Generalized Fractional Matched Filtering and its Applications

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Abstract

Time domain matched filtering is a classic method used in radar and sonar applications to maximize signal to noise ratio (SNR) gain, estimate time delay, and improve range resolution. Fractional Fourier transform, and fractional Fourier domain matched filtering are used to overcome the drawbacks of time domain matched filtering and are shown to have improved performance for linear chirps. This paper presents a generalized fractional matched filtering (GFMF) for estimating higher order chirp parameters with known time delay. It is shown to provide SNR gain equivalent to time domain matched filtering. As an application of GFMF, a novel method to minimize SNR gain degradation due to the range-Doppler coupling effect of quadratic chirps is presented. For higher order chirp with unknown time delay, another method using generalized fractional envelope correlator (GFEC) is proposed, which performs joint estimation of time delay and higher order chirp parameters using a double quadratic chirp.

Main Objectives

1. Derivation of SNR gain and impulse response for GTFT based matched filter called GFMF, which generalizes fractional and time domain matched filtering
2. Demonstration of an application of GFMF to solve SNR gain degradation due to the range-Doppler coupling effect in quadratic chirps
3. Derivation of peak shift between the transmitted and received waveforms using GFEC, which is a generalization of the fractional envelope correlator
4. Joint estimation of time delay and higher order chirp offset parameters using GFEC and double quadratic chirp

Definition of GFMF

Similar to time domain matched filtering, GFMF correlates a GTFT of known signal (replica of the transmitted signal) with GTFT of an unknown signal (received signal) in GTFT domain.

GFMF SNR Gain

$$\text{GFMF SNR gain at } (\alpha, \lambda) = \tau = \text{SNR gain of time domain matched filtering,} \tag{1}$$

Thus, the SNR for GTFT domain matched filtering is comparable to time domain matched filtering. We further discuss the robustness of GTFT matched filter under the effect of Doppler frequency.

GFMF Impulse Response

The Impulse reponse of GFMF giving maximum SNR is obtained for:

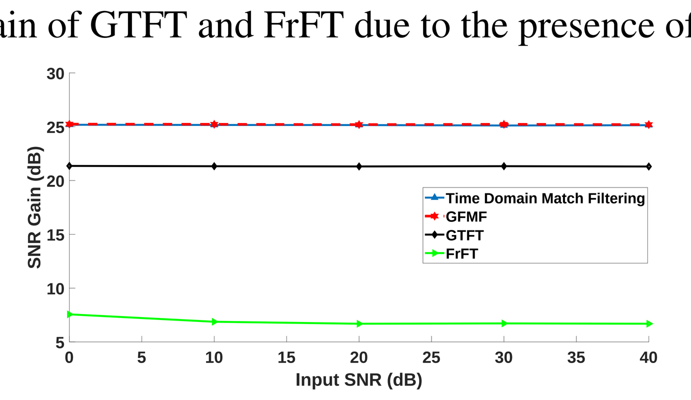
$$H_{\alpha,\lambda}(f_{\alpha,\lambda}) = \frac{X^*_{\alpha,\lambda}(-f_{\alpha,\lambda})}{\frac{\eta}{2}}. \tag{2}$$

Where $\frac{\eta}{2}$ is PSD of AWGN noise, $H_{\alpha,\lambda}(f_{\alpha,\lambda})$ is the transfer function of GFMF, and $X^*_{\alpha,\lambda}(-f_{\alpha,\lambda})$ is GTFT of the original signal at given α and λ.

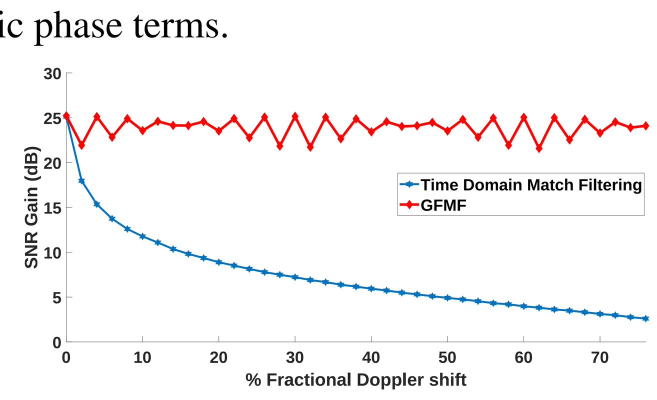
When $\lambda = 0$, from Eq. (2), then impulse response of GFMF becomes impulse response of fractional Fourier matched filter [2]. When $\lambda = 0$ and $\alpha = \frac{\pi}{2}$, then impulse response of GFMF becomes the transfer function of time domain matched filter.

GFMF SNR Comparisons (in absence of doppler)

As shown in figure (a), SNR gain of GFMF is equal to SNR gain of time domain matched filtering. (in absence of range Doppler Coupling effect). Also, SNR gain of GFMF is greater than the SNR gain of GTFT and FrFT due to the presence of quadratic phase terms.



(a) GFMF SNR Comparisons



(b) Range Doppler coupling effect

GFMF Application: Range Doppler Coupling Effect

Time-domain matched filtering gives SNR gain degradation due to a significant mismatch between the transmitted and received waveforms in the case of linear chirps and higher order chirps. This can be confirmed via simulation results, as shown in figure (b). SNR gain in GFMF is almost constant with respect to Doppler frequency because the peak amplitude of GFMF depends on α and λ parameters of GTFT kernel, and thus is independent of Doppler frequency.

Definition and Need of GFEC

In the case of GFMF, for quadratic chirps, peak amplitude's position depends on the α parameter of GTFT kernel, which in turn depends on time delay. Since quadratic chirp contains third order phase terms, a closed-form expression for a peak position of GFMF output is difficult to determine. Hence, this section presents GFEC, which is a generalization of fractional envelope correlator [3]. GFEC is a method to determine time delay and perform joint estimation of higher order chirp offset parameters in case of unknown time delay.

Peak Shift of GFEC

Consider a quadratic chirp transmission waveform $x^{tr}(t)$ as:

$$x^{tr}(t) = rect\left(\frac{t}{\tau}\right) e^{ia\pi t^2 + ic\pi t^3}, \tag{3}$$

Considering an extension of echo model presented in [1], received signal $x^{recv}(t)$ is taken to be:

$$x^{recv}(t) = x(t - t_d).e^{i2\pi f_d t + i\pi a_r t^2 + i\pi J_r t^3}, \tag{4}$$

Peak shift obtained using envelope correlation between the received and transmitted waveform after dimension normalization is given as:

$$\Delta d_n = f_s t_d \cos\alpha + (f_d - 1.5 c t_d^2 + a_r t_d) T_{max} \sin\alpha \tag{5}$$

GFEC Applications: Joint Parameter Estimation

The transmitted signal $x^{tr}(t)$ is a double quadratic chirp, used for joint parameter estimation. A double quadratic chirp contains two quadratic chirps of the same pulse width but different chirp parameters:

$$x^{tr}(t) = rect\left(\frac{t}{\tau}\right).[e^{ic_1\pi t^3 + ia_1\pi t^2} + e^{ic_2\pi t^3 + ia_2\pi t^2}], \tag{6}$$

Broad steps of our techniques are as such: we consider the echo model to obtain the received signal $x^{recv}(t)$. Then we apply GTFT to $x^{recv}(t)$, and then consider cubic and quadratic matched conditions to derive the estimation of the chirp offset parameters.

Optimum λ (for each received wave) is obtained by cubic phase matching condition:

$$\lambda_1 = \frac{c_1 + J_r}{f_0^3} = \frac{(c_1 + J_r) T_{max}^{3/2}}{f_s^{3/2}}, \quad \lambda_2 = \frac{c_2 + J_r}{f_0^3} = \frac{(c_2 + J_r) T_{max}^{3/2}}{f_s^{3/2}}. \tag{7}$$

J_r can be obtain through Eq. (7). Further, using quadratic matched conditions we get:

$$\tan\alpha_1 = \frac{f_s}{T_{max}[3c_1 t_d - (a_1 + a_r)]}, \quad \tan\alpha_2 = \frac{f_s}{T_{max}[3c_2 t_d - (a_2 + a_r)]}. \tag{8}$$

Thus, we can calculate a_r from Eq. (8). Using Eq. (5) for both chirps we get,

$$\Delta d_{1n} = f_s t_d \cos\alpha_1 + (f_d + a_r t_d - 1.5 c_1 t_d^2) T_{max} \sin\alpha_1 \tag{9}$$

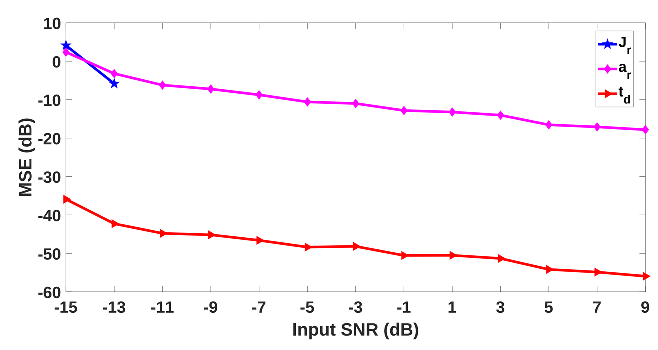
$$\Delta d_{2n} = f_s t_d \cos\alpha_2 + (f_d + a_r t_d - 1.5 c_2 t_d^2) T_{max} \sin\alpha_2 \tag{10}$$

Parameters f_d and t_d can be calculated from the above two equations with the use of the estimated acceleration and jerk parameters.

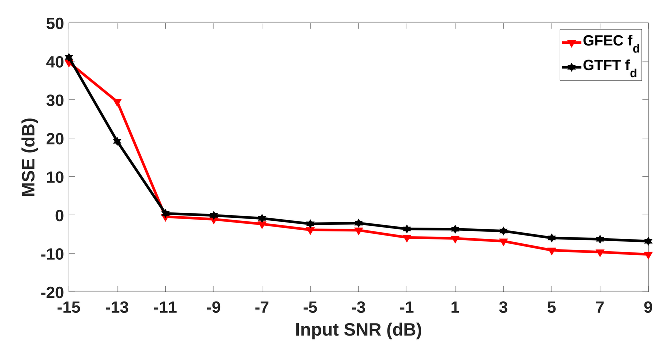
Double Quadratic Chirp Waveform: Estimation Error

Error expressions are calculated for all the estimated parameters and it is observed that the calculated error expressions will be minimum when the fractional Fourier angles (α_1 and α_2) will be near to $\pm 90^0$.

MSE Simulation: Double Quadratic Chirp



(c) MSE for J_r, a_r, and t_d



(d) MSE for f_d

MSE of parameter estimation using GFEC and Double chirp are shown in figure (c-d). The chirps are taken such that optimum fractional Fourier angle of first and second quadratic chirp in simulation are -86.56^0 and 86.56^0 respectively.

Conclusions

- SNR gain comparison is demonstrated to show superior noise performance of GFMF compared to time domain matched filtering, FrFT, and GTFT in the case of known time delay quadratic chirps
- GFMF gives lesser SNR degradation than time-domain matched filtering for non-zero Doppler frequency
- GFEC provides joint estimation of unknown time delay, frequency offset, frequency offset rate, and rate of frequency offset rate with a reasonable accuracy using double quadratic chirp waveform
- MSE of estimated parameters is presented, from which it can be inferred that the MSE of GFEC is lower compared to other existing methods for the same input SNR

Future Work

In the future, higher order waveforms can be analyzed using GFMF by an appropriate selection of $h(\cdot)$ in the kernel. Higher order double chirps can be used for joint estimation of higher order target parameters using GFEC.

References

[1] HU Dexiu, Zhen Huang, Shangyu Zhang, and LU Jianhua. Joint DOA, FDOA and differential Doppler rate estimation: Method and its performance analysis. *Chinese Journal of Aeronautics*, 31(1):137–147, 2018.

[2] Sanjay Kumar and Rajiv Saxena. FrMF: Fractional Fourier matched filter. *Circuits, Systems, and Signal Processing*, 37(1):49–80, 2018.

[3] X. Ning, L. Guo, and X. Sha. Joint time delay and frequency offset estimation based on fractional Fourier transform. In *2012 International Conference on ICT Convergence (ICTC)*, pages 318–322, 2012.