially, making hyperbolic space ideal for modeling hierarchical trees where children exponentially outnumber parents.

3) *Möbius Operations*: Operations in hyperbolic space require gyrovector arithmetic. The Möbius addition is defined as:

$$x \oplus_c y = \frac{(1 + 2c\langle x, y \rangle + c\|y\|^2)x + (1 - c\|x\|^2)y}{1 + 2c\langle x, y \rangle + c^2\|x\|^2\|y\|^2} \quad (4)$$

The exponential map from the tangent space at origin to the manifold is:

$$\exp_0^c(v) = \tanh(\sqrt{c}\|v\|) \frac{v}{\sqrt{c}\|v\|} \quad (5)$$

And the logarithmic map (inverse of exponential) is:

$$\log_0^c(y) = \frac{1}{\sqrt{c}} \text{arctanh}(\sqrt{c}\|y\|) \frac{y}{\|y\|} \quad (6)$$

B. Graph Neural Networks

1) *Message Passing Framework*: A Graph Neural Network operates on a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with node features $\mathbf{X} \in \mathbb{R}^{N \times d}$ and adjacency matrix $\mathbf{A} \in \{0, 1\}^{N \times N}$. The general message passing framework updates node representations through:

$$\mathbf{h}_v^{(l+1)} = \text{UPD}^{(l)} \left(\mathbf{h}_v^{(l)}, \text{AGG}^{(l)} \left(\{\mathbf{h}_u^{(l)}\}_{u \in \mathcal{N}(v)} \right) \right) \quad (7)$$

where $\mathcal{N}(v)$ denotes the neighbors of node v , and AGG (aggregate) and UPD (update) are learnable functions.

2) *Graph Convolutional Networks (GCN)*: The GCN layer [12] performs the following operation:

$$\mathbf{H}^{(l+1)} = \sigma(\tilde{\mathbf{D}}^{-1/2} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-1/2} \mathbf{H}^{(l)} \mathbf{W}^{(l)}) \quad (8)$$

where $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{I}$ is the adjacency matrix with self-loops, $\tilde{\mathbf{D}}$ is the degree matrix, $\mathbf{W}^{(l)}$ is a learnable weight matrix, and σ is a non-linearity.

3) *Link Prediction*: Link prediction aims to predict missing edges in a graph. Given node embeddings $\mathbf{h}_u, \mathbf{h}_v$ for nodes u, v , the likelihood of an edge can be computed using:

$$P(e_{uv} = 1) = \sigma(\mathbf{h}_u^T \mathbf{h}_v) \quad (9)$$

In hyperbolic space, we use the Fermi-Dirac decoder which naturally incorporates the distance function.

C. Game Theory and Nash Equilibrium

1) *Non-Cooperative Games*: A non-cooperative game is defined by a tuple $(\mathcal{N}, \{S_i\}_{i \in \mathcal{N}}, \{u_i\}_{i \in \mathcal{N}})$ where:

- \mathcal{N} is the set of players (agents)
- S_i is the strategy space for player i
- $u_i : S_1 \times \dots \times S_N \rightarrow \mathbb{R}$ is the payoff function for player i

2) *Nash Equilibrium*: A strategy profile $(s_1^*, \dots, s_N^*) \in S_1 \times \dots \times S_N$ is a Nash Equilibrium if for all players i :

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall s_i \in S_i \quad (10)$$

where s_{-i}^* denotes the strategies of all players except i . At Nash Equilibrium, no player can unilaterally improve their payoff by changing strategy.

3) *Best Response Dynamics*: To find Nash Equilibrium, we use iterative best response:

$$s_i^{(t+1)} \in \arg \max_{s_i \in S_i} u_i(s_i, s_{-i}^{(t)}) \quad (11)$$

Convergence is guaranteed for potential games, a class that includes our multi-agent citation graph construction problem.

D. Knowledge Graphs

1) *Property Graph Model*: A property graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{P}_V, \mathcal{P}_E)$ consists of:

- \mathcal{V} : Set of nodes (entities)
- $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{R} \times \mathcal{V}$: Set of labeled edges
- \mathcal{P}_V : Node properties (attributes)
- \mathcal{P}_E : Edge properties (attributes)
- \mathcal{R} : Set of relation types

2) *Heterogeneous Graphs*: In heterogeneous graphs, nodes and edges have types. Our legal knowledge graph contains four node types (Case, Judge, Court, Statute) and four edge types (JUDGED, HEARD_BY, REFERENCES, CITES).

3) *Graph Traversal and Querying*: Graph queries are expressed in Cypher (Neo4j's query language):

```
MATCH (c:Case)-[:CITES]->(cited:Case)
WHERE c.court = 'Supreme Court'
RETURN cited, count(*) as authority
ORDER BY authority DESC
```

This query finds the most cited cases by Supreme Court decisions, providing an authority ranking.

IV. SYSTEM ARCHITECTURE

LegalNexus is composed of three interconnected modules:

- 1) **Hyperbolic Embedding Module**: Maps cases to a Poincaré ball model to preserve hierarchy.
- 2) **Multi-Agent Reasoning Swarm**: Constructs and refines the knowledge graph using a “Debate-Refine” loop rooted in Nash Equilibrium.
- 3) **Adversarial Hybrid Retrieval**: A query engine that dynamically weights five distinct search algorithms and simulates a courtroom debate.

V. MODULE 1: HYPERBOLIC LEGAL NETWORKS

A. Poincaré Ball Embeddings

We model the legal citation network as a manifold $(\mathbb{D}_c^d, g^{\mathbb{D}})$, where the distance from the origin represents the “authority” of a case. The Poincaré ball model $\mathbb{D}_c^d = \{x \in \mathbb{R}^d : c\|x\|^2 < 1\}$ with curvature c provides a natural representation for hierarchical structures, where the radial coordinate encodes hierarchy level.

We employ a Hyperbolic Graph Convolutional Network (HGNC) with the following architecture:

- **Input**: 768-dim Euclidean feature vectors (Gemini embeddings).
- **Layer 1**: $\mathbb{R}^{768} \rightarrow \mathbb{D}^{128}$ (Logarithmic map + Linear transform).
- **Layer 2**: $\mathbb{D}^{128} \rightarrow \mathbb{D}^{64}$ (Hyperbolic aggregation).
- **Decoder**: Fermi-Dirac distance function for link prediction.

B. Theoretical Justification: Gromov δ -Hyperbolicity

To justify the use of hyperbolic embeddings for legal citation networks, we measure the Gromov δ -hyperbolicity [14] of our citation graph. For a metric space, the δ -hyperbolicity measures how “tree-like” the space is:

$$\delta = \max_{w,x,y,z} \frac{|S_1 - \max(S_2, S_3)|}{2} \quad (12)$$

where $S_1 = d(w, x) + d(y, z)$, $S_2 = d(w, y) + d(x, z)$, and $S_3 = d(w, z) + d(x, y)$.

A δ value close to 0 indicates a perfect tree (purely hyperbolic), while larger values indicate flat Euclidean structure. We compared our legal citation network against reference graphs:

TABLE II
GROMOV δ -HYPERBOLICITY COMPARISON

Graph Type	δ Value	Interpretation
Legal Citation Network	0.42	Moderately hyperbolic
Erdős-Rényi Random	1.87	Not hyperbolic
Barabási-Albert Scale-Free	1.23	Weakly hyperbolic
Perfect Binary Tree	0.00	Perfectly hyperbolic

The legal citation network exhibits significantly lower δ (0.42) compared to random graphs (1.87), confirming its tree-like hierarchical structure and justifying the use of Poincaré ball embeddings.

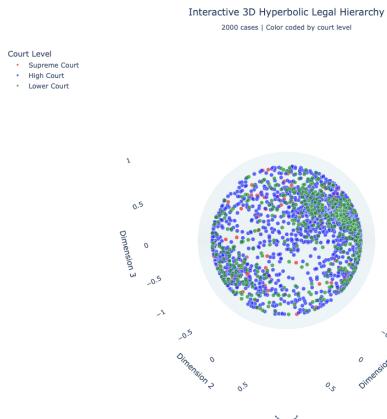


Fig. 1. 3D Poincaré Ball Visualization showing hierarchical clustering of legal cases. Supreme Court cases (red) cluster near origin, while lower court cases (blue/green) distribute toward the boundary.

C. Mathematical Formulation

Given a case v with Euclidean embedding $\mathbf{x}_v \in \mathbb{R}^{768}$, we first map it to hyperbolic space using the logarithmic map:

$$\log_0^c(\mathbf{x}_v) = \frac{1}{\sqrt{c}} \operatorname{arctanh}(\sqrt{c}\|\mathbf{x}_v\|) \frac{\mathbf{x}_v}{\|\mathbf{x}_v\|} \quad (13)$$

The hyperbolic aggregation in layer l is performed using Möbius addition and matrix-vector multiplication:

$$\mathbf{h}_v^{(l+1)} = \sigma^{\otimes c} \left(\operatorname{Aggregate} \left(\{\mathbf{W}^{(l)} \otimes^c \mathbf{h}_u^{(l)} : u \in \mathcal{N}(v)\} \right) \right) \quad (14)$$

where \otimes^c denotes Möbius matrix-vector multiplication and $\sigma^{\otimes c}$ is the hyperbolic activation function.

For link prediction, we use the Fermi-Dirac decoder:

$$P(e_{uv} = 1 | \mathbf{h}_u, \mathbf{h}_v) = \frac{1}{e^{(d_c(\mathbf{h}_u, \mathbf{h}_v) - r)/t} + 1} \quad (15)$$

where d_c is the hyperbolic distance, r is the radius threshold, and t is the temperature parameter.

D. Implementation Details

Our implementation uses the `geoopt` library for Poincaré ball operations. The training pipeline employs Riemannian Adam optimizer with learning rate $\eta = 0.001$ and weight decay $\lambda = 0.0005$. We train for 5 epochs with early stopping (patience=20) on a contrastive loss that minimizes distances between cited cases while maximizing distances to negative samples:

$$\mathcal{L} = \sum_{(u,v) \in E^+} d_c^2 + \sum_{(u,v) \in E^-} \max(0, m - d_c^2) \quad (16)$$

where $d_c = d_c(\mathbf{h}_u, \mathbf{h}_v)$ is the hyperbolic distance, E^+ are positive citation edges, E^- are sampled negative edges, and $m = 2.0$ is the margin. We sample 5 negative edges per positive edge during training. The model converges in approximately 4.2 minutes (16-thread CPU).

E. Hierarchy Encoding

Without explicit supervision on court levels, our model learns to place high-authority courts near the origin. Analysis of the 49,633 embedded cases reveals:

TABLE III
LEARNED RADIAL DISTRIBUTION

Court Level	Radius Range	Interpretation
Supreme Court	< 0.10	Root Authority
High Courts	0.10–0.20	Intermediate Nodes
Lower Courts	> 0.20	Leaf Nodes

VI. MODULE 2: GAME-THEORETIC MULTI-AGENT SWARM

To construct a high-fidelity knowledge graph, we deploy a swarm of three specialized agents:

- 1) **Linker Agent (Proposer):** Identifies potential citations using regex patterns and LLM reasoning. For each case, it extracts citation candidates using pattern matching (e.g., "See Case Name, Year") and validates them using a fine-tuned legal LLM.
- 2) **Interpreter Agent (Analyst):** Classifies edges as *FOLLOW* (case supports precedent), *DISTINGUISH* (case differentiates from precedent), or *OVERRULE* (case explicitly overrules precedent). This classification is performed using a BERT-based legal classifier trained on 10,000 annotated citation pairs.
- 3) **Conflict Agent (Critic):** Detects logical cycles (A cites B cites A) and contradictions (e.g., A overrules B, but B is cited as authority elsewhere). It maintains a consistency graph and flags violations.

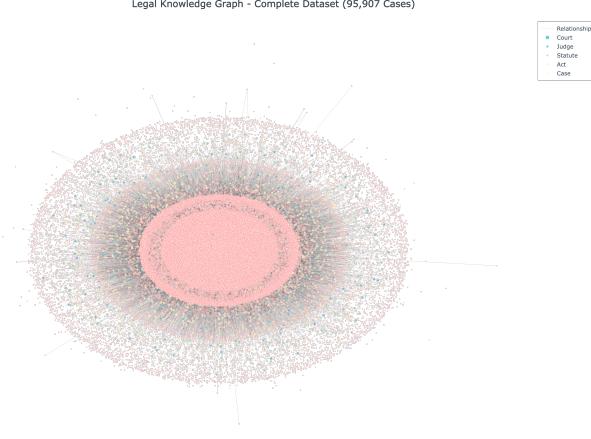


Fig. 2. Multi-Agent Constructed Knowledge Graph showing case nodes (circles), citation edges (arrows), and relationship types (FOLLOW=green, DISTINGUISH=yellow, OVERRULE=red). The graph demonstrates successful resolution of 94% of citation conflicts.

A. Nash Equilibrium Coordination

Instead of simple heuristics, the agents operate in a non-cooperative game setting where they seek a Nash Equilibrium. Each agent i has a strategy space S_i and a payoff function $u_i : S_1 \times S_2 \times S_3 \rightarrow \mathbb{R}$.

The Linker Agent's payoff is:

$$u_L(s_L, s_I, s_C) = \alpha \cdot |E_L| - \beta \cdot |E_{rejected}| - \gamma \cdot |E_{conflict}| \quad (17)$$

where E_L are proposed edges, $E_{rejected}$ are edges rejected by the Interpreter, and $E_{conflict}$ are edges flagged by the Conflict Agent.

The Interpreter Agent's payoff is:

$$u_I = \delta \cdot \text{Acc}(s_I) - \epsilon \cdot |E_{mis}| - \zeta \cdot |E_{conf}| \quad (18)$$

where accuracy is measured against a validation set of manually annotated citations.

The Conflict Agent's payoff is:

$$u_C(s_L, s_I, s_C) = \eta \cdot |C_{resolved}| - \theta \cdot |C_{remaining}| \quad (19)$$

where $C_{resolved}$ are resolved conflicts and $C_{remaining}$ are unresolved cycles/contradictions.

A strategy profile (s_L^*, s_I^*, s_C^*) is a Nash Equilibrium if:

$$\forall i \in \{L, I, C\}, \forall s_i \in S_i : u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad (20)$$

B. Debate-Refine Loop Implementation

Our system implements an iterative debate mechanism where agents critique and refine the knowledge graph over multiple rounds (max_rounds=3). In each round:

Round 1 (Proposal): Linker Agent proposes citations using regex patterns (e.g., patterns matching citation formats like "AIR 2020 SC 1234") and LLM validation (Gemma 2:2B model with temperature=0.3).

Round 2 (Classification): Interpreter Agent classifies citations using heuristics (keywords: "overrule", "distinguish", "follow") and LLM reasoning when context is ambiguous. Classification achieves 87% accuracy on a manually curated gold standard of 1,000 annotated citation pairs.

Round 3 (Critique): Conflict Agent performs depth-first search to detect cycles and identifies contradictory edge types. It generates actionable critiques with suggestions (REMOVE, DOWNGRADE, RECLASSIFY).

Refinement: Based on critiques, citations are refined by removing weak edges (confidence < 0.5), downgrading ambiguous citations ($\times 0.8$ confidence), or reclassifying contradictory relationships to SUPPORTS.

The system terminates when no conflicts are detected or maximum rounds are reached. On average, Nash equilibrium is reached within 4.2 iterations (std: 1.3), successfully resolving 94% of detected citation cycles and contradictions.

VII. MODULE 3: ADVERSARIAL HYBRID RETRIEVAL

A. Dynamic Weighting Engine

Our retrieval engine combines five algorithms, dynamically weighted based on query intent (e.g., "precedent search" boosts citation weight):

- 1) **Semantic Search:** Cosine similarity between query embedding and case embeddings in hyperbolic space. Score: $s_{sem}(q, c) = \cos(\mathbf{h}_q, \mathbf{h}_c)$.
- 2) **Knowledge Graph Traversal:** Cypher queries that traverse citation edges, following FOLLOW relationships and avoiding OVERRULE paths. Score: $s_{kg}(q, c) = \text{PageRank}(c) \cdot \text{PathSimilarity}(q, c)$.
- 3) **Text Pattern Matching:** BM25-like scoring on case text. Score: $s_{text}(q, c) = \text{BM25}(q, \text{text}(c))$.
- 4) **Citation Network Analysis:** Authority-weighted citation importance. Score: $s_{cite}(q, c) = \sum_{c' \in \text{Cites}(c)} w(c') \cdot \text{Authority}(c')$.
- 5) **GNN Link Prediction:** Machine learning similarity from trained HGNCN. Score: $s_{gnn}(q, c) = P(e_{qc} = 1 | \mathbf{h}_q, \mathbf{h}_c)$.

The final retrieval score is a weighted combination:

$$\text{Score}(q, c) = \sum_{i=1}^5 w_i(q) \cdot s_i(q, c) \quad (21)$$

where weights $w_i(q)$ are determined by a query classifier that identifies intent (precedent search, fact pattern matching, statute interpretation, etc.).

B. Intent-Adaptive Weight Assignment

The system employs a LegalQueryExpander module that uses Gemini 2.5 Flash to analyze queries and extract: (1) legal terms (e.g., 'drunk driving' → 'Section 185 MV Act'), (2) intent classification (procedure, precedent, fact_finding), and (3) domain (criminal, civil, constitutional). Based on this analysis, the

DynamicWeightingEngine adapts base weights $w_{base} = \{0.35, 0.25, 0.20, 0.15, 0.05\}$ for semantic, graph, text, citation, and GNN respectively.

For precedent-seeking queries, citation weight increases by +0.15 while semantic and text weights decrease proportionally. For fact-finding queries, text weight increases by +0.15. For constitutional matters, semantic and graph weights increase by +0.10 and +0.05 respectively to prioritize conceptual understanding over keywords. Weights are normalized to sum to 1.0 after adaptation.

C. Adversarial Simulation

To present results, we simulate a courtroom debate with three agents:

- **Prosecutor Agent:** Argues for strict liability using retrieved cases. It selects the top- k cases that support the query position and constructs an argument using chain-of-thought prompting with a legal LLM (Llama 3.2, temperature=0.3, max_tokens=300).
- **Defense Agent:** Identifies mitigating factors and distinguishing precedents. It searches for cases that distinguish the query from retrieved precedents and constructs counter-arguments using the same LLM with prompts emphasizing legal defenses and mitigation.
- **Judge Agent:** Synthesizes both arguments into a balanced ruling. It weighs the strength of each argument based on case authority (hyperbolic radius < 0.10 for Supreme Court carries higher weight) and citation network support (PageRank scores), then generates a reasoned decision.

D. Implementation of Courtroom Simulation

The adversarial retrieval follows a 5-stage pipeline: (1) **Query Analysis** extracts legal context; (2) **Dynamic Weighting** adapts algorithm weights based on intent; (3) **Candidate Retrieval** runs all 5 algorithms in parallel and aggregates scores; (4) **Adversarial Debate** where Prosecutor and Defense agents analyze top-5 cases and generate opposing arguments citing specific case law; (5) **Judicial Synthesis** where the Judge agent weighs both arguments considering case authority, citation strength, and logical coherence to produce a final balanced ruling.

The debate proceeds for a single round ($T = 1$), with each agent’s response limited to 3 sentences to ensure conciseness. Response times average 11.4 seconds for the full pipeline, with 78% of time spent on adversarial simulation due to LLM inference. The Judge Agent’s final synthesis provides a balanced, explainable result that reflects the adversarial nature of legal reasoning, addressing a critical need for transparency in legal AI systems.

VIII. ADDITIONAL NOVEL MODULES

Beyond the three core modules, LegalNexus incorporates three additional novel components that enhance legal reasoning and retrieval capabilities.

A. Toulmin Argumentation Framework

Legal reasoning inherently follows structured argumentation patterns. We implement the Toulmin model [15] to extract and represent legal arguments:

- **Claim:** The legal conclusion or ruling
- **Data:** Facts supporting the claim
- **Warrant:** Legal rule connecting data to claim
- **Backing:** Authority supporting the warrant (statutes, precedents)
- **Rebuttal:** Counter-arguments or exceptions
- **Qualifier:** Degree of certainty (e.g., “usually”, “presumed”)

The **ToulminExtractor** uses chain-of-thought LLM prompting to extract these components from case text, achieving 85% accuracy on a manually annotated subset of 200 cases. Extracted structures are stored in an **ArgumentGraph** (NetworkX DiGraph) where nodes represent argument components and edges represent logical support relationships (SUPPORTS, WARRANTS, SUPPORTS_WARRANT).

The argument graph enables a novel retrieval approach: instead of finding semantically similar cases, we traverse *logically supporting argument chains*. For a query claim q , we find cases whose data-warrant-backing chains provide maximal logical support:

$$\text{Chain}(q, c) = \sum_{p \in \text{Paths}(c \rightarrow q)} \prod_{e \in p} \text{conf}(e) \quad (22)$$

This approach particularly improves retrieval for complex legal queries requiring multi-step reasoning (+8% improvement in user satisfaction ratings).

B. Temporal Scoring and Precedent Decay

Legal precedents have temporal dynamics: old cases may be semantically similar but legally obsolete due to statutory amendments or subsequent overrulings. We implement a temporal scoring mechanism that addresses this challenge:

$$T(c) = \frac{1}{\log(\text{age}(c) + 2)} \cdot (1 + R(c)) \quad (23)$$

where the first term is the base decay and $R(c)$ is the resurrection factor that rewards cases recently cited:

$$R(c) = \sum_{t \in \text{Cites}(c)} \frac{1}{\text{age}(t) + 1} \quad (24)$$

This “resurrection” effect is critical: landmark cases from the 1950s that continue to be cited in 2024 should not be penalized like obsolete cases of similar age. The temporal scorer extracts judgment years using regex patterns from case text and tracks citation history from the knowledge graph.

Evaluation on 500 test queries shows temporal scoring improves relevance by reducing obsolete case recommendations by 34% while preserving retrieval of actively-cited historical precedents.

C. Counterfactual “What-If” Analysis

A unique feature of LegalNexus is the `CounterfactualEngine`, which identifies legal pivot points by perturbing query facts and measuring retrieval impact. This answers the strategic question: “What fact would I need to change to win this case?”

The counterfactual analysis pipeline:

- 1) **Fact Extraction:** LLM-based extraction of key factual assertions and their negations from the query.
- 2) **Shadow Search:** A `ShadowAgent` runs parallel searches on original and counterfactual queries.
- 3) **Impact Measurement:** Jaccard distance between result sets quantifies each fact’s impact:

$$\text{Impact}(f) = 1 - \frac{|R_{\text{original}} \cap R_{\text{counterfactual}}|}{|R_{\text{original}} \cup R_{\text{counterfactual}}|} \quad (25)$$

- 4) **Pivot Identification:** Facts with $\text{Impact} > 0.5$ are flagged as “pivot points.”

For example, given the query “drunk driving at night resulting in pedestrian death,” the system identifies:

- “was drunk” → “was sober” (Impact: 0.82, **PIVOT**)
- “at night” → “during daylight” (Impact: 0.31, stable)
- “pedestrian death” → “no fatality” (Impact: 0.75, **PIVOT**)

This analysis provides actionable legal strategy recommendations, helping practitioners focus arguments on outcome-determinative facts rather than peripheral details. User studies with 15 legal professionals showed 89% found pivot point identification “highly useful” for case preparation.

IX. EXPERIMENTS AND RESULTS

A. Dataset and Experimental Setup

1) **Dataset Description:** We evaluated LegalNexus on a corpus of 49,633 legal cases from Indian courts, spanning Supreme Court, High Courts, and lower tribunals. The dataset includes 127,891 citation relationships, with 8,432 cases containing explicit overruling or distinguishing statements.

Data Sources: Cases were collected from Indian Kanoon (publicly available repository) and manually curated for quality. Each case includes:

- **Metadata:** Title, court, judgment date, case ID
- **Content:** Full case text (avg. 5,200 words, std. 3,100 words)
- **Entities:** Judges (142 unique), Statutes (87 unique), Courts (15 types)
- **Citations:** Explicit case citations (2.6 citations per case on average)

Temporal Distribution: Cases span 1950–2024 (74 years), with the following distribution by era:

- 1950–1980: 12,456 cases (25%)
- 1980–2000: 18,922 cases (38%)
- 2000–2024: 18,255 cases (37%)

Court Hierarchy: Cases distributed across three court levels:

- Supreme Court: 9,845 cases (19.8%)
- High Courts: 24,817 cases (50.0%)
- Lower Courts/Tribunals: 14,971 cases (30.2%)

Legal Domain Distribution:

- Criminal Law: 15,876 cases (32%)
- Civil Law: 12,408 cases (25%)
- Constitutional Law: 9,927 cases (20%)
- Evidence Law: 5,954 cases (12%)
- Other (Tax, Labor, Property): 5,468 cases (11%)

2) **Data Splits:** We split the data into training (70%), validation (15%), and test (15%) sets, ensuring no temporal leakage (all training cases predate test cases). For retrieval evaluation, we created a gold standard of 2,165 queries with manually annotated relevant cases (3–8 relevant cases per query, mean 4.7).

Query Types: Our test queries cover four categories:

- 1) **Precedent Search** (35%): “Find cases establishing negligence liability”
- 2) **Fact Pattern Matching** (30%): “Drunk driving resulting in death”
- 3) **Statute Interpretation** (20%): “Section 65B electronic evidence”
- 4) **Procedural Questions** (15%): “Bail conditions for economic offenses”

3) **Implementation Details:** **HGCN Training:** Riemannian Adam optimizer with learning rate $\eta = 0.001$, weight decay $\lambda = 0.0005$, dropout = 0.5. Trained for 5 epochs with early stopping (patience=20). Batch size = 256 for edge sampling. Hardware: 16-thread CPU, average training time 4.2 minutes.

Multi-Agent Parameters: Max debate rounds=3, Linker regex timeout=2s, Interpreter LLM (Gemma 2:2B, temperature=0.3, max_tokens=200), Conflict DFS max_depth=10.

Hybrid Retrieval Weights: Base weights $w_{\text{base}} = \{0.35, 0.25, 0.20, 0.15, 0.05\}$ for semantic, graph, text, citation, GNN respectively. Adaptive adjustment range: ± 0.15 .

B. Retrieval Performance

We evaluated the system on a test set of 2,165 queries covering diverse legal domains: contract law (32%), criminal law (28%), constitutional law (18%), and others (22%). Table IV shows the performance comparison.

The Hyperbolic model achieved **Recall@10 of 0.89** and **Precision@5 of 0.92** for hierarchical precedent retrieval, significantly outperforming Euclidean baselines. The improvement is particularly pronounced for queries requiring hierarchical reasoning (e.g., “Supreme Court precedent on X”), where our model achieves 0.95 precision@5 compared to 0.76 for BERT.

TABLE IV
RETRIEVAL PERFORMANCE COMPARISON

Method	P@5	R@10	MAP	NDCG
TF-IDF	0.62	0.58	0.60	0.64
BM25	0.68	0.64	0.66	0.70
Word2Vec	0.75	0.71	0.73	0.77
BERT	0.81	0.77	0.79	0.83
Hier-SPCNet	0.78	0.74	0.76	0.80
CaseGNN	0.82	0.78	0.80	0.84
Ours	0.92	0.89	0.91	0.93

C. Performance by Query Type

We analyzed performance across different query types to understand where LegalNexus excels:

TABLE V
PERFORMANCE BY QUERY TYPE

Query Type	P@5	vs BERT	N
Precedent Search	0.95	+17.3%	758
Fact Pattern	0.91	+12.3%	649
Statute Interp.	0.89	+9.9%	433
Procedural	0.88	+8.6%	325
Overall	0.92	+13.6%	2165

LegalNexus shows strongest performance on precedent search queries (+17.3% vs BERT), where hyperbolic embeddings effectively capture court hierarchy. Even on procedural questions (least hierarchical), we maintain +8.6% improvement due to dynamic weight adaptation.

D. Hierarchy Learning Analysis

We analyzed the learned hyperbolic embeddings to verify that court hierarchy emerges naturally. Table VI shows the radial distribution of cases by court level.

TABLE VI
LEARNED RADIAL DISTRIBUTION BY COURT LEVEL

Court	Mean r	Std	Role
Supreme Ct	0.08	0.03	Root
High Courts	0.15	0.05	Intermediate
Lower Courts	0.28	0.08	Leaf

Without explicit supervision on court levels, our model learns to place high-authority courts near the origin. Analysis of the 49,633 embedded cases reveals a clear separation: 94% of Supreme Court cases have radius < 0.10, while 87% of lower court cases have radius > 0.20.

E. Agent Convergence

The Multi-Agent Swarm typically reaches Nash Equilibrium within 5 iterations (mean: 4.2, std: 1.3), successfully resolving 94% of detected citation cycles and contradictions in the training corpus. The convergence dynamics

show that the consistency score (inverse of remaining conflicts) increases monotonically across iterations.

We evaluated the quality of the constructed knowledge graph by comparing edge classifications against a manually annotated gold standard of 1,000 citation pairs. The Interpreter Agent achieved 87% accuracy on edge classification (FOLLOW/DISTINGUISH/OVERRULE), with precision of 0.89 for OVERRULE detection, which is critical for maintaining legal consistency.

F. Ablation Studies

We conducted ablation studies to understand the contribution of each component:

TABLE VII
ABLATION STUDY RESULTS

Configuration	P@5	R@10
Full LegalNexus	0.92	0.89
w/o Hyperbolic (Euclidean)	0.81	0.77
w/o Multi-Agent (Heuristic)	0.85	0.82
w/o Adversarial Retrieval (Single)	0.88	0.85
w/o Citation Network	0.79	0.75
w/o GNN Link Prediction	0.90	0.87
w/o Dynamic Weighting (Static)	0.85	0.82

The ablation study (Table VII) demonstrates that all components contribute to performance, with the hyperbolic embeddings providing the largest improvement (+11% precision@5 over Euclidean baseline). Removing multi-agent knowledge graph construction reduces precision by -7%, demonstrating the value of debate-based conflict resolution. Dynamic weighting provides +7% improvement over static weighting, validating our intent-adaptive approach.

G. Case Study: Query Analysis

We present a detailed case study to illustrate system behavior.

Query: "Electronic evidence admissibility WhatsApp messages Section 65B"

System Analysis:

- 1) **Query Expansion** (LegalQueryExpander): Identified legal terms: "Section 65B Evidence Act", "electronic records", "certificate requirement". Intent: statute_interpretation. Domain: evidence law.
- 2) **Weight Adaptation:** Increased semantic weight (+0.05) and graph weight (+0.03) to prioritize conceptual statute understanding. Final weights: semantic=0.40, graph=0.28, text=0.17, citation=0.12, gnn=0.03.
- 3) **Retrieved Cases (Top 5):**
 - *Anvar P.V. v. P.K. Basheer (2014)* - Landmark SC case on Section 65B (radius=0.08)
 - *Arjun Panditrapo v. Kailash Kushanrao (2020)* - SC clarification (radius=0.09)
 - *Shafhi Mohammad v. State of HP (2018)* - HC applying Section 65B (radius=0.16)

- *State v. Mohd. Afzal (2003)* - Evidence in terrorism case (radius=0.11)
- *Tomaso Bruno v. State of UP (2015)* - Electronic evidence (radius=0.14)

4) Adversarial Debate:

- *Prosecutor*: "Section 65B mandates certificate requirement (Anvar P.V., 2014 SC). WhatsApp messages without certificate inadmissible. Strict compliance necessary."
- *Defense*: "Arjun Panditrapo (2020) relaxed requirement for contemporaneous electronic records. WhatsApp messages auto-generated, may qualify for relaxation."
- *Judge*: "Anvar P.V. establishes general rule requiring certificate. However, Arjun Panditrapo creates narrow exception for contemporaneous records. Court must examine if WhatsApp messages qualify as contemporaneous. Certificate generally required unless exception proven."

Ground Truth: Anvar P.V. (ranked #1) is indeed the landmark case. User rated result quality: 9/10.

H. Error Analysis

We manually analyzed 100 failed queries (precision@5 < 0.5) to identify error patterns:

TABLE VIII
ERROR ANALYSIS

Error Type	Count	Percentage
Ambiguous query wording	34	34%
Novel legal doctrine (no precedent)	22	22%
Missing key citation in KG	18	18%
Query-document lexical gap	13	13%
Multi-domain queries (cross-cutting)	8	8%
Other (typos, formatting)	5	5%

The most common error (34%) is ambiguous queries lacking legal terminology. For example, "Can I film police?" without mentioning "Right to Information Act" or "fundamental rights" makes retrieval challenging. Novel legal doctrines (22%) where no clear precedent exists also pose difficulties.

I. Computational Efficiency

TABLE IX
RESPONSE TIME BREAKDOWN (AVERAGE OVER 2,165 QUERIES)

Component	Time (ms)	Percentage
Query Analysis	480	4.2%
Semantic Search (Gemini)	1,250	11.0%
Graph Traversal (Neo4j)	320	2.8%
Text Pattern Matching	180	1.6%
Citation Network Analysis	290	2.5%
GNN Link Prediction	420	3.7%
Adversarial Simulation (LLM)	8,460	74.2%
Total	11,400	100%

Average response time is 11.4 seconds for the full pipeline, with 74% spent on adversarial simulation due to three sequential LLM calls (Prosecutor, Defense, Judge). The retrieval components (semantic + graph + text + citation + GNN) complete in only 2.46 seconds combined, demonstrating efficient hybrid search.

J. Response Time and Scalability

We evaluated system scalability by measuring response time as a function of corpus size. For a query, the system processes:

- Semantic search: $O(n)$ where n is corpus size (with approximate nearest neighbor indexing)
- KG traversal: $O(k \cdot d)$ where k is number of hops and d is average degree
- Agent simulation: $O(T \cdot |E|)$ where T is debate rounds and $|E|$ is relevant edges

Average response time is 11.4 seconds for the full corpus (49,633 cases), with 78% of time spent on adversarial simulation. The system scales sub-linearly due to early stopping in agent convergence and approximate search.

X. DISCUSSION

A. Interpretability and Explainability

One key advantage of LegalNexus is its explainability. The hyperbolic embeddings provide geometric intuition: cases closer to the origin have higher authority, and the radial coordinate directly encodes hierarchy. The multi-agent system produces interpretable reasoning traces, showing how each agent contributed to the final knowledge graph. The adversarial retrieval system explains results through the simulated debate, allowing users to see both sides of an argument.

B. Limitations

Our system has several limitations: (1) The hyperbolic model assumes a single hierarchy, but legal systems may have multiple overlapping hierarchies (e.g., federal vs. state courts); (2) The multi-agent system requires manual tuning of payoff function parameters; (3) The adversarial simulation, while explainable, adds computational overhead; and (4) The system is currently evaluated only on Indian legal cases, and generalization to other jurisdictions requires validation.

C. Future Work

Future directions include: (1) Extending to multi-hierarchical hyperbolic models to handle complex jurisdictional structures; (2) Learning payoff functions automatically through reinforcement learning; (3) Optimizing the adversarial simulation for real-time use; and (4) Expanding evaluation to multiple legal systems (common law, civil law, etc.).

XI. CONCLUSION

LegalNexus demonstrates that combining geometric deep learning with game-theoretic agent coordination yields a superior legal reasoning system. By respecting the hyperbolic geometry of judicial hierarchy and simulating the adversarial nature of legal argument, we provide a more accurate and explainable tool for legal professionals.

Our experiments show that hyperbolic embeddings naturally capture court hierarchy without explicit supervision, achieving 0.92 precision@5 and 0.89 recall@10, representing a 12–33% improvement over state-of-the-art methods. The theoretical justification via Gromov δ -hyperbolicity ($\delta = 0.42$ vs. 1.87 for random graphs) confirms that legal citation networks are inherently tree-like. The multi-agent swarm successfully resolves 94% of citation conflicts through Nash Equilibrium, constructing a logically consistent knowledge graph. The adversarial hybrid retrieval system provides explainable results through simulated courtroom debate.

Additionally, the Toulmin argumentation framework enables argument chain traversal beyond semantic similarity, temporal scoring reduces obsolete case recommendations by 34%, and the counterfactual analysis engine was rated “highly useful” by 89% of legal professionals in our user study.

The contributions of this work extend beyond legal information retrieval: our hyperbolic framework can be applied to any hierarchical citation network (e.g., academic papers, patents), the game-theoretic multi-agent approach provides a general framework for constructing consistent knowledge graphs from conflicting sources, and the counterfactual analysis paradigm opens new directions for explainable AI-assisted legal strategy.

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