Small omega and Small o Notations

In this lesson, we discuss notations which imply loose bounds.

The small o and small ω are complementary notations to the big O and big Ω notations. For algorithm analysis, the most important notation is the big O. For the sake of completeness, we mention the small o and small ω notations too.

Small o

The small o is **not** an asymptotically tight upper bound. The formal definition is similar to big O, with one important difference. A function f(n) belongs to the set o(g(n)), if the following condition is satisfied.

> $$0 \leq f(n) < cg(n)$$
>
> $for\ any\ positive\ constant\ c,\ there\ exists\ some\ constant\ n_0\ which\ if\ n\ surpasses\ ,\ the\ above\ inequality\ holds$

Note that the inequality should not hold for *some* positive constant c, as is the case for big O; rather, it should hold for all positive constants.

Unlike big O, which may or may not be tight, the small o notation is necessarily not tight.

Small ω

Small ω is similarly **not** a tight lower bound. Big Ω, on the contrary, may or may not be a tight bound. A function f(n) belongs to the set ω(g(n)) if the following inequality holds:

> $$0 \leq g(n) < cf(n)$$

For **any** positive constant c, there exists some constant n_0 which if n surpasses, the above inequality holds. The above inequality should hold for **all constants**. Note that in the case of big omega, the inequality was required to hold for *some constant*.

Explanation

We can see that the small case notations are, in a sense, *relaxed* compared to their upper case notations.

- $2n^2$ is equal to $\omega(n)$ but $2n^2 \neq \omega(n^2)$. To understand the first claim consider the inequality:

  $$cn \leq 2n^2$$

  Let's pick a really big number for c=1000. To make the inequality hold I can set n_0=c=1000, and because of n being squared the right side will be bigger for every value of n greater than n_0.

  Now, to understand why $2n^2 \neq \omega(n^2)$, consider the inequality:

  $$cn^2 < 2n^2$$

  If I pick c = 3 then - no matter what value of n_0 I choose - I can never satisfy the above inequality.

- For small o, $2n^2$ is $o(n^3)$ but $2n^2 \neq o(2n^2)$

1

If f(n) is $\Theta(n^3)$, is f(n) also $o(n^3)$ and $\omega(n^3)$?

2

If $f(n)$ is $O(n^3)$, is $f(n)$ also $o(n^3)$?

Check Answers