ELECTRICAL MACHINES(EE 554)

Chapter-6(Three Phase Synchronous Machines)

Contents

Three Phase Synchronous Generator

- AC generators are usually called alternators because they generate AC currents and voltage.
- Rotating machines that rotate at a speed fixed by the supply frequency and the number of poles are called synchronous machines.

$N_s = \frac{120f}{P}$, runs at synchronous speed

- A synchronous generator is a machine for converting mechanical power from a prime mover to ac electric power at a specific voltage and frequency.
- Rating of generators are expressed in kVA or MVA.
- Generation voltage: 6.3 kV, 6.6kV, 11 kV
- The main parts of a synchronous generator or motor(Synchronous Machines) are:

1. Stator or Armature
2. Rotor
3. Exciter

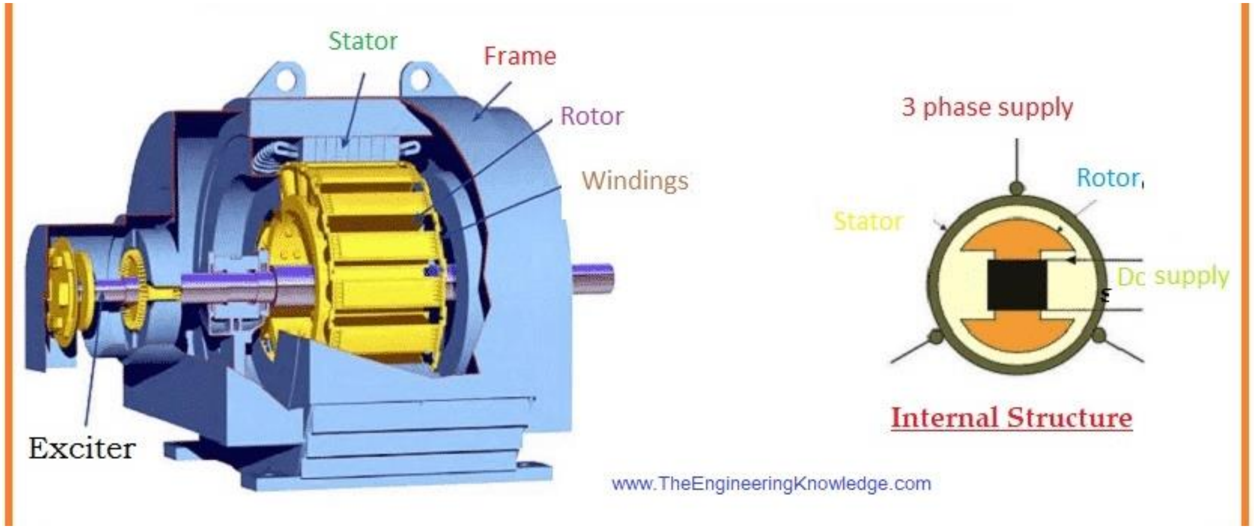
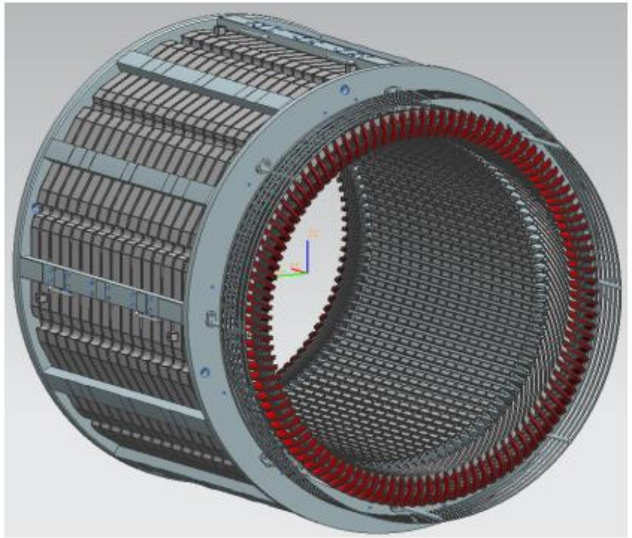
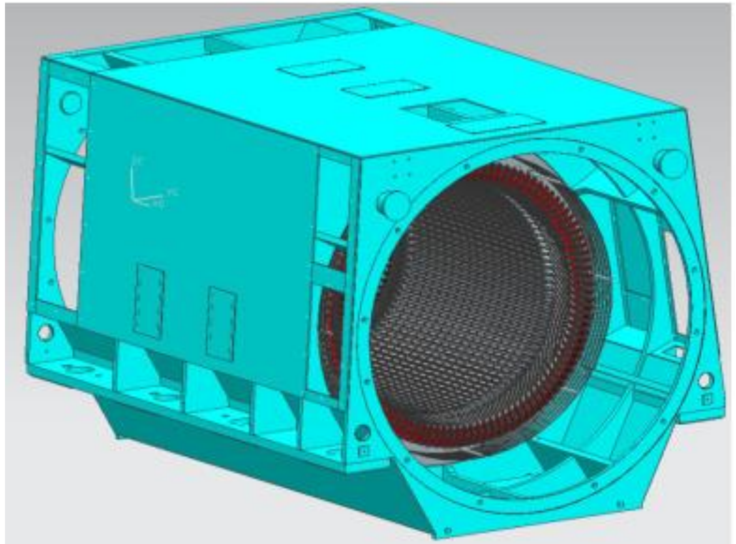


Fig: 3 phase synchronous machines

Stator

- Same as stator of 3-phase induction machines
- It is the stationary part of the machine in cylindrical form with hollow space at the center.
- The stator core is made of laminated silicon steel.
- The inner circumference of the stator core has alternate number of slots and teeth.
- The slots are provided with stator windings made of enamel insulated copper wire.
- In case of three-phase synchronous machine, the stator winding is three-phase uniformly distributed winding with each phase spaced 120^0 electrically apart.
- The windings are insulated from the slots with insulating paper.
- When the stator windings are supplied by three-phase voltage, the winding creates definite number of magnetic poles on the stator core.
- Stator core is protected by outer covering called yoke.
- Yoke houses the stator core and provides mechanical protection of the whole machine.





Rotor

- It is the rotating part of the machine with number of magnetic poles excited by the dc source from exciter.
- The shaft is supported by bearing at both end so that it can rotates freely keeping a small air gap between rotor and stator.
- It is made of laminated silicon steel sheet.
- There are two types of rotor i) Salient pole rotor and ii) Cylindrical type rotor

1) Salient pole rotor

- This type of rotor has got projected magnetic poles as shown in figure below.
- Construction of this type of rotor is easier and cheaper than the cylindrical rotor.
- Salient pole rotors have concentrated winding on the poles
- This type of rotors are generally used in the machines running at lower speed. For eg: water turbine in hydropower station running at lower speed, Diesel engines etc.

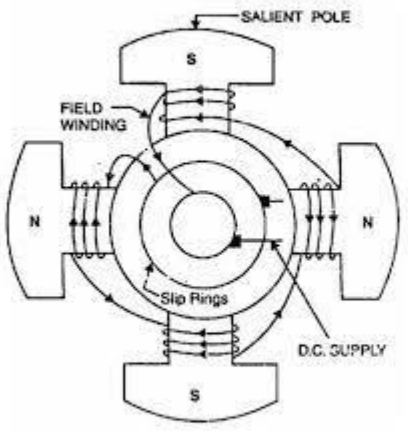


Fig: Salient Pole Machines

2) Cylindrical rotor

- This type of rotor has got smooth magnetic poles in the form of closed cylinder as shown in figure below.
- Construction of this type of rotor is more compact and robust with compare to salient pole.
- This type of rotors are generally used where high speed is required.. For eg: steam turbine driven generators, gas turbine driven generators.

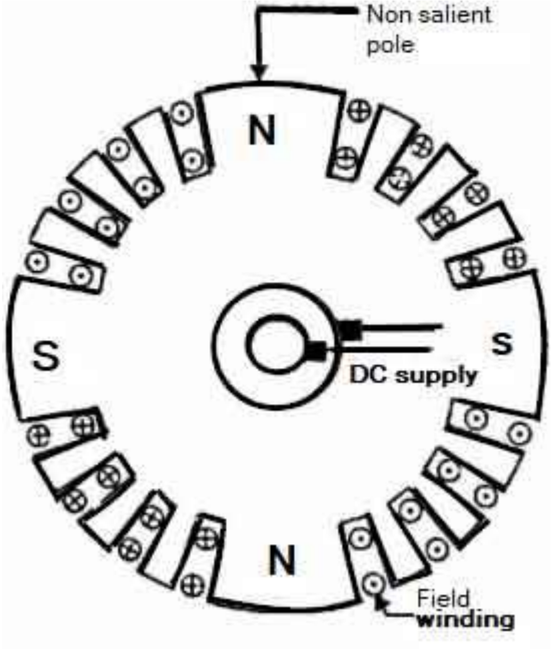
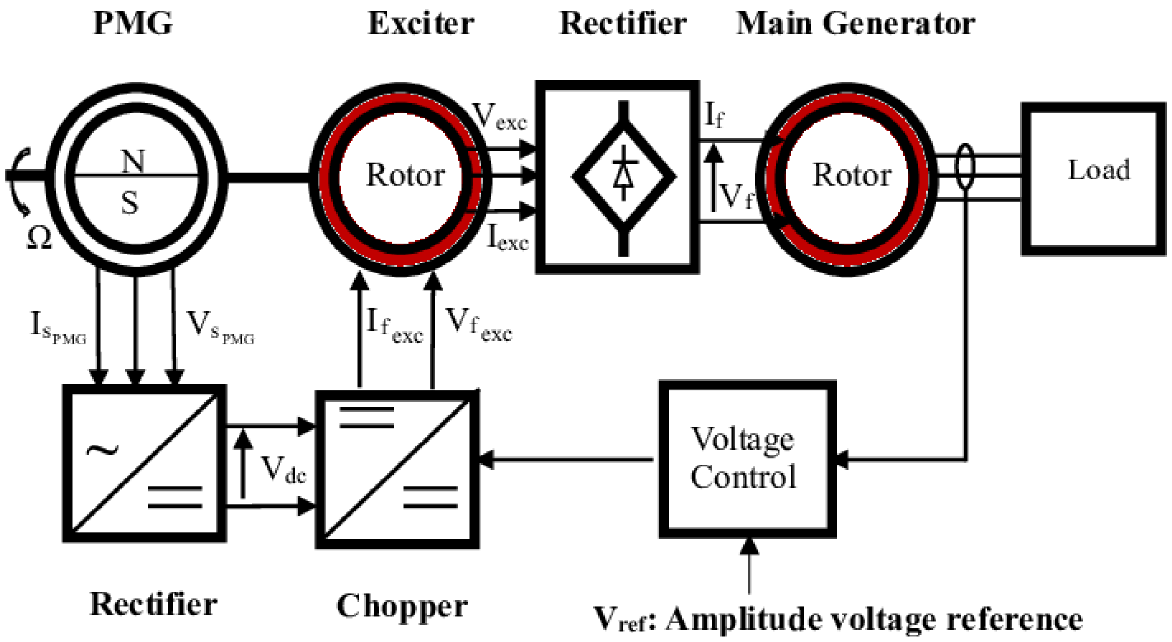


Fig: Cylindrical Rotor

Exciter:

- Exciter is a self excited dc generator or ac generator which then rectified using rectifier arrangement ,mounted on the shaft of the alternators.
- This will provide dc current required to magnetize the magnetic poles of the rotor.
- The dc current generated by the exciter is fed to the field winding of the alternator through slip ring and carbon brush.
- Nowadays, the brushless excitation system is adopted in 3 phase synchronous generators.



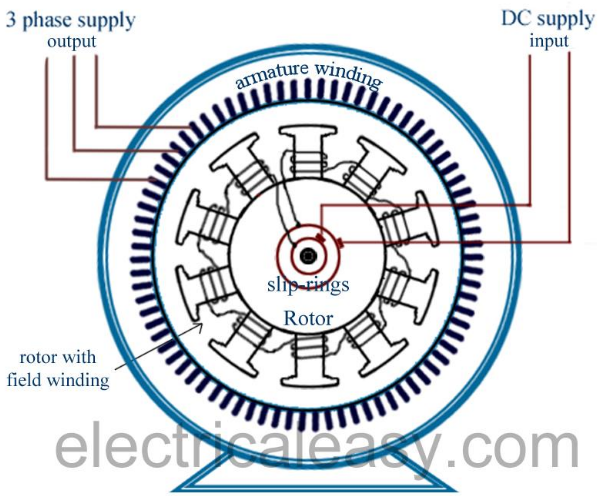
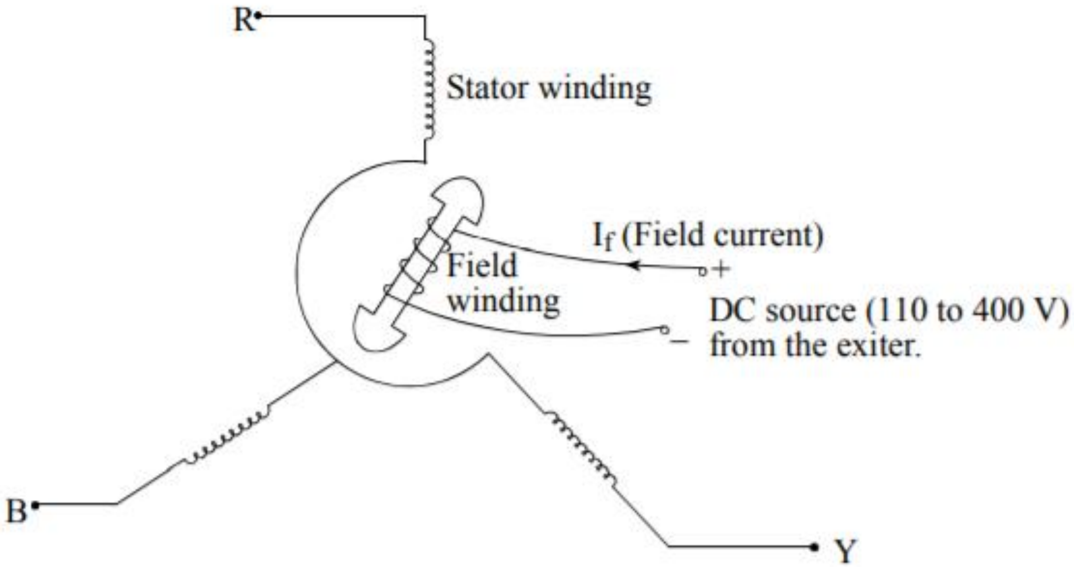


Fig:Brushless excitation system

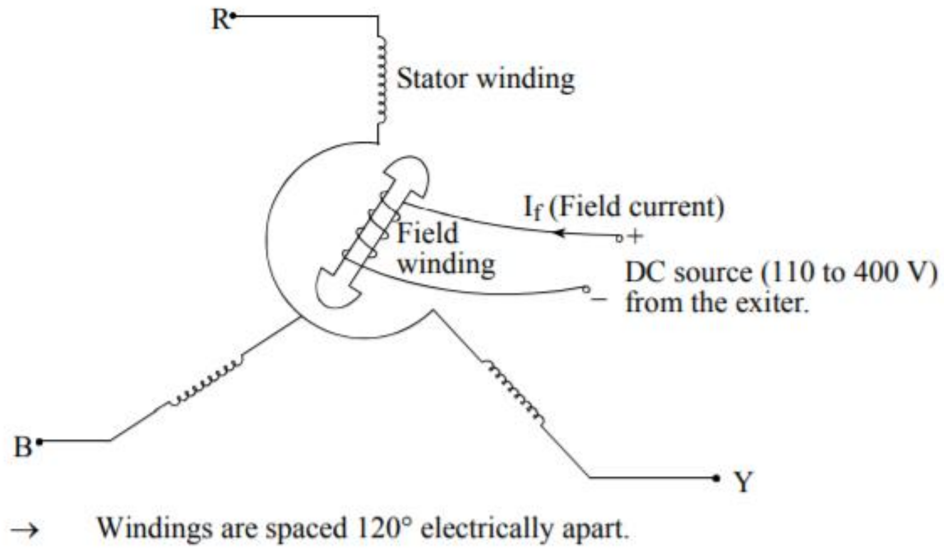
Circuit Representation of 3-phase Synchronous Machine



→ Windings are spaced 120° electrically apart.

Working Principle of three phase synchronous Generator:

- The operation is same as dc generator. The field poles are rotating and armature conductors(stator conductors) are stationary in case of synchronous machine.
- The shaft of the machine is driven by the prime mover at a constant speed equal to the synchronous speed(N_s).
  $N_s = \frac{120f}{P}$
- The excitor build up it's voltage by self excitation and supplies dc current to the field winding of the generator. The magnetic Flux produced by the rotor poles will cut the stationary three phase stator winding. Hence, according to Faraday's law of electromagnetic induction, three phase emf will induce in the stator winding. In an actual power generating station speed governor is used to keep the speed of the machine constant automatically at any load condition so that the frequency. of generated emf is constant.



EMF Equation:

Let Z = No. of conductors or coil sides in series per phase

Or, $Z = 2.T$, where T = No. of turns in series per phase.

P = No. of magnetic pole in the rotor

f = frequency of induced emf

ϕ = magnetic flux per pole

N = speed of the rotor in RPM

In one revolution of the rotor (i.e., in 60/N sec) each stator conductor is cut by a flux of ϕ. P(Webers).

Therefore, average emf induced per conductor = $d\phi/dt = \frac{\Phi P N}{60}$ volts

But, $N = \frac{120f}{P}$

So, Average emf induced per conductor = $\frac{\Phi P}{60} * \frac{120f}{P} = 2f\phi$ volts

Then, Average emf induced per phase = $2f\phi Z = 2f\phi.\, 2T = 4f\phi T$ Volts

We know that form factor for sine wave = $\frac{\text{RMS Valuue}}{\text{Average Value}} = 1.11$

Therefore, RMS value of emf per phase = $1.11 * 4.f.\phi.T = 4.44\, f.\phi.T$, volts

But in actual, there are some factors to affect the emf induced in the stator. They are pitch factor and distribution factor of the stator winding.

Pitch factor

The distance between the two sides of a coil is called the coil span or coil pitch. The angular distance between the central line of one pole to the central line of the next pole is called pole pitch. A pole pitch is always 180 electrical degrees regard less of the number of poles on the machine. A coil having a span equal to 180° electrical is called a full-pitch coil as shown in Fig.1.(a).A coil having a span less than 180° electrical is called a short-pitch coil or, fractional-pitch coil. It is also called a chorded coil. A stator winding using fractional-pitch coils is called a chorded winding. If the span of the coil is reduced by an angle α electrical degrees, the coil span will be (180- α) electrical degrees as shown in Fig. 2(a).In case of a full-pitch coil, the two coil sides span a distance exactly equal to the pole pitch of 180 electrical degrees. As a result, the voltage generated in a full-pitch coil is such. that the coil-side voltages are in phase as shown in Fig. 1(b). Let Ec_1 and Ec_2 be the voltages generated in the coil sides and Ec the resultant coil voltage.

Then $\quad Ec = Ec_1 + Ec_2$

$|Ec_1|=|Ec_2|=E_1$ (say)

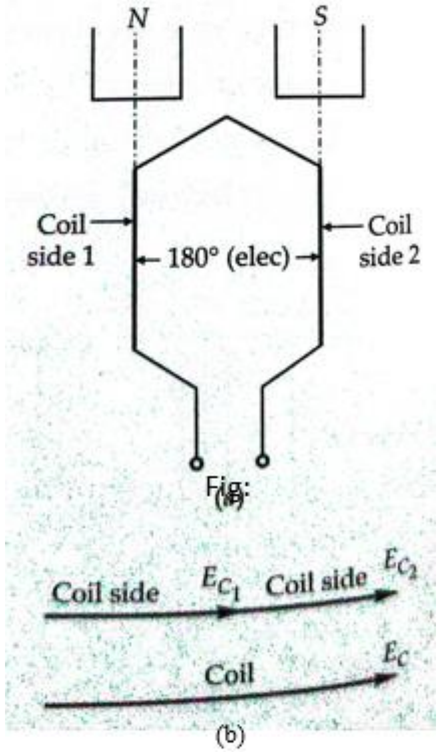


Fig.1: Full-pitch coil

Since Ec_1 and Ec_2 are in phase, the resultant coil voltage Ec is equal to their arithmetic sum.

$\therefore \quad Ec=Ec_1+Ec_2$

If the coil span of a single coil is less than the pole pitch of 180° (elec.), the voltages generated in each coil side are not in phase. The resultant coil voltage Ec is equal to the phasor sum of Ec_1 and Ec_2.

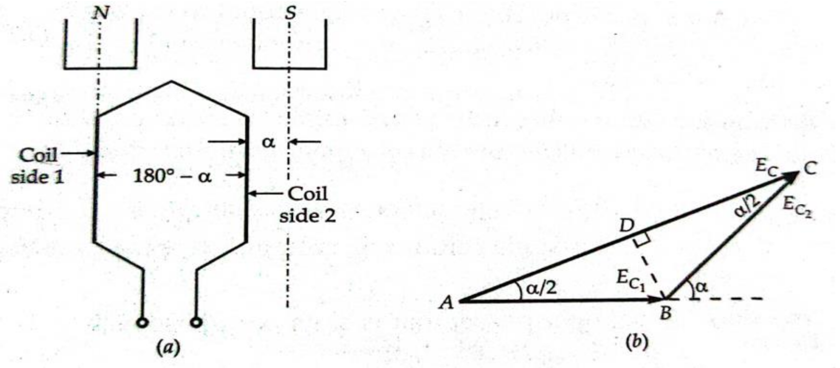


Fig.2.Short-pitch Coil

If the coil span is reduced by an angle α electrical degrees, the coil span is (180- α) electrical degrees. The voltages generated Ec_1 and Ec_2 in the two coil sides will be out of phase with respect to each other by an angle α electrical degrees as shown in Fig. 2(b). The phasor sum of Ec_1 and Ec_2 is Ec(= AC).

The coil span-factor or pitch-factor k_C is defined as the ratio of the voltage generated in the short-pitch coil to the voltage generated in the full-pitch coil. The coil span-factor is also called the chording factor.

Pitch factor(k_C)=$\frac{emf\ generated\ in\ the\ short\ pitched\ coil}{emf\ generated\ in\ the\ full\ pitched\ coil} = \frac{phasor\ sum\ of\ the\ emf\ of\ the\ two\ coil\ sides}{arithmetic\ sum\ of\ the\ emf\ of\ two\ coil\ sides} = \frac{AC}{2AB} = \frac{2E_1\cos(\frac{\alpha}{2})}{2E_1} = \cos(\frac{\alpha}{2})$

Advantages of short-pitched winding are:

They save copper of end connection, They improve the wave-form of generated emf i.e generated emf can be made to approximate to a sine wave more easily and the distorting harmonics can be reduced or totally eliminated. Due to the elimination of high frequency harmonics, eddy current and hysteresis losses are reduced, thereby increasing the efficiency.

Disadvantages of short-pitched winding are:

The total voltage around the coils is somewhat reduced. Because the voltage induced in two sides of short-pitched coil are slightly out of phase.

Distribution Factor:

If all the coil sides of any one phase under one pole are bunched in one slot, the winding obtained is known as concentrated winding and the total emf induced is equal to the arithmetic sum of the emfs induced in all the coils of one phase under one pole. But in practical cases, for obtaining smooth sinusoidal voltage waveform, armature winding of alternator is not concentrated but distributed among the different slots to form polar groups under each pole.

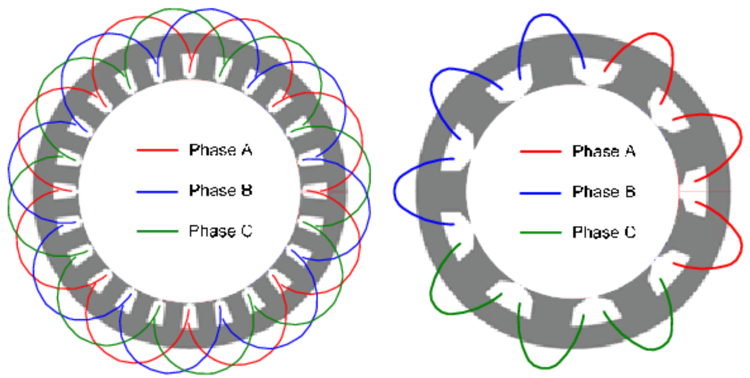


Fig: Distributed and concentrated Winding

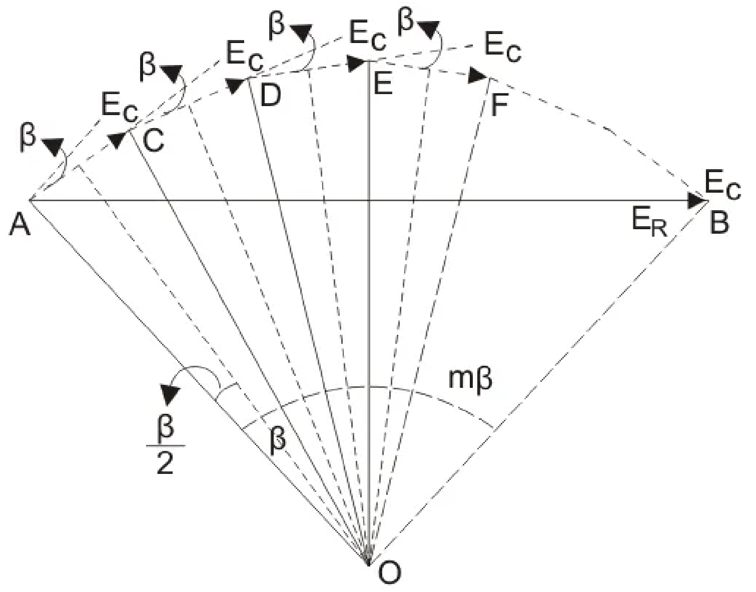
In distributed winding, coil sides per phase are displaced from each other by an angle equal to the angular displacement of the adjacent slots. Hence, the induced emf per coil side is not an angle equal to the angular displacement of the slots. So, the resultant emf of the winding is the phasor sum of the induced emf per coil side. As it is phasor sum, must be less than the arithmetic sum of these induced emfs. Resultant emf would be an arithmetic sum if the winding would have been a concentrated one.

We express it as the ratio of the phasor sum of the emfs induced in all the coils distributed in some slots under one pole to the arithmetic sum of the emfs induced.

Distribution factor (k_d) = $\dfrac{EMF\ \ induced\ in\ distributed\ winding}{EMF\ induced\ if\ the\ winding\ would\ have\ been\ concentrated} = \dfrac{Phasor\ sum\ of\ component\ emfs}{Arithmetic\ sum\ of\ component\ emfs}$

As pitch factor, distribution factor is also always less than unity.
Let the number of slots per pole is n.
The number of slots per pole per phase is m.
Induced emf per coil side is E_c.
Angular displacement between the slots,

$$\beta = \frac{180^0}{n}$$



Let us represent the emfs induced in different coils of one phase under one pole as AC, DC, DE, EF and so on. They are equal in magnitude, but they differ from each other by an angle β.
If we draw bisectors on AC, CD, DE, EF ———. They would meet at common point O.

Emf induced in each coil side, $\mathrm{E} = \mathrm{AC} = 2.\,\mathrm{OA}\sin(\frac{\beta}{2})$

As the slot per pole per phase is m, the total arithmetic sum of all induced emfs per coil sides per pole per phase,

$$\text{Arithmetic sum} = m * 2 * OA\ sin\frac{\beta}{2}$$

The resultant emf would be AB, represented in figure.
Hence, the resultant emf (E_R)= AB = 2*OA $\sin(\frac{\measuredangle AOB}{2})$ = 2*OA $\sin(\frac{m\beta}{2})$

So, Distribution factor (k_d) = $\frac{Phasor\ sum\ of\ component\ emfs}{Arithmetic\ sum\ of\ component\ emfs} = \frac{2*OA\ \sin(\frac{m\beta}{2})}{m*2*OA\ \sin\frac{\beta}{2}} = \frac{\sin(\frac{m\beta}{2})}{m\ \sin\frac{\beta}{2}}$

mβ is also known as the phase spread in electrical degree.

If the windings are concentrated in one slot, extremely deep slots have to be formed to accommodate whole turns of windings, which will increase the armature leakage reactance.

Advantages of Distributed Winding:

1. It reduces harmonics present in the generated emf which also improves the sine waveform.
2. It reduces armature reaction and improves cooling.
3. The coil is distributed over the slots, so the core (copper and iron) is fully used.
4. It improves the mechanical strength of the winding.

Winding factor: It is the product of pitch factor and distribution factor. Winding factor, k_w is given by,

$$k_w = k_p * k_d = \cos(\frac{\alpha}{2}) * \frac{\sin(\frac{m\beta}{2})}{m\ \sin\frac{\beta}{2}}$$

Considering the winding factor, emf equation will be:

$E = 4.44\ k_w\ f.\ \phi.T$, volts

Since, $f = \frac{N_S.P}{120}$

So, $E = k.\ N_s.\phi$

Since, N_s is constant, the emf can be made constant at any load by changing the flux i.e. field current in the rotor.

Alternator with Load

- When the generator is loaded, current will flow through the stators winding and some voltage drop will take place in the stator winding.
- Let, E = emf induced per phase in the stator winding
- At no-load operation, the terminal voltage V will be equal to the emf induced (E).
- The stator of the synchronous generator has three sets of winding on which emfs are induced.
- Usually these three windings are 'star' connected and the neutral is earthed as shown in Fig.
- But at loaded operation, the terminal V will be less than the emf induced (E) due to following three reasons:

  (i) Voltage drop due to armature winding (R_a).

  (ii) Voltage drop due to leakage reactance of armature winding (X_L)

  (iii) Armature reaction (Xa)

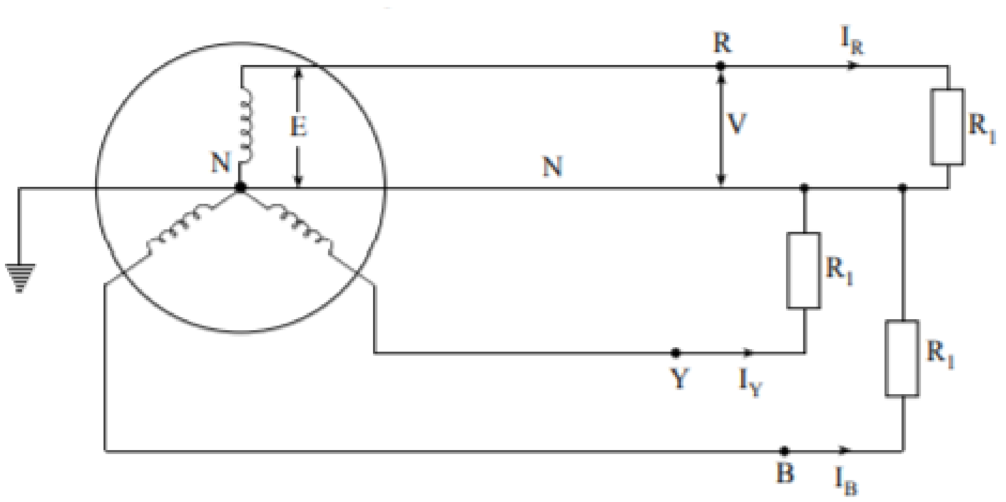


Fig. Alternator with load.

Where, E = per phase no-load voltage.

R = armature resistance.

X_L = leakage reactance of armature.

X_a = reactance corresponding to armature reaction.

- X_L and X_a can be combined and represented by

$X_S = X_L + X_a$

Where, X_s is known as synchromous reactance.

Then, total impedance of the circuit is given by

$Z_s = R_a + j\,(X_L = V_a)$

$\boxed{Z_s = Ra + jX_s}$

Where, Z_s is also called synchromous impedance.

Here,

$E = V + I_aR_a + jI_aX_L + jI_aX_a$

$E = V + I_a(R_a + jX_s)$

$\therefore\ \boxed{E = V + I_aZ_s}$

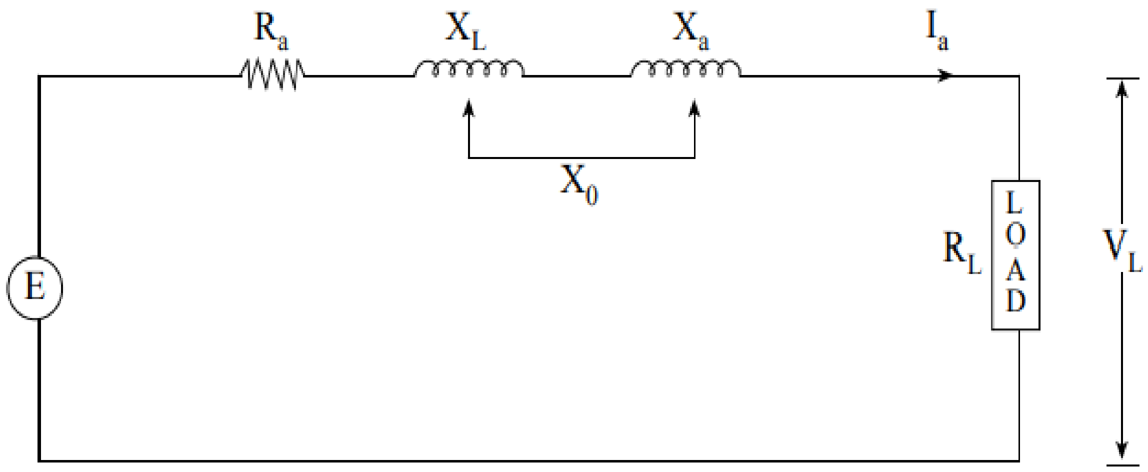


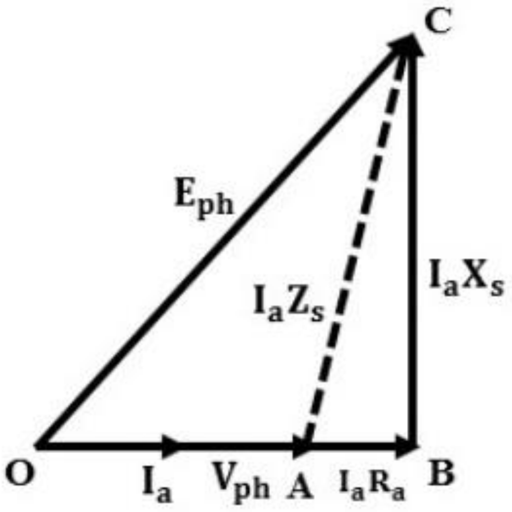
Fig. 4 Equivalent circuit of the synchronous generator (stator side)

Unity power factor load

$$OC^2 = OB^2 + BC^2$$

$$E_{ph}{}^2 = (OA + AB)^2 + BC^2$$

$$E_{ph} = \sqrt{\left(V_{ph} + I_a R_a\right)^2 + (I_a X_s)^2}$$

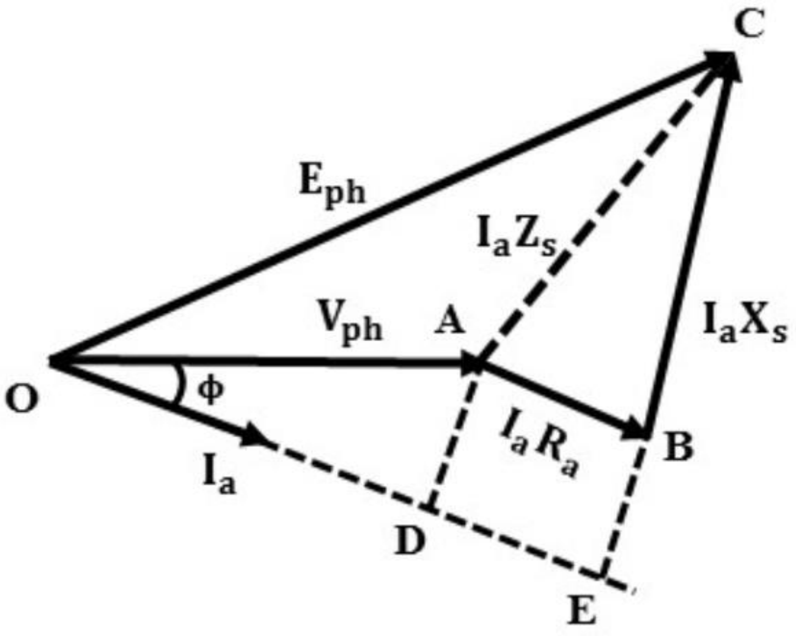


Lagging power factor load

$$OC^2 = OE^2 + EC^2$$

$$E_{ph}{}^2 = (OD + DE)^2 + (EB + BC)^2$$

$$E_{ph} = \sqrt{\left(V_{ph}\cos\phi + I_a R_a\right)^2 + \left(V_{ph}\sin\phi + I_a X_s\right)^2}$$

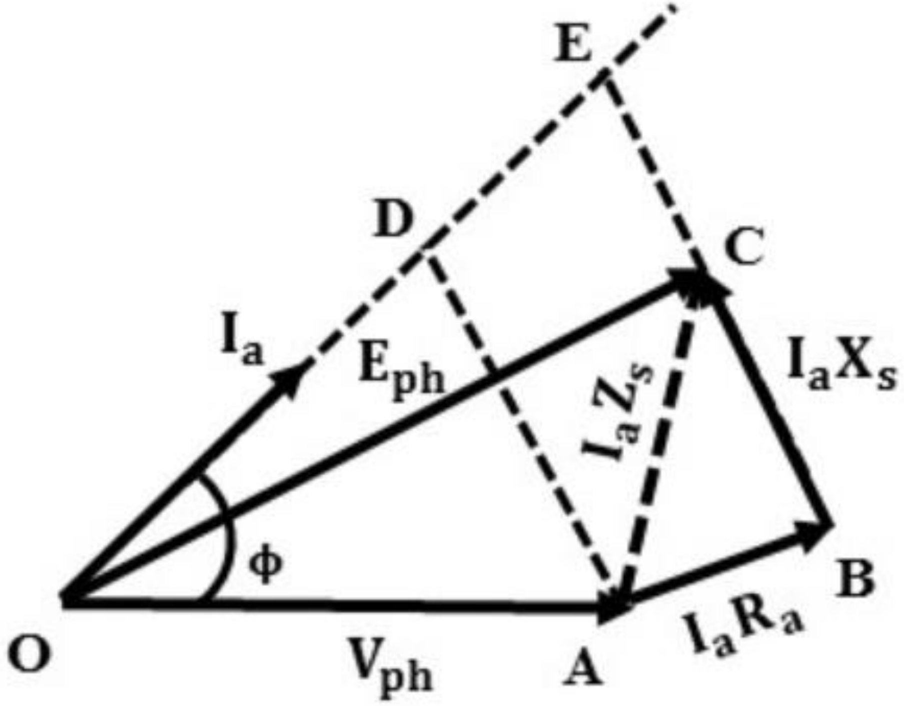


Leading power factor load

$$OC^2 = OE^2 + EC^2$$

$$E_{ph}{}^2 = (OD + DE)^2 + (BE - BC)^2$$

$$E_{ph} = \sqrt{\left(V_{ph}\cos\phi + I_aR_a\right)^2 + \left(V_{ph}\sin\phi - I_aX_s\right)^2}$$

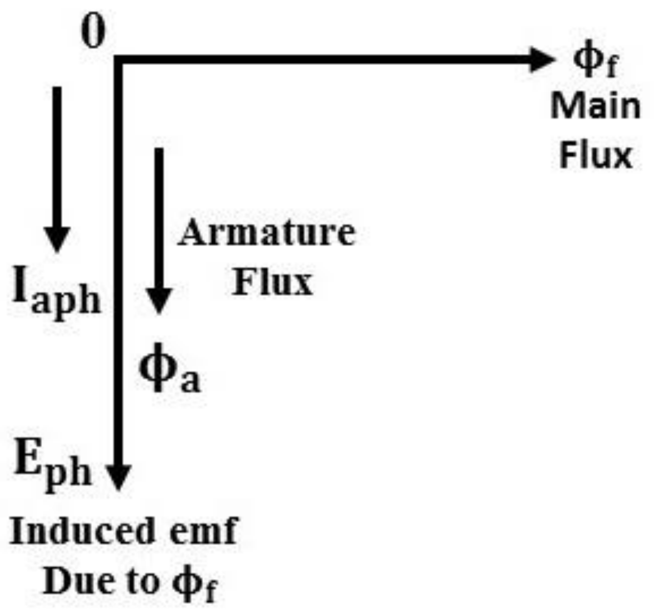


ARMATURE REACTION

- When the synchronous generator is loaded with external load, current will flow through the armature windings. These current carrying armature winding will set up its own magnetic field which is also rotating in nature.
- The effect of this armature field on the field produced by the rotor field poles is known as armature reaction.
- The nature of armature reaction depends on the power factor of the load.

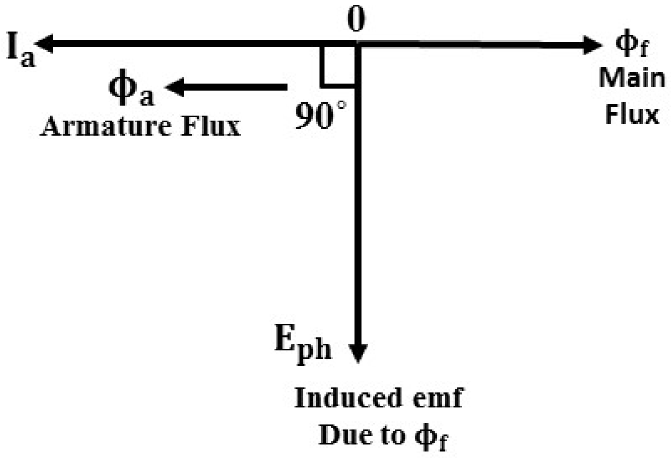
WHEN THE LOAD IS RESISTIVE (UNITY POWER FACTOR)

- At every instant, the armature reaction flux (ϕ_A) try to distort the main flux.
- This type of flux is called cross-magnetizing flux.



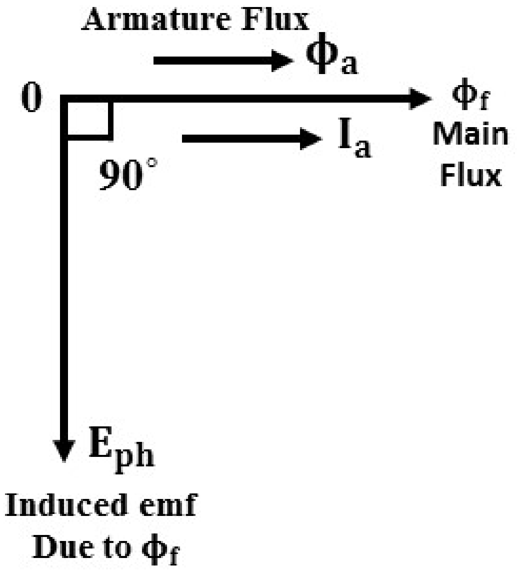
WHEN THE LOAD IS INDUCTIVE (ZERO LAGGING POWER FACTOR)

- At every instant, the armature reaction flux (ϕ_A) try to reduce the main flux.
- This type of flux is called de-magnetizing flux.
- For lagging pf load between 0&1, the effect will be partly demagnetizing and partly cross magnetising



WHEN THE LOAD IS CAPACITIVE (ZERO LEADING POWER FACTOR)

- At every instant, the armature reaction flux (ϕ_A) try to increase the main flux.
- This type of flux is called magnetizing flux.
- For leading pf load between 0&1, the effect will be partly magnetizing and partly cross magnetising



- Thus, the armature flux distorts the main flux and tries to change the magnitude depending on the power factor of the load.
- This causes change in the voltage obtained at the terminal of the generator.
- The I_aX_a represents the voltage drop due to armature reaction.

Three phase synchronous motor

❖A synchronous motor is a machine that converts ac electric power to mechanical power at a constant speed called synchronous speed.

❖A synchronous motor is a "doubly-excited machine" - Its rotor poles are excited by direct current (dc) and its stator windings are connected to the 3 phase ac supply.

❖The air gap flux is, therefore, the resultant of the fluxes due to both rotor current and stator current.

❖In fact, a given synchronous generator can also be used as a synchronous motors.

❖Some characteristic features of a synchronous motor are as follows:

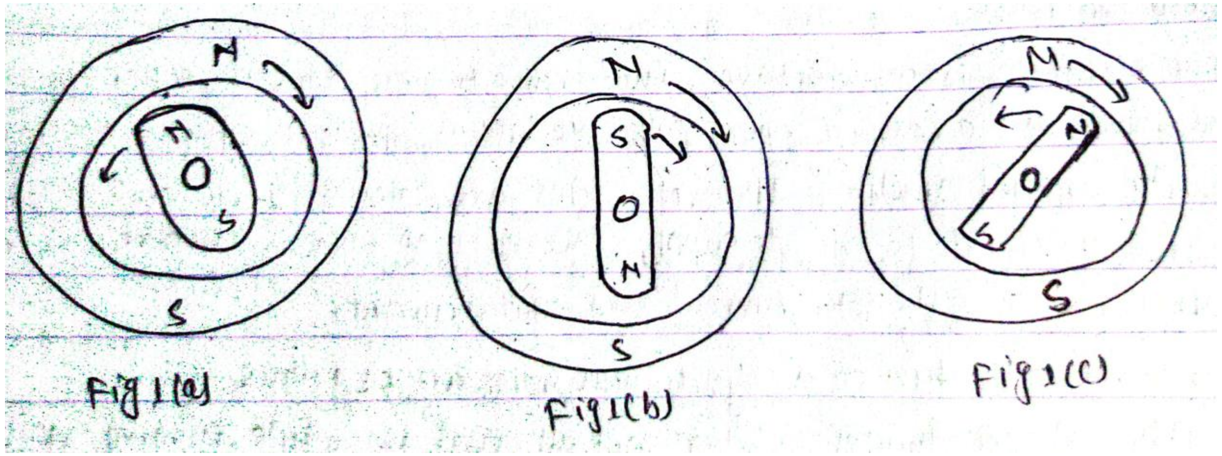
i) It runs either at synchronous speed or not all that is while running it maintains a constant speed equal to the synchronous speed.

ii) It is not self-starting. It has to be run up to synchronous speed by some means before it can be synchronized to the supply.

iii) It can be operated under wide range of power factors both lagging and leading.

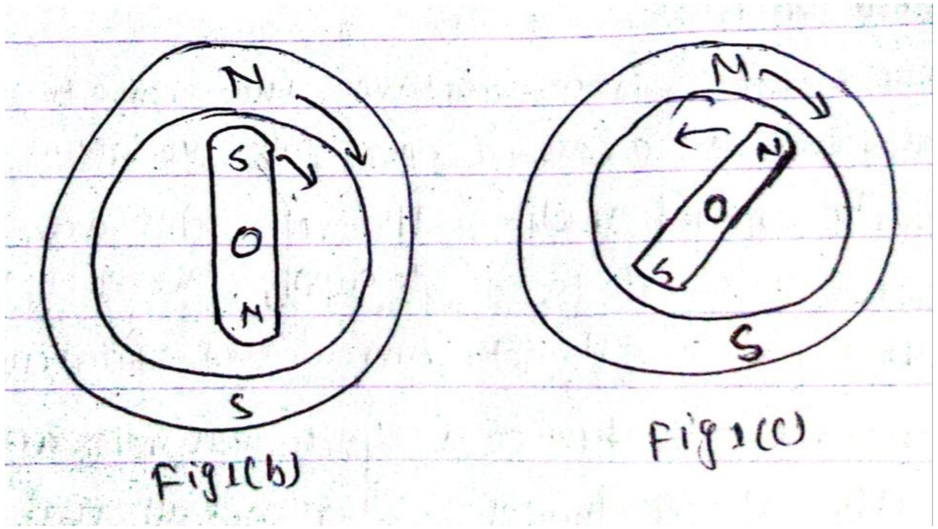
OPERATING PRINCIPLE

Synchronous motor is **not self starting**. When the stator windings are supplied by three phase voltage, the rotating magnetic field will produced in the air gap. The stator field rotates at synchronous speed. At the same time if the rotor field windings are excited by dc current, the rotor poles will get magnetized. But the interaction between stator magnetic field and rotor magnetic field will not be able to produce a continuous rotation. This facts can be explained as follow:



At starting the position of rotor poles could have many alternative positions relative to the stator poles as shown in Fig. 1. If the relative position between rotor poles and stator poles at the starting is as shown in Fig. 1(a), the like poles will get repel and the tendency of the rotor will be to rotate in anti-clockwise direction. But after some time, the N-poles of the stator and s-pole of the rotor comes face to face. Then these opposite poles will try to get attract with each other, then the tendency of the rotor will be to rotate in clockwise direction. But the heavy mass of the rotor cannot response to such a quick reversal of direction of rotation. Hence the rotor remains at rest.

If the relative position between rotor poles and stator poles at the starring is as shown in Fig. 1(b), the unlike poles will get attract and the tendency of the rotor will be to rotate in clockwise direction along with the stators poles. But the heavy mass of the rotor cannot pickup the synchronous speed immediately. Therefore, after some time, N-pole of the stator and N-pole of the rotor comes face to face. Now the like poles repels each other and the tendency of the rotor will be to rotate in anti-clockwise direction. But the heavy mass of the rotor cannot response to such a quick reversal of direction of rotation. Hence the rotor remains at rest.



If the relative position between rotor poles and stator poles at the starting is as shown in Fig. 1(c), the like poles will get repel and the tendency of the rotors will be to rotate in anti-clockwise direction. But after some time, the N-pole of the stator and s-pole of the rotor comes face to face. Then these opposite poles will try to get attract with each other, then the tendency of the rotor will be to rotate in clockwise direction. But the heavy mass of the rotor cannot response to such a quick reversal of direction of rotation. Hence the rotor remains at rest.

Hence, at any position, the motor is not self-starting. If the rotor is rotated up to or near to the synchronous speed, before supplying voltage to the stator, by some auxiliary means without exciting the rotor field winding and then stator and field are excited by their respective supply, the rotor poles will get magnetically locked up into synchronism with the stator poles, then the rotor rotates continuously even the auxiliary means is removed.

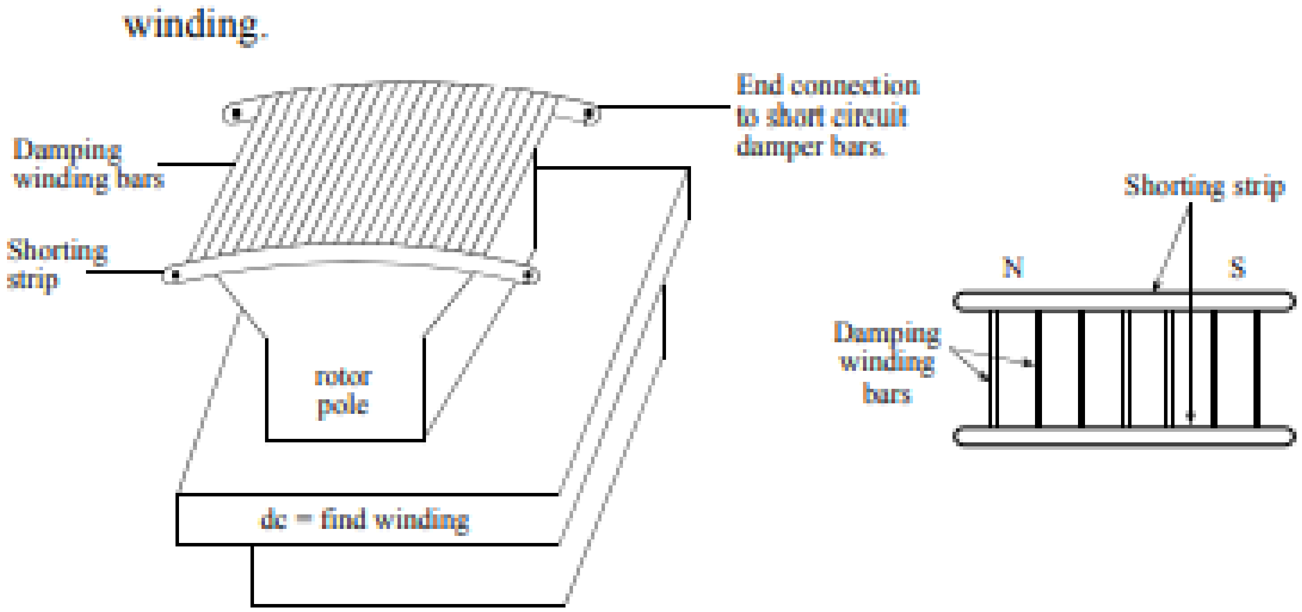
Starting methods

A synchronous motor is not self-starting.

- It can be started by the following methods.

  i) A dc motor coupled to the shaft of synchronous motor.
  ii) Using field exciter of synchronous motor as dc motor..
  iii) Using damper winding as a squirrel cage induction motor.

- ❖ In the first method, the unexcited rotor is rotated by means of a dc motor coupled to the shaft of the synchronous motor. As the speed reaches near to synchronous speed, the field winding of the synchronous motor is excited by the dc current and the dc motor is switched off. Then the motor continuously rotates with synchronous speed.
- ❖ The second method is similar to the first method except that the exciter of the synchronous motor (i.e. a dc shunt generator) is operated as dc motor for the time being and as the speed reaches close to the synchronous speed, the dc machine is again used as exciter.



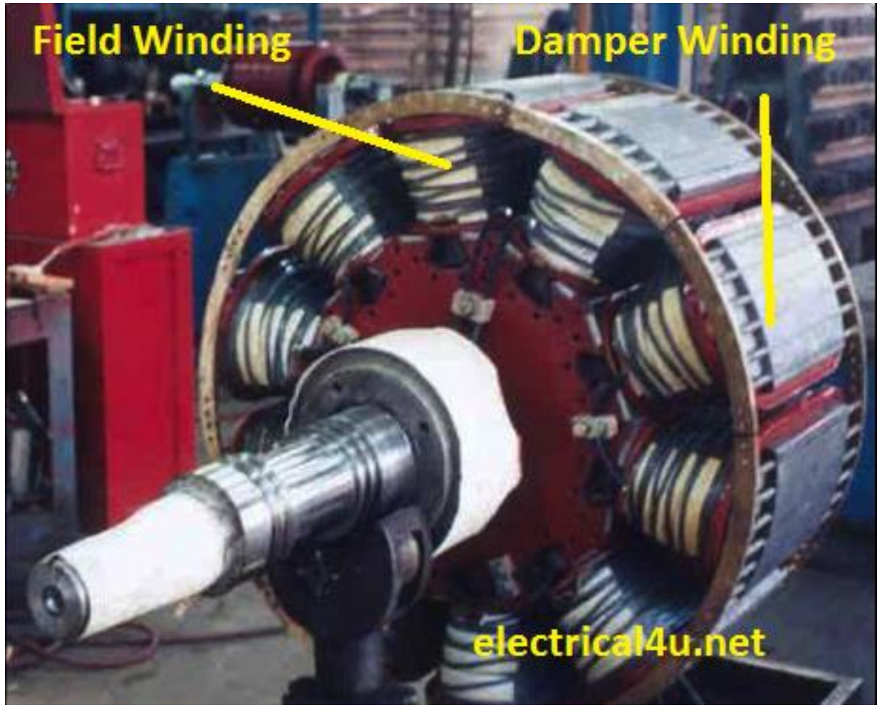


Fig 1: Constructional detail of a rotor pole with damper windings

Today the most widely used method of starting a synchronous motor is to use damper windings.

A damper winding consists of heavy copper bars inserted in slots of the pole faces of the rotor as shown in Fig.1.These bars are short, circuited by end rings at both ends of the rotor. Thus, these short-circuited bars form a squirrel cage winding. When a 3-ϕ supply is connected to the stator, the synchronous motor with damper winding will start as a 3-ϕ induction motor. As the motor approaches synchronous speed, the dc excitation is applied to the field windings.
The rotor will then pull into synchronism with the stator magnetic field and then the synchronous motor runs at synchronous speed.

It is particularly impossible to start a synchronous motor its field excited. Even with un-excited condition, the rapidly rotating magnetic field of the stator will induce extremely high voltage in many turns of the field winding. Therefore, it is better to short circuit dc field winding during the starting period, whatever voltage and current are induced in it may then aid in producing induction motor action.

All the above method shall be started without load. In order to start the synchronous motor with load, phase wound damper winding shall be used so that external resistance can be inserted to produce high starting torque. Fig. 2 shows the schematic diagram of phase would damper winding for starting synchronous motor.

Such motor will have rotor with five slip rings. Two for the dc field excitation and three for a star connected wound damper winding. The motor is started with full external resistance per phase and dc field circuit open. As the motor approaches synchronous speed, the starting resistance is reduced and, when the field voltage is applied, the motor pulls into synchronism.

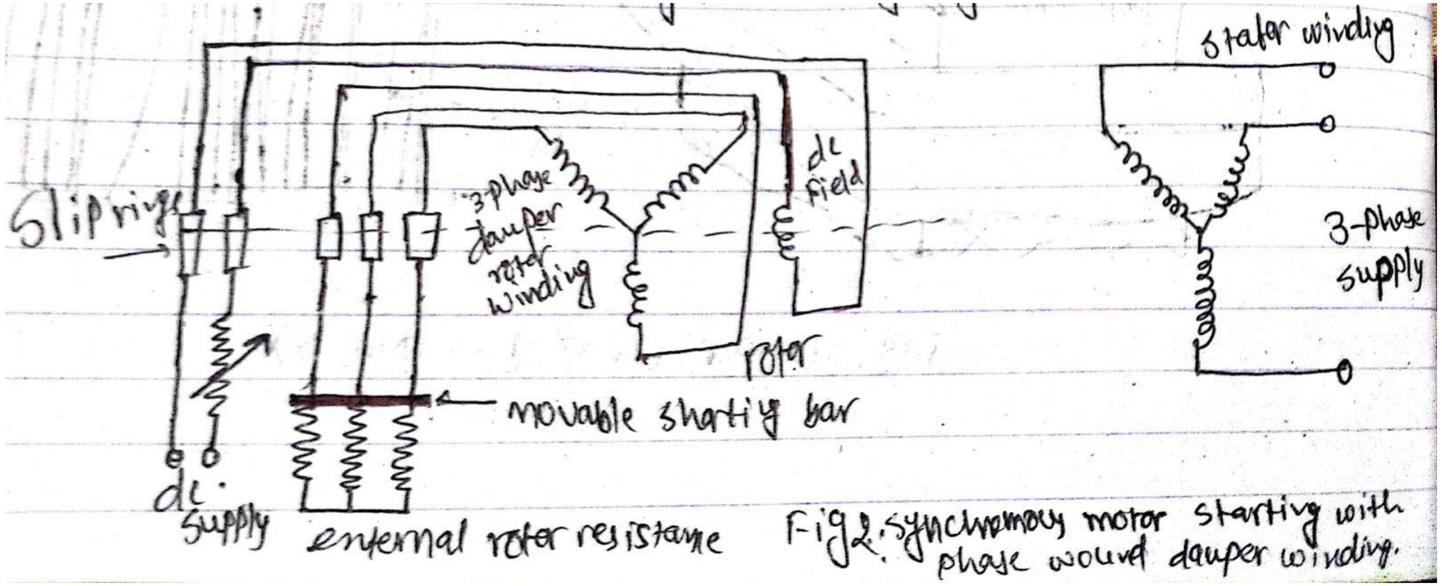


Fig2. Synchronous motor starting with phase wound damper winding.

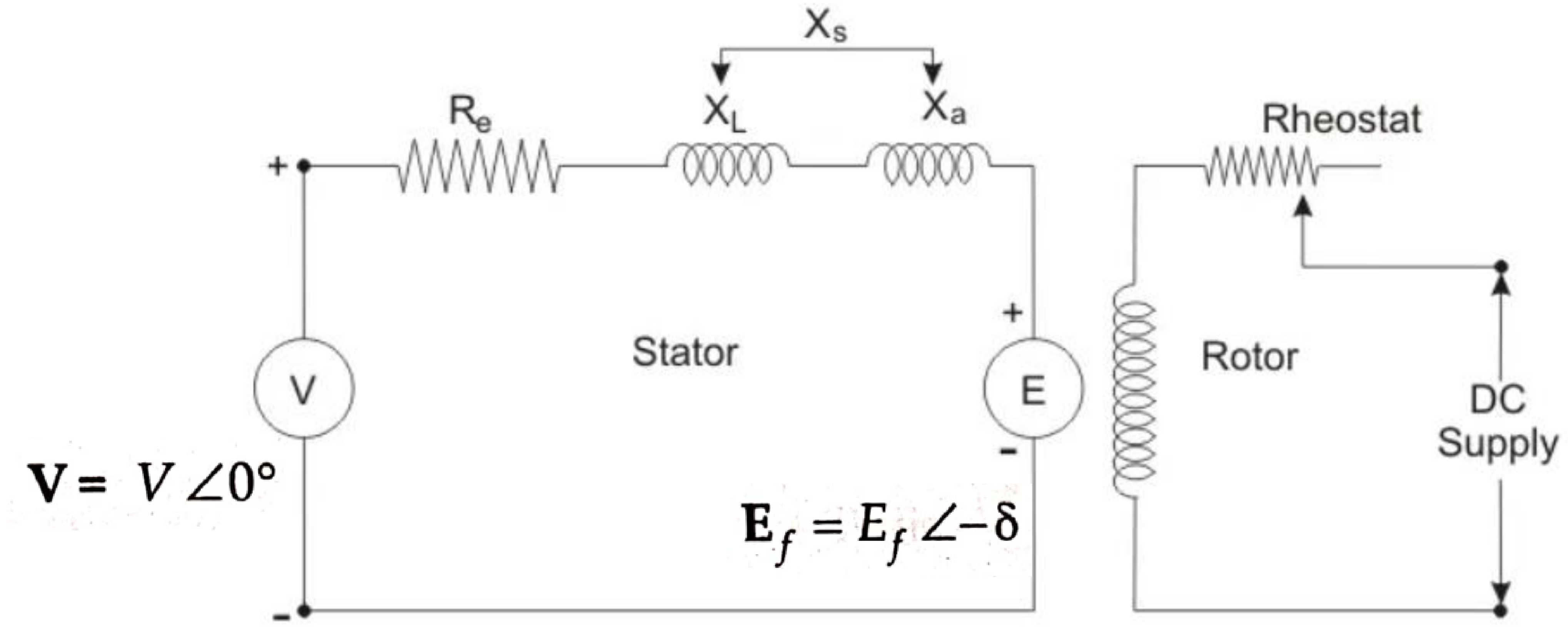
No-load and loaded operation

- A synchronous motor is not self-starting it has to be speeded up to synchronous speed by some auxiliary means.
- The supply to the dc winding of the rotor has to be switched on, then the rotor poles will get magnetically locked up with stator poles.
- However, the engagement between the stator and rotor poles is not absolutely rigid one.
- As the load on the motor increases, the rotor progressively tends to fall back in phase (but not in speed) by some angle, but the motor still continuous to run with the synchronous speed.
- At no-load, if there is no power loss in the motor, the stator poles and rotor poles will be along the same axis and phase difference between the applied voltage 'V' and the back emf 'E_b' (developed in the armature winding) will be exactly 180°.
- But this is not possible in practice, because some power loss takes place due to iron loss and friction loss and there is **δ** angle difference between stator and rotor axis.

Effect of excitation and Power factor Control

- The dc current supply to the rotor field winding is known as excitation in synchronous motor.
- As the speed of synchronous motor is constant, the magnitude of back emf remains constant provided the flux per pole produced by the rotor does not change.
- So the magnitude of back emf can be changed by field excitation.
- By changing the excitation, the motor can be operated at both lagging and leading power factor.
- This fact can be explained by following analysis:
- **The value of excitation for which motor operates at unity pf is known as 100% excitation**.
- If the excitation is more than 100%, then the motor is said to be **over excited** and if the excitation is less than 100%, then the motor is said to be **under excited**.
- **Over excited motor operates at leading p.f whereas under excited motor operates at lagging p.f.**

Equivalent Circuit of synchronous motor



Effect of excitation and Power factor during constant load operation

The effect of field current I_f on synchronous motor power factor can be explained with the help of its phasor diagram. For simplicity, armature resistance R_a is neglected and synchronous reactance X_s and terminal voltage V are assumed constants. The power per phase is given by

$$P = \frac{E_f V}{X_s} \sin \delta = V I_a \cos \phi \tag{5.9.1}$$

Since V and X_s are constants, therefore, for constant power output $E_f \sin \delta$ and $I_a \cos \phi$ should remain constant. That is,

$$E_f \sin \delta = \text{constant} \tag{5.9.2}$$

$$I_a \cos \phi = \text{constant} \tag{5.9.3}$$

Also,

$$\mathbf{E}_f + j\mathbf{I}_a X_s = \mathbf{V} \tag{5.9.4}$$

When the field current increases, the magnitude of $\mathbf{E}_f$ increases, but the component of $\mathbf{E}_f$ normal to $\mathbf{V}$, that is, $E_f \sin \delta$ must remain constant. From Fig. 5.8 it is seen that, as $\mathbf{E}_f$ varies, $\mathbf{I}_a X_s$ and therefore the armature current $\mathbf{I}_a$ also varies subject to the condition that $I_a \cos \phi$ remains constant.

Equations (5.9.1) and (5.9.2) allow us to draw power loci for the phasors $\mathbf{E}_f$ and $\mathbf{I}_a$ on the phasor diagram in Fig. 5.8. When I_f is varied slowly enough to avoid hunting, $\mathbf{E}_f$ varies in magnitude and the tip of $\mathbf{E}_f$ phasor moves along the constant power locus CD so that $E_f \sin\delta$ remains constant.

Since $\quad \mathbf{I}_a (jX_s) = \mathbf{V} - \mathbf{E}_f \qquad (5.9.5)$

and $$\mathbf{I}_a = \frac{\mathbf{V} - \mathbf{E}_f}{jX_s} \qquad (5.9.6)$$

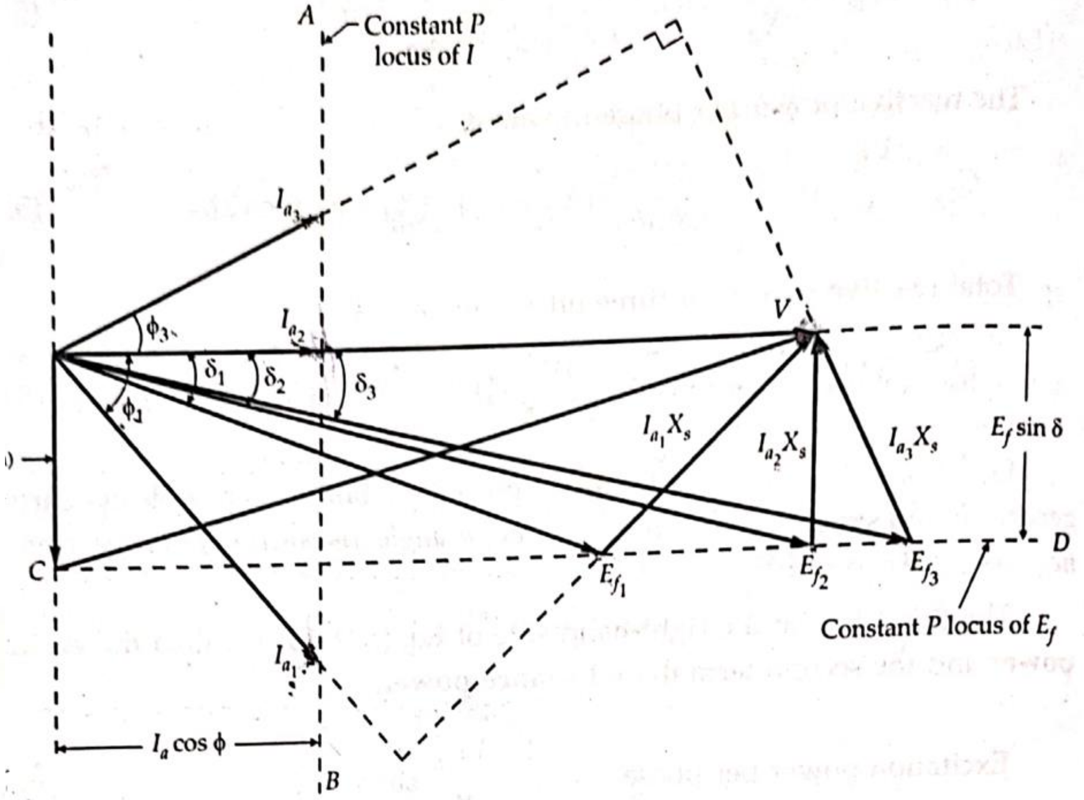
Equation (5.9.6) shows that with the restricted variation of $\mathbf{E}_f$ (that is, $E_f \sin\delta = a$ constant), the armature current $\mathbf{I}_a$ also varies with the constraint that $I_a \cos\phi$ remains constant. The tip of the $\mathbf{I}_a$ phasor moves along the constant power locus AB so that $I_a \cos\phi$ remains constant. Also, the phasor $\mathbf{I}_a$ must always remain perpendicular to $-j\mathbf{I}_a X_s$ drop as the tip of the current phasor $\mathbf{I}_a$ moves along its locus. These *three* constraints

$$E_f \sin\delta = \text{a constant}$$

$$I_a \cos\phi = \text{a constant}$$

and $\quad I_a$ is perpendicular to $-jI_a X_s$

enable us to draw the phasor diagram of a synchronous motor for varying field currents. Fig. 5.8 shows the phasor diagram for lagging pf, unity pf and leading pf.



When excitation voltage is $\mathbf{E}_{f_1}$, the motor is underexcited and the armature current $\mathbf{I}_{a_1}$ lags behind V by pf angle ϕ_1 so that

$$\mathbf{E}_{f_1} + j\mathbf{I}_{a_1} X_s = \mathbf{V}$$

When the excitation voltage is increased to $\mathbf{E}_{f_2}$ by increasing the field current, the torque angle decreases from δ_1 to δ_2 so that $E_{f_1} \sin \delta_1 = E_{f_2} \sin \delta_2$.

Since $\mathbf{E}_f + j\mathbf{I}_a X_s = \mathbf{V}$ is to be satisfied, therefore

$$\mathbf{E}_{f_2} + j\mathbf{I}_{a_2} X_s = \mathbf{V}$$

and the armature current changes to $\mathbf{I}_{a_2}$. Since in Fig. 5.8, $\mathbf{I}_{a_2}$ is in phase with **V**, the motor operates at unity power factor.

Suppose that the excitation voltage is now increased to $\mathbf{E}_{f_3}$. The torque angle decreases from δ_2 to δ_3 so that $E_{f_3} \sin \delta_3 = E_{f_2} \sin \delta_2 = E_{f_1} \sin \delta_1$.

In order to satisfy the voltage relation $\mathbf{E}_f + j\mathbf{I}_a X_s = \mathbf{V}$ again, the armature current $\mathbf{I}_{a_3}$ leads the voltage **V** and the motor operates at a leading power factor as shown in Fig. 5.8.

It is to be noted that the active components of armature currents are equal, that is,

$$I_{a_1} \cos \phi_1 = I_{a_2} \cos \phi_2 = I_{a_3} \cos \phi_3$$

It is seen from Fig. 5.8 that as the value of E_f increases, the magnitude of the armature current first decreases and then increases again. The armature current is minimum at unity *pf* and more at leading or lagging power factors.

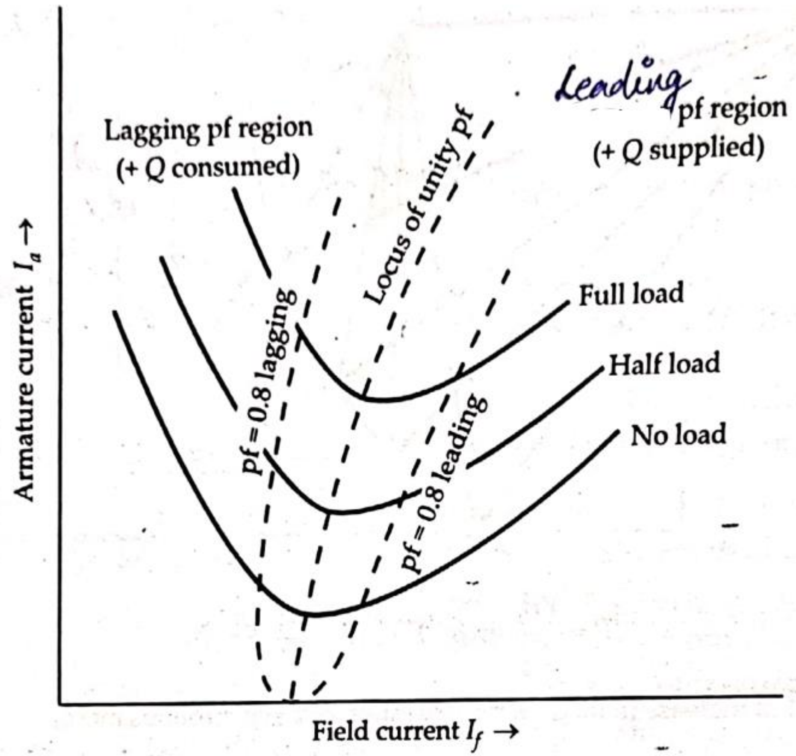
When E_f is small the armature current is lagging and the motor is an inductive load. It acts like an inductor-resistor combination, consuming reactive power Q. As the field current is increased, the armature current $\mathbf{I}_a$ becomes in phase with the terminal voltage **V** and the motor becomes a purely resistor load. As the field current is increased further, the armature current $\mathbf{I}_a$ becomes leading and the motor becomes a capacitive load. It acts like a capacitor-resistor combination, consuming negative reactive power $-Q$, or, alternatively, *supplying* reactive power $+Q$ to the system. Therefore, by controlling the field current of a synchronous motor, the reactive power supplied to or consumed from the power system can be controlled. When $E_f \cos \delta < V$, the synchronous motor has a lagging current and *consumes* Q. Since the field current is small in this case, the motor is said to be *underexcited*.

If $E_f \cos \delta > V$, the synchronous motor has a leading current and *supplies* Q to the system. Since the field current is large in this case, the motor is said to be *overexcited*. The motor diagrams of the underexcited and overexcited motors are shown in Fig. 5.9.

When $E_f \cos \delta = V$, the motor is said to be *normally* excited. Here $Q = 0$, that is, the motor is neither delivering nor absorbing reactive power.

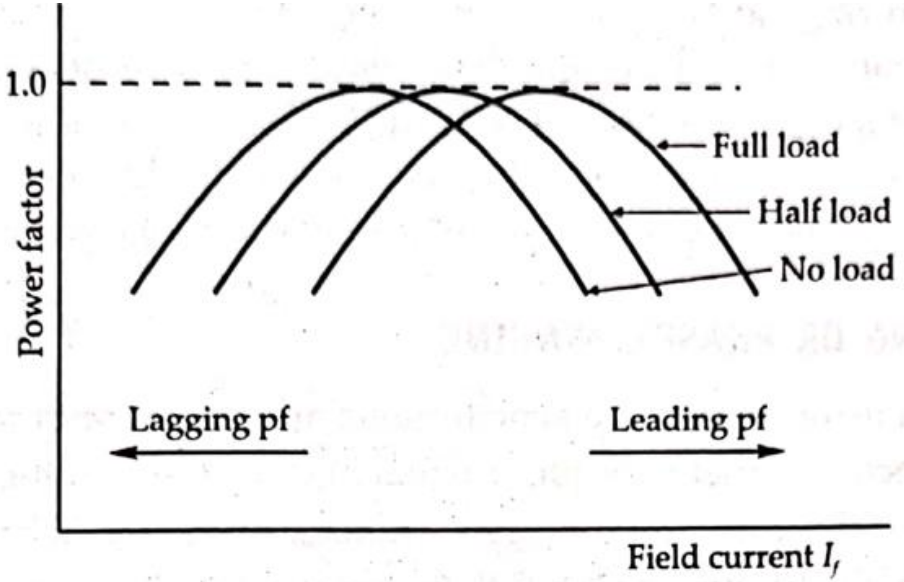
V-curves and inverted V curves

We have seen that the power factor of a synchronous motor can be controlled by variation of field current I_f. It has also been observed that the armature current I_a changes with the change in field current I_f. Let us assume that the motor is operating at no load. If the field current is increased from this small value, the armature current I_a decreases until the armature current becomes minimum. At this minimum armature current the motor is operating at unity power factor. Upto this point the motor was operating at a lagging power factor. If the field current is increased further, the armature current increases again at the motor starts to operate at a leading power factor. If a graph is plotted between armature current I_a and field current I_f at no load the lowest curve in Fig. 5.12 is obtained. If this procedure is repeated for various increased loads, a family of curves is obtained as shown in Fig.



V-curves of a synchronous motor.

A family of curves is obtained by plotting the power factor versus field current. These are inverted V curves as shown in Fig.   The highest point on each of these curves indicates unity power factor. It is to be noted that the field current for unity power factor at full load is more than the field current for unity power factor at no load. Figure. 5.13 also shows that if the synchronous motor at full load is operating at unity power factor then removal of the shaft load causes the motor to operate at a leading power factor.



Power factor versus field current at different loads.

An alternator on open circuit generates 360 V at 60 Hz when the field current 3.6 A. Neglecting saturation, determine the open circuit emf When the frequency 40 Hz and the field current is 2.4 A. (2075 Baishak, BCT)

Emf equation of alternator, $E = 4.44 K_w f \phi T$

where, $\phi \propto I_f$ (field current)

so, $E \propto f \cdot I_f$ (for any machine)

Hence, $\dfrac{E_2}{E_1} = \dfrac{f_2 \cdot I_{f2}}{f_1 \cdot I_{f1}}$

Given,

$E_1 = 360V$, $f_1 = 60 Hz$ & $I_{f1} = 3.6 A$

$f_2 = 40 Hz$, $I_{f2} = 2.4 A$.

Now, $E_2 = E_1 \times \dfrac{f_2 \cdot I_{f2}}{f_1 \cdot I_{f1}} = \dfrac{\overset{10}{\cancel{360}} \times 4\cancel{0} \times \overset{4}{\cancel{2.4}}}{6\cancel{0} \times \cancel{3.6}\, 6}$

$= 160 V$

A 4-pole Alternator has an armature with 25 slots and 8 conductors per slot and rotates at 1500 rpm and flux per pole is 0.05 wb. Calculate the emf generated if winding factor is 0.96 and all conductors are in series.

Given,

No. of pole (P) = 4 , synchronous speed (N) = 1500

Flux per pole (ϕ) = 0.05 wb.

Frequency (F) = $\frac{NP}{120} = \frac{\cancel{4} \times \overset{50}{\cancel{1500}}}{\underset{3}{\cancel{120}}} = 50$ Hz

Number of conductors connected in series, Z = number of slots * number of conductors per slot

= 25 × 8 = 200

Number of Turns, T = $\frac{Zp}{2}$ = 100

winding factor, Kw = 0.96

Generated emf, E = 4.44 Kw ϕ FT , Volts

= 4.44 × 0.96 × 0.05 × 50 × 100

= 1065.6 Volts.

A 3ϕ, 50 Hz, 20 pole salient pole alternator with Stator connected Stator winding has ~~also~~ 180 slots on the stator. Each slot consists of 8 conductors. The flux per pole is 25 mwb and is Sinusoidally distributed. The coils are full-pitched.

calculate: i) the speed of the alternator
ii) winding factor
iii) generated emf per phase
iv) Line voltage

Given,

Flux per pole, $\phi = 25 \text{ mwb} = 25 \times 10^{-3}$ wb.

frequency, $F = 50$ Hz

Number of armature conductors,

$Z =$ No. of slots × no. of conductors per slot

$= 180 \times 8 = 1440$

No. of armature conductors per phase $= \frac{1440}{3} = 480$

No. of turns per phase, $T = \frac{480}{2} = 240$

No. of poles, $P = 20$

i) Speed, $N = \frac{120F}{P} = \frac{120 \times 50}{20} = 300$ rpm.

(ii) Number of slots per pole, $n = \frac{180}{20} = 9$

No. of slots per pole per phase,

$$m = \frac{n}{\text{no. of phases}} = \frac{9}{3} = 3$$

Angular displacement between the slots,

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ \text{ (electrical)}$$

$$\text{Distribution Factor, } K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin(\beta/2)} = \frac{\sin\left(\frac{3 \times 20}{2}\right)}{3 \sin\left(\frac{20}{2}\right)}$$

$$= \frac{\sin 30^\circ}{3 \sin 10^\circ}$$

$$= 0.96$$

pitch factor, $K_p = 1$ for ~~coils are~~ full pitch coil

(ii) winding factor, $K_w = K_p K_d = 0.96$

(iii) Generated emf per phase $= 4.44\, K_w \phi F T$, volts

$= 4.44 \times 0.96 \times 0.025 \times 50 \times 240$

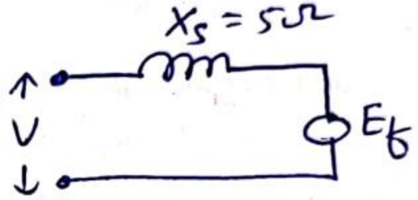
$= 1280$ Volts

(iv) Line voltage $= \sqrt{3} \times 1280 = 2217.025$ Volts

EXAMPLE 5.1 *A 3000 V, 3 phase synchronous motor running at 1500 r.p.m. has its excitation kept constant corresponding to no-load terminal voltage of 3000 V. Determine the power input, power factor and torque developed for an armature current of 250 A if the synchronous reactance is 5 Ω per phase and armature resistance is neglected.*

SOLUTION. Supply voltage per phase

$$V = \frac{3000}{\sqrt{3}} = 1732 \text{ V}$$



Induced e.m.f. per phase

$$E_f = \frac{3000}{\sqrt{3}} = 1732 \text{ V}$$

$$\mathbf{Z}_s = R_a + jX_s = 0 + j5 = 5\angle 90°\,\Omega$$

$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a\mathbf{Z}_s$$

If **V** is taken as reference phasor, then for lagging power factor,

$$\mathbf{I}_a = I_a \angle -\phi$$

$$\mathbf{E}_f = V - (I_a \angle -\phi)(5\angle 90°) = V - 5I_a \angle 90° - \phi$$

$$= V - 5 \times 250 \angle 90° - \phi = V - 1250\,[\cos(90° - \phi) + j\sin(90° - \phi)]$$

$$= V - 1250(\sin\phi + j\cos\phi) = (V - 1250\sin\phi) - j1250\cos\phi$$

$$E_f^2 = (V - 1250\sin\phi)^2 + (1250\cos\phi)^2$$

$$= V^2 - 2\,V \times 1250\sin\phi + (1250\sin\phi)^2 + (1250\cos\phi)^2$$

$$1732^2 = 1732^2 - 2 \times 1732 \times 1250\sin\phi + (1250)^2$$

$$2 \times 1732 \times 1250\sin\phi = (1250)^2$$

$$\sin\phi = \frac{1250}{2 \times 1732} = 0.3608$$

$$\cos\phi = 0.9326 \text{ (lagging)}$$

Input power $P_i = \sqrt{3}\,V_L I_a \cos\phi = \sqrt{3} \times 3000 \times 250 \times 0.9326$

$$= 1211483 \text{ W} = 1211.483 \text{ kW}$$

Also, $P_i = 2\pi n_s \tau = 2\pi \frac{N_s}{60}\tau$

$\therefore$ Torque $\tau = \frac{P_i \times 60}{2\pi N_s} = \frac{1211483 \times 60}{2\pi \times 1500} = 7712.5 \text{ Nm}$