SC 601

Assignment

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Modelling using Levy's Technique.

Suppose that unknown system is $H(s)$

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2 + \cdots}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots}$$

$$H(j\omega) = \frac{(b_0 - b_2\omega^2 + b_4\omega^4 \cdots) + j\omega(b_1 - b_3\omega^2 + b_5\omega^4 \cdots)}{(a_0 - a_2\omega^2 + a_4\omega^4 \cdots) + j\omega(a_1 - a_3\omega^2 + a_5\omega^4 \cdots)}$$

$$H(j\omega) = \frac{\alpha + j\omega\beta}{\sigma + j\omega\gamma} \quad \cdots \text{(notation for above)}$$

$\alpha, \beta, \sigma, \gamma$ are dependent on frequency.

Hence for each frequency ω_k, we have

$\alpha_k, \beta_k, \sigma_k, \gamma_k$

we try to fit the model to the system

so, we try to generate

$$F(\omega_k) = R(\omega_k) + j\,I(\omega_k)$$

The error is

$$E(\omega_n) = F(\omega_n) - \frac{N(\omega_n)}{D(\omega_n)} \quad \dots \text{x}$$

$$\therefore D(\omega_n)\,E(\omega_n) = D(\omega_n)F(\omega_n) - N(\omega_n)$$
$$= A(\omega_n) + jB(\omega_n)$$

now, $D(\omega_n)F(\omega_n) - N(\omega_n) = \left(R(\omega_n) + jI(\omega_n)\right) \cdot (\sigma_k + j\omega\gamma_n) - (\alpha_n + j\omega\beta_n)$

$$= (R_k\sigma_k - \omega_n I_n \gamma_k - \alpha_k) + j(I_n\sigma_k + R_k\gamma_n\omega_k - \omega_n\beta_k)$$

$$= A(\omega_n) + jB(\omega_n)$$

& Error sum is

$$\epsilon = \sum_{n=0}^{m} |E(\omega_n)D(\omega_n)|^2 \quad \because m \text{ is total observed datapoints}$$

$$= \sum_{n=0}^{m} \left(A(\omega_n)^2 + B(\omega_n)^2\right)$$

$$= \sum_{n=0}^{m} (R_k\sigma_k - I_n\gamma_n\omega_n - \alpha_n)^2 + (I_n\sigma_n + \gamma_n R_n\omega_n - \beta_n\omega_n)^2.$$

$$\dots \text{A}$$

note that

$$\alpha_k = b_0 - b_2 \omega_k^2 + b_4 \omega_k^4 \cdots$$

$$\beta_k = b_1 - b_3 \omega_k^4 + b_5 \omega_k^4 \cdots$$

$$a_0 = 1$$

$$\sigma_k = 1 - a_2 \omega_k^2 + a_4 \omega_k^4 \cdots$$

$$\gamma_k = a_1 - a_3 \omega_k^2 + a_5 \omega_k^4 \cdots$$

Consider a 7^{th} order Transfer function as,

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + a_6 s^6 + a_7 s^7}$$

$$\therefore H(j\omega) = \frac{b_0 + b_1 j\omega + b_2 (j\omega)^2}{a_0 + a_1 j\omega - a_2 \omega^2 - j\omega^3 a_3 + \omega^4 a_4 + j\omega^5 a_5 - \omega^6 a_6 - j\omega^7 a_7}$$

$$H(j\omega) = \frac{(b_0 - \omega^2 b_2) + j\omega\, b_1}{(a_0 - \omega^2 a_2 + \omega^4 a_4 - \omega^6 a_6) + j\omega(a_1 - \omega^2 a_3 + \omega^4 a_5 - \omega^6 a_7)}$$

$$= \frac{\alpha + j\omega \beta}{\sigma + j\omega \gamma}$$

where,

$$\alpha_k = b_0 - \omega_k^2 b_2$$

$$\beta_k = b_1$$

$$\sigma_k = a_0 - a_2 \omega_k^2 + a_4 \omega_k^4 - a_6 \omega_k^6$$

$$\tau_k = a_1 - a_3 \omega_k^2 + a_5 \omega_k^4 - a_7 \omega_k^6$$

for levy's technique $a_0 = 1$

now, the error is (from eqn (A))

$$\epsilon = \sum_{k=0}^{m} (R_k \sigma_k - I_k \tau_k \omega_k - \alpha_k)^2 + (I_k \sigma_k + \tau_k R_k \omega_k - \beta_k \omega_k)^2$$

We set the partial derivative of ϵ wrt parameters $b_0, b_1, b_2, a_1, a_2 \ldots, a_7$ equal to 0 & also use following terms for simplification.

$$\Lambda_h = \sum_{k=0}^{m} \omega_k^h \qquad S_h = \sum_{k=0}^{m} \omega_k^h R_k$$

$$T_h = \sum_{k=0}^{m} \omega_k^h I_k \qquad U_h = \sum_{k=0}^{m} \omega_k^h (R_k^2 + I_k^2)$$

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$$\frac{\partial \epsilon}{\partial b_0} = \sum_{n=0}^{m} 2\left(R_n \sigma_n - I_n \gamma_k \omega_k - \alpha_n\right)(-1) = 0$$

$$\Rightarrow \sum_{n=0}^{m} \Big(R_n (a_0 - a_2 \omega_n^2 + a_4 \omega_n^4 - a_6 \omega_n^6) - I_k \omega_k (a_1 - a_3 \omega_k^2 + a_5 \omega_k^4 - a_7 \omega_k^6) - (b_0 - \omega_n^2 b_2) \Big) = 0$$

$$\Rightarrow \sum_{n=0}^{m} R_k \omega_k^0 = \sum_{n=0}^{m} \omega_k^2 R_k a_2 - \sum_{n=0}^{m} \omega_k^4 R_k a_4 + \sum_{n=0}^{m} \omega_n^6 R_k a_6 + \sum_{n=0}^{m} \omega_n I_k a_1 - \sum_{n=0}^{m} \omega_k^3 I_k a_3 + \sum_{n=0}^{m} \omega_n^5 I_k a_5 - \sum_{n=0}^{m} \omega_k^7 I_k a_7 + \sum_{n=0}^{m} b_0 \omega_n^0 - \sum_{n=0}^{m} \omega_n^2 b_2$$

$$S_0 = a_2 S_2 - a_4 S_4 + a_6 S_6 + a_1 T_1 - a_3 T_3 + a_5 T_5 - a_7 T_7 + b_0 \Lambda_0 - b_2 \Lambda_2 \quad \ldots\ldots (1)$$

Similarly,

$$\frac{\partial \epsilon}{\partial b_1} = \sum_{n=0}^{m} 2\left(I_k \sigma_n + \gamma_n R_n \omega_k - \beta_n \omega_n\right)(-\omega_n) = 0$$

$$\Rightarrow \sum_{n=0}^{m} I_n \sigma_n \omega_n + R_n \gamma_k \omega_n^2 - \beta_n \omega_n^2 = 0$$

$$\Rightarrow \sum_{n=0}^{m} I_n \omega_n (a_0 - a_2 \omega_n^2 + a_4 \omega_n^4 - a_6 \omega_n^6)$$

$$+ R_n \omega_n^2 (a_1 - a_3 \omega_n^2 + a_5 \omega_n^4 - a_7 \omega_n^6)$$

$$- \omega_n^2 b_1 = 0$$

$$\Rightarrow \sum_{n=0}^{m} \omega_n I_n a_0 - \sum_{n=0}^{m} \omega_n^3 I_n a_2 + \sum_{n=0}^{m} \omega_n^5 I_n a_4$$

$$- \sum_{n=0}^{m} \omega_n^7 I_n a_6 + \sum_{n=0}^{m} \omega_n^2 R_n a_1 - \sum_{n=0}^{m} \omega_n^4 R_n a_3$$

$$+ \sum_{n=0}^{m} \omega_n^6 R_n a_5 - \sum_{n=0}^{m} \omega_n^8 R_n a_7 - \sum_{n=0}^{m} \omega_n^2 b_1 = 0$$

$$a_0 T_1 - a_2 T_3 + a_4 T_5 - a_6 T_7$$

$$+ a_1 S_2 - a_3 S_4 + a_5 S_6 - a_7 S_8 - b_1 \lambda_2 = 0$$

$$\therefore T_1 = a_2 T_3 - a_4 T_5 + a_6 T_7 - a_1 S_2$$

$$+ a_3 S_4 - a_5 S_6 + a_7 S_8 + b_1 \lambda_2$$

.... (2)

Similarly.

$$\frac{\partial E}{\partial b_2} = \sum_{n=0}^{m} 2(R_n \sigma_n - I_n \tau_n \omega_n - \alpha_n)(\omega_n^2) = 0$$

$$\Rightarrow \sum_{n=0}^{m} R_n \sigma_n \omega_n^2 - I_n \tau_n \omega_n^3 - \alpha_n \omega_n^2 = 0$$

$$\sum_{n=0}^{m} R_n \omega_n^2 (1 - a_2 \omega_n^2 + a_4 \omega_n^4 - a_6 \omega_n^6)$$
$$- I_n \omega_n^3 (a_1 - a_3 \omega_n^2 + a_5 \omega_n^4 - a_7 \omega_n^6)$$
$$- (b_0 - \omega_n^2 b_2) \omega_n^2 = 0$$

- $$S_2 - a_2 S_4 + a_4 S_6 - a_6 S_8$$
$$- a_1 T_3 + a_3 T_5 - a_5 T_7 + a_7 T_9$$
$$- b_0 \Lambda_2 + b_2 \Lambda_4 = 0$$

$$S_2 = a_2 S_4 - a_4 S_6 + a_6 S_8$$
$$+ a_1 T_3 - a_3 T_5 + a_5 T_7 - a_7 T_9$$
$$+ b_0 \Lambda_2 - b_2 \Lambda_4 \quad \ldots\ldots (3)$$

Similarly.

$$\frac{\partial e}{\partial a_1} = \sum_{n=0}^{m} (R_n \sigma_n - I_n \tau_n \omega_n - \alpha_n)(-I_n \omega_n)$$
$$+ \sum_{n=0}^{m} (I_n \sigma_n + \tau_n R_n \omega_n - \beta_n \omega_n)(R_n \omega_n) = 0$$

$$\Rightarrow \sum_{n=0}^{m} - I_n R_n \sigma_n \omega_n + I_n^2 \tau_n \omega_n^2 + I_n \alpha_n \omega_n$$
$$+ \sum_{n=0}^{m} I_n R_n \sigma_n \omega_n + R_n^2 \tau_n \omega_n^2 - R_n \beta_n \omega_n^2 = 0$$

$$\Rightarrow \sum_{n=0}^{m} \omega_n^2 (I_n^2 + R_n^2) \gamma_n - R_n \omega_n^2 \beta_n + I_n \omega_n \alpha_n = 0$$

$$\Rightarrow \sum_{n=0}^{m} \omega_n^2 (I_n^2 + R_n^2) [a_1 - a_3 \omega_n^2 + a_5 \omega_n^4 - a_7 \omega_n^6]$$

$$- \sum_{n=0}^{m} \omega_n^2 R_n [b_1] + \sum_{n=0}^{m} \omega_n I_n [b_0 - \omega_n^2 b_2] = 0$$

$$\Rightarrow a_1 U_2 - a_3 U_4 + a_5 U_6 - a_7 U_8$$

$$- b_1 S_2 + b_0 T_1 - b_2 T_3 = 0 \quad \ldots (4)$$

Similarly.

$$\frac{\partial E}{\partial a_2} = \sum_{n=0}^{m} - R_n^2 \omega_n^2 \sigma_n + I_n R_n \omega_n^3 \gamma_n + R_n \omega_n^2 \alpha_n$$

$$+ \sum_{n=0}^{m} - I_n^2 \omega_n^2 \sigma_n - I_n R_n \omega_n^3 \gamma_n + I_n \omega_n^3 \beta_n = 0$$

$$\Rightarrow \sum_{n=0}^{m} - (R_n^2 + I_n^2) \omega_n^2 [a_0 - a_2 \omega_n^2 + a_4 \omega_n^4 - a_6 \omega_n^6]$$

$$+ \sum_{n=0}^{m} R_n \omega_n^2 [b_0 - \omega_n^2 b_2] + \sum_{n=0}^{m} I_n \omega_n^3 b_1 = 0$$

$$\Rightarrow - U_2 + a_2 U_4 - a_4 U_6 + a_6 U_8$$

$$+ b_0 S_2 - b_2 S_4 + b_1 T_3 = 0 \quad \ldots (5)$$

$$\frac{\partial \epsilon}{\partial a_3} = \sum_{n=0}^{m} 2\left(R_n \sigma_n - I_n \gamma_n \omega_n - \alpha_n\right)\left(-\omega_n^{3} I_n\right)$$

$$+ \sum_{n=0}^{m} 2\left(I_n \sigma_n + R_n \gamma_n \omega_n - \beta_n \omega_n\right)\left(R_n \omega_n^{3}\right) = 0$$

$$\Rightarrow \sum_{n=0}^{m} - I_n R_n \omega_n^{3} \sigma_n + I_n^{2} \omega_n^{4} \gamma_n + I_n \omega_n^{3} \alpha_n$$

$$+ \sum_{n=0}^{m} I_n R_n \omega_n^{3} \sigma_n + R_n^{2} \omega_n^{4} \gamma_n - R_n \omega_n^{4} \beta_n = 0$$

$$\sum_{n=0}^{m} \omega_n^4 (R_n^2 + I_n^2)\left[a_1 - a_3\omega_n^2 + a_5\omega_n^4 - a_7\omega_n^6\right]$$

$$+ \sum_{n=0}^{m} \omega_n^3 I_n \left[b_0 - \omega_n^2 b_2\right] - \sum_{n=0}^{m} \omega_n^4 R_n [b_1] = 0$$

$$\Rightarrow \quad a_1 V_4 - a_3 V_6 + a_5 V_8 - a_7 V_{10}$$

$$+ b_0 T_3 - b_2 T_5 - b_1 S_4 = 0 \quad \cdots (6)$$

$$\frac{\partial \epsilon}{\partial a_4} = \sum_{n=0}^{m} 2\left(\sigma_k R_n - I_n \gamma_n \omega_n - \alpha_n\right)\left(R_n \omega_n^4\right) \quad (11)$$

$$+ \sum_{n=0}^{m} 2\left(I_n \sigma_n + R_n \gamma_n \omega_n - \beta_n \omega_n\right)\left(I_n \omega_n^4\right) = 0$$

$$\Rightarrow \sum_{n=0}^{m} R_n^2 \omega_n^4 \sigma_n - I_n R_n \omega_n^5 \gamma_n - R_n \omega_n^4 \alpha_k$$

$$+ \sum_{n=0}^{m} I_n^2 \omega_n^4 \sigma_k + I_n R_n \omega_n^5 \gamma_n - I_n \omega_n^5 \beta_n = 0$$

$$\Rightarrow \sum_{n=0}^{m} \omega_n^4 \left(I_n^2 + R_n^2\right)\left[1 - a_2 \omega_n^2 + a_4 \omega_n^4 - a_6 \omega_n^6\right]$$

$$- \sum_{n=0}^{m} \omega_n^4 R_n \left[b_0 - \omega_n^2 b_2\right] - \sum_{n=0}^{m} \omega_n^5 I_k \, b_1 = 0$$

$$\Rightarrow U_4 - a_2 U_6 + a_4 U_8 - a_6 U_{10}$$

$$- b_0 S_4 + b_2 S_6 - b_1 T_5 = 0 \quad \text{---} \; (7)$$

$$\frac{\partial \epsilon}{\partial a_5} = \sum_{n=0}^{m} 2\left(R_n \sigma_n - I_n \omega_n \gamma_n - \alpha_n\right)\left(-I_n \omega_n^5\right)$$

$$+ \sum_{n=0}^{m} 2\left(I_n \sigma_n + R_n \gamma_k \omega_k - \beta_n \omega_n\right)\left(R_n \omega_n^5\right) = 0$$

$$\Rightarrow \sum_{n=0}^{m} - I_n R_n \omega_n^5 \sigma_k + I_n^2 \omega_n^6 \gamma_k + I_n \omega_n^5 \alpha_k$$

$$+ \sum_{n=0}^{m} I_n R_n \omega_n^5 \sigma_k + R_n^2 \omega_n^6 \gamma_k - R_n \omega_n^6 \beta_k = 0$$

$$\Rightarrow \sum_{n=0}^{m} \omega_n^6 (R_n^2 + I_n^2)\left[a_1 - a_3 \omega_n^2 + a_5 \omega_n^4 - a_7 \omega_n^6\right]$$

$$+ \sum_{n=0}^{m} I_n \omega_n^5 \left[b_0 - \omega_n^2 b_2\right] \pm \sum_{n=0}^{m} \omega_n^6 R_n b_1 = 0$$

$$\Rightarrow a_1 U_6 - a_3 U_8 + a_5 U_{10} - a_7 U_{12}$$

$$+ b_0 T_5 - b_2 T_7 \pm b_1 S_6 = 0 \quad \ldots (8)$$

$$\frac{\partial E}{\partial a_6} = \sum_{n=0}^{m} 2(\sigma_n R_n - I_n \omega_n \tau_n - \alpha_n)(-R_n \omega_n^6)$$

$$+ \sum_{n=0}^{m} 2(I_n \sigma_n + R_n \omega_n \tau_n - \beta_n \omega_n)(-I_n \omega_n^6) = 0$$

$$\Rightarrow \sum_{n=0}^{m} -R_n^2 \omega_n^6 \sigma_n + R_n I_n \omega_n^7 \tau_n + R_n \omega_n^6 \alpha_n$$

$$+ \sum_{n=0}^{m} -I_n^2 \omega_n^6 \sigma_n - I_n R_n \omega_n^7 \tau_n + I_n \omega_n^7 \beta_n = 0$$

$$\Rightarrow \sum_{n=0}^{m} -\omega_n^6 (R_n^2 + I_n^2)\left[1 - a_2 \omega_n^2 + a_4 \omega_n^4 - a_6 \omega_n^6\right]$$

$$+ \sum_{n=0}^{m} \omega_n^6 R_n \left[b_0 - \omega_n^2 b_2\right] + \sum_{n=0}^{m} \omega_n^7 I_n b_1 = 0$$

$$\Rightarrow -U_6 + a_2 U_8 - a_4 U_{10} - a_6 U_{12}$$

$$+ b_0 S_6 - b_2 S_8 + b_1 T_7 = 0 \quad \ldots (9)$$

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$$\frac{\partial \epsilon}{\partial a_7} = \sum_{n=0}^{m} 2\left(\sigma_n R_n - I_n \omega_n \gamma_n - \alpha_n\right)\left(-I_n \omega_n^7\right)$$

$$+ \sum_{n=0}^{m} 2\left(I_n \sigma_n + R_n \gamma_n \omega_n - \beta_n \omega_n\right)\left(R_n \omega_n^7\right) = 0$$

$$\Rightarrow \sum_{n=0}^{m} - I_n R_n \omega_n^7 \sigma_n + I_n^2 \omega_n^8 \gamma_n + I_n \omega_n^7 \alpha_n$$

$$+ \sum_{n=0}^{m} I_n R_n \omega_n^7 \sigma_n + R_n^2 \omega_n^8 \gamma_n - R_n \omega_n^8 \beta_n = 0$$

$$\Rightarrow \sum_{n=0}^{m} \omega_n^8 \left(R_n^2 + I_n^2\right)\left[a_1 - a_3 \omega_n^2 + a_5 \omega_n^4 - a_7 \omega_n^6\right]$$

$$+ \sum_{n=0}^{m} \omega_n^7 I_n \left[b_0 - \omega_n^2 b_2\right] - \sum_{n=0}^{m} \omega_n^8 R_n b_1 = 0$$

$$\Rightarrow a_1 U_8 - a_3 U_{10} + a_5 U_{12} - a_7 U_{14}$$

$$+ b_0 T_7 - b_2 T_9 - b_1 S_8 = 0$$

...... (10)

We can now use equations (1), (2), ... (10) to determine the parameters

$b_0, b_1, b_2, a_1, a_2 ...... a_7$

From these equations, we can write, in Matrix form,

$$
\begin{bmatrix}
\Lambda_0 & 0 & -\Lambda_2 & T_1 & S_2 & -T_3 & -S_4 & T_5 & S_6 & -T_7 \\
0 & \Lambda_2 & 0 & -S_2 & T_3 & S_4 & -T_5 & -S_6 & T_7 & S_8 \\
\Lambda_2 & 0 & -\Lambda_4 & T_3 & S_4 & -T_5 & -S_6 & T_1 & S_8 & -T_9 \\
T_1 & -S_2 & -T_3 & U_2 & 0 & -U_4 & 0 & U_6 & 0 & -U_8 \\
S_2 & T_3 & -S_4 & 0 & U_4 & 0 & -U_6 & 0 & U_8 & 0 \\
T_3 & -S_4 & -T_5 & U_4 & 0 & -U_6 & 0 & U_8 & 0 & -U_{10} \\
S_4 & T_5 & -S_6 & 0 & U_6 & 0 & -U_8 & 0 & U_{10} & 0 \\
T_5 & -S_6 & -T_7 & U_6 & 0 & -U_8 & 0 & U_{10} & 0 & -U_{12} \\
S_6 & T_7 & -S_8 & 0 & U_8 & 0 & -U_{10} & 0 & U_2 & 0 \\
T_7 & -S_8 & -T_9 & U_8 & 0 & -U_{10} & 0 & U_{12} & 0 & -U_4
\end{bmatrix}
\begin{bmatrix}
b_0 \\ b_1 \\ b_2 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7
\end{bmatrix}
=
\begin{bmatrix}
S_0 \\ T_1 \\ S_2 \\ 0 \\ U_2 \\ 0 \\ U_4 \\ 0 \\ U_6 \\ 0
\end{bmatrix}
$$

To test the Levy's Method, we take a following 7^{th} order System

$$H(s) = \frac{5 + 7s + s^2}{1 + 23s + 15s^2 + 20s^3 + 24s^4 + 16s^5 + 12s^6 + s^7}$$

The levy's Method have been implemented on matlab & I have choosen the significant frequency range as $\omega = 0.01$ to $\omega = 4$ rad/sec

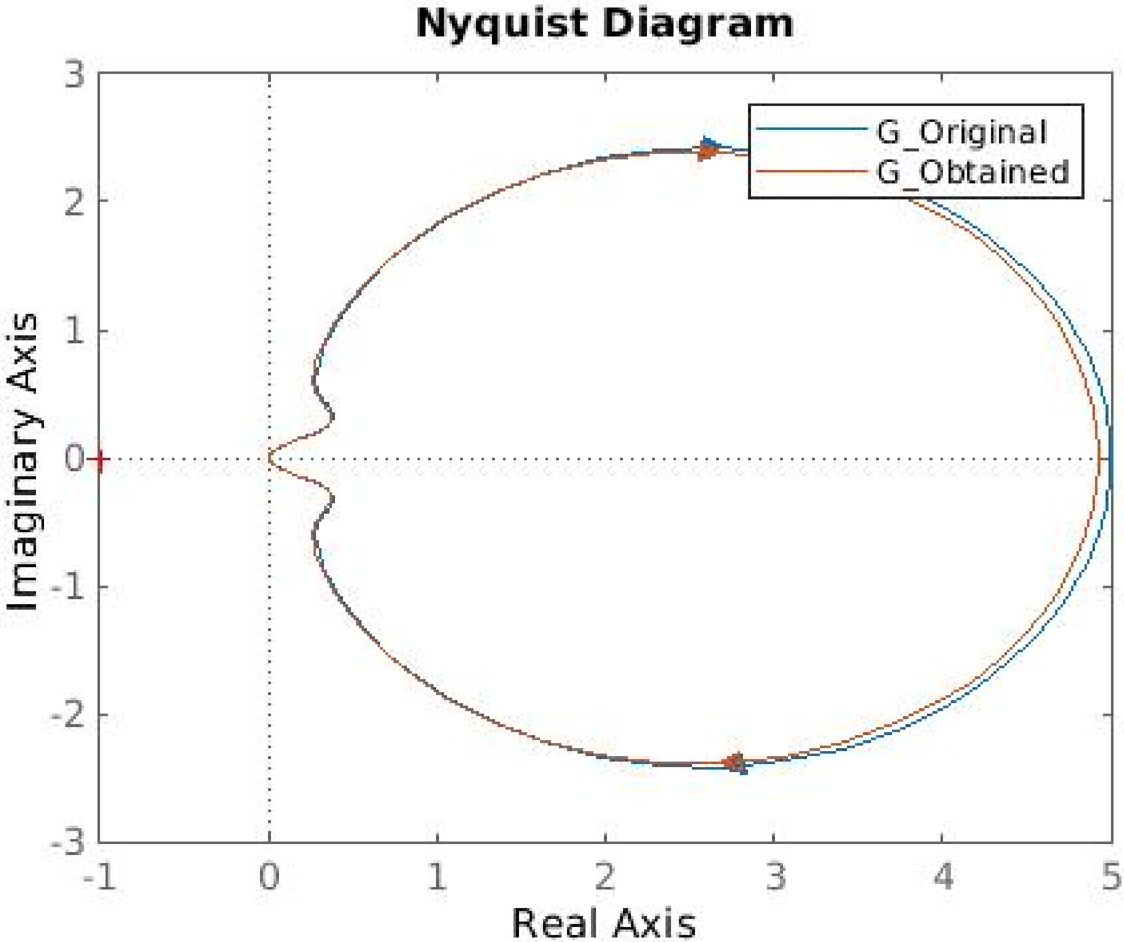
The Transfer function obtained by Levy's technique using the frequency data is,

$$H\_obt(s) = \frac{4.925 - 5.365 - 3.87s^2}{1 + 18.95s - 34.79s^2 + 22.04s^3 - 28.57s^4 - 0.02s^5 - 16.096s^6 - 4.97s^7}$$

The error is calculated using equation (x) (Pg (2)) and the Obtained value is 3.28

& Mean Error is $\frac{3.28}{80} = 0.041$ ... (where, $m = 80$ = no. of datapoints)

Also, polar plot / Nyquist plots of both the systems are almost identical meaning same response can be obtained using both TF's.


Nyquist Diagram
G_Original
G_Obtained
Imaginary Axis
Real Axis
3
2
1
0
-1
-2
-3
-1
0
1
2
3
4
5

Although the coefficients of both the T/F do not match, it is important to see that The frequency response is similar for the frequency range of intrest. In control engineering we are very much intrested in the frequency response of the system rather than the Transfer function itself (eg. a higher order system reduced to smaller order will also give the same frequency response as higher order system, hence frequency response matters).

Result:-

We have used Levy's Technique for identifying the system using only frequency data. It is concluded that the obtained system is similar to original system as the error is 0.041 which is very small. Also, Nyquist plot was obtained to verify the same.