MODELLING THE DUFFING EQUATION WITH AN ANALOG CIRCUIT

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INTRODUCTION

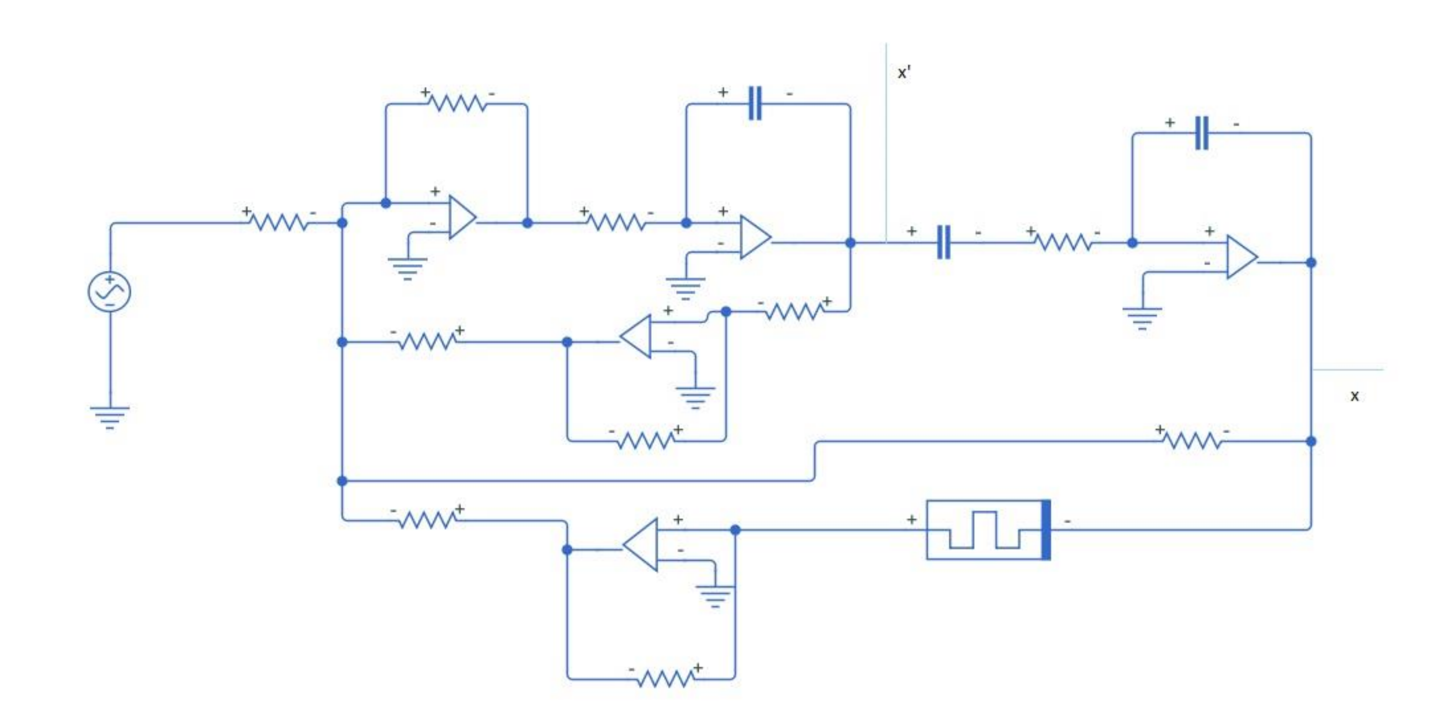
- Aim: To produce a completely analog output for the duffing system.
- The Duffing Equation:

$$\ddot{x} = -\alpha\dot{x} + \delta x - \beta x^3 + \mathrm{F}\sin(2\pi f t + \varphi)$$

- Equivalent equation for Circuit Elements :

$$\ddot{x} = -\alpha 10^5\dot{x} + 2.06 * 10^7 x - 2.136 * 10^8 x^3 + 10^{10}\mathrm{F}\sin(2\pi f t + \varphi)$$

CIRCUIT DIAGRAM



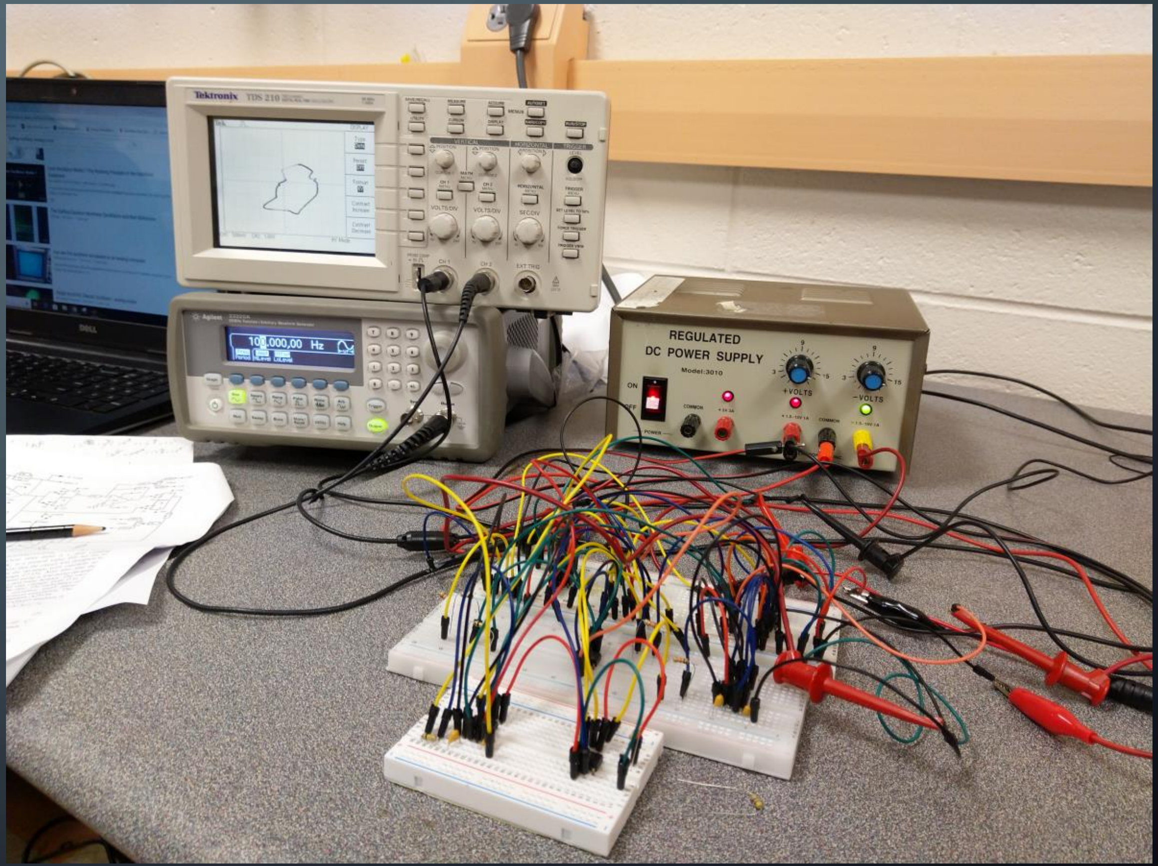
Major Components:

- Op-Amps (UA741)
- Analog Multipliers(AD 633)
- Resistors
- Capacitors
- DC power supply
- Function Generator
- Oscilloscope

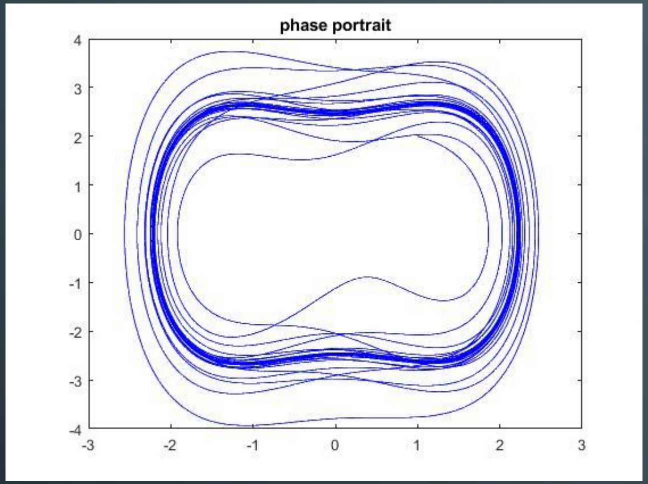
EXPERIMENT SETUP

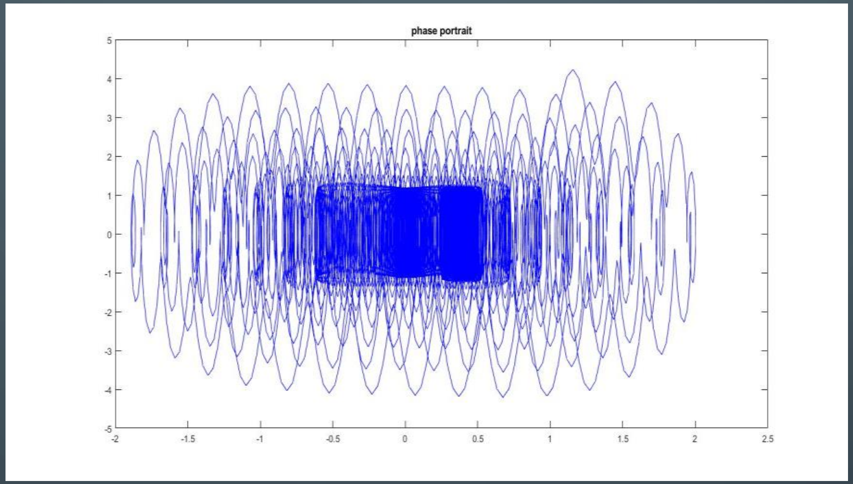
Varying Parameters:

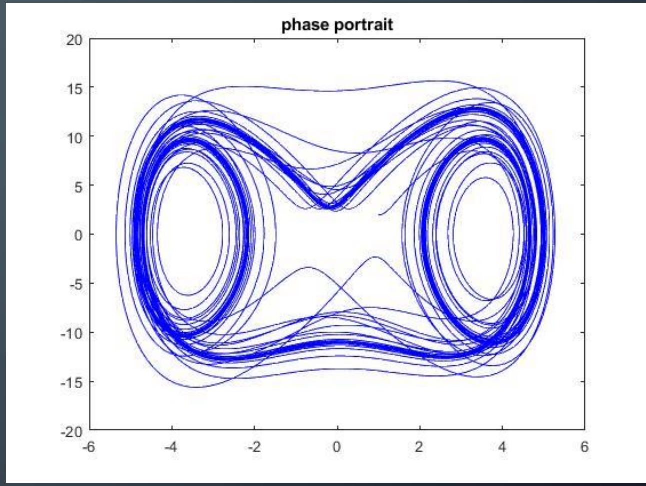
- Input Frequency
- Signal Amplitude
- Damping Coefficient (Resistor Value)



MATLAB SIMULATIONS





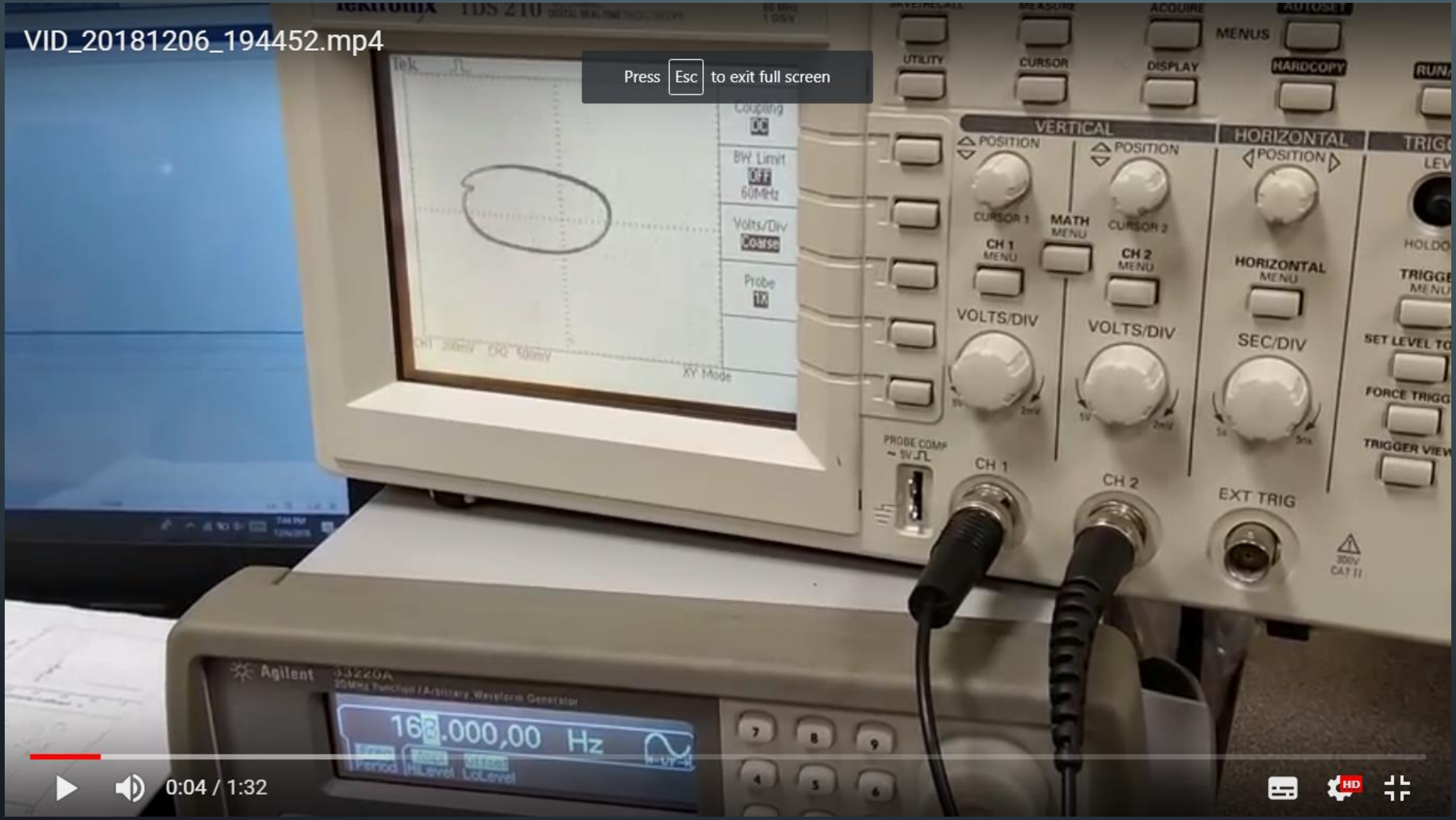


Amplitude: 0.2 V
Frequency: 1 Hz
Damping: 448.9

Amplitude: 0.2 V
Frequency: 628 Hz
Damping: 1000

Amplitude: 0.2 V
Frequency: 1.5 kHz
Damping: 448.9

PERIOD ONE

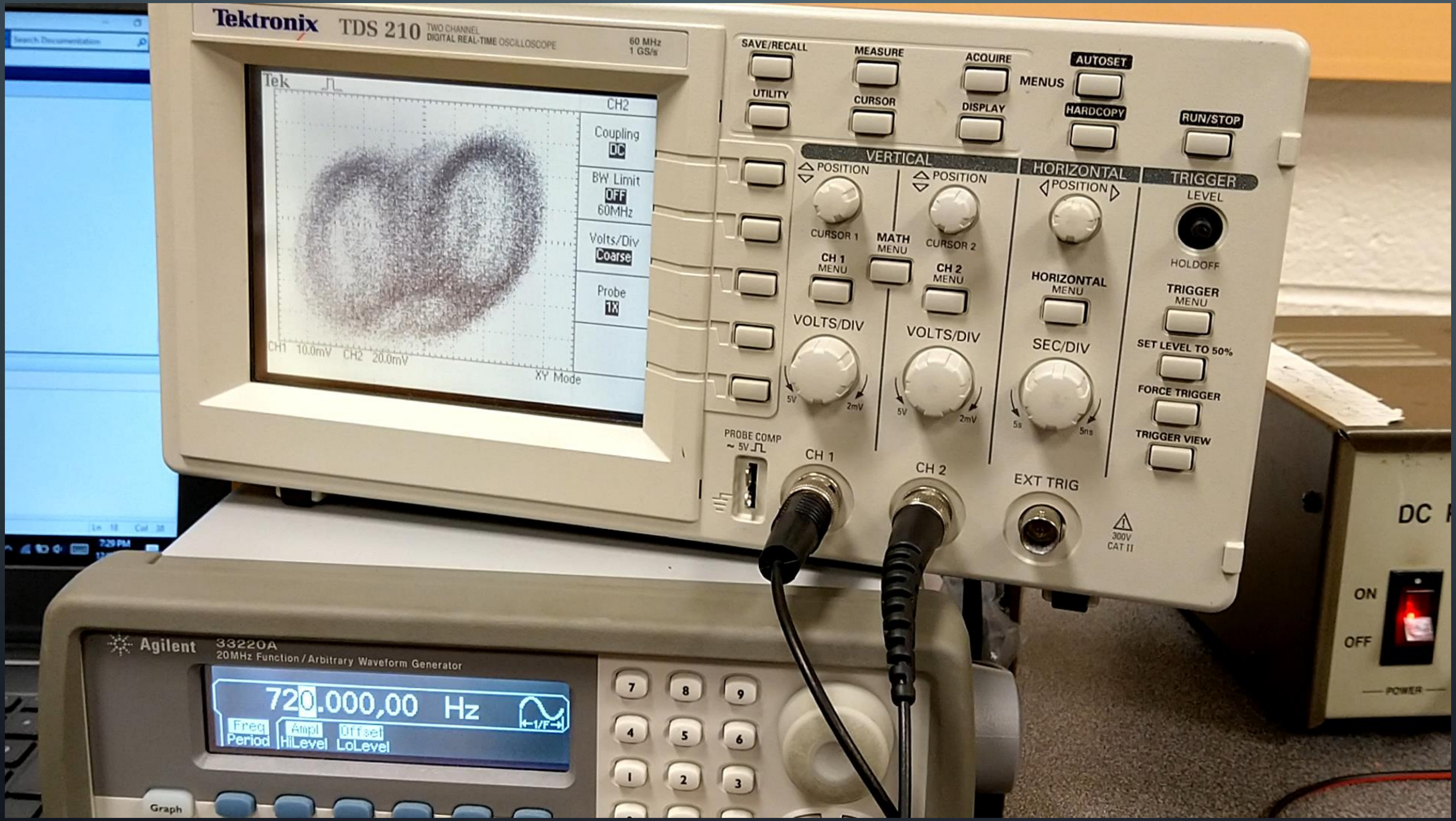


- Amplitude: 0.5 V
- Frequency: 168 Hz
- Damping: 448.9

PERIOD TWO AND PERIOD THREE

- Amplitude: 3.4 V
- Frequency: 5 kHz-35 kHz
- Damping: 448.9

CHAOS



- Amplitude: 0.2 V
- Frequency: 720 Hz
- Damping: 448.9

ROUTE TO CHAOS

<table>
<tr><th>Vrms</th><th>Frequency</th><th>Damping</th><th>Behavior</th></tr>
<tr><td rowspan="4">0.1 V</td><td>1 Hz</td><td rowspan="19">448.9</td><td>1</td></tr>
<tr><td>492 Hz</td><td>2</td></tr>
<tr><td>1.5 kHz</td><td>C</td></tr>
<tr><td>3.4 Khz</td><td>2</td></tr>
<tr><td rowspan="5">0.2V</td><td>1 Hz</td><td>1</td></tr>
<tr><td>300 Hz</td><td>2</td></tr>
<tr><td>600 Hz</td><td>2</td></tr>
<tr><td>720 Hz</td><td>C</td></tr>
<tr><td>900 Hz</td><td>2</td></tr>
<tr><td rowspan="3">0.4 V</td><td>100 Hz</td><td>1</td></tr>
<tr><td>200 Hz</td><td>2</td></tr>
<tr><td>2 kHz</td><td>1</td></tr>
<tr><td>0.75 V</td><td>200 Hz</td><td>1</td></tr>
<tr><td>1 V</td><td>100 Hz</td><td>1</td></tr>
<tr><td>1.5 V</td><td>100 Hz</td><td>1</td></tr>
<tr><td>2 V</td><td>100 Hz</td><td>1</td></tr>
<tr><td>2.5 V</td><td></td><td></td></tr>
<tr><td>3 V</td><td>5 Hz</td><td>2</td></tr>
<tr><td>3.5 V</td><td>5 Hz</td><td>2</td></tr>
<tr><td>0.5 v</td><td>26 Hz</td><td>5000</td><td>1</td></tr>
</table>

Behavior according to varying parameters:

- 1 : Period one
- 2 : Period two
- C : Chaos

PROS AND CONS

Pros	Cons
• Bypassing numerical errors occurring in numerical solvers • Instant parameter variations to study the behavior changes • Reduces solution times	• Low performance amplifiers and multipliers can lead to inaccurate solutions • Any noise in the circuit can get amplified due to presence of amplifiers • Resetting of capacitors to avoid hysterirsis

CONCLUSION

- The duffing equation can be successfully simulated by analog electrical circuit.
- Bifurcations from period one to period two and from period two to three were observed with parameter variation
- Chaotic behavior was verified by observing strange attractor.

IMPROVEMENTS

- With high performance components like AD 534 and Op Amp 356 analog simulation can be better matched with numerical simulations
- With auxiliary circuit for Sample and Hold, bifurcation diagram can be plotted on an oscilloscope.
- With a phase shifter circuit, Poincare sections can also be plotted.

REFERENCES

- ‘Modeling the Duffing Equation with an Analog Computer’ by Matt Schmitthenner. Physics Department, The College of Wooster, Wooster, Ohio 44691, USA
- ‘The Duffing oscillator: A precise electronic analog chaos demonstrator for the undergraduate laboratory’ by Brian Keith Jones and G. Trefan. April 2001 American Journal of Physics 69(4):464-469

THANK YOU