Performance Evaluation of PID, LQG and Linear MPC using Nonlinear System Dynamic Simulation

Amey Samrat Waghmare

203230013

Indian Institute of Technology, Bombay

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Overview

System Model

The Nonlinear Reactor System is governed by following set of ODEs

$$\frac{dX_1}{dt} = \frac{U_1}{100}(D - X_1) - 2k(x_2)X_1^2$$

$$\frac{dX_2}{dt} = \frac{U_1}{100}(275 - X_2) + \alpha_3 k(x_2)X_1^2) - \alpha_4(X_2 - X_3)$$

$$\frac{dX_3}{dt} = \frac{U_2}{10}(250 - X_3) - \alpha_5(X_2 - X_3)$$

$$k(X_2) = \alpha_1 e^{\frac{-\alpha_2}{X_2}}$$

Parameters	Nominal Values
α_1	4.11×10^{13}
α_2	9.2055×10^{3}
α_3	284.1043
α_4	2.8571
α_5	28.5714

Discrete Time Linear Model

$$\Phi = \begin{bmatrix} -0.0747 & -0.0005 & -0.0000 \\ 124.1859 & 0.8718 & 0.0791 \\ 110.8932 & 0.7910 & 0.1063 \end{bmatrix}$$

$$\Gamma_u = \begin{bmatrix} 0.0001 & 0.0000 \\ 0.0177 & -0.0740 \\ 0.0089 & -0.4103 \end{bmatrix}$$

$$\Gamma_d = \begin{bmatrix} 0.0016 \\ 14.8523 \\ 9.2261 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Controller Tuning Parameters

1. PI Controller:-

   Two PI controllers (Ideal Form) with Pairing: $y_1 - u_1$ and $y_2 - u_2$.

   - $k_1 = 8.5396$
   - $T_{i_1} = 0.7278$
   - $k_2 = -1.5703$
   - $T_{i_2} = 0.5286$

2. Pole Placement Controller:-

   Using Innovation Bias

   - Controller Poles $= \begin{bmatrix} 0.7 & 0.4 & 0.2 \end{bmatrix}^T$
   - Controller Poles $= \begin{bmatrix} 0.3 & 0.4 & 0.5 \end{bmatrix}^T$
   - Innovation Filter $(\alpha) = 0.8$

3. Kalman Predictor Parameters:-

   Using State Augmentation

   - $Q_a = \begin{bmatrix} 0.0002 & 0 & 0 \\ 0 & 0.6667 & 0 \\ 0 & 0 & 0.1667 \end{bmatrix}$
   - $R_a = \begin{bmatrix} 0.0400 & 0 \\ 0 & 0.0625 \end{bmatrix}$
   - Note:-

     Corresponding to Project file, $\gamma = \frac{4}{3} = 1.3333$

4. Linear Quadratic Gaussian:-
   - $W_x = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   - $W_u = \begin{bmatrix} 2.0000 & 0 \\ 0 & 0.4000 \end{bmatrix}$
   - Note:- Corresponding to Project file, $\eta = \frac{4}{10} = 0.4$

5. Model Predictive Controller:-
   - p = 50
   - q = 5
   - W_x and W_u same as LQG Controller
   - $W_{\Delta u} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$
   - Note:- Corresponding to Project file, $\eta = \frac{4}{10} = 0.4$

Simulations

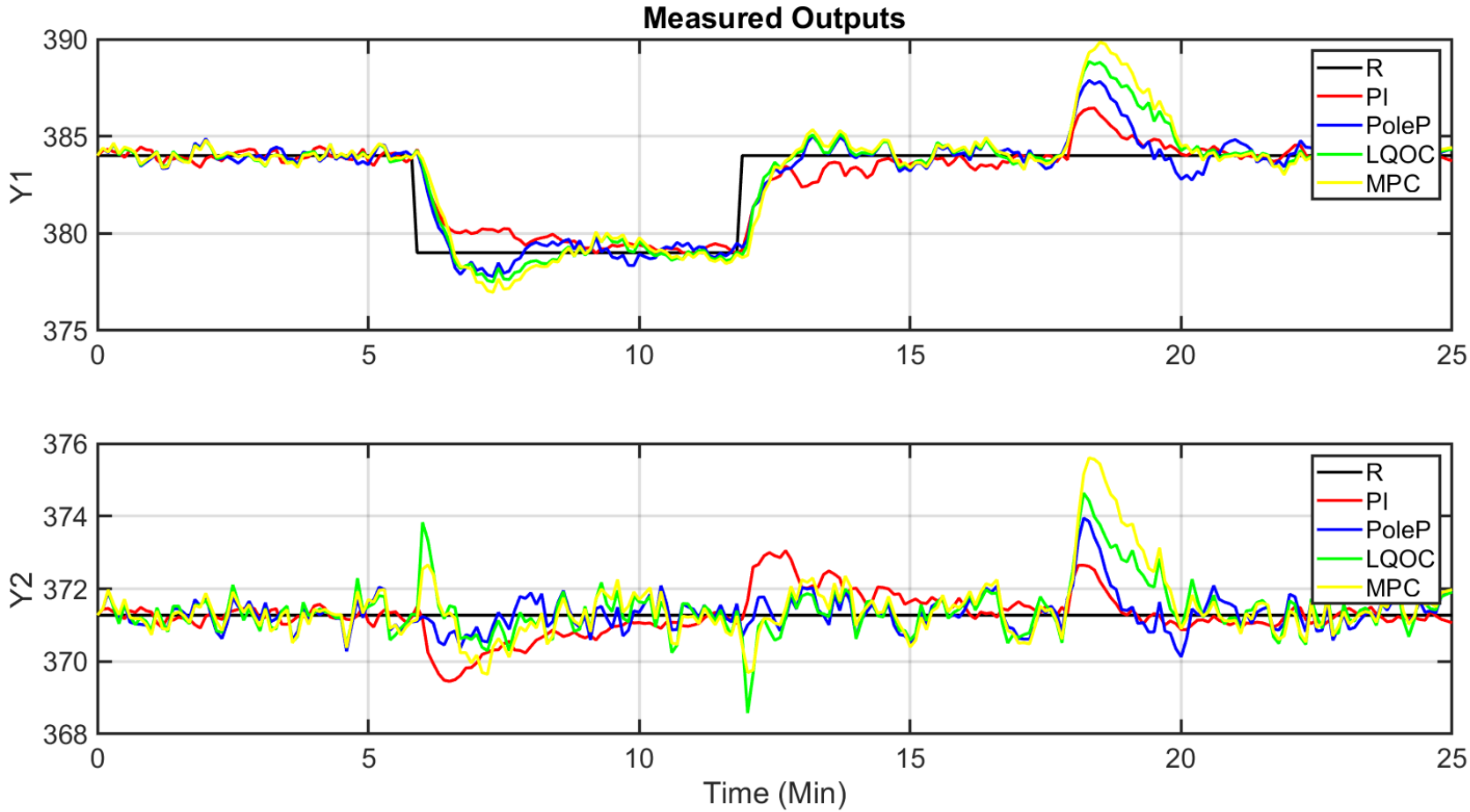

Figure: Comparison of Outputs for Servo and Regulatory Control

Simulations

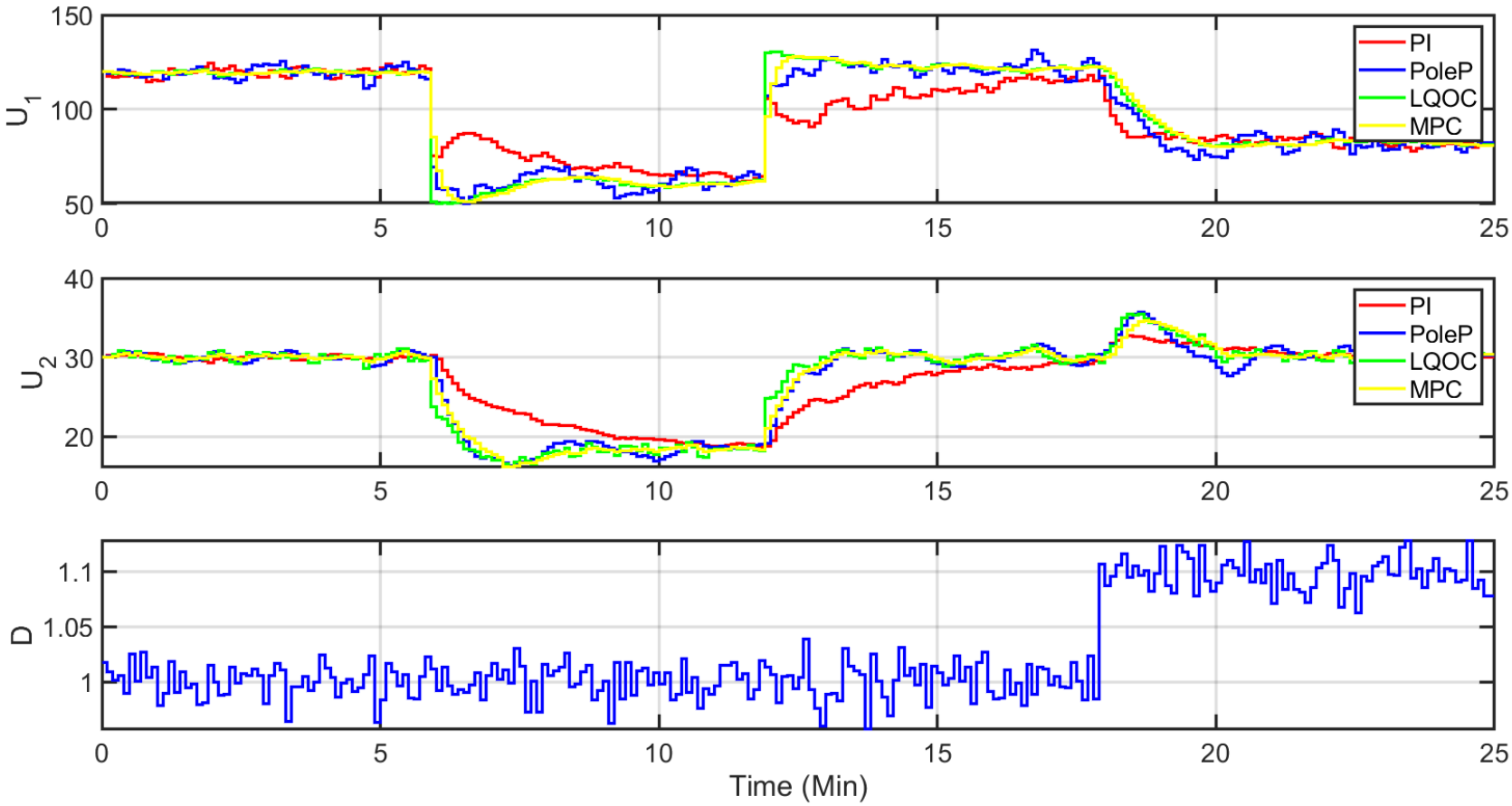

Figure: Comparison of Inputs generated by various controllers and Disturbance

Simulations

SSE		
-	y_1	y_2
PI Controller	223.6515	86.9765
Pole Placement Controller	276.3161	67.4724
LQG Controller	400.8778	131.9852
MPC	570.6810	209.8604

SSMV		
-	u_1	u_2
PI Controller	2.47×10^5	0.0585×10^5
Pole Placement Controller	3.049×10^5	0.0839×10^5
LQG Controller	3.1×10^5	0.0856×10^5
MPC	3.038×10^5	0.0838×10^5

Simulations

1. It is observed that Pole Placement Controller and LQG Controller closely attempts to track the change in reference inputs.
2. When the Disturbance input is applied at 18 min, both the outputs are heavily deviated from reference. Later on, they follow back the reference trajectory.
3. The Given system is a multivariable system. We see the effect of change in input on both the outputs. The highest effect of this multivariable interaction is seen in LQG controller whereas lowest effect is seen in Pole Placement Based controller

Simulations

1. With reference to control efforts, it is observed for U_2 that PI controller was changing slowly whereas all other controllers were quickly trying to mitigate the changes in reference input.
2. Sum of Squared Errors (SSE) are lower for PI and Pole Placement controllers.
3. SSE are higher for MPC and LQG controller.
4. Sum of Squared Manupulated Variables (SSMV) are low for PI Controller among all the controllers.
5. SSMV are almost identical for Pole Placement, LQOC and MPC and are slightly higher than PI Controller.

Conclusion

1. Comparative study was performed for various controllers.
2. It was observed that Pole Placement and LQG controllers did excellent job of tracking the reference.
3. All the controllers deviated when the disturbance input was suddenly increased, however PI Controller was least affected and MPC was most affected.
4. MPC didnt perform very well in terms of disturbance rejection, tuning parameters and constraints might improve this as we had tuning parameters based on roll no, rather than actually penalizing these deviations.

Conclusions

5. Overall Pole Placement and LQOC performed good, however LQOC and MPC could have performed even better if tuning parameters are properly tuned.
6. The Control efforts generated are almost similar for u_2 for Pole Placement, LQOC and MPC.

Thank You