Big O and Big Omega Notations

Discusses the Big O notation with examples

In the previous section, we defined Θ notation. In this section, we'll discuss big O and big Ω notations. The reader would notice the use of the adjective "big" with these notations and rightly suspect that there exist complementary small o and small ω notations, which we'll discuss in the next lesson.

Big O

Big O is expressed as *O(g(n))* and is pronounced as "big oh of g of n". We observed that Θ provides both an asymptotically upper and lower tight bound. Big O only provides an asymptotic upper bound which may not necessarily be a tight bound.

Big O is the most commonly talked-about notation when discussing algorithms in industry settings or in interviews. You won't see Θ or other notations being discussed, and for good reason too. Generally, if we are guaranteed that an algorithm will perform no worse than a certain threshold, and that threshold is acceptable for the problem at hand, then knowing its best-case performance or finding a very tight upper bound may not be productive.

Formal Definition

> Similar to Θ we define O(g(n)) as a set of functions and a function f(n) can be a member of this set if it satisfies the following conditions:
>
> $$0 \leq f(n) \leq cg(n)$$
>
> *where c is a positive constant and the inequality holds after n crosses a positive threshold* n_0

Explanation

- if f(n) = O (g(n)) then it also implies that f(n) = O (g(n))

- If f(n) = Θ (g(n)) then it also implies that f(n) = O (g(n))

- The notation means that the function f(n) is bounded by above to within a constant factor of g(n).
- Note that the inequality requires f(n) to be greater than 0 after n crosses n_0.
- The set O(g(n)) may also contain other functions also that satisfy the inequality. There could be several values of *c* that can be used to satisfy the inequality.

Insertion Sort

Previously, we came up with the following f(n) for the insertion sort algorithm

$$f(n) = [2 * (n+1) + 2n] + [2n + 7[\frac{n(n-1)}{2}]]$$

Now we can attempt to find the O(g(n)) for this expression. As a general rule of thumb, when working with big O we can drop the lower order terms and concentrate only on the higher order terms. The term $n(n-1)/2$ is the highest order term, since it involves a square of n. The above expression is thus quadratic, and we can ignore all the constants and linear terms. Thus a tight bound on the expression would be $O(n^2)$. We can prove it below:

$$\frac{n(n-1)}{2} \leq cn^2$$

$$let\ c = 10\ and\ n_0\ = 10$$

You can always pick a different set of constants as long as it satisfies the inequality for all $n > n_0$.

As a general rule of thumb, for an expression consisting of polynomials the expression/function is *O(the highest polynomial degree in the expression)*.

Big Ω

The complementary notation for big O is the *big omega* notation. The big omega notation provides an asymptotic lower bound and is expressed as Ω(g(n)).

Formal Definition

> As before, Ω(g(n)) is a set of functions and any function f(n) that satisfies the below constraints belongs to this set and f(n) is said to be big omega of g(n).
>
> $$0 \leq g(n) \leq f(n)$$
>
> $where\ c\ is\ a\ positive\ constant\ and\ the\ above\ inequality\ holds\ afte$
> $r\ n\ crosses\ a\ positive\ threshold\ n_0$

Explanation

- Notice that similar to big O, the definition mandates we only consider positive values for f(n).
- Big Ω is not necessarily a tight lower bound.
- If f(n) Θ(g(n)) then it also implies f(n) Ω(g(n)).

Relation to Θ

One can see that if a function **f(n)** is **Θ(g(n))** then it follows that **f(n)** must be **O(g(n))** and **Ω(g(n))**. In the previous lesson we proved that ***f(n)*** = ***2n²-1*** was Θ((n²)). We can simply consider the right and left handsides of the following inequality in isolation to prove that f(n) is also O(g(n)) and Ω(g(n)).

> $$1(n^2 + 2) < 2n^2 - 1 < 2(n^2 + 2)$$

Quiz

1

You are told that a function f(n) is $O(n^3)$, does it also imply that the f(n) is $\Theta(n^3)$?

2

You are told that a function f(n) is O(n) and also $\Omega(n)$, does it also imply that the f(n) is $\Theta(n)$?

Check Answers