Q.1 Premises:

1. "All lions are fierce."

2. "Some lions do not drink coffee."

Conclusion: Can we conclude the following? 3. "Some fierce creatures do not drink coffee."

Solution: Let L(x): "x is a lion."

F(x): "x is fierce." And

C(x): "x drinks coffee."

Premises are-	Conclusion
1. $\forall x\ (L(x) \rightarrow F(x))$	$\exists x\ (F(x) \wedge \neg C(x))$
2. $\exists x\ (L(x) \wedge \neg C(x))$	

Steps	**Reasons**
1. $\exists x\ (L(x) \wedge \neg C(x))$	Premise
2. $L(Foo) \wedge \neg C(Foo)$	Existential Instantiation from (1)
3. $L(Foo)$	Simplification from (2)
4. $\neg C(Foo)$	Simplification from (2)
5. $\forall x\ (L(x) \rightarrow F(x))$	Premise
6. $L(Foo) \rightarrow F(Foo)$	Universal instantiation from (5)
7. $F(Foo)$	Modus ponens from (3) and (6)
8. $F(Foo) \wedge \neg C(Foo)$	Conjunction from (4) and (7)
9. $\exists x\ (F(x) \wedge \neg C(x))$	Existential generalization from (8)

Q.2 Premises:

1. "If x is a lion, then x is carnivorous."

2. "Moo is not carnivorous."

Conclusion: Can we conclude the following? 3. "Moo is not a lion."

Solution: Let L(x): "x is a lion."

C(x): "x is carnivorous."

Premises are:	Conclusion
a) $\forall x\ (L(x) \rightarrow C(x))$	$\neg L(Moo)$
b) $\neg C(Moo)$	

Steps	Reasons
1. $\forall x\,(L(x) \rightarrow C(x))$	Premise
2. $L(Moo) \rightarrow C(Moo)$	Universal instantiation from (1)
3. $\neg C(Moo)$	Premise
4. $\neg L(Moo)$	Modus tollens from (1) and (2)

Q.3 Use rules of inference to show the following statement are valid:

a) Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job.

Solution: $c(x)$: x is in this class

$j(x)$: x knows how to write programs in JAVA,

$h(x)$: x can get a high paying job.

premises	Conclusion
a) c(Doug),	$\exists x\,(c(x) \wedge h(x))$
b) j(Doug),	
c) $\forall x\,(j(x) \rightarrow h(x))$	

Steps	Reasons
1. $\forall x\,(j(x) \rightarrow h(x))$	Given hypothesis
2. $j(Doug) \rightarrow h(Doug)$	Universal instantiation on (1)
3. j(Doug)	Given hypothesis
4. h(Doug)	Modus ponens on 2 and 3
5. c(Doug)	Given hypothesis
6. $c(Doug) \wedge h(Doug)$	Conjunction using (4) and (5)
7. $\exists x\,(c(x) \wedge h(x))$	Existential generalization using (6)

b) “Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.”

Solution :

Let $c(x)$: x is in this class,

$w(x)$: x enjoys whale watching,

$p(x)$: x cares about ocean pollution."

Premises are: Conclusion

a) $\exists x\,(c(x) \land w(x))$ — Conclusion: $\exists x\,(c(x) \land p(x))$

b) $\forall x\,(w(x) \rightarrow p(x))$

Step	Reason
1. $\exists x(c(x) \land w(x))$	Hypothesis
2. $c(y) \land w(y)$	Existential instantiation using (1)
3. $w(y)$	Simplification using (2)
4. $c(y)$	Simplification using (2)
5. $\forall x(w(x) \rightarrow p(x))$	Hypothesis
6. $w(y) \rightarrow p(y)$	Universal instantiation using (5)
7. $p(y)$	Modus ponens using (3) and (6)
8. $c(y) \land p(y)$	Conjunction using (4) and (7)
9. $\exists x(c(x) \land p(x))$	Existential generalization using (8)

c. “Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program.”

Solution:

Let, $c(x)$:x is in this class,

$p(x)$: x owns a PC,

$w(x)$: x can use a word processing program."

Premises are : **Conclusion**

a) c(Zeke) — w(Zeke)

b) $\forall x\,(c(x) \rightarrow p(x))$

c) $\forall x\,(c(x) \rightarrow w(x))$

Step	Reason
1. $\forall x(c(x) \rightarrow p(x))$	Hypothesis
2. $c(\text{Zeke}) \rightarrow p(\text{Zeke})$	Universal instantiation using (1)
3. $c(\text{Zeke})$	Hypothesis
4. $p(\text{Zeke})$	Modus ponens using (2) and (3)
5. $\forall x(p(x) \rightarrow w(x))$	Hypothesis
6. $p(\text{Zeke}) \rightarrow w(\text{Zeke})$	Universal instantiation using (5)
7. $w(\text{Zeke})$	Modus ponens using (4) and (6)

d) “Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean.”

Solution:

Let j(x) : x is in New Jersey,

f(x): x lives within fifty miles of the ocean,

s(x): x has seen the ocean."

Premises are :	Conclusion
a) $\forall x\ (j(x) \rightarrow f(x))$	$\exists x\ (f(x) \wedge \neg s(x))$
b) $\exists x\ (j(x) \wedge \neg s(x))$	

Step	**Reason**
1. $\exists x(j(x) \wedge \neg s(x))$	Hypothesis
2. $j(y) \wedge \neg s(y)$	Existential instantiation using (1)
3. $j(y)$	Simplification using (2)
4. $\forall x(j(x) \rightarrow f(x))$	Hypothesis
5. $j(y) \rightarrow f(y)$	Universal instantiation using (4)
6. $f(y)$	Modus ponens using (3) and (5)
7. $\neg s(y)$	Simplification using (2)
8. $f(y) \wedge \neg s(y)$	Conjunction using (6) and (7)
9. $\exists x(f(x) \wedge \neg s(x))$	Existential generalization using (8)

e) “Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.”

Solution:

Let e(x): x is in the class

r(x): x owns a red convertible

t(x) : x has gotten a speeding ticket.

Premises are:	**Conclusion**
a) e(linda)	$\exists x\ (e(x) \wedge t(x))$
b) r(linda)	
c) $\forall x\ (r(x) \rightarrow t(x))$	

steps	Reasons
1. $\forall x\ (r(x) \rightarrow t(x))$	Given hypothesis
2. r(linda) $\rightarrow$ t(linda)	Universal instantiation on 1
3.r(linda)	Given hypothesis

4. t(linda)	modus ponens on 2 and 3
5. e(linda)	given hypothesis
6. e(linda) Λ t (linda)	Conjunction on 4 and 5
7. ∃x (e(x) Λ t(x))	Existential generalization on 6

f. All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners.”

Solution:

Let J(x): x is the movie produced by John Sayles
W(x): x is wonderful
C(x): x is movie about coal miners.

Premises are:	Conclusion
a) ∀x (J(x)→ W(x))	∃x (W(x) Λ C(x))
b) J(M) Λ C(M)	

steps	Reasons
1. ∀x (J(x)→ W(x))	given hypothesis
2. (J(M)→ W(M)	universal instantiation on 1
3. J(M) Λ C(M)	given hypothesis
4. C(M)	simplification on 3
5. J(M)	simplification on 3
6. W(M)	Modus ponens on 2 and 5
7. W(M) Λ C(M)	Conjunction on 4 and 6
8. ∃x (W(x) Λ C(x))	Existential Generalization on 7

Q. 4. Suppose P → Q; ¬ P → R; Q → S. Prove that ¬ R → S

(1)	$P \rightarrow Q$	Premise
(2)	$\neg P \vee Q$	Logically equivalent to (1)
(3)	$\neg P \rightarrow R$	Premise
(4)	$P \vee R$	Logically equivalent to (3)
(5)	$Q \vee R$	Apply resolution rule to (2)(4)
(6)	$\neg R \rightarrow Q$	Logically equivalent to (5)
(7)	$Q \rightarrow S$	Premise
(8)	$\neg R \rightarrow S$	Apply HS rule to (6)(7)

Q.5 Premises:

a) If it is Saturday today, then we play soccer or basketball.

(b) If the soccer field is occupied, we dont play soccer.

(c) It is Saturday today,

d) The soccer field is occupied.

Prove that : “we play basketball or volleyball”.

First we formalize the problem:

P: It is Saturday today.

Q: We play soccer.

R: We play basketball.

S: The soccer field is occupied.

T: We play volleyball.

Premise:

a) $P \to (Q \vee R)$

b) $S \to \neg Q$

c) P

d) S

Need to prove: $R \vee T$.

(1)	$P \to (Q \vee R)$	Premise
(2)	P	Premise
(3)	$Q \vee R$	Apply MP rule to (1)(2)
(4)	$S \to \neg Q$	Premise
(5)	S	Premise
(6)	$\neg Q$	Apply MP rule to (4)(5)
(7)	R	Apply DS rule to (3)(6)
(8)	$R \vee T$	Apply Addition rule to (7)

Q. 6.

Premises:

a) All natural numbers are integers;

b) There exists a natural number;

Prove that there exists an integer

Solution:

We can formalize this problem as follows

$N(x)$: x is a natural number.

$I(x)$: x is an integer.

Premise:	Conclusion
a)∀ x (N(x) → I(x)),	∃ x I(x)
b) ∃ x N(x)	

(1)	$\exists\, x\, N(x)$	Premise
(2)	$N(c)$	Apply existential instantiation rule to (1)
(3)	$\forall\, x\, (N(x) \to I(x))$	Premise
(4)	$N(c) \to I(c)$	Apply universal instantiation rule to (3)
(5)	$I(c)$	Apply MP rule to (2)(4)
(6)	$\exists\, x\, I(x)$	Apply existential generalization rule to (5)

Q.7 Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

SOLUTION

Let us assume:

$p =$"Randy works hard"
$q =$"Randy is a dull boy"
$r =$"Randy will get the job"

We can then rewrite the given statements (1), (2) and (3) using the above interpretations.

	Step	Reason
1.	p	*Premise*
2.	$p \to q$	*Premise*
3.	$q \to \neg r$	*Premise*
4.	q	Modens ponens from (1) and (2)
5.	$\neg r$	Modens ponens from (3) and (4)

Q.8 : Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

- $R =$ "It rains"
- $F =$ "It is foggy"
- $S =$ "The sailing race will be held"
- $D =$ "Life-saving demonstrations will go on"
- $T =$ "The trophy will be awarded"

We can now proceed to prove the claim:

(1)	$\neg T$	hypothesis
(2)	$S \to T$	hypothesis
(3)	$\neg S$	modus tollens from (1), (2)
(4)	$\neg S \vee \neg D$	addition
(5)	$\neg(S \wedge D)$	DeMorgan
(6)	$(\neg R \vee \neg F) \to (S \wedge D)$	hypothesis
(7)	$\neg(\neg R \vee \neg F)$	moduls tollens from (5),(6)
(8)	$R \wedge F$	DeMorgan
(9)	R	simplification

Q.9 ***Given the premises:***

p

$p \to q$

$s \vee r$

$r \to \neg q$

Arrive at the conclusion:

$s \vee t$

Step	**Reason**
1) p	**Premise**
2) $p \to q$	**Premise**
3) q	**Modus Ponens (using 1, 2)**
4) $r \to \neg q$	**Premise**
5) $q \to \neg r$	**Contrapositive statement of 4**
6) $\neg r$	**Modus Ponenes (using 3, 5)**
7) $s \vee r$	**Premise**
8) s	**Disjunctive Syllogism (using 6, 7)**
9) $s \vee t$	**Disjunctive Amplification (using 8)**

Q.10

If I get my Christmas bonus AND my friends are free, I will take a road trip with my friends.. If my friends don't find a job after Christmas, then they will be free. I got my Christmas bonus and my friends did NOT find a job after Christmas. Therefore, I will take a road trip with my friends!

Assign the statements as follows:

p = "I get my Christmas bonus."

q = "My friends are free."

r = "I will take a road trip with my friends."

s = "My friends find a job after Christmas."

The argument, symbolically is as follows:

Premises: **Conclusion: r**

1. $(p \wedge q) \to r$
2. $\neg s \to q$
3. p
4. $\neg s$

Steps	**Reasons**
1) $\neg s \to p$	Premise
2) $\neg s$	Premise
3) p	Modus Ponens (using 1, 2)
4) q	Premise
5) $p \wedge q$	Conjunction
6) $(p \wedge q) \to r$	Premise
7) r	Modus Ponens (using 5, 6)