For Non-Homogeneous Linear Eqn:

$c_0 a_n + c_1 a_{n-1} + \dots + c_k a_{n-k} = f(n)$

G.S. : $a_n = a_n^{h} + a_n^{P}$

Case I: $\boxed{\text{If } f(n) = \text{constant}}$

1. find P by putting $\boxed{(a_n = P)}$ in eqn.
2. If fails put $\boxed{a_n = nP}$
3. Again if fails put $\boxed{a_n = n^2 P}$

⋮

Case II: If $f(n)$ is Polynomial of degree m.

i.e. $f(n) = c_0 + c_1 n + c_2 n^2 + \dots + c_m . n^m$.

Then,

1. put $a_n = d_0 + d_1 n + d_2 n^2 + \dots + d_m . n^m$.
2. Find $d_0, d_1, \dots, d_m$ by comparing coefficients.

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Case III: If $f(n)$ is exponential fun

i.e. $f(n) = p.a^n$.

Then,

$\boxed{\text{P.S. is : } a_n = d a^n}$

If homo. sol^n contains a term

Then, $s(k) = n(d.a^n)$

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• Sol^n to Linear - Non - Homogenous Eq^n:

Eg:

Case I: $\Rightarrow$ with initial conditions, $a_0 = 2$ & $a_1 = 5$

$a_n - 2a_{n-1} + a_{n-2} = 1$ ——— (1)

Let the homogonous eq^n. be.

$a_n - 2a_{n-1} + a_{n-2} = 0$

put $a_n = r^n$

$r^n - 2r^{n-1} + r^{n-2} = 0$

or, $r^2 - 2r + 1 = 0$ [$\because$ divide by r^{n-2}]

or, $r^2 - r - r + 1 = 0$

or, $r(r-1) - r(r-1) = 0$

or, $(r-1)(r-1) = 0$

$\therefore r = 1, 1$

Then, its, homogeneous sol^n. is,

$$\boxed{a_n^h = (C_1 + C_2 n) \cdot 1^n}$$

Now,

P.S.: put $a_n = P$ in (1),

$P - 2P + P = 1 \Rightarrow 0 = 1$

Rule fails,

So, put $a_n = nP$ in (1).

$n(P) - 2(n-1)P + (n-2)P = 1$

or, $nP - 2nP + 2P + nP - 2P = 1$

$\Rightarrow 0 = 1$ $\therefore$ Rule fails.

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Put, $a_n = n^2 p$

Then,

$n^2p - 2(n-1)^2 p + (n-2)^2 p = 1$

$2p = 1, \ p = \frac{1}{2}$

$$\boxed{a_n^p = n^2 \frac{1}{2}}$$

Then,

$$\boxed{a_n = (C_1 + C_2 n) 1^n + \frac{1}{2} n^2} \quad \text{--- (ii)}$$

put $n = 0$ in (ii),

$a_0 = (C_1 + 0) 1^0 + \frac{1}{2} 0^2$

or, $a_0 = C_1$

$\therefore \boxed{C_1 = 2}$

put $n = 1$ in (ii),

$a_1 = (C_1 + C_2 . 1) 1^1 + \frac{1}{2}$

or, $5 = (2 + C_2) 1 + \frac{1}{2}$

or, $5 = \frac{4 + 2C_2 + 1}{2}$

or, $10 = 4 + 2C_2 + 1$

or, $\boxed{C_2 = \frac{5}{2}}$

$$\boxed{\therefore a_n = \left(2 + \frac{5}{2} n\right) 1^n + \frac{1}{2} n^2}$$

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Case II: If $f(n)$ is Polynomial

$a_n - 8a_{n-1} = 5 + 14n$ ——— (1)

sol^n:

→ let it's homogenous eq^n. be:

$a_n - 8a_{n-1} = 0$

put $a_n = r^n$

$r^n - 8r^{n-1} = 0$

$r - 8 = 0$ [∵ divide by r^{n-1}]

∴ $r = 8$

$\boxed{a_n^h = c_1 8^n}$

P.S.

let $a_n = d_0 + d_1 n$

Then, from (1),

$d_0 + d_1 n - 8(d_0 + d_1(n-1)) = 5 + 14n$

$d_0 + d_1 n - 8d_0 - 8d_1 n + 8d_1 = 5 + 14n$

$-7d_0 - 7d_1 n + 8d_1 = 5 + 14n$

Comp. coefficients,

$-7d_1 \cancel{n} = 14\cancel{n}$

or, $\boxed{d_1 = -2}$

Also,

$-7d_0 + 8d_1 = 5$

or, $-7d_0 - 16 = 5$

or, $-7d_0 = 21$

∴ $\boxed{d_0 = -3}$

∴ $a_n^p = -3 + (-2)n = -3 - 2n$

$\therefore a_n = a_n^h + a_n^p$

$$\boxed{a_n = C_1 . 8^n - 3 - 2n}$$

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Case III: If $f(n)$ is in exponential form:

$a_n - 8a_{n-1} = 5.2^n$ —(1)

let its homogeneous eqn. be:

$a_n - 8a_{n-1} = 0$

put $a_n = r^n$

Then,

$r^n - 8r^{n-1} = 0$

or, $r - 8 = 0$ [∵ divide both side by r^{n-1}]

∴ $r = 8$

$$\boxed{\therefore a_n^h = c_1 . 8^n}$$

P.S. $f(n) = 5 . 2^n$

, $a_n = d . 2^n$

put in eqn. (1),

$d . 2^n - 8d . 2^{n-1} = 5 . 2^n$

or, $d . 2^n - \cancel{8}^4 d . \frac{2^n}{2} = 5 . 2^n$

or, $d . 2^n - 4d . 2^n = 5 . 2^n$

or, $2^n (d - 4d) = 5 . 2^n$

or, $\cancel{2^n} (-3d) = 5 . \cancel{2^n}$

or, $\boxed{d = -5/3}$

$$\therefore \boxed{a_n^p = -5/3 . 2^n}$$

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G.S. : $a_n = a_n^h + a_n^p$

$$\boxed{a_n = c_1 8^n - \frac{5}{3} 2^n}$$