Module 2 Practice Quiz

Due No due date **Points** 10 **Questions** 10
Available after Jan 22 at 12am **Time Limit** None
Allowed Attempts Unlimited

Take the Quiz Again

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	11 minutes	10 out of 10

Submitted Feb 5 at 11:54am

Question 1

1 / 1 pts

Which of the following statements about “term” in first-order logic is true?

Correct!

- (selected) “school(tom)” is a term if “tom” is a term and “school” is a function constant of arity 1.

  This is correct according to the definition of term in Module "Syntax of First Order Logic".

- Object variables such as x, y, z are not terms.
- A term is meant to represent a statement that is either true or false.
- “Father(tom)” is a term if “tom” is a term and “Father” is a predicate constant of arity 1.

Question 2

1 / 1 pts

Assume that the signature consists of the object constant Me, the unary predicate constant Male, and the binary predicate constant Parent, and nothing else. Which of the following first-order logic formulas express the following English sentence?

"I have a brother"

Choose all that apply.

Correct!

- [x] ∃x∃y (Male(x) ∧ Parent(y, x) ∧ Parent(y, Me) ∧ x ≠ Me)

This is correct since x cannot be Me, but x is a male, and x and Me have the same parent y.

- [ ] ∃x∃y (Male(y) ∧ (Parent(x, y) = Parent(x, Me)) ∧ ¬(x = Me))

- [ ] ∃x∃y (Male(y) ∧ Parent(x, y) ∧ Parent(x, Me))

Correct!

- [x] ∃x∃y (Male(y) ∧ Parent(x, y) ∧ Parent(x, Me) ∧ y ≠ Me)

This is correct since y cannot be Me, but y is a male, and y and Me have the same parent x.

Question 3

1 / 1 pts

Let P be the only predicate constant that is unary, and I an interpretation such that the universe is the set of all ASU students. For any ξ ∈ |I|,

$$P^I$$

(ξ) = t iff ξ has taken CSE 579. Which of the following first-order logic formulas express the following English sentence?

"There exists exactly two students who took CSE 579."

Choose all that apply.

Correct!

- [x] $\exists x \exists y(P(x) \wedge P(y) \wedge x \neq y \wedge \forall z\ (P(z) \rightarrow (z=x \vee z=y)))$

This is correct since x and y are different persons and every student must be either x or y.

Correct!

- [x] $\exists x \exists y \forall z[P(x) \wedge P(y) \wedge x \neq y \wedge ((x \neq z \wedge y \neq z) \rightarrow \neg P(z))]$

This is correct since x and y are different persons and for any other person z, z does not take CSE579.

- [ ] $\exists x \exists y[P(x) \wedge P(y)]$
- [ ] $\neg(\exists x \exists y \exists z(P(x) \wedge P(y) \wedge P(z))) \wedge \exists x \exists y(P(x) \wedge P(y))$

Question 4

1 / 1 pts

Let the underlying signature be {a, P, Q}, where a is an object constant, P is a unary predicate constant, and Q is a binary predicate constant. Assume object variables range over the set N of nonnegative integers, and the signature is interpreted as follows:

- a represents the number 10,
- P(x) represents the condition “x is a prime number,”
- Q(x, y) represents the condition “x is less than y.”

Which of the following first-order logic formulas express the following English sentence?

"There are infinitely many prime numbers."

- ○ $\forall x \exists y[P(x) \wedge Q(x, y) \wedge P(y)]$

Correct!

- ◉ $\exists x\ P(x) \wedge \forall x[P(x) \rightarrow \exists y\ (P(y) \wedge Q(x, y)]$

This is correct since we first say there exist at least one prime number x, then we say "we can always find a bigger prime number y given x", indicating that the number of prime numbers is infinite.

- ○ $\exists x P(y)$

- ∀xP(x)

Question 5

1 / 1 pts

Let the underlying signature be {a, P, Q}, where a is an object constant, P is a unary predicate constant, and Q is a binary predicate constant. Assume object variables range over the set N of nonnegative integers, and the signature is interpreted as follows:

- a represents the number 10,
- P(x) represents the condition “x is a prime number,”
- Q(x, y) represents the condition “x is less than y.”

Which of the following first-order logic formulas express the following English sentence?

"x equals 8."

Choose all that apply.

- [ ] ∀y [P(y) ∧ Q(y,a) → Q(y,x)] ∧ ¬∃y,z [Q(x,y)∧Q(y,a)∧Q(x,z)∧Q(z,a)∧y≠z]

Correct!

- [x] ∀y [P(y) ∧ Q(y,a) → Q(y,x)] ∧ ∃y [Q(x,y) ∧ Q(y,a)]

∃y [Q(x,y) ∧ Q(y,a)] means that there is a number y between x and a, thus x can only be {0,1,2, ...,7,8}.

∀y [P(y) ∧ Q(y,a) → Q(y,x)] means that for all prime number y that is smaller than 10, y must be smaller than x. The possible values of y are {2,3,5,7}, and since they are all smaller than x, x can only be 8.

Correct!

- [x] ∃y [Q(x,y) ∧ Q(y,a)] ∧ ¬∃y,z [Q(x,y)∧Q(y,a)∧Q(x,z)∧Q(z,a)∧y≠z]

∃y [Q(x,y) ∧ Q(y,a)] means that there is a number y between x and a, thus x can only be {0,1,2, ...,7,8}.

¬∃y,z [Q(x,y)∧Q(y,a)∧Q(x,z)∧Q(z,a)∧y≠z] means that we cannot find 2 different numbers y and z between x and a. Thus x now can only be 8.

- [ ] $\neg P(x) \land Q(x,a) \land \exists y\, [Q(x,y) \land Q(y,a)]$

- We are asked to select the formula(s) that state "x equals 8"

  Considering the formula:
  $\forall y\, [P(y) \land Q(y,a) \rightarrow Q(y,x)] \land \exists y\, [Q(x,y) \land Q(y,a)]$

  This formula is stating:
  FOR ALL y: [IF "y is prime" AND "y < a", THEN "y < x"], AND FOR SOME y: ["x < y" AND "y < a"]

  So, we can reduce this to the following two statements:
  1) when y is prime and y < a, then y < x
  2) x < y < a

  Using our requirement of x=8 (and assumption of a=10), we see that to satisfy the second statement, y must be equal to nine (because 8 < y < 10). So, if y=9, this means that P(y) is FALSE (because nine is not prime) making our first statement TRUE. Checking the first statement for all valid (y<a) values of y, we find:

  y=0: F ∧ T → T == T
  y=1: F ∧ T → T == T
  y=2: T ∧ T → T == T
  y=3: T ∧ T → T == T
  y=4: F ∧ T → T == T
  y=5: T ∧ T → T == T
  y=6: F ∧ T → T == T
  y=7: T ∧ T → T == T
  y=8: F ∧ T → F == T
  y=9: F ∧ T → F == T

  We see that the statement is satisfied and thus, for x=8, this option is valid. But what about for x>8 or x<8? Well, we can observe that x cannot be greater than eight because it would not satisfy the second statement. Therefore, we just need to check that this formula is FALSE for every (valid) x less than eight.

  Let us examine the first statement for the case where x=7:

  y=0: F ∧ T → T == T
  y=1: F ∧ T → T == T
  y=2: T ∧ T → T == T
  y=3: T ∧ T → T == T
  y=4: F ∧ T → T == T
  y=5: T ∧ T → T == T

y=6: F ∧ T → T == T

Question 6

1 / 1 pts

Is the following first-order formula satisfiable?

a = b

Unsatisfiable

Correct!

Satisfiable

We can find an interpretation I below that satisfies a=b.

First, the universe of I, denoted by |I|, is {apple}.

Second, $a^I = apple,\ b^I = apple$

Question 7

1 / 1 pts

Is the following first-order formula satisfiable?

∀xy(x ≠ y)

Satisfiable

Correct!

Unsatisfiable

No matter what interpretation I we define, its universe must be non-empty, let's say the universe is

|I| = {apple, ...}

Then the formula ∀xy(x ≠ y) is true indicates that at least the following formula

apple ≠ apple

is true, while it's not.

Question 8

1 / 1 pts

Let σ be the signature {a, b, P, Q} where a, b are object constants and P, Q are unary predicate constants. Choose all Herbrand interpretations of σ that satisfy the formula $\exists x(P(x) \to Q(x))$.

Correct!

- [x] {P(a), P(b), Q(a), Q(b)}

The formula $\exists x(P(x) \to Q(x))$ can be seen as

$(P(a) \to Q(a)) \vee (P(b) \to Q(b))$

whose left-hand side and right-hand side are true under the given Herbrand interpretation.

Correct!

- [x] {P(a), P(b), Q(a)}

The formula $\exists x(P(x) \to Q(x))$ can be seen as

$(P(a) \to Q(a)) \vee (P(b) \to Q(b))$

whose left-hand side is true under the given Herbrand interpretation.

Correct!

- [x] {P(a), Q(b)}

The formula $\exists x(P(x) \to Q(x))$ can be seen as

$(P(a) \to Q(a)) \vee (P(b) \to Q(b))$

whose right-hand side is true under the given Herbrand interpretation.

- [ ] {P(a), P(b)}

Correct!

- [x] {P(a)}

The formula $\exists x(P(x) \to Q(x))$ can be seen as

$(P(a) \to Q(a)) \vee (P(b) \to Q(b))$

whose right-hand side is true under the given Herbrand interpretation.

Correct!

- [x] ∅ (empty set)

The formula $\exists x(P(x) \rightarrow Q(x))$ can be seen as

$(P(a) \rightarrow Q(a)) \lor (P(b) \rightarrow Q(b))$

whose left-hand side and right-hand side are true under the given Herbrand interpretation.

Question 9

1 / 1 pts

Suppose p and q are atoms, is the following formula a tautology?

((p -> q) -> p) -> q

- ( ) Yes
- (•) No — Correct!

This is not a tautology and we can give a counter-example: an interpretation I that does not satisfy this formula.

I = {p}

Question 10

1 / 1 pts

What are the free variables in the following formula?

$$\exists x(P(x,y) \rightarrow \forall y P(y,x))$$

- ( ) Both x and y
- ( ) No free variable
- ( ) x
- (•) y — Correct!

An **occurrence** of a variable v in a formula F is **free** if v is not bounded by any quantifier. A variable v is a **free variable** of F if v has at least 1 free occurrence in F.

In this formula, both x are bounded by $\exists x$ and only the y in P(x,y) is a free occurrence, thus only y is a free variable.