SC 626

SYSTEMS AND CONTROL ENGINEERING LABORATORY

MODEL ESTIMATION USING STEP TEST

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Aim

To determine/identify the following system models using Step Test data

1. First Order Transfer Function Model.
2. Second Order Transfer Function Model.
3. State Space Models (Controllable Canonical Form and Observable Canonical Form)

System Overview

Single Board Multi Heater System (SBHS) is an experimental setup and is in the form of Multi Input Multi Output (MIMO) system. The inputs to this system are various heaters and the measured output is the temperature. The plant consists of an aluminium plate which is virtually divided into four equal square parts and heating elements are mounted on each of the plate. Five temperature sensors are mounted below the plate. A fan is placed below the plate for cooling purpose.
There are 4 Manipulated Inputs (heaters) and 5 Controlled Outputs (temperature sensors) and 1 Disturbance Input (Fan) to the system.

Step Test

In Industry, various physical systems are used. However, mathematical models of these processes are not available. A mathematical model is required for the analysis and for designing controllers. Hence, development of Process Model is a fundamental task for Control engineers. These models takes the following form,

1. **White Box Model:**
   These models are developed using laws of physics. It is a tedious process to design these models and requires a lot of expertise in various domains to develop such models. The obtained models are precise models of the actual system.
2. **Black Box Model:**
   These models are developed completely based on the input and output data obtained from the system. Inputs are given to the system and corresponding outputs are recorded. These i/o data helps to develop model and such models are called as Black Box Models. As opposed to white box models, domain knowledge is not required.

3. **Grey Box Models:**
   These models are partially developed from both knowledge of the physics of the system and the i/o data of the system, i/o data obtained from exciting the system. This is the preferred method to obtain model of the system in Industry.

Based on the information available about the system, we may assume the type of the model of our system and then estimate the parameters of the system by formulating an objective Function and performing Linear regression for Data Driven Models. For example, we may assume that our system model takes a form of FOPTD, First Order Plus Time Delay, so in this case we need to estimate three parameters, a pole, Gain and Time delay.

0.1 DATASET

Two datasets for training and validation were provided. Step inputs were provided to the SBHS and the temperature was recorded at an sampling rate of 3.0930 sec. This process was carried out for around 4786 sampling instants for training data and 2310 instants for Validation data. For each sampling instant 4 manipulated input, 1 disturbance input and measured output from sensors were recorded. This formed the dataset.
The above dataset is a raw data and needs to be preprocessed before it is used for the estimation of a model. Few samples, 25 from initial and 40 from end time were discarded in order to ensure that the dataset does not contains abnormal datapoints. Then the data was linearized around the equilibrium point, so as to obtain perturbation data. This data was supplied to System Identification Toolbox to estimate models.

MODEL ESTIMATION

0.1.1 First Order Model

We can assume that our model takes the following form,

$$G(s) = \frac{k_p}{1 + s\tau} e^{-sT_d} \tag{1}$$

Then, our objective is to determine the parameters k_p, τ and T_d for this system by using the Training data. To measure the performance of our model, we will use the Validation data. As our system consists of 4 inputs, 1 Disturbance (total 5 inputs) and 5 outputs, we will obtain a Transfer Function Matrix of the System.

The preprocessed training data was supplied to System Identification App (Alternatively, data can be supplied using iddata command) and a FOPTD model was estimated. This model is shown below.

$$G(s) = \begin{bmatrix} \frac{1.004}{1+44.4867s}e^{-3.0899} & \frac{0.5139}{1+60.3066s} & \frac{0.3266}{1+61.0168s} & \frac{0.2993}{1+37.652s}e^{-5.2612s} & \frac{-0.2372}{1+69.7026s} \\ \frac{0.3081}{1+117.375s} & \frac{1.0627}{1+29.13s} & \frac{0.3651}{1+55.1s} & \frac{0.178}{1+22.29s}e^{-31.8769s} & \frac{-0.2232}{1+67.26s} \\ \frac{0.2796}{1+80.93s}e^{-5.2086s} & \frac{0.3982}{1+54.65s} & \frac{0.9589}{1+25.2969s}e^{-1.0362s} & \frac{0.2551}{1+s}e^{-21.651s} & \frac{-0.1817}{1+37.57s} \\ \frac{0.5095}{1+69.73s}e^{-0.8568s} & \frac{0.3697}{1+49.93s} & \frac{0.3119}{1+45.986s} & \frac{0.9}{1+26.1158s}e^{-2s} & \frac{-0.2042}{1+40.639s}e^{-0.9557s} \\ \frac{0.4319}{1+43.08s}e^{-10.1419s} & \frac{0.5202}{1+86.249s} & \frac{0.4403}{1+55.577s} & \frac{0.3135}{1+47.21s} & \frac{-0.1698}{1+72.2265s} \end{bmatrix} \tag{2}$$

The above model was simulated on both training data and validation data so as to asses the model. Root mean Squared Error was used as a Performance Index to assess the model. Following values were obtained,

First Order Model		
-	Training Data	Validation Data
$RMSE_{Y1}$	0.5772	1.3846
$RMSE_{Y2}$	0.6395	1.2224
$RMSE_{Y3}$	0.996	1.4937
$RMSE_{Y4}$	0.7814	1.3411
$RMSE_{Y5}$	0.5376	1.1837

Figure (1) shows LTI Response of the above model.

0.1.2 Second Order Model

We can assume that our model takes the following form,

$$G(s) = \frac{k_p}{1 + 2\zeta T_w s + (T_w s)^2} e^{-sT_d} \tag{3}$$

Then, our objective is to determine the parameters k_p, ζ, T_w and T_d for this system by using the Training data. To measure the performance of our model, we will use the Validation data. As our system consists of 4 inputs, 1 Disturbance (total 5 inputs) and 5 outputs, we will obtain a Transfer Function Matrix of the System.

The preprocessed training data was supplied to System Identification App (Alternatively, data

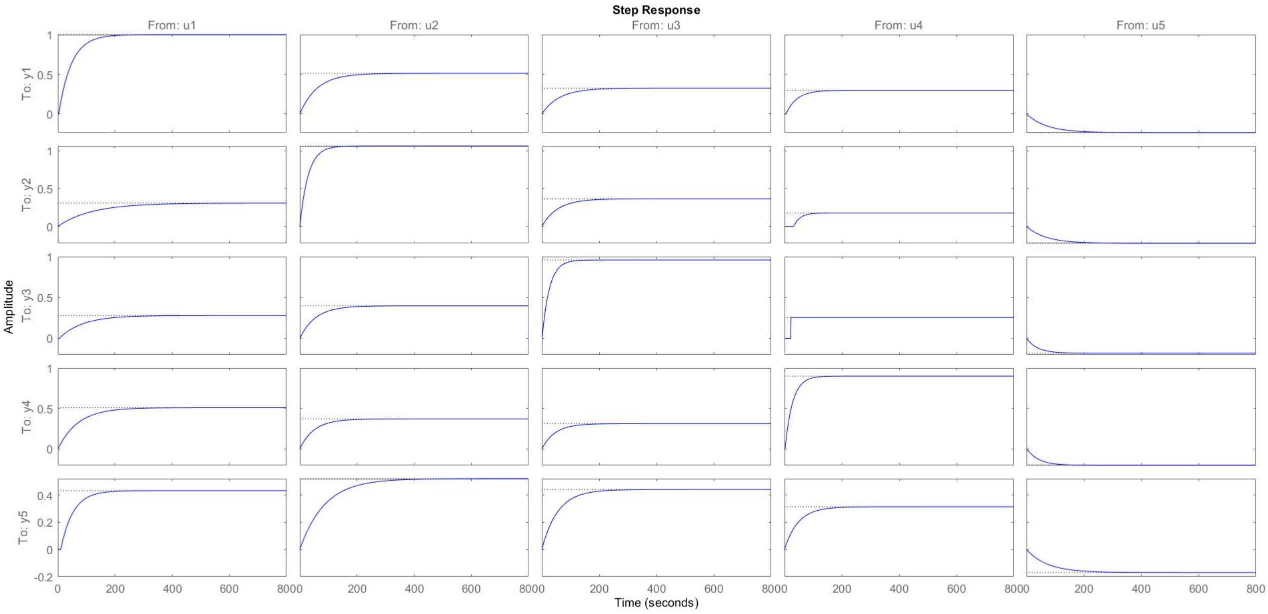

Figure 1: LTI Response for First order model

can be supplied using iddata command) and a FOPTD model was estimated. This model is shown below.

from u1 to u3,

$$G(s) = \begin{bmatrix} \frac{0.6588}{1+683.73s+(96.3s)^2}e^{-0.003s} & \frac{0.7398}{1+71.54s+(3.99s)^2}e^{-1s} & \frac{0.256}{1+116.206s+(29.2s)^2} \\ \frac{0.29}{1+14.361s+(0.2572s)^2}e^{-36.624s} & \frac{1.0633}{1+42.77s+(4s)^2}e^{-3.414s} & \frac{0.3695}{1+63.4436s+(2s)^2}e^{-2.8548s} \\ \frac{015.026}{1+(100s)^2}e^{-0.003s} & \frac{0.3766}{1+25.189s+(21.149s)^2}e^{-14.144s} & \frac{0.958}{1+23.97s+(10.058s)^2}e^{-0.9959s} \\ \frac{0.515}{1+106.8s+(4.0656s)^2}e^{-17.4s} & \frac{0.377}{1+81.023s+(22.4354s)^2} & \frac{0.3139}{1+58.38s+(21.74s)^2} \\ \frac{0.4025}{1+56.756s+(0.7373s)^2}e^{-9.2388s} & \frac{0.5164}{1+64.267s+(1.5719s)^2} & \frac{0.4275}{1+29.267s+(2.056s)^2}e^{-0.02s} \end{bmatrix} \tag{4}$$

u4 to u5

$$\begin{bmatrix} \frac{0.0722}{1+119.612s+(51.619s)^2} & \frac{-0.2468}{1+114.719s+(36.597s)^2} \\ \frac{-0.159}{1+12.164s+(760.28s)^2}e^{-92.79s} & \frac{-0.2087}{1+35.732s+(1.134s)^2}e^{-5.6s} \\ \frac{-0.57}{1+149.6s+(1550s)^2}e^{-92.79s} & \frac{-0.1944}{1+79.3866s+(0.0923s)^2}e^{-54.38s} \\ \frac{0.8999}{1+27.76s+(3.192s)^2}e^{-1.54s} & \frac{-0.2045}{1+38.645s+(38.36s)^2} \\ \frac{690}{1+509.57s+(254.78s)^2} & \frac{-0.1695}{1+68.3523s+(4.424s)^2} \end{bmatrix} \tag{5}$$

The above model was simulated on both training data and validation data so as to asses the model. Root mean Squared Error was used as a Performance Index to assess the model. Following values were obtained,

First Order Model		
-	Training Data	Validation Data
$RMSE_{Y1}$	1.9663	2.4515
$RMSE_{Y2}$	0.6835	1.3732
$RMSE_{Y3}$	1.4171	2.1392
$RMSE_{Y4}$	0.8485	1.571
$RMSE_{Y5}$	0.8435	1.2927

Figure (2) shows LTI Response of the above model.

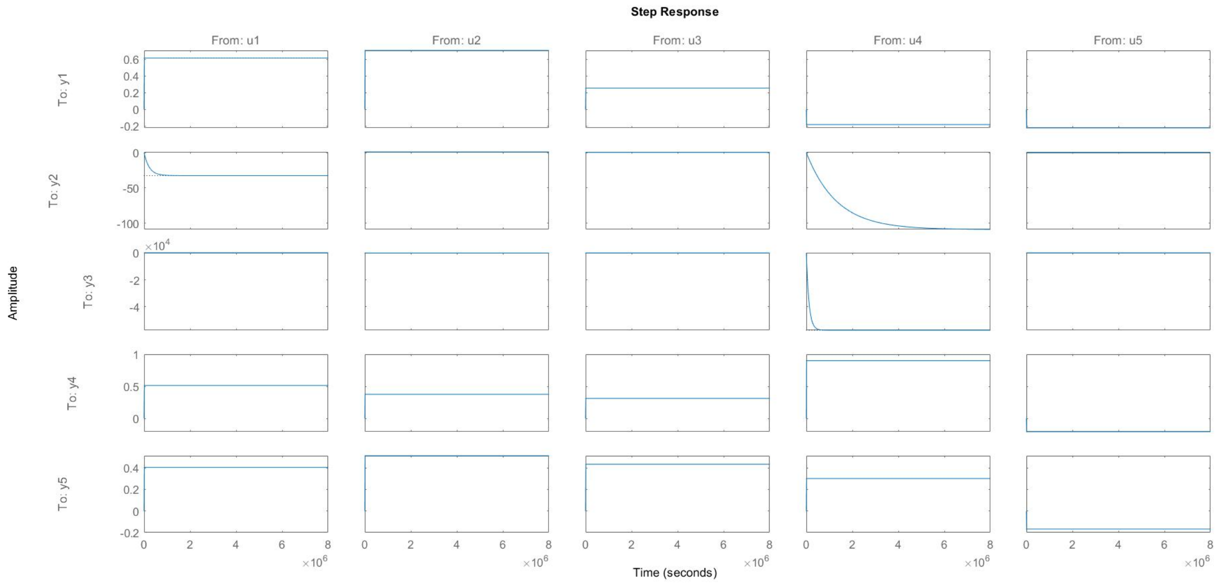

Figure 2: LTI Response for Second order model

0.1.3 State Space Models

We can assume that our model takes the following form,

$$\dot{x} = Ax + Bu + Le$$

$$y = Cx + Du + e$$

Where e is the error signal. Then, our objective is to determine the parameters A,B,C,D and L for this system by using the Training data. To measure the performance of our model, we will use the Validation data. We can determine the State space model in Controllable canonical

form as well as Observable canonical form. For the given data, we have chosen 4th order system (states = 4).
The preprocessed training data was supplied to System Identification App (Alternatively, data can be supplied using iddata command) and a FOPTD model was estimated. The parameters of this model is shown below. Observable Canonical Form:

$$A_{obs} = \begin{bmatrix} -0.0230 & 0.0059 & -0.0000 & 0.0022 \\ 0.0073 & -0.0225 & -0.0118 & 0.0051 \\ 0.0009 & -0.0161 & -0.0236 & 0.0182 \\ 0.0065 & -0.0086 & 0.0161 & -0.0326 \end{bmatrix}$$

$$B_{obs} = \begin{bmatrix} 0.0207 & 0.0028 & 0.0061 & 0.0036 & -0.0034 \\ 0.0001 & 0.0205 & 0.0173 & -0.0000 & -0.0040 \\ 0.0013 & 0.0158 & 0.0253 & -0.0076 & -0.0034 \\ 0.0079 & 0.0105 & -0.0033 & 0.0249 & -0.0041 \end{bmatrix}$$

$$C_{obs} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.1364 & 0.4531 & 0.0462 & 0.2213 \end{bmatrix}$$

$$L_{obs} = \begin{bmatrix} 0.0641 & 0.0405 & 0.0118 & 0.0280 & 0.0073 \\ 0.0065 & 0.1161 & 0.0204 & 0.0144 & 0.0197 \\ -0.0087 & 0.0328 & 0.1202 & 0.0470 & -0.0066 \\ 0.0048 & 0.0007 & 0.0449 & 0.1015 & 0.0156 \end{bmatrix}$$

Similarly, Controllable Canonical Form,

$$A_{CCF} = \begin{bmatrix} -0.0230 & 0.0073 & 0.0009 & 0.0065 \\ 0.0059 & -0.0225 & -0.0161 & -0.0086 \\ -0.0000 & -0.0118 & -0.0236 & 0.0161 \\ 0.0022 & 0.0051 & 0.0182 & -0.0326 \end{bmatrix}$$

$$B_{CCF} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.1364 \\ 0 & 1 & 0 & 0 & 0.4531 \\ 0 & 0 & 1 & 0 & 0.0462 \\ 0 & 0 & 0 & 1 & 0.2213 \end{bmatrix}$$

$$C_{CCF} = \begin{bmatrix} 0.0207 & 0.0001 & 0.0013 & 0.0079 \\ 0.0028 & 0.0205 & 0.0158 & 0.0105 \\ 0.0061 & 0.0173 & 0.0253 & -0.0033 \\ 0.0036 & -0.0000 & -0.0076 & 0.0249 \\ -0.0034 & -0.0040 & -0.0034 & -0.0041 \end{bmatrix}$$

The above model was simulated on both training data and validation data so as to asses the model. Root mean Squared Error was used as a Performance Index to assess the model. Following values were obtained,

State Space Model		
-	Training Data	Validation Data
$RMSE_{Y1}$	0.6845	0.8673
$RMSE_{Y2}$	0.7816	1.047
$RMSE_{Y3}$	1.2655	1.21
$RMSE_{Y4}$	0.957	0.9665
$RMSE_{Y5}$	0.5419	0.7329

Figure (3) shows LTI Response of the above model.

Figure (4) shows the performance of the Developed models on the validation data.

CONCLUSION

Various Models were Obtained using the System Identification Toolbox. To asses these models, Root Mean Square Error was used as a Performance Index. It was observed that the State space Model had the least RMSE values as Compared to other models. This was also evident from the comparison of simulation of validation data. In the Process Models, the First order Model Outperformed the Second order model.

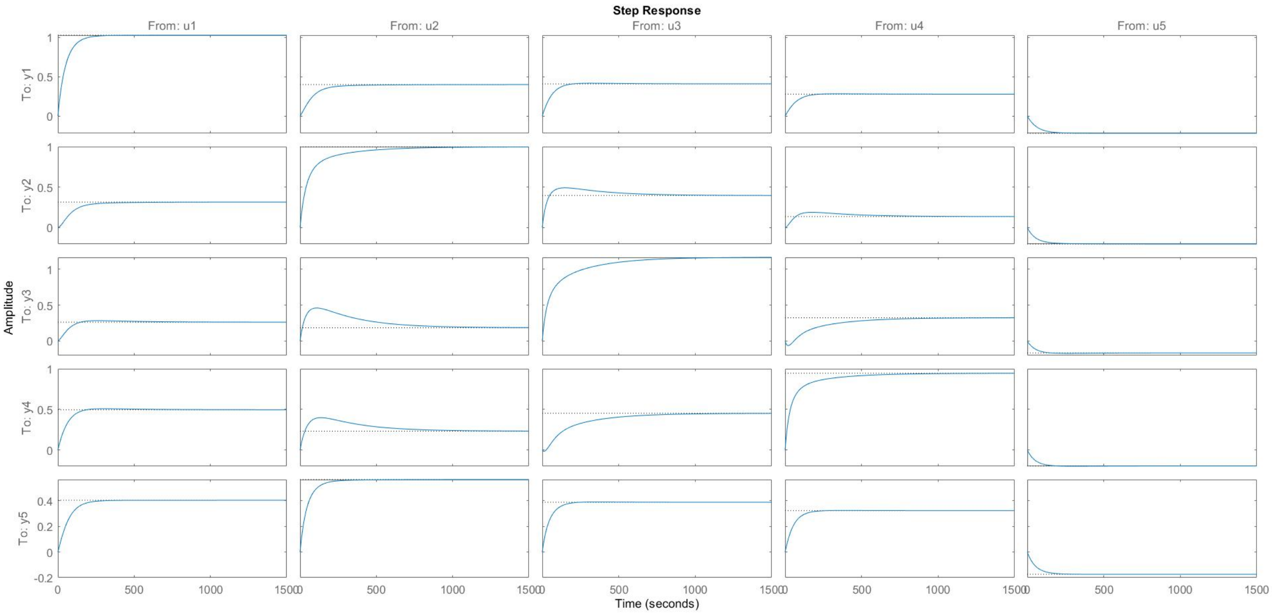

Figure 3: LTI Response for State Space model

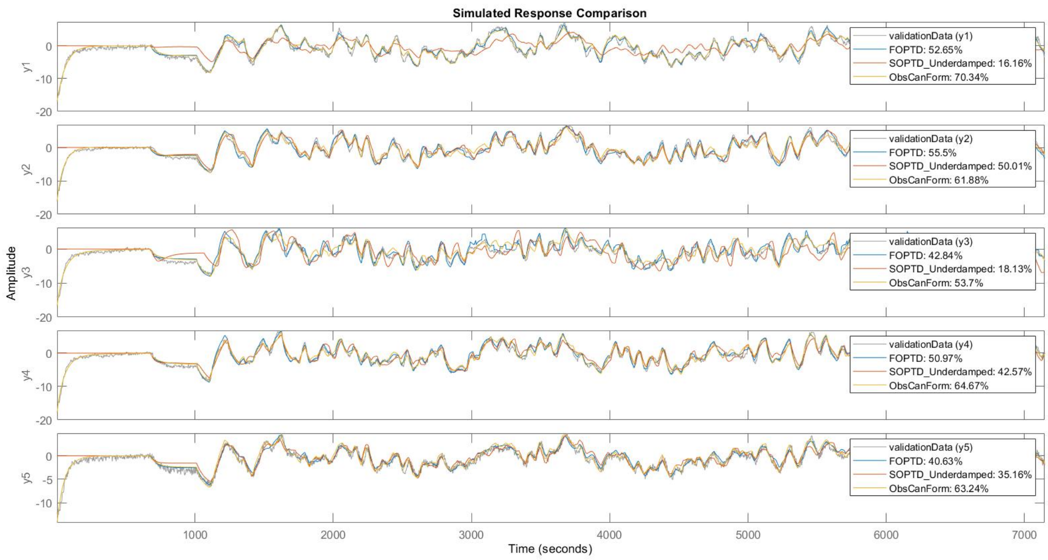

Figure 4: Comparison of Models on Validation Data

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MODEL ESTIMATION USING PSEUDO RANDOM BINARY SIGNAL DATA

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Aim

To determine/identify the following system models using Step Test data

1. ARMAX Model.
2. FIR Model.

ARMAX Model

ARMAX stands for Auto Regressive Moving Average Model. A MISO ARMAX Model takes the following form,

$$A(q^{-1})y(k) = B_1(q^{-1})u_1(k) + B_2(q^{-1})u_2(k) + e(k)$$

where,

$$A(q^{-1}) = 1 + a_1q^{-1} + a_2q^{-2} + a_3q^{-3}$$

$$B_1(q^{-1}) = b_{11}q^{-1} + b_{12}q^{-2} + b_{13}q^{-3}$$

$$B_2(q^{-1}) = b_{21}q^{-1} + b_{22}q^{-2} + b_{23}q^{-3}$$

The order of the A polynomial is na, and similarly the order of the B parameter is nb.The above model is for a system with two inputs. For MIMO Models, we first estimate MISO Models for each output and then concatenate the obtained models. The ARMAX model in state Space form is,

$$x(k+1) = \phi x(k) + \Gamma u(k) + Le(k)$$

$$y(k) = Cx(k) + e(k)$$

where,

$$\phi = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \\ b_{13} & b_{23} \end{bmatrix}$$

$$L = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The dataset given was preprocessed and supplied as input to ARMAX model estimator in System Identification Toolbox. Therefore five MISO models were obtained as our System have 5 Outputs. These models are shown below,

MISO Y1

$$\phi_1 = \begin{bmatrix} 0 & 0 & -0.6374 \\ 0.5 & 0 & -0.1454 \\ 0 & 1 & 1.449 \end{bmatrix}$$

$$\gamma_1 = \begin{bmatrix} -0.1055 & -0.009518 & 0.0005348 & 0.01236 \\ 0.075 & 0.005024 & 0.01844 & -0.006039 \\ -0.008335 & 0.005923 & -0.01563 & 0.004507 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -0.6374 \\ -0.1454 \\ 0.6987 \end{bmatrix}$$

MISO Y2

$$\phi_2 = \begin{bmatrix} 0 & 0 & -0.6796 \\ 0.5 & 0 & -0.1972 \\ 0 & 1 & 1.463 \end{bmatrix}$$

$$\gamma_2 = \begin{bmatrix} 0.02443 & -0.06895 & 0.01033 & 0.01138 \\ -0.01237 & 0.03797 & 0.01159 & -0.001242 \\ 0.002979 & 0.0184 & -0.01189 & -0.002094 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.5796 \\ -0.1972 \\ 0.8875 \end{bmatrix}$$

MISO Y3

$$\phi_3 = \begin{bmatrix} 0 & 0 & -0.5491 \\ 0.5 & 0 & -0.2028 \\ 0 & 1 & 1.456 \end{bmatrix}$$

$$\gamma_3 = \begin{bmatrix} 0.01541 & 0.01947 & -0.09086 & 0.04205 \\ 0.002308 & -0.01087 & 0.06997 & -0.02867 \\ -0.007731 & 0.007424 & -0.003236 & 0.01292 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -0.5491 \\ -0.2028 \\ 0.7709 \end{bmatrix}$$

MISO Y4

$$\phi_4 = \begin{bmatrix} 0 & 0 & -0.6017 \\ 0.5 & 0 & -0.1741 \\ 0 & 1 & 1.457 \end{bmatrix}$$

$$\gamma_4 = \begin{bmatrix} -0.004513 & 0.005018 & 0.02539 & -0.09395 \\ 0.01608 & -0.01046 & -0.008681 & 0.05328 \\ -0.007877 & 0.01258 & 0.0001038 & 0.01113 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} -0.6017 \\ -0.1741 \\ 0.757 \end{bmatrix}$$

MISO Y5

$$\phi_5 = \begin{bmatrix} 0 & 0 & -0.1291 \\ 0.5 & 0 & 0.373 \\ 0 & 1 & 7734 \end{bmatrix}$$

$$\gamma_5 = \begin{bmatrix} 0.191 & 0.404 & 0.7135 & 0.4382 \\ 0.01811 & 0.02538 & -0.02194 & 0.01061 \\ -0.008249 & -0.007032 & -0.01435 & -0.01152 \end{bmatrix}$$

$$C_5 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$L_5 = \begin{bmatrix} -0.1291 \\ 0.373 \\ 0.4637 \end{bmatrix}$$

The Overall ARMAX model is the combination of all the MISO systems given above.

$$\phi = \begin{bmatrix} \phi_1 & [0] & [0] & [0] & [0] \\ [0] & \phi_2 & [0] & [0] & [0] \\ [0] & [0] & \phi_3 & [0] & [0] \\ [0] & [0] & [0] & \phi_4 & [0] \\ [0] & [0] & [0] & [0] & \phi_5 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \end{bmatrix}$$

$$C = \begin{bmatrix} C_1 & [0] & [0] & [0] & [0] \\ [0] & C_2 & [0] & [0] & [0] \\ [0] & [0] & C_3 & [0] & [0] \\ [0] & [0] & [0] & C_4 & [0] \\ [0] & [0] & [0] & [0] & C_5 \end{bmatrix}$$

$$L = \begin{bmatrix} L_1 & [0] & [0] & [0] & [0] \\ [0] & L_2 & [0] & [0] & [0] \\ [0] & [0] & L_3 & [0] & [0] \\ [0] & [0] & [0] & L_4 & [0] \\ [0] & [0] & [0] & [0] & L_5 \end{bmatrix}$$

Figures below show the Outputs of the above model for validation data input.

Few residue plots are shown below,

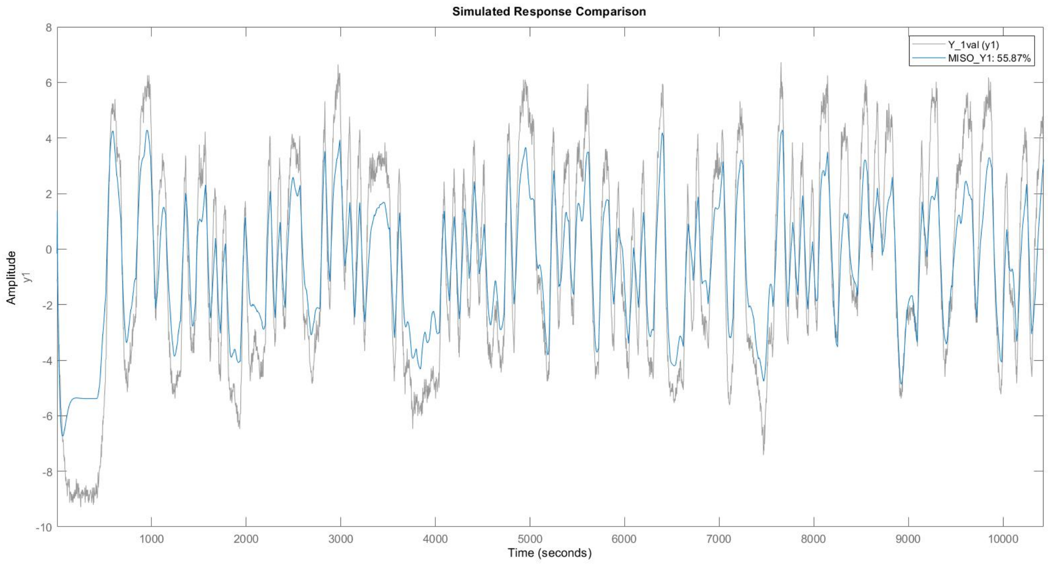

Figure 1: Output Y1

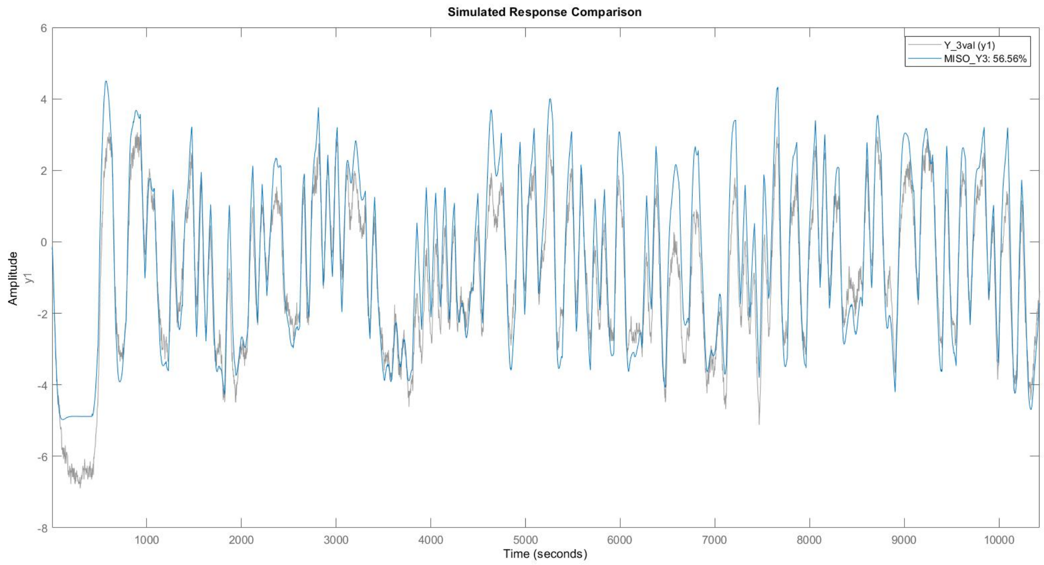

Figure 2: Output Y3

The following table gives comparison of RMS values for training set and Validation set

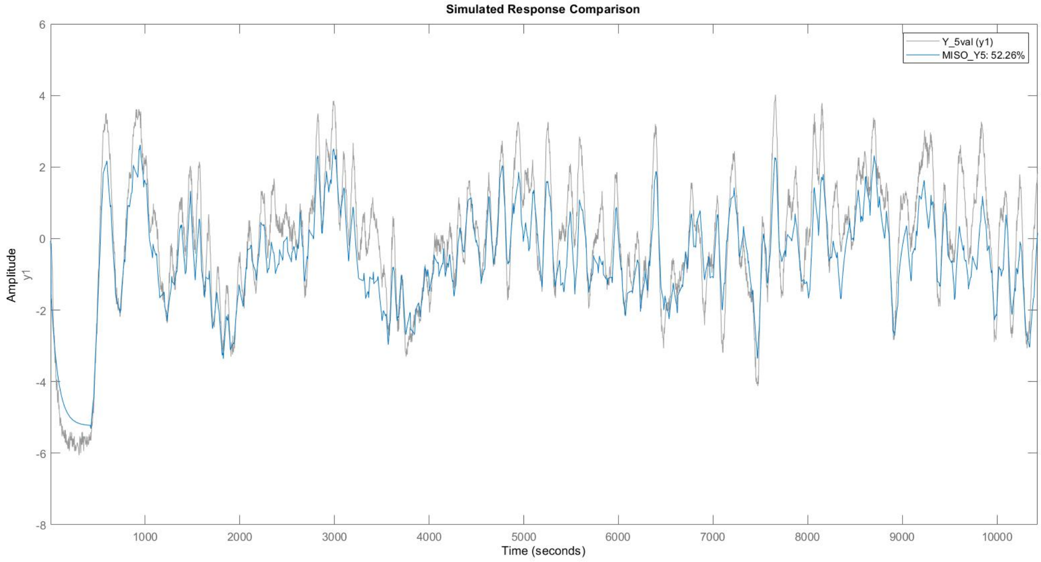

Figure 3: Output Y5

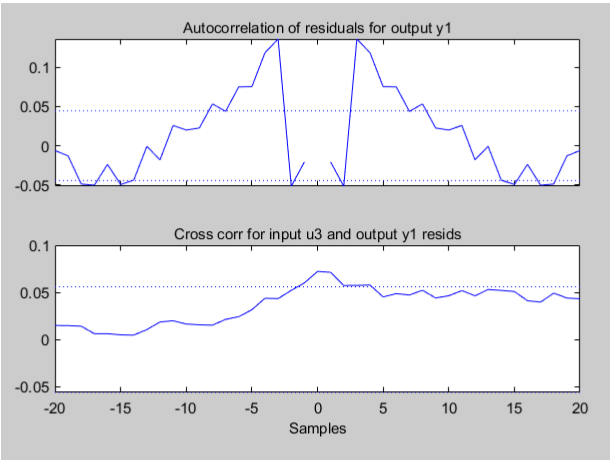

Figure 4: Corresponding to u3-Y1

ARMAX Model		
-	Training Data	Validation Data
$RMSE_{Y1}$	1.6449	1.6284
$RMSE_{Y2}$	1.5095	0.9418
$RMSE_{Y3}$	1.3027	0.9
$RMSE_{Y4}$	1.4698	1.1034
$RMSE_{Y5}$	1.1663	0.89

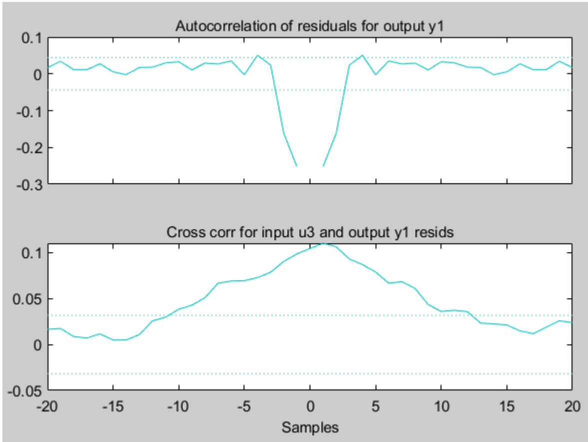

Figure 5: Corresponding to u3-Y4

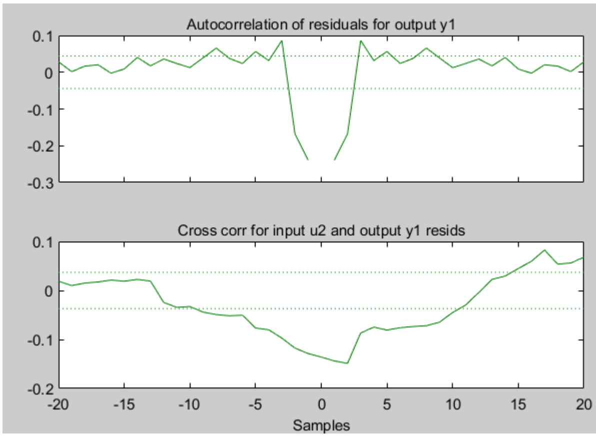

Figure 6: Corresponding to u2-Y2

FINITE IMPULSE RESPONSE MODEL

For model estimation, we can apply control impulse and collect the data. However it is difficult to apply short impulse big enough such that the response is larger than the noise. A better way is to apply multiple steps. We can identify impulse response by applying multiple steps. For this Pseudo Random Binary Signal can be used as input. Finite Impulse Response model takes the form,

$$y(t) = \sum_{k=1}^{K} h(k)u(t-k) + e(t)$$

Command *impulseest* is used for the estimation of the above model. Preprocessed data was supplied. The Identification toolbox automatically estimates the best order of the polynomial (K in model). This was found out to be 70 for given dataset. However this is not a good value as it leads to large number of parameters, 70x5 corresponding to each input (5 comes from number of outputs in our case) and we have total 4 inputs. So total parameters become 1400! Hence, a lower order of 20 is chosen, which still leads to a large number of parameters and practically impossible to indicate in this report (mat file attached). Figure (7) shows the comparison for 70th order model and 20th order model. It is observed that the models are closer and even 20th order is a good representation of the system.

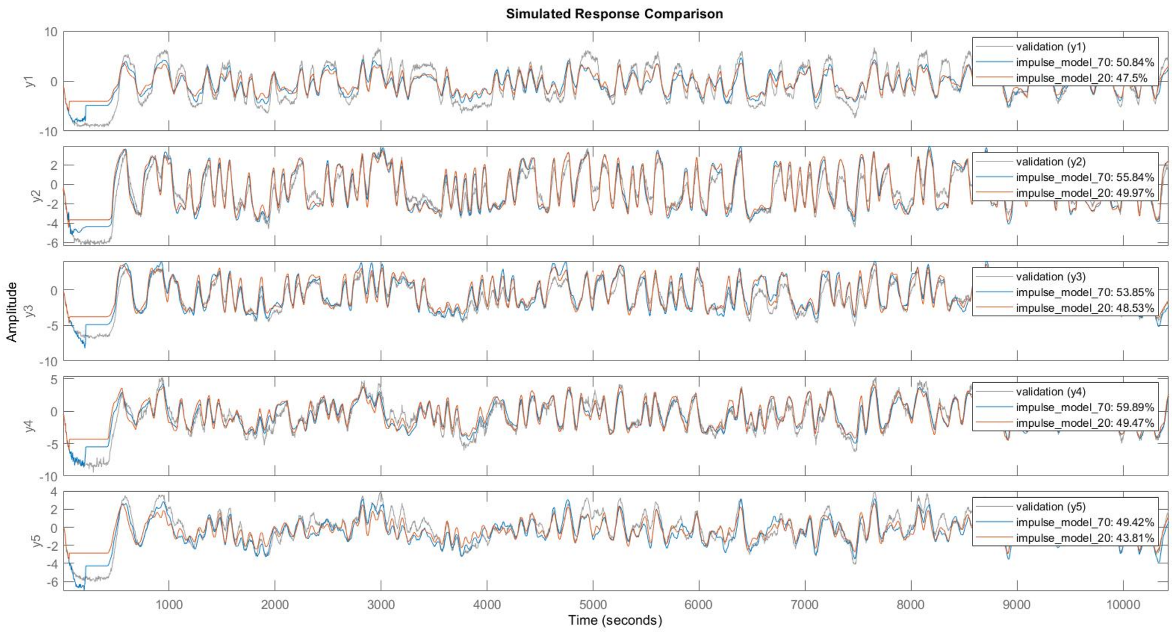

Figure 7: Comparison of 70th and 20th order FIR models

Following is the Root mean squared error obtained for validation data for both 70th order model, 20th order model and 15th order model.

FIR Model (Validation Data)			
-	Order 70	Order 20	Order 15
$RMSE_{Y1}$	1.8142	1.9372	2.2241
$RMSE_{Y2}$	0.8944	1.0134	1.1793
$RMSE_{Y3}$	0.9565	1.0666	1.2507
$RMSE_{Y4}$	1.032	1.3004	1.5756
$RMSE_{Y5}$	0.9428	1.0473	1.2378

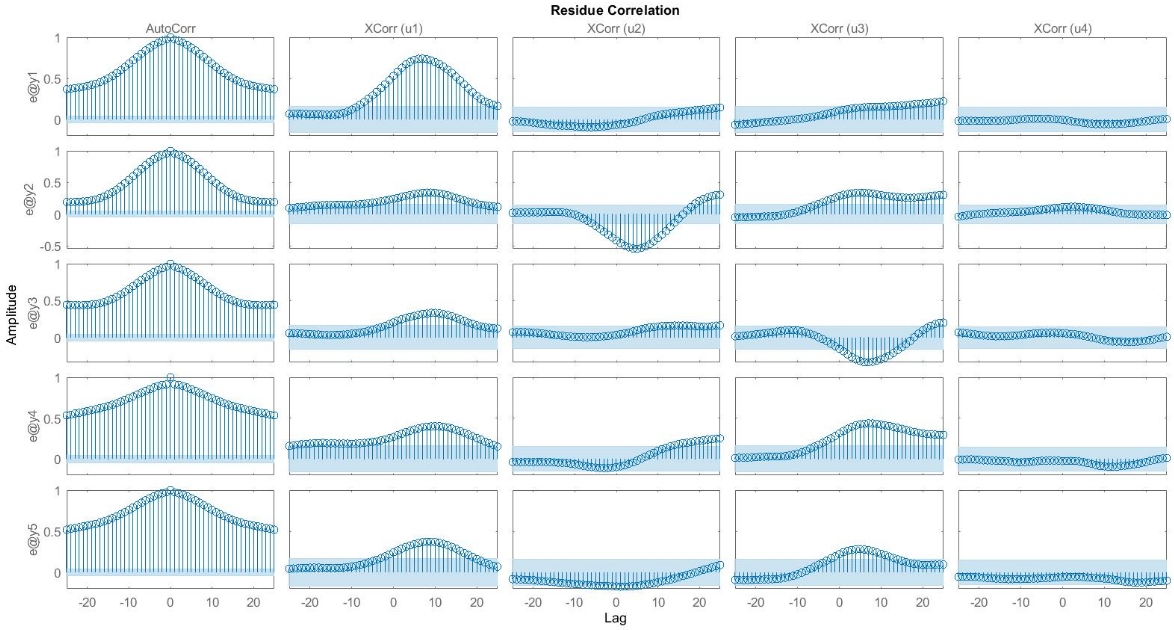

Figure 8: Residual Analysis 70th order

CONCLUSION

1. ARMAX model Identification was studied and corresponding model was developed by using PRBS data.

2. The obtained ARMAX model performed good on Validation data indicating a good model as the Validation data was unseen to the training process.

3. Finite Impulse Response Model was also studied and estimated.

4. FIR Model estimation is based on PRBS data and 70th order, 20th order and 15th order models were developed.

5. It was found that 15th order model also performed well on validation data thereby reducing overall number of parameters required to represent the model and hence this model can be used for further analysis and controller design.