Homework1 Solution

1 Part 1

Given, $\frac{P(y=1 \mid x)}{P(y=0 \mid x)}$ $\begin{matrix} \text{y = 1 if } > 1 \\ \text{y = 0 if } \leq 1 \end{matrix}$

So, this infers that there are only two class labels 0 and 1.
$\Rightarrow P(y=1 \mid x) + P(y=0 \mid x) = 1$ (1)

Also given, $\frac{P(y=1 \mid x)}{P(y=0 \mid x)} = \mathrm{e}^z$

$\frac{P(y=1 \mid x)}{1 - P(y=1 \mid x)} = \mathrm{e}^z$ (from Eq (1))

$P(y=1 \mid x) = \mathrm{e}^z \ (1 - P(y=1 \mid x))$

$P(y=1 \mid x) = \mathrm{e}^z$ - $e^z P(y=1 \mid x))$

$P(y=1 \mid x) + e^z P(y=1 \mid x)) = \mathrm{e}^z$

$P(y=1 \mid x) \ (1 + e^z) = \mathrm{e}^z$

$P(y=1 \mid x) = \frac{e^z}{(1+e^z)}$ (multiply with e^{-z})

$P(y=1 \mid x) = \frac{e^z * e^{-z}}{(1+e^z) * e^{-z}}$ $(e^z * e^{-z} = e^0 = 1)$

$$\boxed{P(y=1 \mid x) = \frac{1}{(1+e^{-z})}}$$