Trajectory Generation using Inverse Kinematics for a five bar Parallel Planar Manipulator (DexTAR)

Ameya Deshpande, Kartik Prakash and Mukund Viswanathan

[1] ***Abstract*—This paper addresses the formulation of Forward Kinematics, Inverse Kinematics, Jacobian, Inertia matrix and Mass Matrix of a five bar parallel planar manipulator. The use of equal lengths for proximal and distal links increases the usable workspace since it tends to not have holes in the workspace[1]. One such commercial manipulator in the market is DexTAR (Dexterous Twin Arm Robot). This paper discusses about the kinematics for DexTAR and how it can be used to obtain various trajectories using inverse kinematics.**

***Index Terms*—Forward Kinematics,inverse Kinematics, Parallel manipulator, five-bar, singularities, DexTAR.**

I. Introduction

Parallel manipulators have smaller workspace than similar sized serial manipulators [2]. This is because there are more constraints in a parallel manipulator than in a serial manipulator. A five bar mechanism parallel robot is comparable with a two link serial manipulator. A serial manipulator has more freedom in motion as the end effector is connected to just one link and can traverse more workspace. On the other hand for a parallel robot the end effector is connected to at least two links and hence its motion is bounded by the interactions between those two links.

This is one part of the picture which discusses about reduction of workspace from serial to parallel manipulators. However, a five bar mechanism may or may not have the ability to access the entirety of its bounded workspace, depending on the lengths of proximal links and distal links. Distal refers to the links that are connected to the end effector and proximal refers to the links which are connected to the ground.

Although there are some disadvantages over the serial manipulators a very important advantage is that the errors of a parallel manipulators are averaged out in parallel chains. However for serial manipulators they get accumulated, and hence parallel robots are more accurate[3]. Parallel robots become useful when it can be designed optimally to use the available workspace, and if the workspace that can be traversed by the robot is good enough for the task at hand

DexTAR: Dexterous Twin Arm Robot commonly referred to as DexTAR (Fig. 1) is a five bar parallel planar manipulator. It has two base joints which are actuated by motors and they are used to control the links directly connected to them. Generally they also have a linear actuator which is mounted on the end-effector which is used for pick and place operations. This makes DexTAR a 3 DOF parallel robot.

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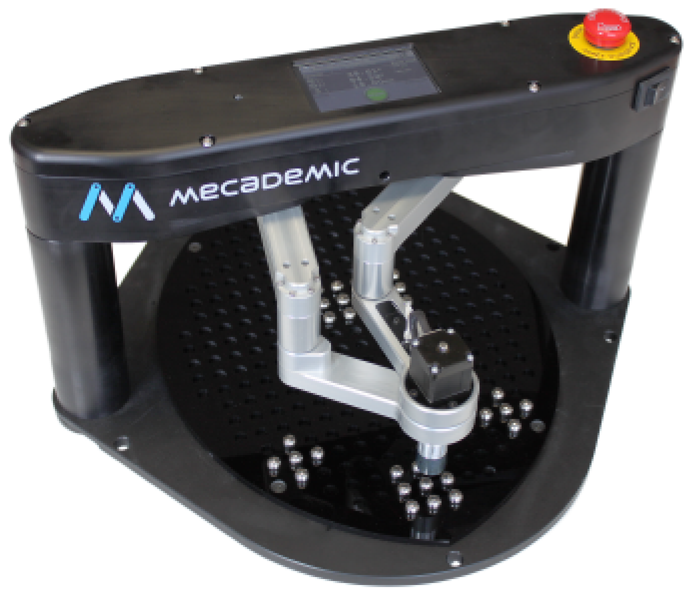

Fig. 1. DexTAR robot [4]

It is interesting to note that DexTAR is essentially a double SCARA robot. The major advantage of DexTAR over a serial SCARA comes from the placement of motors. As the motors are placed at the base the moving links can be much lighter than a SCARA robot. Due to lighter moving parts and lower inertia with respect to serial manipulators, it can reach high speeds, and is extremely precise. Since there are two links connected to the end effector it has more rigidity compared to a two link serial manipulator and thus has more strength. This without a doubt increases its payload capacity over its serial counterpart.

The DexTAR robot is driven by just two actuators at the base has lesser restrictions compared to other parallel robots like the Delta robot. As a result of this, it has more workspace. In Fig.1 one can see the workspace available represented by the black oval region. Furthermore as DexTAR uses equal link lengths it has most optimal workspace when compared to other five bar robots.

II. FORWARD KINEMATICS

From the geometry of the problem and Figure 2, we can get expressions for end-effector position as follows,

$$x_p = l_1 c_1 + l_3 c_3 = l_2 c_2 + l_4 c_4 + l_5 \tag{1}$$

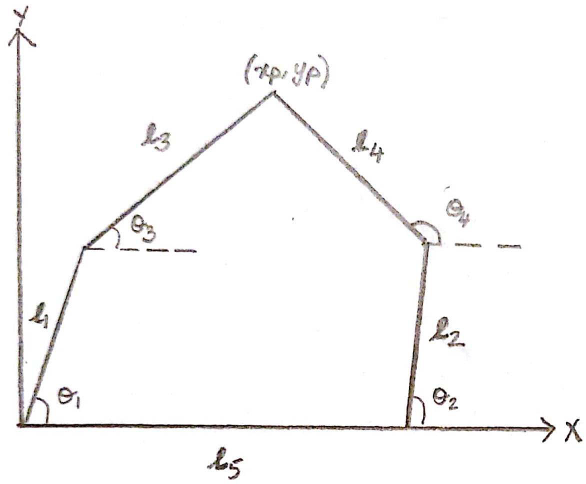

Fig. 2. Representation of five bar mechanism

$$y_p = l_1s_1 + l_3s_3 = l_2s_2 + l_4s_4 \tag{2}$$

To obtain the equations of forward kinematics in terms of independent variables, we can substitute θ_3 and θ_4 from inverse kinematics, which is discussed in the following section.

III. INVERSE KINEMATICS

$\theta_1$and θ_2 are independent. θ_3 and θ_4 can be calculated in terms of $\theta_1$and θ_2 from FK.

$$\theta_4 = 2tan^{-1}((A_1 \pm \sqrt{(A_1^2 + B_1^2 - C_1^2)}/(B_1 - C_1)) \tag{3}$$

where,

$$A_1 = 2l_2l_4sin\theta_2 - 2l_1l_4cos\theta_1$$

$$B_1 = 2l_4l_5 + 2l_2l_4cos\theta_2 - 2l_1l_4cos\theta_1$$

$$C_1 = l_1^2 - l_3^2 + l_4^2 + l_2^2 - l_5^2 - l_1l_2sin\theta_1sin\theta_2$$
$$-2l_1l_5cos\theta_1 + 2l_2l_5cos\theta_2 - 2l_1l_2cos\theta_1cos\theta_2$$
$$\theta_3 = sin^{-1}((l_4sin\theta_4 + l_2sin\theta_2 - l_1sin\theta_1)/l_3) \tag{4}$$

By using FK and eliminating θ_3 and θ_4,

$$\theta_1 = 2tan^{-1}(-B_2 \pm \sqrt{(A_2^2 + B_2^2 - C_2^2)})/(-A_2 - C_2) \tag{5}$$

where,

$$A_2 = x_p; B_2 = y_p; C_2 = (l_1^2 - l_3^2 + x_p^2 + y_p^2)/(2l_1)$$

$$\theta_2 = 2tan^{-1}(-B_3 \pm \sqrt{(A_3^2 + B_3^2 - C_3^2)})/(-A_3 - C_3) \tag{6}$$

where,

$$A_3 = x_p - l_5; B_3 = y_p; C_3 = (l_2^2 - l_5^2 - l_4^2 - 2x_pl_5 + x_p^2 + y_p^2)/(2l_4)$$

IV. JACOBIAN

For Jacobian, we will divide the system into 2 parts(2-link planar manipulators) and then calculate the Jacobian for the two systems individually.

The Jacobian for the first 2-link manipulator system will be,

$$J_1 = \begin{bmatrix} -A_{11} & -A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{7}$$

where,

$$A_{11} = l_1sin\theta_1 + l_3sin(\theta_1 + \theta_6)$$

$$A_{12} = l_3sin(\theta_1 + \theta_6)$$

$$A_{21} = l_1cos\theta_1 + l_3cos(\theta_1 + \theta_6)$$

$$A_{22} = l_3cos(\theta_1 + \theta_6)$$

Similarly for the second system,

$$J_2 = \begin{bmatrix} -B_{11} & -B_{12} \\ B_{21} & B_{22} \end{bmatrix} \tag{8}$$

$$B_{11} = l_2sin\theta_2 + l_4sin(\theta_2 + \theta_7)$$

$$B_{12} = l_4sin(\theta_2 + \theta_7)$$

$$B_{11} = l_2cos\theta + l_4cos(\theta_2 + \theta_7)$$

$$B_{12} = l_4cos(\theta_2 + \theta_7)$$

Using the constraints that the velocities of the end effector are equal, and eliminating θ_6 and θ_7 we can get the Jacobian for DexTAR,

$$J = \begin{bmatrix} -C_{11} & -C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{9}$$

where,

$$C_{11} = A_{11} + A_{21}((B_{12}A_{21} - A_{11}B_{22})/(A_{12}B_{22} - B_{12}A_{22}))$$

$$C_{12} = A_{12}((B_{11}B_{22} - B_{21}B_{12})/(A_{12}B_{22} - B_{12}A_{22}))$$

$$C_{21} = A_{21} + A_{22}((B_{12}A_{21} - A_{11}B_{22})/(A_{12}B_{22} - B_{12}A_{22}))$$

$$C_{22} = A_{22}((B_{11}B_{22} - B_{21}B_{12})/(A_{12}B_{22} - B_{12}A_{22}))$$

V. DYNAMICS

For getting the inertia matrix we will use kinetic energy of the system. The kinetic energy, K can be defined as.

$$K = \frac{1}{2}\sum I_i \dot{}_i + \dot{m}_i(x_i^2 + y_i^2) \tag{10}$$

$$K = \frac{1}{2}\dot{Q}_{12}^T I_{12}\dot{Q}_{12} + \frac{1}{2}\dot{Q}_{34}^T I_{34}\dot{Q}_{34} + \frac{1}{2}\dot{X}_{12}^T M_{12}\dot{X}_{12}$$

$$+\frac{1}{2}\dot{Y}_{12}^T M_{12}\dot{Y}_{12} + \frac{1}{2}\dot{X}_{34}^T M_{34}\dot{X}_{34} + \frac{1}{2}\dot{Y}_{34}^T M_{34}\dot{Y}_{34} \tag{11}$$

where,

$$\dot{X}_{12} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}; \dot{Y}_{12} = \begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix}; \dot{X}_{34} = \begin{bmatrix}\dot{x}_3\\ \dot{x}_4\end{bmatrix}; \dot{Y}_{34} = \begin{bmatrix}\dot{y}_3\\ \dot{y}_4\end{bmatrix};$$

$I_{12} = diag\begin{bmatrix}I_1 & I_2\end{bmatrix}; I_{34} = diag\begin{bmatrix}I_3 & I_4\end{bmatrix};$
$M_{12} = diag\begin{bmatrix}M_1 & M_2\end{bmatrix}; M_{34} = diag\begin{bmatrix}M_3 & M_4\end{bmatrix}$
Differentiating FK,

$$l_1c_1\dot{q}_1 + l_3c_3\dot{q}_3 - l_2c_2\dot{q}_2 - l_4c_4\dot{q}_4 = 0 \tag{12}$$

$$l_1s_1\dot{q}_1 + l_3s_3\dot{q}_3 - l_2s_2\dot{q}_2 - l_4s_4\dot{q}_4 = 0 \tag{13}$$

Writing the above equations in vector form gives,

$$\dot{Q}_{34} = A_{34}^{-1}A_{12}\dot{Q}_{12} \tag{14}$$

where,

$$A_{12} = \begin{bmatrix}l_1s_1 & -l_2s_2\\ l_1c_1 & -l_2c_2\end{bmatrix}$$

$$x_1 = l_{c1}c_1; y_1 = l_{c1}s_1$$

$$x_2 = l_{c2}c_2; y_2 = l_{c2}s_2$$

$$x_3 = l_{c3}c_3 + l_1c_1; y_3 = l_{c3}s_3 + l_1s_1$$

$$x_4 = l_{c4}c_4 + l_2c_2; y_4 = l_{c4}s_4 + l_2s_2$$

Differentiating the above equations w.r.t. time, we get relation between $x_i$and q_i and writing them in vector form we note,

$$\dot{X}_{12} = -\begin{bmatrix}l_{c1}s_1 & 0\\ 0 & l_{c2}s_2\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = -A_s\dot{Q}_{12} \tag{15}$$

$$\dot{Y}_{12} = \begin{bmatrix}l_{c1}c_1 & 0\\ 0 & l_{c2}c_2\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = A_c\dot{Q}_{12} \tag{16}$$

$$\dot{X}_{34} = -(A_{s34}A_{34}^{-1}A_{12} + A_s)\dot{Q}_{12} \tag{17}$$

$$\dot{Y}_{34} = (A_{c34}A_{34}^{-1}A_{12} + A_c)\dot{Q}_{12} \tag{18}$$

where,

$$A_{s34} = \begin{bmatrix}l_{c3}s_3 & 0\\ 0 & l_{c4}s_4\end{bmatrix}; A_{c34} = \begin{bmatrix}l_{c3}c_3 & 0\\ 0 & l_{c4}c_4\end{bmatrix}$$

Substituting the equations obtained in the kinetic energy equation, we note that inertia matrix is,

$$D = I_{12} + (A_{34}^{-1}A_{12})^T I_{34}A_{34}^{-1}A_{12} + A_s^T M_{12}A_s$$
$$+(A_{s34}A_{34}^{-1}A_{12} + A_s)^T M_{34}(A_{s34}A_{34}^{-1}A_{12} + A_s)$$

$$+A_c^T M_{12}A_c + (A_{c34}A_{34}^{-1}A_{12} + A_c)^T M_{34}(A_{c34}A_{34}^{-1}A_{12} + A_c) \tag{19}$$

By using the Jacobian and Inertia matrix we note that mass matrix will be,

$$M = J^{-T}DJ^{-1} \tag{20}$$

VI. SINGULARITIES

A two link serial manipulator has only one kind of singularity where the end effector loses one degree of freedom. This happens when the arm is fully stretched or fully folded. In case of DexTAR, and any parallel robot in general, there are two kinds of singularities [1].

A. Type 1 Singularity

Type I singularities are commonly referred to as serial singularities, and are common to both parallel and serial manipulators. In this case the end effector loses one or several degrees of freedom.

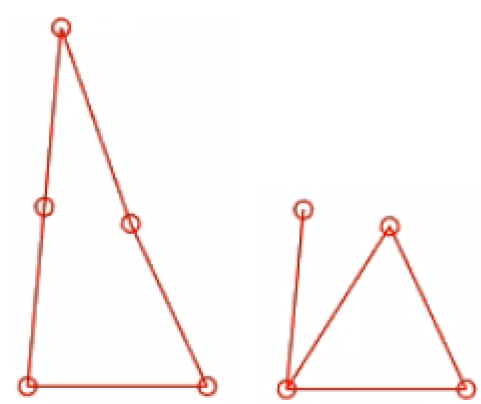

Fig. 3. Type I singularity configurations

B. Type 2 Singularity

Type II singularities are referred to as parallel singularities and are common in parallel robots. Here the actuators are not capable of resisting a force/moment applied to the end effector. They also lead to segmentation of the workspace into various singularity–free regions.

In the following text we will be dealing only with Type II singularity.

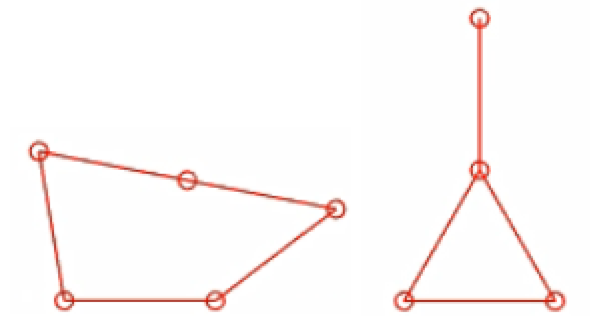

Fig. 4. Type II singularity configurations

VII. Modes of Operation

To eliminate the Type 2 singularity mentioned above, the concept of switching modes is used. The modes as shown

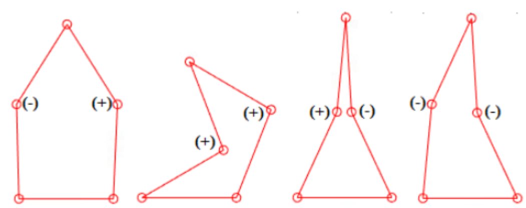

Fig. 5. Modes of operation

in Figure 4 definite workspaces which are singularity free. This fact can be utilised to map different geometries and trace different paths without encountering the singularity. If our task space overlaps between two or more such singularity free zones, the approach is to switch between the appropriate working modes.

VIII. Trajectory Generation

In the previous sections we have already calculated the inverse kinematics for the model. So for a given end effector position we know the angles theta1 and theta2 for our actuator input. As there can be multiple solutions for the given end effector position we can have different values of our actuator inputs theta1 and theta2. But for the purpose of obtaining trajectories we will make some assumptions with respect to the kind of configuration we want to be working with.

A. Tracing a Line

Assembly mode: Positive
Working mode: (+) (+)
Equation of line: y= x+10

B. Tracing a Rectangle

Assembly mode: Negative
Working mode: (-)(+)
Corner points of rectangle: {(2,2),(6,2),(6,6),(2,6)} in the order ABCD

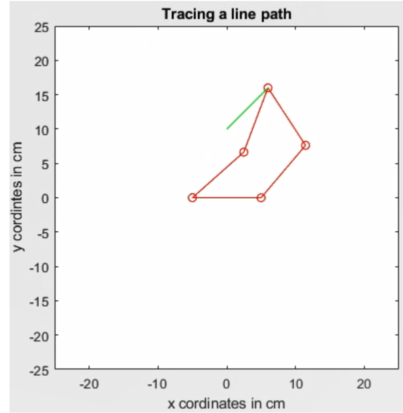

Fig. 6. Tracing a line

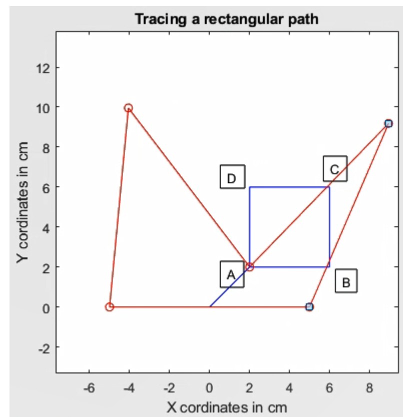

Fig. 7. Tracing a rectangle

C. Tracing a Circle

Assembly mode: Positive
Working mode: (-)(+)
Centre: (0,15)
Radius: 3 cm

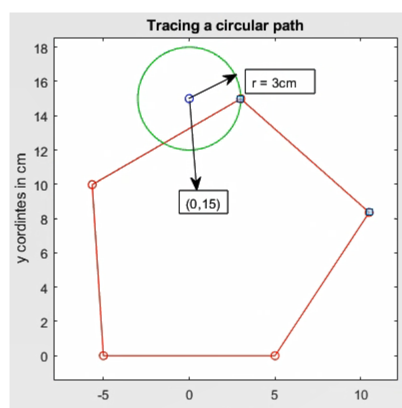

Fig. 8. Tracing a circle

D. Tracing an Involute Spiral

Assembly mode: Positive

Working mode: (-)(+)

Centre : (0,15)

Equation of spiral: $X=X_c+$ aspiral*(cos(t)+t*sin(t)) and $Y = Y_c+$ aspiral*(sin(t) - t*cos(t));

where t is a paramter, (X_c,Y_c) is the centre point and aspiral is the scaling factor.

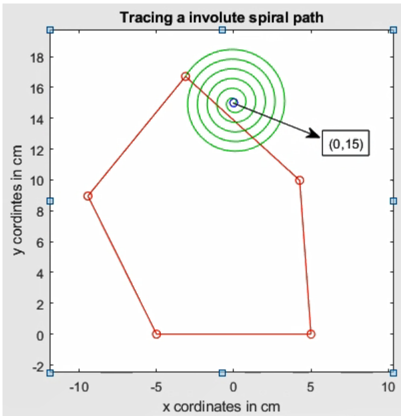

Fig. 9. Tracing an involute spiral

IX. CONCLUSION

The five bar equal length parallel manipulator DexTAR was studied in detail through this article. The forward kinematics and inverse kinematics equations, jacobian and the formulations for Inertia matrix (Joint Space) and Mass Matrix (Cartesian space) were calculated. The use of equal link lengths makes optimal use of workspace and is free of any holes[1], but has the drawback of more singularities, and hence reduced degrees of freedom in such scenarios. Using the concept of modes of operation and associated singularity free zones, the end effector position could be traced along four different paths using a MATLAB script without encountering any Type II singularities.

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