Birla Insititute of Technology & Science – Pilani, K.K. Birla Goa Campus
First Semester 2020-21

PHY F313 **Lab test-2 (Open Book)**

Computational Physics 7 th November 2020

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1. In this problem you will get to see a classic example of chaos. You will be numerically solving the Lorenz equations. Picture depicting chaos in Lorenz equations are famous and are invariably shown to attract interest of people to the field of chaos.

   Lorenz equations were set up by Lorenz (ofcourse !) in an attempt to describe fluid flow moving due to convection. The experimental attempting to be described involved a fluid in motion between two plates with a temperature difference between them. Here are the Lorenz equations

$$\frac{dX}{dt} = p(Y - X) \tag{1}$$

$$\frac{dY}{dt} = -XZ + rX - Y \tag{2}$$

$$\frac{dZ}{dt} = XY - bZ \tag{3}$$

   The variables Y and Z are both related to the temperature of the fluid in motion. In absence of convection, due to conduction the temperature variation from bottom to top plate will be linear. However, when convection sets in temperature variation from top to bottom plate is not linear. The variable Z is related to the deviation of temperature variation from linearity. Y is related to tempeature difference between rising and falling fluid (due to convection) X is related to the time dependence of the so called stream function. Velocity of the fluid can be found from the stream function. You needn't bother to understand more than this right now.

   p, r, b are parameters that can be varied in an experiment. p is the Prandtl number (ratio of kinetic diffusivity to thermal diffusivity). r is proportional to the Rayleigh number which is a dimensionless measure of temperature difference between top plate and bottom plate. b is related to the vertical height of fluid layer.

   For small values temperature differences between bottom and top plate (expressed by r) there is no convection (you know that) and only conduction. Hence for low r values$Y = 0, Z = 0$ (see description of Y, and Z above) and $X = 0$ (since this is related to velocity which is zero in absence of convection). As we raise the temperature difference between the bottom and top plate (by increasing$r$) convection will start up

   (a) Write a Runge-Kutta **3rd order (RK-3)** code to solve the Lorenz equations for chosen values of parameters p, r, b and initial values of X, Y and Z. You are allowed to modify your own previously written RK-2 code to a RK-3 code .

   (b) Test your RK-3 code using SHM equations. Plot displacement vs time result & upload your result.

   (c) After testing your code successfully carry out your simulations of Lorenz equations for the following parameter values

      i. $r = 0.5,\ p = 10,\ b = 8/3$
      ii. $r = 2,\ p = 10,\ b = 8/3$

iii. $r = 25$, $p = 10$, $b = 8/3$

iv. $r = 160$, $p = 10$, $b = 8/3$

Plot Z vs t, and Z vs X for each of the above parameters.
Describe what you find through these plots.