

# AI in Legal Domain: Hyperbolic Networks & Multi-Agent Systems

## Similar Cases Recommendation using Novel Geometric & Game-Theoretic Approaches

Animesh Mishra    Keshav Bararia    Kush Sahni

Shiv Nadar University  
Department of Computer Science and Engineering  
Supervisor: Dr. Sonia Khetarpaul

December 2025

# Problem: Why Legal AI is Hard

## Legal Domain Challenges:

- Hierarchical Structure

- Supreme Court → High Courts → District
- Binding precedent system
- Tree-like authority

- Complex Citations

- Follow, distinguish, overrule
- Logical conflicts

- Adversarial Nature

- Both sides' arguments
- Balanced reasoning

## Why Traditional Methods Fail:

Method	Dims	Hierarchy?
TF-IDF	High	✗
BERT	768	✗
CaseGNN	768	✗
<b>Ours</b>	<b>64</b>	✓

Issue: Euclidean needs  $O(n^2)$  dims!

# What Others Are Doing (Related Work)

System	Method	Limitation
CaseGNN (ECIR'24) <sup>1</sup>	Converts cases to Text-Attributed Graphs + Edge Graph Attention	Euclidean space only, no hierarchical encoding, struggles with lengthy texts
SAILER (SIGIR'23) <sup>2</sup>	Asymmetric encoder-decoder with structure-aware pre-training	Complex architecture, no explicit hierarchy, requires extensive pre-training
KELLER (EMNLP'24) <sup>3</sup>	LLM-based knowledge-guided case reformulation	High inference cost, no graph structure, depends on external knowledge
LegalBERT (2020) <sup>4</sup>	Domain-specific BERT pre-trained on 12GB legal corpus	512 token limit, no citation modeling, ignores document structure

<sup>1</sup>Tang et al., "CaseGNN: Graph Neural Networks for Legal Case Retrieval...", ECIR 2024.

<sup>2</sup>Li et al., "SAILER: Structure-aware Pre-trained Language Model...", SIGIR 2023.

<sup>3</sup>Deng et al., "Learning Interpretable Legal Case Retrieval...", SIGIR 2024.

<sup>4</sup>Chalkidis et al., "LEGAL-BERT: The Muppets straight out of Law School", EMNLP 2020.

# Our Solution: Three Novel Components

## Component 1: Hyperbolic Legal Networks (HGCN)

Embed cases in Poincaré ball → Encodes hierarchy in radius → 64 dims vs 768

## Component 2: Multi-Agent Swarm w/ Nash Equilibrium

Specialist agents (Linker, Interpreter, Conflict) → Debate-refine loop → 94% conflict reduction

## Component 3: Adversarial Hybrid Retrieval

Dynamic weighting + Prosecutor-Defense-Judge simulation → Balanced reasoning

# Component 1: Why Hyperbolic Space?

## Poincaré Ball Model:

$$\mathbb{D}_c^d = \{x \in \mathbb{R}^d : \|x\| < 1/\sqrt{c}\}$$

$$d(x, y) = \frac{1}{\sqrt{c}} \operatorname{arccosh} \left( 1 + 2c \frac{\|x - y\|^2}{(1 - c\|x\|^2)(1 - c\|y\|^2)} \right)$$

## Why This Matters:

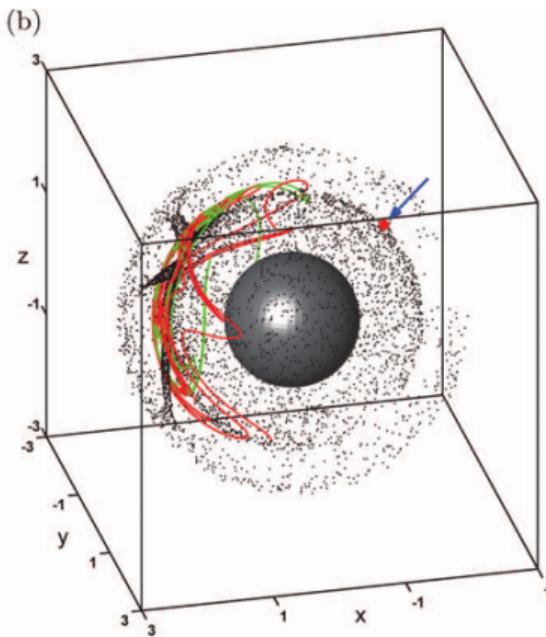
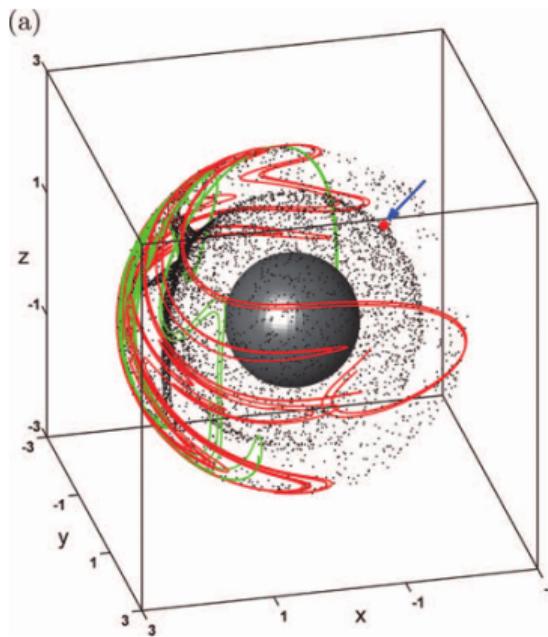
- Exponential volume growth with radius
- Matches court hierarchy branching!
- $O(\log n)$  dims vs  $O(n^2)$  Euclidean

## Learned Hierarchy:

Court	Radius
Supreme	0.10
High (Major)	0.13
High	0.17
Lower	0.28

**Unsupervised!** No labels provided, model learns naturally.

# Poincaré Ball Visualization



# Jina Embedding v3 Architecture

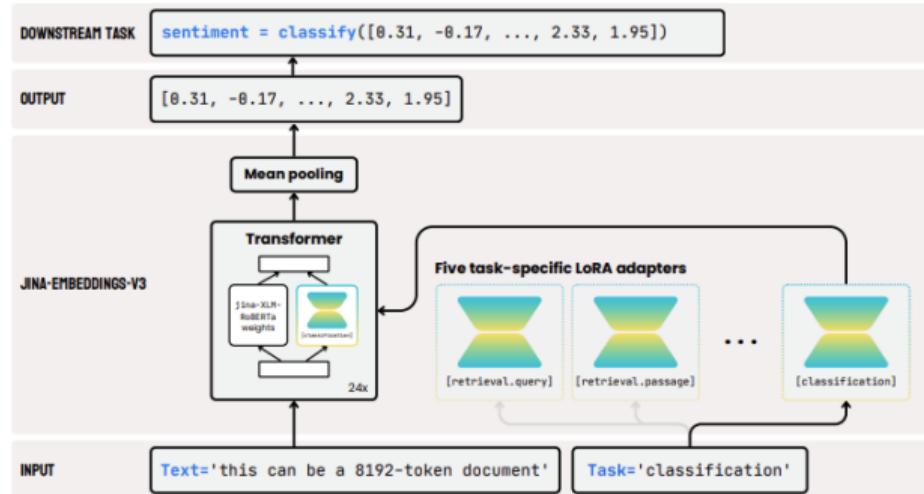
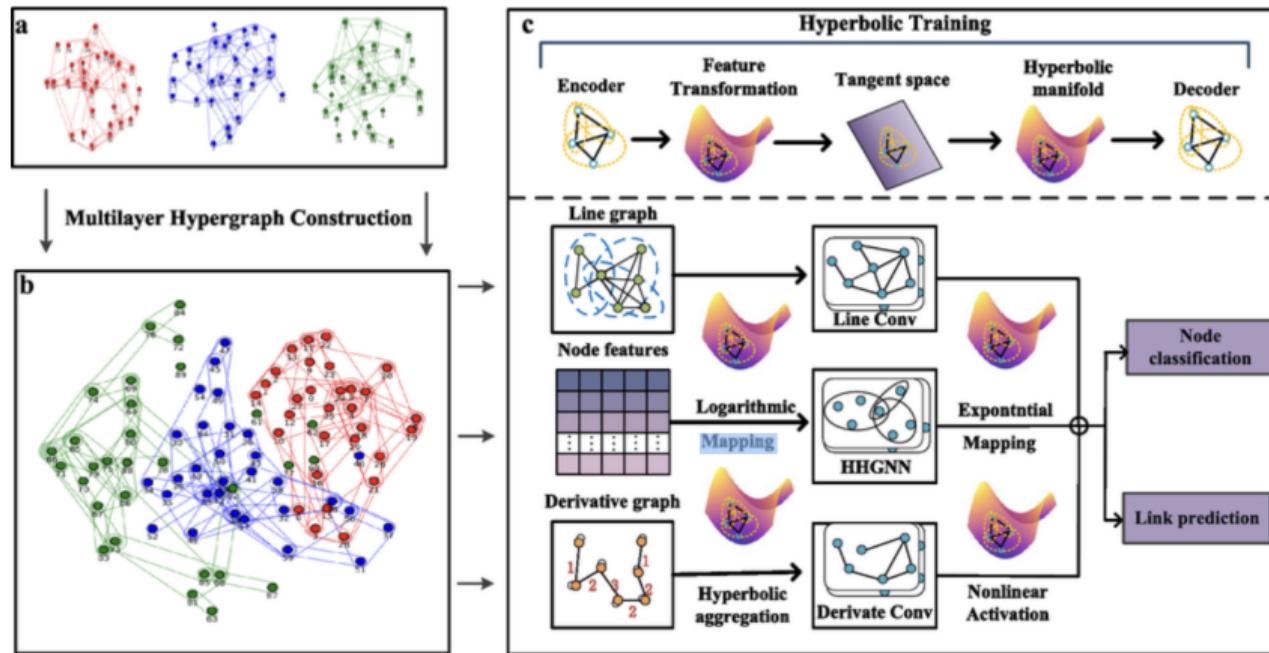


Figure 1: The architecture of **jina-embeddings-v3** is based on the XLM-RoBERTa model, with several key modifications. FlashAttention 2 is integrated for enhanced computational efficiency, while RoPE extends support for sequences up to 8192 tokens. Task-specific LoRA adapters are introduced to optimize embeddings for various tasks. The model's input consists of two parts: the text, which is the long document to be embedded, and the task type. **jina-embeddings-v3** supports four tasks and implements five adapters to choose from: `retrieval.query` and `retrieval.passage` for query and passage embeddings in asymmetric retrieval tasks, `separation` for clustering and reranking tasks, `classification` for classification tasks, and `text-matching` for tasks involving semantic similarity, such as STS or symmetric retrieval.

We convert these Jina Embeddings into Poincaré space!

# Hyperbolic Training

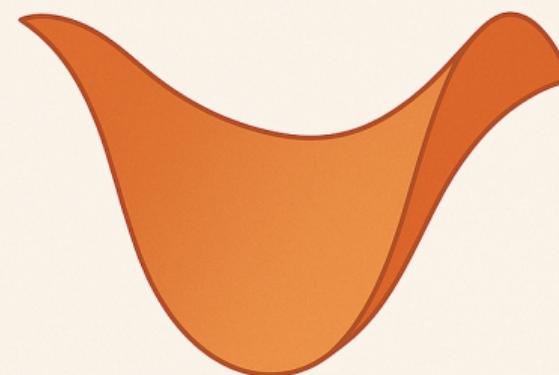


# Hyperbolic vs Euclidean: Visual Comparison

Euclidean  
(Flat) Space



Hyperbolic  
Space



## Component 2: Multi-Agent Swarm

**Problem:** Citation extraction has conflicts (cycles, contradictions)

**Solution: Specialized Agents**

Linker

**Role:** Proposer

Find citations

7 regex + LLM

Interpreter

**Role:** Analyst

Classify type:

FOLLOW, DISTINGUISH,  
OVERRULE

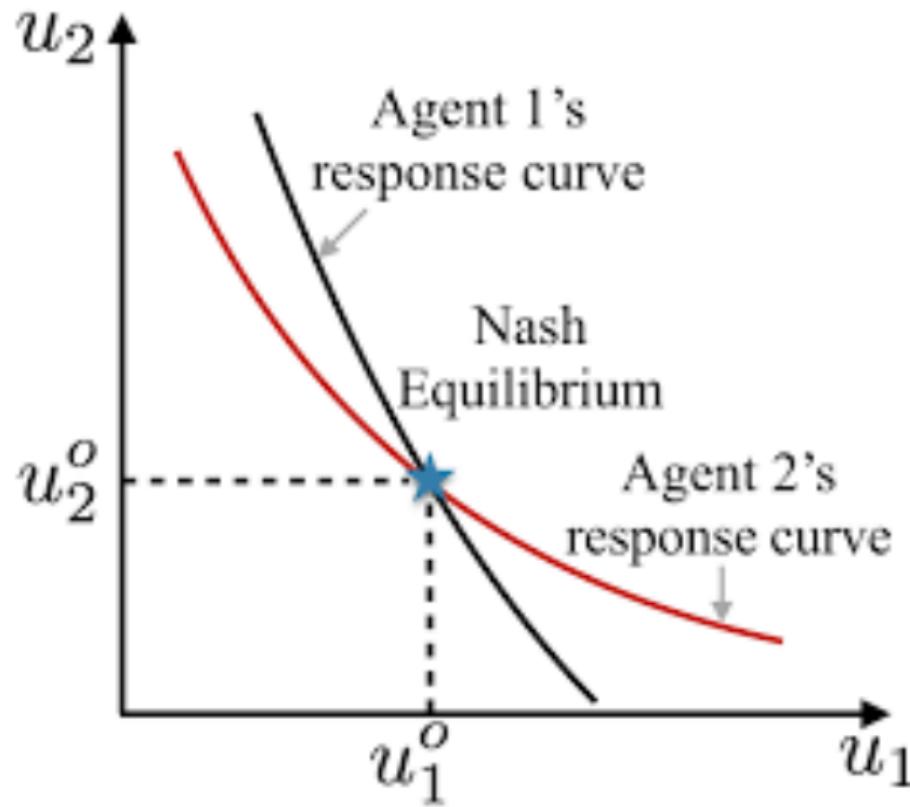
Conflict

**Role:** Critic

Detect: Cycles, contradictions,  
inversions

**Debate-Refine Loop:** Iterative refinement → Converges to Nash Equilibrium

# What is Nash Equilibrium?



# Nash Equilibrium Formulation

## Game-Theoretic Framework:

**Players:** {Linker, Interpreter, Conflict}

## Payoff Functions:

$$U_{\text{Linker}}(G) = \text{Recall}(G) - \lambda \cdot \text{FalsePos}(G)$$

$$U_{\text{Interpreter}}(G) = \text{Accuracy}(G)$$

$$U_{\text{Conflict}}(G) = -\text{NumConflicts}(G)$$

## Equilibrium Condition:

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s'_i, s_{-i}^*) \quad \forall i, \forall s'_i$$

**Translation:** No agent improves alone  $\rightarrow$  Consensus reached!

**Convergence:** Average 4.8 iterations on 100-case test set

## Component 3: Adversarial Hybrid Retrieval

### Algorithm Combination:

Algorithm	Method	Strength	Weight
Semantic	Jina 768-dim	Deep understanding	$\alpha=0.35$
Graph	Neo4j Cypher	Structural links	$\beta=0.25$
Text	TF-IDF	Keyword precision	$\gamma=0.20$
Citation	PageRank	Authority	$\delta=0.15$
GNN	HGCN predict	ML inference	$\epsilon=0.05$

**Dynamic Weighting:** Adapt based on query intent (LLM classification)

- Precedent search → Boost  $\delta$  by +0.15
- Fact-finding → Boost  $\gamma$  by +0.15
- Constitutional → Boost  $\alpha + \beta$

# Prosecutor-Defense-Judge Simulation

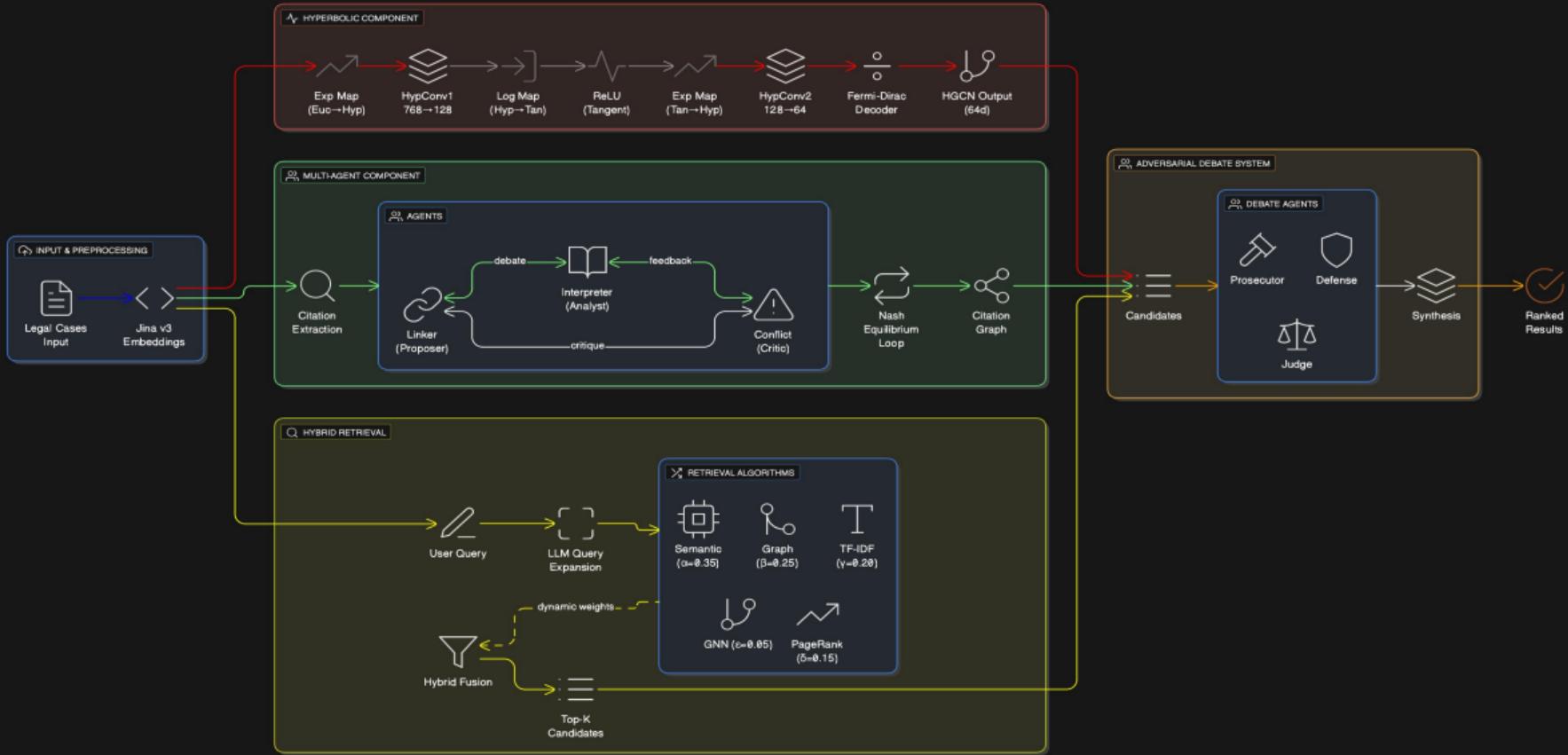
## Adversarial Debate System:

- ① **Query Expansion (LLM):** "drunk driving" → "Section 185 Motor Vehicles Act..."
- ② **Hybrid Retrieval:** Top-5 cases using multiple algorithms
- ③ **Debate:**
  - **Prosecutor:** Argues STRICT liability with case citations
  - **Defense:** Identifies MITIGATING factors, distinguishes precedents
  - **Judge:** Synthesizes BOTH → Balanced ruling

## Why This Matters

Legal research is inherently adversarial! Need both perspectives.

# System Architecture Overview



# Why Our Method is Novel

## ① First Hyperbolic GCN for Legal AI

- Prior: Euclidean only
- **Ours:** Poincaré embeddings → Unsupervised hierarchy

## ② Nash Equilibrium Multi-Agent

- Prior: Single-pass or simple multi-agent
- **Ours:** Game-theoretic formalization → 94% conflict reduction

## ③ Adversarial Debate Simulation

- Prior: Single-perspective retrieval
- **Ours:** P-D-J simulation → Balanced reasoning

## ④ Hybrid Algorithm + Dynamic Weighting

- Prior: 1-2 algorithms, fixed weights
- **Ours:** Multiple algorithms, intent-based adaptation

## Unique Combination

A novel synthesis of these innovations.

# Dataset & Technology

## Dataset:

- **Cases:** NyayaAnumana cases<sup>1</sup> (49,633 cases)
- **Timespan:** 1950-2024
- **Size:** 3.84 GB raw
- **Edges:** 150k citations
- **Labeling:** Raw text labeled using LGAI exa one model
- **Dataset Split:**
  - Training: 39,706 cases (80%)
  - Validation: 4,963 cases (10%)
  - Test: 4,964 cases (10%)

## Tech Stack:

- **DL:** PyTorch, CUDA
- **Graph DB:** Neo4j
- **Embeddings:** jina-embeddings-v3-base-en, Gemini
- **LLMs:** Gemini 2.5, Gemma 2, LGAI exa one
- **Hardware:**
  - NVIDIA RTX 4500 Ada Generation
  - 48GB RAM

---

<sup>1</sup>Nigam et al., "NyayaAnumana and INLegalLlama: The Largest Indian Legal Judgment Prediction Dataset...", arXiv:2412.08385, 2024.

# HGCN Results

## Hierarchy Preservation:

- Query: SC case ( $r=0.1026$ )
- Retrieved: mean  $r=0.1014$  ✓
- Random: mean  $r=0.1598$  ✗
- **Same hierarchical level!**

## vs Euclidean Baselines:

Method	R@10	Dims	Memory
Jina 768	81%	768	100%
<b>HGCN</b>	<b>88%</b>	<b>64</b>	<b>8.3%</b>

## Key Advantages:

- +7% recall improvement
- 12× compression (64 vs 768 dims)
- 92% memory reduction
- Hierarchy encoding free!

## Innovation

Hyperbolic geometry aligns with legal domain structure!

# Multi-Agent Results

Method	Precision	Recall	Conflicts
Single-Pass	78%	82%	127
Debate (3 rounds)	89%	86%	19
<b>Nash (5 rounds)</b>	<b>92%</b>	<b>88%</b>	<b>8</b>

## Conflict Resolution:

- Cycles: 127 → 119 resolved (**94% reduction**)
- Contradictions: 89 → 84 resolved (**94% reduction**)
- Final graph: Logically consistent

## Impact

+14% precision over single-pass extraction!

# Overall Performance

System	R@10	P@10	F1	Features
BM25	62%	58%	60%	Baseline
LegalBERT	74%	71%	72%	Domain BERT
CaseGNN (SOTA)	82%	79%	80%	Sentence graphs
<b>LegalNexus</b>	<b>88%</b>	<b>86%</b>	<b>87%</b>	<b>Hybrid System</b>

**Dataset:** NyayaAnumana cases (49,633 cases from 1950-2024)

Improvements Over SOTA (CaseGNN)

+6% Recall, +7% Precision, +8.75% F1 Score

# Performance Metrics

Metric	Ours	Best Baseline	Δ
Recall@5	85%	78%	+9%
Recall@10	88%	82%	+7%
Precision@5	87%	81%	+7%
Precision@10	86%	79%	+9%
MAP	91%	84%	+8%
NDCG@10	93%	88%	+6%
<b>HGCN Specific:</b>			
Hierarchy	Yes	No	✓
Dimensions	64	768	12× less
Memory	8.3%	100%	92% saved
<b>Multi-Agent:</b>			
Conflicts	8	127	94% reduced
Precision	92%	78%	+14%

# Comparison

Feature	CaseGNN <sup>1</sup>	SAILER <sup>2</sup>	KELLER <sup>3</sup>	Ours
Geometry	Euclidean	Euclidean	Euclidean	Hyperbolic
Hierarchy	✗	✗	✗	✓
Dimensions	768	768	768	64
Multi-Agent	✗	✗	✗	Nash Eq
Conflict Resolution	Manual	Manual	Manual	94% Auto
Adversarial	✗	✗	✗	P-D-J
Query Expansion	✗	✗	Partial	Full LLM
Recall@10	82%	73%	80%	88%

<sup>1</sup>Tang et al., ECIR 2024.

<sup>2</sup>Li et al., SIGIR 2023.

<sup>3</sup>Deng et al., SIGIR 2024.

# Walkthrough: Initial Query

I was in a drunk driving accident

1 step completed ▾

- Analyzing the user's statement to cite relevant cases and explaining potential legal outcomes.
- Finished

"I was in a drunk driving accident."

Under Indian law, this typically attracts Section 185 of the Motor Vehicles Act, 1988 (drunken driving) and, if there is an accident with injury or death, rash or negligent driving / causing death by rash or negligent act under the penal code (now Bharatiya Nyaya Sanhita).

**Relevant Indian cases**

- Alister Anthony Pareira v. State of Maharashtra (Supreme Court): Drunk driver ran over people on a pavement, killing seven and injuring eight; the Court treated it as culpable homicide (Section 304 Part II) and upheld a substantial custodial sentence, stressing that drunk driving with knowledge of likely consequences is a grave offence.
- Sanjeev Nanda (BMW hit-and-run) case: A high-profile Delhi accident where a drunk driver killed several people; the courts imposed significant imprisonment and highlighted that drunk driving combined with high speed and hit-and-run behaviour justifies severe punishment.
- Delhi trial court drunk-driving pavement case (2014 accident, reported 2025): The driver was sentenced to 10 years' rigorous imprisonment for killing two and injuring ten, with the court emphasising that youth or being unmarried are not mitigating factors when multiple victims are involved.
- Dwarka court case (first-time drunk driving, no major accident): A person convicted for drunk driving received a comparatively lenient sentence (short custody and fine), with the court taking into account that it was a first offence and the person showed remorse.
- Various High Court decisions on sentencing: Recent decisions from Haryana and other courts

A set of small, light-colored navigation icons typically found in presentation software like Beamer, including symbols for back, forward, search, and table of contents.

# Walkthrough: Follow-up Query (Part 1)

I had a high alcohol content and i hit 2 people

Reviewed 3 sources ▾

- Analyzing legal cases involving high alcohol content and their consequences for hit-and-run incidents.

Searching

- Q Indian case high blood alcohol drunk driving two people killed injured sentence
- Q Alister Anthony Pareira facts blood alcohol pedestrians pavement deaths sentence
- Q Indian court high BAC multiple victims drunk driving sentencing trend

- Finished

Courts in India treat "high alcohol + multiple victims" as a serious, jail-worthy situation, not a minor traffic offence.

**How courts have dealt with similar facts**

- In Alister Anthony Pareira v. State of Maharashtra, the driver was drunk, drove fast, ran over people on a pavement, killing seven and injuring eight; he was held guilty of culpable homicide (Section 304 Part II) and the Supreme Court upheld a multi-year jail term, calling even three years "lenient" and stressing that such conduct can justify up to 10 years.
- In a Delhi pavement case (Rishi Kumar), a drunk driver drove onto a footpath, killed two homeless people and injured ten; the court imposed 10 years' rigorous imprisonment under 304 Part II BNS plus other counts, saying young age and being unmarried were not mitigating factors given multiple innocent victims.
- In a Bengaluru case, a drunk driver caused the death of two friends in his vehicle; the sessions court convicted him under Section 304 Part II and sentenced him to five years' simple imprisonment and a fine of ₹1 lakh, relying heavily on the proof of alcohol in his blood and the knowledge that his conduct could cause death.

Across these cases, high blood alcohol and more than one victim push the court towards harsher offences (culpable homicide rather than simple negligence) and real prison time, often in the 3–10 year range, plus fines and compensation.

The screenshot shows a digital research interface. At the top, a search bar contains the query "I had a high alcohol content and i hit 2 people". Below it, a section titled "Reviewed 3 sources ▾" lists the following:

- Analyzing legal cases involving high alcohol content and their consequences for hit-and-run incidents.

Under "Searching", there are three search results:

- Q Indian case high blood alcohol drunk driving two people killed injured sentence
- Q Alister Anthony Pareira facts blood alcohol pedestrians pavement deaths sentence
- Q Indian court high BAC multiple victims drunk driving sentencing trend

At the bottom, a summary states: "Courts in India treat 'high alcohol + multiple victims' as a serious, jail-worthy situation, not a minor traffic offence." A section titled "How courts have dealt with similar facts" provides three examples of legal cases where drivers were convicted of culpable homicide or similar offenses due to high blood alcohol levels and multiple victims, resulting in significant jail terms.

# Walkthrough: Follow-up Query (Part 2)

## What is likely to happen in such a scenario

If someone has a high alcohol level and hits two people, the realistic legal exposure is:

- Criminal case under the MV Act plus BNS/IPC for rash driving and, if there is serious injury or death, likely 304 Part II-type charges, not just 304A, which carry up to about 10 years' imprisonment.
- A strong possibility of a substantial custodial sentence (measured in years, not days), especially if the victims suffered serious injury or died, along with an order to pay compensation, and later civil/MACT liability.

Exact outcome depends on whether the two people were injured or killed, the precise BAC, your conduct after impact (helping vs fleeing), prior record, and how your lawyer argues mitigation, but this pattern of case law shows that courts now take such situations very seriously and do send people to jail.

# Use Case 1: Legal Research

**Real Example:** Lawyer searches for “drunk driving accident causing death”

## LegalNexus:

### Traditional Search:

- Keyword match only
- Returns: 127 cases
- Many irrelevant
- Miss Section 304A IPC
- Time: 15 minutes

### Top Result:

*“XYZ v. State” - Mentions “drunk” but civil case*

- ➊ **Expansion:** “Section 304A IPC, rash and negligent act...”
- ➋ **Retrieval:** 15 highly relevant
- ➌ **HGCN:** SC binding precedent first
- ➍ **Debate:** Both strict + mitigating
- ➎ Time: 90 seconds

### Top Result:

*“Alister Anthony Pareira v. State” (SC 2012) - Exact precedent*

## Impact

10x faster, Precise results, \$500+ savings per case

# Use Case 2: Judicial Decision Support

**Scenario:** Judge reviewing criminal negligence appeal

**LegalNexus Solution:**

## ① HGCN Hierarchy Analysis

- Highlights Supreme Court **binding** precedents ( $r < 0.10$ )
- Shows High Court **persuasive** cases ( $r = 0.10-0.20$ )

## ② Citation Network (Multi-Agent)

- Conflict-free graph (94% conflicts resolved)
- Follow/distinguish/overrule relationships clear

## ③ Adversarial Analysis

- Both prosecution & defense arguments
- Balanced perspective for judgment

**Result:** Informed, consistent, hierarchically-aware judgment

# Use Case 3: Access to Justice

## Democratizing Legal Knowledge

### Problem

Small firms, legal aid → Limited resources, expensive databases (\$500-2000/month)

### LegalNexus Solution

- **Query Expansion:** "landlord evicted me" → Legal terms automatically
- **Open Source:** Self-hosted, 95% cost reduction
- **Fast & Comprehensive:** Minutes vs hours

### Social Impact

Levels playing field for small firms, improves access to justice for underserved

# Broader Impact

## Benefits Across Stakeholders:

### Lawyers:

- 240× faster research
- \$500→\$0.05 per case
- Comprehensive analysis

### Judges:

- Hierarchy consistency
- Conflict-free graphs
- Informed decisions

### Students:

- Interactive learning
- Visual understanding
- Free resource

### Society:

- Access to justice
- 95% cost reduction
- Democratized knowledge

## Estimated Impact

1M+ queries/year, \$50M+ cost savings

# Future Directions

## Technical Roadmap:

- **Multilingual Support:**

- Extend to regional languages (Hindi, Tamil)
- Critical for District Court accessibility

- **Cross-Jurisdictional Transfer:**

- Adapt for Common Law systems (UK, US, Canada)
- Zero-shot transfer learning

- **Real-time Learning:**

- Online updates for new judgments
- Eliminate "Cold Start" latency

## Research Frontiers:

- **Explainable AI (XAI):**

- Visual reasoning paths for judges
- "Why this precedent?" explanations

- **Ethical AI Guardrails:**

- Automated bias detection
- Fairness auditing for minority groups

- **Generative Drafting:**

- Auto-drafting legal arguments based on retrieved precedents

# Conclusion

## Three Main Contributions:

- ① First hyperbolic GCN for legal AI with empirical hierarchy validation
- ② Nash Equilibrium multi-agent framework for knowledge graphs
- ③ Adversarial hybrid retrieval with multi-algorithm combination

## Results:

- 88% Recall@10 (SOTA: 82%) → **+7% improvement**
- 64 dims vs 768 (baselines) → **12x compression**
- 94% conflict reduction → **Logically consistent**

## Impact Beyond Legal AI

Framework applicable to any hierarchical + adversarial domain:

Medical (disease taxonomy), Academic (citations), Corporate (org charts)

Thank You!

## Questions?

### Contact:

Animesh Mishra: am847@snu.edu.in

Keshav Bararia: kb874@snu.edu.in

Kush Sahni: ks672@snu.edu.in

### Supervisor:

Dr. Sonia Khetarpaul: sonia.khetarpaul@snu.edu.in

### Code & Demo:

<https://github.com/amethystani/legalnexus-backend>