

TRIBHUVAN UNIVERSITY INSTITUTE OF ENGINEERING THAPATHALI CAMPUS

Instrumentation I

BEX, BCT, BEL

Chapter 3: Transducers

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Chapter 3. Transducer (8 hours/16 Marks)

- 3.1 Introduction
- 3.2 Classification
- 3.3 Application
 - 3.3.1 Measurement of mechanical variables: displacement, strain, velocity, acceleration and vibration
 - 3.3.2 Measurement of process variables: temperature pressure, level, fluid flow, chemical constituents in gases or liquids, pH and humidity.
 - 3.3.3 Measurement of bio-physical variables, blood pressure and bioelectric potentials, myoelectric potentials

Introduction to sensor and transducer

- A Transducer / sensor is a device which converts• Potentiometer is one of the example of passive one form of energy into the another form of sensor whereas Thermocouple & Piezo-electric energy.
- Sensor converts non electrical energy in to electrical energy
- Transducers are the devices which converts non electrical energy into the electrical form and viceversa.
- For example, Thermo couple which converts heat energy into electrical energy (small voltage).
- There are many ways, a sensor can be classified. On the basis of physical principle involved sensor Sensor may be broadly classified as
 - a. Passive sensor and
 - b. Active sensor.
- Passive sensors are those which acquires some external power supply for their operation, where as active sensors do not need any such external power supply for their function in sensing energy.

crystal device are the examples of the active sensors.

- On the basis of applications, sensor may be classified as
 - a. Displacement sensor,
 - Temperature sensor,
 - Resistive sensor,
 - d. Humidity sensor etc.

may be classified as.

- Resistive sensor
- Inductive sensor
- Capacitive sensor C.
- Thermo-electric sensor
- e. Piezo electric sensor

Classification of Transducer/sensor

- In various ways transducers may be classified ,such as on the basis of electrical principles involved, methods of applications, methods of energy conversion used, nature of output signal etc.
- Generally the transducers of all kind may fall in following broad categories.
 - a) Primary and Secondary transducers
 - b) Active and Passive transducers
 - c) Analog and Digital transducers
 - d) Transducers and Inverse Transducers

Primary and Secondary transducer

- When the input signal is directly sensed by the transducer and physical phenomenon is converted into the electrical form directly then such a transducer is called the Primary transducer.
- Ex- Thermistor used to sense the temperature.
- But when the input signal is sensed first by some detector or sensor and then its output being of some form other than input signals is given as input to a transducer for conversion into electrical form, then such a transducer falls in the category of Secondary transducers.
- Ex- when the pressure is measured with Bourdon tube & LVDT then LVDT falls into the secondary transducer group.

Active and passive transducer

- On the basis of methods of energy conversion used, transducers may be classified into active & passive transducers.
- Self generating type transducers i.e. the transducers ,which develop their output in the form of electrical voltage or current without any auxiliary source are called the active transducers.
- Such transducers draw energy from the systems under measurement
- Ex: Tacho-generators, Thermo couples, piezo electric crystal etc.
- Transducers, in which electrical parameters i.e. resistance, inductance or capacitance changes with the change in input signal, are called the passive transducers. These transducers require external power supply source for energy conversion
- Ex: Resistive transducers, Inductive transducers & Capacitive transducers etc.

Analog & Digital Transducers

- Transducer on the basis of nature of output signal, may be classified into analog & digital transducers
- Analog transducer converts input signal into output signal, which is a continuous function of time such as thermistors, strain gauge, LVDT, thermo-couple etc.
- Digital transducer converts input signal into the output signal in the form of discrete output signal.
- These are noise immune & distortion less in nature.
- Sometimes an analog transducer combined with an ADC is called a digital transducer. A real example of digital transducer is Quartz crystal used in Quartz watch

Transducers & Inverse transducer

- Transducer is a device that convert a non- electrical physical quantity into an electrical quantity
- Normally a transducer and associated circuit has a non- electrical input and an electrical output.
- Ex: Thermocouple, Photo conductive cell, Pressure gauge, Strain gauge etc.
- An Inverse transducer is a device that converts an electrical quantity into a non-electrical quantity.
- It is a precision actuator for having an electrical input and a low-power non electrical output.
- Ex: Piezo electric crystal
- Translational & angular moving coil elements can be employed as an inverse transducers, data indicating and recording devices are basically inverse transducers
- Ex: loudspeakers

Classification of Transducer:

On the basis of physical principle involved the sensor/transducer may be classified as

- 1. Resistive sensor
- 2. Inductive sensor
- 3. Capacitive sensor
- 4. Thermo-electric sensor
- 5. Piezo-electric sensor
- 6. Hall effect sensor
- 7. Electro magnetic sensor

Resistive sensor:

The input being measured is transformed in to change in resistance Eg. Potentiometer, Strain gauze, Photo-conductive cell, resistance thermometer etc.



inductive sensor:

Eg. LVDT (linear variable differential transformer)



Eg. Capacitive displacement sensor, capacitive liquid level sensor, capacitive hygrometer



Thermo-electric sensor:

The input is temperature and the output is Emf. Eg. Thermocouple



Piezo-electric sensor:

A piezo electric material is one in which an electric potential appears across certain surfaces of the crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. The effect is reversible.

Eg. Rochelle salt, Ammonium dihydrogen phosphate, lithium sulphate, quartz and ceramics.

Hall effect sensor:

The action of a magnetic field on a flat plate carrying an electric current generate a potential difference which is the measure of the strength of a magnetic field. Eg. Hall effect device, Metal detector

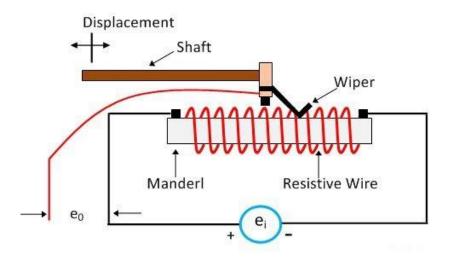
Electro-magnetic sensor:

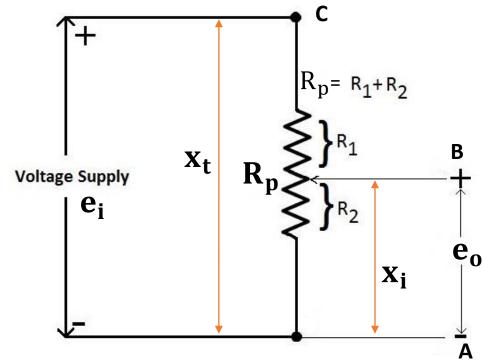
The sensor based on Faraday's law of Electromagnetic induction, with the input is measured giving rise to induced Emf. Eg. Tacho-generator

Resistive Sensor

1. Potentiometer (POT):

- Potentiometer is one of the example of resistive sensor used for the measurement of displacement. The displacement may be either liner or rotary (angular). Hence accordingly there are two types of potentiometer
 - Linear Potentiometer
 - 2. Angular/Rotary Potentiometer





1.Potentiometer (POT)

Let us consider a linear potentiometer

 $e_i = input \ voltage$

 e_0 = output voltage

 $x_t = total length of potentiometer$

 R_p = total resistance of potentiometer

Resistance per unit length = $\frac{R_p}{x_t}$

Resistance of displacement AB is $= R_{AB}$

$$\therefore R_{AB} = \frac{R_p}{x_t} \times x_i$$

$$R_{AB} = \frac{x_i}{x_t} \times R_p$$

$$R_{AB} = K \times R_p$$

Where $K = (\frac{x_i}{x_t})$ and $0 \le K \le 1$

If
$$x_i = x_t$$
 then $K = 1$

The ideal voltage drop across the displacement is given by,

$$e_{o} = \frac{R_{AB}}{R_{AB} + R_{BC}} \times e_{i}$$

$$e_{o} = \frac{K \times R_{P}}{R_{P}} \times e_{i}$$

$$e_{o} = K. e_{i}$$

$$e_{o} = \frac{x_{i}}{x_{t}} \cdot e_{i} \dots (i)$$

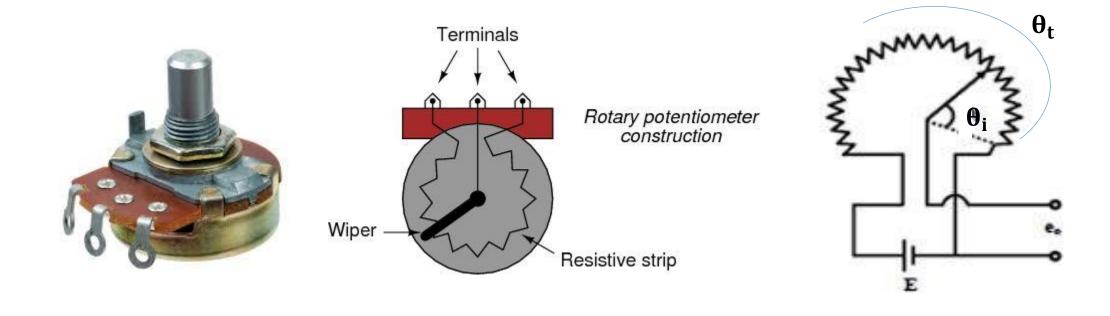
$$\frac{e_{o}}{e_{i}} = \frac{x_{i}}{x_{t}} \dots (ii)$$

$$\frac{e_{o}}{x_{i}} = \frac{e_{i}}{x_{t}} \dots (iii)$$
Displacement
$$\frac{x_{i}}{x_{t}} \text{ ratio}$$

From above equations we conclude that there exist a linear relationship between output and input, which can be represented in figure as, The sensitivity of a device is given by,

• Sensitivity
$$S = \frac{Output}{Input} = \frac{e_o}{x_i} = \frac{e_i}{x_t} = Constant$$

• The above relations are applicable for rotary potentiometer as well, if we replaced x_i by θ_i and x_t by θ_t

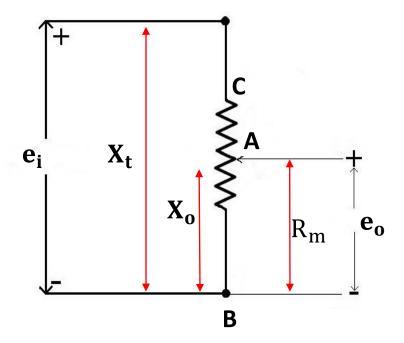


Loading effect:

- If the resistance across the output terminal is infinite, we get a linear relationship between the output and input voltage given by $e_0 = K.e_i = \frac{x_i}{x_t}.e_i$
- However, in practice the output terminal of POT is connected to a device whose internal resistance (R_m) is finite.
- Thus an electrical instrument is connected across the output terminal, the indicated voltage is less than that given by above equation. This effect is known as loading effect.
- Due to the loading effect there exists nonlinear relationship between output and input.
- The loading effect is caused by the internal resistance of the output device.

Now new R_{AB} becomes,

•
$$R_{AB} = (KR_P | | R_m) = \frac{KR_PR_m}{KR_P + R_m}$$



The output voltage measured by the voltmeter is given by,

$$\begin{split} e_{om} &= \frac{R_{AB}}{R_{AB} + R_{BC}} \times e_{i} \\ &= \frac{\frac{KR_{P}R_{m}}{KR_{P} + R_{m}}}{\frac{KR_{P}R_{m}}{KR_{P} + R_{m}} + (R_{p} - KR_{p})} \times e_{i} \\ &= \frac{KR_{P}R_{m}}{KR_{P}R_{m} + KR_{p}^{2} + R_{P}R_{m} - K^{2}R_{p}^{2} - KR_{P}R_{m}} \times e_{i} \\ &= \frac{KR_{P}R_{m}}{KR_{p}^{2} + R_{P}R_{m} - K^{2}R_{p}^{2}} \times e_{i} \\ &= \frac{KR_{P}R_{m}}{R_{P}R_{m}[K_{Rm}^{R} + 1 - K^{2}\frac{R_{P}}{R_{m}}]} \times e_{i} \\ &= \frac{K}{[K_{P}^{R}R_{m} + 1 - K^{2}\frac{R_{P}}{R_{m}}]} \times e_{i} \end{split}$$

Let us assume
$$\frac{R_m}{R_p} = \alpha$$
, then
$$e_{om} = \frac{K}{\left[\frac{K}{\alpha} - \frac{K^2}{\alpha} + 1\right]} \times e_i$$

$$e_{om} = \frac{\alpha K}{\left[K - K^2 + \alpha\right]} \times e_i$$

$$e_{om} = \frac{\alpha K}{\left[\alpha + K(1 - K)\right]} \times e_i$$

Thus from above expression we conclude that there exist non-linear relationship between output and input due to loading effect.

Loading Error

1. Relative Error (ε_r):

$$\begin{split} \epsilon_r &= \frac{\text{output without loading effect} - \text{output with loading effect}}{\text{output without loading effect}} \\ &= \frac{\frac{\kappa e_i - \frac{\alpha K}{[\alpha + K(1-K)]} \times e_i}{[\alpha + K(1-K)]}}{\frac{K e_i}{\alpha + K(1-K)}} \\ &= \frac{K e_i \left[1 - \frac{\kappa e_i \alpha}{\alpha + K(1-K)}\right]}{\frac{K e_i}{\alpha + K(1-K)}} \\ &= \frac{\alpha + K(1-K) - \alpha}{\alpha + K(1-K)} \\ &\vdots \epsilon_r = \frac{K(1-K)}{\alpha + K(1-K)} \end{split}$$
 Where $K = \frac{x_i}{\alpha}$ and $K = \alpha$

Loading Error

2. Absolute Loading Error (ε_a):

output without loading effect — output with
$$\varepsilon_{a} = \frac{\text{loading effect}}{\text{input voltage}}$$

$$= \frac{\text{Ke}_{i} - \frac{\alpha \text{K}}{[\alpha + \text{K}(1 - \text{K})]} \times \text{e}_{i}}{\text{e}_{i}}$$

$$= \frac{\text{e}_{i}[\text{K} - \frac{\alpha \text{K}}{\alpha + \text{K}(1 - \text{K})}]}{\text{e}_{i}}$$

$$= \frac{\alpha \text{K} + \text{K}^{2}(1 - \text{K}) - \alpha \text{K}}{\alpha + \text{K}(1 - \text{K})}$$

$$\therefore \boxed{\varepsilon_{a} = \frac{\text{K}^{2}(1 - \text{K})}{\alpha + \text{K}(1 - \text{K})}}$$

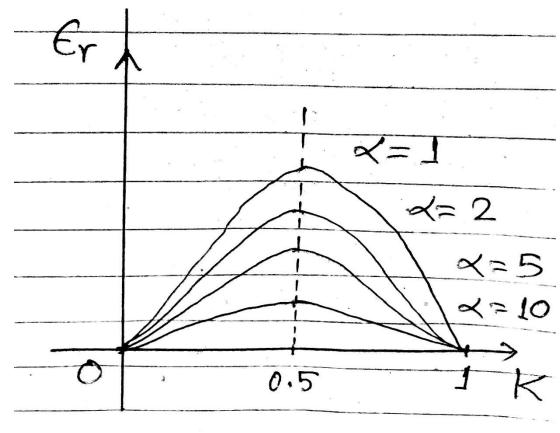
From this expression, we know that error depends upon value of K. let us find the value of K at which error is maximum.

$$\begin{aligned} &\text{We know for that } \frac{dE}{dK} = 0 \\ &= \frac{d[\frac{K - K^2}{\alpha + K - K^2}]}{dK} \\ &= \frac{(\alpha + K - K^2) \cdot (1 - 2K) - (K - K^2) \cdot (1 - 2K)}{(\alpha + K - K^2)^2} \\ &= \frac{(1 - 2K)\{\alpha + K - K^2 - K + K^2\}}{(\alpha + K - K^2)^2} \\ &= \frac{\alpha(1 - 2K)}{(\alpha + K - K^2)^2} \\ &= \frac{\alpha(1 - 2K)}{(\alpha + K - K^2)^2} \end{aligned}$$
 Equating with zero,
$$\alpha(1 - 2K) = 0$$
 Since $\alpha \neq 0$, so
$$(1 - 2K) = 0$$

$$\therefore K = \frac{1}{2} = 0.5$$

- Thus the error will be maximum when wiper is at the center of the potentiometer
- In actual practice error is maximum at K=0.67
- Let us plot the variation of error with the variation of K for different value of α .
- From the graph, we conclude that as α increases, then relative error ϵ_r decreases.

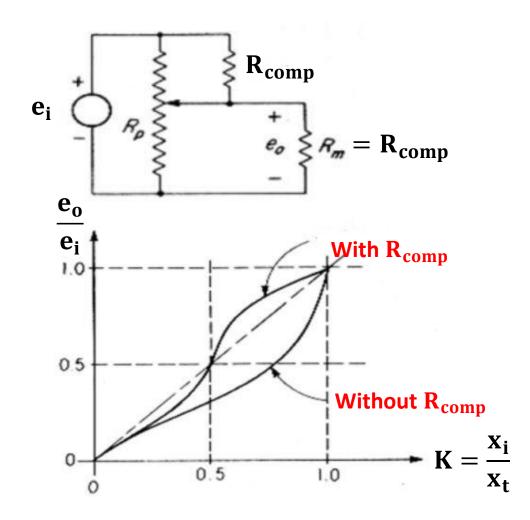
Displacement ratio K	Relative error ε_{r}
0	0
0.1	Increases
•••••	
0.5	Maximum
0.6	Decreases
1	0



Methods to reduce Loading error effect

To decrease loading error, we have to increase value of α i.e. $R_m \gg R_p$

- 1. Use the digital meters instead of analog meter as digital meter have higher impedance.
- 2. Use buffer circuit
- 3. Modify the construction of potentiometer
- 4. By connecting a compensating resistor $R_{com}=R_{m}$ across the remaining portion of the potentiometer as shown in figure



Power rating of potentiometer:

It is the safe value of heat dissipating capacity.

$$e_{imax} = \sqrt{p \times R_p}$$
 since $P = \frac{e_{imax}^2}{R_p}$

The resistance of potentiometer R_{p} can not be made low because it results in the higher power dissipation.

Linearity and sensitivity of potentiometer:

For linearity, $\alpha = \infty$

$$\frac{R_{\rm m}}{R_{\rm p}} = \infty$$
 implies $\frac{R_{\rm p}}{R_{\rm m}} = \frac{1}{\infty} = 0$

i.e.
$$R_p \ll R_m$$

For linearity R_p should be as minimum as possible.

For sensitivity,

$$S = \frac{\text{output}}{\text{input}} = \frac{e_o}{X_i} = \frac{e_i}{X_t}$$

For better sensitivity; input voltage e_i should be high and this calls for a high value of resistance $R_{\rm p}$.

Prove that linearity and sensitivity are two conflicting requirements of the potentiometer

We know that output voltage during loading

$$e_{om} = \frac{\alpha K}{[\alpha + K(1 - K)]} \times e_i$$

The relation shows that non linear relationship between output voltage ${
m e_{om}}$ and input ${
m e_i}$

For linearity,
$$\alpha = \infty$$
 and $\frac{R_m}{R_p} = \infty$ i.e. $R_p \ll R_m$

i.e. R_p should be as low as possible for better linearity,

But for sensitivity,

$$S = \frac{\text{output}}{\text{input}} = \frac{e_o}{X_i} = \frac{e_i}{X_t}$$

The relation shows, for higher sensitivity; input voltage e_i should be high and this calls for a high value of resistance R_{p}

But we know that power rating $P = \frac{e_i max^2}{R_p}$

i.e. we should keep the value of P within certain limit for that it is good if e_i is as low as possible. The low value of input voltage e_i leads to low output voltage e_o resulting in lower sensitivity. So we can see from the above relation that linearity and sensitivity are two conflicting requirements of the potentiometer.

A linear POT is 50mm long and is uniformly wound with a wire having a resistance of $10 \mathrm{K}\Omega$. Under normal conditions the slider is at the center of the POT. Find the linear displacement when the resistance of the POT as measured by Wheatstone bridge for two cases if (i)3850 Ω (ii)7560 Ω are the two displacement in the same direction. If it is possible to measure a minimum value of 10Ω resistance with the above arrangements. Find the resolution of the POT.

Given:

- For Total length of potentiometer $(x_t) = 50 \text{ mm}$
- Total resistance of potentiometer (R_P)= 10 K Ω = 10000 Ω
- \triangleright Minimum measurable resistance = 10 Ω
- > We know resistance per unit length

$$= \frac{\text{Total Resistance}}{\text{Total Length}} = \frac{10000 \,\Omega}{50 \,\text{mm}} = 200 \,\Omega \text{mm}^{-1}$$

➤ Resolution of POT

$$= \frac{\text{Minimum measurable resistance}}{\text{Resistance per unit length}} = \frac{10 \,\Omega}{200 \,\Omega \text{mm}^{-1}} = 0.05 \,\text{mm}$$

Therefore, Resolution of POT = $\frac{1}{20}$ = 0.05 mm

> Resistance of POT at normal position,

$$\succ$$
 i.e. at center position, $R_{AB} = \frac{10000}{2} = 5000 \; \Omega$

Case I:

- \triangleright Resistance of POT measured by Wheatstone bridge = 3850 Ω
- Change of the resistance from normal position

$$= 5000\Omega - 3850\Omega = 1150\Omega$$

➤ Now linear displacement from normal position

$$= \frac{1150\Omega}{200 \ \Omega \text{mm}^{-1}} = \boxed{5.75 \ \text{mm along BA}} (\downarrow \text{from central position})$$

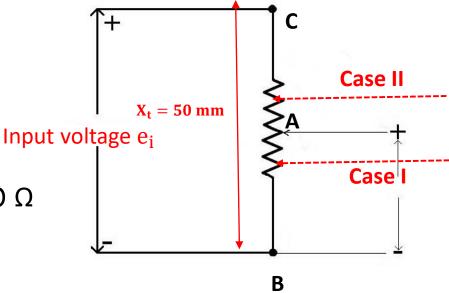
Case II:

- \triangleright Resistance of POT measured by Wheatstone bridge = 7560 Ω
- Change of the resistance from normal position

$$= 7560Ω - 5000Ω = 2560 Ω$$

➤ Now linear displacement from normal position

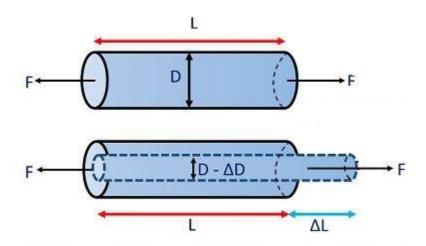
$$= \frac{2560 \,\Omega}{200 \,\Omega \text{mm}^{-1}} = \boxed{12.8 \,\text{mm along BC}} (\uparrow \text{ from central position})$$



Resistive Sensor

2. Strain Gauge

- The strain gauge are used for measurement of strain and associated stress in experimental stress analysis.
- Strain gauges are the metallic conductor and are stretched or compressed, their resistance change on account of the fact that both length and diameter of them change.
- Also there is a change in the value of resistivity of the conductor when it is strained and this property is called Piezo-resistive effect.
- Therefore resistance strain gauges are also known as Piezo-resistive gauges.
- It is to be noted that strain gauges are subjected to either tensile or compressive forces.
- Let us consider a strain gauge made of circular wire. Assuming L be the length, A be the cross sectional area, D be the diameter of the wire before being strained. Say, the material of the wire has a resistivity ρ



Resistance of unstrained gauge $R = \frac{\rho L}{A}$(i)

Let Tensile force F having the stress of S is provided to the strain gauge, then the effect of tensile stress S is shown by differentiating equation (i) with respect to S (partially)

$$\frac{\partial R}{\partial S} = \frac{\partial \left[\frac{\rho L}{A}\right]}{\partial S}$$

$$\frac{\partial R}{\partial S} = \frac{\rho}{A} \cdot \frac{\partial L}{\partial S} - \frac{\rho L}{A^2} \cdot \frac{\partial A}{\partial S} + \frac{L}{A} \cdot \frac{\partial \rho}{\partial S} \dots (ii)$$

Dividing equation (ii) by (i)

$$\frac{1}{R} \cdot \frac{\partial R}{\partial S} = \frac{1}{L} \cdot \frac{\partial L}{\partial S} - \frac{1}{A} \cdot \frac{\partial A}{\partial S} + \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial S} \dots (iii)$$

If the variation is very-very small, then equation (iii) becomes,

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}....(iv)$$

We know that the area of the solid circular wire is given by,

$$A = \frac{\pi D^2}{4}$$
....(v)

Differentiating partially,

$$\frac{\partial A}{\partial S} = \frac{\pi}{4} \cdot 2D \cdot \frac{\partial D}{\partial S}$$
 (vi)

Dividing equation (vi) by (v), we get

$$\frac{1}{A} \cdot \frac{\partial A}{\partial S} = \frac{2}{D} \cdot \frac{\partial D}{\partial S}$$
 (vii)

From equation (iii) and (vii)

$$\frac{1}{R} \cdot \frac{\partial R}{\partial S} = \frac{1}{L} \cdot \frac{\partial L}{\partial S} - \frac{2}{D} \cdot \frac{\partial D}{\partial S} + \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial S} \dots \dots (viii)$$

If the variation is very-very small, then equation (viii) becomes,

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - 2.\frac{\Delta D}{D} + \frac{\Delta \rho}{\rho}.$$
 (ix)

Now let us define, Poisson's ratio

$$\mu = \frac{\text{Lateral Strain}}{\text{Longituidinal strain}} = -\frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}$$

$$\frac{\Delta D}{D} = -\mu \cdot \frac{\Delta L}{L} = -\mu \epsilon(x)$$

Where $\epsilon = \frac{\Delta L}{L} =$ Strain or Longituidinal Strain

From equation (ix) and (x)

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2. \,\mu \epsilon + \frac{\Delta \rho}{\rho}$$
 (xi)

Now let us define a gauge factor (G),it is defined as the ratio of per unit change in resistance to per unit change in length

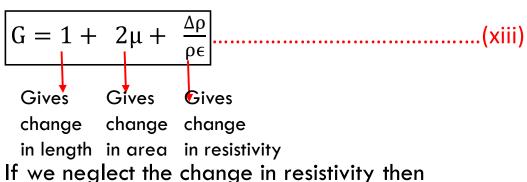
$$G = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}}$$

$$\frac{\Delta R}{R} = G \cdot \frac{\Delta L}{L} = G \cdot \epsilon ... (xii)$$

From equation (xi) and (xii)

G.
$$\epsilon = \frac{\Delta L}{L} + 2$$
. $\mu \epsilon + \frac{\Delta \rho}{\rho}$

$$G. \epsilon = \epsilon + 2. \mu \epsilon + \frac{\Delta \rho}{\rho}$$



$$G = 1 + 2\mu \tag{xiv}$$

This is the required expression for the gauge factor of the strain gauge

Note:

 The strain is unit less quantity but it is generally expressed in micro-strain

1 micro-strain =
$$\frac{1\mu m}{1m} = \frac{\Delta L}{L}$$

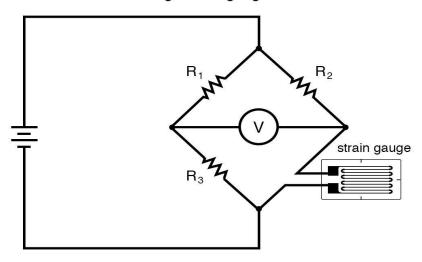
- Tensile stress = +Ve
- Compressive stress = -Ve

Resistive Sensor

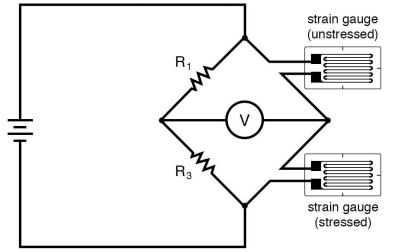
3. Dummy Gauge

- Dummy gauge is to cancel out any temperature related resistance variation in the active gauge.
- Any variation of temperature will now affect both dummy and active gauges.
- So any variation in resistance in the active gauge due to temperature also appears in the dummy gauge and hence there will be no effect on the bridge balanced condition
- In this way variation due temperature can be eliminated as a source of error.

Quarter-bridge strain gauge circuit



Quarter-bridge strain gauge circuit with temperature compensation



A compressive force is applied to a structure members. The strain is 5μ strain. Two separate Strain gauge are attached to the structural member, one is nickel wire strain gauge having gauge factor of -12.1 and other is nichrome wire gauge having gauge factor of 2. Calculate the value of resistance of the gauges after they are strained. The resistance of strain gauges before being strained is 120 Ω .

- > Solution:
- > Strain $\epsilon = \frac{\Delta L}{L}$ and we know 1 micro-strain = $\frac{1 \mu m}{1 m}$
- > 5 micro-strain = $-\frac{5\mu m}{1m} = -5 \times 10^{-6}$ (compressive)
- ➤ Case I:
- \triangleright Gauge factor of Nickel (G_{Ni}) = -12.1
- \triangleright Resistance of gauges before strain (R) = 120 Ω
- We know gauge factor $G = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = \frac{\frac{\Delta R}{R}}{\epsilon}$
- $\therefore \frac{\Delta R}{R} = G. \epsilon \text{ and } \Delta R = G. \epsilon. R$
- Arr $\Delta R = (-12.1).(-5 \times 10^{-6}).(120)$

- ightharpoonup Resistance will increase by $R + \Delta R$
- $> 120 + 7.26 \times 10^{-3} = 120.00726 \Omega$
- Case II:
- \triangleright Gauge factor of Nichrome (G_{NC}) = 2
- \triangleright Resistance of gauges before strain (R) = 120 Ω
- We know gauge factor $G = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = \frac{\frac{\Delta R}{R}}{\epsilon}$
- $\Rightarrow \qquad \therefore \frac{\Delta R}{R} = G. \in \text{and } \Delta R = G. \in R$
 - Arr $\Delta R = (2). (-5 \times 10^{-6}). (120)$

 - > New resistance after strain = $R + \Delta R = 120 1.2 \times 10^{-3} \Omega$
 - $> = 119.9988 \Omega$

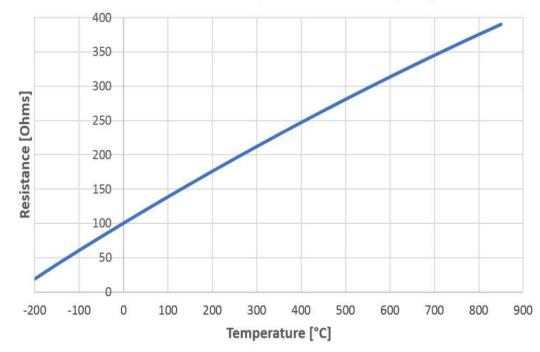
Resistive Sensor

4. Resistive Thermometer

- The resistive thermometers are based on the electrical resistance changing.
- When temperature changes, the resistance of metal generally increase with temperature, hence the change in resistance
- It is linear with temperature i.e. the change in resistance with temperature is linear with temperature over range of -100 °C to 800°C
- In general the relation is given by, $R_t = R_o(1 + \alpha t + \beta t^2 + \gamma t^3 + \cdots)$



Resistance vs. Temperature - Pt100 (385)



Resistive Thermometer....

Where,

 R_t = resistance at t°C

 $R_o = resistance at 0°C$

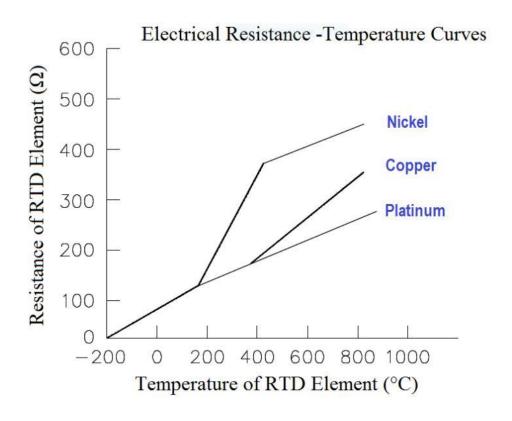
 $\alpha,\beta,\gamma=$ temperature coefficient of resistance with $\alpha>\beta>\gamma$

• For an ideal linear relationship we have

$$R_t = R_o(1 + \alpha t)$$

- The resistance thermometers are made by winding Pt (platinum) wire or Nickel (Ni) wire or Copper (Cu) wire
- Platinum has closely linear relation between resistance & temperature and has long term stability.
- Platinum (Pt) has temperature range of about
 200 °C to 850 °C is relatively inert.
- It is more expensive than the other metals but is, however most widely used.

- Nickel & copper are cheaper but have less stability and can not be used for over such range of temperature.
- Nickel(Ni) has range of about -80 °C to 300 °C and for copper (Cu) -200 to 250 °C



Resistive Sensor

5. Photo conductive cells (or LDR):

- Photo conductive cells or light dependent resistors (LDR) are semiconductors used for their property of changing resistance when electromagnetic radiation is incident on such materials.
- When radiation of very high frequency is incident on such materials, electrons are excited from the valence-band to the conduction - band, with a result, a hole is produced in the valence - band for conduction and an electron for the conduction in the conduction band.
- The result is an increase in the number of charge carriers and hence a drop in resistance.

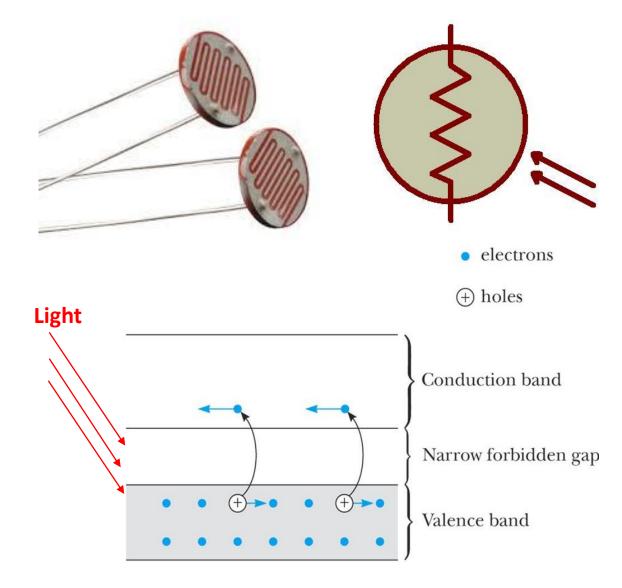
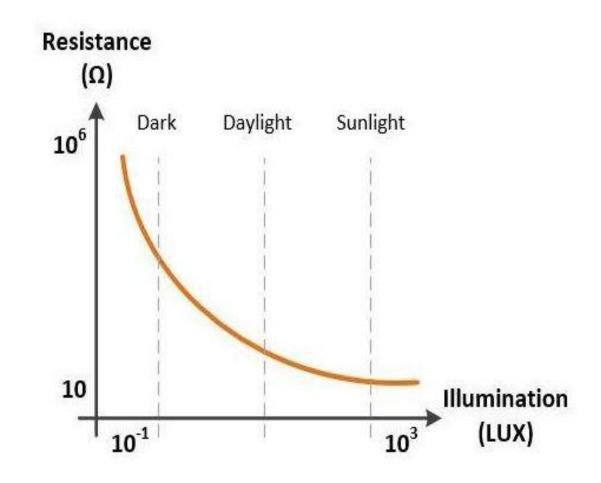


Photo conductive cells (or LDR)....

- As the intensity of radiation falling on the cell increases, the number of charge carriers available for conduction is increased and the resistance decreases
- Cadmium sulphide generally doped with atoms of other elements is a commonly material for photo conductive cells.
- Fig, given below, shows the resistance versus the Intensity (in lux) curve that of Cadmium Sulphide (CdS_2) photo conductive cell.



Inductive sensor/transducer:

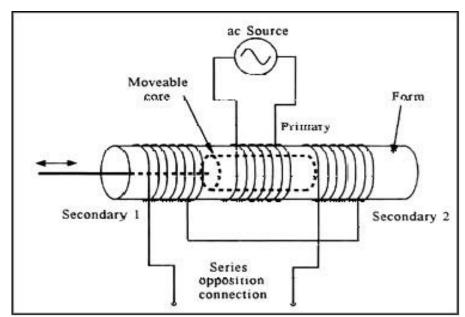
- These are analog passive transducers.
- These transducers work generally upon one of the following three principles
 - 1) Variation of self inductance of the coil
 - Variations of mutual inductance of the coils &
 - 3) Production of Eddy currents
- The general block diagram of the inductive sensor is drawn below

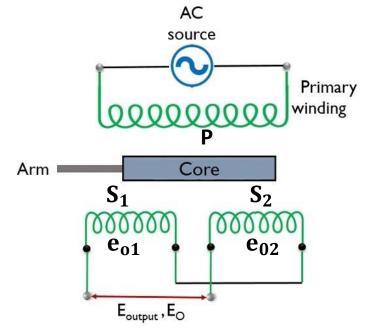
- There are two types of inductive sensors to be used to translate the linear motion and rotational motion in to the electrical signal
- These are listed below
- Linear variable differential transformer (LVDT)
- Rotary variable differential transformer (RVDT)



1. Linear variable differential transformer (LVDT)

- Generally the change in self inductance ΔL or mutual inductance ΔM is sufficient for deflection for subsequent stages of instrumentation system.
- However, instead of responding (L+ Δ L) or (M+ Δ M) and giving the output voltage corresponding to it, if the instrumentation system responds to Δ L or Δ M only, then the sensitivity and accuracy will be much higher because Δ L & Δ M are very small in comparison to L & M.
- This can be achieved by splitting the coils into two halves and taking the output across the whole windings as shown in the figure. The arrangement can be drawn as a simple circuit diagram as well.





LVDT....

- As we can see that a Linear variable differential transformer is composed of a primary winding P along with 2 secondary windings $S_1 \& S_2$
- The ac supply is provided to the primary winding of the transformer while the two secondary windings are kept equidistant on both the sides of the primary winding. A cylindrical core is placed in the structure, which is moved linearly within the former.
- The movement of the core in different directions produces a differential voltage at the output.
- Now a question comes in our mind that how only due to movement of the core, a voltage is generated?
- So let us see the reason behind it.
- Basically, the produced differential output does not solely depend on the movement of the core but also on the applied ac voltage at the primary winding of the transformer.
- Because due to the applied ac signal, an alternating magnetic field is generated. Due to this magnetic field, an ac voltage gets induced in the two secondary windings. But the variation in the induced voltage generates 3 different cases.

LVDT....

- We consider 3 different cases in order to understand the operation of an LVDT;
- Firstly, when the core is present at the initial or normal position. Then the voltage induced in both the secondary windings $S_1 \& S_2$ will be equal.
- As the two windings are serially joined. Thus the differential voltage at the output will be zero
- Secondly, when the core is moved to the left of the normal position, then, in this case, the linking flux in S_1 will be more than that of S_2 . So, the voltage at S_1 will be greater than S_2 . So, the differential output voltage is given as

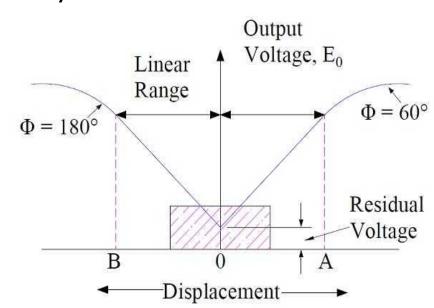
$$e_0 = e_{01} - e_{02}$$

• Thirdly, when the core is moved to the right side form the central position, then this time the linking flux in S_2 will be more than S_1 . So the voltage produced as S_1 will be less. Then the differential output voltage will be

$$e_0 = e_{02} - e_{01}$$

• Thus we can say that change in output voltage in case of LVDT is proportional to the displacement of the core.

- The figure given below shows the variation of output voltage against displacement for various positions of core.
- The curve is practically linear for small displacements. Beyond this range of displacement the curve starts to deviate from straight line.
- Here for the displacement range 0 to A & 0 to B, the behavior of the LVDT is linear & beyond A & B it is non-linear in nature.



Advantages of LVDT

- 1. It has high range
- 2. It provides the friction-less & electrical isolation
- 3. It has the immunity for external effects
- 4. It has high sensitivity
- The equipment is ruggedness and preserves themselves from adverse effect providing stable output
- Low hysteresis loss & low power consumption

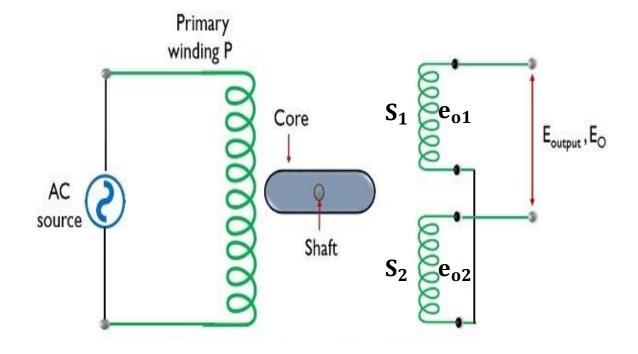
<u>Disadvantages of LVDT</u>

- 1. Only high displacements are sensed
- 2. Sensitive to Stray magnetic fields
- 3. Output is influenced by vibrations.
- 4. If dc output is required then conversion of ac to dc is needed

2. Rotary variable differential transformer (RVDT)

- RVDT is an abbreviation used for the rotary variable differential transformer.
- It is a transducer that has the ability to sense angular displacement and converts this displacement into an electrical signal.
- Generally, the operation of RVDT is somewhat similar to an LVDT. However, in the case of RVDT, a cam-shaped core is taken that is allowed to be rotated between the windings of the transformer with the help of a shaft.
- Here we can clearly see, that the core is placed between the primary and two secondary windings of the transformer.
- The external supply is provided at the primary winding of the transformer and the differential output is taken from the two secondary windings.

 As we have already mentioned that the operation of RVDT is similar to LVDT. Thus here also the externally supplied ac source and rotation of the core generates a differential voltage at the output.



RVDT....

- Here, also we will consider 3 conditions in order to understand the operation performed by the RVDT.
- Firstly, when the core is at the centrally placed or at the null position. Then due to the externally supplied ac voltage, a magnetic field gets generated and this magnetic field induces an equal voltage at both the windings.
- So, this resultantly causes the differential voltage to be 0. As here e_{01} will be equal to e_{02}
- Secondly when the core is rotated in a clockwise direction then, the greater part of the core will come across the secondary winding. This will cause more flux linkage across S_1 rather than S_2 .
- Therefore, the differential voltage at the output will be

$$e_0 = e_{01} - e_{02}$$

- Likewise, in the third case when the core is rotated in an anti-clockwise direction then, the greater portion of the core will be present at the secondary winding S_2 . Thus more flux will link across S_2 than S_1 .
- Hence, now the differential output voltage will be

$$e_0 = e_{02} - e_{01}$$

• So, we can say the more the angular displacement will be the greater will be the differential output.

Capacitive sensor/Transducer

the parallel plate capacitor with the cross sectional area of A and distance between the two plates d be given by

 Capacitive sensor consists of two parallel plates, one is fixed and the other is moveable. Hence, capacitance is given by equation (i)

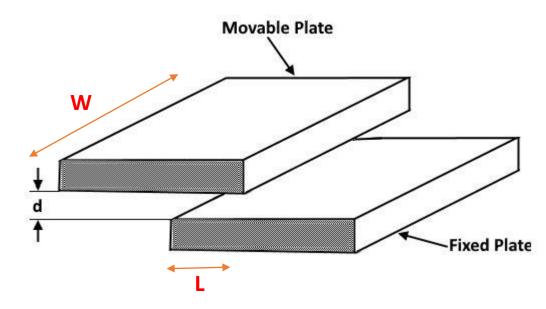


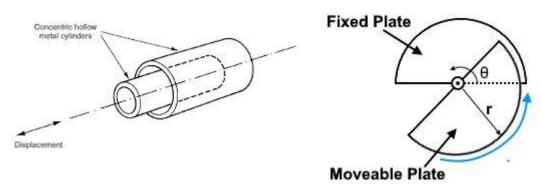
- The expression for the capacitance of
 As input being measured is transformed into change in capacitance. Thus the input variable must change anyone of the three variable on right side of the equation (i) to make a change in capacitance.
 - Hence, accordingly, there are three principles on which capacitive sensor or transducer works.
 - Capacitive sensor working on change in overlapping area A
 - Capacitive sensor working on change in separation distance d &
 - Capacitive sensor working on change in permittivity ($\varepsilon \& \varepsilon_r$)

Capacitive sensor working on change in overlapping area A

- It consists of two plates, one is fixed and the other is movable.
- The body whose displacement is to be measured is coupled with the moveable plate.
- Thus there will be two types of the capacitive displacement sensor, i.e. one for the linear displacement & the other for the rotary or angular displacement.
- The general arrangement of both are shown in the figure
- For the linear displacement

- Thus, we can see that there exist a linear relationship between input displacement and output capacitance
- By measuring capacitance, we can measure the displacement or length as well.

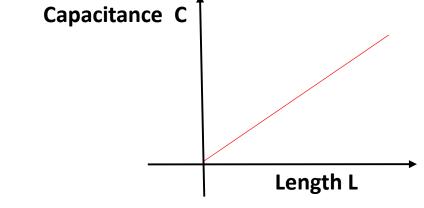




This sensor can be used for the measurement of linear displacement from few mm to several cm. Now the sensitivity is given by,

$$S = \frac{small\ change\ in\ output}{small\ change\ in\ input} = \frac{\Delta C}{\Delta L} = \frac{dC}{dL}$$

$$S = \frac{dC}{dL} = \frac{d}{dL} \left[\frac{\epsilon(W \times L)}{d} \right] = \frac{\epsilon W}{d} = constant$$



For the case of rotary displacement:

The overlapping areas will be maximum when the plate fully overlap i.e. $\theta=\pi$

Hence the maximum overlapping area be $\frac{\pi r^2}{2}$

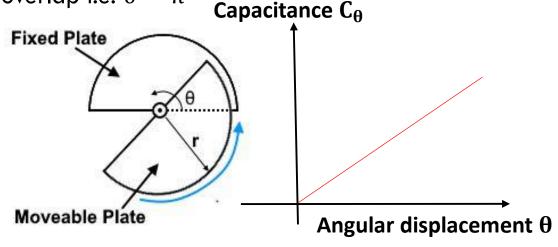
So the maximum capacitance is given by,

$$C_{\text{max}} = \frac{\varepsilon A}{d} = \frac{\varepsilon . \pi r^2}{2d}$$

Thus a capacitance corresponding to overlapping angle θ is

given by,
$$C_{\theta} = \frac{\epsilon.\theta r^2}{2d}$$

i.e. $C_{\theta} \propto \theta$



Here θ must be in radian. From the expression, there exist a linear relationship between $C_{\theta} \& \theta$. Thus by measuring capacitance we can measure the angular displacement.

$$S = \frac{\text{small change in output}}{\text{small change in input}} = \frac{\Delta C_{\theta}}{\Delta \theta} = \frac{dC_{\theta}}{d\theta}$$

$$S = \frac{dC_{\theta}}{d\theta} = \frac{d}{d\theta} \left[\frac{\epsilon \cdot \theta r^2}{2d} \right] = \frac{\epsilon \cdot r^2}{2d} = constant$$

For the case of cylindrical capacitor:

For cylindrical capacitor

$$C = \frac{2\pi\varepsilon L}{\ln[\frac{D_2}{D_1}]}$$



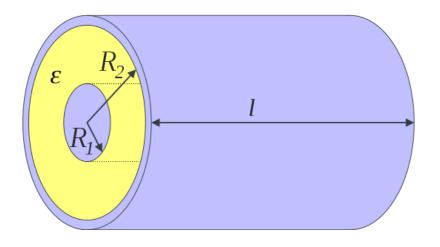
C = capacitance

Epsilon (ε) = permittivity

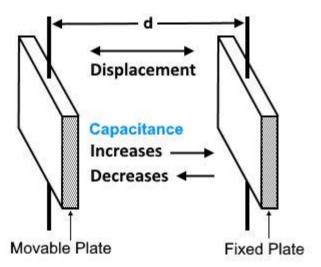
L = length of conductor

 D_2 = outer conductor diameter

 D_1 = inner conductor diameter



Capacitive sensor working on change in separation distance d



The capacitance of the capacitor for this case is given by

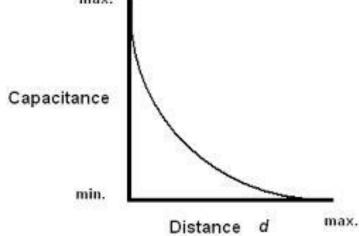
Let us differentiate equation (i) with respect to separation distance d, to get sensitivity S

$$S = \frac{\text{small change in output}}{\text{small change in input}}$$

$$S = \frac{\Delta C}{\Delta d} = \frac{dC}{dd} = \frac{d}{dd} \left[\frac{\epsilon A}{d} \right] = -\frac{\epsilon A}{d^2} \dots \dots \dots \dots (ii)$$

- ➤ But separation distance d can be made small up to certain limit which is determined by the breakdown strength of the dielectric material, between the plates.
- Equation (ii) shows that sensitivity does not remain constant throughout the whole range of device.
- The non-linear relation of input and output is also shown by equation (ii).

To obtain a linear relationship, the separation distance must be as small as possible.



Differential arrangement

To obtain linear relationship a differential arrangement can be used.

At normal position, when the moveable plate M at the mid-point between the fixed plate P_1 and P_2 , then,

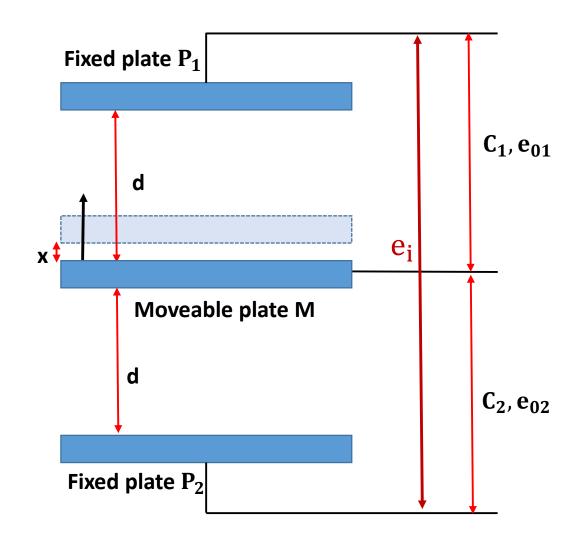
$$C_1 = C_2 = \frac{\varepsilon A}{d}$$
 From figure, if $C_1 = C_2$
$$e_{01} = \frac{C_2}{C_1 + C_2} \times e_i = \frac{e_i}{2}$$

Similarly,

$$e_{02} = \frac{C_1}{C_1 + C_2} \times e_i = \frac{e_i}{2}$$

There is equal voltages, so the differential output is given by,

$$\Delta e_0 = e_{01} - e_{02} = \frac{e_i}{2} - \frac{e_i}{2} = 0$$



Let a moving body is attached to the plate M which is displaced a small distance x upward as shown in above figure

$$C_1 = \frac{\varepsilon A}{d-x} \text{ and}$$

$$C_2 = \frac{\varepsilon A}{d+x}$$

$$e_{01} = \frac{C_2}{C_1+C_2} \times e_i \text{ and}$$

$$e_{02} = \frac{C_1}{C_1+C_2} \times e_i$$

Now,

$$\Delta e_0 = e_{02} - e_{01}$$

$$\Delta e_0 = \frac{C_1}{C_1 + C_2} \times e_i - \frac{C_2}{C_1 + C_2} \times e_i = \frac{C_1 - C_2}{C_1 + C_2} \times e_i$$

From equation (i) and (ii)
$$C_1 - C_2 = \varepsilon A \left[\frac{1}{d-x} - \frac{1}{d+x} \right] = \varepsilon A \left[\frac{2x}{d^2 - x^2} \right]$$

$$= \frac{2\varepsilon Ax}{d^2 - x^2}$$

Again

$$C_1 + C_2 = \varepsilon A \left[\frac{1}{d - x} + \frac{1}{d + x} \right] = \varepsilon A \left[\frac{2d}{d^2 - x^2} \right]$$
$$= \frac{2\varepsilon dx}{d^2 - x^2}$$

Substituting the values of $[C_1 - C_2] \& [C_1 + C_2]$ in equation (iii)

$$\Delta e_0 = \frac{C_1 - C_2}{C_1 + C_2} \times e_i = \frac{\frac{2\epsilon Ax}{d^2 - x^2}}{\frac{2\epsilon dx}{d^2 - x^2}} \times e_i = \frac{x}{d} \times e_i$$

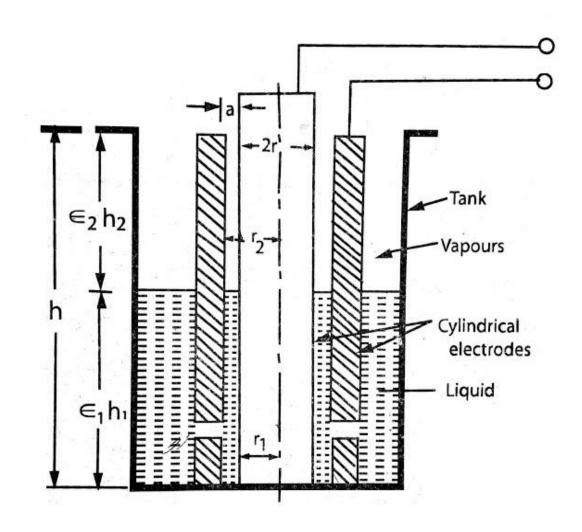
$$\therefore \Delta \mathbf{e}_0 = \frac{\mathbf{x}}{\mathbf{d}} \times \mathbf{e}_{\mathbf{i}}$$

$$\therefore \boxed{\Delta e_0 \propto x}$$

Thus there exist a linear relationship between output Δe_0 and displacement x which is the function of change in capacitance

Variation of dielectric constant for measurement of displacement

- Liquid level measurement of non-conducting liquid
- The third principle used in capacitive transducers is the variation of capacitance due to change in dielectric constant.
- This principle can be used for the measurement of level of liquid levels
- But for this purpose the liquid must be nonconducting liquid
- The electrodes are two concentric cylinders and the non-conducting liquid acts as the dielectric.
- At the lower end of the outer cylinder there are holes which allow passage of liquid.
- In case these holes are small, they provide mechanical damping of the surface vibration



Variation of dielectric constant for measurement of displacement

The value of capacitance for this capacitor is,

$$C = 2\pi\varepsilon_0 \frac{\varepsilon_1 h_1 + \varepsilon_2 h_2}{\log_e[\frac{r_2}{r_1}]}....(i)$$

Where,

 h_1 =height of liquid in meter m

 h_2 =height of cylinder above liquid in meter m

 ε_1 =relative permittivity of liquid

 ε_2 =relative permittivity of vapor above liquid

 r_1 =outside radius of inner cylinder in meter m

 r_2 =inside radius of outer cylinder in meter

 ϵ_0 =permittivity of free space

Equation (i) is based on the assumption that

h >>
$$r_2$$
 and $r_2 \gg (r_2 - r_1) \gg a$

Now from figure,

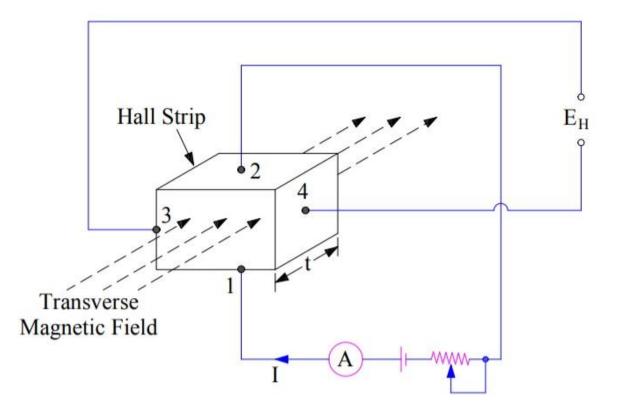
$$\begin{split} r_2 &= r + a \text{ and } r_1 = r \text{ then} \\ C &= 2\pi\epsilon_0 \frac{\epsilon_1 h_1 + \epsilon_2 h_2}{\log_e[\frac{r+a}{r}]} \\ C &= 2\pi\epsilon_0 \frac{\epsilon_1 h_1 + \epsilon_2 h_2}{\log_e[1 + \frac{a}{l}]}.....(ii) \end{split}$$

The equation (ii) can be used as a measure of height of the liquid level

Hall effect sensor/transducer

- Hall Effect Transducer is a device which is used for the measurement of magnetic field strength.
- This transducer uses a conducting strip to convert magnetic field into proportional potential difference across the opposite faces of strip using Hall Effect.
- Hall Effect is basically the process of development of potential difference across the two faces of a current carrying strip when the strip is kept in a magnetic field.
- The magnitude of voltage depends upon the current, strength of magnetic field and the property of conducting material.
- The Hall Effect is found in conducting material and semiconductor in varying amount depending upon the density and mobility of current carrier.





- The current through the strip and the magnetic field are perpendicular to each other.
- Flow of current means the flow of positive charges in the direction of current. This means that, magnetic field will exert a force on the moving positive charges as per

$$F = q(v \times B)$$

- where v & B are the velocity and strength of magnetic field and in vector form.
- Since v and B are perpendicular to each other, the magnitude of force on the moving positive charges will be

$$F = qvB$$

• The direction of force F will be perpendicular to both the v and B as per the law of cross product of two vectors. This essentially means that F will be directed from edge 3 to 4 in the above figure.

- Due to this force on the positive charges, these charges will continue to accumulate on the face 3 which in turn will create an Electric Field.
- The direction of electric filed will be opposite to the direction of F i.e. from edge 3 to 4.
- Therefore, after some time, the magnitude of force exerted by electric filed E and F will become equal and hence there will not be any further movement of the charges.

F = qE (qE is the force on the positive charge due to electric filed)

$$qE = qvB$$

$$\therefore E = vB$$

Due to set-up of electric filed E in the conducting strip across edge 3 and 4, a potential difference will be produced across this face.

Assuming the thickness of strip to be "t", Hence, from information (2), the strength of potential difference across 3 & 4 is given as

$$E_{H} = Et$$

$$= vBt(1)$$

Since the current density through a material is directly proportional to the velocity of carriers, therefore

$$v = K_H J....(2)$$

where J is the current density through the strip and K_H is a constant of proportionality called the Hall Effect Coefficient

But
$$J = \frac{I}{A}$$

where A is the surface area, therefore,

$$J = \frac{I}{t^2}$$

$$v = \frac{K_H I}{t^2}$$

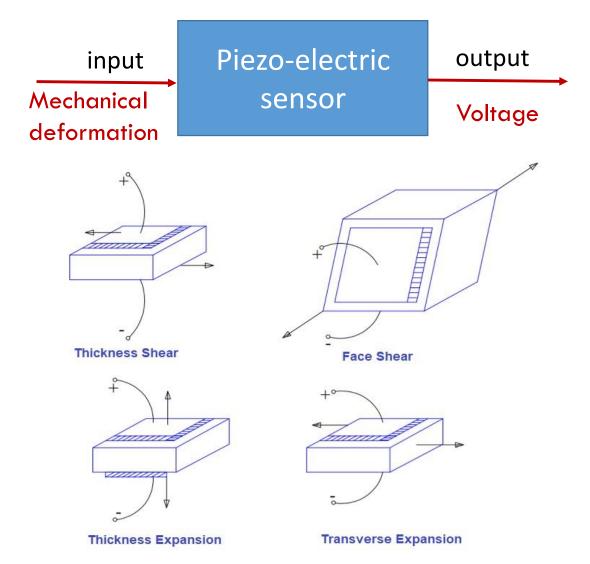
From information (1),

$$E_{H} = \frac{K_{H}BI}{t}$$

- >The above expression gives the voltage developed due to Hall Effect.
- This voltage is called the hall effect emf and used to either measure the magnitude of current or magnetic field strength.
- The magnitude of Hall Effect emf is very small in conductors and hence very difficult to measure. However, its value is quite sufficient semiconductors and can easily measured by sensitive moving coil instruments

Piezo-electric sensor/transducer

- A transducer utilizing piezoelectric element to convert mechanical motion into electrical signal is called a piezoelectric transducer.
- In certain solid materials, an electric field appears across certain surface of the crystal, if the dimension of the crystal structure is deformed by the application of a mechanical force
- This transducer works on the principle of Piezoelectric Effect.
- It is an active transducer and is reversible
- The piezo electric effect can be made to response to mechanical deformation of the material in different modes, such as thickness expansion , transverse expansion, thickness shear and face shear as shown in the figure



Piezo-electric sensor....

- The potential is produced as these materials generate within them an electric charge when deformed.
- This effect is known as Piezo-electric effect and is reversible. i.e. if the varying potential is applied to the proper axis of crystal, it will change the dimension of the crystal there by deforming it
- There are 2 types of piezo-electric materials
 - a) Natural materials Quartz and ceramicsA & B
 - b) Synthetic materials Lithium sulphate, Ammonium dihydrogen phosphate
- The ceramics materials are polycrystalline in nature and they are basically made of Barium Titanate.

- In a piezoelectric transducer, a piezoelectric crystal is sandwiched between the two electrodes.
- When a mechanical deformation takes place, it generates charge and hence it acts as a capacitor.
- A voltage is developed across the electrodes of the transducer which can be measured and calibrated with the deforming force to directly measure the mechanical deforming force.
- The piezo-electric effect is direction sensitive. This means that, the polarity of charge will not be same for a tensile and compressive force.
- The polarity of voltage induced due to a tensile force will be opposite to the polarity of voltage produced due to a compressive force.

Piezo-electric Transducer....

- Figure shows a simple piezoelectric transducer.
- The charge generated is proportional to the force, i.e.

 $Q \propto F$ and charge

$$Q = d \times F$$
 coulomb

- Where d = charge sensitivity of the crystal which depends upon the material of the crystal
- As metal electrodes behave like a two plates of the capacitor and between them crystal behave like di-electric.
- Thus so formed capacitance is given by,

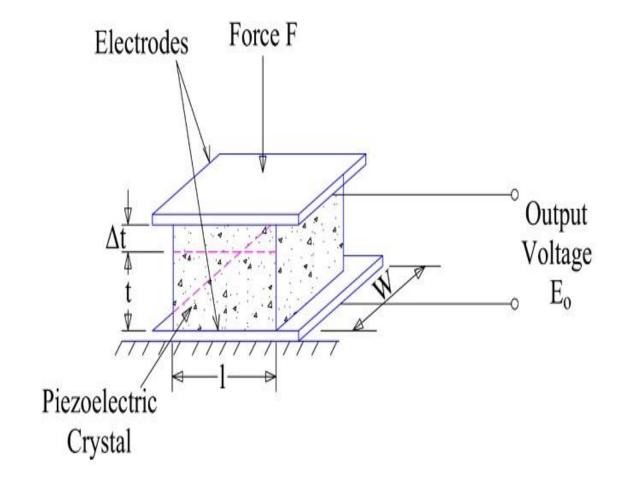
$$C_{P} = \frac{\varepsilon A}{t} = \frac{\varepsilon_{0} \varepsilon_{r} A}{t}$$
....(i)

• Where,

 ε = Permitivity of crystal

A = Area of crystal &

t =Thickness of the crystal



The force F causes a change in thickness of the crystal,

$$\therefore F = \frac{AE}{t} \times \Delta t$$

Where

A = Area of crystal

t = Thickness of crystal

E = Young's modulus

Young's modulus

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{A}}{\frac{\Delta t}{t}}$$

$$\therefore E = \frac{Ft}{A\Delta t}$$

And area $A = W \times L$

Where

W = width of the crystal L = length of the crystal As the charge developed in two electrodes, there will be a potential difference between the two plates and given by,

$$E_{o} = \frac{Q}{C_{P}} = \frac{dF}{\left[\frac{\varepsilon_{0}\varepsilon_{r}A}{t}\right]} = \left[\frac{d}{\varepsilon_{0}\varepsilon_{r}}\right]\frac{Ft}{A} = gPt$$

$$\therefore E_{o} = gPt$$

Where,

$$g = \left[\frac{d}{\epsilon_0 \epsilon_r}\right] = \text{voltage sensitivity}$$

$$P = mechanical pressure = \frac{F}{A}$$

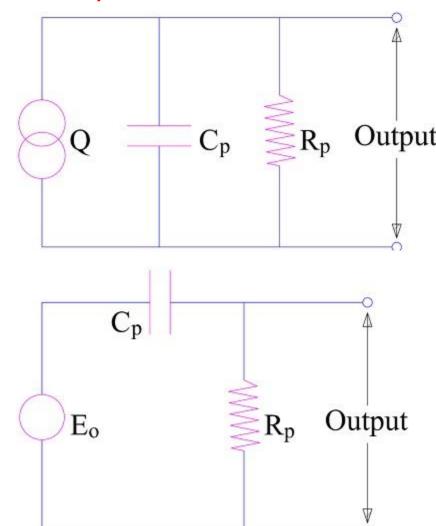
Let us define voltage sensitivity,

$$g = \frac{E_o}{Pt} = \frac{(\frac{E_o}{t})}{P} = \frac{\text{Electric field intensity}}{\text{Mechanical pressure}}$$

$$\therefore g = \frac{E_o}{Pt} = \text{volt meter/Newton}$$

Equivalent circuit of piezo-electric sensor/Transducer

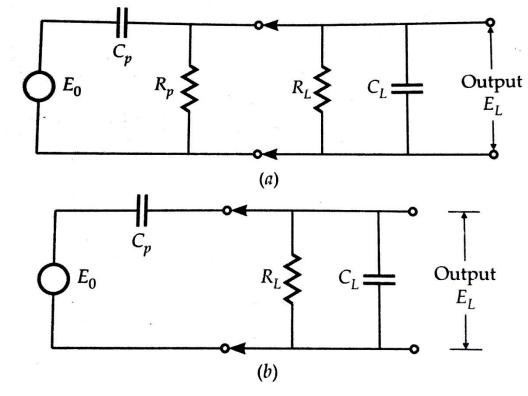
- As piezo-electric sensor is taken as the charge generator, the basic electrical equivalent circuit of a piezoelectric transducer is shown in figure aside.
- Following points may be noted from the equivalent circuit of piezoelectric transducer:
 - i. Source is a charge generator whose value is equal to $\mathbf{Q} = \mathbf{d} \times \mathbf{F}$
 - ii. The charge generated is across the capacitance C_P of the crystal and leakage resistance R_P .
 - iii. The charge generator can be replaced by an equivalent voltage source having a voltage of $E_o = \frac{Q}{C} = \frac{\mathrm{d} F}{C}$



Loading effect and frequency response

- When output terminal is open circuit(OC) then voltage appearing at the output terminal will be $E_{\rm 0}$ but in actual practice when a certain load is connected having capacitance $C_{\rm L}$ and resistance $R_{\rm L}$, then the voltage appearing at the output terminal is always less than $E_{\rm 0}$
- Let the transducer be loaded by a capacitance C_L and a load resistance R_L .
- The capacitance $C_{\rm L}$ is the combination of the capacitance of the load , the capacitance of the cable and the stray capacitance. The diagram showing the load connected to a piezo-electric transducer is given in figure (a)
- The value of leakage resistance R_P of the crystal is very large (in the order of 0.1 \times $10^{12}~\Omega)$

• The value of load resistance R_L is considerably smaller than R_P , and hence the equivalent circuit of the crystal under load conditions is shown in the figure (b) in which the leakage resistance R_P of the crystal has been dropped.



The voltage output of the transducer under no load conditions is therefore $E_{\rm o}$. Under conditions of load, impedance of load becomes,

$$Z_{L} = \frac{\left[\frac{1}{j\omega C_{L}}\right] \times R_{L}}{R_{L} + \frac{1}{j\omega C_{L}}} = \frac{R_{L}}{1 + j\omega R_{L}C_{L}}$$
$$Z_{P} = \frac{1}{j\omega C_{P}}$$

Total impedance of the circuit

$$Z_{t} = Z_{L} + Z_{P}$$

$$= \frac{R_{L}}{1+j\omega R_{L}C_{L}} + \frac{1}{j\omega C_{P}}$$

$$= \frac{1+j\omega R_{L}(C_{P}+C_{L})}{(j\omega C_{P})(1+j\omega R_{L}C_{L})}$$

Hence the voltage across the load,

$$E_{L} = \frac{Z_{L}}{Z_{t}} \times E_{O}$$

$$= \frac{\frac{R_{L}}{1+j\omega R_{L}C_{L}}}{\frac{1+j\omega R_{L}(C_{P}+C_{L})}{(j\omega C_{P})(1+j\omega R_{L}C_{L})}} \times E_{O}$$

$$= \left(\frac{R_{L}}{1 + j\omega R_{L}C_{L}}\right) \left(\frac{(j\omega C_{P})(1 + j\omega R_{L}C_{L})}{1 + j\omega R_{L}(C_{P} + C_{L})}\right) \times E_{O}$$

$$= \frac{j\omega R_{L}C_{P}}{1 + j\omega R_{L}(C_{P} + C_{L})} \times E_{O}$$

In magnitude,

$$E_{L} = \frac{\omega R_{L} C_{P}}{\sqrt[2]{1 + [\omega R_{L} (C_{P} + C_{L})]^{2}}} \times E_{O}$$

If the frequency is high, then

$$E_{L} = \frac{\omega R_{L} C_{P}}{\omega R_{L} (C_{P} + C_{L})} \times E_{O}$$
$$= \frac{C_{P}}{(C_{P} + C_{L})} \times E_{O}$$

And we know, $E_o = \frac{Q}{C_P}$ and $Q = d \times F$

$$E_{L} = \frac{\frac{Q}{E_{O}}}{(C_{P} + C_{L})} \times E_{O} = \frac{Q}{(C_{P} + C_{L})} = \frac{d \times F}{(C_{P} + C_{L})}$$

$$\therefore E_{L} = \frac{dF}{(C_{P} + C_{L})}$$

This is the expression for the load voltage when the piezo-electric crystal is used

Application / Usages of Piezo-electric Material and Transducer:

- The application or usages of piezoelectric material and transducer are listed below:
- ➤ Quartz is commonly used for stabilizing electronic oscillators due to its high stability.
- Piezoelectric Transducer is mainly used in dynamic measurements. The voltage developed by application of strain is not held under static condition. Therefore, these transducers are used in the measurement of quantities such as surface roughness, acceleration (called accelerometer) and vibrations.
- ➤ Ultrasonic generator also uses Barium titanate which is a piezoelectric material. Such materials are used in industrial cleansing apparatus and also in underwater detection system known as <u>sonar</u>

Measurement of Humidity

- Humidity is the measure of water vapour present in a gas.
- It is usually measured as absolute humidity, relative humidity or dew point temperature.
- Absolute humidity is the mass of water vapour present per unit volume.
- The relative humidity is the ratio of actual water vapour pressure to the maximum water vapor pressure required for saturation at a given temperature.
- The ratio or relative humidity is expressed in percentile scale. The relative humidity always depends on the temperature
- To measure the humidity, a class of transducer or sensor device used named as the hygrometer.
- The physical properties of the material changes by the effect of the humidity and this principle use in hygrometer for measurement.
- Generally, the output of a hygrometer is used to indicate the relative humidity.
- There are following types of sensor devices to measure the relative humidity
 - a) Resistive hygrometer
 - b) Capacitive hygrometer
 - c) Aluminum oxide hygrometer
 - d) Micro-wave hygrometer
 - e) Crystal hygrometer

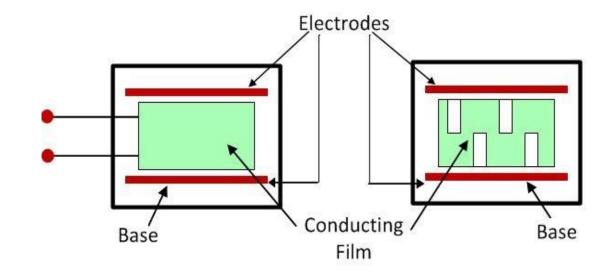
1. Resistive hygrometer

- Some hygroscopic salts exhibit a change in resistivity with humidity. The most common is lithium chloride.
- This, with a binder, may be coated on a wire or on electrodes.
- Resulting resistance changes cover a wide range e.g. 10^4 to $10^9\Omega$ as the humidity changes from 100 to 0 %.
- This makes it impractical to design a single element to operate from 1 to 100 % scale of relative humidity.
- Instead of using single, several elements are used, each in narrow range, with provision for switching elements.
- Resistance is measured either with a Wheatstone bridge or by a combination of current and voltage measurements.
- Most of the case, 100% humidity damages the device, so it is operated at a constant temperature environment

Resistive hygrometer....

- The construction of the resistive
 hygrometer is shown in figure
- These are constructed using the lithium chloride and carbon
- The resistance of the element changes when it is exposed to variations in humidity.
- The higher the relative humidity, the more moisture the lithium chloride will absorb, and the lower will be its resistance.
- The resistance of the sensing unit is a measure of the relative humidity
- The obstructions occur in the flows of current shows the value of resistance or the value of relative humidity.

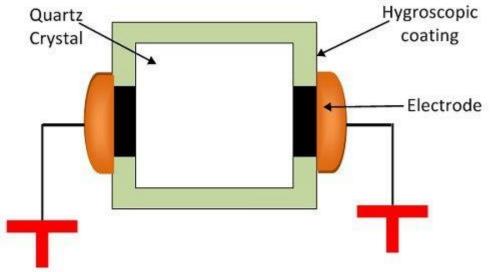
- Resistance should be measured by applying ac to the Wheatstone bridge, dc Voltage is not applied because it tends to break down the lithium chloride to it's lithium and chlorine atoms.
- The current flow is a measure of the resistance and hence of the humidity.



2. Crystal hygrometer

- When the crystal absorbs the drops of the water, the mass of the crystal changes.
- The change in mass is proportional to the total water absorbed by the crystals.
- Some crystals are hygroscopic, and others may be coated with a hygroscopic material.
- The coating materials are hygroscopic polymers.
- The crystals are used as frequency determination elements in electronic oscillators.

- Frequency shifts with humidity as the mass of the crystal changes with amount of water absorbed by coating are measured electronically.
- These transducers are useful if a telemetry system is needed because the frequency range can be chosen as a standard telemetry frequency



3. Capacitive Hygrometer

- The change in capacitance of the capacitor shows the surrounding humidity.
- The capacitive hygrometer gives the very accurate result. It is made by placing the hygroscopic material between the metal electrodes.
- The hygroscopic material can quickly absorb the water.
- The material absorbs water because of which the capacitance of the capacitor decreases.
- The electronic circuit measures the change in capacitance.

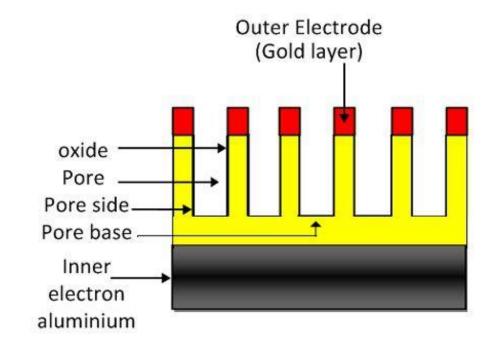
4. Microwave Refractometer

- The microwave refractometer measures the refractive index of the moist air when their humidity is change.
- The refractive index means the ratio of the velocity of air in one medium to that of another medium.
- The refractive index of humid air is measured either by measuring the dielectric constant using the capacitor or by measuring the change in frequency.

5. Aluminum Oxide Hygrometer

- In this hygrometer, the aluminum oxide is coated with the anodized aluminum.
- The dielectric constant and the resistance of the aluminum changes by the effects of the humidity.
- The aluminum oxide hygrometer uses the aluminum as their one electrode and the gold layer as the second electrode.
- The second electrode is porous for absorbing the air vapor mixture.
- The changes occur in the capacitance and resistance of the material because of the humidity.
- The change in properties changes the impedances of the material.

- The impedance measures with the help of the bridge.
- This hygrometer is the essential component of the electronic system.

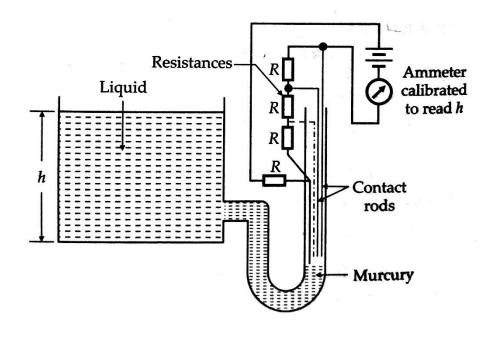


Measurement of Liquid level

- The measurement of liquid level is widely carried out for monitoring as well as measuring quantitatively the liquid content in tanks, vessels, reservoirs, or the height of the liquid column in open-channel streams and various other similar cases in industrial processes.
- Following are some common methods:

1. Resistive method:

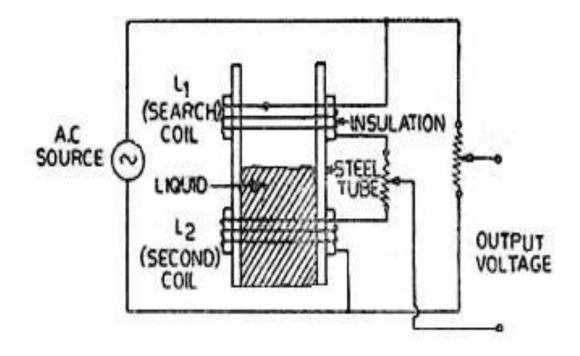
- > It is also known as contact point type.
- A number of resistances of suitable values are placed at various levels of liquid as shown in figure.
- In this method mercury column is operated by liquid column.
- As the level of liquid rises, the mercury level also rises and shorts the successive resistances and so the equivalent resistance decreases or current through indicator increases.



2. Inductive method:

- Based on variable permeability method.
- Consists of two coils $(L_1\&L_2)$ wound on steel tube consisting conducting liquid.
- Coils are in series aided through resistor
- Arrangement is excited by ac system
- The inductance of each coil is initially equal. One of the coils acts as the search coil and is set at a predetermined level.
- The inductance of the search coil changes rapidly as the conducting liquid moves into the plane of the coil.
- The method works well because the tape material is weakly magnetic and the liquid metal is a conductor which allows eddy currents to flow in it.

- The relationship between the output voltage and the liquid level is essentially non-linear.
- When level off liquid changes permeability changes and hence inductance and hence output voltages



3. Capacitive methods:

• Change in capacitance with change in area of plates, distance between plates and the dielectric constant has already been explained.

$$C = \frac{\varepsilon A}{d}$$

Where,

A = Overlapping area of the plates

d = separation distance between the plates

 $(\varepsilon \& \varepsilon_r) = Permittivity$

C = Capacitance in pico-farad

 All these principles are employed in detecting the level of liquids and solids in a container

a. Variable area method

- The variable area capacitive transducer is used for measurement of levels of both solids and liquids.
- The electrical conducting container containing the materials is used as one connection point of the transducer.
- The other point is a metal rod completely covered by insulating material inside the container.

• The insulating material acts as the dielectric medium and the capacitance varies linearly with the

height of the material.

• The relationship is given by

$$C = \frac{2\pi\varepsilon h}{\ln[\frac{D_2}{D_1}]}$$

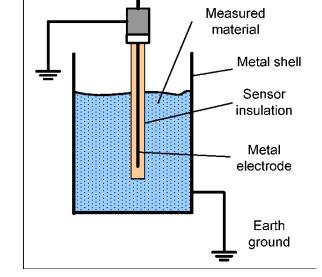
Where,

 $\varepsilon = \text{Permittivity}; F/m$,

h = Height of material; m,

 d_1 = Diameter of the metal rod; m, and

 d_2 = External diameter of the insulator; m.



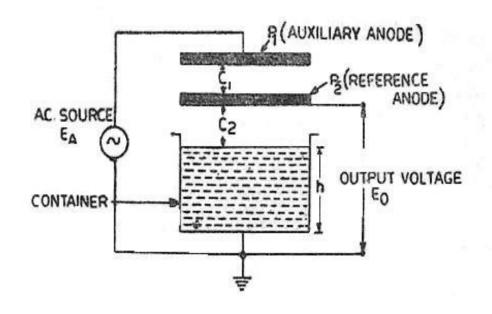
 The container should be earthed to avoid any danger of electric shock to the personnel and to prevent any errors due to external metallic objects

b. Capacitive voltage divider method

- In case, the conductivity of a liquid is high, its surface can be used as one electrode of the capacitor.
- The other electrode is a fixed reference plate parallel to the surface of the liquid.
- A system incorporating these features is shown.
- This uses an auxiliary electrode P_1 placed at a fixed distance above the reference electrode P_2 .
- The two electrodes P_1 and P_2 are electrically insulated from each other.
- An AC voltage is applied between the liquid and the electrode P_1 .
- ullet Potential of electrode P_2 with respect to earth

$$E_{O} = \frac{C_{1}}{C_{1} + C_{2}} \times E_{1}$$

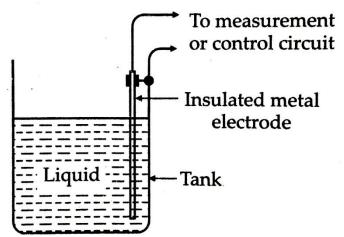
- Capacitance C_2 is inversely proportional to the distance between the liquid surface and P_2 .
- Thus the output voltage decreases with rise of liquid level and therefore the relationship between them is non-linear



c. Variable dielectric constant method

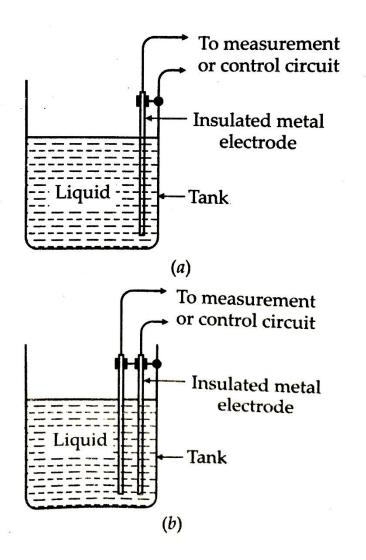
- If the liquid is non-conducting it can be used as a dielectric in a capacitor.
- The arrangement for measurement of liquid level for non-conducting liquids is illustrated.
- An insulated metal electrode firmly
 fixed near and parallel to the metal wall of the tank.
- If the liquid is non-conductive, the electrode and the tank wall form the plates of a parallel plate capacitor with the liquid in between them acting as the dielectric.

- If the liquid is conductive the rod and the liquid form the plates of the capacitor, and the insulation between them is the dielectric.
- Where the tank is not of metal, two parallel insulated rods or electrodes, kept at a fixed distance apart are used as shown in fig (b)
- The two rods act as two plates of a parallel plate capacitor as shown in fig (b)



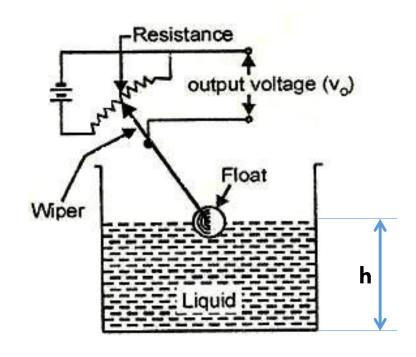
Variable dielectric constant method....

- The capacitance of this capacitor depends, among other factors, upon the height of the dielectric between the plates.
- The higher the liquid level, the greater is the capacitance.
- The lesser the height, the smaller is the capacitance.
- Thus, the capacitance is proportional to the height of the liquid in the tank.
- The capacitance in the above cases may be measured and this measured capacitance is an indication of liquid levels.



d. Measurement of liquid level using Float

- A float operated voltage potential divider is shown in figure
- As one liquid level rises in the tank, the float, which is generally a hollow ball, is raised.
- Its arm causes the wiper to move over the potential divider whose output terminals are connected to a voltmeter.
- As a float rises, a greater part of the potential divider is included in the output circuit giving an increased output voltage.
- Therefore, the output voltage is proportional to the liquid level 'h'.
- The output terminals from the potential divider may also be taken to a remote for display and control.



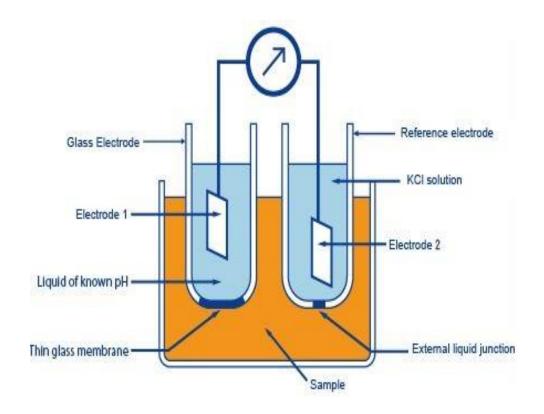
Measurement of pH value

- In the glass-electrode method, the known pH of a reference solution is determined by using two electrodes, a glass electrode and a reference electrode, and measuring the voltage (difference in potential) generated between the two electrodes.
- The difference in pH between solutions inside and outside the thin glass membrane creates electromotive force in proportion to this difference in pH.
- This thin membrane is called the electrode membrane.
- Normally, when the temperature of the solution is 30°C, if the pH inside is different from that of outside by 1, it will create approximately 60 mV of electromotive force.
- The liquid inside the glass electrode usually has a pH of 7.
- Thus, if one measures the electromotive force generated at the electrode membrane, the pH of the sample can be found by calculation.

Measurement of pH value....

- A second electrode is necessary when measuring the electromotive force generated at the electrode membrane of a glass electrode.
- This other electrode, paired with the glass electrode, is called the reference electrode.
- The reference electrode must have extremely stable potential.
- Therefore, it is provided with a pinhole or a ceramic material at the liquid junction
- In other words, a glass electrode is devised to generate accurate electromotive force due to the difference in pH.

 And a reference electrode is devised not to cause electromotive force due to a difference in pH



Measurement of liquid flow

1. Turbine flow meters

- Turbine flow meters are volumetric flow meters and are available in wide ranges.
- The output is usually in the form of a digital electrical signal whose frequency is directly proportional to flow rate and whose total count is proportional to the total quantity, as each pulse represents a discrete volume.
- A feature of this turbine meter is a hydraulically supported turbine rotor.
- A permanent magnet sealed inside the rotor body is polarized at 90° to the axis of the rotation.
- As the rotor rotates so does the magnet and therefore rotating magnetic field is produced.
- This produces an a.c voltage pulse in the pickup coil located external to the meter housing.
- The frequency of this voltage is directly proportional to the rate of flow.

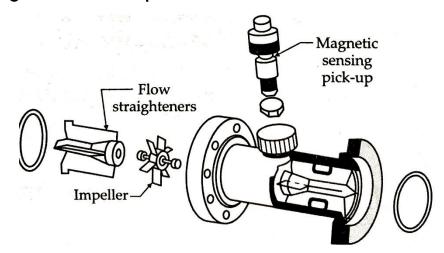
 The pulse can be totalized by a counter to give the value of total flow over a particular interval of time

Advantages:

- They are used for all types of liquids.
- There are no moving or rotationary parts to be replaced or maintained.

Disadvantages:

• They are clogged due to accumulation of solids on the turbine blades. This leads to degradation in performance



2. Electromagnetic flow meters

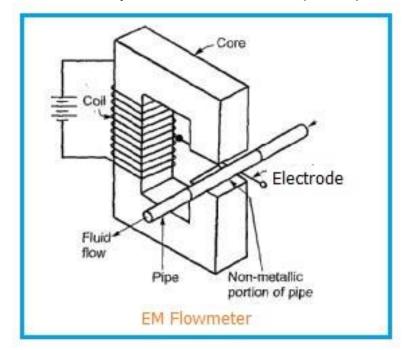
- The electromagnetic flow meters are particularly suitable for the flow measurements of slurries, sludge and electrically conducting liquid
- A schematic diagram of an electromagnetic flow meter is shown in the figure
- It consists basically of a pair of insulated electrodes buried flush in the opposite sides of a non-conducting, non-magnetic pipe carrying the liquid which flow is to be measured
- The pipe is surrounded by electromagnet which produces magnetic field.
- This arrangement is analogous to a conductor moving across a magnetic field
- Therefore, voltage is induced across the electrodes.

The voltage is given by,

E = Blv volts

Where, B = flux \times density; Wb/m²

l = length of conductor = diameter of pipe; m v = velocity of a conductor (flow); m/s



Electromagnetic flow meters....

- Thus assuming a constant magnetic field, the magnitude of the voltage appearing across the electrodes will be directly proportional to velocity
- Non conducting pipe has to be used as the output voltage gets short circuited if metallic pipes are used
- This is true when liquids of low conductivity are measured
- But when liquids of high conductivity are measured the short circuiting has no effects
- Stainless steel pipes can then be used
- The voltage produced are small specially at low flow rates
- Therefore the meter relies greatly on a high gain amplifier to convert the induced voltage in to a usable form

Advantages:

- Unaffected by the temperature, pressure, density, or viscosity of the liquid.
- The electromagnetic flow meters may be manufactured to measure flow in pipes of any size provided powerful magnetic field can be produced
- Able to detect liquids that include contaminants (solids, air bubbles)
- > There is no pressure loss.
- No moving parts (improves reliability)

Limitations:

- The operating costs are high particularly if heavy slurries are handled
- Can not detect gases and liquids without electrical conductivity.
- > A short section of straight pipe is required.

Electromagnetic flow meters....

If a fixed magnetic field strength (constant B) and an electrode spacing equal to the fixed diameter of the pipe (constant l=d), the only variable capable of influencing the magnitude of induced voltage is velocity (v).

The voltage is given by,

$$E = Blv$$
 volts

We may state the relationship between volumetric flow rate (Q) and motional EMF (E) more precisely by algebraic substitution

$$Q = A v$$
 and $v = \frac{Q}{A}$

If d is the diameter of pipe then, cross sectional area

$$A = \frac{\pi d^2}{4}$$

Substituting

$$v = \frac{Q}{A} = \frac{Q}{\frac{\pi d^2}{A}} = \frac{4Q}{\pi d^2}$$

So Emf produced $E = Blv = Bd \frac{4Q}{\pi d^2}$

$$\therefore E = \frac{4BQ}{\pi d}$$

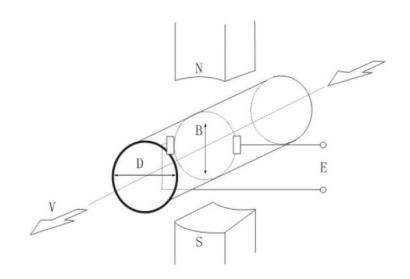
Where

Q = Volumetric flow rate (cubic meters per second)

E = Motional EMF (volts)

B = Magnetic flux density (Tesla)

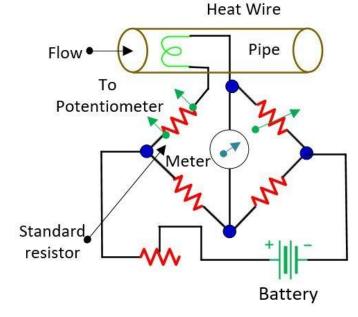
d = Diameter of flow tube (meters)



3. Hot wire Anemometers

- The Hot Wire Anemometer is a device used for measuring the velocity and direction of the fluid.
- This can be done by measuring the heat loss of the wire which is placed in the fluid stream. The wire is heated by electrical current.
- The hot wire when placed in the stream of the fluid, in that case, the heat is transferred from wire to fluid, and hence the temperature of wire reduces. The resistance of wire measures the flow rate of the fluid.
- The hot wire anemometer is used as a research tool in fluid mechanics. It works on the principle of transfer of heat from high temperature to low temperature.
- The hot wire anemometer consists two main parts.
 - a. Conducting wire
 - b. Wheat stone bridge.

In hot wire anemometer, the heat is supplied electrically to the wire which is placed in the fluid stream. The Wheatstone bridge is used for measuring the temperature of wire regarding their resistance. The temperature of the wire remains constant for measuring the heating current. Thus, the bridge remains balanced



- The standard resistor is connected in series with the heating wire. The current across the wire is determined by knowing the voltage drop across the resistor. And the value of voltage drop is determined by the potentiometer.
- The equation determines the heat loss from the heated wire

$$= a(v\rho + b)^{\frac{1}{2}} J/s$$

Where,

v = velocity of heat flow, $\rho =$ the density of fluid a and b are constants

Their value (a and b) depends on the dimension and the physical properties of the fluid and wire.

Suppose I, is the current of the wire and the R is their resistance

In equilibrium condition,

Heat generated = Heat Lost

$$I^2R = a(\nu\rho + b)^{\frac{1}{2}}$$

$$v = \frac{\left[\frac{I^4 R^2}{a^2} - b\right]}{\rho}$$

Thus the resistance and temperature of the instrument are kept constant for measuring the rate of the fluid by measuring the current I

Measurement of Temperature:

1. Thermoelectric effect sensors (Thermocouples)

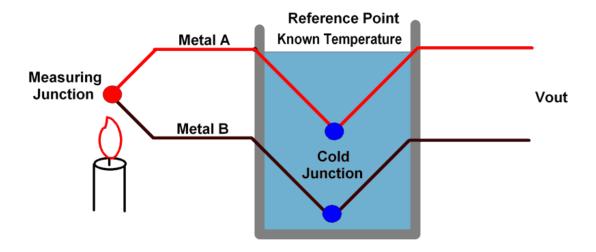
- Thermocouples consist of two dissimilar metals (wires), metal A and metal B. These metals are joined at an end called as measuring junction, while the other end is called as reference point as shown in above figure.
- Note that measuring Junction point is used to measure the temperature. The reference point in figure is a known temperature.
- As per Seebeck effect, thermoelectric voltage is generated which is proportional to the temperature difference between two junctions. This voltage can be measured at reference point.
- The Seebeck effect states that when two different or unlike metals are joined together at two junctions, an electromotive force (EMF) is generated at the two junctions

 The emf produced is the function of the difference in temperature of hot junction and cold junctions and is given by,

$$E = a\Delta\theta + b(\Delta\theta)^2$$

Where

- $\Delta\theta$ =temperature difference between two junctions
- a,b = constants and depends upon the type of materials



2. Bimetallic thermometer

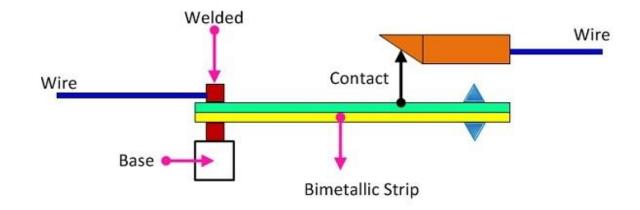
Defination

- The bimetallic thermometer uses the bimetallic strip which converts the temperature into the mechanical displacement.
- The working of the bimetallic strip depends on the thermal expansion property of the metal.
- The thermal expansion is the tendency of metal in which the volume of metal changes with the variation in temperature.
- Every metal has a different temperature coefficient. The temperature coefficient shows the relation between the change in the physical dimension of metal and the temperature that causes it.
- The expansion or contraction of metal depends on the temperature coefficient, i.e., at the same temperature the metals have different changes in the physical dimension.

Working Principle of Bimetallic Thermometer

The working principle of bimetallic thermometer depends on the two fundamental properties of the metal.

- a) The metal has the property of thermal expansion, i.e., the metal expand and contract concerning the temperature.
- b) The temperature coefficient of all the metal is not same. The expansion or contraction of metals is different at the same temperature



Construction

- The bimetallic strip is constructed by bonding together the two thin strips of different metals.
- The metals are joined together at one end with the help of the welding.
- The bonding is kept in such a way that there is no relative motion between the two metals.
- The physical dimension of the metals varies with the variation in temperature.
- Since the bimetallic strip of the thermometer is constructed with different metals.
- Thereby, the length of metals changes at different rates.
- When the temperature increases, the strip bends towards the metal which has a low-temperature coefficient.
- And when the temperature decreases, the strip bends towards the metal which has a high-temperature coefficient.

Construction

- The figure below shows the bimetallic strip in the form of the straight cantilever beam. The strip fixed at one end and deflects at the other end.
- The range of deflection of bimetallic strip depends on the type of metals used for construction.
- The deflection of the metal is directly proportional to the length of the strip and the variation of temperature and is inversely proportional to the thickness of the strips.

Fixed End

Advantages

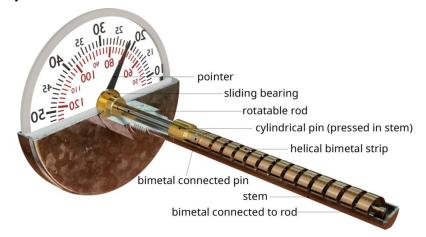
• The thermometer is simple in construction, robust and less expensive.

Disadvantages

• The thermometer gives the less accurate result while measuring the low temperature.

Applications of Bimetallic Thermometer

 The bimetallic thermometer is used in household devices likes oven, air conditioner, and in industrial apparatus like refineries, hot wires, heater, tempering tanks etc. for measuring the temperature.



Deflection

Bimetallic

Strip



End of chapter 3