# CS 517: Problem Set 1

Due: Thursday Apr 11; **typed** and **submitted electronically**.

1. Cantor's proof shows that if $f : X \to \mathcal{P}(X)$, then there exists at least one value $D \in \mathcal{P}(X) \setminus \text{range}(f)$.

   Prove that if $X$ is infinite, then for any $f : X \to \mathcal{P}(X)$, there are **infinitely many** values in $\mathcal{P}(X) \setminus \text{range}(f)$.

   In the class of April 3, I showed two different ways to construct a "diagonal" set. You might find this useful as inspiration, but you should appreciate that there is no guarantee that those two diagonal sets are different!!

   So, to be clear: given any $f$, I want you to clearly describe a collection of sets $D_1, D_2, D_3, \ldots$ (their definition should depend on $f$), and I want you to be able to clearly identify why $D_i \neq f(j)$ for all $i, j$ ($D_i$ not in range($f$)) and why $D_i \neq D_j$ for all $i \neq j$ (all the $D_i$'s are different).

   It's fine if you want to describe your answer just in the special case of $X = \mathbb{N}$.

2. Prove that a language $L$ is decidable **if and only if** both $L$ and its complement $\overline{L}$ are recognizable.

   You must prove both directions:

   - Assuming $L$ is decidable, show that $L$ and its complement are recognizable.
   - Assuming $L$ and $\overline{L}$ are recognizable, show that $L$ is decidable.

   You show that things are decidable/recognizable by constructing a suitable algorithm (and arguing that it satisfies the suitable properties).