TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
THAPATHALI CAMPUS

INSTRUMENTATION I

Chapter 2: Theory of Measurement

BEL, BEX, BCT II-II

Er. Umesh Kanta Ghimire

Department of Electronics & Computer Engineering

IOE ,Thapathali Campus

ukg@tcioe.edu.np

+977-9843082840

23 August, 2020

Ch.2: Theory of measurement (10 hrs./16 Marks)

2.1 Static performance parameters - accuracy, precision, sensitivity, resolution and linearity

2.2 Dynamic performance parameters - response time, frequency response and bandwidth

2.3 Error in measurement

2.4 Statistical analysis of error in measurement

2.5 Measurement of voltage & current (moving coil & moving iron instruments)

2.6 Measurement of low, high & medium resistances

2.7 AC bridge & measurement of inductance and capacitance

Introduction

- Measurement is the act of process of comparison between the unknown physical quantity to its predefined standard in order to find the value of unknown Quantity.
- The measurement is said to be proper if the measured value is less erroneous.
- To make the error free measurement, the user must know the characteristics & operation of instruments ,basic laws to handle instruments and causes of errors in measurement.
- Hence in theory of measurement we are studying about the characteristics of the instruments, errors and analysis, statistical analysis of error and minimization of that error or controlling of error etc.
- Measurement System Performance
- Characteristics of instruments show the performance of instruments to be used. It determines how faithfully the system measures the desired output. Quantitatively it relates the degree of approach to perfection.
- The treatment of instrument and measurement system characteristics can be divided into two distinct categories:
    a) Static characteristics
    b) Dynamic Characteristics

Static characteristics/Static performance parameters

- Static characteristics of a measurement system are, in general, those that must be considered when the system or instrument is used to measure a condition not varying with time.
- Ex: accuracy, precision, linearity, hysteresis etc.

Dynamic Characteristics/Dynamic performance parameters

- Many measurements are concerned with rapidly varying quantities, and therefore, for such cases we must examine the dynamic relations which exist between the output and the input.
- This is normally done with the help of differential equations.
- Performance criteria based on dynamic relations constitute the dynamic performance criteria (Characteristics) or the dynamic characteristics.
- Ex: Frequency response, bandwidth, time response, dynamic error. etc.

Static performance parameters

- Static performance Parameters are those criteria of a measurement system, those that must be considered when the system or instrument is used to measure a condition not varying with time.
- Some of the important static performance parameters are listed below:
    a) Accuracy
    b) Precision
    c) Sensitivity
    d) Resolution
    e) Linearity
    f) Reproducibility
    g) Repeatability
    h) Drift
    i) Dead time
    j) Dead space
    k) Static Error
    l) Instrument hysteresis
    m) Tolerance
    n) Range of span
    o) Threshold
    p) stability
    q) Bias

Accuracy

- Accuracy is the closeness with which an instrument reading approaches the true value (TV) of the variable being measured.
- It is the degree of closeness or conformity to the true value of the quantity under measurement.
- Deviation from the true value is the indication of how accurately a measurement has been done
- For analog instrument, accuracy refers to their full scale reading unless specified
- The accuracy may be specified in terms of inaccuracy or limits of error and can be expressed in the following ways
    I. Point accuracy
    II. Accuracy as 'Percentage of scale range'
    III. Accuracy as 'Percentage of true value'

Point accuracy

- This is the accuracy of the instrument only at one point on it scale
- The specification of this accuracy does not give any information about the accuracy at other points on the scale or the general accuracy of the instrument

Accuracy as "Percentage of scale Range"

- When an instrument has uniform scale, its accuracy may be expressed in terms of scale range
- Ex: The accuracy of thermometer having range of 500℃ may be expressed as 0.5% of scale range
- But while the reading of thermometer be 25°C then the error is high as $\frac{500}{25} \times (\pm 0.5\%) = \pm 10\%$
- So, accuracy as percentage of scale range specification is highly misleading

Accuracy as "percentage of true value"

- The best way to conceive the idea of accuracy is to specify it in terms of the true value of the quantity being measured
- If accuracy is specified as 5% of true value, then this statement means that as the readings get smaller so do the errors.

If a voltmeter having accuracy of 1% of its full scale reading of 100 V is used to measure (i) 80V, (ii) 12V ;
How accurate will the reading be ?

- When True value (T.V.) = 80 V

  Measured value (MV) = 80 V $\pm$ 1% of 100 V
  = 80 V $\pm$ 1V
  i.e. 81 V or 79 V
  So error = TV – MV = 80-79 = 1V
  % Error = $\frac{TV-MV}{TV} \times 100\% = \frac{1}{80} \times 100\% = 1.25\ \%$

- When True value (T.V.) = 12V

  Measured value (MV) = 12 V $\pm$ 1% of 100 V
  = 12V $\pm$ 1V
  i.e. 13 V or 11 V
  So error = TV – MV = 12-11 = 1 V
  % Error = $\frac{TV-MV}{TV} \times 100\% = \frac{1}{12} \times 100\% = 8.333\ \%$

Precision

- Precision is the measure of the reproducibility of the measurements
- i.e. it gives a fixed value of a variable
- Precision is a measure of the degree to which successive measurements differ from one another.
- Precision also refers to the degree of agreement within a group of measurements or instruments
- Precision is composed of two characteristics:
- Conformity & the number of significant figures to which a measurement may be made.
- To differentiate between accuracy & precision ; Consider an instrument that has a difference in its operation. The instrument may be giving a result that is highly repeatable from measurement to measurement yet far from true value (T.V)

Precision....

- The data obtained from this instrument would be highly precise but inaccurate.
- Therefore, precision doesn't guarantee accuracy ; Although accuracy requires precision
- In short, more significant figures in measured value the greater the precision of measurement, And higher degree of closeness or conformity to the true value of the measurand guarantees the accuracy
- Ex: For measuring 4.0V (True Value TV)
- 3.99 V -------------accurate
- 3.989879 V-------precise (if repeated)

Sensitivity

- The static sensitivity of an instrumentation system is the ratio of the magnitude of the output signal or response to the magnitude of input signal or the quantity being measured
- Its units are millimeter per microampere, counts per volt etc. depending upon the type of input and output
- If 'S' be the sensitivity, Δq_o be change in output & Δq_i be change in input then

$$\text{Sensitivity (S)} = \frac{\text{small change in output signal}}{\text{small change in input signal}} = \frac{\Delta qo}{\Delta qi}$$

Sensitivity….

- When a calibration curve is linear as shown in the fig (a), the sensitivity of the instrument will be constant over the entire range
- But when the calibration curve is not a straight line ,the sensitivity varies with the input as shown in figure (b)

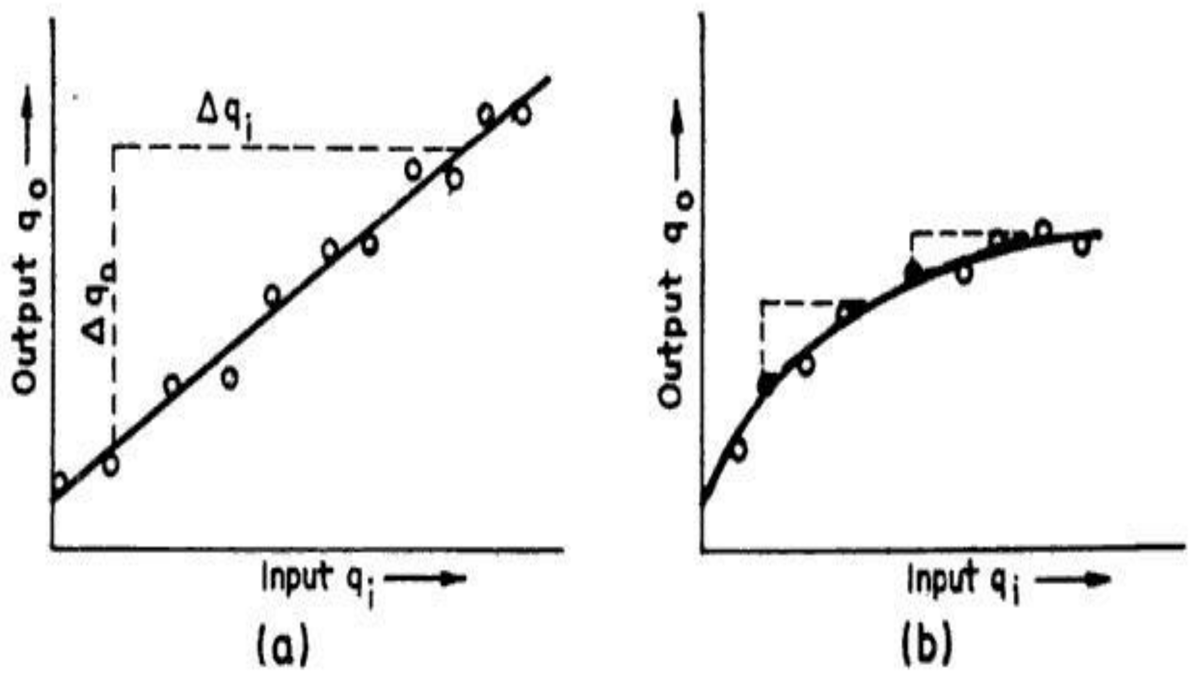


Fig. Definition of static sensitivity

Resolution

- It is the smallest change in measured value to which the instrument will respond.
- If an input is given to an instrument and is slowly increased, it will be observed that the output does not change until a certain increment is increased, this increment is termed as <u>resolution</u>
- Thus it is defined as the smallest change in the input (i/P) which results in a detectable output (o/p)
- In the case of analog instrument it is the significant of the smallest division in the scale where as in the case of digital meters, it is the significance of the least significant Bit (LSB)
- Higher resolution means finer detail i.e. smaller division

Linearity

- Linearity means constant sensitivity throughout the whole measurement range.
- It is defined as the ability to produce the input (i/p) characteristics symmetrical and this can be expressed by the equation in slope intercept form as

  $y = mx + c$;

  where y = o/p of the system or instrument

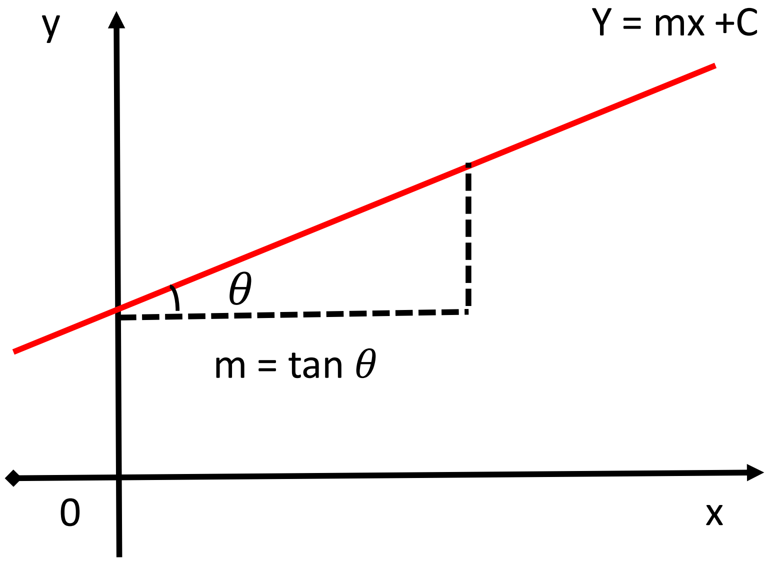
  x = input given to instrument

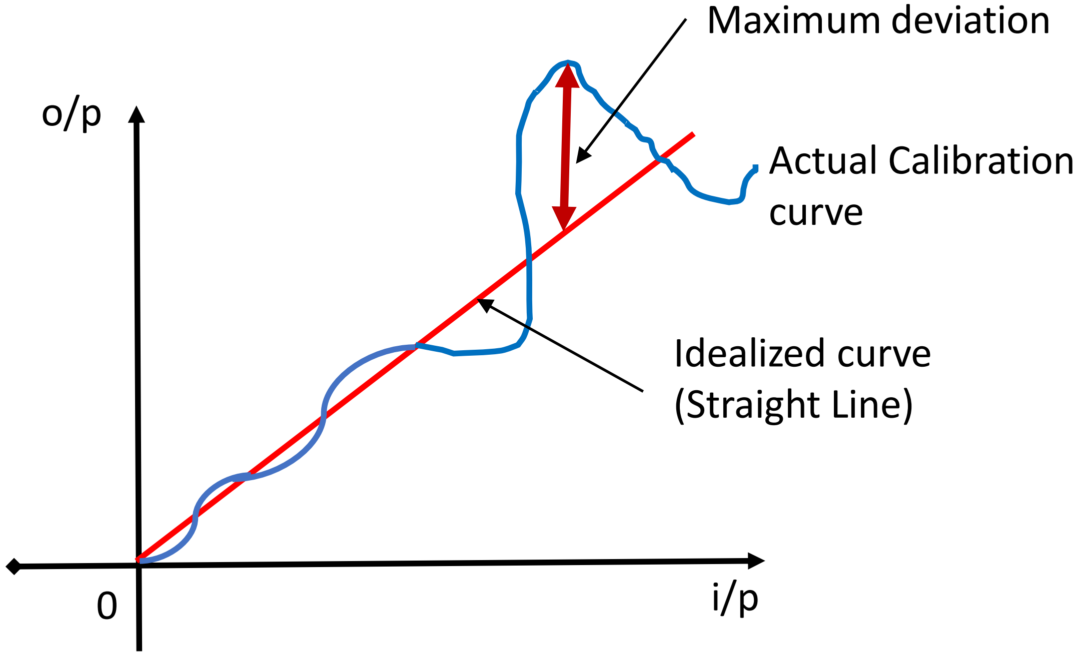
  m= slope found in characteristics (response)

  C = the intercept in the output axis

Linearity….

- The linearity is simply a measure of maximum deviation of any of the calibration points from the straight line
- linearity tells us how well the instrument measurement corresponds to reality



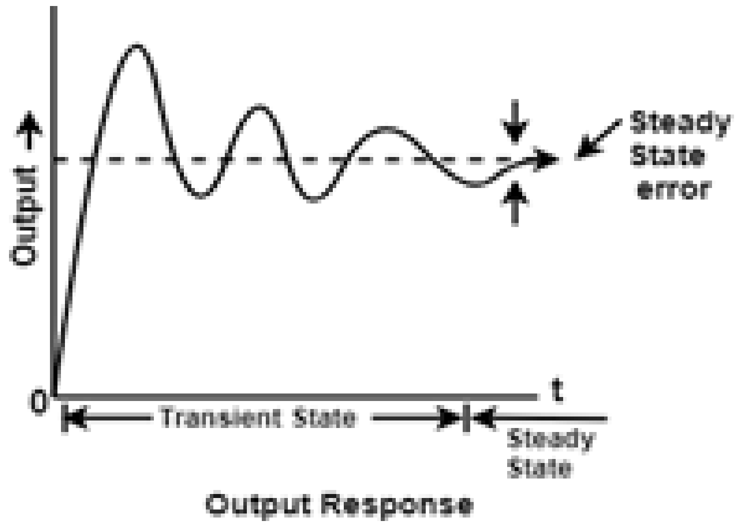


Dynamic performance parameters

- When an input is applied to an instrument or a measurement system, the instrument or the system can't take up its final steady state position immediately
- The system goes through a transient state before it finally settle to it final steady state position
- When the quantity under measurement changes rapidly with time, it is necessary to study the dynamic relations existing between input and output which is expressed as differential equations. The set of criteria defined based on such dynamic differential equation is called dynamic characteristics.
- The term dynamic characteristics is used for the study of behaviors of the system between the time that the output(o/p) value changes and the time, the value has settled down to its steady state value.

Dynamic performance parameters….

- The dynamic characteristics of any measurement system are listed below
  - Delay time
  - Rise time
  - Peak time
  - Settling time
  - Maximum overshoot
  - Speed of response
  - Response time
  - Frequency responses
  - Bandwidth
  - Signal to Noise Ratio (SNR)
  - Damping factor
  - Fidelity
  - Dynamic error



Dynamic performance parameters….

Delay Time (t_d)

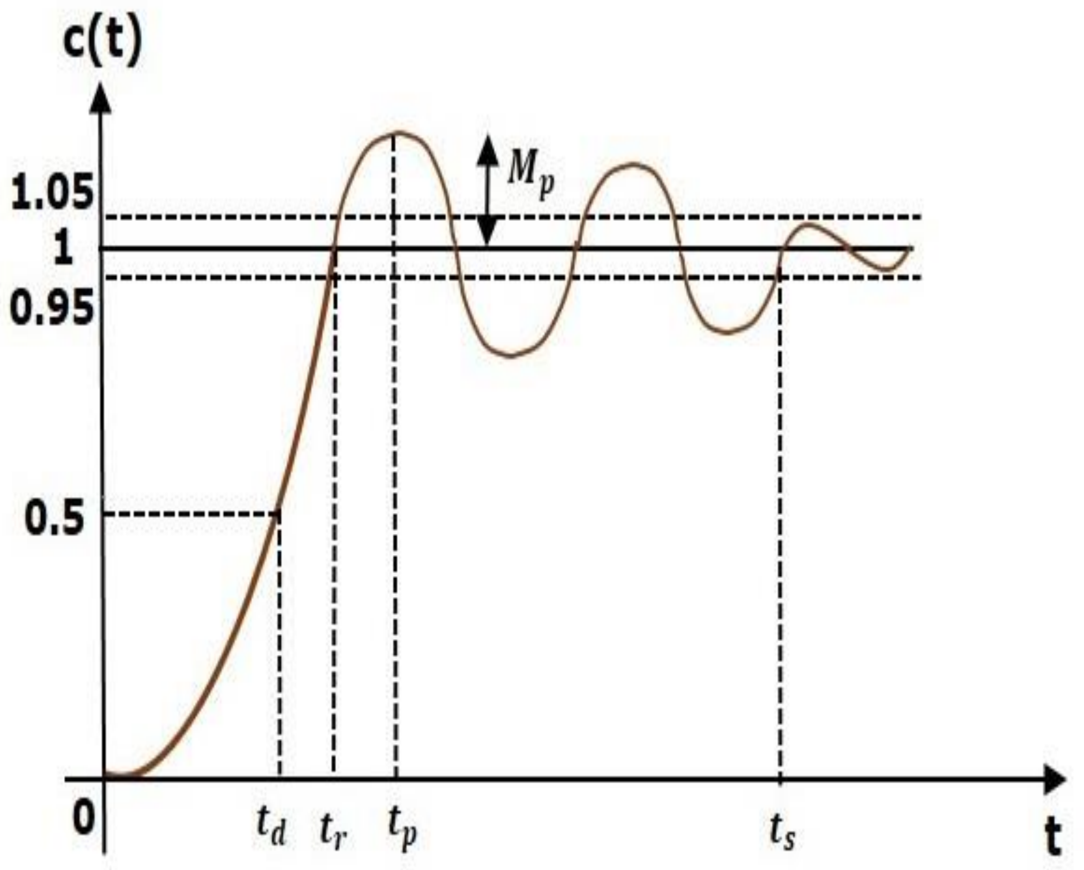
It is the time required for the response to reach half of its final value from the zero instant

Rise Time (t_r)

It is the time required for the response to rise from zero instant 90% or 100% of its final value

Peak Time (t_p)

It is the time required for the response to reach the peak value for the first time.



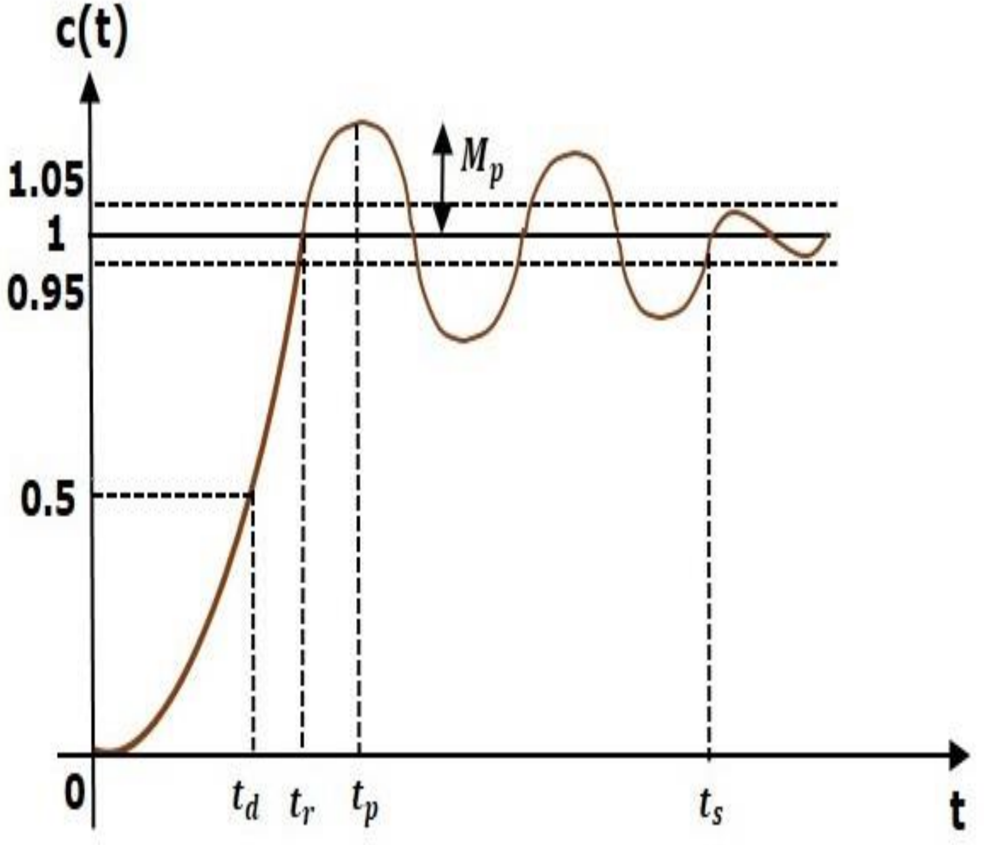
Dynamic performance parameters….

Peak Overshoot (M_p)

It is defined as the deviation of the response at peak time from the final value of response. It is also called the maximum overshoot

Settling time (t_s)

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%



Dynamic performance parameters….

Speed of response

It indicates how fast the input given to the measurement or an instrument brings the output

It is defined as the rapidity with which a measurement system responds to changes in the measured quantity

Response time

It is defined as the time required by the system to settle to it final steady state position after the application of input

Measuring lag

An instrument doesn't immediately react to a change in input.

So measuring lag is the retardation or delay in the response of a measurement system to changes in the measured quantity.

The measuring lags are of two types:

a) Retardation type

   The response of the measurement begins immediately after a change in measured quantity has occurred.

b) Time delay type

   The response of the measurement system begins after a dead time after the application of the input

Dynamic performance parameters….

Frequency responses

- If $v_2(s)$ is o/p and $v_1(s)$ as i/p then

  $$TF=T(s)=\frac{v_2(s)}{v_1(s)}$$

  As $S \rightarrow j\omega$

- $T(j\omega)=\frac{v_2(j\omega)}{v_1(j\omega)}=Re[T(j\omega)]+Im\ j\ [T(j\omega)]$

  $|T(j\omega)| + <\theta(j\omega)$

- As Transfer function TF constitutes of both real and imaginary parts , frequency response involves studying both magnitude and phase response of the system
- As Transfer function TF constitutes of both real and imaginary parts , frequency response involves studying both magnitude and phase response of the system
- The plot of Transfer function $T(j\omega)$ against the frequency (ω) is called frequency response of the system
- The plot of magnitude $|T(j\omega)|$ vs. frequency is called magnitude plot
- The plot of phase $\theta(j\omega)$ vs. frequency is called phase plot

Dynamic performance parameters….

Bandwidth

- The frequency range over which an instrument or measurement system is designed to operate to provide the output receiving the input signal or quantity with a constant gain is known as the bandwidth of the corresponding instrument or measurement system.
- In another word, It is defined as the quantity of the measurement system taken in the interval between those frequencies where the power gain of the systems has dropped to one half of its mid-frequency value
- It is calculated from magnitude plot (gain vs. frequency curve) of the instrument or measurement system and its unit is in Hz

Dynamic performance parameters….

Bandwidth….

- Range of frequencies that belongs to half power points
- Range of frequencies that belongs to -3.02 dB gain
- Range of frequencies that belongs to 0.707 voltage gain
- Range of frequencies where magnitude is larger than 0.707 times of its maximum magnitude
- BW = $|f_2 - f_1|$

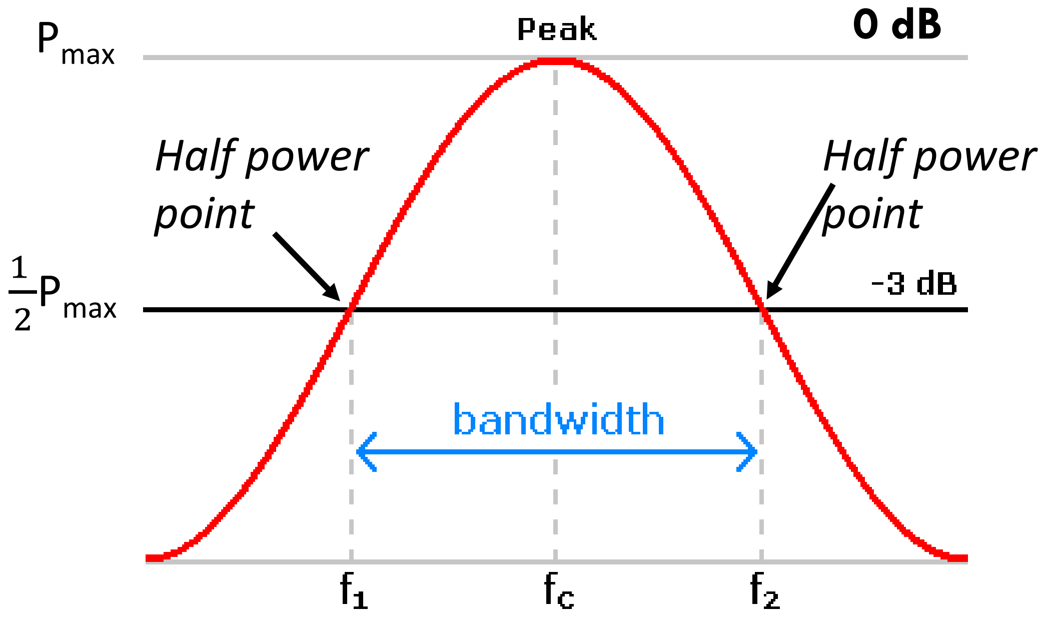


Fig: gain vs. frequency curve

Errors in measurement

- Error may be defined as the deviation of the measured value (MV) from the true value (TV) of the quantity under measurement.
- Diagrammatically it is shown below

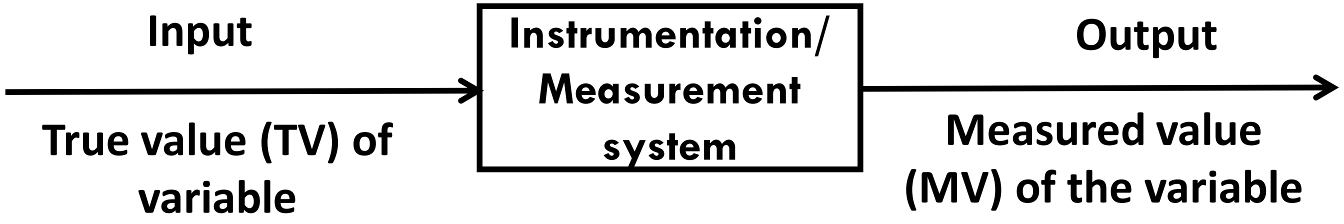


Fig: A simple block representation of instrumentation/ measurement system

- Absolute Error (δE)= TV-MV
- $\text{Percentage Error}(\%\delta E) = \frac{\text{Absolute error}}{\text{True value}} X\ 100\%$

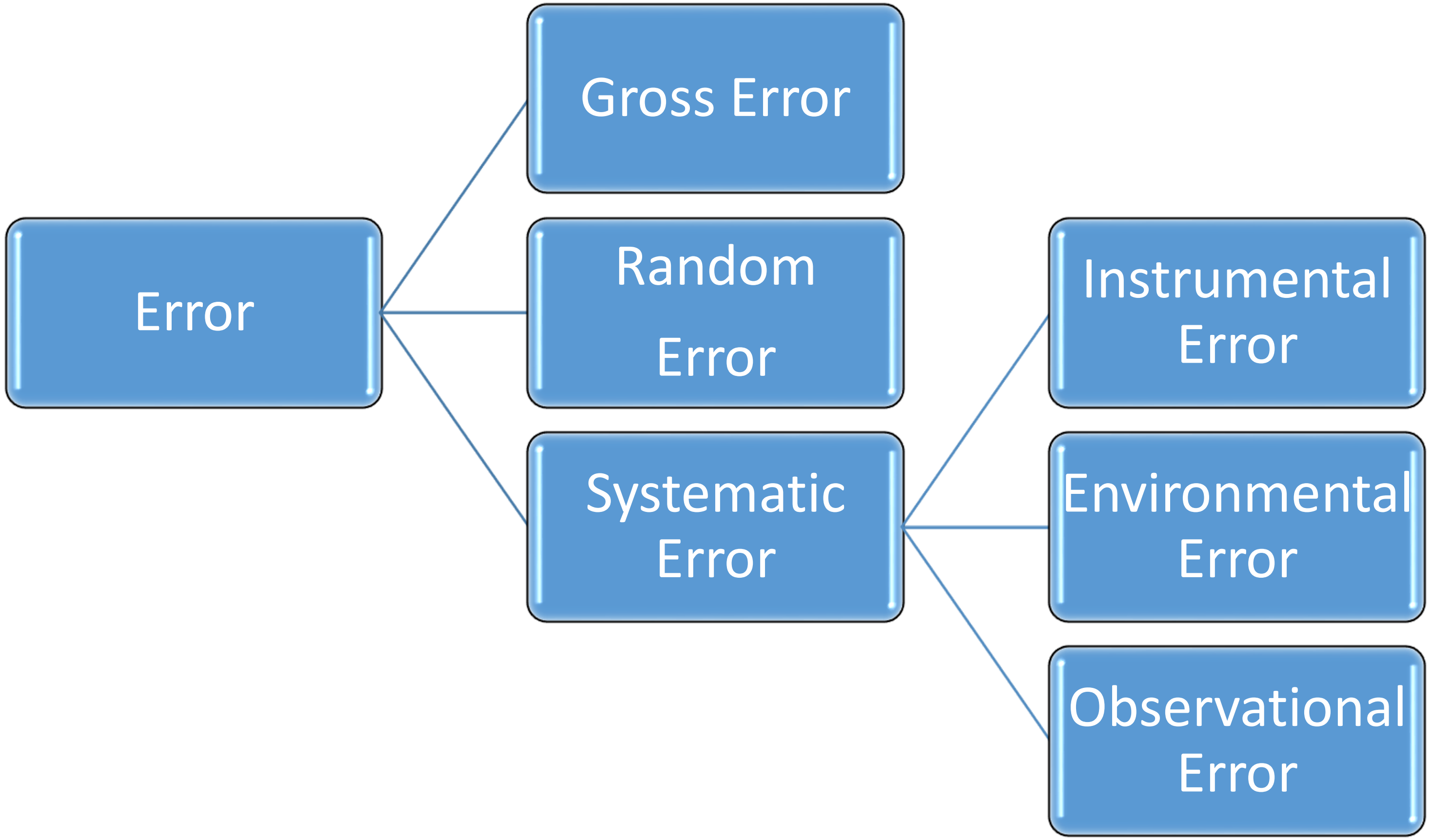
Errors in measurement

- ❖ $\text{Relative Error} = \frac{\text{Absolute error}}{\text{True value}}$
- ❖ Accuracy = 1 - Relative Error
- ❖ Percentage Accuracy = (1-Relative Error) x 100%

- ➢ No measurement can be made without perfect accuracy but it is important to find out what the accuracy actually is and how different errors have entered in to the measurement
- ➢ A study of error is the first step in finding ways to reduce them
- ➢ Such study also allows us to determine the accuracy of the final test result
- ➢ Errors come from different source and usually classified under three main headings

Types of Error

The algebraic difference between the indicated value and the true value of the quantity to be measured is called an **Error.**



Gross Error/ Human Error

- This category of errors includes all the human mistakes while reading, recording and calculating the results.
- It can occur due to incorrect adjustments or wrong connection of instruments.
- Ex: while taking the reading from the meter one may read 12 as 21.

To reduce Gross error /corrective measures:

- Take care while reading, recording and calculating results.
- At least 3 observation of the same quantity should be taken by the same instrument preferably by the different observer

Systematic Error

- A constant uniform deviation in operation of an instrument is known as systematic error
- All the error due to the short-comings of the instrument such as less accuracy in the scale calibration, defective or worn parts, and effects of the environment on the equipment or the user are systematic error

Corrective measures:

- ➢Use proper instrument for proper application
- ➢Apply corrective factor
- ➢Don't use the defective or worn instruments
- ➢Calibrate the instrument with the standard instrument

Types of Systematic Error

1. Instrumental Error

- These errors may be due to their mechanical structure like wrong construction, calibration of the measuring instruments.
- These types of error may be arises due to friction or may be due to hysteresis.
- These types of errors also include the loading effect and misuse of the instruments.
- These errors also arise due to short comings and characteristics of the material used in instrument like worn parts, ageing effects etc.
- In order to minimize the instrumental errors in measurement various correction factors must be applied and in extreme condition instrument must be recalibrated carefully.

Types of Systematic Error….

2. Environmental Error

- This type of error arises due to the surrounding environment to the instrument.
- External condition includes temperature, pressure, humidity or it may include external magnetic field ,electrostatic field and radio frequency

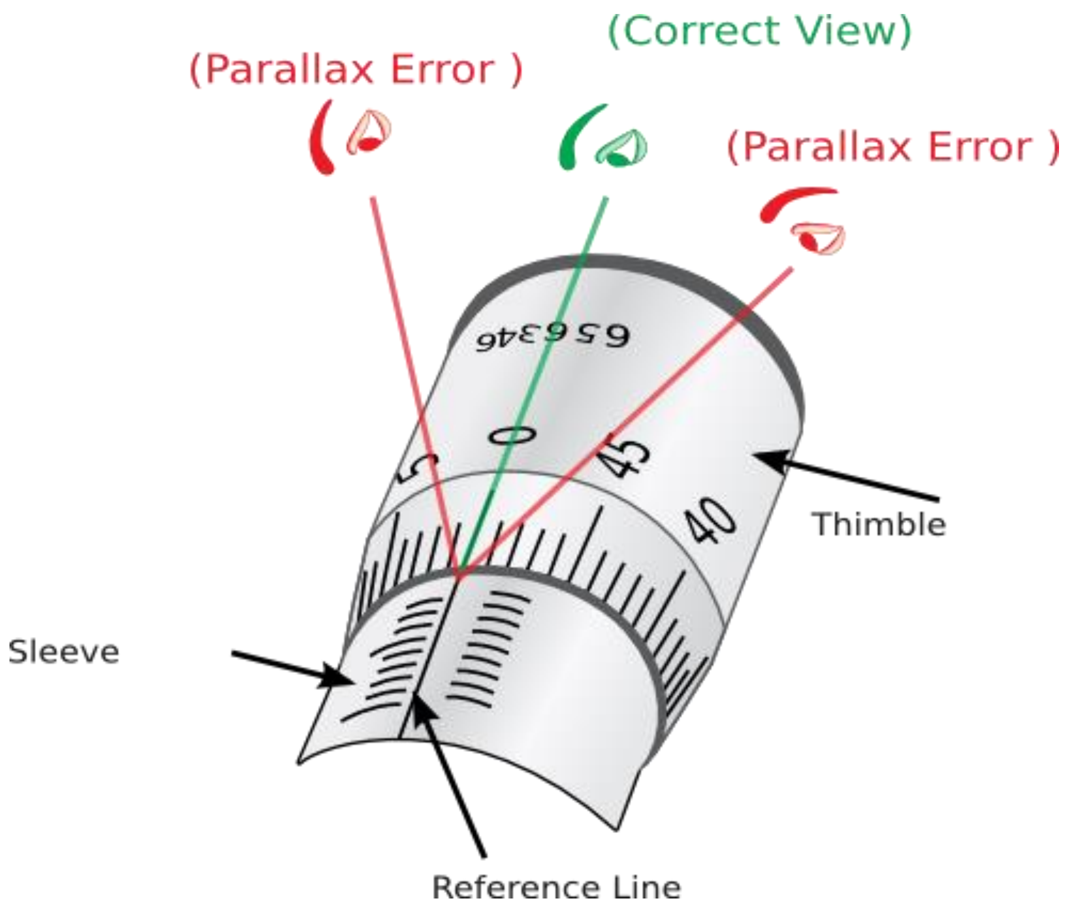
Corrective measures:

- Use proper correction factors given by the manufacturer
- Seal the components [casting should be proper]to avoid dust, humidity
- Provide magnetic or electrostatic shields
- Maintain controlled temperature (like in Air conditioned AC room) ,pressure and humidity of the laboratory

Types of Systematic Error….

3. Observational Error

- As the name suggests these types of errors are due to wrong observations.
- The wrong observations may be due to PARALLAX.
- In order to minimize the PARALLAX error highly accurate meters are required, provided with mirrored scales.



Random/Residual Error

- After calculating all systematic errors, it is found that there are still some errors in measurement are left.
- These errors are known as random errors.
- These errors are unrepeatable, inconsistent errors, resulting in scatter in the output data. Some of the reasons of the appearance of these errors are known but still some reasons are unknown. Hence we cannot fully eliminate these errors.
- The effect of random error is minimized measuring the given quantity many times under same conditions and calculating the arithmetic mean of obtained value i.e. minimized using Statistical analysis

Assignment

An analog voltmeter is used to measure 50V across a resistor, the reading is 49V. Find

i. Absolute error

ii. Relative error

iii. Accuracy

iv. Percentage accuracy

Solution,

True value (TV) = 50V

Measured value (MV) = 49V

Absolute error (E) = 50V - 49V = 1V

Relative error = E/TV = 1/50 = 0.02

Accuracy (A) = 1 - Relative error (E) = 1 – 0.02 = 0.98

% accuracy = 0.98 $\times$ 100% = 98%

Statistical analysis of Error in Measurement

- The random errors are coming from the unknown sources
- So it is very difficult to correct the random error
- Statistical approaches are normally preferred to minimize the random error
- A number of measurements of a quantity have data scattered around a central value.
- Statistical tools can be employed to reach best approximation to the true value of the quantity
- The techniques are
  a) Arithmetic mean
  b) Deviation from the mean(mean deviation)
  c) Average deviation
  d) Standard deviation

Arithmetic mean

- Let $X_1, X_2, X_3, \ldots, X_n$ are n different measurements of same quantity (samples)
- Most probable value from various observations of value can be obtained using arithmetic mean

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \ldots\ldots\ldots + Xn}{n}$$

Where ,

X_i = different observed values of the variables

n = number of observations

In general form , the arithmetic mean be

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Mean deviation

- Deviation from mean
- It is the departure of the observed value/readings from the arithmetic mean of the group of readings

$$d_1 = X_1 - \bar{X}$$
$$d_2 = X_2 - \bar{X}$$
$$d_3 = X_3 - \bar{X}$$
$$\ldots\ldots\ldots\ldots$$
$$d_n = X_n - \bar{X}$$

- d_i may be positive or negative
- The algebraic sum of the mean deviation must be zero

$$\sum_{i=1}^{n} d_i = \sum_{i=1}^{n} [X_i - \bar{X}] = 0$$

Average deviation

- Is an indication of the precision of the instruments used in making the measurements
- Sum of the absolute values of deviations divided by the number of reading

$$D = \frac{|d_1|+|d_2|+|d_3|+\ldots|d_n|}{n}$$

- In general form,

$$D = \frac{1}{n}\sum_{i=1}^{n}|d_i|$$

Standard deviation

- Root mean square (RMS) deviation
- Square root of the sum of the individual deviations squared divided by the number of readings

❖ For reading n>20,

$$S.D.\ (\sigma) = \sqrt{\frac{d_1^{\ 2}+d_2^{\ 2}....+dn^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n} d_i^{\ 2}}{n}}$$

❖ For reading n<20,

$$S.D.\ (\sigma) = \sqrt{\frac{d_1^{\ 2}+d_2^{\ 2}....+dn^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} d_i^{\ 2}}{n-1}}$$

Variance

it is the square of standard deviation

$$\therefore V = (\sigma)^2$$

❖For n>20,

$$V = (\sigma)^2 = \frac{d_1^{\ 2} + d_2^{\ 2} \ldots + dn^2}{n} = \frac{\sum_{i=1}^{n} d_i^{\ 2}}{n}$$

❖For n<20,

$$V = (\sigma)^2 = \frac{d_1^{\ 2} + d_2^{\ 2} \ldots + dn^2}{n-1} = \frac{\sum_{i=1}^{n} d_i^{\ 2}}{n-1}$$

Range

The range of a set of data is the difference between the largest and smallest values

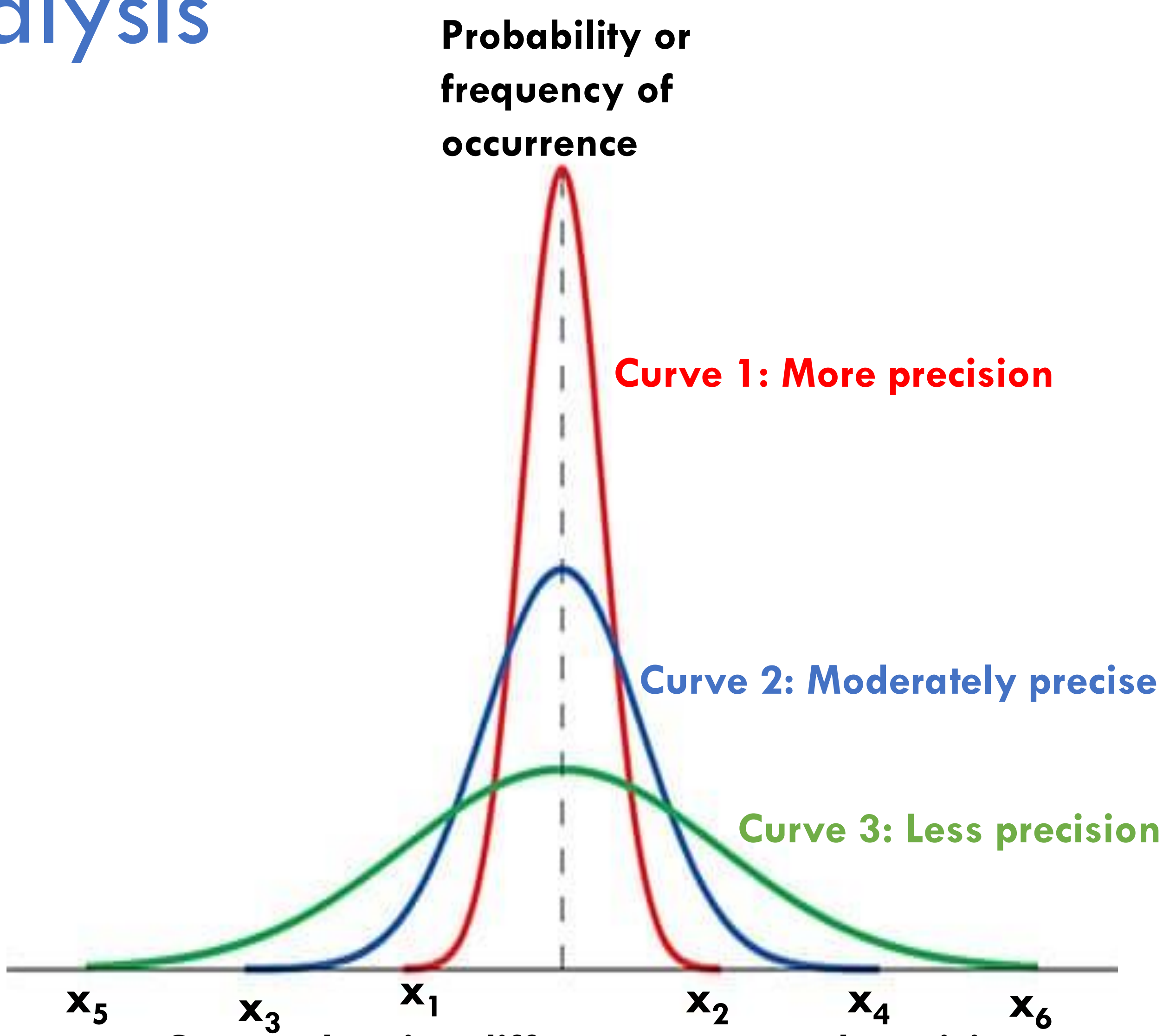
Mode

The mode of a set of data values is the value that appears most often

Basics of Statistical Analysis

Dispersion(Spread or Scatter)

- The property which denotes the extent to which samples are dispersed around a central value (mean) .
- Curve 1: Data spreads in the range x_1 to x_2. The observations are less disperse (more precise)
- Curve 2: Data spreads in the range x_3 to x_4. The observations are moderately disperse (moderately precise)
- Curve 3: Data spreads in the range x_5 to x_6. The observations are more disperse (less precise)



Curves showing different ranges and precision

Normal or Gaussian Curve

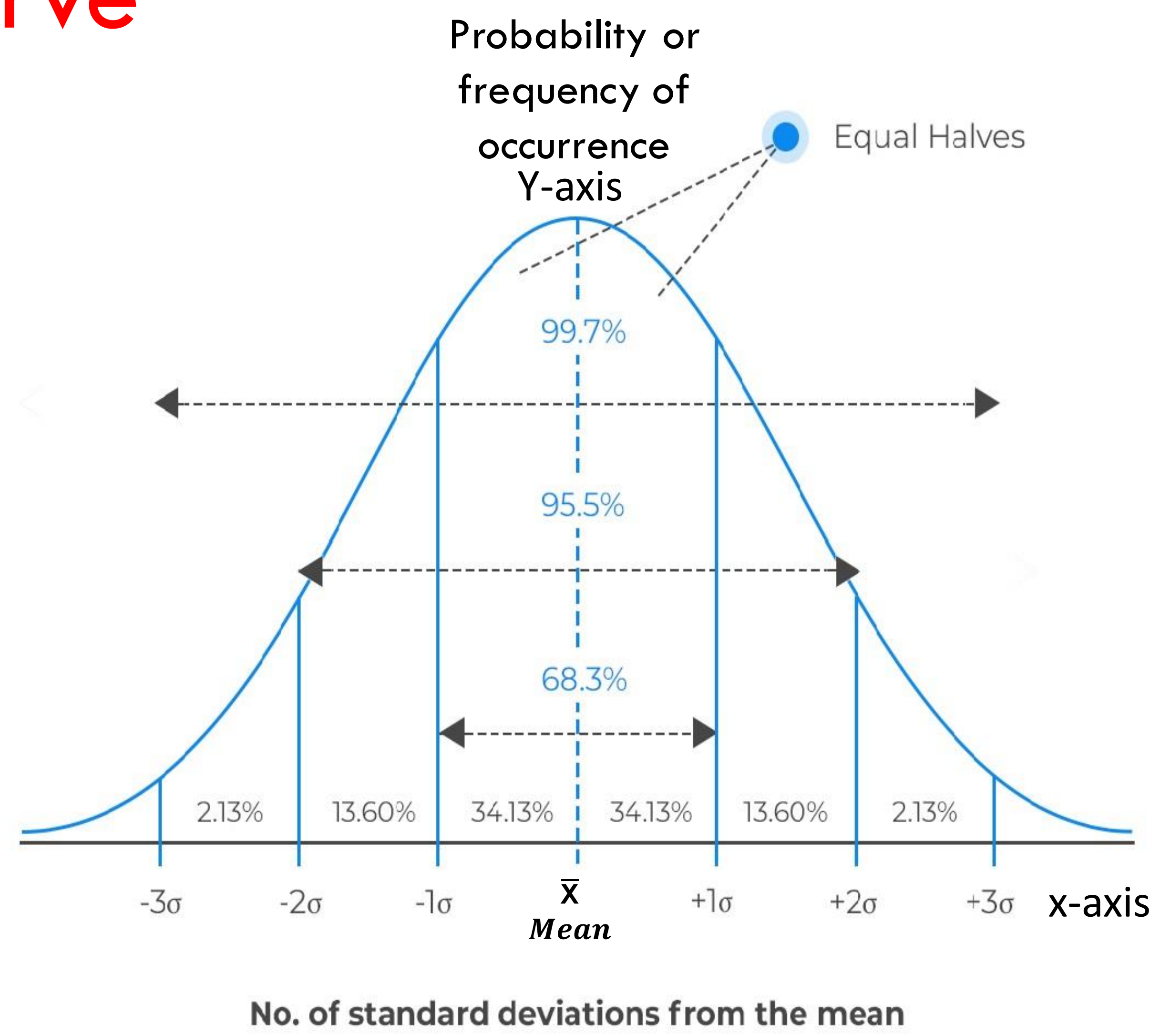
- Gaussian curve is symmetric about arithmetic mean and area under the curve is one. Therefore, data is normalized to be zero mean
- Most of natural events, measurements having some amount of randomness follows Gaussian curve
- For Normal curve, probability y is given as

$$Y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\bar{X}}{\sigma}\right)^2}$$

X = Normalized value (deviation) i.e. $\bar{X}$=0

$\bar{X}$ = mean

σ = Standard deviation



Normal or Gaussian Curve….

- Gaussian distribution (also known as normal distribution) is a bell-shaped curve, and it is assumed that during any measurement values will follow a normal distribution with an equal number of measurements above and below the mean value.
- In order to understand normal distribution, it is important to know the definitions of “mean,” “median,” and “mode.”
- If a distribution is normal, then the values of the mean, median, and mode are the same.
- However, the value of the mean, median, and mode may be different if the distribution is skewed (not Gaussian distribution).

Normal or Gaussian Curve….

❖ **Characteristics of Gaussian distributions**

➢ Mean ± 1 SD contain 68.2% of all values.

➢ Mean ± 2 SD contain 95.5% of all values.

➢ Mean ± 3 SD contain 99.7% of all values.

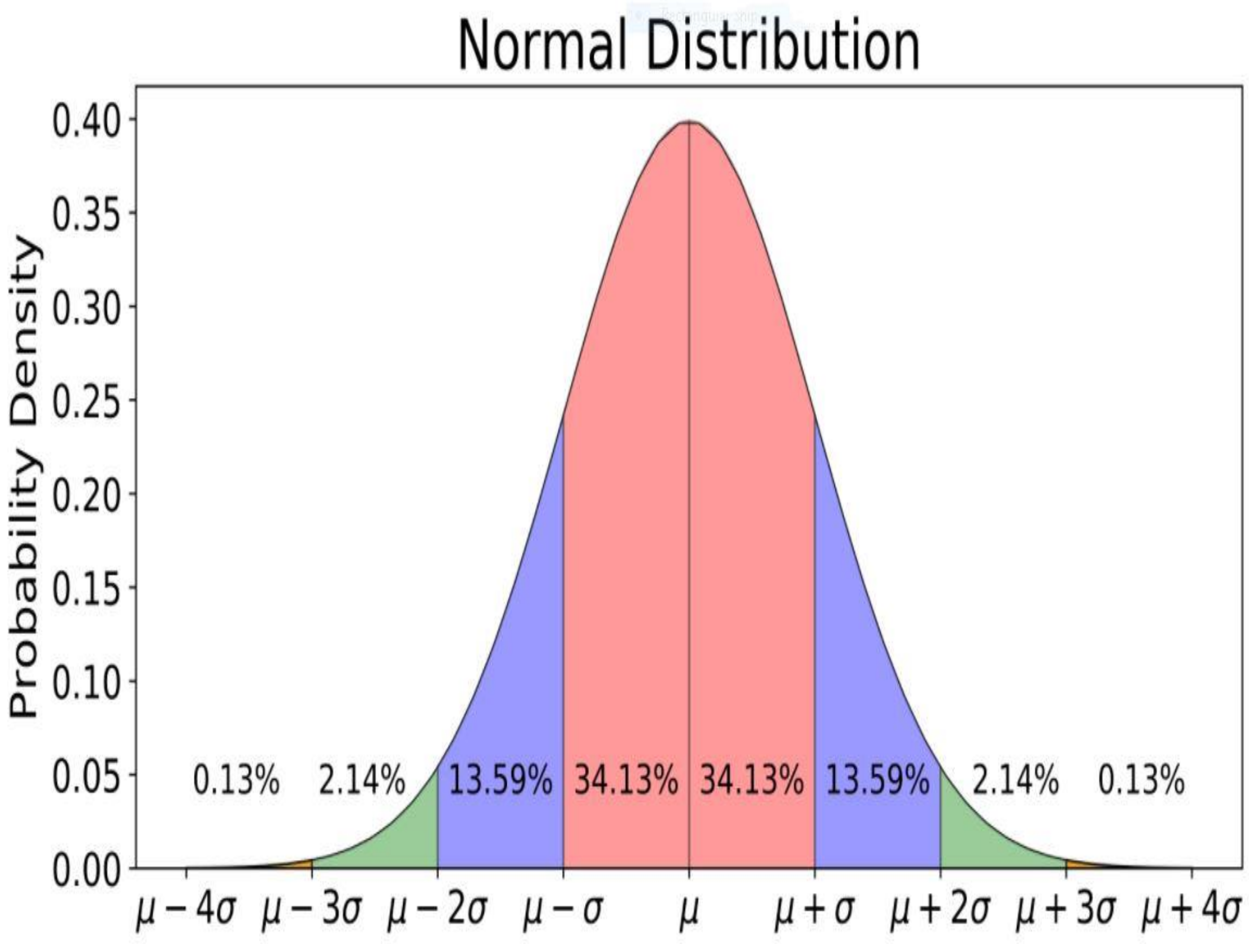
❖ **Properties of a Normal distribution**

➢ The mean, mode and median are all equal.

➢ The curve is symmetric at the center (i.e. around the mean, μ).

➢ Exactly half of the values are to the left of center and exactly half the values are to the right.

➢ The total area under the curve is 1



Essential features of indicating instruments

- Indicating instrument consists, essentially a pointer moving over a calibrated scale and attached to the moving system pivoted on jeweled bearings.
- For satisfactory operation electromechanical indicating instrument, three torques are necessary.
  a) Deflecting torque/ Operating Torque
  b) Controlling/ Balancing/ Restoring torque
  c) Damping torque

Deflecting torque/ Operating Torque

- When there is no input signal to the instrument, the pointer will be at its zero position.
- To deflect the pointer from its zero position, a force is necessary which is known as deflecting force.
- The deflecting (T_d) is produced by utilizing magnetic, electrostatic, electrodynamics, thermal or chemical effect.
- A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

Essential features of indicating instruments

Controlling Torque/ Restoring torque/ Balancing Torque

- The deflection of the moving system would be indefinite, if there were no controlling (or restoring) torque.
- To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force.
- This force is known as controlling force. A system which produces this force is known as a controlled system.
- When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position.
- This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

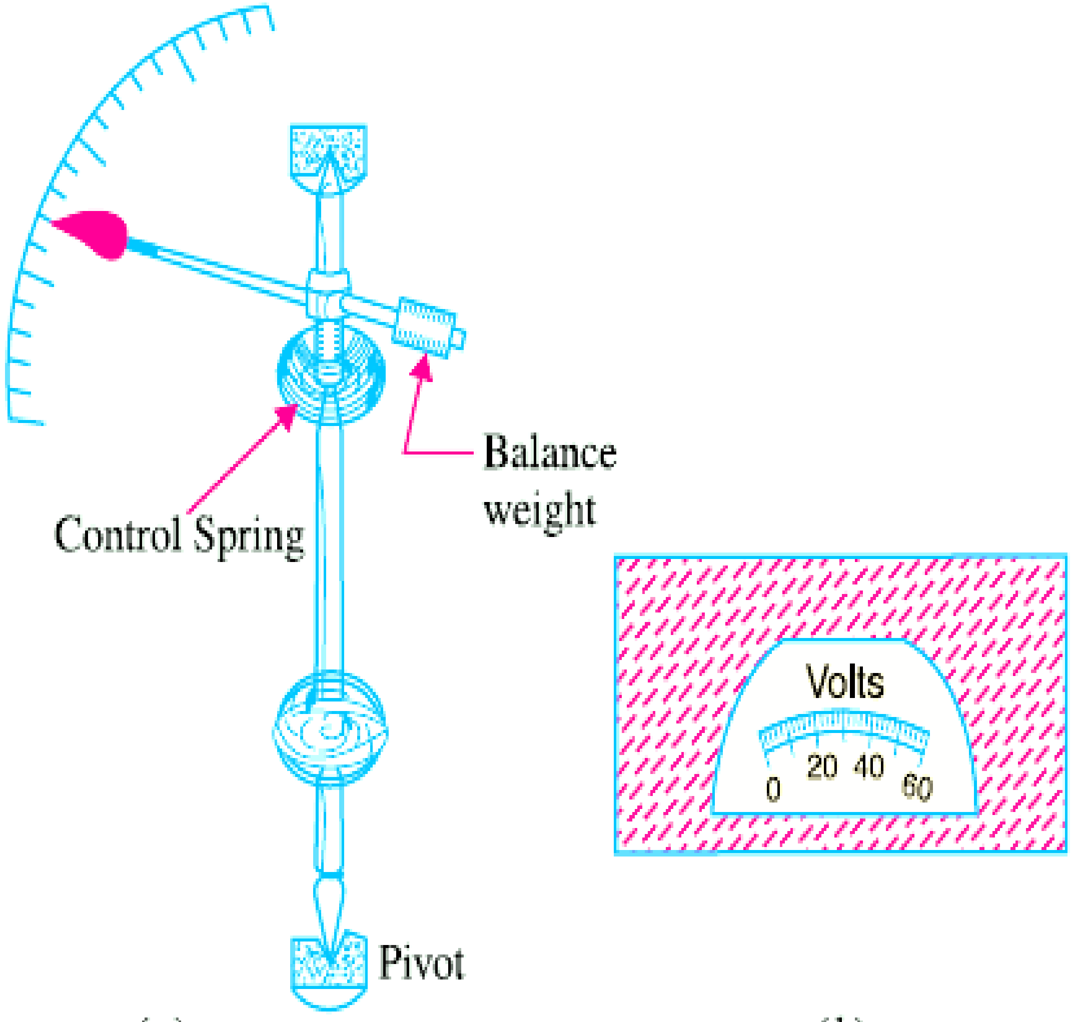
  i.e. $T_d = T_C$

Controlling Torque/ Restoring torque/ Balancing Torque….

The controlling torque in indicating instruments is either obtained by spring control or gravity control

Spring control

- A hair-spring control is shown in figure.
- With the deflection of the pointer, the spring is twisted in opposite direction.
- This twist in the spring produces restoring torque, which is directly proportional to the angle of deflection of moving system.
- The pointer comes to rest when deflection torque and restoring torque are equal.

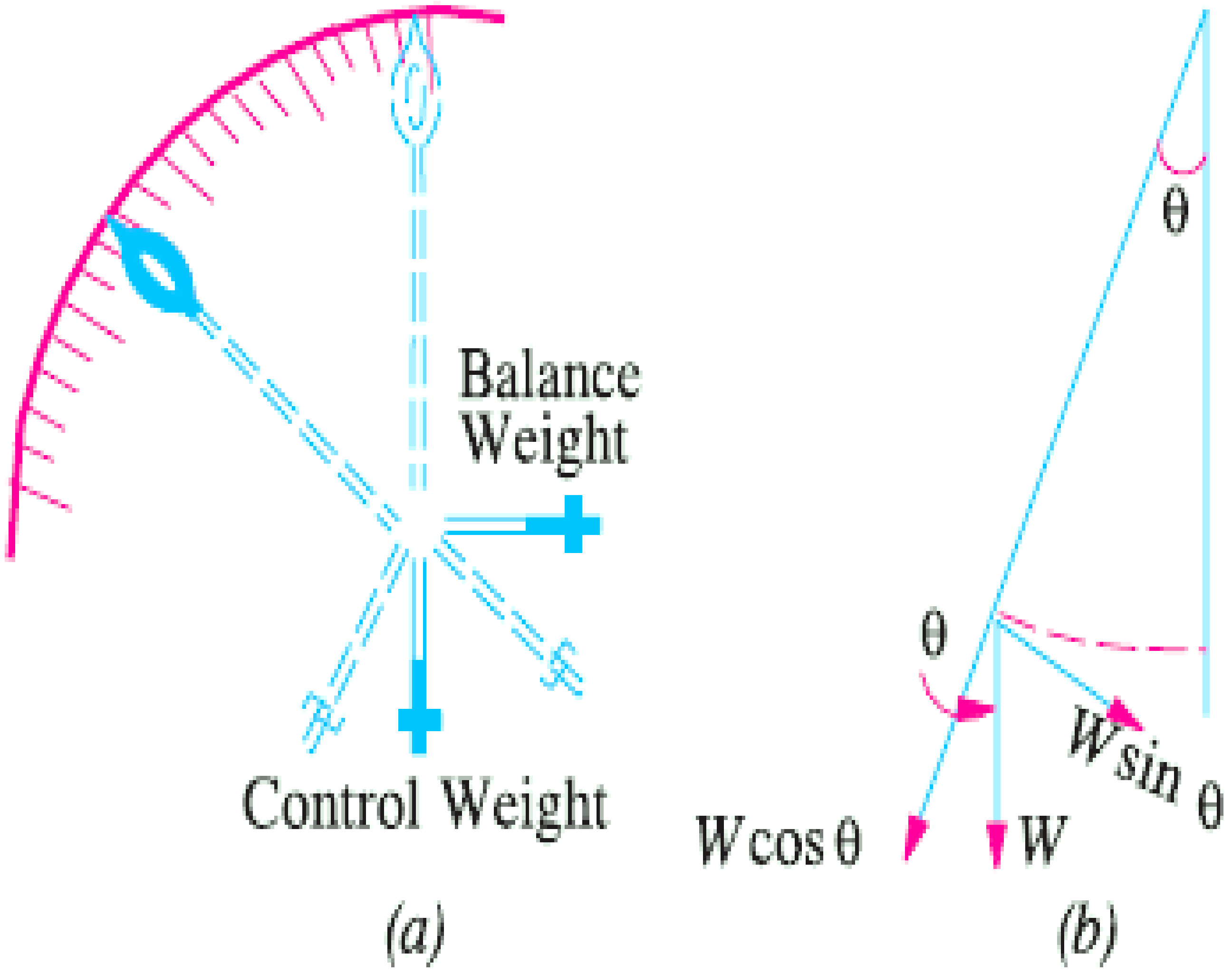


(a) (b)

Controlling Torque/ Restoring torque/ Balancing Torque….

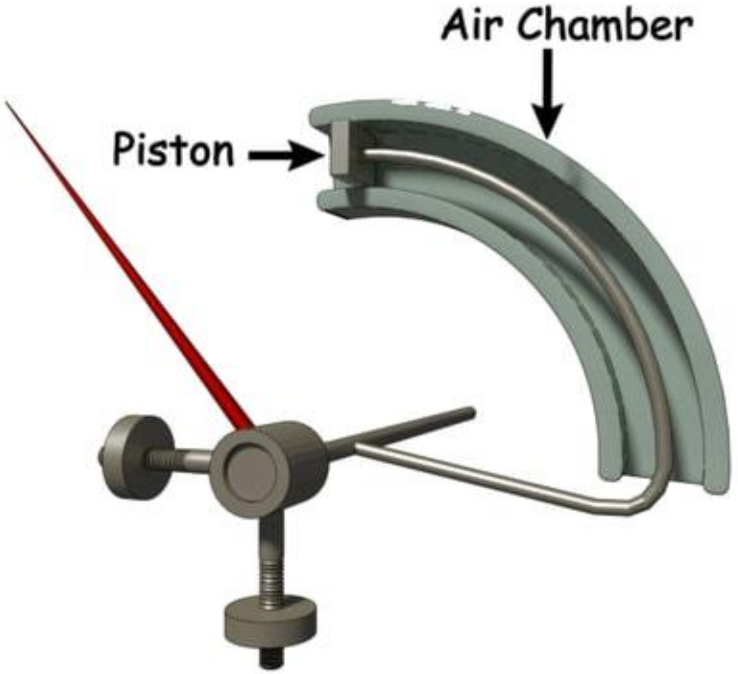
Gravity control

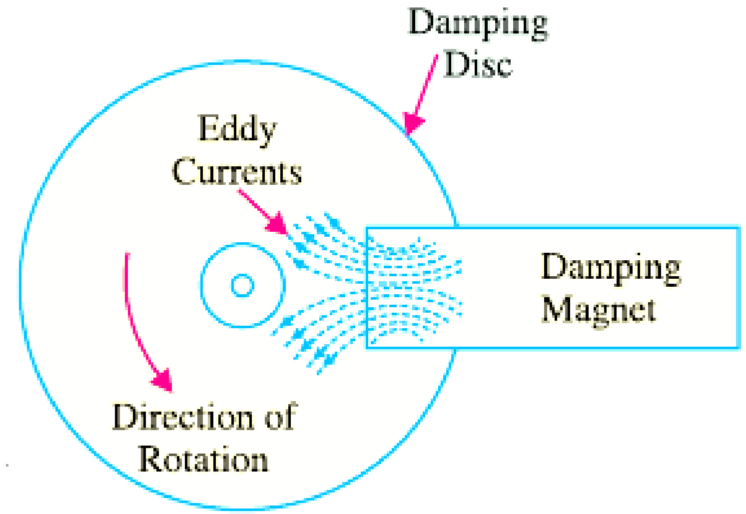
- Gravity control is obtained by attaching a small adjustable weight to some part of the moving system, such that the two torques exert force in opposite directions as show in figure.



Damping Torque

- The deflection torque and controlling torque produced by systems are electro mechanical.
- Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest.
- The time required to take the measurement is more as it takes time for pointer to indicate steady value.
- To damp out the oscillation quickly, a damping force is necessary. This force is produced by different systems.
  - (a) Air friction damping
  - (b) Fluid friction damping
  - (c) Eddy current damping





Measurement of Voltage and Current

- Voltage and current are measured using voltmeter and ammeter. There are two classes of these meters

Analog Ammeter

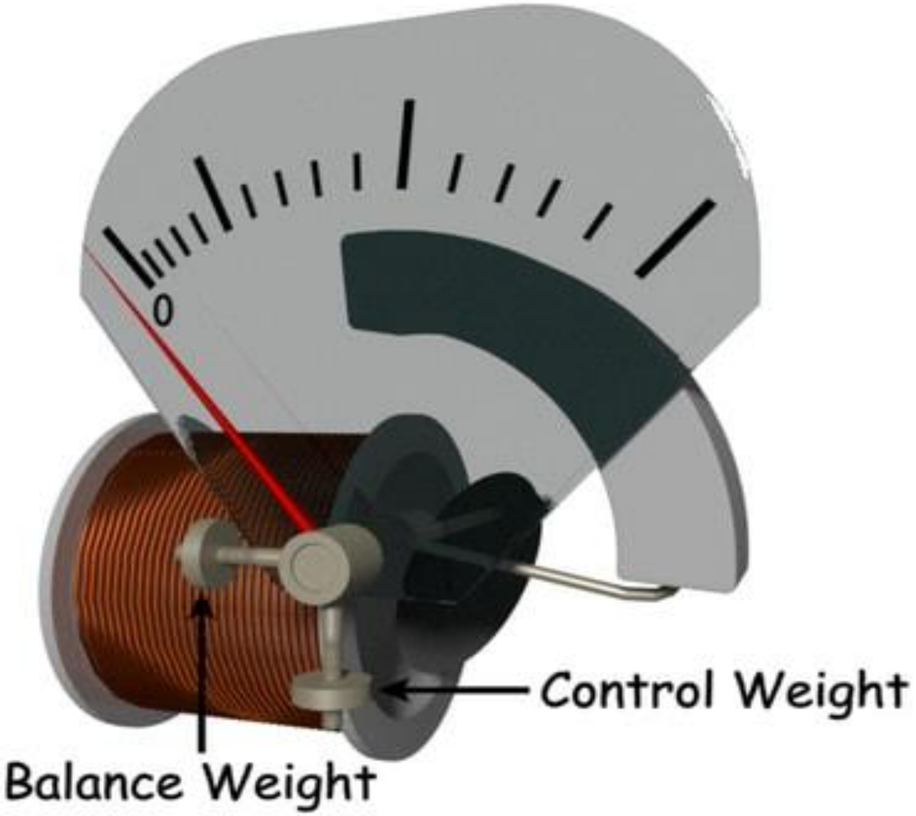
- These are connected in series with the circuit carrying the current under measurement
- Must be of very low resistance so that the voltage drop across the ammeter and power absorbed from the circuit are as low as possible

Analog Voltmeter

- These are connected in parallel with the circuit across which the voltage is to be measured
- Must be of very high resistance so that the current flowing through the voltmeter and the power absorbed from the circuit are minimum possible

Measurement of Voltage and Current….

- Ammeters and voltmeters except electrostatic voltmeters operate on the same principle
- Incase of analog or pointer indicating ammeter and voltmeters there are two classes of instruments depending up on the operations
  a) Moving iron instruments (voltmeter/ammeter)
  b) Moving coil instruments (voltmeter/ammeter)



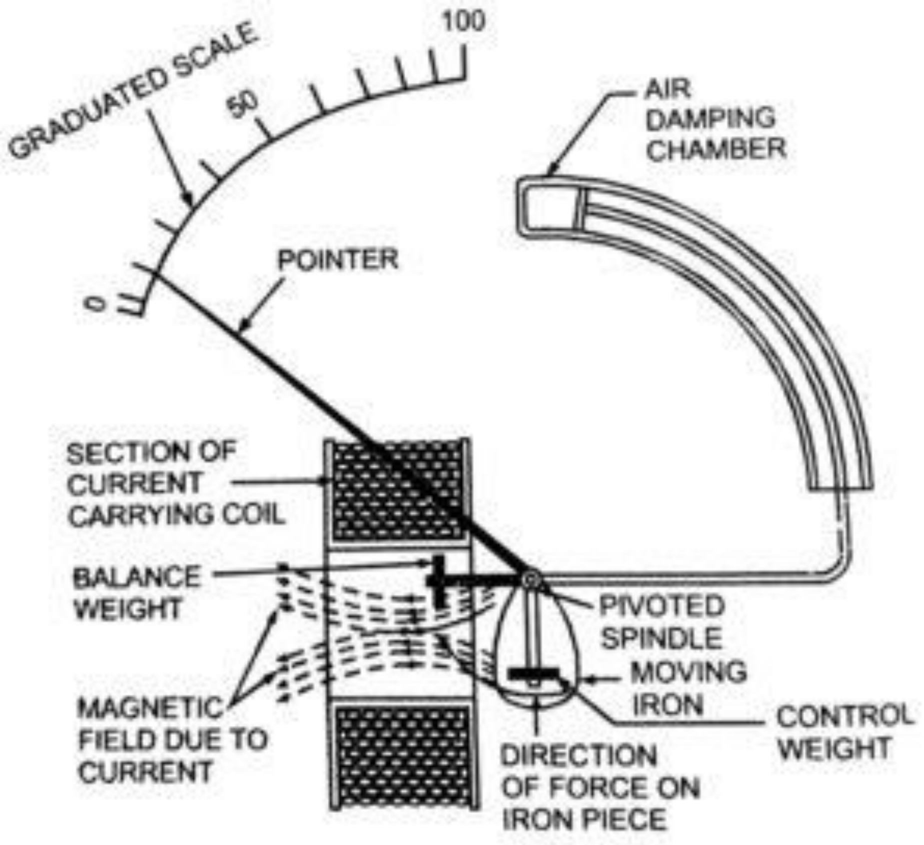
Moving-iron type

- One of the most accurate and cheap indicating instruments used for both AC and DC measurement
- Two types of moving iron instrument.
  a) the attraction type
  b) the repulsion type
- Operation of the attraction type depends on the attraction of a single piece of soft iron into a magnetic field
- Operation of repulsion type depends on the repulsion of two adjacent pieces of iron magnetized by the same magnetic field.
- For both types of these instruments, the necessary magnetic field is produced by the ampere-turns of a current-carrying coil
- In case when the instrument is to be used as an ammeter, the coil has comparatively fewer turns of thick wire so that the ammeter has low resistance because it is connected in series with the circuit.
- In case when it is used as a voltmeter, the coil has high impedance so as to draw as small current as possible since it is connected in parallel with the circuit. The current through the coil is small, so, it has large number of turns in order to produce sufficient ampere-turns.

1.Attraction type Moving Iron instrument

Construction

The moving iron disc of oval shape fixed to the spindle is kept near the hollow fixed coil. The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing as shown in figure. Here air friction damping is used.



Principle of operation

An attraction-type moving-iron instrument works on the principle that, if a piece of soft iron is brought up near either of the two ends of a current-carrying coil, it would be attracted into the coil in the same way as it would be attracted by the pole of a bar magnet.

Attraction type Moving Iron instrument

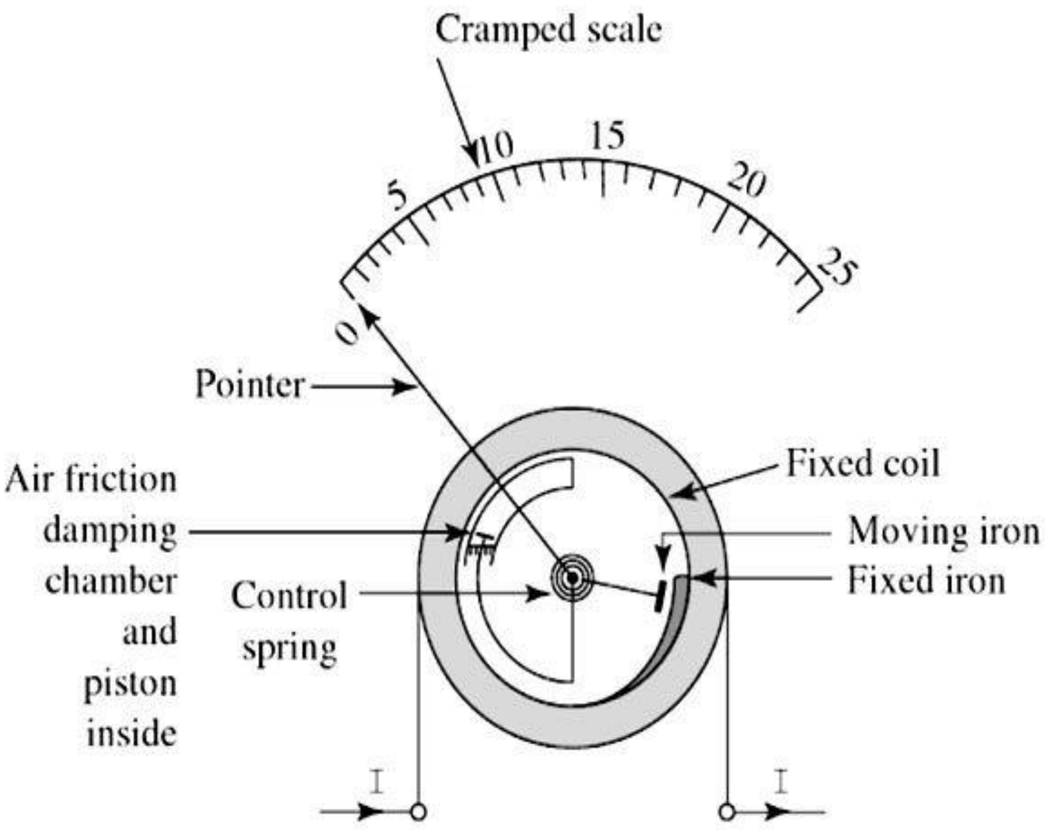
Principle of operation….

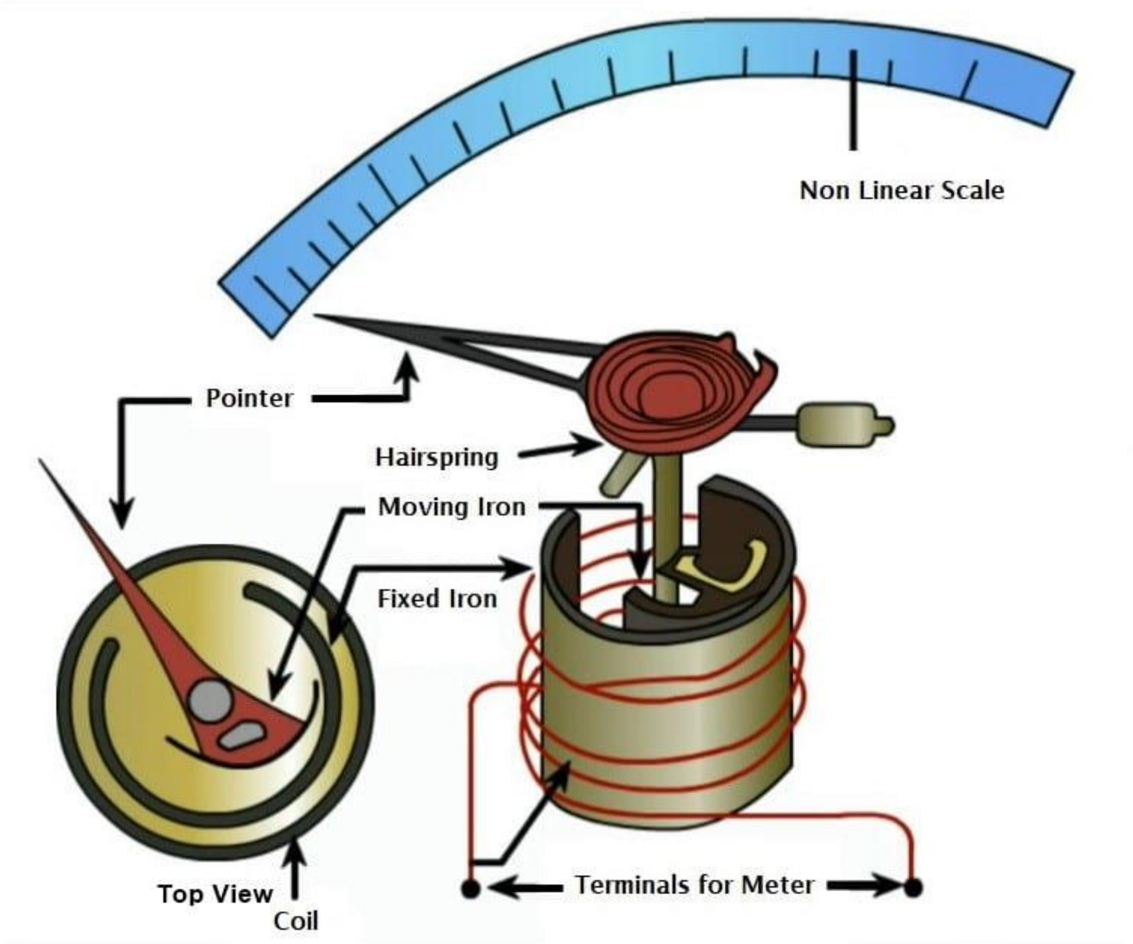
- The current to be measured is passed through the fixed coil
- As the current flows through the fixed coil, a magnetic field is produced
- By magnetic induction the moving iron gets magnetized. The north pole of moving iron is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction
- Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale.
- The force of attraction depends on the current flowing through the coil
- Another point worth noting is that whatever the direction of current through the coil, the iron disc would always be magnetized in such a way that it is pulled inwards. Hence, such instruments can be used both for DC as well as AC current
- Air-friction damping is provided

2.Repulsion type Moving Iron instrument

Construction

- The repulsion type instrument consists of a fixed coil inside which are placed two soft-iron rods parallel to one another and along the axis of the coil
- One of them is fixed and the other is movable
- The moving iron is connected to the spindle
- The pointer is also attached to the spindle supported with jeweled bearing





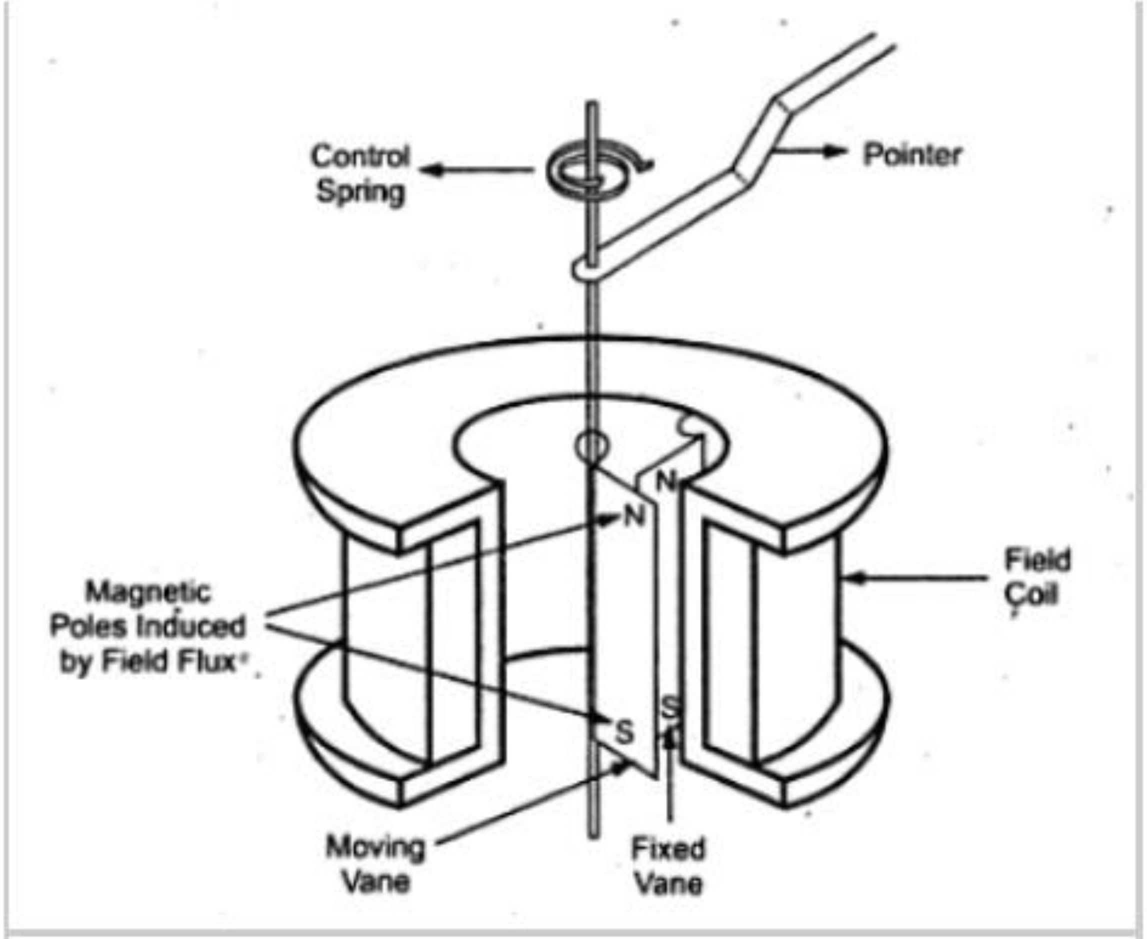


Fig. Radial Van Repulsion Type Instrument

Repulsion type Moving Iron instrument

Principle of operation:

- When the current flows through the coil, a magnetic field is produced by it
- So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field
- Similar poles of fixed and moving iron get repelled
- Thus the deflecting torque is produced due to magnetic repulsion
- Since moving iron is attached to spindle, the spindle will move
- So that pointer moves over the calibrated scale

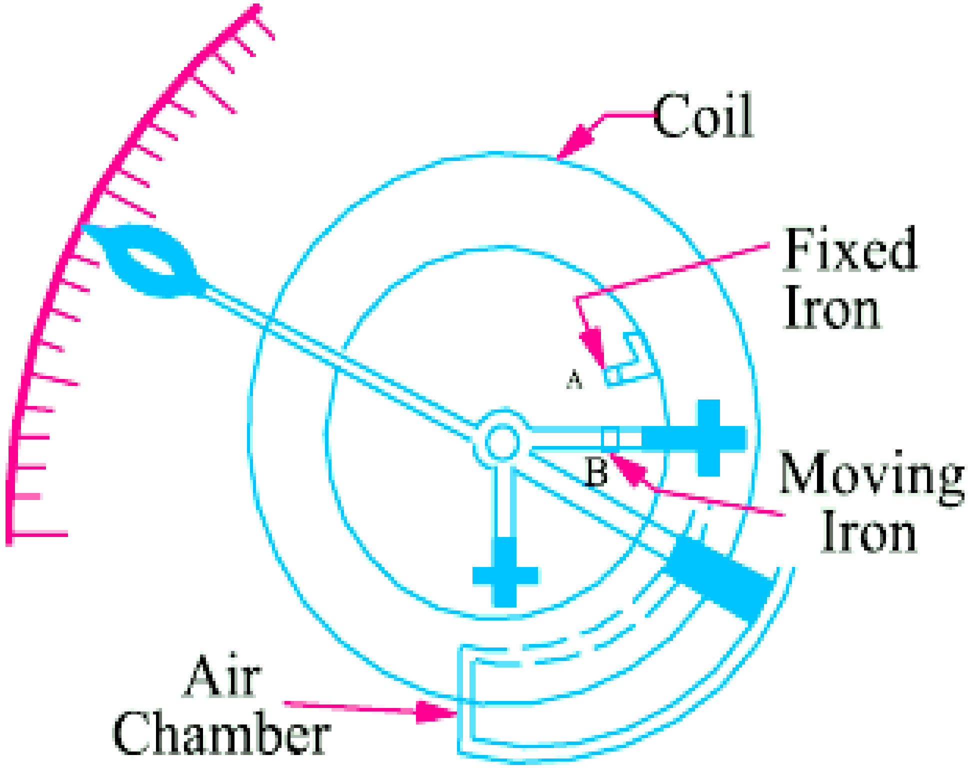


Fig.b Deflection type MI

Moving-coil Instruments

There are two types of such instruments :

Permanent-magnet type

- The operation of a permanent-magnet moving-coil type instrument is based upon the principle that when a current-carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it to one side and out of the field.

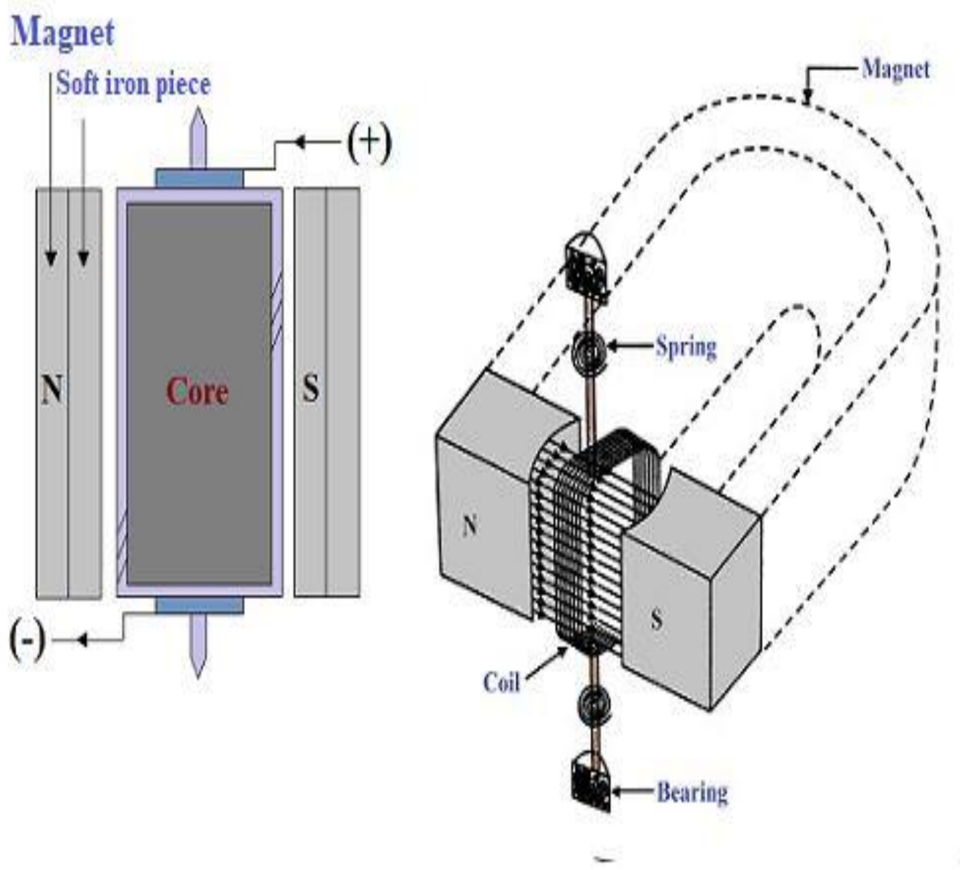
Electrodynamics or dynamometer type

- An electrodynamics instrument is a moving-coil instrument in which the operating field is produced, not by a permanent magnet but by another fixed coil
- This instrument can be used either as an ammeter or a voltmeter but is generally used as a wattmeter.

Permanent-magnet moving coil (PMMC) instrument

Construction

- PMMC instrument consists of a permanent magnet and a rectangular coil of many turns wound on a light aluminum or copper former inside which is an iron core.
- The powerful U-shaped permanent magnet is made of Alnico and has soft-iron end-pole pieces which are bored out cylindrically.
- Between the magnetic poles there is a fixed soft iron cylinder whose functions are
    i. To make the field radial and uniform
    ii. To decrease the reluctance of the air path between the poles and hence increase the magnetic flux



Permanent-magnet moving coil (PMMC) instrument

Construction….

- Surrounding the core is a rectangular coil of many turns wound on a light aluminum frame which is supported by delicate bearings and to which is attached a light pointer.
- The aluminum frame not only provides support for the coil but also provides damping by eddy currents induced in it.
- The sides of the coil are free to move in the two air gaps between the poles and core.
- Control of the coil movement is affected by two phosphor-bronze hair springs, one above and one below, which additionally serve the purpose of lending the current in and out of the coil.
- The two springs are spiraled in opposite directions in order to neutralize the effects of temperature changes.
- Damping: Eddy current damping is used which is produced by aluminum former.
- Control: Spring control is used

Permanent-magnet moving coil (PMMC) instrument

Working Principle:

- When D.C. supply is given to the moving coil, D.C. current flows through it.
- When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates.
- The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale.
- When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction.
- Therefore the polarity should be maintained with PMMC instrument.
  If A.C. is supplied, a reversing torque is produced.
  This cannot produce a continuous deflection.
- Therefore this instrument cannot be used in A.C.

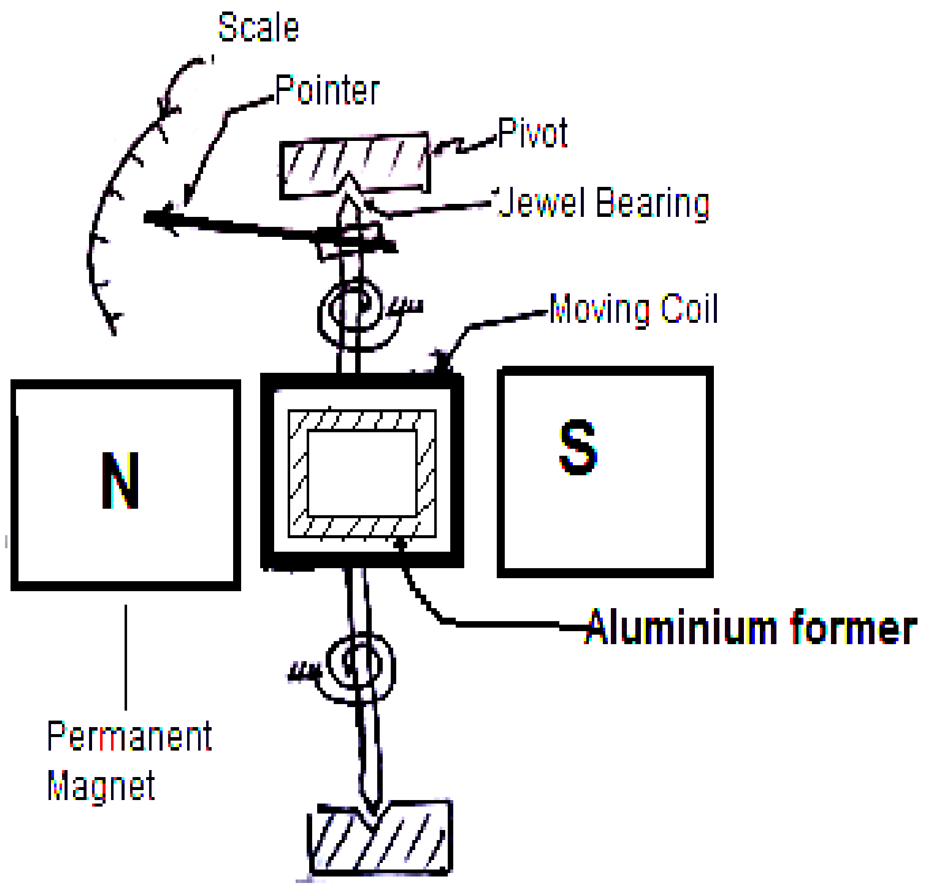
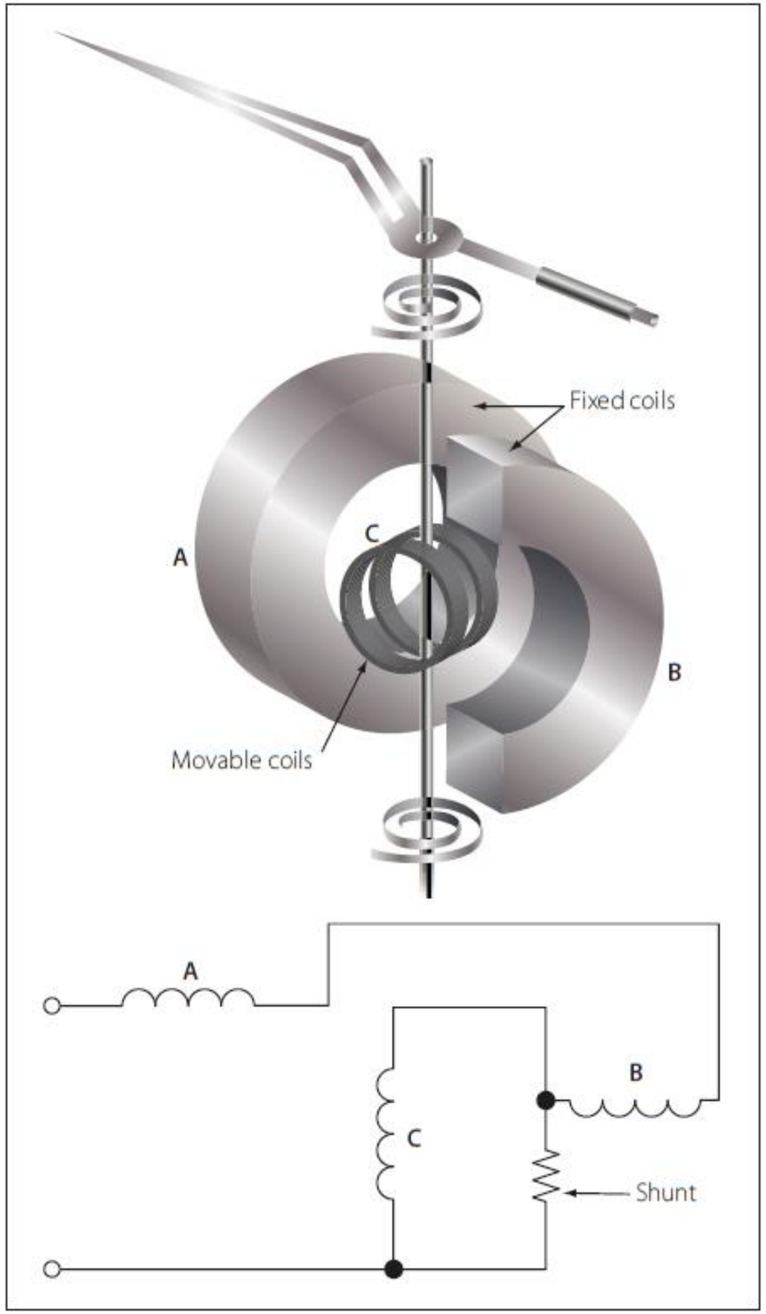


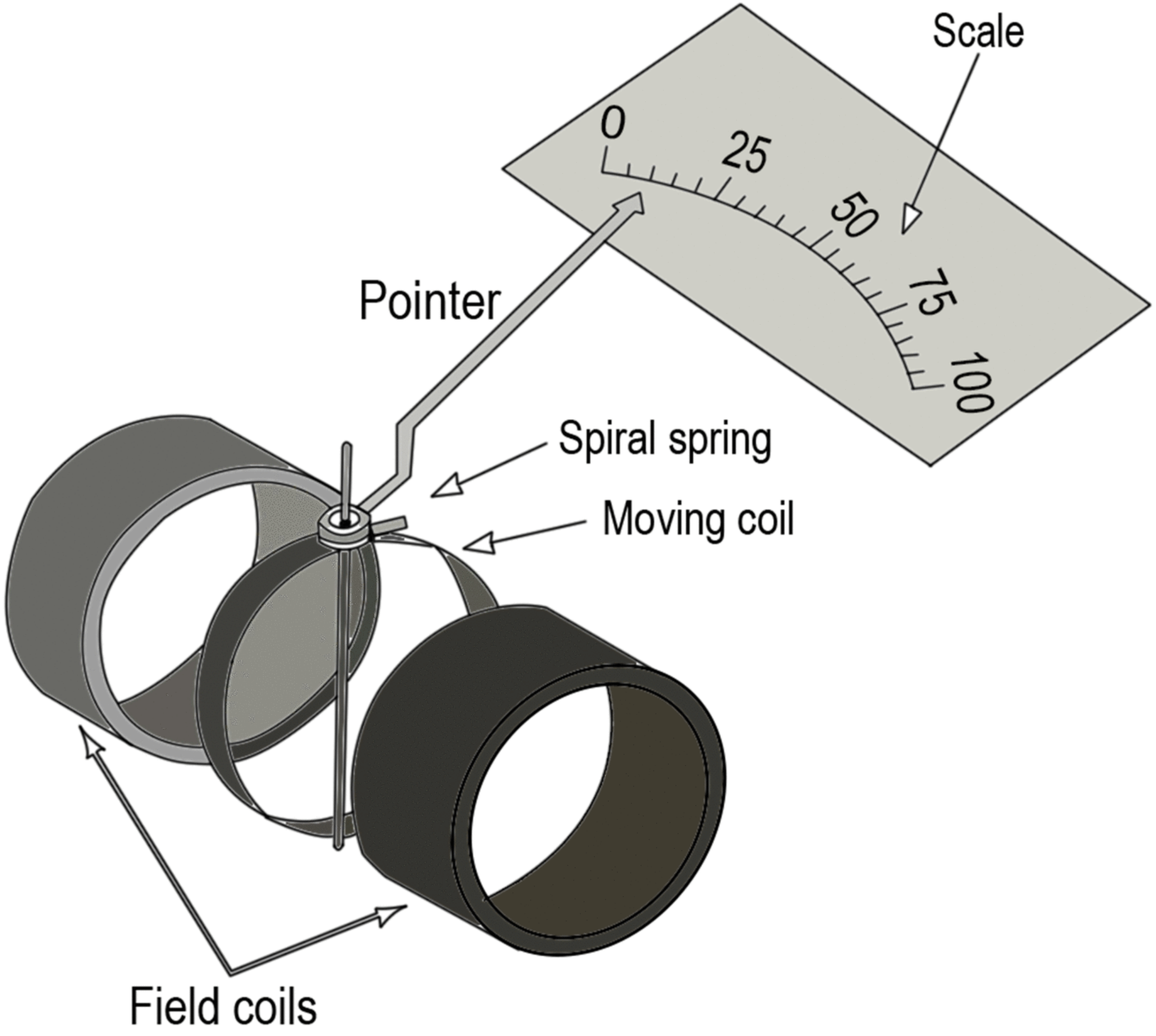
Fig.c. PMMC

Electrodynamics or Dynamometer Type Instruments (EMMC- Electromagnetic moving coil)

Construction

- The fixed coil is usually arranged in two equal sections F_1 and F_2 placed close together and parallel to each other.
- The moving coil is placed between the two half of the fixed coil.
- Both the fixed and moving coils are air cored, so that the hysteresis effect will be zero.
- The pointer is attached with the spindle.
- In a non metallic former the moving coil is wounded.
- The moving coil is spring-controlled and has a pointer attached to it
- Control: Spring control is used.
- Damping: Air friction damping is used.

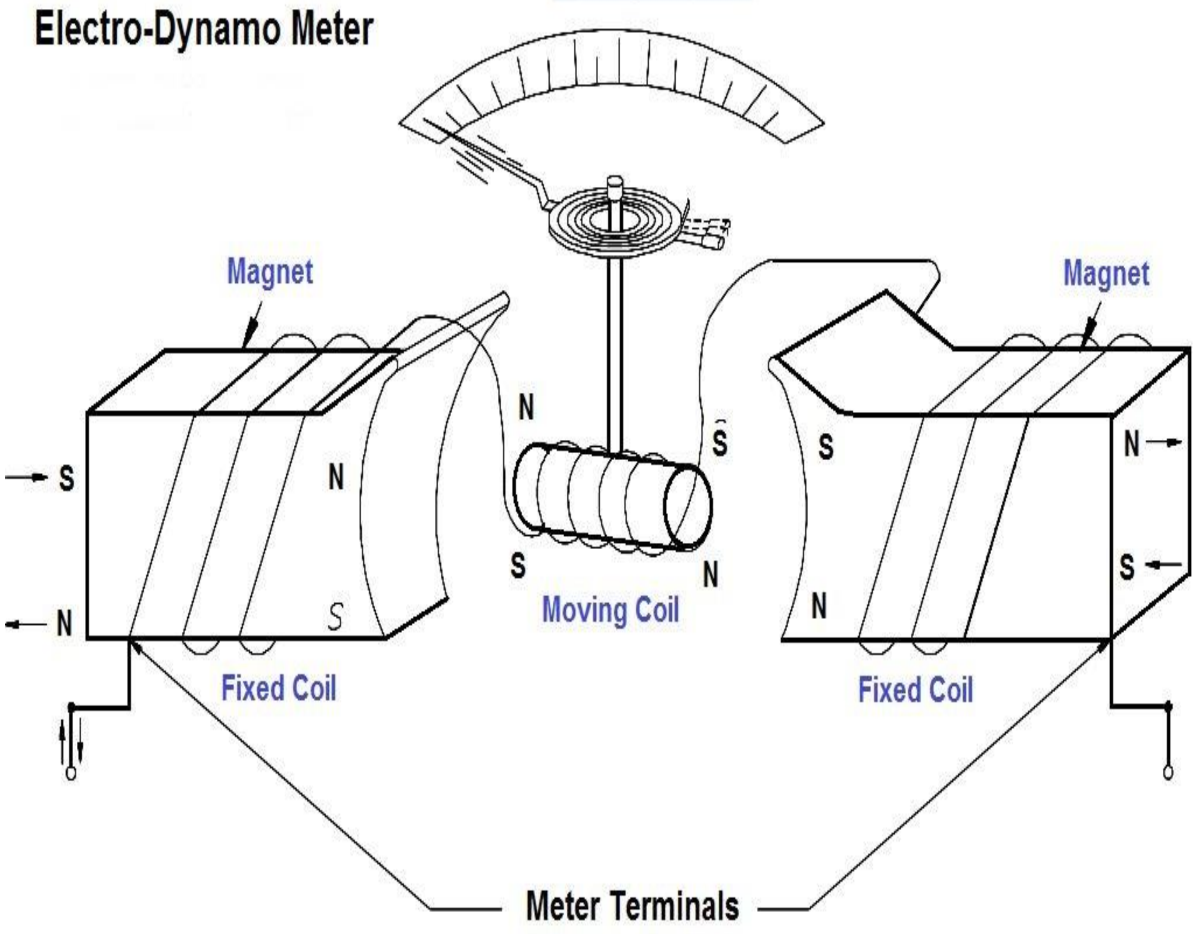

Fixed coils
A
C
B
Movable coils
A
B
C
Shunt


Scale
0
25
50
75
100
Pointer
Spiral spring
Moving coil
Field coils

(EMMC- Electromagnetic moving coil)….

Principle of operation:

- When the current flows through the fixed coil, it produced a magnetic field, whose flux density is proportional to the current through the fixed coil.
- When the current passes through the moving coil kept between the fixed coils, a magnetic field is produced by this coil too.
- The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale.
- This instrument works on both AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction, since the current i_1 and i_2 reverse simultaneously.



What will happen if we connect a PMMC instrument for the measurement of AC quantity?

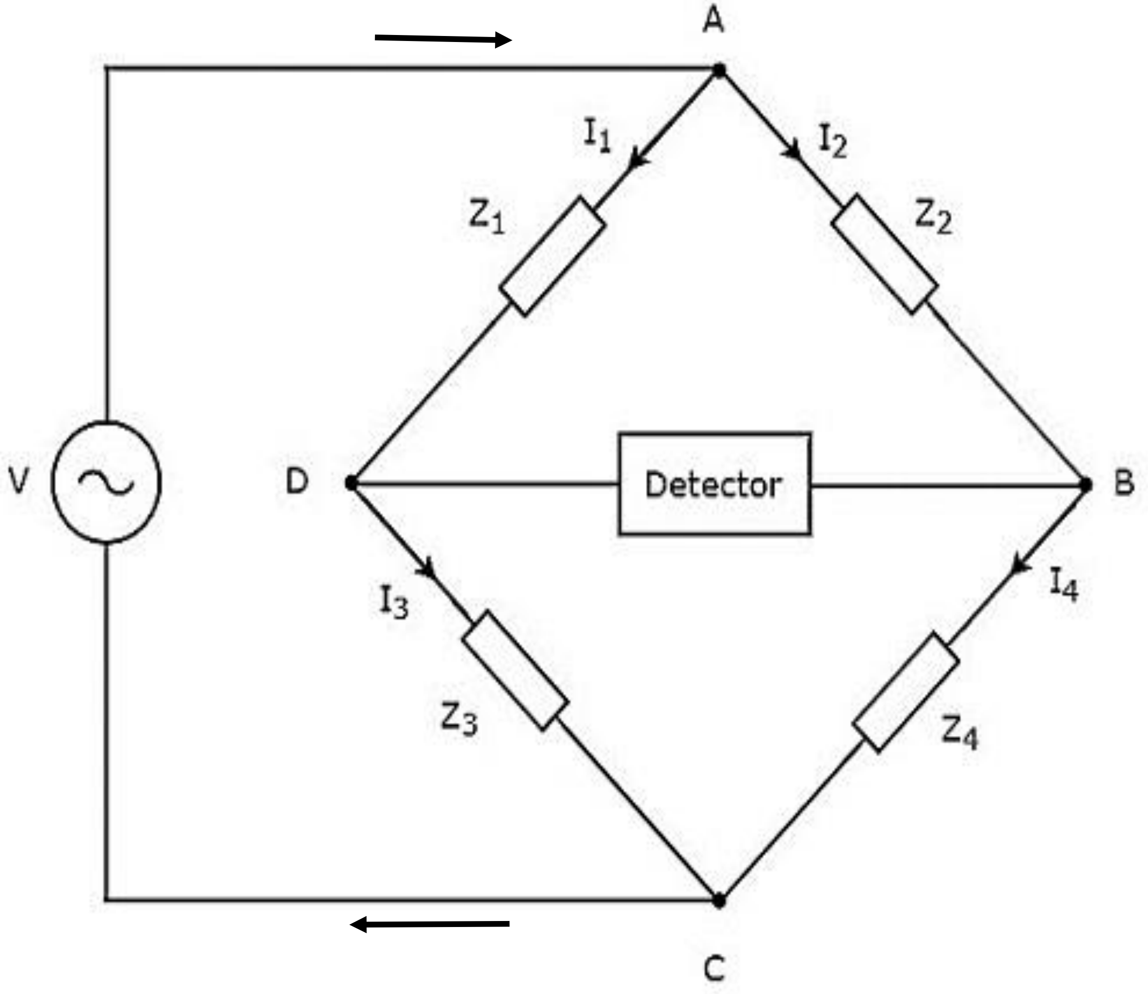
- Obviously, your answer will be there will be no deflection in the needle of PMMC instrument. Why?
- Since the direction of current is changing in each half cycle, there will be a corresponding change in the direction of magnetic flux. Because of this change of magnetic flux, the torque on moving coil will also change in each half cycle of AC quantity to be measured. Suppose the torque on moving coil for positive half cycle of AC quantity is clockwise then it will change to anticlock wise direction in the negative half cycle. Thus ideally the needle of PMMC instrument shall swing back and forth around zero position. But there is something called inertia, you better know. Due to the inertia of instrument, the needle cannot follow the rapid change in direction of torque at power frequency and therefore merely vibrates around zero position.
- Now suppose, if we were able to reverse the direction of magnetic flux each time the direction of AC quantity changes through the moving coil then we can have a unidirectional torque. Thus we can measure the AC quantity. Actually this method of reversing the magnetic flux is used in Electrodynamic or Electrodynamometer type instrument.
- If you carefully observe the figure above, you will notice that Fixed Coil as well as Moving coil is in series. Therefore, the direction of current in the moving coil is same as that in the fixed coil. This means that as the direction of magnetic field changes, the direction of current in moving coil also changes. Thus the torque on the moving coil will not change rather it will be unidirectional.

AC bridge and measurement of Inductance and Capacitance

- A bridge is a special class of instrument consisting of four arms where three arms have the known values of the parameter & one arm is used to find the value of unknown parameter.
- The value of unknown is found in terms of known values of the other three parameters.
- Normally bridges are used for the measurement of the values of passive circuit components such as resistance of the resistor, inductance of the inductor & capacitance of the capacitor.
- Generally bridges are classified on the basis of input bias,
  a) DC bridge
  b) AC bridge
- **AC Bridge:**
- When the excitation voltage or current provided to the bridge circuit is alternating in behavior then the respective bridges are called the AC bridges.
- Measurement of inductance, capacitance, loss factor may be easily done using AC bridge networks
- These circuit measures not only unknown impedances and parameters like inductance, capacitance but also used in communication system and complex electronic circuits.

AC Bridge

- The simple AC bridge circuit is shown
- Detectors commonly used are: Head phones, Vibration galvanometer & Tunable amplifies detector
- The Condition for balance of AC bridge circuit require that there should be no current through the detector
- At this condition the potential difference(pd.) across AD arm will be equal to pd. across AB arm.



General equation for AC bridge balanced condition

Let Z_1, Z_2, Z_3 and Z_4 are the impedance of bridge arms and E be the Ac voltage of frequency f.

For balanced condition,

Pd. across AD = Pd. across AB

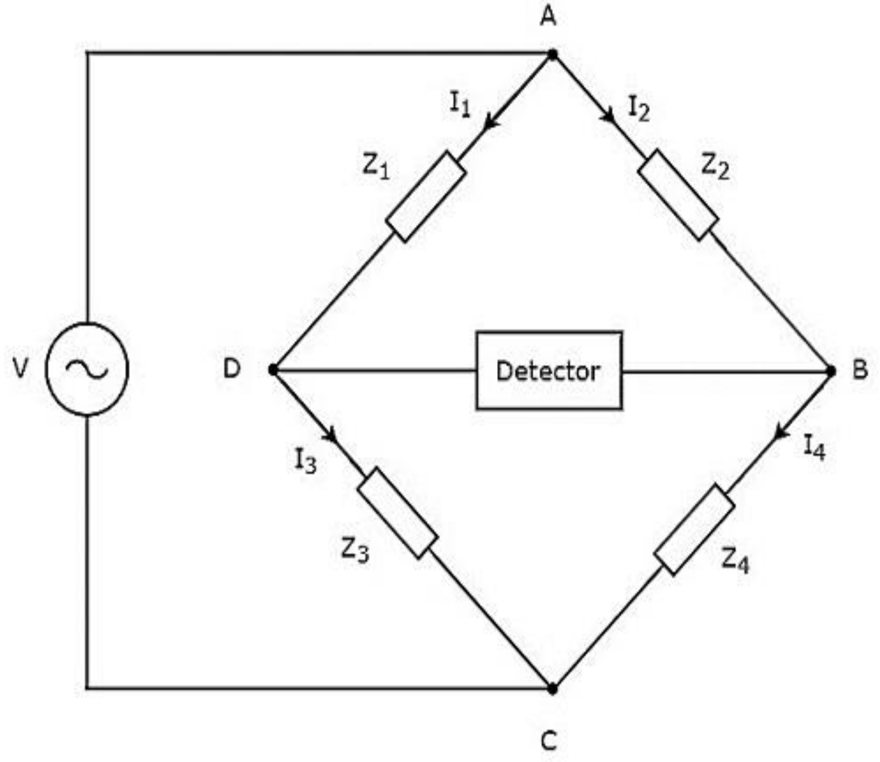
$I_1Z_1 = I_2Z_2$……………………………(i)

Also at balanced condition

$$I_1 = I_3 = \frac{E}{Z_1+Z_3} \text{…………………………(ii)}$$

And $$I_2 = I_4 = \frac{E}{Z_2+Z_4} \text{…………………………(iii)}$$

Putting the values of I_1 from equation (ii) & value of I_2 from in equation (iii) in equation (i), we get



General equation for AC bridge balanced condition….

$I_1Z_1 = I_2Z_2$

$$\frac{E}{Z_1+Z_3} \times Z_1 = \frac{E}{Z_2+Z_4} \times Z_2$$

$Z_1(Z_2 + Z_4) = Z_2(Z_1 + Z_3)$

$Z_1Z_2 + Z_1Z_4 = Z_2Z_1 + Z_2Z_3$

$Z_1Z_4 = Z_2Z_3$……………..(iv)

Now considering the polar form of impedances and putting in equation (iv)

$Z_1\angle\theta_1 \times Z_4\angle\theta_4 = Z_2\angle\theta_2 \times Z_3\angle\theta_3$

$Z_1Z_4 \angle(\theta_1+\theta_4) = Z_2Z_3\angle(\theta_2+\theta_3)$

There are two conditions for AC bridge to be balanced

1. The product of magnitude of any two opposite arms impedance must be equal to the product of magnitude of other two arm's impedances

   i.e. $Z_1Z_4 = Z_2Z_3$

2. The sum of phase angle of any two opposite arm must be equal to that of other two's

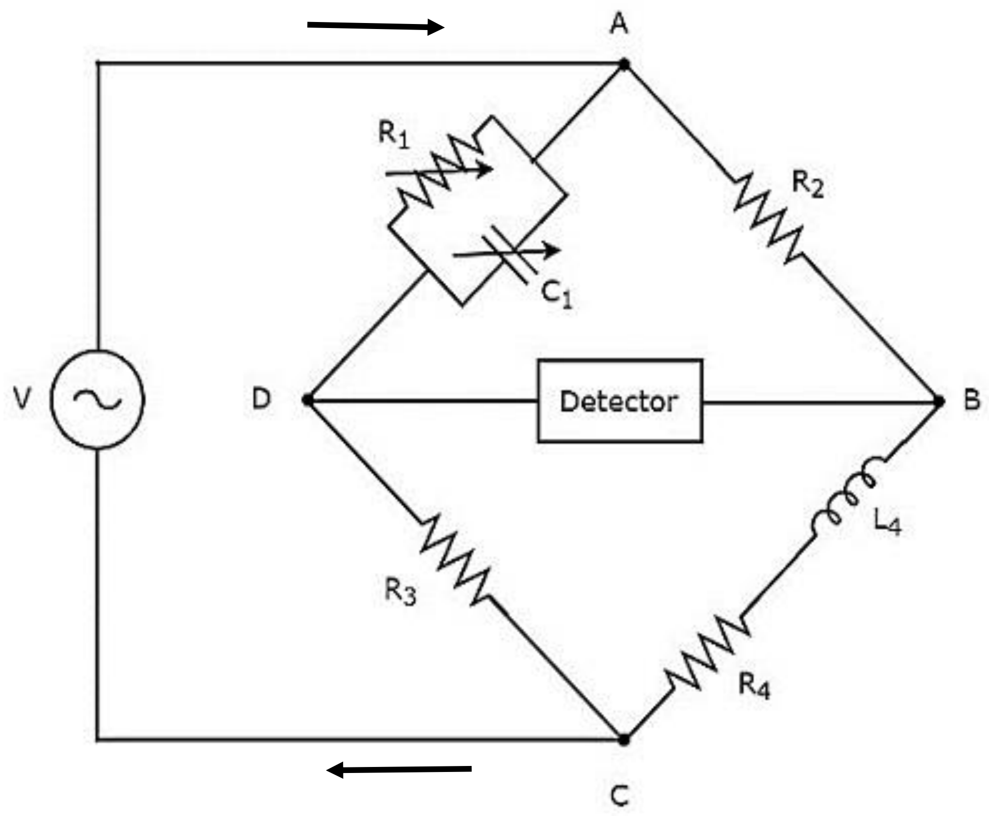
   i.e. $\angle(\theta_1+\theta_4) = \angle(\theta_2+\theta_3)$

Maxwell's Inductance Bridge(Maxwell's Inductor capacitor bridge)

- In this bridge, an inductance of the inductor is measured by comparison with a standard variable capacitance of the capacitor
- Known values = R_3, R_2, R_1&C_1
- Unknown values =R_4,L_4=?
- We know the bridge will be balanced when,

$$Z_1Z_4 = Z_2Z_3$$
$$Z_4 = \frac{1}{Z_1} \times Z_2Z_3$$
$$Z_4 = Y_1Z_2Z_3$$

- Here we have to find impedances of different arm.



For arm AD, the impedance is Z_1 and component R_1 & C_1 in parallel so,

<u>Impedances</u>:

$$Z_1 = \frac{R_1 \times X_{C1}}{R_1 + X_{C1}} = \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

So,

Admittance (Y_1) $= \frac{1 + j\omega R_1 C_1}{R_1} = \frac{1}{R_1} + j\omega C_1$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4 + X_{L4} = R_4 + j\omega L_4$$

Say, the bridge is balanced ,

then the balanced condition will be,

$$Z_1 Z_4 = Z_2 Z_3 \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(i)}$$

We have to find value of Inductance ,so

$$Z_4 = \frac{1}{Z_1} \times Z_2 Z_3$$

$$\therefore Z_4 = Y_1 \times Z_2 Z_3$$

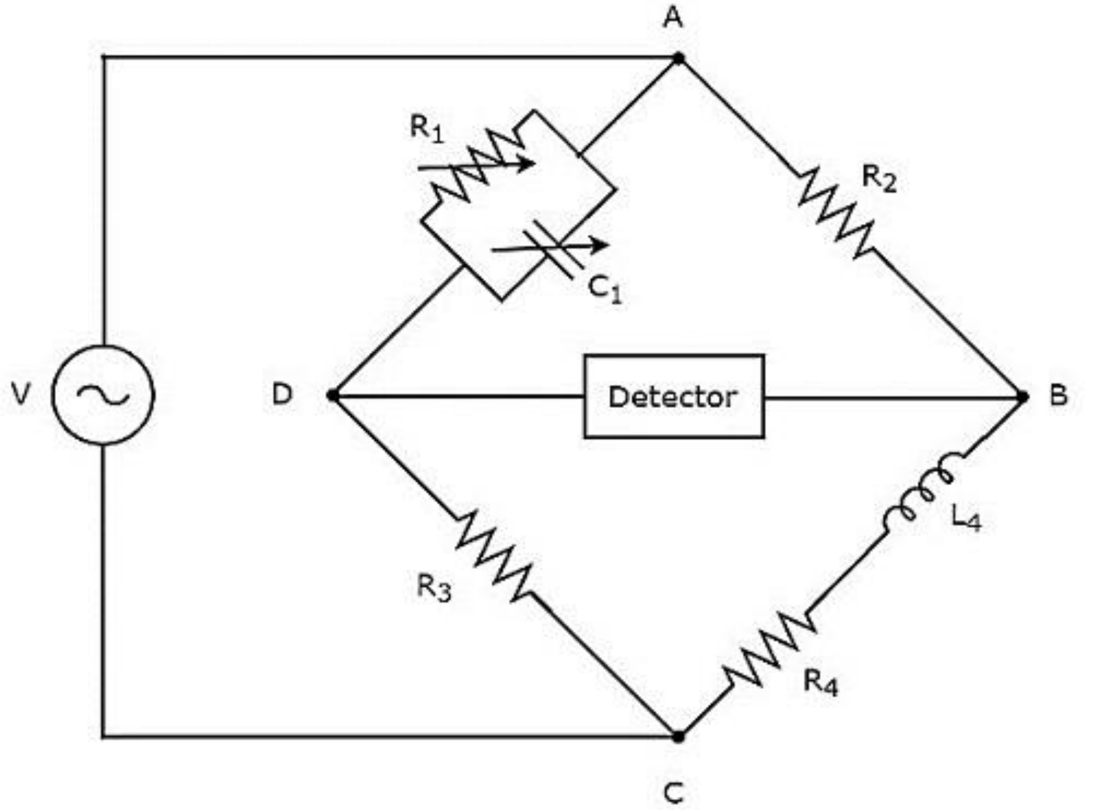


Fig: Maxwell's Inductance Bridge

Substituting values of Y_1, Z_4, Z_2 & Z_3

$$Z_4 = Y_1 \times Z_2 Z_3$$

$$R_4 + j\omega L_4 = [\frac{1}{R_1} + j\omega C_1]R_2R_3$$

$$R_4 + j\omega L_4 = \frac{R_2R_3}{R_1} + j\omega R_2R_3C_1$$

Comparing Real and Imaginary part

$$R_4 = \frac{R_2R_3}{R_1} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(ii)}$$

$$j\omega L_4 = j\omega R_2R_3C_1$$

$$L_4 = R_2R_3C_1 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(iii)}$$

So using above expressions we can measure unknown resistance (R_4)and unknown inductance (L_4)

"Maxwell bridge is not suitable for the measurement of high Q-coils". Justify the statement.

Quality factor (Q-factor)

$$Q = \frac{\omega L_4}{R_4} = \omega \frac{R_2R_3C_1}{[\frac{R_2R_3}{R_1}]} = \omega R_1C_1$$

- The above expression shows that for higher quality factor, the values of R_1 must be very high (in order of 10^5 or 10^6)
- The cost of such a high value decade resistance is very high
- So it becomes impracticable for the measurement of the inductance of the coil having high quality factor.
- Limited to measurement of low Q coil ($1 < Q < 10$) also unsuitable for very low Q coil ($Q < 1$)

Advantages and disadvantages of Maxwell bridge

Advantages

- Used to measure the self inductance of a coil having medium Q-value
- If DC excitation is given and the DC null detector is used then it behaves as the Wheatstone bridge, suitable to measure the resistance of a resistor
- Simple headphones may be used as the null detector
- Common tuning of R_1 & R_3 makes it easy to find the bridge balanced condition
- Balance equations are independent if R_1 and C_1 are variable.
- Frequency does not affect the calculation of unknown variable.
- Simple expressions for unknown L_4 and R_4 in terms of known bridge elements.
- Maxwell's inductance-capacitance bridge is very useful for measurement of wide range of inductance at power and audio frequency.

Disadvantages

- Variable capacitor is expensive and less accurate. So, fixed capacitor is sometimes used. In that case balance adjustments are done by:

➢ Varying R_1 and R_3 , which is difficult.

➢ Putting an additional resistance in series with inductance under measurement and varying this resistance and R_1.

- Restricted to measure the value of inductance of medium Q-coil ((1< Q< 10)
- Low Q-coil and High Q-coil inductance measurement fails to observed by this bridge

Hay's bridge

- This is the modification of the Maxwell bridge and designed to measure the self inductance of a coil having high Q-factor
- From the circuit diagram as shown

In AD arm, R_1 and C_1 in series so,

$$Z_1 = R_1 + X_{C1} = R_1 + \frac{1}{j\omega C_1} = R_1 - \frac{j}{\omega C_1}$$

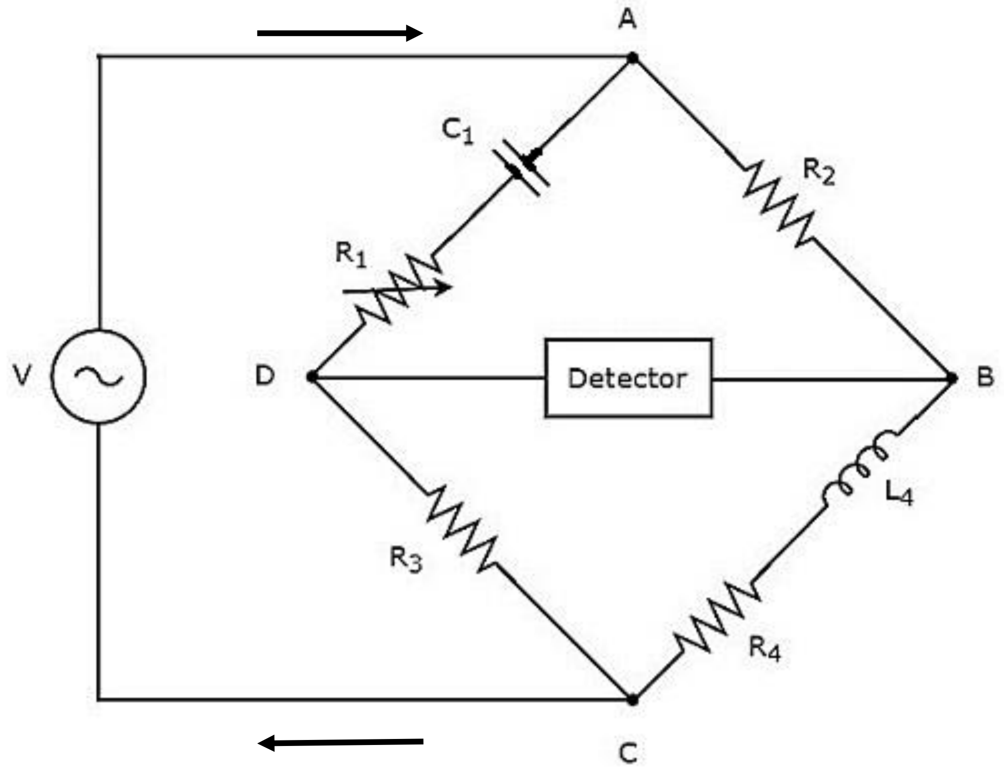
$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4 + X_{L4} = R_4 + j\omega L_4$$

Under the bridge balanced condition

$$Z_1 Z_4 = Z_2 Z_3$$



Substituting the values we get,

$Z_1Z_4 = Z_2Z_3$…………………………………………(i)

$(R_1 - \frac{j}{\omega C_1})(R_4 + j\omega L_4) = R_2R_3$

$R_1R_4 + j\omega R_1L_4 - j\frac{R_4}{\omega C_1} - j^2\frac{\omega L_4}{\omega C_1} = R_2R_3$

$R_1R_4 + j(\omega R_1L_4 - \frac{R_4}{\omega C_1}) + \frac{L_4}{C_1} = R_2R_3$

$R_1R_4 + \frac{L_4}{C_1} + j(\omega R_1L_4 - \frac{R_4}{\omega C_1}) = R_2R_3$

Equating the real and imaginary part,

$R_1R_4 + \frac{L_4}{C_1} = R_2R_3$ ………………………………………(ii)

and.,

$j(\omega R_1L_4 - \frac{R_4}{\omega C_1}) = 0$

$\omega R_1L_4 = \frac{R_4}{\omega C_1}$……………………………………….(iii)

From equation (ii)

$R_1R_4 + \frac{L_4}{C_1} = R_2R_3$

Substitute value of L_4 from eq^n (ii),

$R_1R_4 + \frac{1}{C_1}[\frac{R_4}{\omega^2R_1C_1}] = R_2R_3$

$R_1R_4 + \frac{R_4}{\omega^2R_1C_1^{\,2}} = R_2R_3$

$R_4[R_1 + \frac{1}{\omega^2R_1C_1^{\,2}}] = R_2R_3$

$R_4 = R_2R_3\frac{\omega^2R_1C_1^{\,2}}{1+\omega^2R_1^{\,2}C_1^{\,2}}$

$\therefore R_4 = \frac{\omega^2R_1R_2R_3C_1^{\,2}}{1+\omega^2R_1^{\,2}C_1^{\,2}}$…………………….(iv)

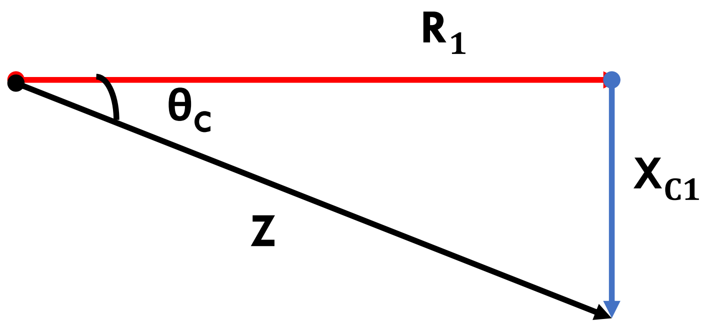
Similarly substituting value of R_4 from eq^n (ii), $L_4 = \frac{R_4}{\omega^2R_1C_1}$

From equation (iv),

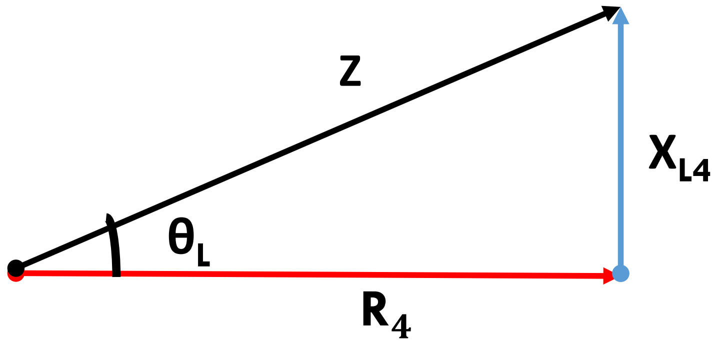
$L_4 = \frac{R_4}{\omega^2R_1C_1} = \frac{1}{\omega^2R_1C_1} \times \frac{\omega^2R_1R_2R_3C_1^{\,2}}{1+\omega^2R_1^{\,2}C_1^{\,2}}$

$\therefore L_4 = \frac{R_2R_3C_1}{1+\omega^2R_1^{\,2}C_1^{\,2}}$……………………….(v)

- Impedance diagram of 1st arm 'AD'



- Impedance diagram of 4th arm 'BC'



For balanced condition

$$\theta_L = \theta_C$$

$$\text{i.e. } \tan\theta_L = \tan\theta_C$$

Now Quality factor (Q-factor)

$$Q = \frac{X_{C1}}{R_1} = \frac{\frac{1}{\omega C_1}}{R_1} = \frac{1}{\omega R_1 C_1}$$

The expression for inductance and capacitance contains frequency terms. So frequency should be known accurately.

Then equation (v) becomes,

$$L_4 = \frac{R_2R_3C_1}{1+\omega^2R_1{}^2C_1{}^2} = \frac{R_2R_3C_1}{1+(\omega R_1C_1)^2} = \frac{R_2R_3C_1}{1+(\frac{1}{Q})^2}$$

As, Quality factor (Q)>10, then the fraction $(\frac{1}{Q})^2$ can be neglected. So,

$$\therefore L_4 = R_2R_3C_1$$

Hay's bridge….

Advantages

- Fixed capacitor is cheaper than variable capacitor.
- This bridge is best suitable for measuring high value of Q-factor (Q>10)
- The expression of Q is simple
- Value of series resistance R_1 is very small for Hay bridge but for Maxwell it is very large

Disadvantages

- Equations of L_4 and R_4 are very complicated.
- Measurements of R_4 and L_4 require the value of frequency.
- Only suitable for the measurement of inductance of high Q-coils (Q>10). This bridge cannot be used for measuring low Q- factor.
- The term $(\frac{1}{Q})^2$ can be neglected only for Q>10

Schering bridge

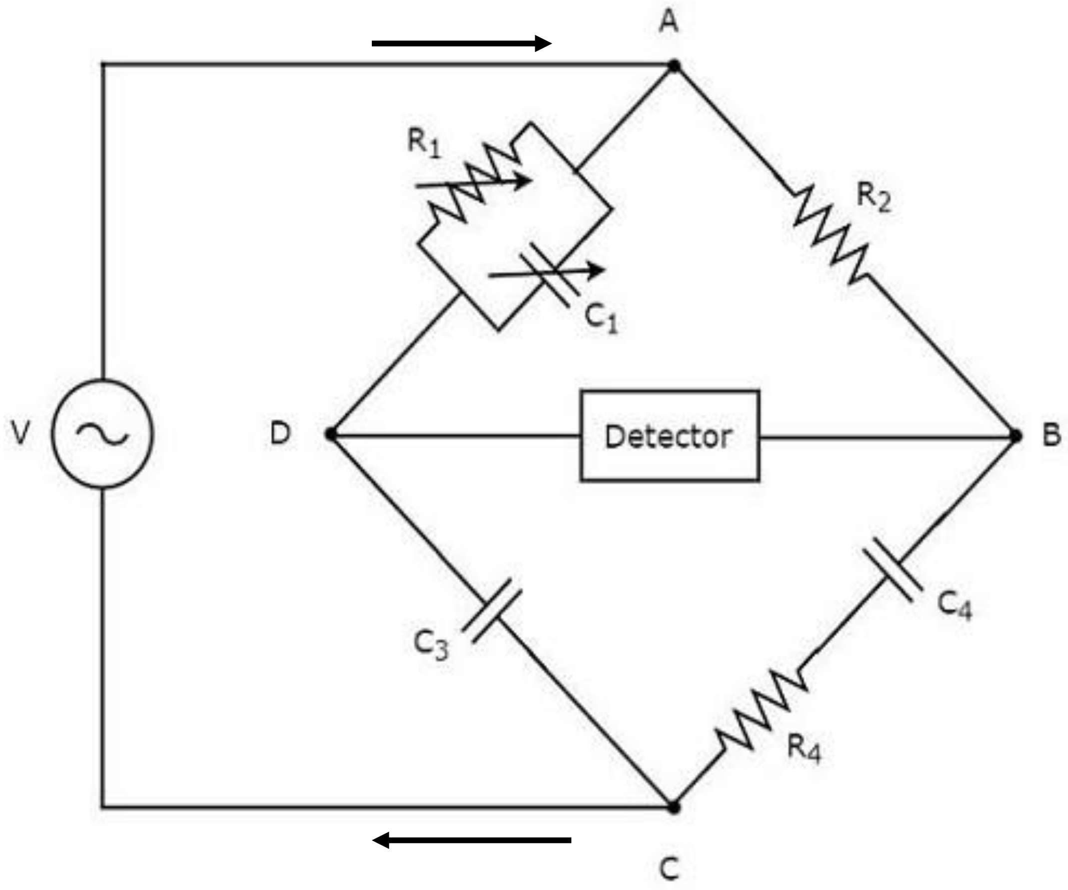
- Value of capacitor with unknown capacitance is determined
- Known values = $C_3, R_2, R_1 \& C_1$
- Unknown values $= R_4, C_4 = ?$
- We know the bridge will be balanced when,

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_4 = \frac{1}{Z_1} \times Z_2 Z_3$$

$$Z_4 = Y_1 Z_2 Z_3$$

- Here we have to find impedances of different arm.



For arm AD, the impedance is Z_1 and component R_1 & C_1 in parallel so,

<u>Impedances</u>:

$$Z_1 = \frac{R_1 \times X_{C1}}{R_1 + X_{C1}} = \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

So, Admittance $(Y_1) = \dfrac{1 + j\omega R_1 C_1}{R_1} = \dfrac{1}{R_1} + j\omega C_1$

$$Z_2 = R_2$$

$$Z_3 = X_{C3} = \frac{1}{j\omega C_3} = -j\frac{1}{\omega C_3}$$

$$Z_4 = R_4 + X_{C4} = R_4 + \frac{1}{j\omega C_4} = R_4 - j\frac{1}{\omega C_4}$$

Say, the bridge is balanced ,

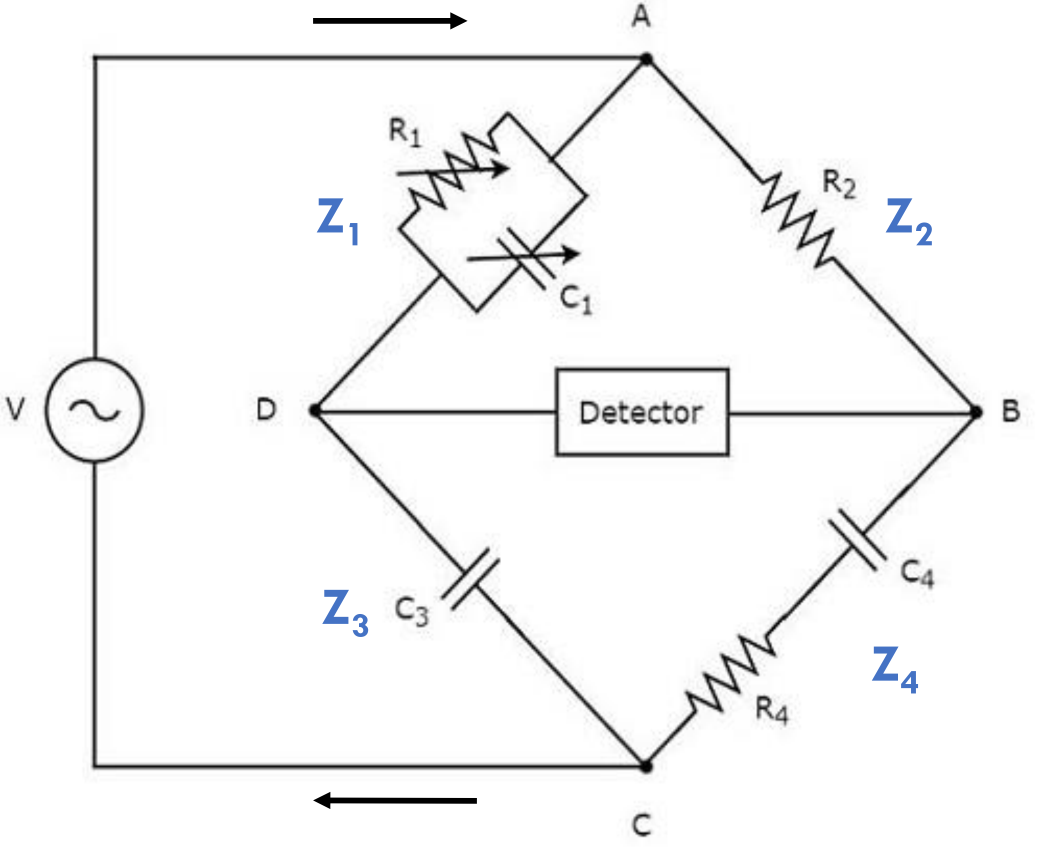
then the balanced condition will be,

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{(i)}$$

We have to find value of Inductance ,so

$$Z_4 = \frac{1}{Z_1} \times Z_2 Z_3$$

$$\therefore Z_4 = Y_1 \times Z_2 Z_3$$



Substituting values of Y_1, Z_4, Z_2 & Z_3

$$Z_4 = Y_1 \times Z_2 Z_3$$

$$R_4 - j\frac{1}{\omega C_4} = \left[\frac{1}{R_1} + j\omega C_1\right] \times R_2 \times \left[-j\frac{1}{\omega C_3}\right]$$

$$R_4 - j\frac{1}{\omega C_4} = -j\frac{R_2}{\omega R_1 C_3} - j^2\frac{\omega R_2 C_1}{\omega C_3}$$

$$R_4 - j\frac{1}{\omega C_4} = \frac{R_2 C_1}{C_3} - j\frac{R_2}{\omega R_1 C_3} \quad \text{(ii)}$$

Comparing Real and imaginary part

$$\therefore R_4 = \frac{C_1}{C_3} R_2 \quad \text{(iii)}$$

And, $\frac{1}{\omega C_4} = \frac{R_2}{\omega R_1 C_3}$ implies,

$$\therefore C_4 = \frac{R_1}{R_2} C_3 \quad \text{(iv)}$$

Advantages

- In this type of bridge, the value of capacitance can be measured accurately.
- It can measure capacitance value over a wide range.
- It can measure dissipation factor accurately.

Disadvantages

- It requires extra two capacitors.
- Variable standard capacitor is costly.

Assignment

1. An Ac bridge is working at 1000 Hz. Arm AB has 0.2μF pure capacitance, arm BC has 500 Ω pure resistance, arm CD contains an unknown impedance and arm DA has 300 Ω resistance in parallel with 0.1 μF capacitor. Find the constant of arm CD considering it as a series circuit. [2075 Baisakh, 2071 Magh]
2. A balanced Ac bridge has following constant:
   a) Arm AB: R=1000 Ω in parallel with C=0.5 μF
   b) Arm BC: R=1000 Ω in series with C=0.5 μF
   c) Arm CD: R=200 Ω in series with L=30 mH

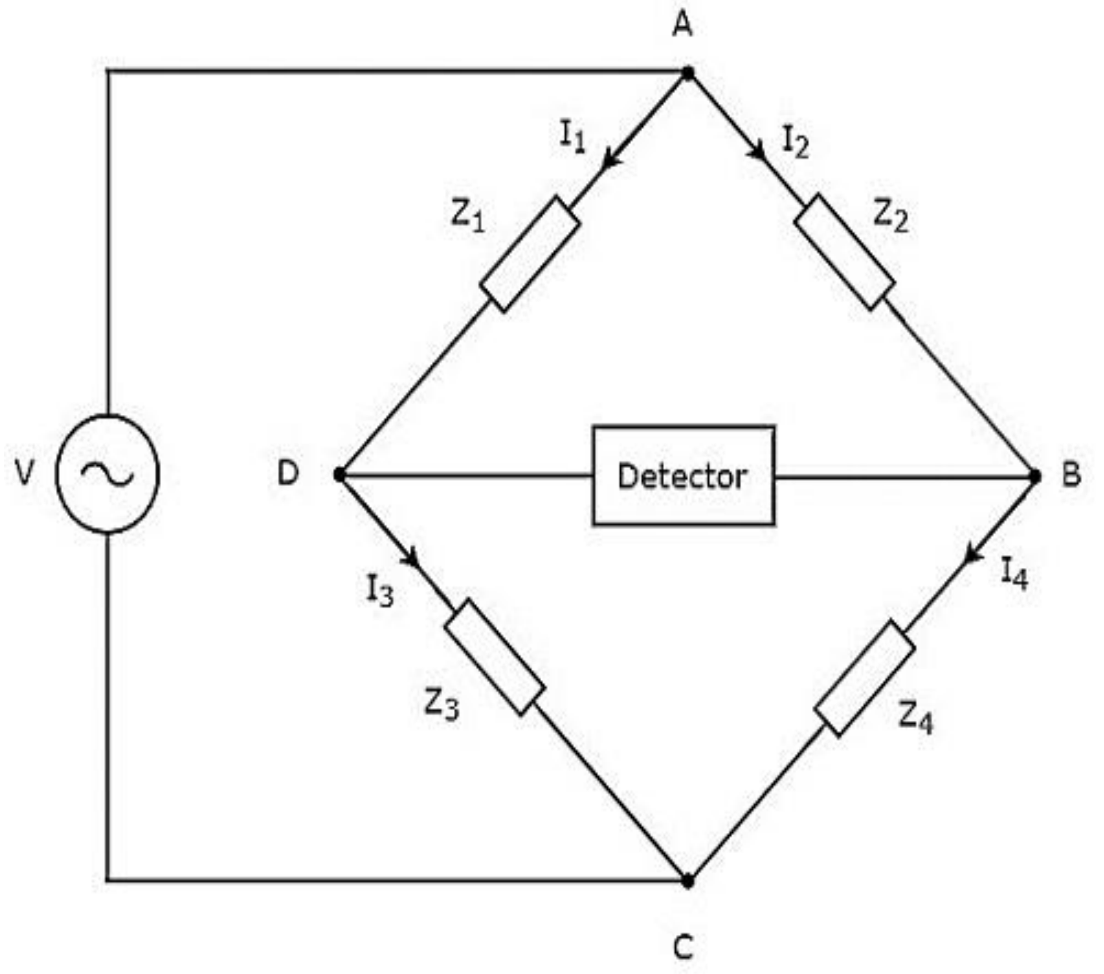
   Find the constant of arm DA. Express the result as a pure R in parallel with C or L. [IOE 2070 Bhadra]
3. A balanced Ac bridge has following constant:
   a) Arm AB: R=2000 Ω in parallel with C=0.047 μF
   b) Arm BC: R=1000 Ω in series with C=0.47 μF
   c) Arm CD: unknown R
   d) Arm DA: C=0.5 μF

   The frequency of oscillator is 1000Hz. Find constant of unknown arm CD. [2069 Bhadra]

An Ac bridge is working at 1000 Hz. Arm AB has 0.2μF pure capacitance, arm BC has 500 Ω pure resistance, arm CD contains an unknown impedance and arm DA has 300 Ω resistance in parallel with 0.1 μF capacitor. Find the constant of arm CD considering it as a series circuit. [2075 Baisakh, 2071 Magh]

Solution:

Let us design a circuit as shown in the figure



Impedances:

For arm AB $Z_2 = X_{C2} = \frac{1}{j\omega C_2} = -\frac{j}{\omega C_2}$ where $C_2 = 0.2\mu F$

For arm BC $Z_4 = R_4 = 500\ \Omega$

For arm CD Z_3 = ? (series circuit)

Let R_3 and C_3 in series connection, Then

$Z_3 = R_3 + X_{C3} = R_3 + \frac{1}{j\omega C_3} = R_3 - j\frac{1}{\omega C_3}$

For arm DA, the impedance is Z_1 and component $R_1 = 300\ \Omega$ & $C_1 = 0.1\ \mu F$ in parallel so,

$$Z_1 = \frac{R_1 \times X_{C1}}{R_1 + X_{C1}} = \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1} = \frac{300}{1 + j\omega 30\mu}$$

Say, the bridge is balanced , then the balanced condition will be,

$Z_1 Z_4 = Z_2 Z_3$ ______________________________(i)

From equation (i) we have to find value of unknown impedance Z_3,So

$$Z_1Z_4 = Z_2Z_3$$

$$\frac{300}{1+j\omega 30\mu}.\ R_4 = -\frac{j}{\omega C_2}\times[R_3\text{- }j\frac{1}{\omega C_3}]$$

$$\frac{300R_4}{1+j\omega 30\mu} = -\frac{j}{\omega C_2}\times[R_3\text{- }j\frac{1}{\omega C_3}]$$

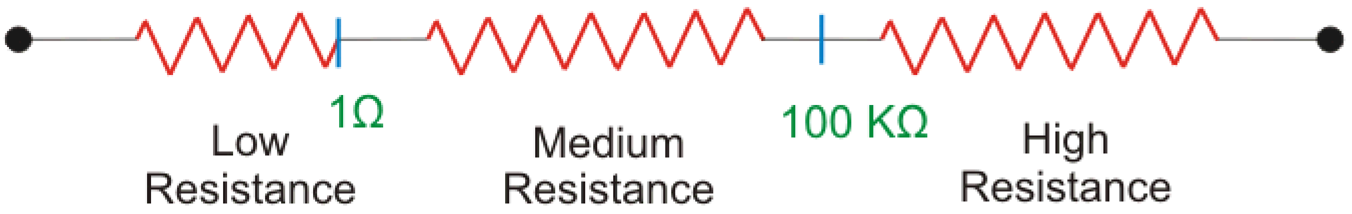
$$\frac{300R_4}{1+j\omega 30\mu} = -j\frac{R_3}{\omega C_2}+\frac{j^2}{\omega^2 C_2C_3}$$

$$\frac{j300R_4\omega C_2}{1+j\omega 30\mu} = -\frac{1}{\omega^2 C_2C_3} - j\frac{R_3}{\omega C_2}$$

Comparing real and imaginary term,

Measurement of Resistance

- Resistance is one of the most basic elements encountered in electrical and electronics engineering.
- The value of resistance in engineering varies from very small value like, resistance of a transformer winding, to very high values like, insulation resistance of that same transformer winding.
- Although a multimeter works quite well if we need a rough value of resistance, but for accurate values and that too at very low and very high values we need specific methods.
- In this chapter we will discuss various methods of resistance measurement.
- For this purpose we categories the resistance into three classes



Measurement of low, medium and high resistances:

From measure point of view, resistances are classified into

1. Low resistance: ($<1\Omega$)

   These include resistance of 1Ω or below. Following are few methods used for measurement of low resistance values-

   a) Kelvin's Double Bridge Method
   b) Potentiometer Method
   c) Ducter Ohmmeter.

2. Medium resistance: ($1\Omega - 100k\Omega$)

   These include resistance in the range of 1Ω to $0.1M\ \Omega$. Following are few methods used for measurement of medium resistance values-

   a) Ammeter-Voltmeter Method
   b) Wheatstone Bridge Method
   c) Substitution Method
   d) Carey- Foster Bridge Method
   e) Ohmmeter Method

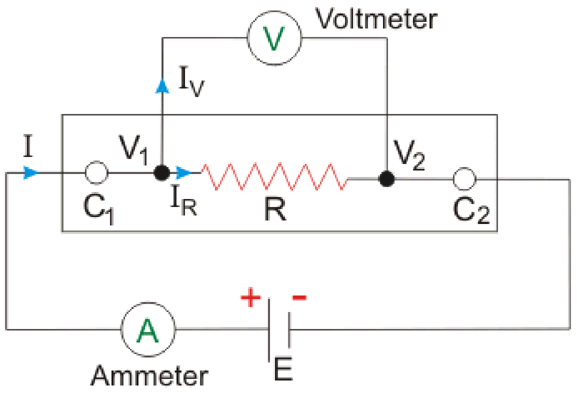
3. High resistance: ($>100k\Omega$)

   These include resistance of $0.1M\Omega$ and above. Following are few methods used for measurement of high resistance values-

   a) Loss of Charge Method
   b) Megger
   c) Megohm bridge Method
   d) Direct Deflection Method

1. Measurement of Low Resistance (<1Ω)

- The major problem in measurement of low resistance values is the contact resistance or lead resistance of the measuring instruments, though being small in value is comparable to the resistance being measured and hence causes serious error.
- Thus to eliminate this issue small valued resistance are constructed with four terminals. Two terminals are current terminals and other two are potential terminals.

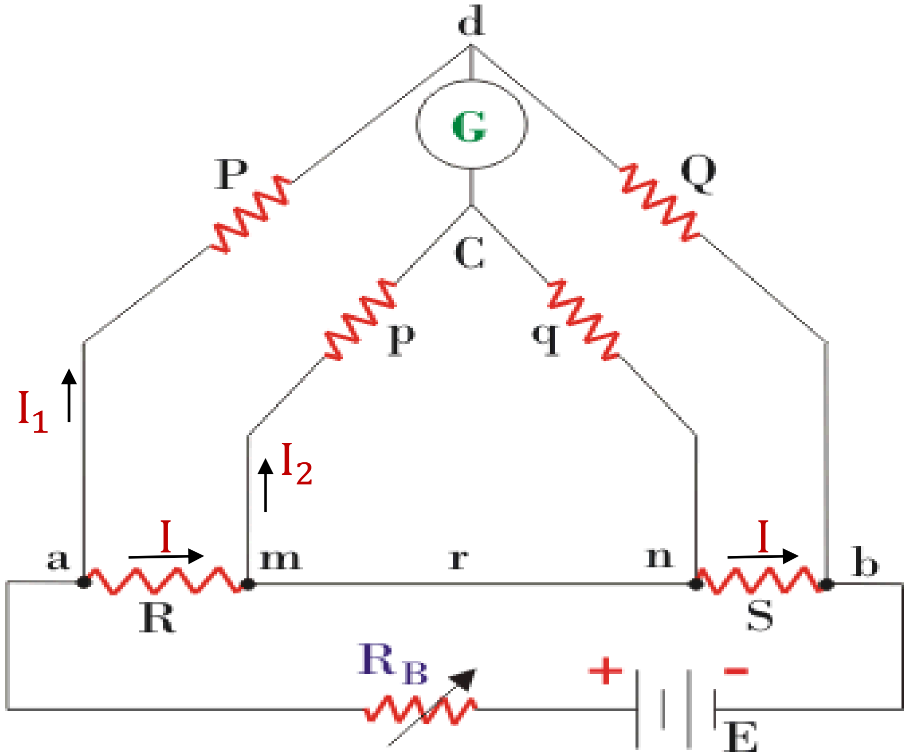


Low Resistance

- The current is flown through current terminals C1 and C2 while the potential drop is measured across potential terminals V1 and V2. Hence we can find out the value of resistance under experiment in terms of V and I as indicated in the above figure. This method helps us to exclude the contact resistance due to current terminals and though contact resistance of potential terminals still comes into picture, it is very small fraction of high resistance potential circuit and hence induces negligible error.

i) Kelvin's Double Bridge

- Kelvin's double bridge is a modification of simple Wheatstone bridge.



Kelvin Double Bridge

- As we can see in the above figure there are two sets of arms, one with resistances P and Q and other with resistances p and q.
- R is the unknown low resistance and S is a standard resistance.
- Here r represents the contact resistance between the unknown resistance and the standard resistance, whose effect we need to eliminate.
- For measurement we make the ratio $\frac{P}{Q}$ equal to $\frac{p}{q}$ and hence a balanced Wheatstone bridge is formed leading to null deflection in the galvanometer.
- Hence for a balanced bridge we can write

➢ $\frac{P}{Q} = \frac{p}{q}$ ————(1)

Kelvin's Double Bridge….

Applying KVL in loop 'adcma'

$$I_1P - I_2p - I.R = 0$$

$$I.R = I_1P - I_2p \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

Similarly applying KVL in loop 'dbncd'

$$I_1Q - I_2q - I.S = 0$$

$$I.S = I_1Q - I_2q \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

Dividing equation (2) by (1) and solving

$$\frac{I.R}{I.S} = \frac{I_1P - I_2p}{I_1Q - I_2q}$$

$$\frac{R}{S} = \frac{P[I_1 - I_2\frac{p}{P}]}{Q[I_1 - I_2\frac{q}{Q}]}$$

From equation (1);

$$\frac{P}{Q} = \frac{p}{q} \text{ so } \frac{q}{Q} = \frac{p}{P}$$

So we can write

$$\frac{R}{S} = \frac{P[I_1 - I_2\frac{p}{P}]}{Q[I_1 - I_2\frac{p}{P}]}$$

$$\frac{R}{S} = \frac{P}{Q} \text{ implies}$$

$$\therefore R = \frac{P}{Q}.S$$

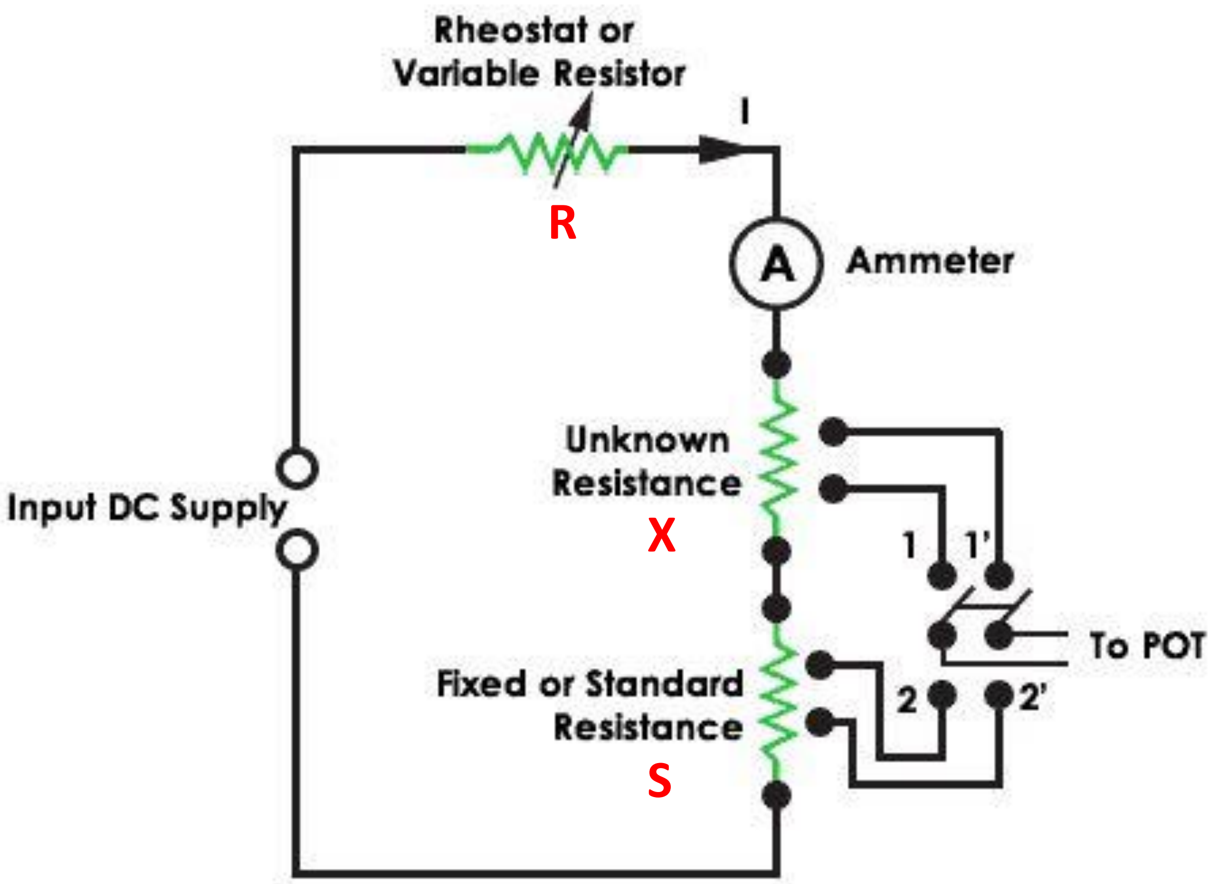
Hence we see that by using balanced double arms we can eliminate the contact resistance completely and hence error due to it. To eliminate another error caused due to thermo-electric Emf, we take another reading with battery connection reversed and finally take average of the two readings. This bridge is useful for resistances in range of 0.1μΩ to 1.0 Ω.

ii) Potentiometer method

- Potentiometer method is used for measuring very low resistance in laboratory or test rooms.
- It is based on comparison of one resistance against another by an indirect method.
- In this method unknown resistance X, a rheostat R (just to limit current), an ammeter and a standard resistor S are connected in series with a low voltage high current supply source. The current through the circuit is adjusted so that potential difference across each resistors is of about 1 volt.
- The voltage across both resistors is measured using dc-potentiometer.
- Here voltage drop across each resistors is proportional to their resistance.

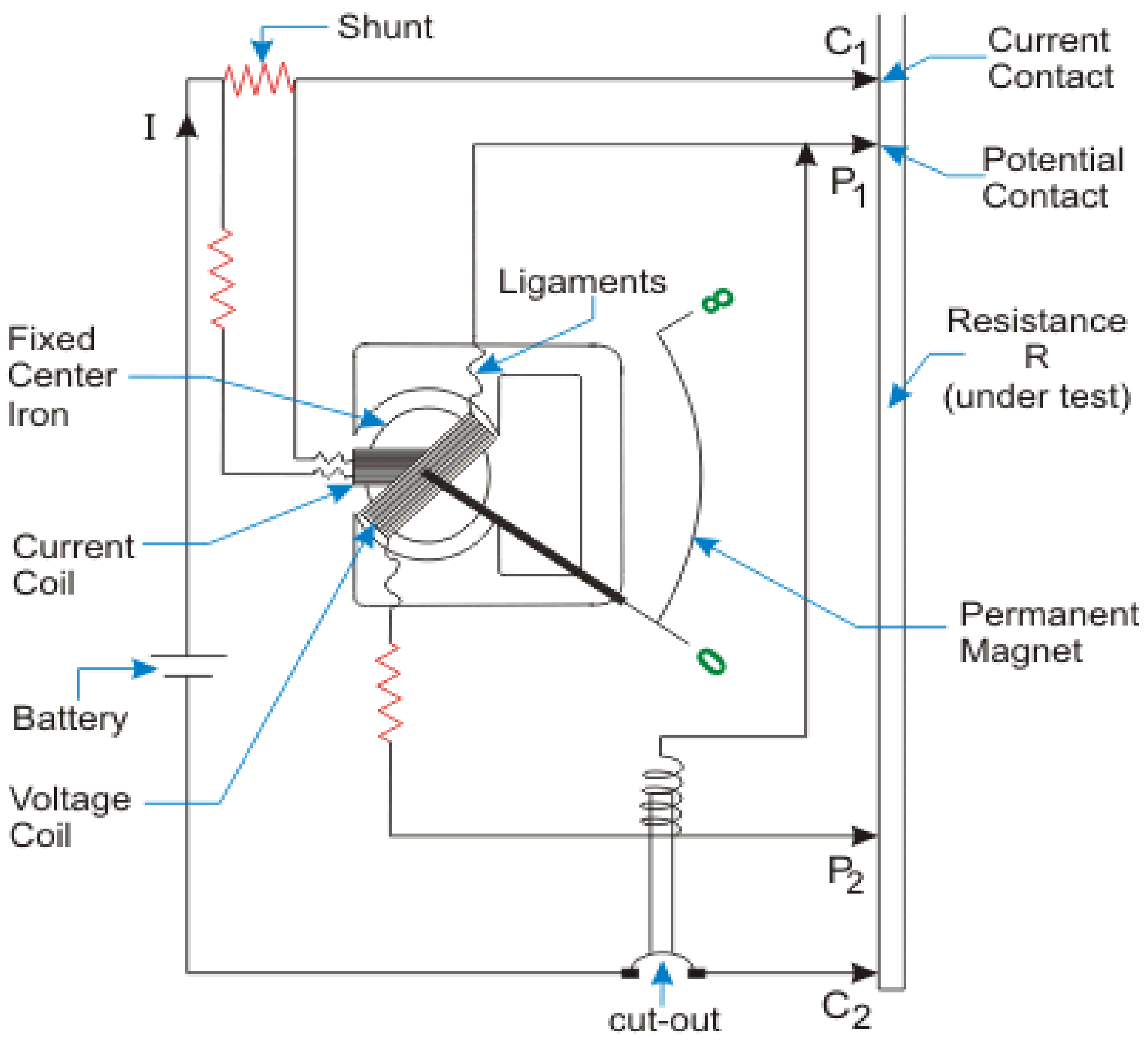
$$\text{So, } \frac{X}{S} = \frac{V_X}{V_S}$$

$$\therefore \boxed{X = \frac{V_X}{V_S} . S}$$



iii) Ducter Ohmmeter

- It is an electromechanical instrument used for measurement of low resistances.
- It comprises of a permanent magnet similar to that of a PMMC instrument and two coils in between the magnetic field created by the poles of the magnet.
- The two coils are at right angles to each other and are free to rotate about the common axis.
- Figure aside shows a Ducter Ohmmeter and the connections required to measure an unknown resistance R.



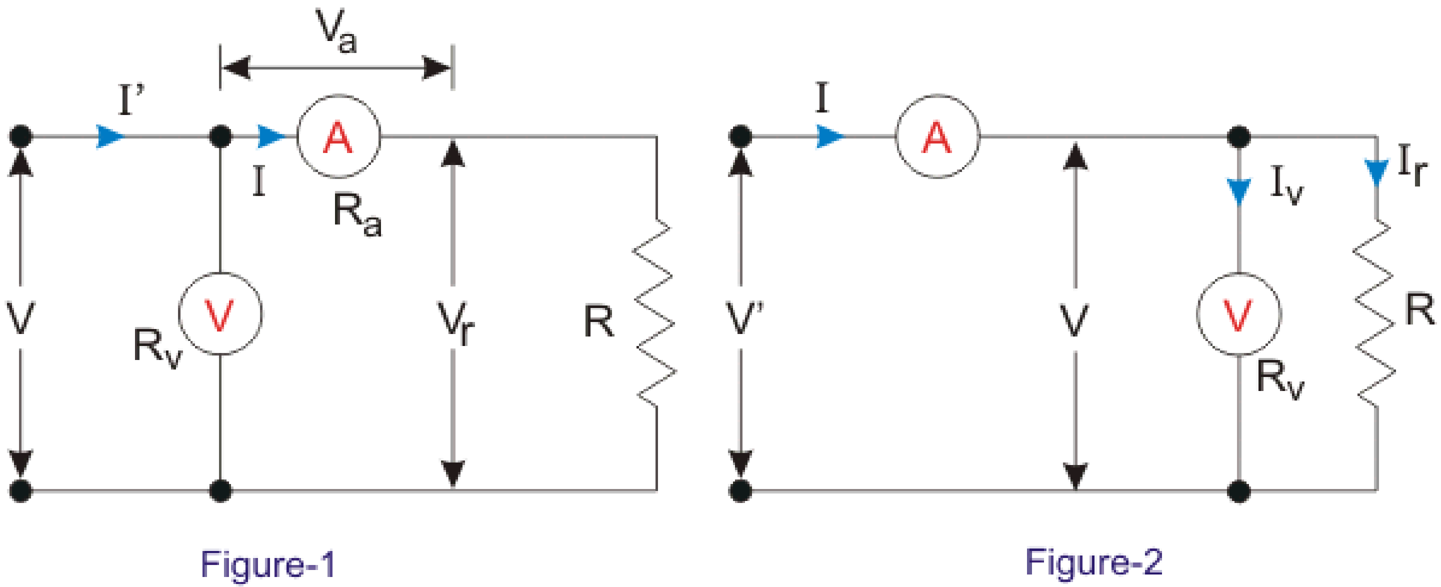
Ducter Ohmmeter….

- One of the coil called current coil, is connected to current terminals C_1 and C_2, while the other coil called, voltage coil is connected to potential terminals V_1 and V_2.
- Voltage coil carries current proportional of the voltage drop across R and so is its torque produced.
- Current coil carries current proportional to the current flowing through R and so is its torque too.
- Both the torque acts in opposite direction and the indicator come to halt when the two are equal.
- This instrument is useful for resistance in range 100μΩ to 5Ω.

2. Measurement of Medium Resistance (1Ω to 100KΩ)

i) Ammeter Voltmeter Method

- This is the most crude and simplest method of measuring resistance. It uses one ammeter to measure current, I and one voltmeter to measure voltage, V and we get the value of resistance as R= $\frac{V}{I}$
- Now we can have two possible connections of ammeter and voltmeter, shown in the figure below.



Ammeter Voltmeter Method

Now in figure 1, the voltmeter measures voltage drop across ammeter and the unknown resistance, hence

$$R_1 = \frac{V_a+V_r}{I} = \frac{IR_a+IR}{I} = R_a + R$$

Hence, the relative error will be,

$$\epsilon = \frac{R_1-R}{R} = \frac{R_a}{R}$$

For connection in figure 2, the ammeter measures the sum of current through voltmeter and resistance, hence

$$R_2 = \frac{V}{I_V+I_r} = \frac{V}{[\frac{V}{R_V}+\frac{V}{R}]} = \frac{R}{[1+\frac{R}{R_V}]}$$

Hence, the relative error will be,

$$\epsilon = \frac{R_2-R}{R} = \approx -\frac{R}{R_V}$$

It can be observed that the relative error is zero for $R_a = 0$ in first case and $R_V = \infty$ in second case.

Now the questions stand that which connection to be used in which case. To find out this we equate both the errors

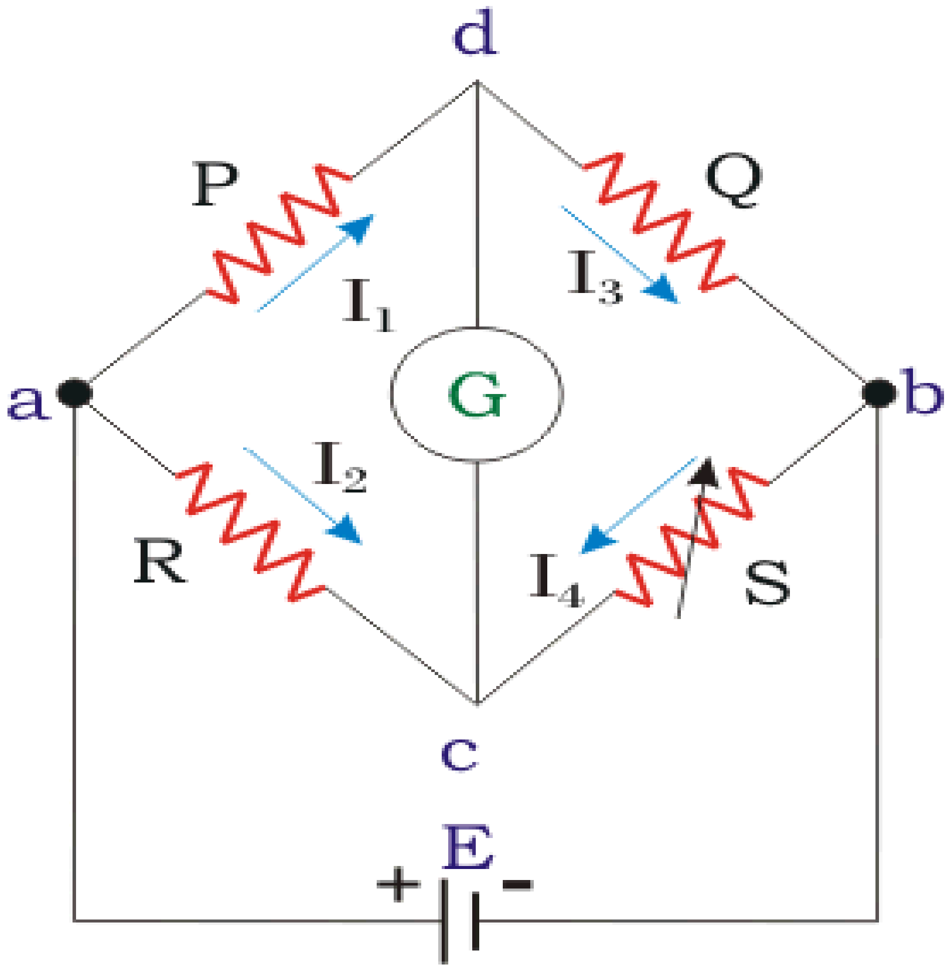
$$\frac{R_a}{R} = \frac{R}{R_V}$$

$$R = \sqrt{R_a R_V}$$

Hence for resistances greater than that given by above expression we use the first method and for less than that we use second method.

ii) Wheatstone Bridge Method

- This is the simplest and the most basic bridge circuit used in measurement studies.
- It mainly consists of four arms of resistance P, Q, R and S.
- R is the unknown resistance under experiment, while S is a standard resistance.
- P and Q are known as the <u>Ratio arms</u>.
- An EMF source is connected between points a and b while a galvanometer is connected between points c and d.



Wheatstone Bridge

A bridge circuit always works on the principle of null detection, i.e. we vary a parameter until the detector shows zero and then use a mathematical relation to determine the unknown in terms of varying parameter and other constants.

Here also the standard resistance, S is varied in order to obtain null deflection in the galvanometer.

This null deflection implies no current from point c to d, which implies that potential of point c and d is same. Hence

$$I_1 P = I_2 R$$ Also,

$$I_1 = I_3 = \frac{E}{P+Q}$$ and

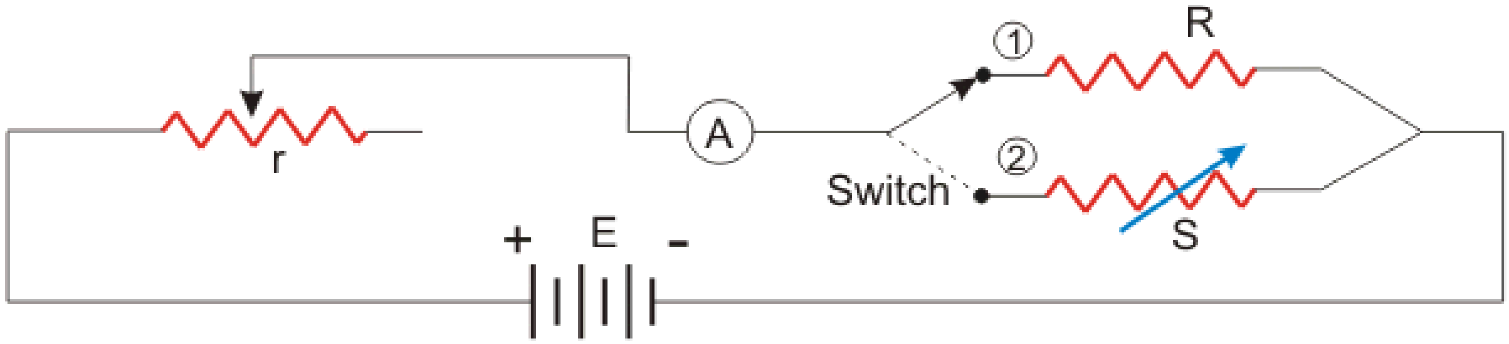
$$I_2 = I_4 = \frac{E}{R+S}$$

Combining the above two expressions we get the famous equation

$$R = \frac{P}{Q} . S$$

iii) Substitution Method

- The figure below shows the circuit diagram for resistance measurement of an unknown resistance R.
- S is a standard variable resistance and r is a regulating resistance.



Substitution Method

- First the switch is place at position 1 and the ammeter is made to read a certain amount of current by varying r. The value of ammeter reading is noted.
- Now the switch is moved to position 2 and S is varied in order to achieve the same ammeter reading as it read in the initial case.
- The value of S for which ammeter reads same as in position 1, is the value of unknown resistance R, provided the EMF source has constant value throughout the experiment.

iv) Carey Foster Method

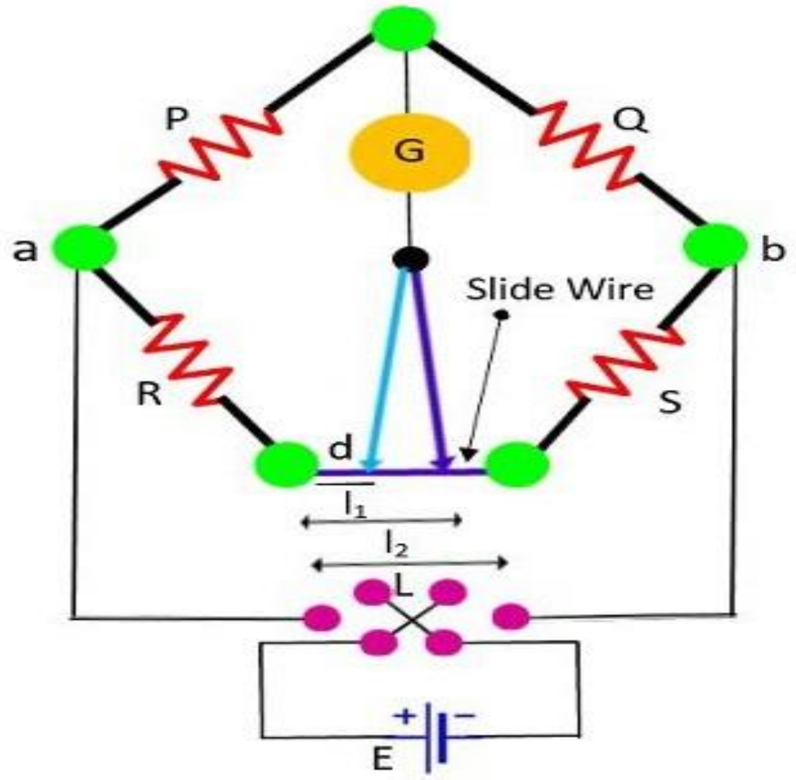
- It is an elaboration of wheat stone bridge. In figure P & Q are equal ratio arms and adjusted in such a way that the bridge is in balanced condition.
- R is resistance under test and S is a standard resistance. A slide-wire of length l and uniform cross section is included between R & S as shown.
- First balance point is obtained by sliding contact on the slide-wire. Let l_1 be the distance of the sliding contact from left hand end of the slide wire.
- Then for the first balance

$$\frac{P}{Q} = \frac{R+l_1 r}{S+(l-l_1)r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

- Where r is the resistance per unit length of the slide wire
- The resistors R & S are interchanged, and a new balance point is obtained. Let l_2 be the new balance point is obtained. Then at balance

$$\frac{P}{Q} = \frac{S+l_2 r}{R+(l-l_2)r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

- Comparing expression(i) & (ii)
- We get, $\frac{R+l_1 r}{S+(l-l_1)r} = \frac{S+l_2 r}{R+(l-l_2)r}$
- $\frac{R+l_1 r}{S+(l-l_1)r} + 1 = \frac{S+l_2 r}{R+(l-l_2)r} + 1$



Carey Foster Method….

- After simplification

$$S-R= (l_1-l_2)r……………………………………………..…. \quad (iii)$$

- Thus the difference between standard resistance S and unknown resistance R is obtained in terms of resistance of slide wire between two balance points.
- The slide-wire is calibrated. i.e. r is obtained by shunting either S or R by a known resistance by a known high resistance and again determining the difference in length $(l_1^1-l_2^1)$
- Suppose that S is known and reducing its effective value to S^1 after shunting. The new balance points l_1^1 and l_2^1 are obtained by repeating above procedure.

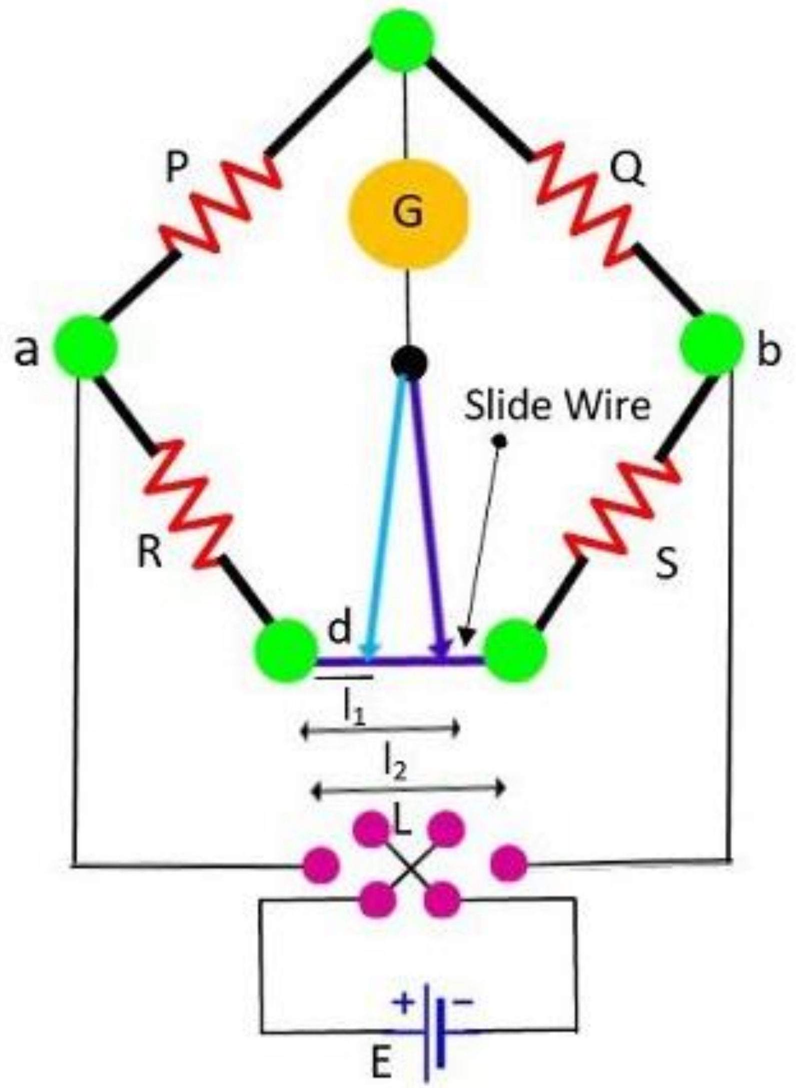
$$S^1-R = (l_1^1-l_2^1)r…………………………………..…..(iv)$$

from equation (iii)and (iv)

$$\frac{S-R}{s^1-R} = \frac{l_1-l_2}{l_1^1-l_2^1}$$

$$R = \frac{S(l_1^1-l_2^1)-s^1(l_1-l_2)}{l_1^1-l_2^1-l_1+l_2}$$

- Thus this expression gives unknown resistance, R in terms of standard resistance and lengths only.



v) Ohmmeter method

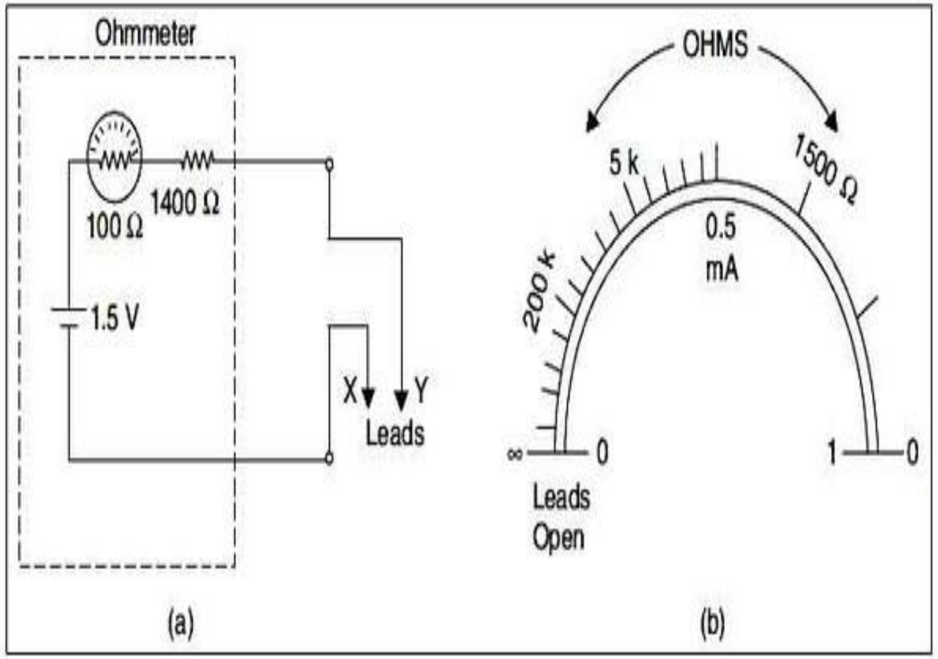
- Ohmmeter is a direct reading instrument for measurement of resistance. Though this method of measuring resistance is simple, convenient and fast, it has a lower accuracy. Unlike the ammeter or voltmeter, the ohmmeter circuit does not receive the energy necessary for its operation from the circuit under test. In the ohmmeter this energy is supplied by a self-contained source of voltage, such as a battery.

The Analog Ohmmeter

- Analog ohmmeter can be designed simply by adding a battery and a variable resistor in series with the moving coil instrument as shown in Figure.
- The unknown resistance is connected to the terminals of the device to complete the electrical circuit.
- The output terminals are shorted together with the leads (wires) used in connecting the external resistor. The variable resistance is adjusted until the full-scale deflection current passes through the coil. This is marked as the "0" resistance.
- When the leads are separated from each other, no current flows indicating an open-circuit which means "infinite " resistance

The Analog Ohmmeter….

- For measuring resistance, the ohm-meter leads X-Y are connected across the unknown resistance after switching off the power in the circuit under test.
- Only in that case, the ohmmeter battery can provide current for the meter movement.
- Since the amount of current depends on the amount of external resistance, the meter scale can be calibrated in ohms (instead of mA).
- When the leads X-Y are shorted, meter current is $=\frac{1.5\text{ V}}{(100+1400)\Omega}$ = 1 mA.
- The meter shows full-scale deflection to the right.
- The ohmmeter reading corresponds to 1 because external resistance is zero (i.e. short circuited)
- When leads X-Y are open circuited i.e. do not touch each other, then the external resistance corresponds to infinite and hence meter current is zero.
- Hence, it corresponds for measureable resistance

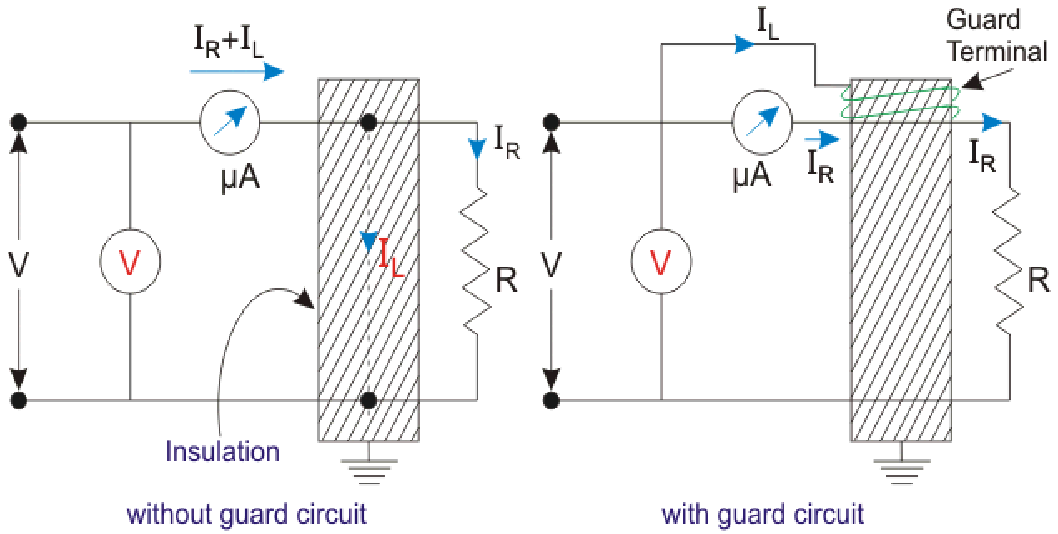


3. Measurement of High Resistance (>100kΩ)

- Following are few methods used for measurement of high resistance values-
  a) Loss of Charge Method
  b) Megger
  c) Megohm bridge Method
  d) Direct Deflection Method
- We normally utilize very small amount of current for such measurement, but still owing to high resistance chances of production of high voltages is not surprising. Due to this we encounter several other problems such as-
  a) Electrostatic charges can get accumulated on measuring instruments
  b) Leakage current becomes comparable to measuring current and can cause error
  c) Insulation resistance is one of the most common in this category; however a dielectric is always modeled as a resistor and capacitor in parallel. Hence while measuring the insulation resistance (I.R.) the current includes both the component and hence true value of resistance is not obtained. The capacitive component though falls exponentially but still takes very long time to decay. Hence different values of I.R. are obtained at different times.
  d) Protection of delicate instruments from high fields.

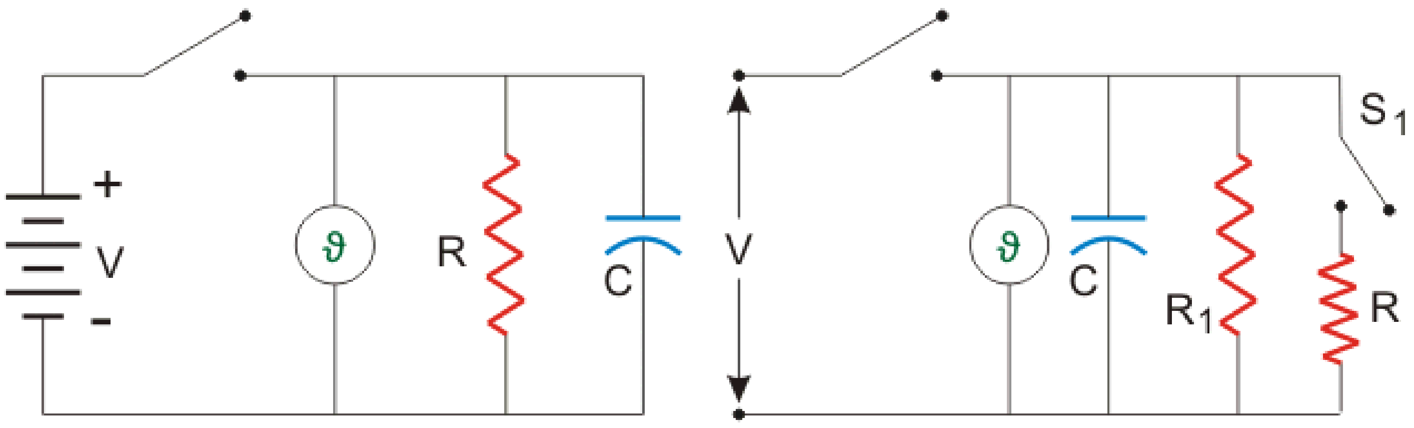
Measurement of High Resistance (>100kΩ)....

- Hence to solve the problem of leakage currents or capacitive currents we use a guard circuit.
- The concept of guard circuit is to bypass the leakage current from the ammeter so as to measure the true resistive current.
- Figure below shows two connections on voltmeter and micro ammeter to measure R, one without guard circuit and one with guard circuit
- In the first circuit the micro ammeter measures both capacitive and the resistive current leading to error in value of R, while in the other circuit the micro ammeter reads only the resistive current.



i) Loss of Charge Method

- In this method we utilize the equation of voltage across a discharging capacitor to find the value of unknown resistance R. Figure below shows the circuit diagram and the equations involved are-



Loss of Charge Method

$$v = Ve^{\frac{-t}{RC}}$$

$$R = \frac{0.4343t}{C\, log_{10}V/v}$$

- However the above 1st case assumes no leakage resistance of the capacitor. Hence to account for it we use the circuit shown in the figure 2nd
- R_1 is the leakage resistance of C and R is the unknown resistance.

In this method the insulation resistance R to be measured is connected in parallel with a capacitor C and a electrostatic voltmeter. During measurement of resistance, the capacitor is charged to some suitable voltage by means of supply voltage V and then allowed to discharge through the resistance. The time taken t, for the potential difference to drop from V1 to V2 is observed.

Voltage across capacitor at any instant, t

$$\mathcal{v} = Ve^{-\frac{t}{CR'}}$$

$$\frac{V}{\mathcal{v}} = e^{\frac{t}{CR'}}$$

$$\frac{t}{CR'} = \ln\frac{V}{\mathcal{v}}$$

$$R' = \frac{t}{C.\ln\frac{V}{\mathcal{v}}} = \frac{t}{2.303\ C.\log\frac{V}{\mathcal{v}}} = \frac{0.4343\ t}{C.\log\frac{V}{\mathcal{v}}}$$

We follow the same procedure but first with switch S1 closed and next with switch S1 open. For the first case we get

$$R' = \frac{0.4343\ t}{C.\log\frac{V}{\mathcal{v}}} \quad \text{Where } R' = \frac{RR_1}{R+R_1}$$

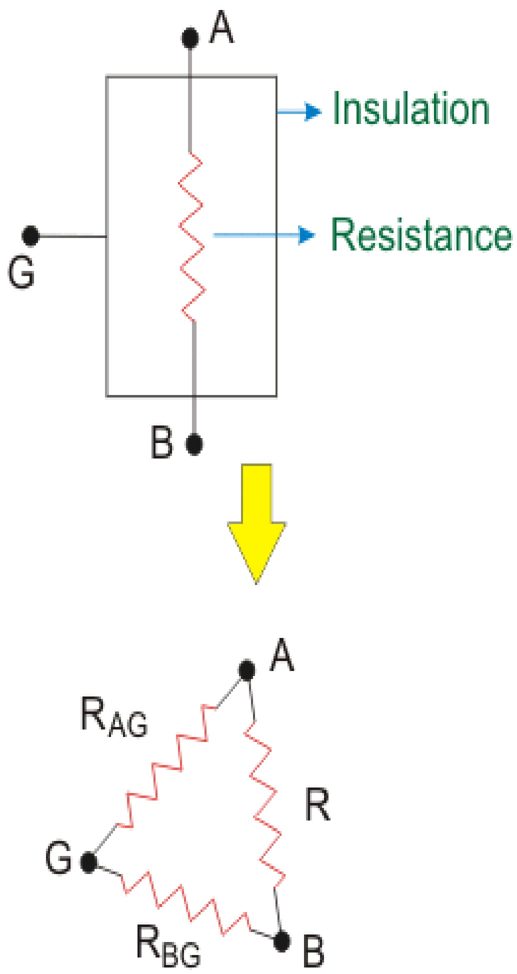
For second case with switch S1 open we get

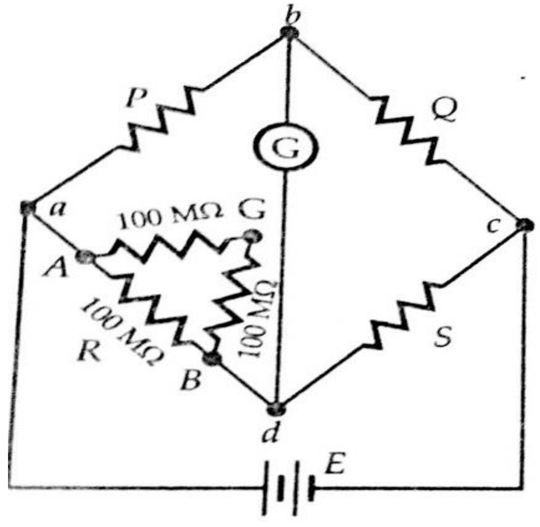
$$R_1 = \frac{0.4343\ t}{C.\log\frac{V}{\mathcal{v}}}$$

Using R_1 from above equation in equation for R' we can find R.

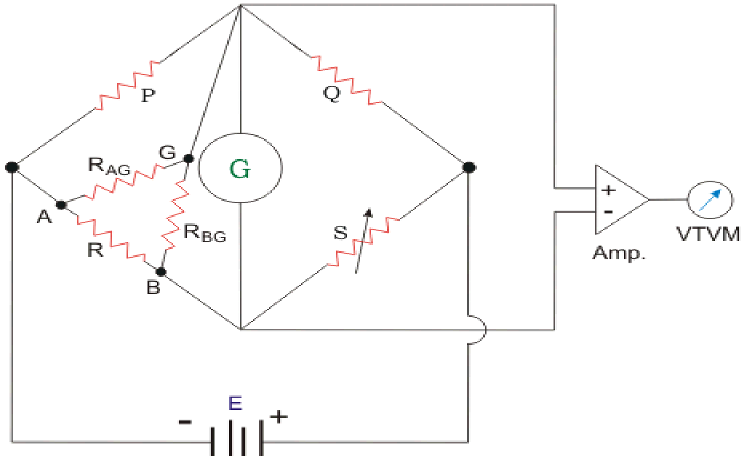
ii) Megohm Bridge Method

- In this method we use the famous Wheatstone bridge philosophy but in a slightly modified way. A high resistance is represented as in the figure aside.
- G is the guard terminal. Now we can also represent the resistor as shown in the adjoining figure, where R_{AG} and R_{BG} are the leakage resistances. The circuit for measurement is shown in the figure below.
- It can be observed that we actually obtain the resistance which is parallel combination of R and R_{AG}. Although this causes very insignificant error.
- It has a range of 0.1 MΩ to 1,000,000 MΩ





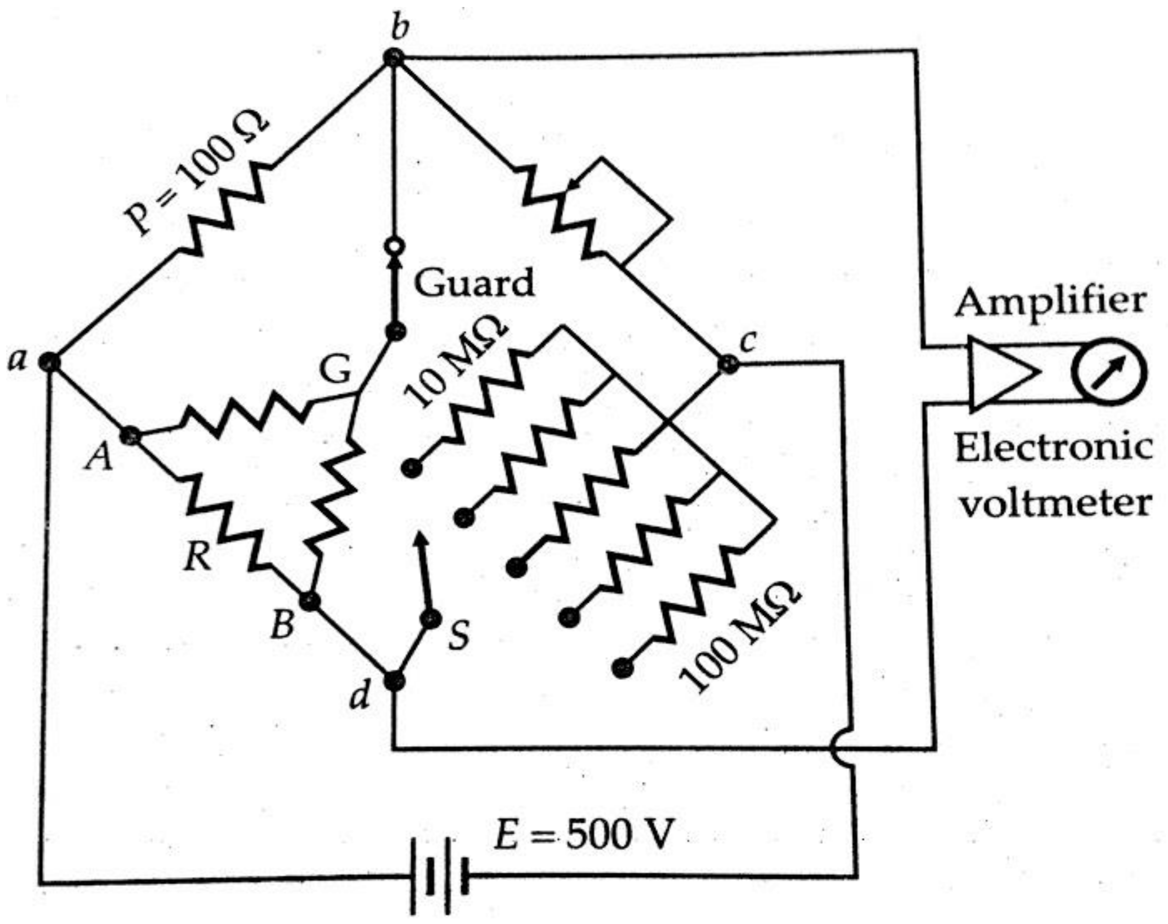
(a)



Megohm Bridge

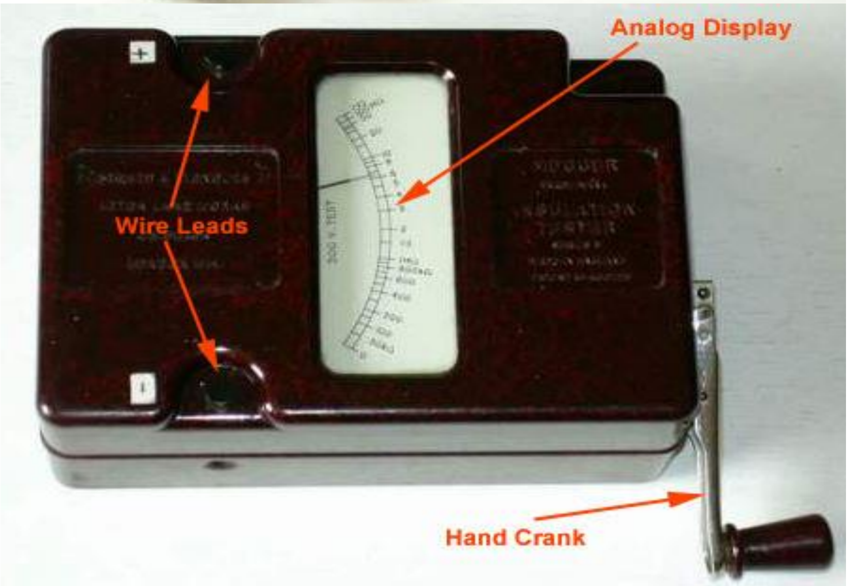
Megohm Bridge Method….

- Let us consider the hypothetical case of a 100 MΩ resistance.
- i.e. $R_{AG} = R_{BG} = 100\ \text{M}\Omega$
- Let this resistance is measured by ordinary Wheatstone bridge
- $R_{Equivalent} = \frac{100 \times 200}{100 + 200} = 67\ \text{M}\Omega$
- It measures 67 instead of measuring 100 MΩ, thus giving an error of 33%
- However if the same resistance is measured by modified Wheatstone bridge with the Guard connection G, the error is considerably reduced.
- Figure aside shows the circuit of completely self contained Megaohm bridge which includes power supplies, bridge a members, amplifiers, and indicating instrument



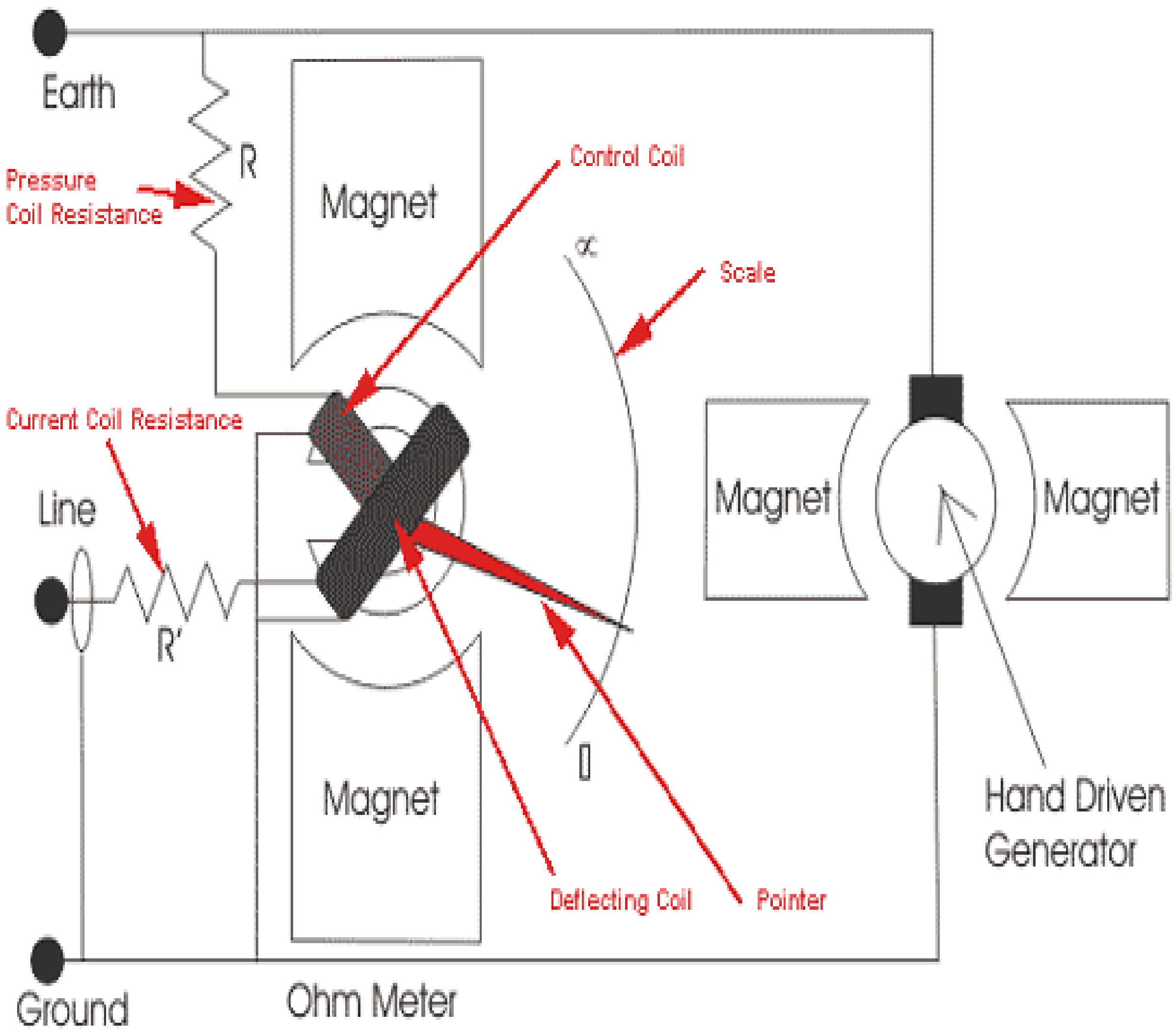
iii) Megger

- Megger is one of the most important measuring device used by electrical engineers and is essentially used for measuring insulation resistance only.
- The Megger insulation tester is a small, portable instrument that gives a direct reading of insulation resistance in ohms or megohms.
- It consists of a generator which can be hand driven or nowadays we have electronic megger.
- This can be separated into mainly two categories:-
  - a) Electronic Type (Battery Operated)
  - b) Manual Type (Hand Operated)
- Megger insulation tester essentially consists of a high-range resistance meter (ohmmeter) with a built-in direct-current generator. This meter is of special construction with both current and voltage coils, enabling true ohms to be read directly, independent of the actual voltage applied.



Construction of Megger

- Deflecting and Control coil : Connected parallel to the generator, mounted at right angle to each other and maintain polarities in such a way to produced torque in opposite direction.
- Permanent Magnets : Produce magnetic field to deflect pointer with North-South pole magnet.
- Pointer : One end of the pointer connected with coil another end deflects on scale from infinity to zero.
- Scale : A scale is provided in front-top of the megger from range ‘zero’ to ‘infinity’, enable us to read the value.
- D.C generator or Battery connection : Testing voltage is produced by hand operated DC generator for manual operated Megger. Battery / electronic voltage charger is provided for automatic type Megger for same purpose.
- Pressure Coil Resistance and Current Coil Resistance : Protect instrument from any damage because of high external electrical resistance under test.

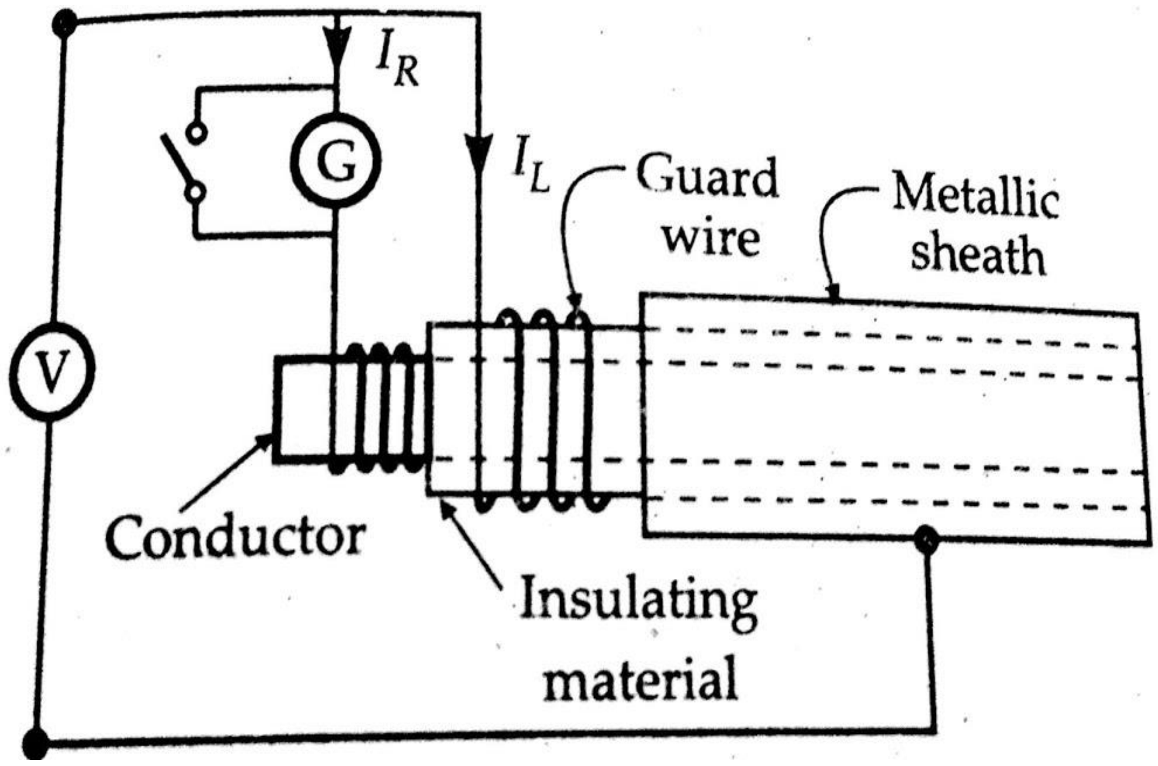


Principle of operation

- A battery is used for electronic tester
- By rotation of crank in case of hand operated type i.e. armature arranges to move in permanent magnetic field or vice versa
- As the voltage increases in external circuit the deflection of pointer increases and deflection of pointer decreases with a increases of current.
- Hence, resultant torque is directly proportional to voltage and inversely proportional to current
- Produced torque shall be in opposite direction if current supplied to the coil.
  a) <u>High Resistance</u> = No Current :- No current shall flow through deflecting coil, if resistance is very high i.e. infinity position of pointer.
  b) <u>Small Resistance</u> = High Current :- If circuit measures small resistance allows a high electric current to pass through deflecting coil, i.e. produced torque make the pointer to set at 'ZERO'.
  c) <u>Intermediate Resistance</u> = Varied Current :- If measured resistance is intermediate, produced torque align or set the pointer between the range of 'ZERO to INIFINITY'.

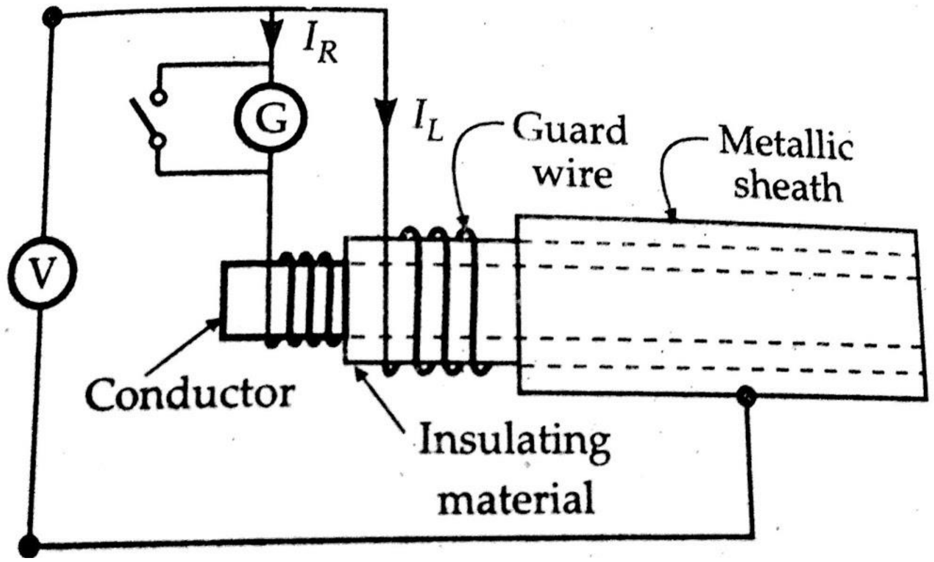
iv) Direct deflection method:

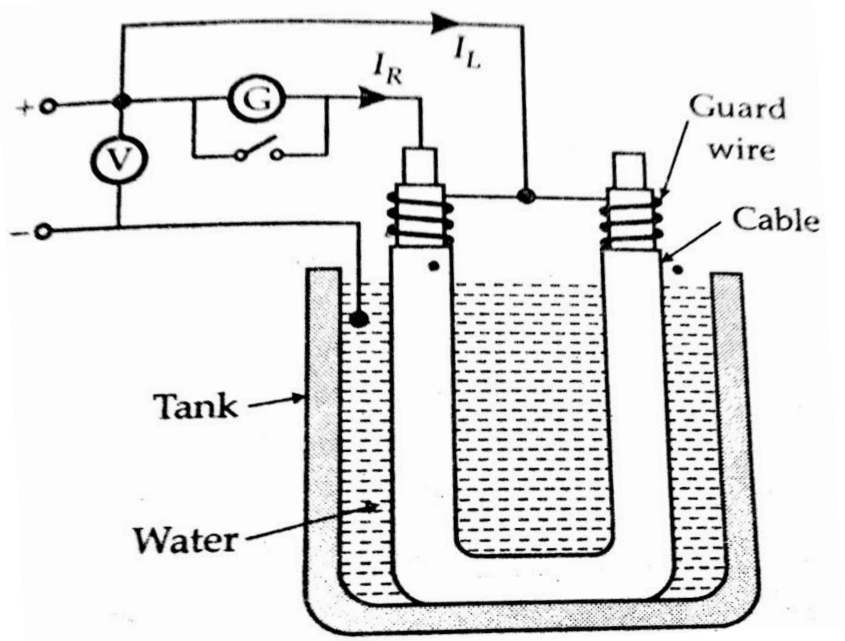
- This is a simple method of measurement of insulation resistance.
- In this method a very sensitive and high resistance (>1000 ohm) galvanometer is connected in series to the resistance to be measured and a battery supply.
- The deflection of the galvanometer gives a measure of the insulation resistance. Though it is a simple method, it is very rough method of measurement of resistance.
- A method for measuring insulation resistance using direct deflection method is shown in figure



Direct deflection method....

- Galvanometer G measures current I_R between the conductor and metal sheath.
- Leakage current I_L over the insulating material is carried by guard wire wound on the insulation
- Cables without metal sheaths can be tested in a similar way by immersing the cable ends, on which connection are made, in water in a tank. The water and tank then form the return path for the current.
- The ratio of voltage applied between the core and metal sheath and the current flowing between them gives the insulation resistance of the cable.
- Insulation resistance of cable, $R = \frac{V}{I_R}$





THANK YOU !

End of chapter 2