ELECTRICAL MACHINES(EE 554)

Chapter-5(Three Phase Induction Machines)

5.1 Introduction

Induction machine is a rotating electrical machine operated by AC voltage source or which generates AC voltage. It also known as asynchronous machine, because it never operates at synchronous speed. It can be used as generator as well as motor. Because of its simple construction and better operating characteristics with compare to a dc motor, induction machines are widely used in various industrial and domestic applications.

5.2 Basic constructional details of induction machine

An induction machine has three major parts namely- stator, rotor and yoke. Fig.5.1 shows a cross-sectional view of a dc machine. The various parts of the machine are described below:

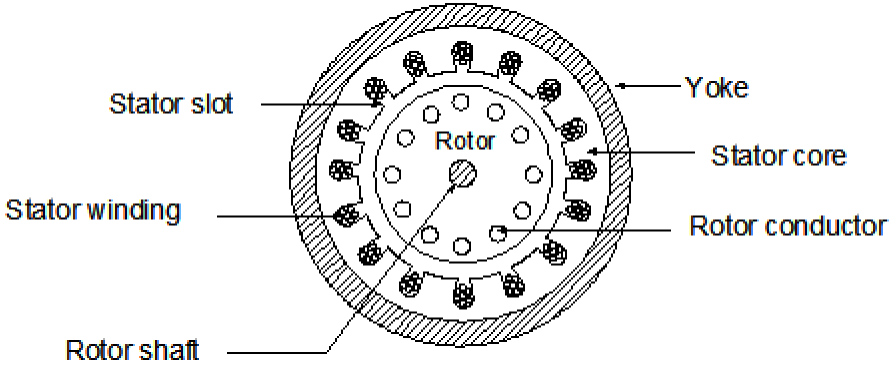


Fig.5.1 Cross-sectional view of induction machine

a) Stator: It is the stationary part of the machine in cylindrical form with hallow space at the center. The stator core is made of laminated silicon steel. The inner circumference of the stator core has alternate number of slots and teeth. The slots are provided with stator windings made of enamel insulated copper wire. In case of three-phase induction machine, the stator winding is three-phase distributed winding with each phase spaced 120^0 electrically apart. The windings are insulated from the slots with insulating paper. When the stator windings are supplied by three-phase voltage, the winding creates definite number of magnetic poles on the stator core.

Example of a three-phase stator winding:

Let number of slots = 12 Number of magnetic poles = 2

$$\text{Coil Span} = \frac{\text{No. of slots}}{\text{No. of poles}} = \frac{12}{2} = 6. \qquad \text{No of slots per phase} = \frac{\text{No. of slots}}{\text{No. of phase}} = \frac{12}{3} = 4.$$

Coil span is the number of teeth between two side of a coil.

Here, 360^0 electrical = 12 slots $\therefore 120^0 = \frac{12}{360} \times 120 = 4\,\text{slots}$

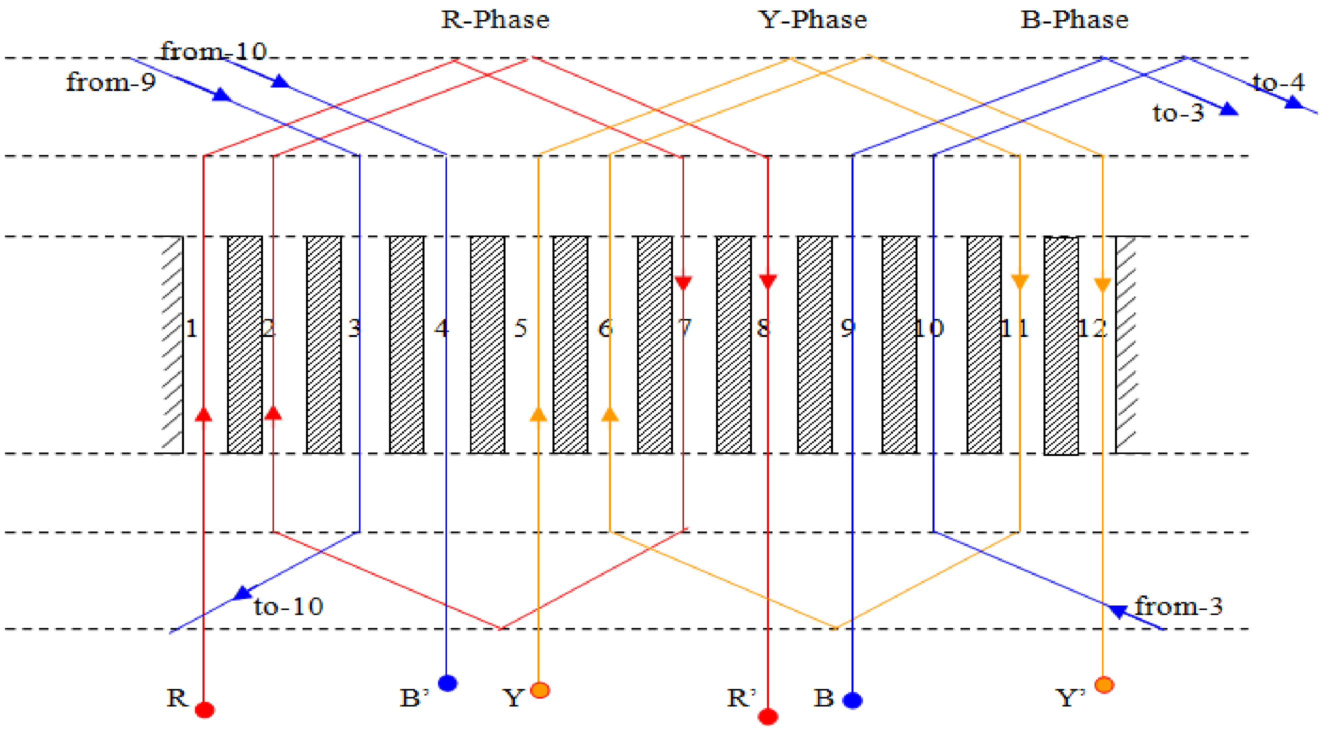


Fig.5.2 Example of three-phase stator winding

b) Rotor: It is the rotating part of the machine. It is cylindrical in shape with a central shaft. The shaft is supported by bearing at both end so that it can rotates freely keeping a small air gap of about 1 to 4mm between rotor and stator. It is made of laminated silicon steel sheet. There are two types of rotor i) Squirrel cage rotor and ii) Phase wound rotor

i) Squirrel cage rotor: This type of rotor is made of cylindrical laminated core with parallel slots near by outer circumference as shown in Fig.5.3. These parallel slots carry rotor conductors and ends of these conductors are short circuited by copper rings known as end rings.

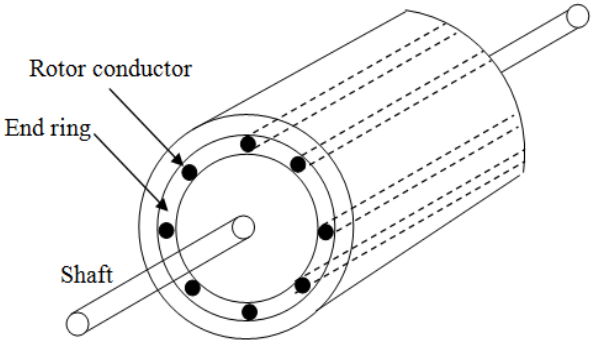


Fig.5.3 Squirrel cage rotor

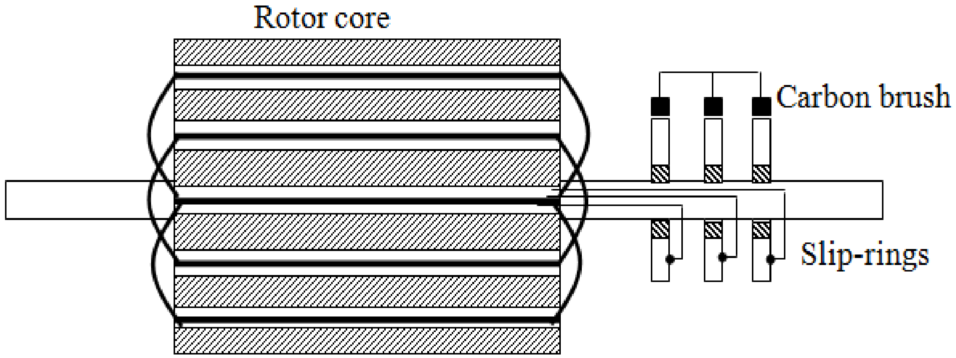
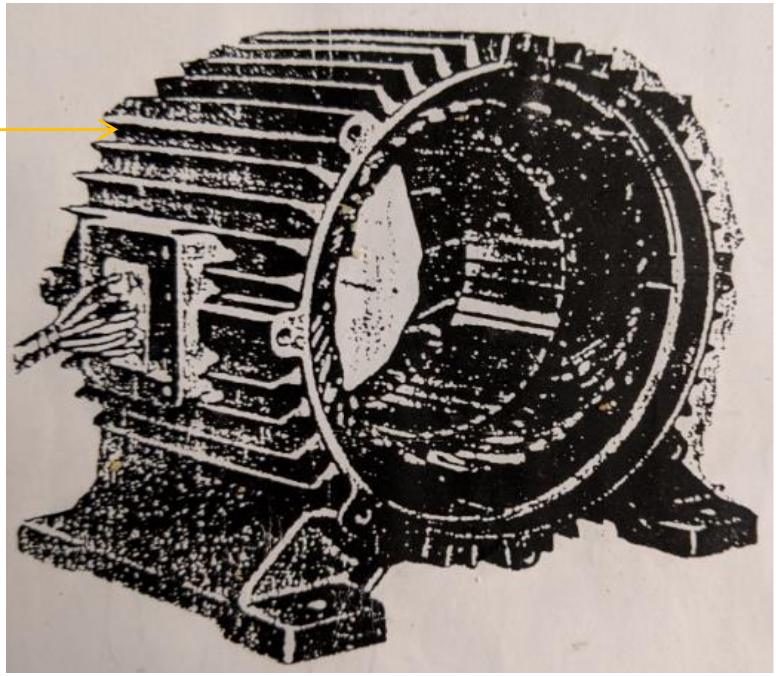


Fig.5.4 Phase wound rotor

ii) Phase wound rotor: This type of rotor is also made of cylindrical laminated silicon steel core, but it has open slots along the outer circumference. Three phase windings are wounded on these slots which creates same number of magnetic poles as produced by the stator windings. The winding principle is same as that of the stator windings. The three-phase rotor windings are connected in star and three ends of rotor windings are connected to the three separate slip-rings mounted on the shaft and the slip-rings are short circuited by the carbon brushes with or without external resistance as shown in Fig.5.4. The slip-rings are electrically insulated from the shaft. The slip-rings rotates along with the shaft, but the carbon brushes are fixed and always touching over the slip-rings.

c) Yoke: It is the outermost frame of the machine. It houses the stator core and provides mechanical protection of the whole machine.

Yoke



4.3 Operating principle of three-phase induction motor

When the three phase stator windings are supplied by three phase balanced ac voltage source, three phase currents will flow through the stator windings. The stator winding will magnetized the stator core. Let us study the nature of magnetic field produced by these three phase stator currents.

Let us consider a two pole machine and suppose that each phase winding is concentrated in a slot as shown in Fig.5.5. R and R’ represents the starting and finishing ends of the R-phase winding. The cross mark (×) indicate the current entering inside and the dot mark (•) indicates the current coming out. Let us assume that the current is positive when it is flowing inside.

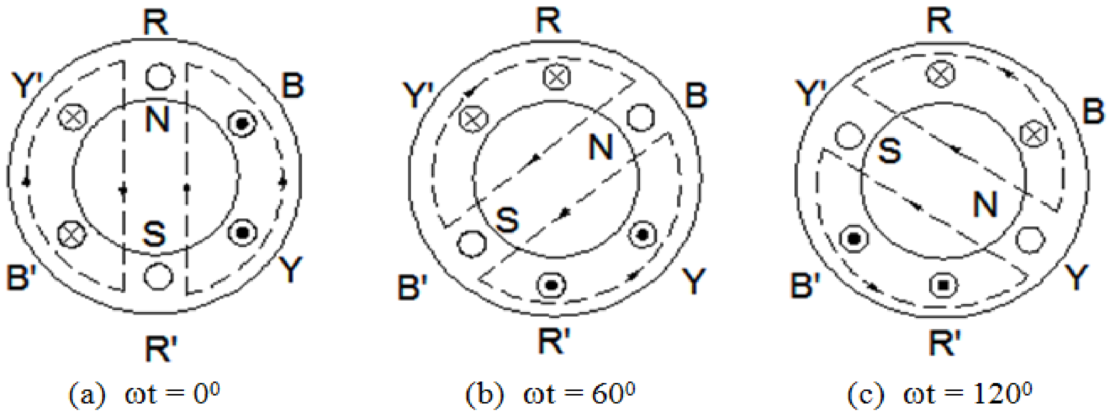


Fig.5.5 Distribution of magnetic flux due to three phase stator current

The three-phase stator currents produce their own magnetic flux, whose nature will be same that of three phase stator currents. That means, the magnetic flux produced by each phase current will be alternating in nature and given by following mathematical equations:

$$\left.\begin{aligned} \phi_R &= \phi_m \operatorname{Sin}\omega t \\ \phi_Y &= \phi_m \operatorname{Sin}(\omega t - 120^0) \\ \phi_B &= \phi_m \operatorname{Sin}(\omega t - 240^0) \end{aligned}\right\} \quad (5.1)$$

Fig.5.6 shows the waveform and Fig.5.7 shows the phasor diagram of the magnetic flux produced by three-phase stator currents.

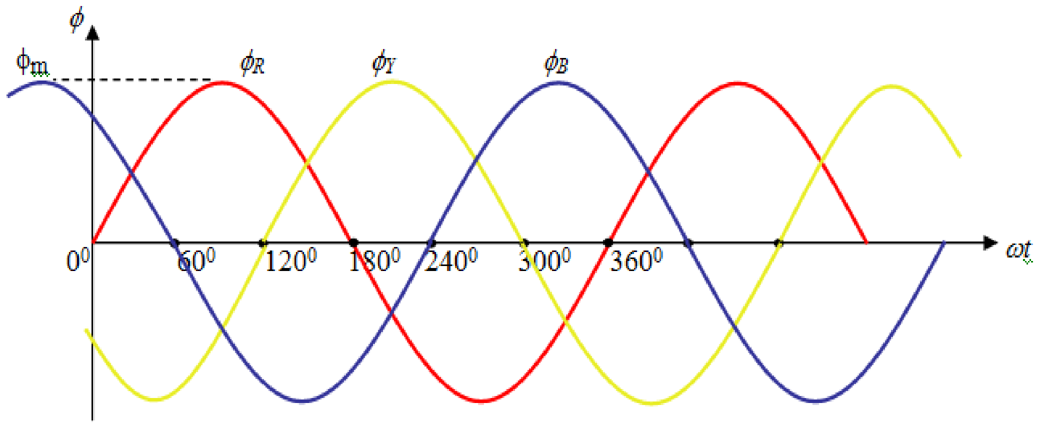


Fig.5.6 Waveform of three-phase stator magnetic flux

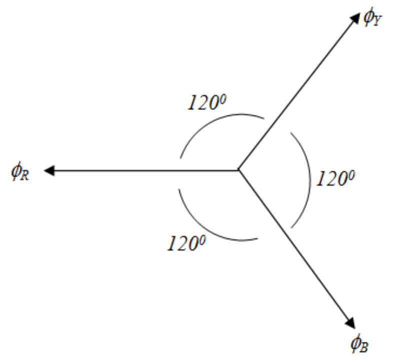


Fig.5.7 Phasor diagram of three-phase flux

The net air-gap magnetic flux is the sum of three individual magnetic flux produced by individual phase current. Let us study the nature of net air-gap magnetic flux at any instant.

When $\omega t = 0^0$,

$\phi_R = \phi_m \text{Sin}\omega t = 0$

$\phi_Y = \phi_m \text{Sin}(\omega t - 120^0) = \phi_m \text{Sin}(0^0 - 120^0) = -\dfrac{\sqrt{3}}{2}\phi_m$

$\phi_B = \phi_m \text{Sin}(\omega t - 240^0) = \phi_m \text{Sin}(0^0 - 240^0) = +\dfrac{\sqrt{3}}{2}\phi_m$

At this instant, flux produced by R-phase current is zero and net air-gap flux is sum of ϕ_Y and ϕ_B. Their sum is shown in Fig.5.8.

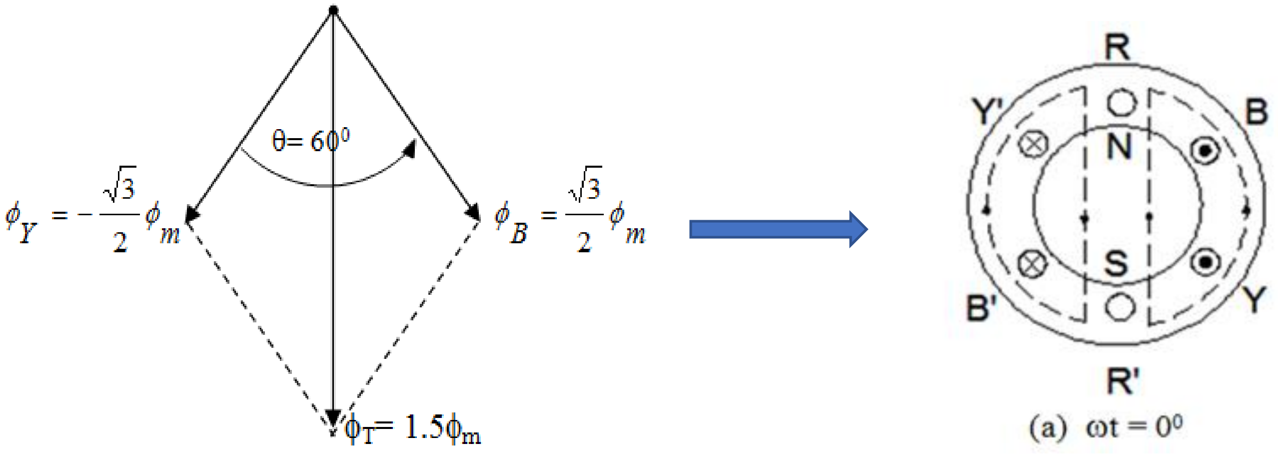


Fig.5.8 Total air gap flux at $\omega t = 0^0$

The total air-gap flux is given by:

$$\phi_T = \sqrt{\phi_Y^2 + \phi_B^2 + 2.\phi_Y.\phi_B.Cos\theta}$$

$$\text{OR}\ \ \phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\phi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\phi_m\right)^2 + 2.\frac{\sqrt{3}}{2}\phi_m\frac{\sqrt{3}}{2}.\phi_m.Cos60^0} = 1.5\phi_m$$

Hence, net air-gap flux at $\omega t = 0^0$ has a magnitude of 1.5 ϕm and direction is downward.

When $\omega t = 60^0$,

$$\phi_R = \phi_m \, Sin\omega t = \phi_m \, 60^0 = +\frac{\sqrt{3}}{2}\phi_m$$

$$\phi_Y = \phi_m \, Sin(\omega t - 120^0) = \phi_m \, Sin(60^0 - 120^0) = -\frac{\sqrt{3}}{2}\phi_m$$

$$\phi_B = \phi_m \, Sin(\omega t - 240^0) = \phi_m \, Sin(60^0 - 240^0) = 0$$

At this instant, flux produced by B-phase current is zero and net air-gap flux is sum of ϕ_R and ϕ_Y. Their sum is shown in Fig.5.9.

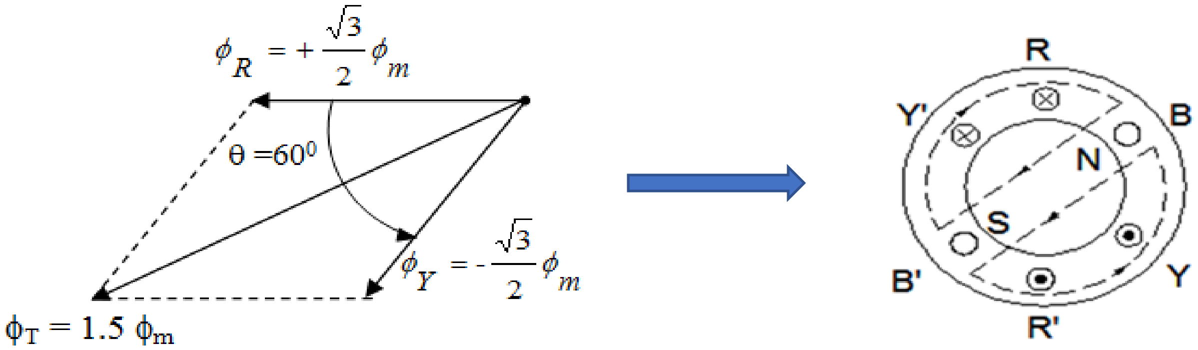


Fig.5.9 Total air gap flux at $\omega t = 60^0$

(b) $\omega t = 60^0$

The total air-gap flux is given by: $$\phi_T = \sqrt{(\frac{\sqrt{3}}{2}\phi_m)^2 + (\frac{\sqrt{3}}{2}\phi_m)^2 + 2.\frac{\sqrt{3}}{2}\phi_m.\frac{\sqrt{3}}{2}.\phi_m.Cos60^0} = 1.5\phi_m$$

Hence, the net air-gap flux at $\omega t = 60^0$ has a magnitude of 1.5 ϕm (same as before) and direction is as shown in Fig.5.9. Which has rotated by 60^0 in clockwise direction with compare to that for $\omega t = 0^0$.

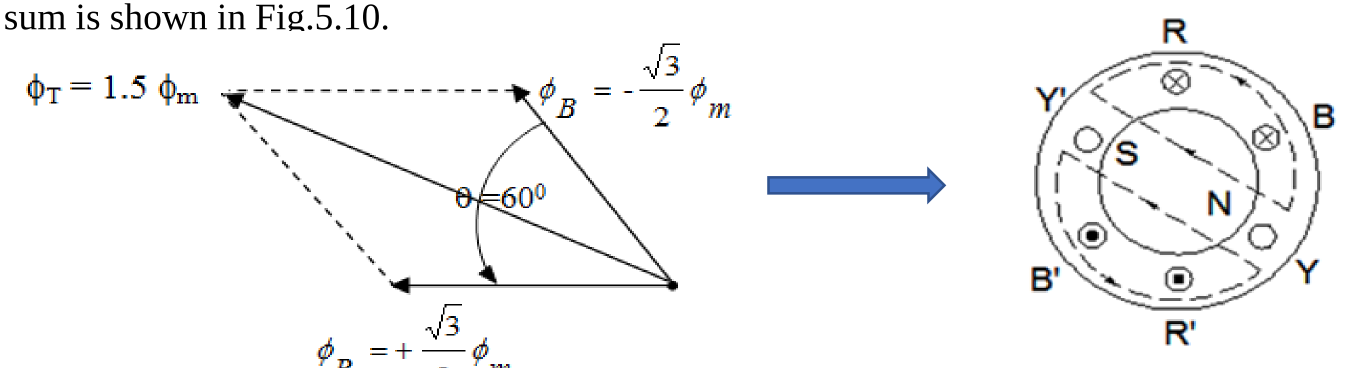
When $\omega t = 120^0$,

$$\phi_R = \phi_m \mathrm{Sin}\omega t = \phi_m \mathrm{Sin}120^0 = +\frac{\sqrt{3}}{2}\phi_m$$

$$\phi_Y = \phi_m \mathrm{Sin}(\omega t - 120^0) = \phi_m \mathrm{Sin}(120^0 - 120^0) = 0$$

$$\phi_B = \phi_m \mathrm{Sin}(\omega t - 240^0) = \phi_m \mathrm{Sin}(120^0 - 240^0) = -\frac{\sqrt{3}}{2}\phi_m$$

At this instant, flux produced by Y-phase current is zero and net air-gap flux is sum of ϕ_R and ϕ_B. Their sum is shown in Fig.5.10.



(c) $\omega t = 120^0$

Fig.5.10 Total air gap flux at $\omega t = 120^0$

The total air-gap flux is given by: $\phi_T = \sqrt{(\frac{\sqrt{3}}{2}\phi_m)^2 + (\frac{\sqrt{3}}{2}\phi_m)^2 + 2.\frac{\sqrt{3}}{2}\phi_m.\frac{\sqrt{3}}{2}.\phi_m.Cos60^0} = 1.5\phi_m$

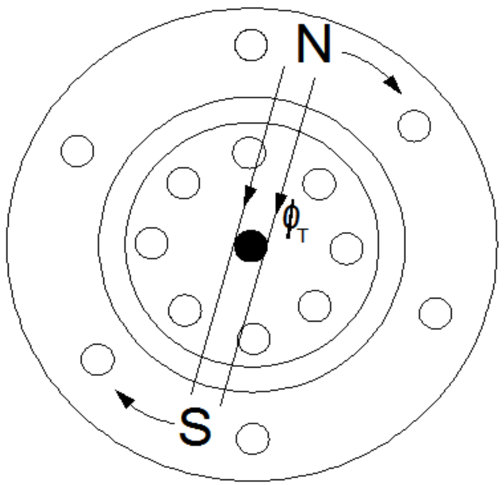
Hence, it is clear from the above mathematical and phasor analysis that the net air gap flux has a constant magnitude of 1.5 ϕm at any instant and its direction is rotating in clockwise direction with a constant speed. Such a magnetic field is known as rotating magnetic field. The speed of the rotating magnetic field is known as synchronous speed and is given by:

$$N_S = \frac{120.f}{P}$$

f = Frequency of voltage applied to the stator winding

P = Number of magnetic pole for which the stator winding is wound.

The rotating magnetic field produced by the stator currents cuts the rotor conductor (which are at rest at starting). Therefore, according to Faraday’s law of electromagnetic induction, emf will induced in the rotor conductors. As the rotor conductors are short circuited by end rings at both ends, there is a closed path and current will circulate through the rotor conductors. Now the current carrying rotor conductors are lying in the magnetic field, hence force will develop on the rotor conductors. Under the action of the force so developed, the rotor starts rotating.



Now the question is, in which direction the rotor will rotates. This can be determined with the help of Lenz’s law. According to Lenz’s law, the direction of induced current will be such that the magnetic field produced by the induced current opposes the cause by which it was induced. Here, the main cause of induce of rotor current is the relative speed between magnetic field and rotor conductor. Hence the direction induced current in the rotor will be in such a direction which will produced force in the conductors in such a direction that the rotor rotates to reduce the relative speed between magnetic field and rotor conductors. That means, the rotor rotates in the same direction as the rotating magnetic field.

The rotor will try to catch-up the synchronous speed of the rotating magnetic field, but it never success to do so. If the rotor rotates with synchronous speed, there will no relative speed between magnetic field and rotor conductors, hence no emf and current will induce in the rotor conductors and no force will develop on the rotor conductors. Therefore, the rotor always rotates at a speed less than the synchronous speed. That is why an induction motor is also known as asynchronous motor.

Let N = speed of the rotor (in RPM)

$\frac{Ns - N}{Ns}$ is a factor indicating the fraction by which the speed of the rotor is less than the synchronous speed. This factor is known as 'Slip'.

$$\therefore \ \text{Slip} \ s = \frac{Ns - N}{Ns} \qquad (5.3)$$

For example: If number of magnetic pole P = 4 and f = 50 Hz

Then $N_S = \frac{120.f}{P} = \frac{120 \times 50}{4} = 1500$ RPM If the rotor rotates at N = 1470 RPM

$$\text{Then slip} \ s = \frac{1500 - 1470}{1500} = 0.02\, pu$$

That means, the speed of the rotor is 2% less than the synchronous speed. The slip of the motor changes with load on the shaft. If the load on the shaft of the motor increases, the speed of the rotor decreases and the value of slip increases.

At starting, rotor speed is zero. The relative speed between magnetic field and rotor conductors (N_S – N) is maximum. Therefore, maximum emf will induce in the rotor conductors at the starting and maximum current will flow in the rotor conductors and accordingly the stator will draw maximum current at starting. At starting, the electric circuit of induction motor is similar to that of a transformer. The stator winding acts as primary winding and rotor conductors acts as secondary winding. Hence, the frequency of emf induced in the rotor at starting is equal to the frequency of voltage applied to the stator winding and the magnitude of emf induced in the rotor circuit at starting is given by:

$$E_2 = \frac{N_2}{N_1} E_1 \qquad (5.4)$$

Where,

E_1 = emf induced in the stator winding per phase

N_1 = Numbers of turn per phase in the stator winding

N_2 = Numbers of turn per phase in the rotor winding

At starting, the frequency of emf induced E_2, will be equal to the frequency of applied input voltage just like in transformer.

$$N_S = \frac{120.f}{P} \quad \text{OR} \quad f = \frac{\mathrm{Ns.P}}{120}$$

When the rotor rotates, the relative speed between rotating magnetic field and rotor conductors decreases. Therefore, the magnitude of emf induced in the rotor conductors reduces to:

$$E_R = s.E_2 \qquad (5.5)$$

Frequency of emf induced in rotor circuit also decreases to:

$$f_R = \frac{(\mathrm{Ns.} - \mathrm{N})\mathrm{P}}{120} \quad \text{OR} \quad \frac{f_R}{f} = \frac{(N_s - N)P/120}{N_s.P/120} = \frac{(N_s - N)}{N_s.} = s \qquad \text{OR} \quad f_R = s.f \qquad (5.6)$$

5.4 Analysis of stand-still condition of induction motor

Stand still condition is the instant of starting, when the speed of rotor is zero. At this instant, the relative speed (N_S – N) is maximum and slip is maximum (s =1) and emf induced in the rotor circuit is maximum. This condition is similar to the transformer operation with stator winding as primary winding and rotor circuit as secondary winding. Hence, the electrical equivalent circuit of three-phase induction motor at stand-still is same as that of a transformer. Fig.5.11 shows the per phase equivalent circuit of a three-phase induction motor at standstill.

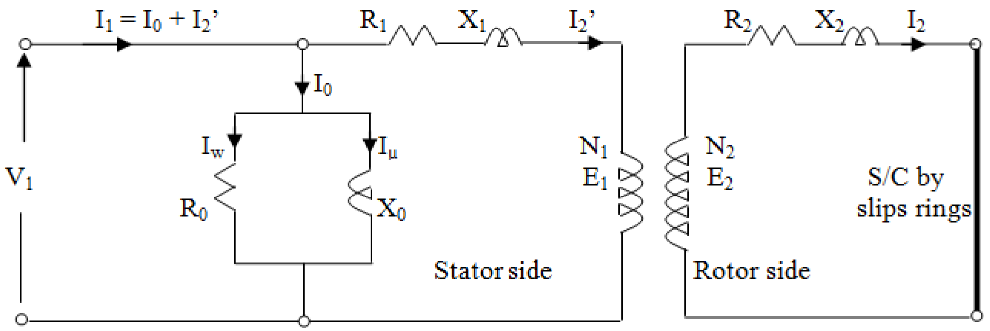


Fig.5.11 Per phase equivalent circuit of three phase induction motor at standstill

Here, V_1 = Supply voltage to stator winding per phase

I_1 = Stator current per phase

I_0 = No-load stator current per phase

E_1 = Stator emf per phase

E_2 = Rotor emf at stand-still per phase

R_1 = Stator winding resistance per phase

X_1 = Stator winding leakage reactance per phase

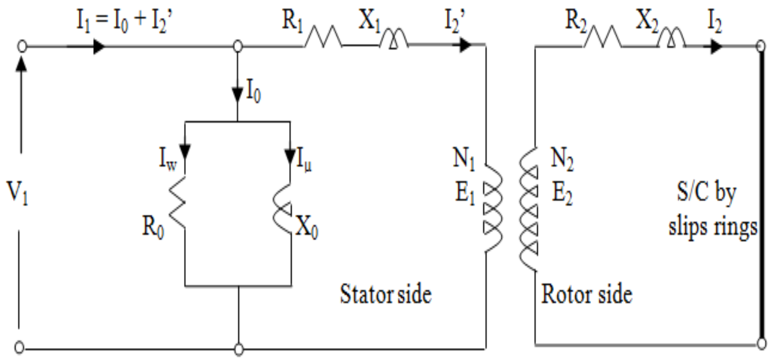
R_2 = Rotor winding resistance per phase

X_2 = Rotor winding leakage reactance per phase at standstill

I_2 = Rotor current per phase at stand-still

Rotor current per phase at stand-still is given by:

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \qquad (5.7)$$



I_2 lags E_2 by a phase angle of ϕ_2.

$$\text{Where,} \quad Cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \qquad (5.8)$$

The torque developed by the rotor at stand-still (T_S) is proportional to the flux per pole and active component of I_2.

Hence, $T_S = K.\ \phi.\ I_2\ Cos\phi_2$, Where ϕ = Flux per pole.

Like in transformer, the magnetic flux per pole (ϕ) remains constant irrespective of change in I_1 and I_2. It only depends on $I\mu$ Or V_1 or E_1.

Since, $E_2 \propto E_1 \quad \therefore \quad \phi \propto E_2$

$$\therefore \quad T_S = K_1.E_2 \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$\boxed{\text{OR} \quad T_S = \frac{K1.E_2^2 R_2}{R_2^2 + X_2^2} \qquad (5.9)}$$

5.5 Analysis of running condition of induction motor

When the rotor rotates, the relative speed between rotating magnetic field and the rotor conductors decreases, there by reducing the rate of cutting the flux by the conductor. Therefore, the magnitude of emf induced in the rotor decreases with compare to emf induced at stand still condition. The frequency of emf induced in the rotor circuit also reduces accordingly. The magnitude and frequency of emf induced in the rotor at running condition are given by:

$E_R = s.\ E_2$ (5.10) and $f_R = s.f$ (5.11)

Since the reactance of an inductor depends on frequency, the value of leakage reactance of rotor circuit 'X_2' also reduces in running condition and is given by:

$X_R = s.\ X_2$ (5.11)

Hence, the situation of running condition is drastically different from the standstill condition and the equivalent circuit of the three-phase induction motor at running condition is shown in Fig.5.12.

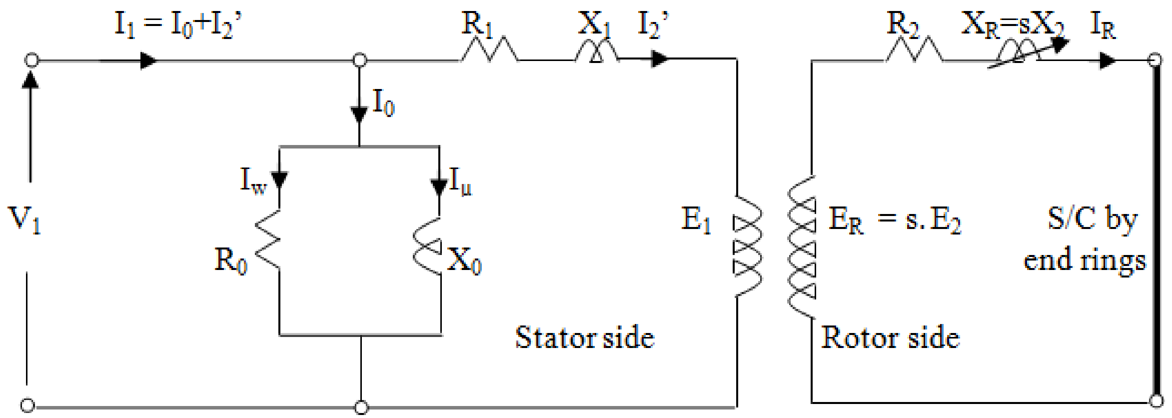
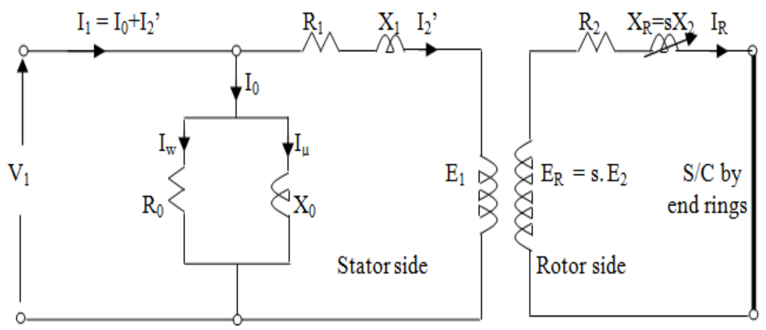


Fig.5.12 Per phase equivalent circuit of induction motor at running condition

Here, X_R is shown variable, because it value changes with speed. The rotor current at running condition is given by:



$$I_R = \frac{s.E_2}{\sqrt{R_2^2 + (s.X_2)^2}} \qquad (5.13)$$

I_R lags E_R (i.e. $s.E_2$) by a phase angle of ϕ_R.

Where, $$Cos\phi_R = \frac{R_2}{\sqrt{R_2^2 + (s.X_2)^2}} \qquad (5.14)$$

The torque developed by the rotor at running condition (T_R) is proportional to the flux per pole and active component of I_R.

Hence, $T_R = K.\ \phi.\ I_R\ Cos\phi_R$, Where ϕ = Flux per pole.

$$\therefore \quad T_R = K_1.E_2 \frac{s.E_2}{\sqrt{R_2^2 + s^2X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + s^2X_2^2}}$$

$$\boxed{\text{OR} \quad T_R = \frac{K_1 s\, E_2^2 R_2}{R_2^2 + s^2 X_2^2} \qquad (5.15)}$$

Illustrative example 4.1:

A 400V, 4-pole, 50 Hz, 3-phase, 10 HP, star-connected induction motor has no-load slip of 0.01 and full load slip of 0.04 Calculate:

a)Speed of rotating magnetic field b) No-load speed c) Full load speed

d) Frequency of rotor current at full load d) Full load torque

Solution:

a) Speed of rotating magnetic field : $N_S = \frac{120 \times f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$

b) Slip is given by: $s = \frac{N_s - N}{N_s}$ Or $N = N_s(1\text{-}s)$

No-load slip $s_{nl} = 0.01$ $\therefore$ No-load speed $N_0 = N_s(1\text{-}s_{nl})$

$N_0 = N_s(1\text{-}s_{nl}) = 1500\,(1 - 0.01) = \underline{1485 \text{ rpm}}$

c) Full load slip $s_{fl} = 0.04$ $\therefore$ Full-load speed $N_{fl} = N_s(1\text{-}s_{fl})$

Or $N_{fl} = N_s(1\text{-}s_{fl}) = 1500\,(1 - 0.04) = \underline{1440 \text{ rpm}}$

d) Frequency of rotor current at full load = $f_r = s.\,f = 0.04 \times 50 = \underline{2 \text{ Hz}}$

e) Full load torque:

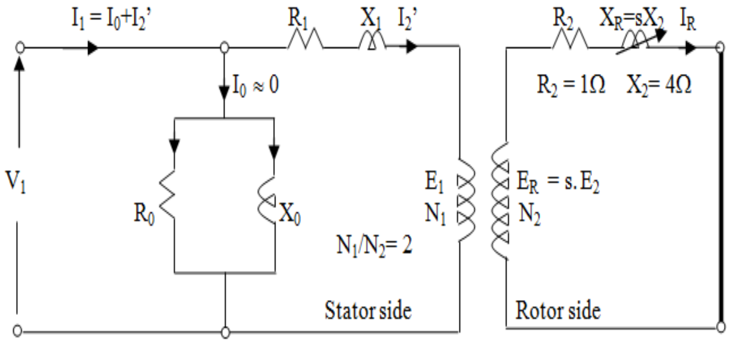
Full load power = 10 HP = 10 ×735.5 = 7355 watts

The relation between power and torque is given by: $P = \frac{2.\pi.N.T}{60}$

$\therefore$ Full load torque $T_{FL} = \frac{P_{fl} \times 60}{2.\pi.N_{fl}} = \frac{7355 \times 60}{2 \times \pi \times 1440} = 48.77 \text{ N - m}$

<u>Illustrative example 4.2</u>:

A three-phase, 4-pole, 50Hz slip-ring induction motor has star-connected stator windings and rotor windings. The rotor winding impedance is (1 + j4) ohms per phase at standstill. The rotor circuit has induced emf of 400V between slip rings at standstill. The stator to rotor turn ratio is 2. Assuming that there is no power loss, calculate: rotor current and power factor and stator current a) at starting b) when running at 1440 rpm.



Solution:

The per phase equivalent circuit of three-phase induction motor is shown below:

a) At standstill:

$$N_S = \frac{120 \times f}{P} = \frac{120 \times 50}{4} = 1500 \text{ Speed of rotor } N = 0$$

$$\therefore \text{ Slip } s = \frac{N_s - N}{N_s} = \frac{1500 - 0}{1500} = 1$$

Emf induced in rotor circuit at stand still (line to line) = 400V

Emf induced in rotor circuit at standstill (E_2 per phase) = $\frac{400}{\sqrt{3}} = 230.9V$

Then rotor circuit current per phase, $I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} = \frac{230.9}{\sqrt{1^2 + 4^2}} = 56A$

Rotor circuit power factor $Cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{1}{\sqrt{1^2 + 4^2}} = 0.243 \text{ lagging}$

Asssuming no power loss, $\frac{I_1}{I_2} = \frac{N_2}{N_1}$ Or $I_1 = \frac{N_2}{N_1} \times I_2 = I_1 = \frac{1}{2} \times 56 = 28A$

b) At running condition:

$$s = \frac{N_s - N}{N_s} = \frac{1500-1440}{1500} = 0.04$$

Emf induced in rotor circuit (per phase) $E_R = s\,E_2 = 0.04 \times 230.9 = 9.23V$

Rotor circuit current per phase $$I_R = \frac{s.E_2}{\sqrt{R_2^2 + (s.X_2)^2}} = \frac{0.04 \times 230.9}{\sqrt{1^2 + (0.04 \times 4)^2}} = 9.12A$$

Rotor circuit power factor, $$Cos\phi_R = \frac{R_2}{\sqrt{R_2^2 + (s.X_2)^2}} = \frac{1}{\sqrt{1^2 + (0.04 \times 4)^2}} = 0.987 Lagging$$

Stator current $I_1 = \frac{N_2}{N_1} \times I_R$ OR $I_1 = \frac{1}{2} \times 9.12 = 4.56$ A

5.6 Torque-slip characteristic of three-phase induction motor

The torque- slip characteristic is curve showing the torque developed by the motor at different values of slip or speed.

$$\text{The torgue develpoed by the motor is given by} : T_R = \frac{K_1 s.E_2^2 R_2}{R_2^2 + s^2 X_2^2}$$

From this equation, it is clear that the torque developed by the motor depends upon the slip, provided R_2 X_2 and E_2 are constants. However, the relation between torque and speed is not linear.

Within the normal operating range (i.e. no-load to full-load), the value of slip is very small. Therefore the term $s^2X_2^2$ is very small with compare to R_2^2. Therefore, the torque equation for normal operating range can be simplified as follow:

$$T_R = \frac{K_1 s.E_2^2}{R_2} \quad \text{Or} \quad T_R \,\alpha\, \frac{s}{R_2} \quad \text{Or} \quad T_R \,\alpha\, s \quad \text{for constant } R_2$$

Hence, torque increases proportionately with increase in slip (or with decrease in speed). This part of characteristic curve is represented by a straight line AB on the curve shown in Fig.5.13. The torque goes on increasing with slip, but there is a limit up which the torque increases with slip. After that limit, the torque decreases with increase in slip.

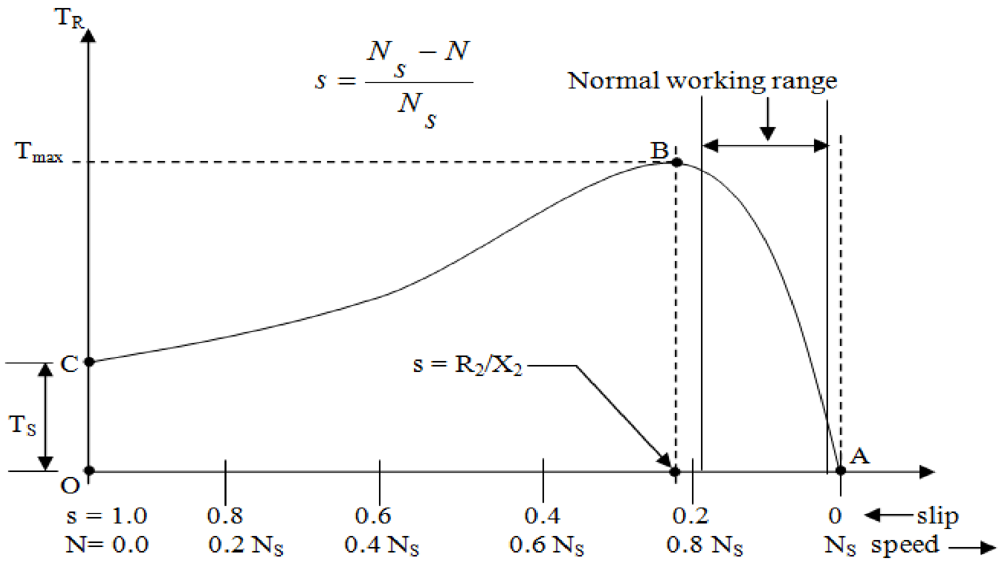


Fig.5.13 Torque-slip characteristic of 3-phase induction motor.

The value of slip at which the torque will be maximum, can be determined as follow:

$$T_R = \frac{K_1 s..E_2^2 R_2}{R_2^2 + s^2 X_2^2} \qquad \text{Let } Y = \frac{1}{T_R}$$

$$\text{Then } Y = \frac{R_2^2 + s^2 X_2^2}{K_1 s.E_2^2 R_2} = \frac{R_2^2}{K_1 s.E_2^2 R_2} + \frac{s^2 X_2^2}{K_1 s.E_2^2 R_2} \qquad = \frac{R_2}{K_1 s.E_2^2} + \frac{s X_2^2}{K_1 E_2^2 R_2}$$

$$\text{OR } \frac{dY}{ds} = \frac{-R_2}{K_1 s^2.E_2^2} + \frac{X_2^2}{K_1 E_2^2 R_2} \qquad \text{Y will be minimum, when } \frac{dY}{ds} = 0, \quad \text{Or TR will be maximum when } \frac{dY}{ds} = 0$$

$$Or \quad \frac{R_2}{K_1 s^2 . E_2^2} = \frac{X_2^2}{K_1 E_2^2 R_2} \qquad \text{Or} \quad s^2 = \frac{R_2^2}{X_2^2} = 0 \qquad \text{Or} \quad s = \frac{R_2}{X_2} = 0 \ \text{(Condition for maximum efficiency)}$$

At this particular value of slip (speed), the torque will be maximum as shown in Fig.5.14.

If the slip is further increased beyond this value of s = R_2/X_2, $s^2X_2^2$ will be no more small with compare to R_2^2. But R_2^2 becomes small with compare to $s^2X_2^2$. This is the overload condition. Hence, the torque equation during overload condition can be simplified as follow:

$$T_R = \frac{K_1 s..E_2^2 R_2}{R_2^2 + s^2 X_2^2} \qquad \text{Or} \quad T_R = \frac{K_1 . R_2 . E_2^2}{s.X^2}$$

$$\text{Or} \quad T_R \propto \frac{R_2}{s} \qquad \text{Or} \quad T_R \propto \frac{1}{s} \qquad \text{for } R_2 \text{ constant}$$

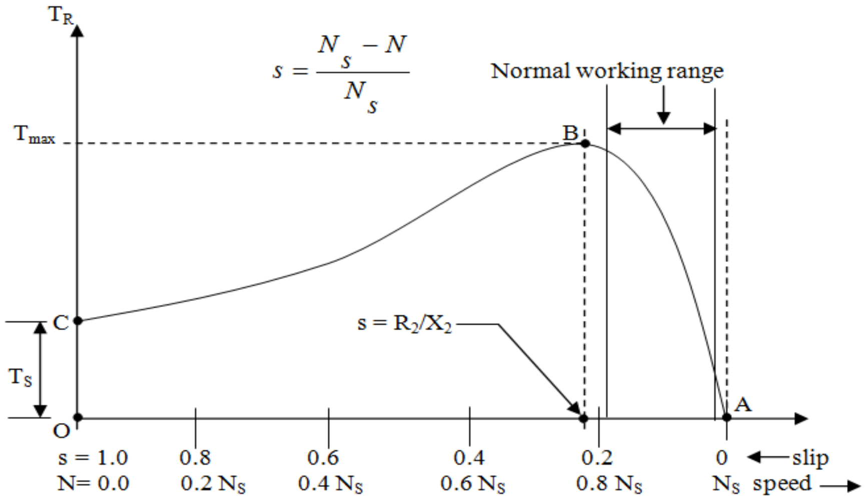


Fig.5.14 Torque-slip characteristic of 3-phase induction motor.

Hence, torque decreases with increase in slip (or with decrease in speed). This part of characteristic curve is represented by a the line BC on the curve shown in Fig.5.13. At point ‘C’ speed is zero (s=1), the torque developed by the motor is equal to OC, which known as starting torque of the motor. Hence, the expression for starting torque can be obtained by setting s =1 in the eqn(5.15) as follow:

$$T_S = \frac{K_1 . E_2^2 R_2}{R_2^2 + X_2^2} \qquad (5.16)$$

5.7 Effect of rotor resistance on T-S characteristic

It has been shown in the previous section that:

$T_R \propto \frac{s}{R_2}$ for normal operating range And $T_S \propto \frac{R_2}{s}$ for over-load and starting conditions.

If some resistance (R_{ex}) per phase is added in series with the rotor winding (that can be done in case of phase wound rotor), the starting torque will increase and running torque will decrease. Now the maximum torque will developed at new value of slip $s_m = (R_2+R_{ex})/X_2$. Fig.5.15 shows the effect of rotor resistance on T-S characteristic.

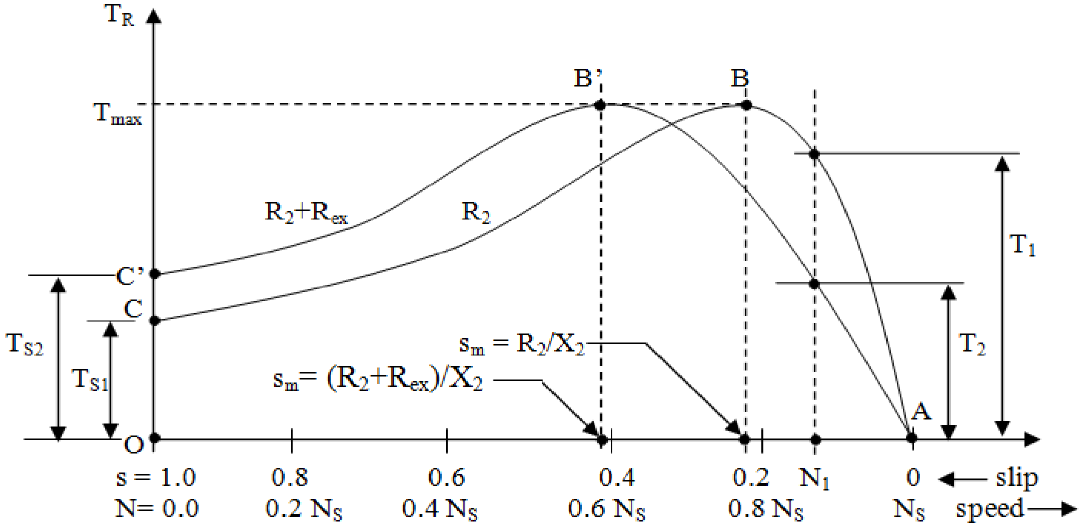
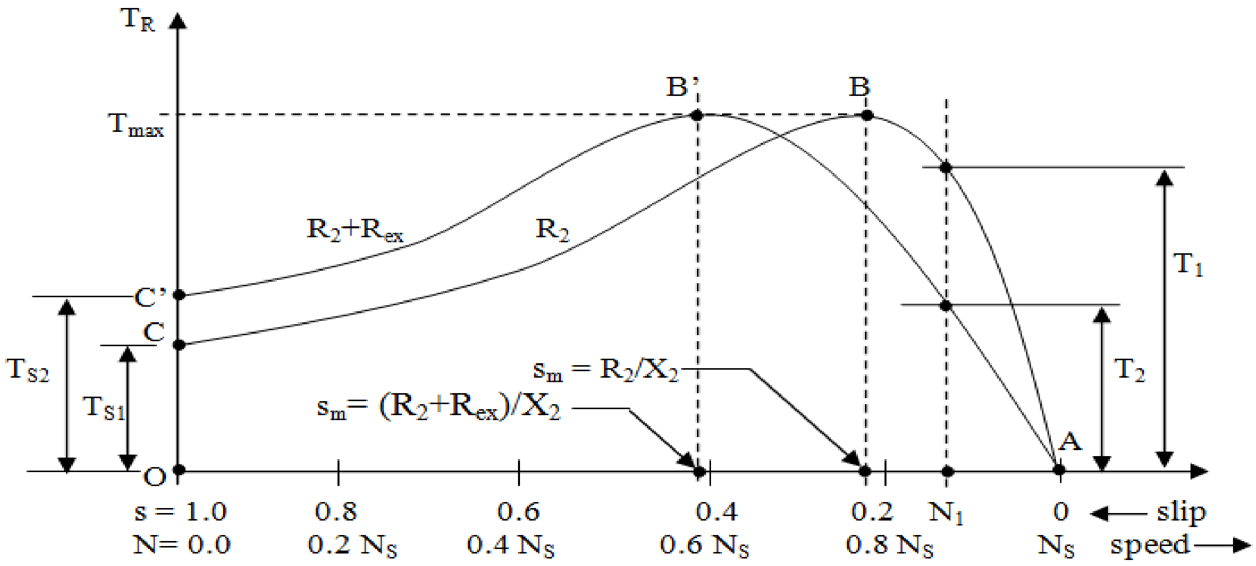


Fig.5.15 Effect of rotor resistance on T-S characteristic.



Curve ABC is the T-S characteristic with rotor resistance R_2 and Curve AB'C' is the T-S characteristic with rotor resistance R_2+R_{ex}. At a particular speed N_1, with rotor resistance R_2, the running torque is equal to T_1. At the same speed with rotor resistance equal to R_2+R_{ex} , the running torque is onlyT_2. However, the starting T_{S2} with rotor resistance R_2+R_{ex} is greater than the starting torque T_{S1} with rotor resistance R_2. Hence, external resistance is used in series with rotor winding, where high starting is required such as in electric lift, electric train etc. In such a case, the external resistance shall be reduce again during the normal running period, otherwise running torque will be low.

Illustrative example 4.3:

A three-phase, 4-pole, 50Hz slip-ring induction motor has star-connected stator windings and rotor windings. The rotor winding impedance is (1+j4) ohms per phase at standstill. The rotor circuit has induced emf of 400V between slip rings at standstill. The stator to rotor turn ratio is 2. The motor produces a starting torque of 25N-m. Calculate the torque produced by the motor at 1440 rpm

Solution:

Synchronous speed $\mathrm{N_S} = \frac{120\times f}{P} = \frac{120\times 50}{4} = 1500\,\mathrm{rpm}$

At starting, speed of rotor N = 0 $\therefore$ slip at starting $s_0 = \frac{N_s - N}{N_s} = \frac{1500-0}{1500} = 1$

Emf induced in rotor circuit (line to line) = 400V

Emf induced in rotor circuit (per phase) $= \mathrm{E_2} = \frac{400}{\sqrt{3}} = 230.9V$

Starting torque is given by : $T_S = \frac{K_1.E_2^2 R_2}{R_2^2 + X_2^2}$ Or $25 = \frac{K_1.E_2^2 R_2}{R_2^2 + X_2^2}$

Or $K_1 = \frac{25\times(R_2^2 + X_2^2)}{E_2^2 R_2} = \frac{25\times(1+4^2)}{(230.9)^2\times 1} = 1.933\times 10\text{-}3$

When the rotor speed N = 1440 rpm Slip $s = \frac{N_s - N}{N_s} = \frac{1500-1440}{1500} = 0.04$

Torque developed by rotor at any speed is given by : $T_R = \frac{K_1 s..E_2^2 R_2}{R_2^2 + s^2 X_2^2}$

$\therefore\ T_R = \frac{1.933\times 10^{-3}\times 0.04\times(230.9)^2\times 1}{1^2+(0.04\times 4)^2} = 3.55\,\mathrm{N\text{-}m}$

5.7 Losses and efficiency of three phase induction motor:

Induction motor takes electrical power input from the electrical source and the shaft of the motor gives mechanical power output. The output power is always less than the input power, because there are power losses within the motor at various stages. The different power transformation stages in induction motor are shown below in Fig.5.16.

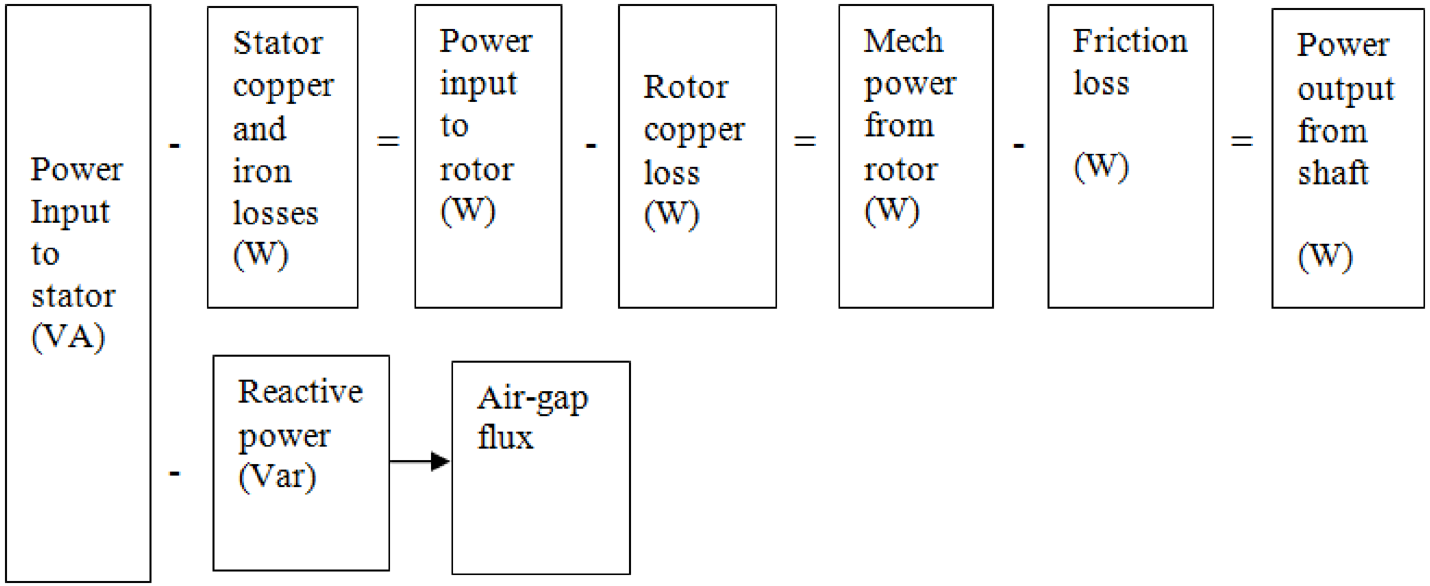


Fig.5.16 Block diagram of power stages in 3-phase induction motor

The power stages diagram also can be presented in pictorial format as shown in in Fin.5.17

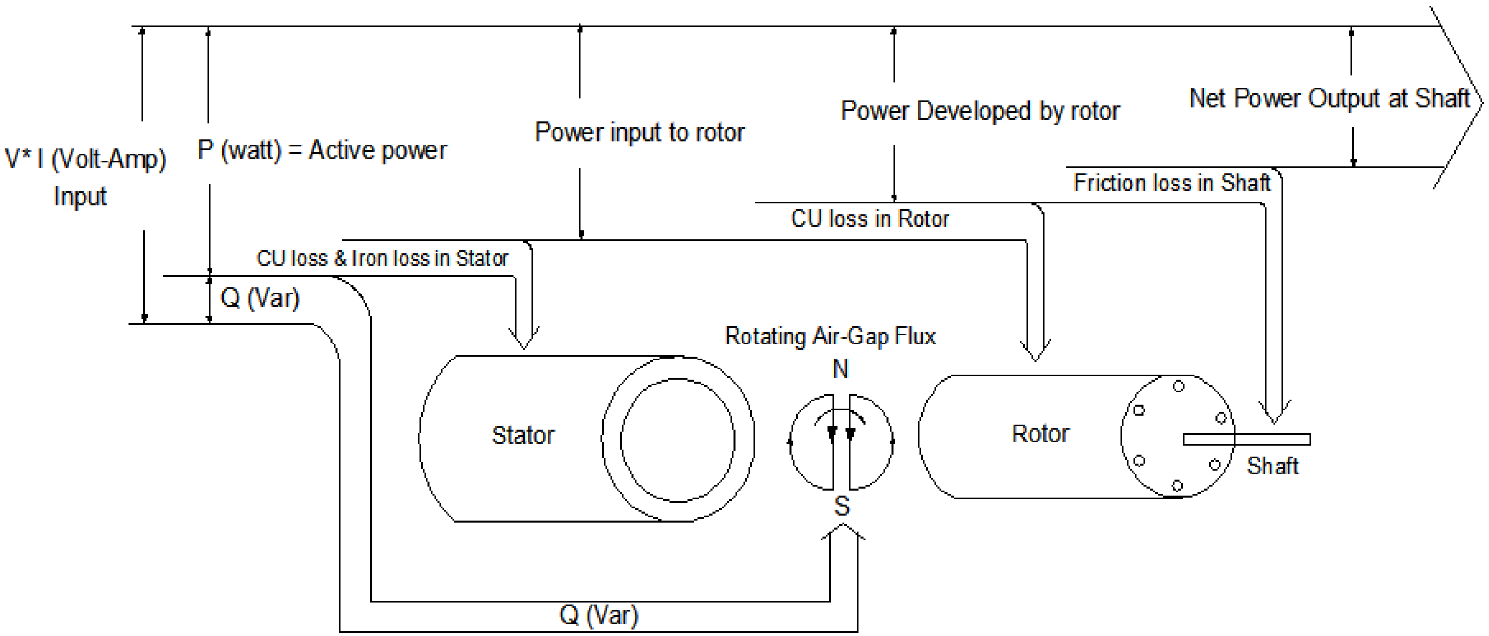
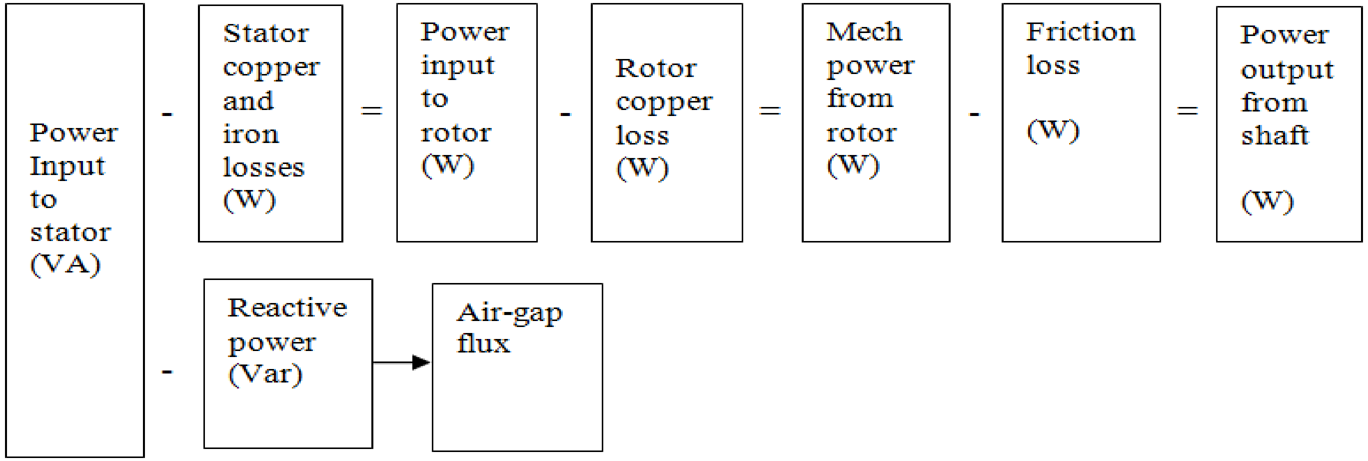


Fig.5.17 Pictorial format of power stages in 3-phase induction motor

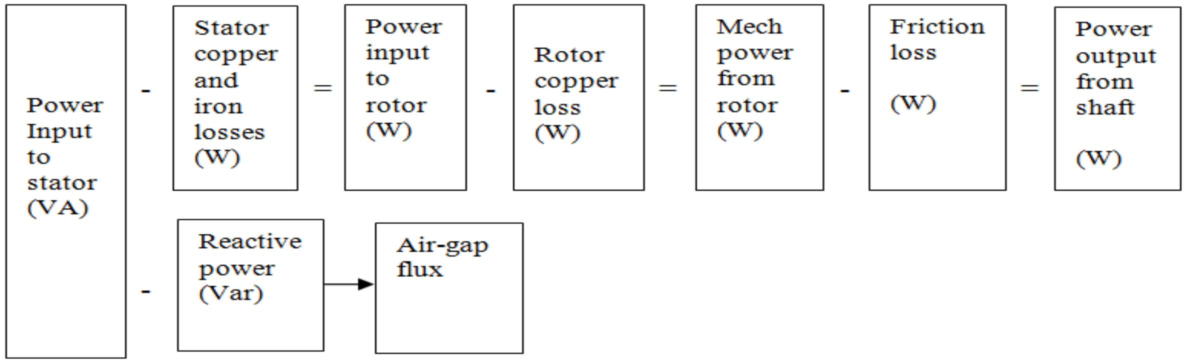
- Total power input to the stator is the sum of the active power and reactive power consumed by the motor.
- The reactive power will be utilized to establish air-gap magnetic flux in the machine. Whereas, the active power will be utilized to give mechanical power at the end.
- Stator copper loss is the power loss (I^2R) due to the internal resistance of the stator winding, which will be converted into heat energy.
- Stator iron loss is the power loss in the stator iron core due to eddy current loss and hysteresis loss.
- Hence, the active power input to the rotor will be less than input active power by an amount of stator copper and iron loss



- Frequency of emf induce in the rotor circuit is very low ($f_r = sf$).
- Hence, the iron loss in rotor circuit is negligible.
- If the copper loss in the rotor circuit is subtracted from the power input to the rotor, it gives mechanical power developed by the rotor.
- There will be some power loss in the shaft due to the friction.
- Hence, the net power output at shaft is obtained by subtracting the friction loss from the mechanical power developed by the rotor.

$$\text{The overall efficiency of the motor : } \eta_o = \frac{\text{Net Power Output from Shaft}}{\text{Avtive Power Input to Stator}} \qquad (5.17)$$

Let torque developed by the rotor at a speed of 'N' RPM = T_R



Then power developed by the rotor $P_R = \dfrac{2.\pi.N.T_R}{60}$

If there were no copper loss in the rotor circuit, <u>all the power input to the rotor</u> would be converted into mechanical power and in such a case the rotor would rotate at synchronous speed.

In such a case the power developed by the rotor would be $\dfrac{2.\pi.Ns.T_R}{60}$

Hence, the rotor copper loss $= \dfrac{2.\pi.Ns.T_R}{60} - \dfrac{2.\pi.N.T_R}{60} = 2.\pi.T_R \dfrac{Ns - N.}{60}$

$$\text{Then } \frac{\text{Rotor copper loss}}{\text{Rotor input power}} = \frac{2.\pi.T_R (Ns - N)}{2.\pi.T_R Ns} = \frac{Ns - N}{Ns} = s\ (\text{slip})$$

Therefore, Rotor copper loss = s × Rotor input power (5.18)

EXAMPLE 4.6 *The power input to a 3 phase induction motor is 60 kW. The stator loses total 1 kW. Find the total mechanical power developed and the rotor copper loss per phase if the motor is running with a slip of 3%.*

SOLUTION. Stator input $P_{is} = 60$ kW, $s = 3\% = \frac{3}{100} = 0.03$ pu

Stator losses $= 1$ kW ; Stator output $= 60 - 1 = 59$ kW

Rotor input = stator output $= 59$ kW

Total rotor copper loss $= s \times$ rotor input $= 0.03 \times 59 = 1.77$ kW

Rotor copper loss per phase $= \frac{1}{3} \times 1.77 = 0.59$ kW

Mechanical power developed = rotor input − rotor copper loss

$= 59 - 1.77 = 57.23$ kW

EXAMPLE 4.7 *A 6-pole, 50 Hz, 3-Φ induction motor running on full load develops a useful torque of 150 Nm at a rotor frequency of 1.5 Hz. Calculate the shaft power output. If the mechanical torque lost in friction be 10 Nm, determine (a) rotor copper loss, (b) the input to the motor, and (c) the efficiency.*

The total stator loss in 700 W.

SOLUTION. $N_s = \frac{120 f_1}{P} = \frac{120 \times 50}{6} = 1000$ r.p.m.

$$s = \frac{f_2}{f_1} = \frac{1.5}{50} = 0.03 \text{ or } 3\%$$

$$N_r = (1 - s) N_s = (1 - 0.03) \times 1000 = 970 \text{ r.p.m.}$$

$$\omega_r = 2\pi n_r = \frac{2\pi \times 970}{60} = 101.58 \text{ rad/s}$$

Shaft power output, $P_0 = \tau_0 \omega_r = 150 \times 101.58 = 15237 \text{ W} = 15.237 \text{ kW}$

Mechanical power developed

$$P_{md} = (150 + 10) \times 101.58 = 16253 \text{ W} = 16.253 \text{ kW}$$

(*a*) Rotor copper loss $p_{rc} = \left(\frac{s}{1-s}\right) P_{md} = \frac{0.03}{1-0.03} \times 16253 = 502.6 \text{ W} = 0.5026 \text{ kW}$

(*b*) Input to motor, $P_i = P_{md} + p_{rc} + p_{sc} = 16.253 + 0.5026 + 0.700 = 17.4556$ kW

(*c*) Efficiency $= \frac{P_0}{P_i} = \frac{15.237}{17.4556} = 0.8729 \text{ pu} = 87.29\%$.

5.12 <u>Induction Generator</u>:

An induction motor also can be used as generator driving it by some prime mover (turbine) at a speed above the synchronous speed. However, the induction generator needs reactive power to establish the air-gap magnetic flux and turbine can not support to generate the reactive power. Hence, some external capacitor has to be connected across the stator terminal to generate reactive power as shown in Fig.5.23.

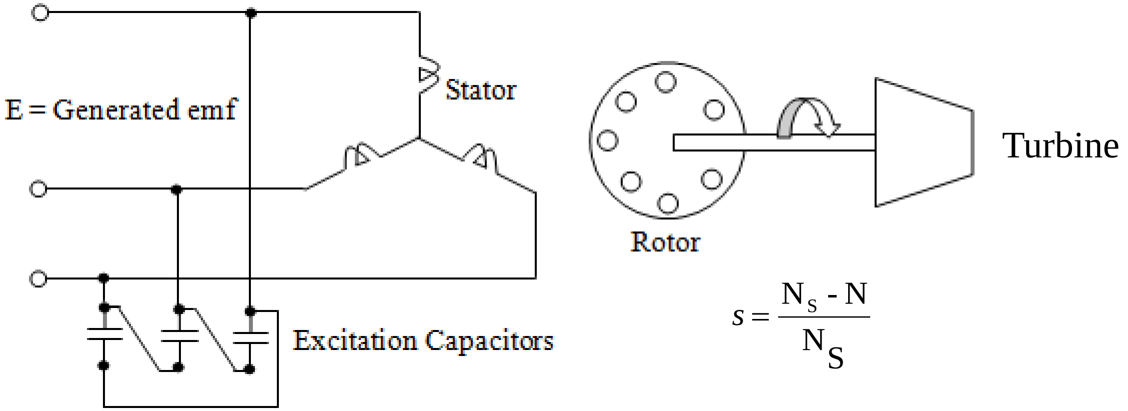


Fig.5.23 Induction generator with excitation capacitors

When the rotor is driven at speed above the synchronous speed, the slip become negative as shown in torque-slip characteristic in Fig.5.24. The torque is also shown as negative, which signify that the torque is consumed by the generator from the turbine rather than generating the torque.

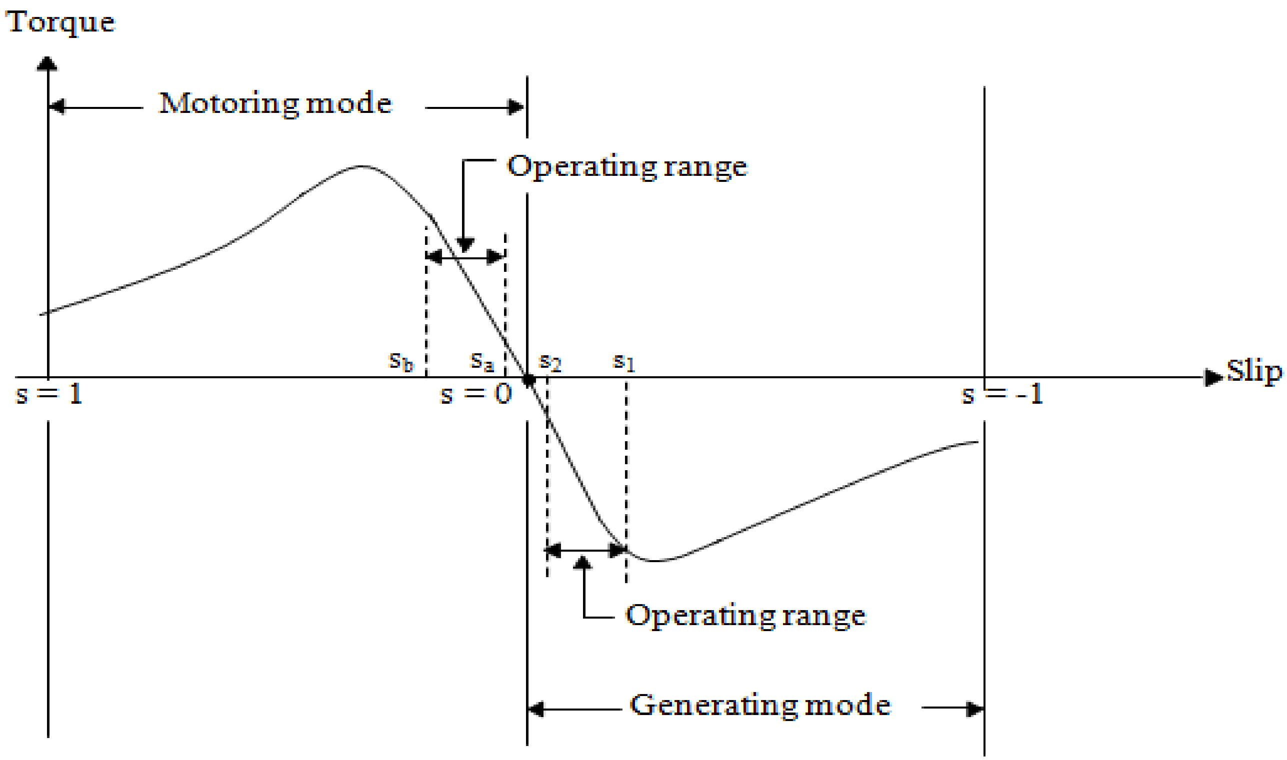


Fig.5.24 Torque-Slip characteristic of induction machine

Torque developed by motor is considered as positive, whereas torque given to induction generator is considered as negative.

It is clear from the T-S characteristic that the induction generator operates at different slip (speed) at different loading conditions. At full load it operates at slip ‘s_1’, whereas at no-load it operates at slip ‘s_2’.

The power stages in an induction generator is shown on Fig.5.25 and equivalent circuit for generating mode is shown in Fig.5.26.

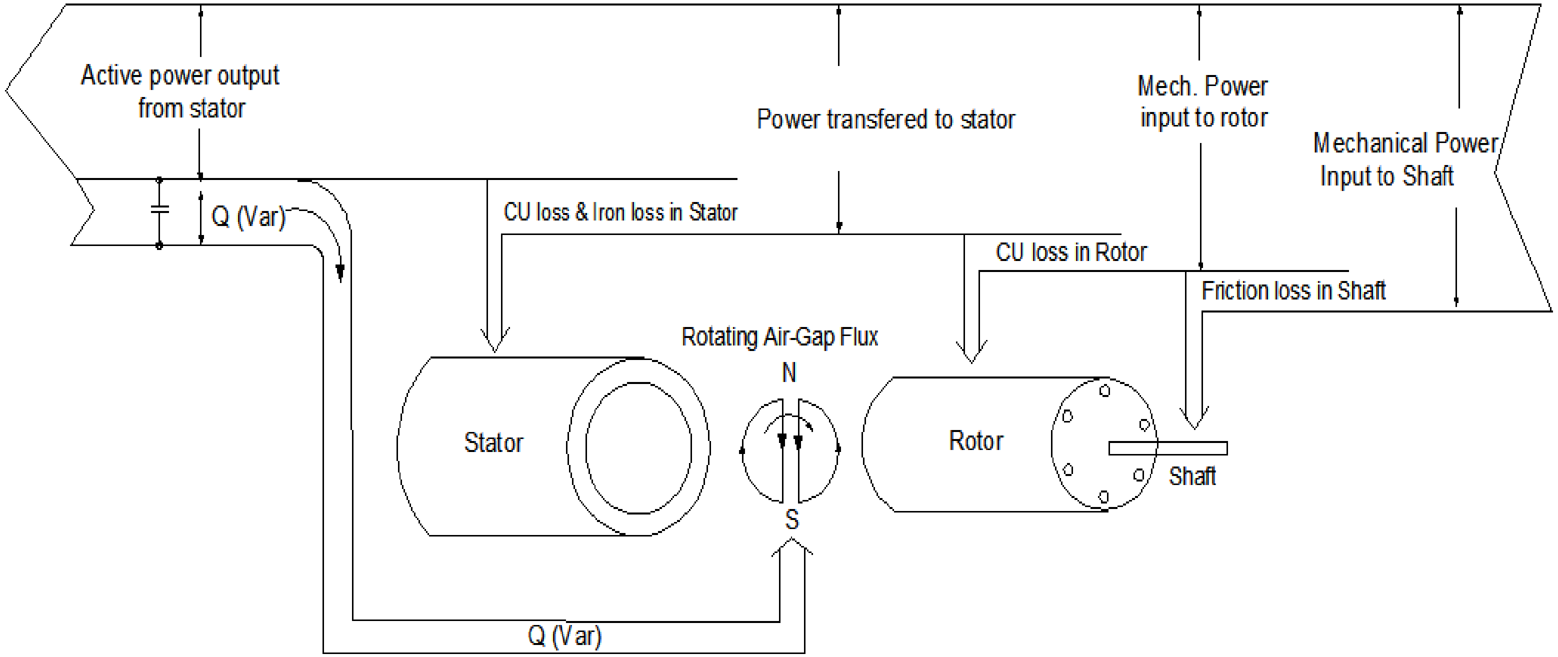


Fig.5.25 Power stages in induction generator

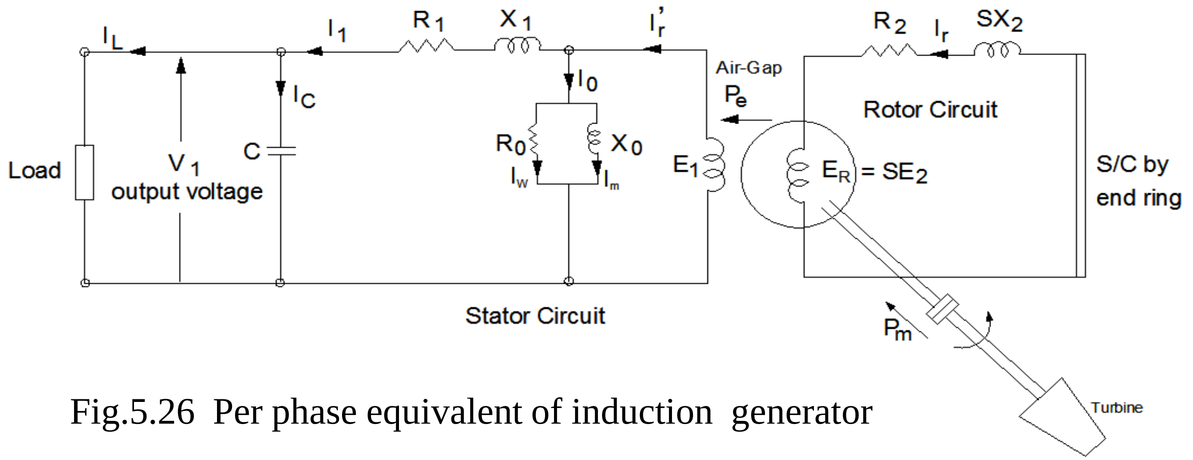


Fig.5.26 Per phase equivalent of induction generator

There will be some residual magnetic flux in the air gap between stator core and rotor core. When the rotor is rotated a speed above its synchronous speed, the rotor conductor cuts the magnetic flux. Therefore, as per Faraday's law of electromagnetic induction, a small emf E_R will induce in the rotor circuit. By transformer action a small value of emf E_1 will induce in stator winding. The emf E_1 will circulate current I_C through the capacitor, provided no load is connected across the stator terminals. The capacitor generates some reactive power which helps to produce more air gap magnetic flux. With increase magnetic flux, emf E_R and E_1 increases and with increased E_1 more current will circulate through the capacitor. In this way, the generated voltage builds up. When the voltage builds up its rated value, then load can be connected.

The voltage build up process is very much similar to the voltage build up process in DC generator. The voltage build-up process in a dc machine depends upon the residual magnetism in the field poles and the final steady terminal voltage is determined by the resistance of the field winding. Whereas in case of induction generator, the final steady terminal voltage depends on the capacitive reactance X_C of the capacitor.

The equivalent circuit during voltage build up and the no-load voltage build-up of induction generator is shown in Fig.5.27 and Fig.5.28 respectively.

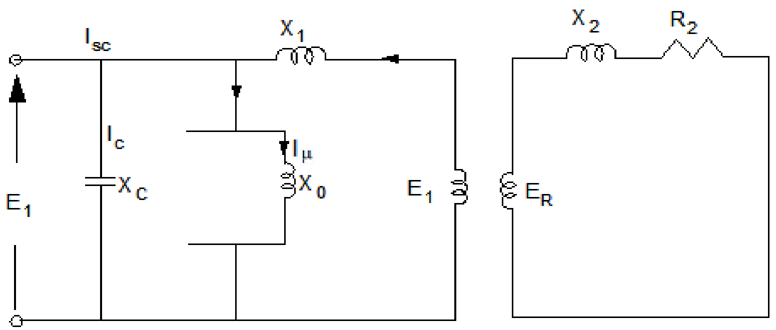


Fig.5.27 Equivalent circuit during voltage build up

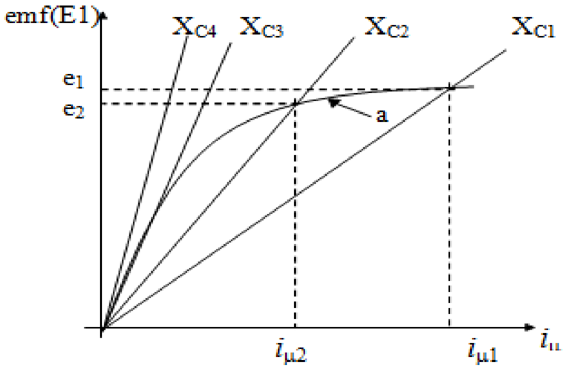


Fig.5.28 Voltage build up curve

$X = X_0 + X_1$ Total inductive reactance

i_μ = Magnetizing current

X_c = Capacitive reactance

Curve 'a' shown in Fig.5.28 is the magnetization curve of the machine showing emf (E_1) at different values of magnetizing current i_μ and the lines X_{C1}, X_{C2}, X_{C3}, and X_{C4} represents the volt-amp characteristics of different rating capacitors used for excitation.

Let us assume that the reactance of the capacitor is X_{C1}. For this value of XC, the voltage build up stops at $i_{\mu 1}$ and final steady emf is e_1.

if the capacitive reactance is increased to X_{C2} (i.e. capacitance C is decreased to C_2) , then the final steady emf is e_2.

If the capacitive reactance is increased to X_{C4}, then the voltage build up process fails.

X_{C3} is the critical value of capacitive reactance at which voltage build up process will be just successful. Therefore the value of C should be large enough so that X_C is little less than (X_1+X_0)