

# MODELLING THE DUFFING EQUATION WITH AN ANALOG CIRCUIT

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# INTRODUCTION

- Aim: To produce a completely analog output for the duffing system.
- The Duffing Equation:

$$\ddot{x} = -\alpha\dot{x} + \delta x - \beta x^3 + F \sin(2\pi f t + \varphi)$$

- Equivalent equation for Circuit Elements :

$$\ddot{x} = -\alpha 10^5 \dot{x} + 2.06 * 10^7 x - 2.136 * 10^8 x^3 + 10^{10} F \sin(2\pi f t + \varphi)$$

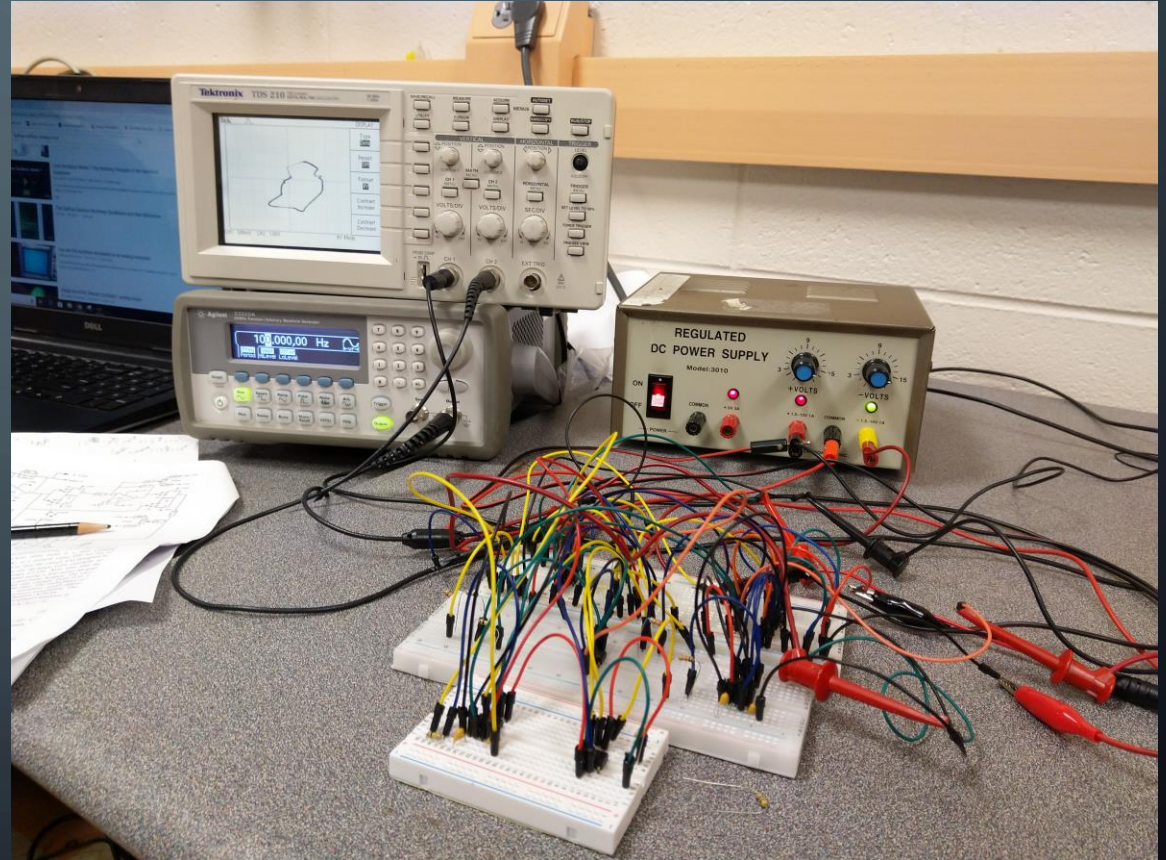
The circuit diagram illustrates a three-stage operational amplifier (op-amp) system. The first stage is a non-inverting amplifier where the input signal is divided by a voltage divider (two resistors) and fed into the non-inverting input of an op-amp. The op-amp's output is fed back to its inverting input through a resistor. The second stage is a differential amplifier consisting of two op-amps. The outputs of the first stage are connected to the non-inverting inputs of these two op-amps. Each op-amp has a resistor in its feedback path and its inverting input is connected to ground. The outputs of the second stage are connected to the inputs of a third stage, which is an inverting amplifier. This third stage uses a resistor for feedback and a capacitor for integration. The final output of the system is labeled 'x'.

- Op-Amps (UA741)
- Analog Multipliers(AD 633)
- Resistors
- Capacitors
- DC power supply
- Function Generator
- Oscilloscope

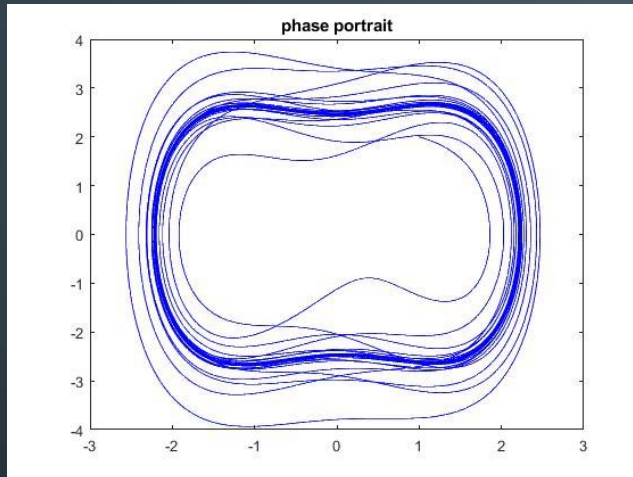
# EXPERIMENT SETUP

Varying Parameters:

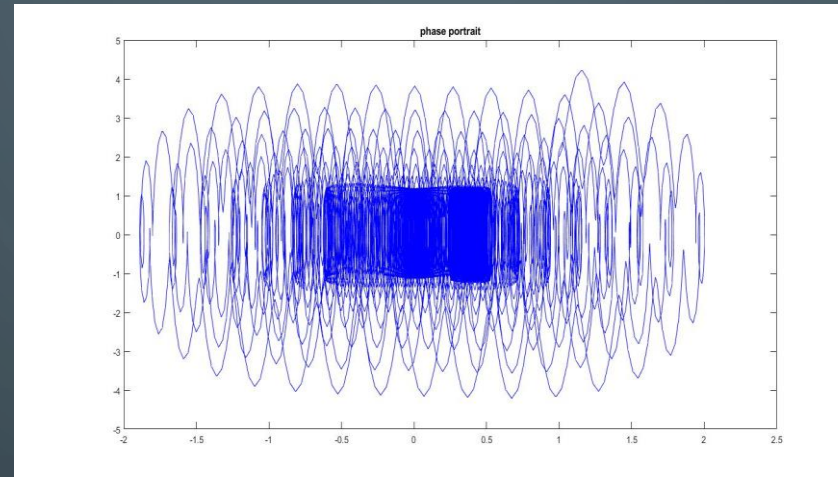
- Input Frequency
- Signal Amplitude
- Damping Coefficient (Resistor Value)



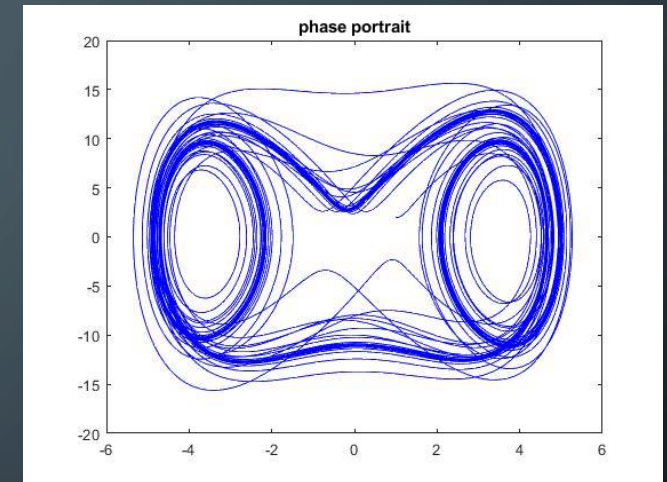
# MATLAB SIMULATIONS



Amplitude: 0.2 V  
Frequency: 1 Hz  
Damping: 448.9



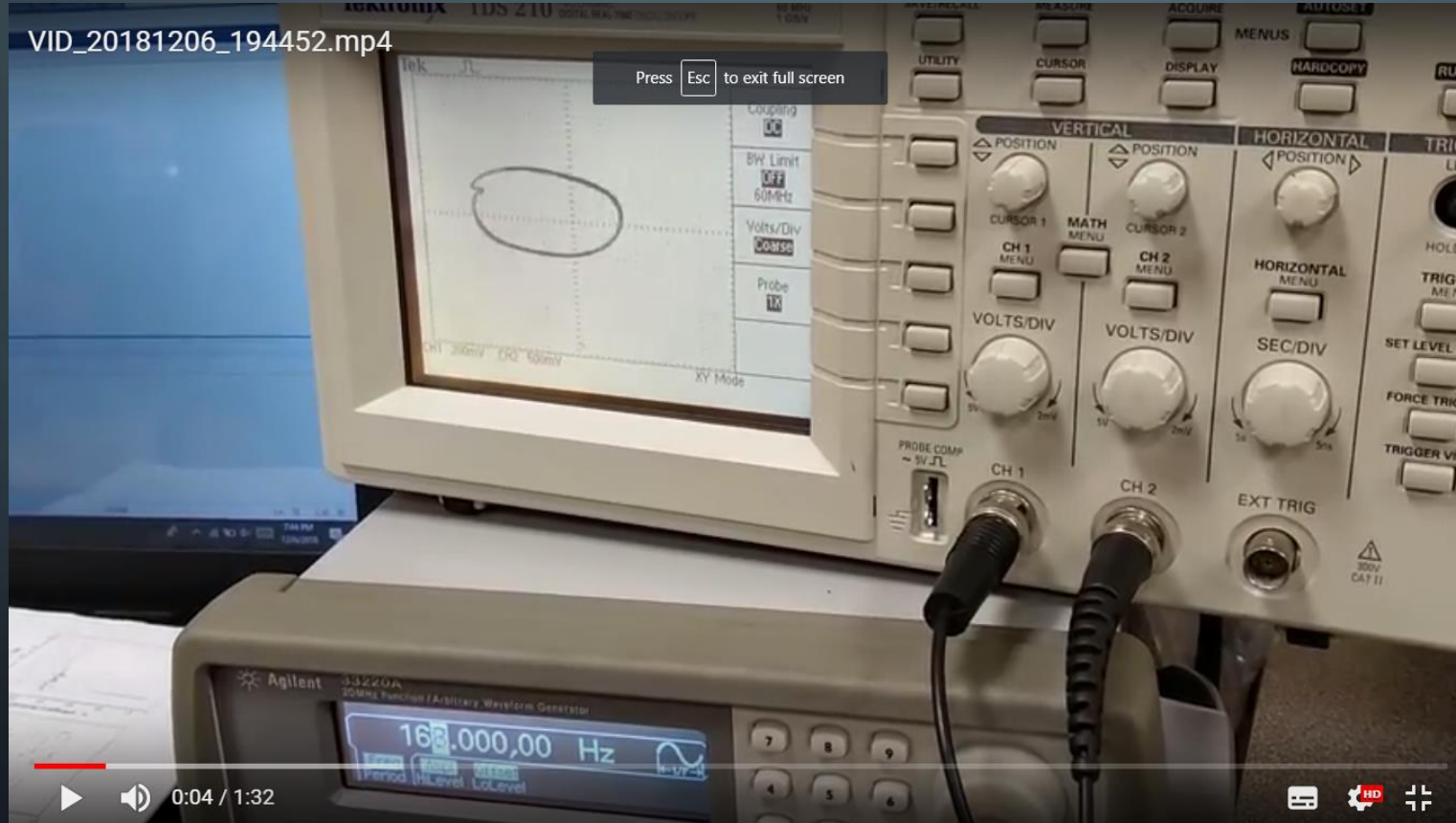
Amplitude: 0.2 V  
Frequency: 628 Hz  
Damping: 1000



Amplitude: 0.2 V  
Frequency: 1.5 kHz  
Damping: 448.9



# PERIOD ONE

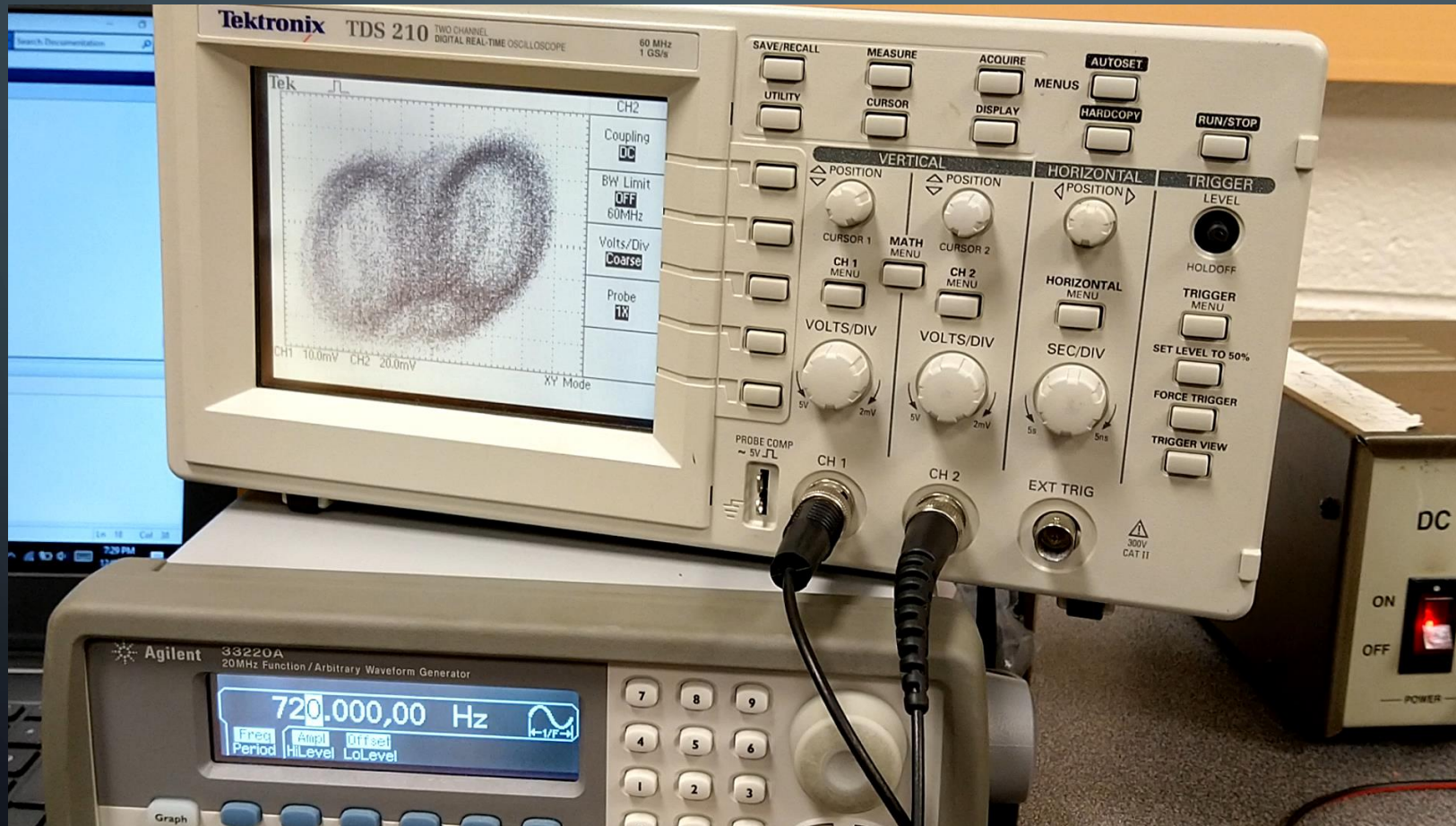


- Amplitude: 0.5 V
- Frequency: 168 Hz
- Damping: 448.9

# PERIOD TWO AND PERIOD THREE

- Amplitude: 3.4 V
- Frequency: 5 kHz-35 kHz
- Damping: 448.9

# CHAOS



- Amplitude: 0.2 V
- Frequency: 720 Hz
- Damping: 448.9



# ROUTE TO CHAOS

Vrms	Frequency	Damping	Behavior
0.1 V	1 Hz	448.9	1
	492 Hz		2
	1.5 kHz		C
	3.4 Khz		2
0.2V	1 Hz		1
	300 Hz		2
	600 Hz		2
	720 Hz		C
	900 Hz		2
0.4 V	100 Hz		1
	200 Hz		2
	2 kHz		1
0.75 V	200 Hz		1
1 V	100 Hz		1
1.5 V	100 Hz		1
2 V	100 Hz		1
2.5 V			
3 V	5 Hz		2
3.5 V	5 Hz		2
0.5 v	26 Hz	5000	1

Behavior according to varying parameters:

- 1 : Period one
- 2 : Period two
- C : Chaos

# PROS AND CONS

Pros	Cons
<ul style="list-style-type: none"><li>• Bypassing numerical errors occurring in numerical solvers</li><li>• Instant parameter variations to study the behavior changes</li><li>• Reduces solution times</li></ul>	<ul style="list-style-type: none"><li>• Low performance amplifiers and multipliers can lead to inaccurate solutions</li><li>• Any noise in the circuit can get amplified due to presence of amplifiers</li><li>• Resetting of capacitors to avoid hysteresis</li></ul>

# CONCLUSION

- The duffing equation can be successfully simulated by analog electrical circuit.
- Bifurcations from period one to period two and from period two to three were observed with parameter variation
- Chaotic behavior was verified by observing strange attractor.

# IMPROVEMENTS

- With high performance components like AD 534 and Op Amp 356 analog simulation can be better matched with numerical simulations
- With auxiliary circuit for Sample and Hold, bifurcation diagram can be plotted on an oscilloscope.
- With a phase shifter circuit, Poincare sections can also be plotted.



# REFERENCES

- 'Modeling the Duffing Equation with an Analog Computer' by Matt Schmitthenner. Physics Department, The College of Wooster, Wooster, Ohio 44691, USA
- 'The Duffing oscillator: A precise electronic analog chaos demonstrator for the undergraduate laboratory' by Brian Keith Jones and G. Trefan. April 2001 American Journal of Physics 69(4):464-469

The background is a dark blue gradient with a large, faint, light blue circle in the center. In the four corners, there are white, stylized circuit board traces and nodes (small circles) extending from the edges towards the center.

THANK YOU