

ELECTRICAL MACHINES(EE 554)

Chapter-1 (Magnetic Circuits and Induction)

General

- -Electric machines such as generators and transformers are major components in a power system.
- -Electric motors are most widely used in different applications such as manufacturing industries, agriculture, electric drives, domestic electrical equipments etc.
- Types of electric generators:

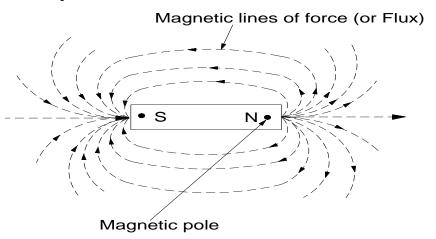
 - i) DC generator ii) AC generator
- -Types of Electric Motors:
- i) DC Motor
 - ii) AC motor
- **Transformers** operates only on AC system
- Operating principle of all the electric machines are based on theories of electro-magnetism

Some Basic Terminologies

- ☐ Magnet: A substance, which has a property of attracting small pieces of iron towards it. There are two types of magnets. They are:
 - i) Natural Magnet: Load Stone (a permanent magnet consisting of magnetite that possess polarity and has the power to attract as well as to be attracted magnetically).
 - ii) Artificial Magnet: Electromagnets (coils wound around an iron core)

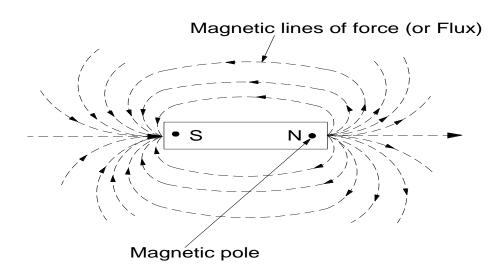
☐ Magnetic field:

- Magnetic field is space around a magnet within which the iron particles gets attracted toward the magnet.
- The attracting effect is most pronounced at certain points of the magnet known as its poles.
- Fig. shows a magnetic field around a bar magnet which actually three dimensional



☐ Magnetic Lines of Force :

- These are imaginary lines along which the small iron particles gets attracted toward the pole. Stronger magnetic field is represented by dense number of lines meeting toward a pole.
 - The magnitude of lines of force emitted by a pole is measured in Weber (wb).
 - It is also known as Magnetic flux emitted by a pole.
 - Magnetic pole strength is also measured in Weber.



Opposite pair of magnetic poles attracts each other, whereas like pair of magnetic poles repels each other as shown in Figure below.

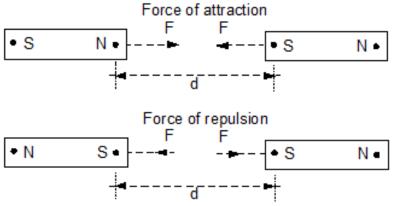


Fig Force of attraction and repulsion between magnetic poles

The force of attraction or repulsion between two magnetic poles is given by:

$$F = \frac{1}{4\pi\mu_o\mu_r} \; \frac{m_1\times m_2}{d^2} \; \; \text{(Newton)}$$
 Where,
$$m_1 = \text{Magnetic pole strength of the first pole (wb)}$$

$$m_2 = \text{Magnetic pole strength of the second pole (wb)}$$

$$d = \text{Distance between two magnetic poles (m)}$$

$$\mu_o = \text{Permeability of free space} = 4\pi \times 10^{-7} \, \text{(H/m)}$$

$$\mu_r = \text{Relative permeability of the medium on which the two poles are placed.}$$

☐ Magnetic field intensity:

Magnetic field intensity at any point in a magnetic field is defined as the force experienced by a unit north-pole placed at that point. Here, the unit north-pole can be assumed as the reference test pole to examine magnetic field intensity at any point.

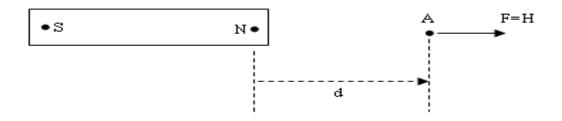


Fig. Magnetic field intensity at any point in a magnetic field

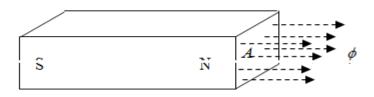
Magnetic field intensity at point 'A' is given by:

$$H_A = \frac{1}{4\pi\mu_o\mu_r} \times \frac{m \times 1}{d^2} \text{ (Newton)}$$

$$H_A = \frac{m}{4\pi\mu_o\mu_r d^2} \tag{1.2}$$

☐ Magnetic Flux Density:

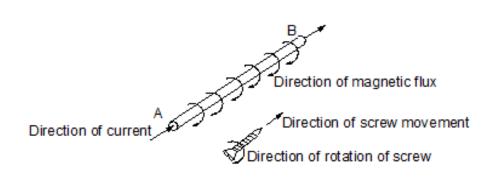
It is defined as magnetic flux per unit area. It is denoted by **B**.



Mathematically:
$$\mathbf{B} = \frac{\phi}{A}$$
 (1.3)

Electromagnet

A current carrying conductor produces magnetic field around it. This magnetic field can be used to magnetise a piece of iron to make an electromagnet. The direction of magnetic flux so produced can be determined by right hand thumb rule.



Conductive wire Iron Core Magnetic Flux Right hand thumb rule

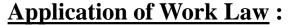
Fig. Electromagnet

□ Work law

The net work in Joules done by a unit N pole in moving around any single closed path in a magnetic field is equal to the 'Amp-turns' linked with the closed path.

Work done in moving a unit N-pole around the circle = Force \times distance = $H_a \times 2\pi r$

Therefore, according to work law,
$$H_a \times 2\pi r = NI$$
 OR $H_a = \frac{N.I}{2.\pi.r}$ (1.4)



Calculation of Magnetic Field Inside the core of Electro-Magnet:

Let L = Length of the core

I = Coil current

H = magnetic field intensity inside the coil.

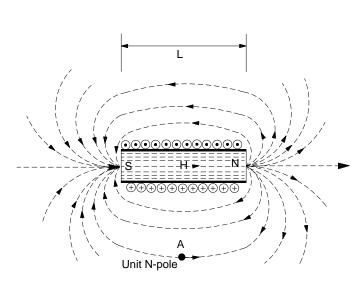
Assumption: 'H' inside the core remains constant throughout the length and value of 'H' outside the core is negligible with compare to that inside the core.

If the unit N-pole is taken around the closed path as shown in the figure in the direction opposite to that of 'H', then the work done against the magnetizing force 'H' is given by:

Work done =
$$H \times L$$

Therefore, according to work law, $H \times L = NI$

$$OR H_a = \frac{N.I}{I} \qquad (1.4)$$



☐ Magnetic hysteresis

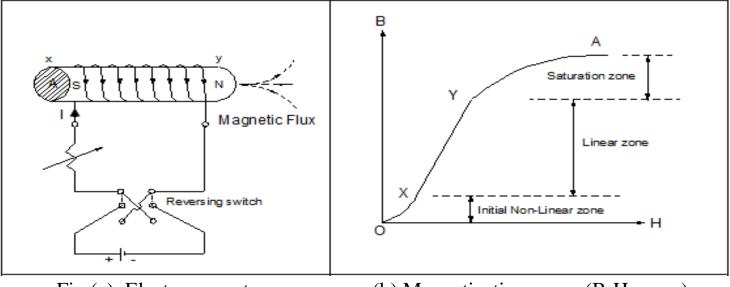


Fig.(a) Electromagnet

(b) Magnetization curve (B-H curve)

An unmagnetized iron bar is wound by a conductive wire and when dc current is passed it gets magnetized.

Unmagnetized magnetic material consist of very tiny magnets called *molecular magnets*, oriented randomly. So, Net magnetic field is zero.

When the material is magnetized by some means, the molecular magnets start to rearrange in particular direction and magnet is formed.

Molecular theory of magnetism:

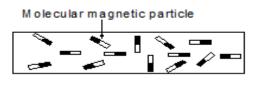


Fig. (a) Un-magnetized

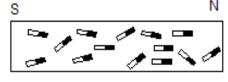


Fig.(b) Partially magnetized

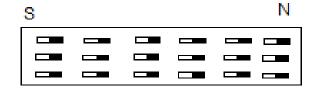
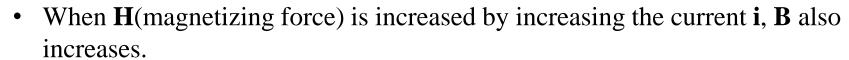


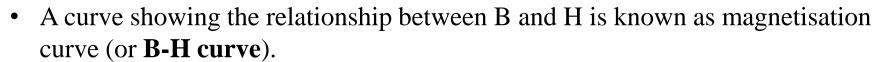
Fig.(c) Fully magnetized (Saturated)

☐ Magnetic hysteresis

Magnetic field intensity inside the core, $\mathbf{H} = \frac{Ni}{l} \propto i$

Magnetic flux density in the core, $\mathbf{B} = \frac{\phi}{A} = \frac{Ni}{Rel X A} \propto i$





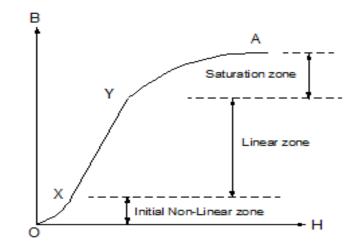
- In the beginning, the relation between B and H is not linear.
- From X to Y in the magnetisation curve, the relation is linear and is given by

$$\mu = \frac{B}{H}$$
 where μ is permeability of iron core (The property of the material to allow the magnetic flux to pass through it)

• Permeability of a material also can be expressed in terms of permeability of the free space as

$$\mu_r = \frac{\mu}{\mu_0}$$
 Where, μ = Permeability of any medium μ_0 = Permeability of air or vacuum or, $\mu = \mu_0 \mu_r$

• After linear region, B doesn't increase in proportion to the H as most of the molecular magnets has been re-oriented in a particular direction which is also called as **magnetic saturation**.



☐ Magnetic hysteresis

After magnetizing core up to saturation zone, if magnetising force (H) is reduced by decreasing the current(i), the curve follows the path A-C such that OC = Amount of magnetic flux density (B) remained in the core after the current is reduced to zero which is called **Residual Flux** -H **Density** or **retentivity** of the core.

To demagnetize the core to zero flux density, we have to apply the magnetising force (H) in the opposite direction. OD = Magnitude of magnetizing force required to demagnetize the core and called **Coercivity**

If we magnetise the core in reverse direction the process is similar and the curve follows **D-E-F-G-A**. This loop formed is called *Hysteresis Loop* and the energy wasted this process in demagnetizing the residual flux density called **Hysteresis Loss**.

Retaintivity

-H

Coercivity

-B

+B .

Fig. Hysteresis loop of an electromagnet

Area under hysteresis loop is proportional to the energy loss per cycle per unit volume.

Mathematical Proof:

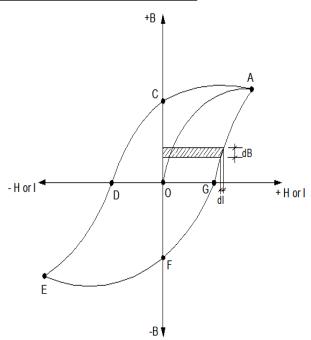


Fig. Hysteresis loop of a magnetic material

The magnetic flux in the core at any instant is given by:

$$\varphi = B.A$$

The magnetizing force at any instant is given by:

$$H = \frac{N.I}{l}$$
 OR $I = \frac{H.l}{N}$

The instantaneous value of emf induced in the coil due to time varying magnetic flux is given by:

$$e = N \frac{d\varphi}{dt} = N \frac{d}{dt}(B.A) = NA \frac{d}{dt}(B)$$

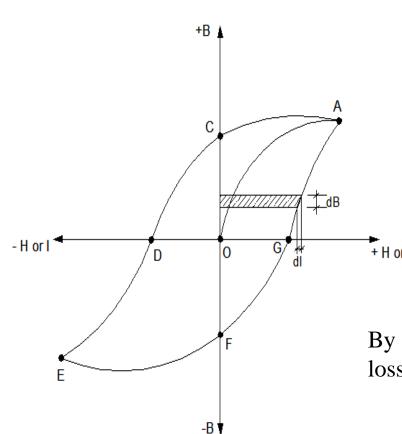
The source has to expend some energy to push the current against this emf.

The instantaneous power or rate of expenditure of energy in maintaining the current "I" against the emf induced is given by:

$$p = \text{e.I} = \text{NA} \frac{d}{dt}(B) \times \frac{Hl}{N} = AlH \frac{d}{dt}(B)$$

Then energy spend in this small time **dt** is given by:

$$dW = p. dt = AlH \frac{d}{dt}(B) \times dt = AlH. dB$$



Hence, the total energy spent in a cycle of magnetization is given by:

$$W = Al \oint H. dB$$

Here, H.dB = Area of shaded part

Therefore, $\oint \mathbf{H} \cdot d\mathbf{B} = \mathbf{Area}$ of whole loop

Therefore,
$$\oint H \cdot dB = \frac{W}{A \cdot l} = \frac{\text{total energy spent in a cycle of magnetization}}{\text{Volume of iron core}}$$

If f is the no. of cycle of magnetism and demagnetism made per second then Hysteresis loss/m³ = area of one hysteresis loop * f (J/S or watt)

By series of experiment, Steinmetz's found that for a sinusoidal flux, hysteresis loss in the magnetic material **per unit volume** is expressed as:

$$\mathbf{W}_{h} / \mathbf{m}^{3} = \eta \mathbf{B}_{m}^{1.6} f$$
or, $\mathbf{W}_{h} = \eta \mathbf{B}_{m}^{1.6} f V$

Where, $W_h = Hysteresis loss in Watts$

 B_m = Peak value of magnetic flux density in the core (Wb/m²)

f =Frequency of the applied ac voltage (Hz)

 $V = \text{Volume of iron core } (\text{m}^3)$

 η = Steinmetz constant, whose value depends upon the grade of iron core.

Different grade of iron core will have different shape of hysteresis loop as shown in Fig.

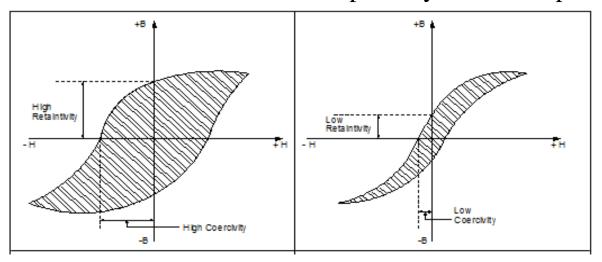


Fig. Hysteresis loop of hard magnetic material and soft magnetic material

- Materials having larger hysteresis loop area are known as hard magnetic materials.
 - They have high retentivity, coercivity. So, suitable to make permanent magnets.
 - It has high energy loss therefore not suitable for making core of electric machine.
 - Examples : Alloys of Aluminum, nickel and steel.
- Materials having smaller hysteresis loop area are known as soft magnetic materials.
 - They have low retentivity, coercivity and low energy loss.
 - Therefore suitable for making core of electric machine.
 - Examples: Silicon Steel

Eddy Current Loss:

If a.c. is passed through the winding, the iron core will get magnetized and magnetic flux will flow through the iron core. Since magnitude of a.c. is changing w.r.t. time, the magnitude of flux in the core will change w.r.t. time. Therefore, according to the faraday's law of electromagnetic induction, some emf will induced inside the core and due to this emf, local current will circulate within the core, which is known as eddy current. The current will produce heat loss which is equal to

i_{eddy}^2 *resistance of the path followed

by eddy current

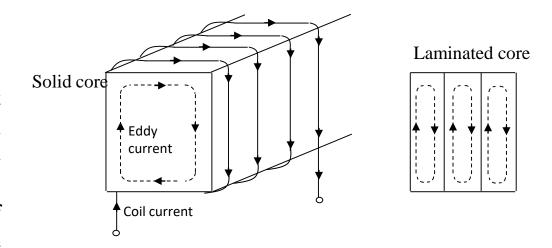


Fig. Eddy current in solid core

Fig. Eddy current in laminated core

- The power loss due to heat produced in the iron core due to eddy current is known as eddy current loss.
- Eddy current loss can be reduced by laminated core.
- After lamination, the cross sectional area of each lamination will be very small and it offers very high resistance to eddy current therefore the magnitude of eddy current in each lamination will be very small.

Eddy Current Loss:

Eddy current loss in the core is given by:

$$\mathbf{W}_{\mathrm{e}} = \mathbf{K} \, \mathbf{B}_{\mathrm{m}}^{2} f^{2} \, \mathbf{t}^{2} \, V \, \, (\text{Watts})$$

Where, $B_m = \text{Peak value of magnetic flux density in the core (Wb/m}^2)$

f =Frequency of the applied voltage (Hz)

t = Thickness of each lamination (in mm)

 $V = \text{Volume of iron core } (m^3)$

K = Constant depending upon the grade of iron core

Electromagnetic Induction:

In 1831, English scientist Michael Faraday discovered the relationship between magnetism and electricity. He observed the momentary induced current in a circuit, when the magnetic flux linking with the circuit changes momentary with respect to time. He had stated two laws known as Faraday's Law Of Electromagnetic Induction as follows:

i) **First law**: "Whenever the <u>magnetic flux-linkage</u> in a conductor changes with respect to time, an emf will induced on it"

Flux linkage = $N \phi$

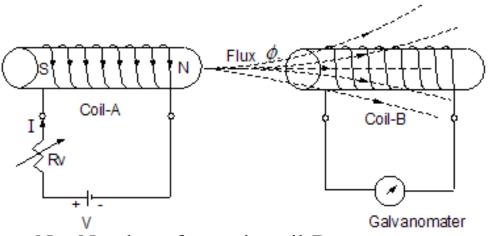
ii) **Second law**: "The magnitude of emf induced is equal to the <u>time rate of change of magnetic flux-Linkage</u>"

$E=Nd\phi/dt$

The magnetic flux-linkage could be changed in two different ways. Therefore, emf can be produced in two different ways:

- i) Statically induced emf
- ii) Dynamically induced emf

Statically Induced emf:



No Relative motion between conductor and magnetic flux

Let N = Number of turns in coil-B

 ϕ = Magnetic flux passing through the N turns of conductive coil

Then the product $N \times \phi = Magnetic Flux-Linkage in coil-B$

If the magnetic flux in the coil-B changes from ϕ_1 to ϕ_2 in a small time interval from t_1 to t_2 , then according to second law given by Faraday's law of electromagnetic induction, magnitude of emf induced in a turn of coil-B is given by:

e (per turn) =
$$\frac{\varphi_2 - \varphi_1}{t_2 - t_1} = \frac{d\varphi}{dt}$$

If there is 'N' number of turns in the coil-B, then total emf induced across the coil is given by:

$$e = N \frac{d\varphi}{dt}$$

Statically Induced emf:

The direction of statically induced emf and direction of current due to this emf can be determined by Lenz's law.

Lenz's law statement: "Direction of induced current in the conductor will be such that the magnetic field set up by the induced current opposes the cause by which the current was induced."

Example-1: Emf due to increasing current in coil-A

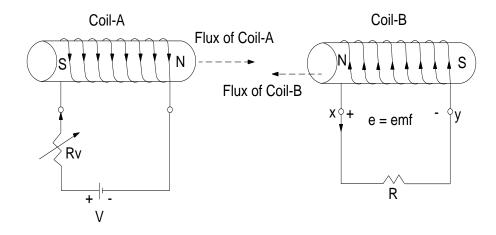


Fig. Illustration of Lenz's law (case-I)

Example-2: Emf due to decreasing current in coil-A

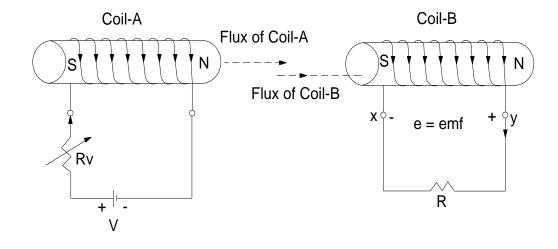
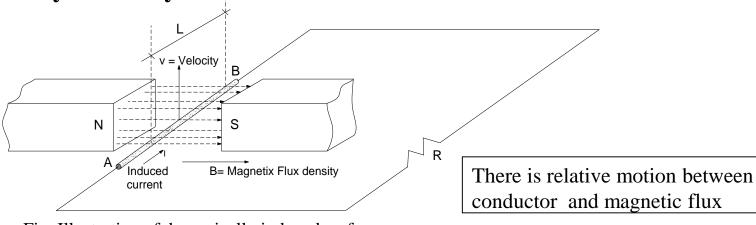


Fig. Illustration of Lenz's law (case-II)

Dynamically Induced emf:



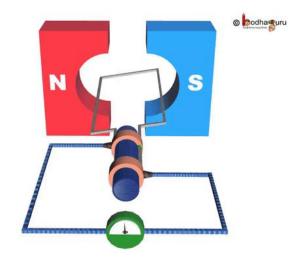


Fig. Illustration of dynamically induced emf

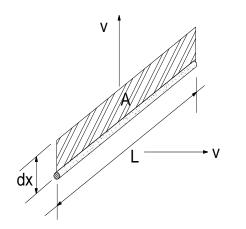


Fig. Detail of area swept by the conductor

The distance moved 'dx' by the conductor in a small time interval of 'dt' is given by:

$$dx = v \times dt$$

Area swept by the conductor in a small time interval of 'dt' is given by:

$$\mathbf{A} = L \times dx = L \times v \times dt$$

Total flux swept by the conductor in a small time interval of 'dt' is given by:

$$d\varphi = B \times A = B \times L \times v \times dt$$

Hence, emf induced in the conductor is given by:

$$e = \frac{d\varphi}{dt} = \frac{B \times L \times v \times dt}{dt} = BLv$$

or
$$e = BLv$$

Dynamically Induced emf:

The direction of dynamically induced emf and current in the conductor is determined by Fleming's Right Hand Rule. To use this rule, we shall use our right hand as shown in Figure below.

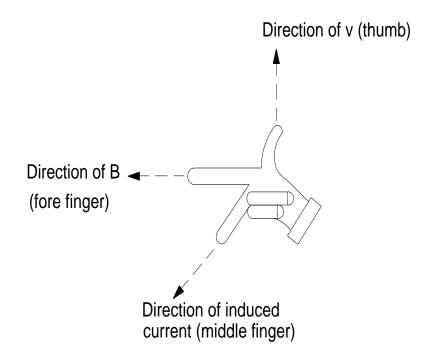
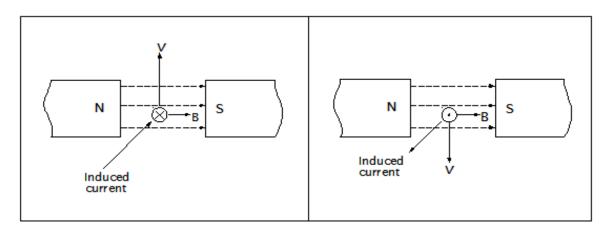


Fig. Illustration of Fleming's right hand rule

Dynamically Induced emf:



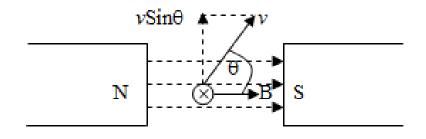


Fig. Dynamically induced emf with inclined direction of motion of conductor

(a) Conductor motion upward (b) Conductor motion downward Fig. Direction of dynamically induced current

If the direction of motion is inclined with the direction of magnetic flux density (B) as shown in Fig., then the magnitude of emf induced is given by:

$$e = B.L.v.sin\theta$$

Where, θ = Angle between direction of motion and direction flux density. $v. \sin \theta$ = Component of v in the direction perpendicular to direction of B

Force developed on current carrying conductor:

When electric current is passed through a conductor lying in the magnetic field, a force will develop on the conductor.

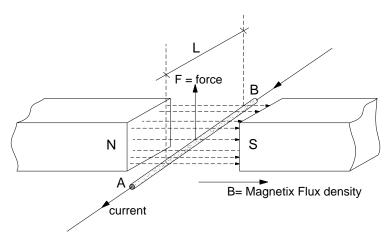


Fig. Force developed on current carrying conductor

The magnitude of the force so developed is given by:

F = B.I.L (Newton)

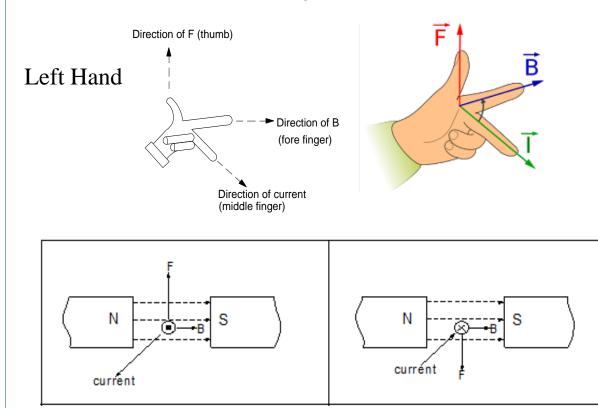
Where,

B = Magnetic flux density (wb/m²)

I = Current passing through the conductor (Amp)

L = Length of the conductor lying within the magnetic field (m)

The direction of force so developed is determined by Fleming's Left Hand Rule. To use this rule, we shall use our left hand as shown in Fig.



(a)

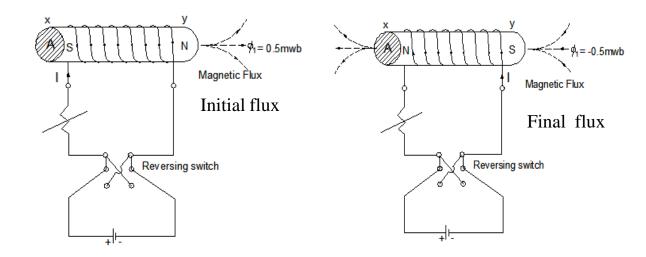
(b)

Numerical illustration:

Illustrative Example 1

A magnetic flux of 0.5 mWb links a coil of 400 turns. If the flux is reversed in 0.2 sec, determine the average value of the induced emf in the coil.

Solution:



Initial flux in the coil $\phi_1 = 0.5$ mWb = 0.5×10^{-3} wb Final flux in the coil $\phi_2 = -0.5$ mWb = -0.5×10^{-3} wb

Magnitude of change in flux = $d\phi = |\phi_2 - \phi_1| = |0.5 \times 10^{-3} - (-0.5 \times 10^{-3})| = 1 \times 10^{-3} \text{ wb}$

Average value of induced emf(e) = N $\frac{d\varphi}{dt} = \frac{1 \times 10^{-3}}{0.2} = 2V$

Magnetic circuit

The path followed by the magnetic flux is known as magnetic circuit. The path a-b-c-d-a shown in Fig. is a magnetic circuit consisting of iron core and winding.

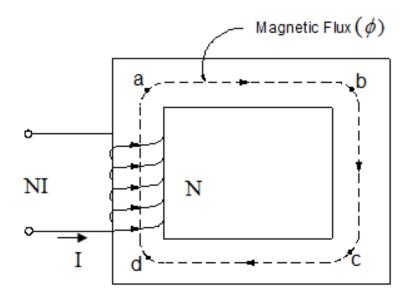
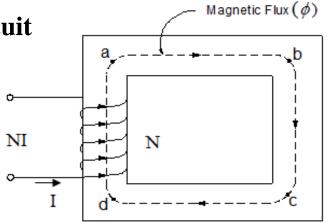


Fig. Magnetic circuit

Magnetic circuit



Let I = Current through the exciting winding (Amp)

N = Number of turns in exciting winding

 ϕ = Magnitude of magnetic flux (wb)

A = Cross-sectional area of the iron core (m^2)

L = Mean length of the magnetic flux path (a-b-c-d-a)

Magnetic flux density in the core is given by: $B = \frac{\phi}{A}$

And, magnetic field intensity in side the core is given by: $H = \frac{N.I}{I}$

For linear part of the magnetization curve: $\frac{B}{H} = \mu$

Or
$$B = \mu.H$$
 OR $\frac{\phi}{A} = \frac{\mu.N.I}{L}$ OR $\phi = \frac{N.I}{L/\mu.A} = \frac{mmf}{Reluctance}$ (ohm's Law)

Where, N.I = mmf = magnetomotive force, which push the magnetic flux in the magnetic circuit.

Rel =
$$\frac{L}{\mu . A}$$
 = Reluctance of magnetic circuit

Magnetic circuit

Table 1.1 Comparison between electric circuit and magnetic circuit

Electric Circuit	Magnetic Circuit	
emf (V)	mmf (NI)	
Electric current (I)	Magnetic flux (ϕ)	
Resistance (R)	Reluctance (Rel)	
$R = \rho \frac{L}{A}$	$Rel = \frac{L}{\mu . A}$	
Conductance = $\frac{1}{R} = \frac{A}{\rho . L}$	Permeance = $\frac{1}{\text{Re }l} = \frac{\mu A}{L}$	

Series Magnetic circuit

Series magnetic circuit is such circuit, where the same magnetic flux passes through the all sections of

the magnetic circuit as shown in Fig.

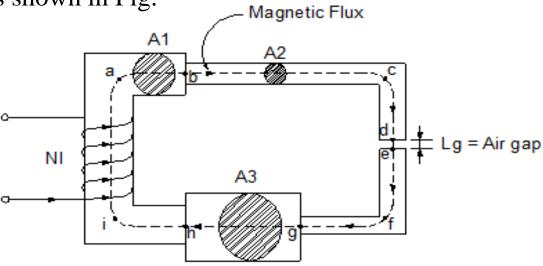


Fig.Series magnetic circuit

The magnetic circuit has four different sections with different length and cross-sectional area as follow:

Section-1: $L_1 = ba + ai + ih$,	Area = A_1 ,	Permeability = μ_1
Section-2: $L_2 = bc + cd + ef + fg$,	Area = A_2	Permeability = μ_2
Section-3 : $L_3 = gh$,	Area = A_3	Permeability = μ_3
Section-4 : $L_g = de$,	$Area = A_g = A_2$	Permeability = μ_0

Series Magnetic circuit

The Total reluctance of the magnetic circuit = sum of the reluctances of all the sections.

Therefore, Total Rel =
$$\sum \frac{L}{\mu.A} = \frac{L_1}{\mu._1 A_1} + \frac{L_2}{\mu._2 A_2} + \frac{L_3}{\mu._3 A_3} + \frac{L_g}{\mu._0 A_2}$$

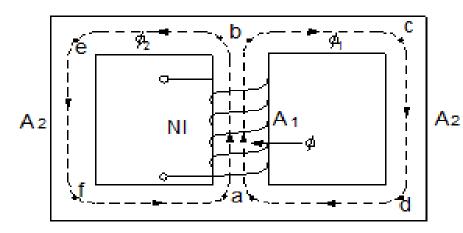
Hence, Magnetic Flux in circuit,
$$\varphi = \frac{N.I}{\sum L/\mu.A}$$

• Note that the air gap offers very high reluctance. It reduces the magnetic flux in the circuit. It is quite similar to addition of very high resistance in series with low resistance in case of series electric circuit.

Parallel Magnetic circuit

If the magnetic flux divides into two or more parallel paths, such magnetic circuit is known as parallel magnetic

circuit.



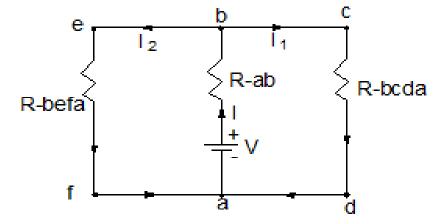


Fig.1.18(a) Parallel magnetic circuit

Fig.1.18(b) Corresponding electric circuit

For electric circuit,

$$I = I_1 + I_2$$
 If $R_{bcda} = R_{befa}$, Then $I_1 = I_2$ Or $I_1 = I/2$

Writing KVL for loop a-b-c-d:
$$V = I \times R_{ab} + I_1 \times R_{bcda}$$
 OR $V = I \times R_{ab} + 0.5I \times R_{bcda}$ OR $I = \frac{V}{(R_{ab} + 0.5R_{bcda})}$

Similarly, for magnetic circuit:

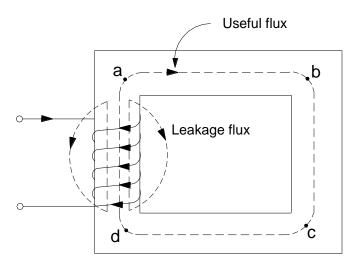
 $\phi = \phi_1 + \phi_2$ If Reluctance of path-1 = Reluctance of path-2, then $\phi_1 = \phi_2 = 0.5 \ \phi$

Therefore,
$$N.I = \phi \times Rel_{(ab)} + \phi_1 \times Rel_{(bcda)}$$

$$OR \quad N.I = \phi \times Rel_{(ab)} + 0.5 \phi \times Rel_{(bcda)}$$

$$\phi = \frac{N.I}{(Rel_{(ab)} + 0.5 Rel_{(bcda)})}$$

Leakage flux



- Leakage flux is not useful in electric machine.
- It's effect is undesirable in electric machine

Some Numerical illustrations:

Illustrative Example 2

A coil has 750 turns. When a current of 10 Amp is passed through the coil, it produces a magnetic flux of 1.2 mWb. Calculate the self inductance of the coil. If the current in the coil is reversed in 0.01 sec, calculate the emf induced in the coil.

Solution:

$$N = 750 \text{ turns}, I = 10 \text{ Amp}, \phi = 1.2 \text{ mWb} = 1.2 \times 10^{-3} \text{ Wb}$$

The self inductance is given by:
$$L = \frac{N.\phi}{I} = \frac{750 \times 1.2 \times 10^{-3}}{10} = 0.09H$$

Initial current in the coil $I_1 = 10 \text{ Amp}$

Final current in the coil $I_2 = -10$ Amp

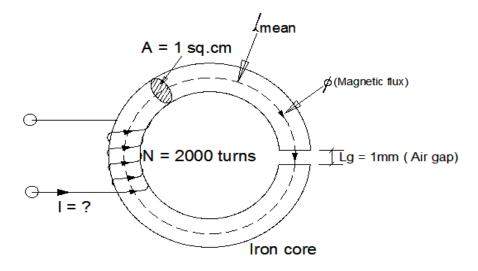
Magnitude of change in current in 0.01 sec dI = |-10 - 10| = 20 Amp

Emf induced
$$e = L \frac{dI}{dt} = 0.09 \times \frac{20}{0.01} = 180V$$

Illustrative Example 3

A magnetic core, in the form of a closed ring, has a mean length of 20 cm and cross-sectional area of 1 sq.cm. An air gap of 1 mm is cut on the ring. The core is wound with 2000 turns of winding. What DC current will be needed in the coil to create a flux density of 2 Wb/m². Given that the relative permeability of the core is 2400.

Solution:



Here, $l_{\rm mean}=20~{\rm cm}=0.2~{\rm m}$, Air gap length Lg = 2mm = 0.002 m Length of iron core $l=l_{\rm mean}-{\rm Lg}=0.2-0.002=0.198~{\rm m}$ Cross-sectional area of the iron core A = 1 sq.cm = 1×10⁻⁴ sq.m Number of turns in winding = 2000 turns Relative permeability of the core $\mu_{\rm r}=2400$ Required magnetic flux density in the core $B = 2 \text{ wb/ m}^2$

Therefore required magnetic flux in the core $\phi = B \times A = 2 \times 1 \times 10^{-4} \text{ wb}$

Reluctance of iron core : Rel(core) =
$$\frac{l}{\mu_0 \mu \, rA} = \frac{0.198}{4\pi \times 10^{-7} \times 2400 \times 1 \times 10^{-4}} = 6.5648 \times 10^5$$

Reluctance of air gap: Rel(airgap) =
$$\frac{Lg}{\mu_0 A} = \frac{0.002}{4\pi \times 10^{-7} \times 1 \times 10^{-4}} = 159.15 \times 10^{5}$$

Total Reluctance of the magnetic circuit:

$$Rel_{(total)} = 6.5648 \times 10^5 + 159.15 \times 10^5 = 165.684 \times 10^5$$

Magnetic flux in the core is given by: $\phi = \frac{N.I}{Rel_{\text{(total)}}}$

OR
$$I = \frac{\phi \times \text{Re l}_{\text{(total)}}}{N} = \frac{2 \times 10^{-4} \times 165.684 \times 10^{5}}{2000} = 1.656 \text{Amp}$$

References

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- [2] A. E. Fitzgerald, C. Kingsley and S. D. Umans, Electric Machinery, 6 ed., New Delhi: McGraw Hill, 2009.
- [3] D. P. Kothari and I. J. Nagarth, Electric Machines, 5 ed., Chennai: McGraw Hill, 2020.