Robotic Path Planning: Steering Limitations

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Overview

Objective

Figure: Auriga-α robot with 2 trailers

Following Problems arise to control such robot,

- Trailer Motion Instability
- Inter-unit Collision
- Off Tracking

To overcome these issues, we can restrict the steering control of the driving unit during forward motion of the robot.

Kinematic Model for n-trailer robot

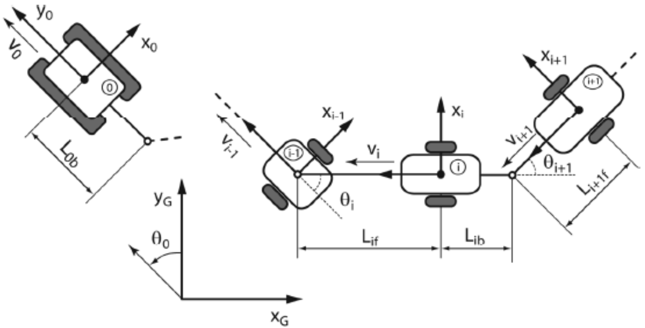

Figure: Kinematic Chain

- Total n+1 units for n-trailer system.
- L_f and L_b are the distances between hitch/axle of consecutive units.
- θ_i is relative angle of ith trailer w.r.t (i-1)th unit.
- The control inputs for the system are, Speed v_0 and Curvature γ_0

Kinematic Model

- ϕ_i is heading of ith trailer w.r.t global co-ordinate system.

$$\phi_i = \sum_{j=0}^{i} \theta j$$

- ω_i is Angular Velocity of i^{th} unit,

$$\omega_i = \frac{d\theta_i}{dt}$$

- Ω_i is Absolute Angular Velocity is calculated as,

$$\Omega_i = \sum_{j=0}^{i} \omega_j$$

- γ_i is the curvature, and is given by

$$\gamma_i = \frac{d\phi_i}{ds_i} = \frac{\Omega_i}{v_i}$$

where s_i is the distance travelled and v_i is the speed.

Kinematic Model

$$\begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} = v_{i-1} \begin{bmatrix} sin\theta_i \\ cos\theta_i \end{bmatrix} + L_{i-1b}\Omega_{i-1} \begin{bmatrix} cos\theta_i \\ -sin\theta_i \end{bmatrix} + L_{if}\omega_i \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{1}$$

Transversal Component $v_{ix} = 0$
Longitudinal Component $v_{iy} = v_i$
(Assuming Rolling without slipping motion)

$$v_i = v_{i-1}[cos\theta_i - \gamma_{i-1}L_{i-1b}sin\theta_i] \tag{2}$$

$$\omega_i = -v_{i-1}\left(\frac{sin\theta_i}{L_{if}} + \frac{\gamma_{i-1}L_{i-1b}cos\theta_i}{L_{if}}\right) \tag{3}$$

From Equation (2) and Equation (3), we obtain

$$\gamma_i = \begin{cases} \frac{\gamma_{i-1}L_{i-1b}cos\theta_i + sin\theta_i}{\gamma_{i-1}L_{i-1b}L_{if}sin\theta_i - L_{if}cos\theta_i} & L_{i-1b} \neq 0 \\ -\frac{tan\theta_i}{L_{if}} & L_{i-1b} = 0 \end{cases} \tag{4}$$

- Observations: γ_i depends on θ_i and γ_{i-1}

Steady Response

We set v_{os} and γ_{os} as the constant set points for speed and curvature of the tractor unit.

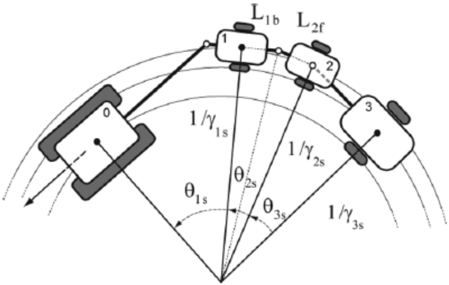

Figure: Constant Tractor set points

$$\gamma_{is} = \frac{\gamma_{i-1s}}{\sqrt{1 + \gamma_{i-1s}^2 (L_{i-1b}^2 - L_{if}^2)}} \tag{5}$$

Equilibrium Limit

$$\gamma_{is} = \frac{\gamma_{i-1s}}{\sqrt{1 + \gamma_{i-1s}^2 (L_{i-1b}^2 - L_{if}^2)}}$$

- This equilibrium exists only when γ_{is} is real valued, $L_{i-1b} > L_{if}$ or $\gamma_{i-1} \leq \frac{1}{\sqrt{L_{if}^2 - L_{i-1b}^2}}$
- In Forward Motion, this Equilibrium Limit is set as

$$\gamma_{i-1m_1} = \begin{cases} \frac{1}{\sqrt{L_{if}^2 - L_{i-1b}^2}} & L_{i-1b} < L_{if} \\ \infty & L_{i-1b} \geq L_{if} \end{cases} \tag{6}$$

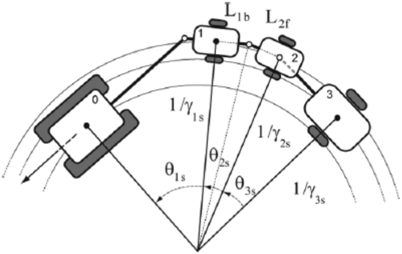

Figure: Constant Tractor set points

- Two equilibrium points for mechanical limit are geometrically possible,

$$\theta_{is}^{(1)} = -arctan(\gamma_{i-1s}L_{i-1b}) - arctan(\gamma_{is}L_{if})$$

$$\theta_{is}^{(2)} = \pi - arctan(\gamma_{i-1s}L_{i-1b}) + arctan(\gamma_{is}L_{if})$$

- During Forward motion, $\theta_{is}^{(2)}$ is unstable as it is near jack-knife position ($\pm\pi$).
- During reverse motion, $\theta_{is}^{(1)}$ is unstable as it is near jack-knife position

Mechanical Limit

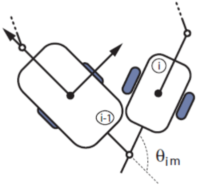

Figure: Physical Limit of Angle θ_i

- The joint between every pair of consecutive units (i-1) and i have an inherent mechanical limit θ_{im}.
- If this is surpassed, then there can be inter-unit collisions or Linkage breakage.
- Hence, the mechanical limit obtained from Mechanical Equilibrium must be imposed,

$$\gamma_{i-1m_2} = \frac{sin\theta_{im}}{L_{if} + L_{i-1b}cos\theta_{im}} \tag{7}$$

Backpropogated Limit

- The Steady state curvature is given by,

$$\gamma_{is} = \frac{\gamma_{i-1s}}{\sqrt{1 + \gamma_{i-1s}^2 (L_{i-1b}^2 - L_{if}^2)}}$$

- The (i-1)$^{\text{th}}$ unit should take into account the maximum steering limitations of the following i$^{\text{th}}$ unit.

$$\gamma_{i-1m_3} = \frac{\gamma_{im}}{\sqrt{1 + \gamma_{im}^2 (L_{if}^2 - L_{i-1b}^2)}} \tag{8}$$

Among all the limits in Equation (6), Equation (7) and Equation (8), the most constraining one is chosen,

$$\gamma_{i-1m} = min(\gamma_{i-1m_1}, \gamma_{i-1m_2}, \gamma_{i-1m_3}) \tag{9}$$

Algorithm

Algorithm 1: Tractor Curvature Limitation

for $i = n$ *to* 1 **do** *(Steady analysis)*

  Compute *equilibrium limit* $\gamma_{i-1\,m_1}$ (17)

  Compute *mechanical limit* $\gamma_{i-1\,m_2}$ (18)

  Compute *backpropagation limit* $\gamma_{i-1\,m_3}$ (19)

  $\gamma_{i-1\,m} = \min(\gamma_{i-1\,m_1}, \gamma_{i-1\,m_2}, \gamma_{i-1\,m_3})$

if *any hitch from 1 to* $n-1$ *is off-axle* **then**

  repeat *(Dynamic analysis)*

    $\gamma_{0\,m} \leftarrow (\gamma_{0\,m} - \Delta\gamma)$

    Evaluate step response from $\mp\gamma_{0\,m}$ to $\pm\gamma_{0\,m}$

  until *equilibrium is reached without inter-unit collision*

Result: γ_{0m}

Implementation on Auriga-α

- The motion control system consists of
  - Path Generator
  - Path Tracker
  - Kinematic Model
  - Track Speed Controller
- The first 3 systems are implemented on CPU/Computer, whereas the Last system is implemented on onboard digital signal processor.
- Path Generator Generates Obstacle free list of points to be followed by Tractor.
- Path Tracker generates speed and curvature references in order to follow the path.
- Kinematic model and Track speed controller consists of two independent PID controllers which controls track speed.
- PID parameters provides overdamped track speed response so that overdamped response is obtained for curvature setpoints

Simulation

- The algorithm was implemented to determine the steering limitations for a 2 Trailer System with off-axle joints.
- The Axle Joints have maximum angle limits of $\theta_{1m} = 68^o$ and $\theta_{2m} = 43.6^o$
- The Steering limit obtained by using the Algorithm is 0.41m^{-1}.

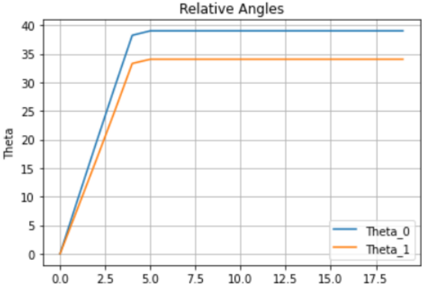

Figure: Response after Implementing Steering Limits

- The angle limits from above response is $\theta_{1m} = 39^o$ and $\theta_{2m} = 34^o$

Simulation

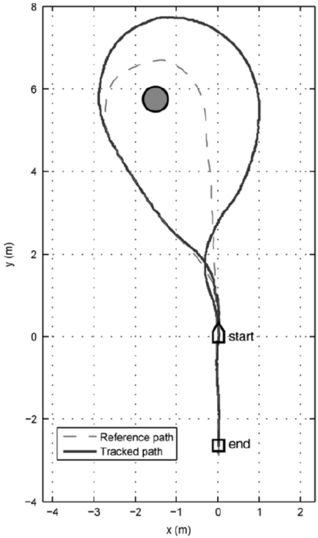

Figure: Path Tracking

Conclusion

- The Algorithm can be used for avoiding instability and inter-unit collision.
- The Algorithm relies on the existence of equilibrium of trailers.
- The Algorithm takes into account the type of joint/ hitch used for connecting the trailer units.
- The Steering limitations of the tractor can be imposed on Path Generator
- In such case, the tracking errors are negligible.

References

1] Jorge L. Martínez, Jesús Morales, Anthony Mandow, and Alfonso García-Cerezo "Steering Limitations for a Vehicle Pulling Passive Trailers". IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, Vol. 16, no. 4. pp 809-818, 2008
2] J.L. Martínez, A. Mandow, J. Morales, "Forward path tracking for Mobile Robots with several trailers". 5th IFAC/EURON Symposium on Intelligent Autonomous Vehicles Instituto Superior Técnico, Lisboa, Portugal, 2004.

Thank You