

The optimisations described in this report have been implemented on a quantum computer using the resources provided by DWave Systems.

0.1 Problem Description

Given N stocks, their covariance matrix and average returns over a time period (computable using classical computers too since it can be done beforehand, or offline), an (optional) expected average return amount, (optional) desired number of stocks, to come up with a portfolio that satisfies these conditions to different degrees and also minimises the investment risk.

0.2 Boolean Variables

Define a boolean variable x_i for the i^{th} stock. $x_i = 1$ means that we include this stock in our portfolio, $x_i = 0$ means that we do not include this stock in our portfolio.

0.3 Objective Function

The objective function we want to minimise is the investment risk R incurred. It is defined as follows-

$$R = \sum_{i=1}^n \sigma_{ii} x_i^2 + \sum_{i=1}^n \sum_{j=i+1}^n \sigma_{ij} x_i x_j$$

where σ_{ij} is the covariance between the i^{th} and the j^{th} stocks. (σ_{ii} is the variance of the i^{th} stock.)

0.4 Fixed number of Stocks Constraint

We want to allow exactly f stocks, hence the following constraint-

$$\left(\sum_{i=1}^n x_i - f \right)^2 = 0$$

This gives a quadratic form equation. To satisfy this, we need exactly f many x_i set to 1.

0.5 Desired Return Constraint

We want to ensure that the Returns we get are as close to the Desired Returns as possible, therefore-

$$\left(\sum_{i=1}^n r_i x_i - \mu_p \right)^2 = 0$$

where r_i denotes the average returns of the i^{th} stock and μ_p is the Desired Returns.