SC-626 Systems and Control Laboratory

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INTRODUCTION

In most of the MIMO systems, couplings between input and output signals are present which may complicate the feedback controller design.
Each output is affected by each input in this case and this may cause the response to deviate from the actual reference.
So we have designed 3 decouplers to reduce this multiloop Interactions.

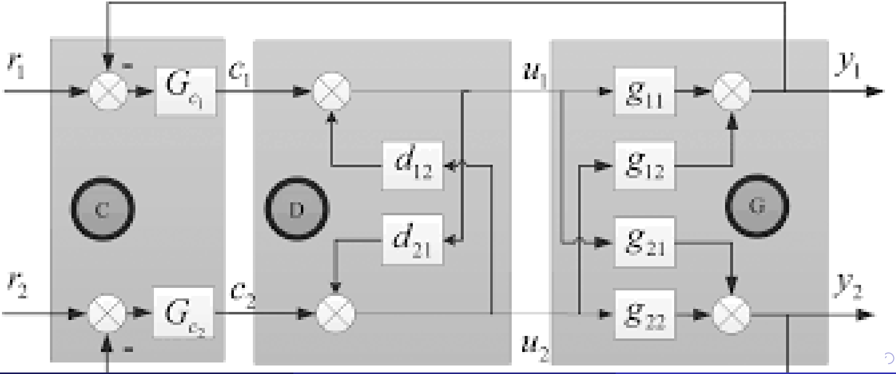
- Simplified Decoupling.
- Normalized Decoupling.
- Inverted Decoupling Internal Model Control.

Simplified Decoupling

In simplified decoupling we need to design decoupler transfer function so that decoupled system is of diagonal form which will reduce interloop interactions.

The decoupler takes the following form

$$\mathrm{D(s)} = \begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix}$$



MIMO (SIMPLIFIED DECOUPLER DESIGN)

RGA Analysis is required to decide the paring and it was found that diagonal pairing is the best pairing (i.e u1-y1,u4-y4).So we have designed decouplers for u1-y1 and u4-y4.
Our MIMO Transfer function matrix is

$$G_p = \begin{bmatrix} \frac{1.004}{44.4867s+1} e^{-3.0899s} & \frac{0.2993}{37.652s+1} e^{-5.2612s} \\ \frac{0.5095}{69.7322s+1} e^{-0.8568s} & \frac{0.9}{26.1158s+1} e^{-2.0074s} \end{bmatrix}$$

The resultant system after adding decoupler

$$G_p(s)D(s) = \begin{bmatrix} g_{11}^{*} & 0 \\ 0 & g_{22}^{*} \end{bmatrix}$$

MIMO (SIMPLIFIED DECOUPLER DESIGN)

where $g_{11}^*(s)$ and $g_{22}^*(s)$ were obtained to be,

$$g_{11}^*(s) = \frac{1.004}{44.4867s+1}e^{-3.0899s} - \frac{0.1694(26.1158s+1)}{(37.652s+1)(69.7322s+1)}e^{-5.2612s}$$

$$g_{22}^*(s) = \frac{0.9}{26.1158s+1}e^{-1.1506s} - \frac{0.1518(44.4867s+1)}{(69.7322s+1)(69.7322s+1)}e^{-5.2612s}$$

$g_{11}^*(s)$ and $g_{22}^*(s)$ are first reduced to FOPTD Model using model reduction by step input for designing PI Controllers.
The FOPTD Models obtained are

$$g_{11}^*(s) = \frac{0.836}{1+38.404s}e^{-3.334s}$$
$$g_{22}^*(s) = \frac{0.751}{1+22.018s}e^{-3.326s}$$

MIMO (SIMPLIFIED DECOUPLER DESIGN)

The Decoupler obtained is

$$D(s) = \begin{bmatrix} 1 & \frac{-0.2987(44.4867s+1)}{37.652s+1} e^{-2.1713s} \\ \frac{-0.5661(26.1158s+1)}{69.7322s+1} & e^{-1.1506s} \end{bmatrix}$$

The PI Parameters for FOPTD model($g_{11}^*(s)$ & $g_{22}^*(s)$) is obtained by Lambda tuning

$$k = \frac{1}{k_p} \frac{T}{L+T_c l}$$

$$T_i = T \quad \& \quad T_{cl} = \frac{T}{3} \text{ as a design parameter}$$

The PI parameters are designed for FOPTD g_{11s}^* and g_{22s}^* and the values obtained are

- $k_{11} = 2.847$
- $k_{22} = 7.332$
- $T_{i11} = 38.404$
- $T_{i22} = 22.018$

Response

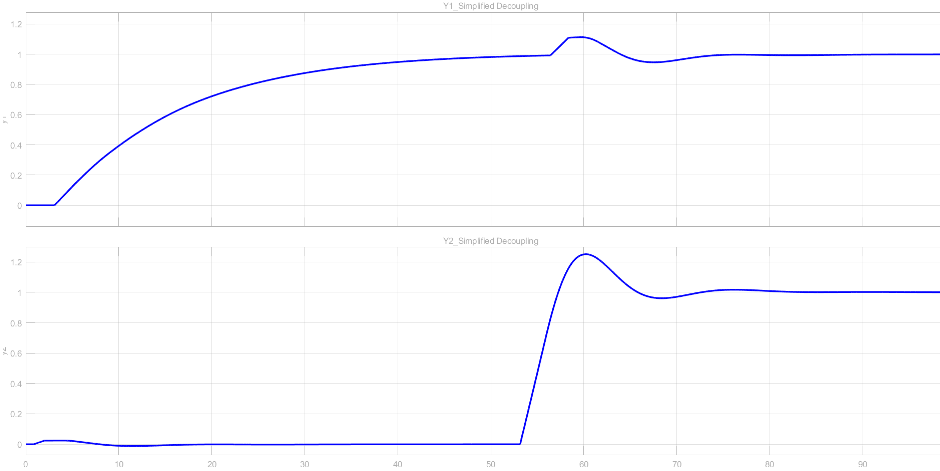


Figure: Response: Simplified Decoupler

Normalized Decoupling

- The objective is to find $G_I(s)$ which we can insert between controller $G_c(s)$ and the plant $G(s)$ in order to get the decoupled process such that if seen from controller output must be diagonal.

$$G_I = G^{-1}(s)G_R(s)$$

But the drawbacks is:

- This may result in a complicated $G_I(s)$.
- Even if we use simplified decoupling or ideal decoupling, we get complex $G_R(s)$.

- The solution to this problem is designing the decoupler using Equivalent Transfer Function (ETF) which acts as an approximate inverse for the plant transfer function matrix.
- Then the decoupler Transfer Function are appropriately selected to maintain stability, causality and properness.

Normalized Gain

$$k_{N,ij} = \frac{k_{ij}}{T_{ij}+\theta_{ij}}$$

Defining RNGA similar to RGA as

$$RNGA = K_N \otimes K_N^{-T}$$

- It captures combined changes in both steady state and dynamic when other loops are open.
- When all the loops are closed, to capture the dynamic behaviour we determine the Relative Average Residence Time Array (RARTA).

$$RARTA = RNGA \odot \Lambda$$

By using the above arrays, we can determine the elements of ETF, which takes the FOPTD form as follows,

$$\hat{K} = K \odot \Lambda$$

$$\hat{T} = RARTA \otimes T$$

$$\hat{L} = RARTA \otimes L$$

Inverse of ETF matrix is given by,

$$\hat{G}(s) = \begin{bmatrix} \frac{1}{\hat{g_{11}}(s)} & \frac{1}{\hat{g_{12}}(s)} \\ \frac{1}{\hat{g_{21}}(s)} & \frac{1}{\hat{g_{22}}(s)} \end{bmatrix}$$

Where $G_I(s)$ is,

$$G_I(s) = \hat{G}^T(s) G_R(s)$$

Where the elements of $G_R(s)$ are taken as FOPTD model.

$$G_{I,ij} = \begin{bmatrix} \frac{1}{\hat{g_{11}}(s)} & \frac{1}{\hat{g_{12}}(s)} \\ \frac{1}{\hat{g_{21}}(s)} & \frac{1}{\hat{g_{22}}(s)} \end{bmatrix} x \begin{bmatrix} g_{R,11}(s) & 0 \\ 0 & g_{R,22}(s) \end{bmatrix}$$

Decoupler matrix

$$G_{I,ij} = \begin{bmatrix} \frac{g_{R,11}(s)}{\hat{g_{11}}(s)} & \frac{g_{R,22}(s)}{\hat{g_{12}}(s)} \\ \frac{g_{R,11}(s)}{\hat{g_{21}}(s)} & \frac{g_{R,11}(s)}{\hat{g_{22}}(s)} \end{bmatrix}$$

- Elements of $G_R(s)$ are chosen such that elements of $G_I(s)$ are stable, proper and causal.
- For causality : $\theta_{R,ii} - \hat{\theta}_{ij} \geq 0$

  ensured by $\theta_{R,ii} = Max_{j=1,2}\hat{\theta}_{ij} \geq 0$

$$\text{RGA} = \begin{bmatrix} 1.2030 & -0.2030 \\ -0.2030 & 1.2030 \end{bmatrix} \qquad K_N = \begin{bmatrix} 0.0211 & 0.0070 \\ 0.0072 & 0.0320 \end{bmatrix}$$

$$\text{RNGA} = \begin{bmatrix} 1.0805 & -0.0805 \\ -0.0805 & 1.0805 \end{bmatrix} \qquad \text{RARTA} = \begin{bmatrix} 0.8982 & 0.3967 \\ 0.3967 & 0.8982 \end{bmatrix}$$

The resultant **ETF** is shown below,

$$G^{\hat{T}}(s) = \begin{bmatrix} \frac{39.9577s+1}{0.8346} e^{2.7753s} & \frac{27.6652s+1}{-2.5094} e^{0.3399s} \\ \frac{14.9379s+1}{-1.4745} e^{2.0873s} & \frac{23.4571s+1}{0.7481} e^{1.803s} \end{bmatrix}$$

Diagonal Forward **Transfer Function** becomes,

$$G_R(s) = \begin{bmatrix} \frac{1}{39.9577s+1} e^{-2.7753s} & 0 \\ 0 & \frac{1}{23.4571s+1} e^{-1.803s} \end{bmatrix}$$

$$G_I(s) = \begin{bmatrix} \frac{1}{0.8346} & \frac{27.6652s+1}{-2.5094(1+23.4571s)}e^{-1.4631s} \\ \frac{14.9379s+1}{-1.4745(39.9577s+1)}e^{-0.688s} & \frac{1}{0.7481} \end{bmatrix}$$

PI parameters

- $k_{11} = 2.4827$ $\qquad T_{i_{11}} = 39.9577$
- $k_{22} = 2.438$ $\qquad T_{i_{22}} = 23.4571$

Result

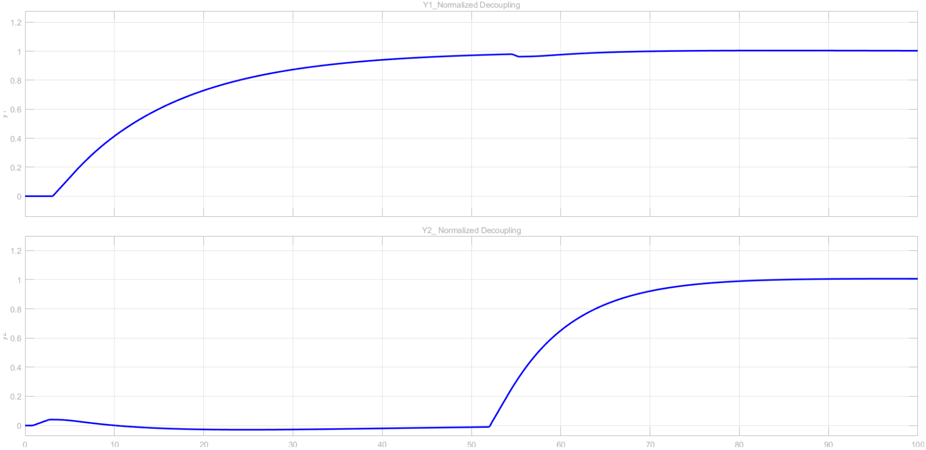


Figure: Response: Normalized Decoupling

InvertedDecoupling Internal Model Control

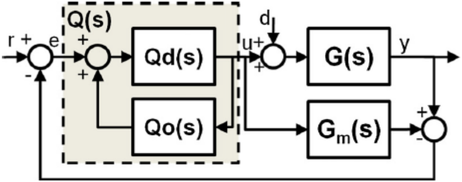


Figure: Block Diagram ID-IMC

1. Let T(s) be a Diagonal, Closed Loop transfer function matrix from references to outputs.
2. $T(s) = G(s)Q(s)[I + (G(s) - G_m(s))Q(s)]^{-1}$
3. $Q_d^{-1} - Q_o(s) = T^{-1}(s)G(s)$

ID-IMC

- $$T(s) = \begin{bmatrix} t_1(s) & 0 \\ 0 & t_2(s) \end{bmatrix}$$
- $$Q_d^{-1} - Q_o(s) = T^{-1}(s)G(s)$$
- $\begin{bmatrix} qd_{11}(s) & 0 \\ 0 & qd_{22}(s) \end{bmatrix}^{-1} - \begin{bmatrix} 0 & qo_{12}(s) \\ qo_{21}(s) & 0 \end{bmatrix} = \begin{bmatrix} \frac{g_{11}(s)}{t_1(s)} & \frac{g_{12}(s)}{t_1(s)} \\ \frac{g_{21}(s)}{t_2(s)} & \frac{g_{22}(s)}{t_2(s)} \end{bmatrix}$
- Hence, the elements of the decoupler comes out to be,
  - $qd_{11}(s) = \frac{t_1(s)}{g_{11}(s)}$
  - $qd_{22}(s) = \frac{t_2(s)}{g_{22}(s)}$
  - $qo_{12}(s) = \frac{-g_{12}(s)}{t_1(s)}$
  - $qo_{12}(s) = \frac{-g_{21}(s)}{t_2(s)}$

ID-IMC

1. The elements of the Diagonal Transfer Function Matrix T(s) takes the form,

$$t_i(s) = \frac{e^{-\theta_i s}}{\lambda_i s + 1}$$

2. λ_i is the tuning parameter
3. The elements of the Decoupler must be realizable.
   - Avoid RHP Poles
   - Avoid Time Advance Elements
4. For realizability, θ_i is chosen as minimum delay in the particular row of Transfer function matrix G(s)

ID-IMC for SBMH System

$$G(s) = \begin{bmatrix} \frac{1.004}{44.4867s+1}e^{-4.2405s} & \frac{0.2993}{37.652s+1}e^{-5.2612s} \\ \frac{0.5095}{69.7322s+1}e^{-2.0074s} & \frac{0.9}{26.1158s+1}e^{-2.0074s} \end{bmatrix}$$

The transfer function $T(s)$ have elements,

$$t_1(s) = \frac{e^{-4.2405s}}{\lambda_1 s + 1}$$

$$t_2(s) = \frac{e^{-2.0074s}}{\lambda_2 s + 1}$$

- $qd_{11} = \frac{44.4867s+1}{1.004(\lambda_1 s+1)}$
- $qd_{22} = \frac{26.1158s+1}{0.9(\lambda_2 s+1)}$
- $qo_{12} = \frac{-0.2993(\lambda_1 s+1)}{37.652s+1}e^{-1.0207s}$
- $qo_{21} = \frac{-0.5095(\lambda_2 s+1)}{69.7322s+1}$

λ

ID-IMC Results

- $\lambda_1 = 2.5$
- $\lambda_2 = 7.8$

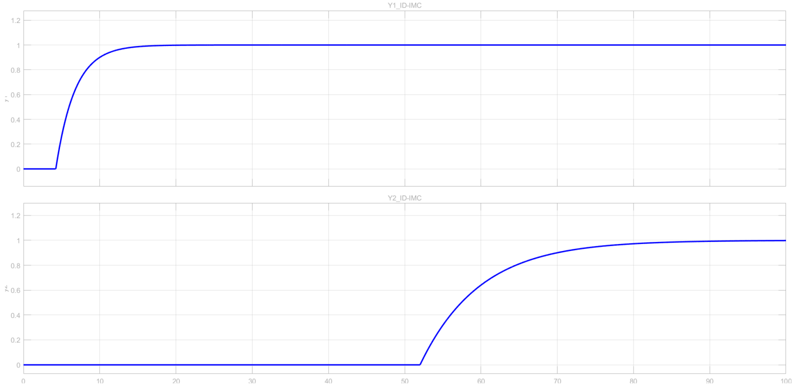


Figure: Response: ID-IMC

Comparative Study of Decouplers

The Decoupled System takes the following form,

1. Simplified Decoupler
$$g^*_{11}(s) = \frac{1.004}{1+44.4867s}e^{-3.0899s} - \frac{0.1694(26.1158s+1s)}{(37.652s+1s)(69.7322s+1)}e^{-5.2612s}$$
$$g^*_{22}(s) = \frac{0.9}{26.1158s+1}e^{-1.1506s} - \frac{0.1518(44.4867s+1s)}{(69.7322s+1)(37.652s+1)}e^{-5.262s}$$
2. Normalized Decoupler
$$G_R(s) = \begin{bmatrix} \frac{1}{39.9577s+1}e^{-2.7753s} & 0 \\ 0 & \frac{1}{23.4571s+1}e^{-1.803s} \end{bmatrix}$$
3. Inverted Decoupling Internal Model Controller
$$T(s) = \begin{bmatrix} \frac{1}{\lambda_1 s+1}e^{-4.2405s} & 0 \\ 0 & \frac{1}{\lambda_2 s+1}e^{-2.0074s} \end{bmatrix}$$

Comparative Study of Decouplers

The obtained Decoupler Matrices are

1. Simplified Decoupler

$$D(s) = \begin{bmatrix} 1 & \frac{-0.2981(44.4867s+1)}{(37.652s+1)} e^{-2.1713s} \\ \frac{-0.5661(26.1158s+1)}{(69.7322s+1)} & e^{-1.1506s} \end{bmatrix}$$

2. Normalized Decoupler

$$G_I(s) = \begin{bmatrix} \frac{1}{0.8346} & \frac{27.6652s+1}{-2.5094(23.4571s+1)} e^{-0.3399s} \\ \frac{14.9379s+1}{-1.4745(39.9577s+1)} e^{-2.0873s} & \frac{1}{0.7481} \end{bmatrix}$$

3. Inverted Decoupling Internal Model Control

- $qd_{11} = \frac{44.4867s+1}{1.004(\lambda_1 s+1)}$
- $qd_{22} = \frac{26.1158s+1}{0.9(\lambda_2 s+1)}$
- $qo_{12} = \frac{-0.299(\lambda_1 s+1)}{37.652s+1} e^{-1.0207s}$
- $qo_{21} = \frac{-0.5095(\lambda_2 s+1)}{69.7322s+1}$

Comparison of Response

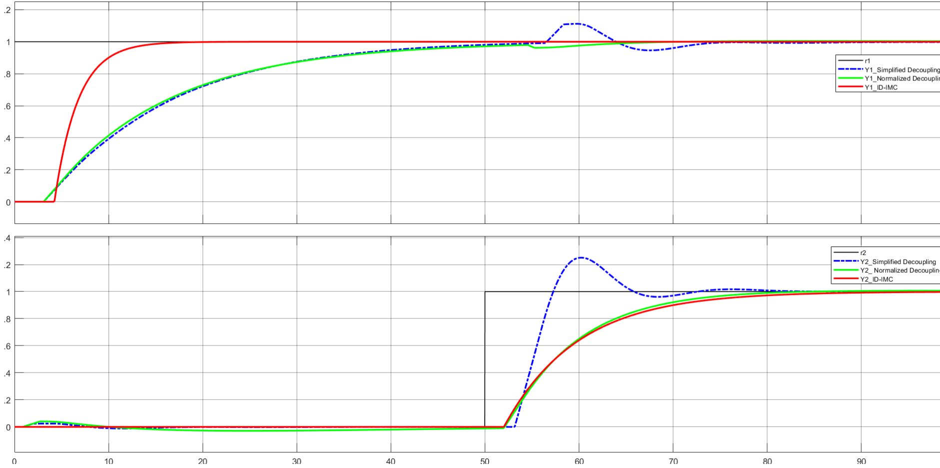


Figure: Comparison of Response

Comparison of Control Efforts

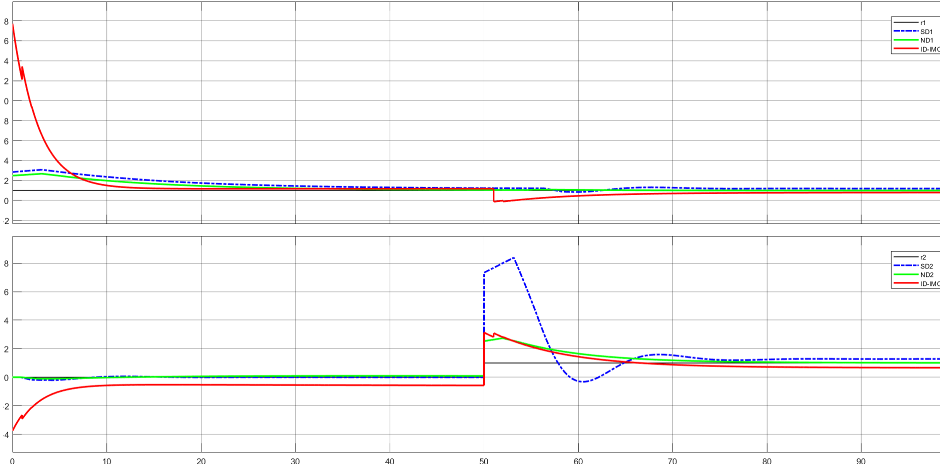


Figure: Control Inputs

Root Mean Squared Errors

RMSE		
-	y_1 (^oC)	y_2 (^oC)
Simplified Decoupling	0.3151	0.2174
Normalized Decoupling	0.3104	0.2451
ID-IMC	0.2346	0.2432

Conclusions

1. Various Decouplers were designed for a 2x2 system.
2. Designing the PI/PID controllers for Simplified decoupling is very difficult as it resulted in very higher order system.
3. Normalized Decoupling avoided the above problem of simplified decoupling by the use of Effective Transfer Function.
4. There was no need to design PI/PID controllers for ID-IMC controller.
5. The decoupler matrices required for ID-IMC are 2 ($Q_d(s)$ and $Q_o(s)$) as compared to 1 in other decouplers.
6. The control efforts required were lowesst for Normalized Decoupling.

References

1. SC645: Intelligent Feedback and Control Course, IIT Bombay
2. Normalized Decoupling New Approach for MIMO Process Control System Design,2008. Wen-Jian Cai, Wei Ni, Mao-Jun He, and Cheng-Yan Ni
3. Inverted decoupling internal model control for square stable multivariable time delay systems,2014. Juan Garridoa, Francisco Vázqueza, Fernando Morilla

Thank You

ID-IMC: Choice of Tuning Parameter

1. λ is the tuning parameter and is chosen based upon the desired rise time of the system $t_i(s)$.
2. Inverse Laplace transform,

$$y_i(t) = 1 - e^{\frac{-(t-\theta_i)}{\lambda_i}}$$

3. for rise time i.e time taken to reach to 90%,

$$0.9 = 1 - e^{\frac{-(t_{ri}-\theta_i)}{\lambda_i}}$$

4. $$\lambda_i = \frac{t_{ri} - \theta_i}{2.3026}$$

λ