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✿ 5kM six :-

⇒ Difference

* Fourier Series & Fourier Transform (10 marks)

Periodic function cahinxa

auto simple contour example :- $\int_{-1}^{1}$, $1 \to 8$

Non periodic

$\boxed{f(x) = 1 \quad -1 \leq x \leq 0}$

- Fourier Cosine Transform : e^{mx}, e^{-x}, $\boxed{f(x) = e^{-x}, x > 0}$

  " Sine "

  cosine ⇒ $\alpha / 1 + \alpha^2$

  sine ⇒ $1 / 1 + \alpha^2$

→ Parsival → square wala

⇓

cosine & sine wale

→ verify convolution theorem

* Z - transform (20 marks)

FT	ZT	LT
- LT ka particular case	- is the generalisation of FT because of FT doesnot converge of every sequence	- Discontinuous → contineous (ckt problem)
- Discrete sequence		
- Time domain - Freq. domain in complex		

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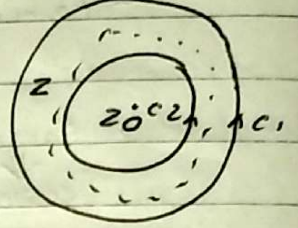
extension of Tayler's series

Laurent series :- By using Laurent series.

$$f(z) = \sum_{n=0}^{\infty} b_n (z - z_0)^{-n}$$

The Region of convergence (ROC) is obtained by inlarging the circle c_1 & contracting the circle c_2.

A pt. where function z ceases to be analytic is called singular point.



Distingues between singular pt. & pole :

If $\sum_{n=0}^{m}$ → pole

Anular region :

⇒ Remark → ROC doesn't include the pole.

$$X(n) = \sum_{n=0}^{\infty} x(n) z^{-n} \qquad [\because \text{Laurent's series}]$$

ROC of z-transform ⇒ provided that $\sum_{n=0}^{\infty} |x(n) z^{-n}| < \infty$

If $x(n) = a^k$.

$$|X(n)| = \sum_{n=0}^{\infty} |a^n z^{-n}| < \infty$$

$$= 1 + az^{-1} + (az^{-1})^2 + \dots$$

$$= \frac{1}{1 - az^{-1}}$$

Sign.

$$= \frac{z}{z-a}, \quad \left|\frac{a}{z}\right| < 1$$

$$\Rightarrow \left|\frac{z}{a}\right| > 1$$

$$\Rightarrow |z| > a \quad (ROC)$$

Initial value theorem } → $x(0)$, $x(1)$, $x(2)$ # Find z-transform
Final value theorem # Find inverse ⇒ Partial

Solve the difference eqn.

✡ Partial diff. eqn. (PRK six) → 20 marks

⇒ vibrating string

→ Theorem → derive wave eqn.
& solve completely

$\boxed{lx - x^2}$

→ Example → 10 marks - solve all 3 case

→ $\boxed{\text{F Series}}$ or $\boxed{\text{compare}}$ → Trig.
↓
algebric

→ Derive one dime. heat eqn. & solve completely

→ " two " " "

→ Laplace eqn → ~~pot~~

→ Polar form of Laplace eqn.

→ Problem :

solve the related problem.

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Complex analysis (30 marks)

Derivation
Cauchy's Reman eqn. in cartesial 4 [UVC]
(Polar)

Analytic fn. : u given v = ?
v given u = ? } (UVC)

* Find u or v under the given condition v = --
u - -

& $f(z) = u+iv$ is analytic

② Harmonic fxn
→ That satisfy laplacian eqn.
→ derivative gar, integration gar

③ Complex integration vvvvvI (sure)
~~4~~ $\boxed{\log z}$ ⇒ integration (not analytic at o)
e^z
$\sin z$ }

⇒ $\oint \log z \, dz$

$c : |z| = 1$
$\Downarrow$
$z = 1e^{i\theta}$

- - - - - ⇒ $2\pi i$

Sign.

Discrete → Theren & proof ? diff

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proof $\boxed{\boxed{\text{CIF}}}$ ~~CIT~~ → jaha $\boxed{0}$ x a } one statement & one proof

Residue → pole

Power series :·

$$\sum_{n=a}^{\infty} a_n (x-a)^n$$ → Infite series with center a converging to some pt.

⇓

Taylor's series

$$\sum_{n=0}^{\infty} a_n (z-a)^n$$

$$a_n = \frac{f^n(a)}{n!} = \frac{1}{2\pi i} \oint_c \frac{f(z^*)\, dz^*}{(z^*-a)^{n+1}}$$

$\boxed{\text{CRT}}$ → state & proof Taylor's series.
state Laurent's series → solve the problem

⇓

$\boxed{\text{No pole, no CRT}}$

Isolated pole → a pole is said to be if its neberhood doesnot contain any further pole in that region.

xxx don't care # $\frac{\sin z}{z}$ → exponential me change ger

Sign.

* $\boxed{\text{CRT}}$ → Evolute

Contour integration (OR) $\boxed{\begin{matrix}\sin\theta \\ \cos\theta\end{matrix}}$

$$\int_{-\infty}^{\infty} R(x)\,dx = 2\pi i \sum_{im\,P>0} Res\{f(z), P\}$$

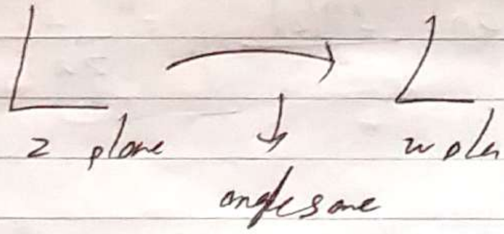
Conformal mapping - 5 marks $\boxed{\text{100\% score}}$

A mapping $w = f(z)$ is said to be conformal if it preserves the same mag. & same sense of direction.

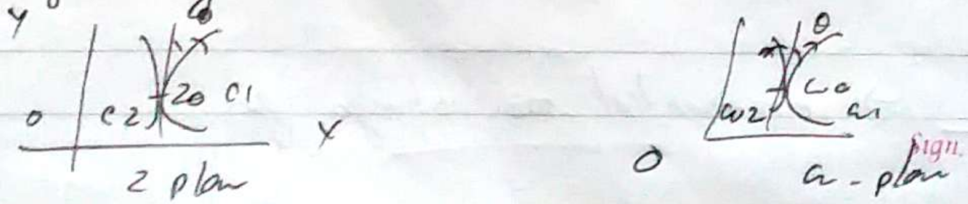
↓

Isogonal

⇓



If the curves c_1 & c_2 ~~ins~~ intersects at z_0 in the z-plane and the corresponding curves ω_1 & ω_2 intersects at ω_0 then $\omega_0 = f(z)$ is conformal only when ~~mag~~ it preserves same ~~mg~~ mag. & same sense of dir^n.



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$f(z) = e^z \rightarrow$ analytic function

$$f'(z) = e^z \neq 0$$

$\therefore$ $f(z) = e^z$ is also conformal

$f(z) = \sin z \rightarrow$ analytic function

$$f'(z) = \cos z = 0, \quad z = \pm \pi/2$$
$$= \pm 3\pi/2$$
$$= \vdots$$

Then, $w = f(z) = \sin z$ is not conformal at $z = \pm \pi/2, \pm 3\pi/2, \ldots$

☆ linear fnx.

$$\boxed{w = az + b}$$

$$\boxed{\boxed{w = az}}$$

Inversal fn :-

$$w = \frac{1}{z}$$

$$u + iv = \frac{x - iy}{x + iy} * \frac{1}{x - iy}$$

~~u+iv =~~ $u = \dfrac{x}{x^2 + y^2}$

$$v = -y/x^2 + y^2$$

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If $x^2+y^2=a^2$ then $u^2+v^2=\frac{1}{a^2}$.

Imp Bilinear Transformation :- Definition

A transformation $w=f(z)=\frac{az+b}{cz+d}$ provided that $ad-bc\neq 0$ is called bilinear transformation.

$$w_i=\frac{az_i+b}{cz_i+d}, \quad i=1,2,3$$

$\boxed{\begin{matrix} z_1 \to w_1 \\ z_2 \to w_2 \\ z_3 \to w_3 \end{matrix}}$

Sign.