CS : 736

Assignment 3

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7th March, 2018

Question 1

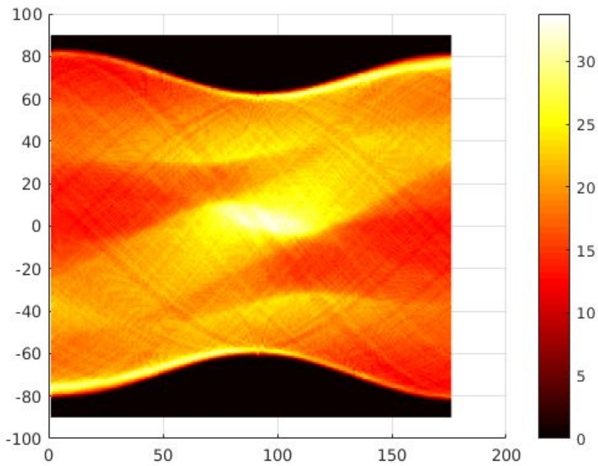
a) Image Interpolation refers to the resampling of image using linear interpolation schemes so as to increase or decrease the size of the image. In MATLAB “inter2p()” function helps us to do so.

(Original Image of the Phantom) (Interpolated image of the Phantom)

In this part, interpolation scheme increases the sample points about 1.4 times and hence the resolution of image is increased to about 1.4 times. A higher value of sample points might lead to artifacts due to linear interpolation whereas a lesser value might not be useful. This seems to be a good choice as in part (b) we have to plot radon transform for the domain of “t” from [-90,90] and hence it would be good to cover the the entire phantom within that range.

Δs was chosen to be approximately √2 pixel-width unit. This will ensure that about any line, two different points (separated by a step size) will lie within different pixels.

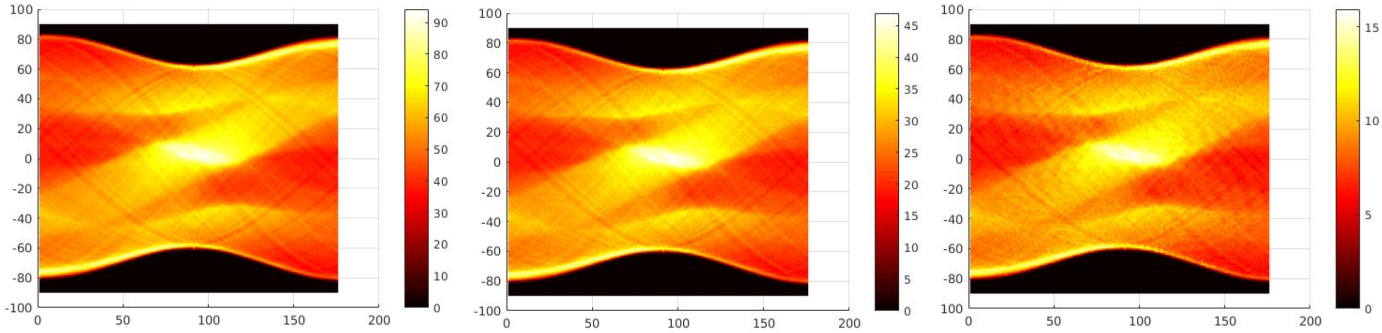
b) Here we have to draw the radon transform of phantom. T ranges from [-90:90] and θ from [0:175]



(Original Image of the Phantom) (Radon transform of the Image)

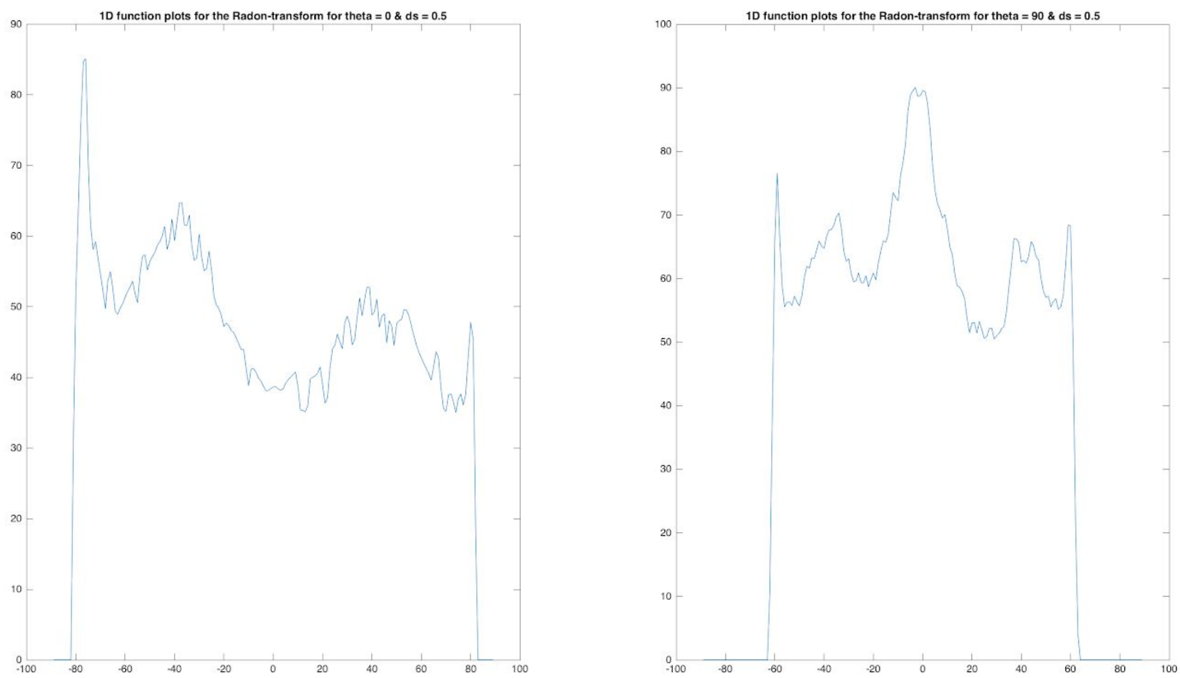
NOTE: In the above radon transform the origin was assumed at the center of the geometrical original image.

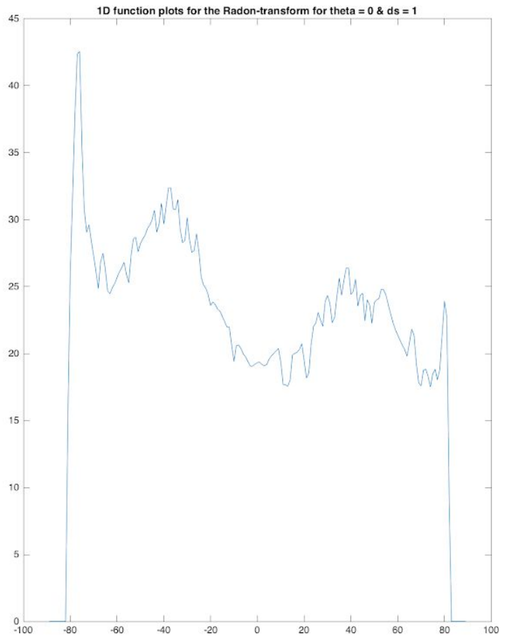
c) Here we have to draw the radon transform of phantom with following step sizes.

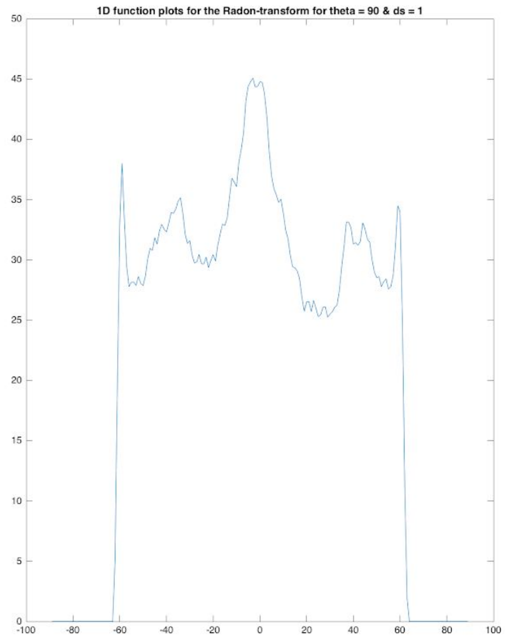


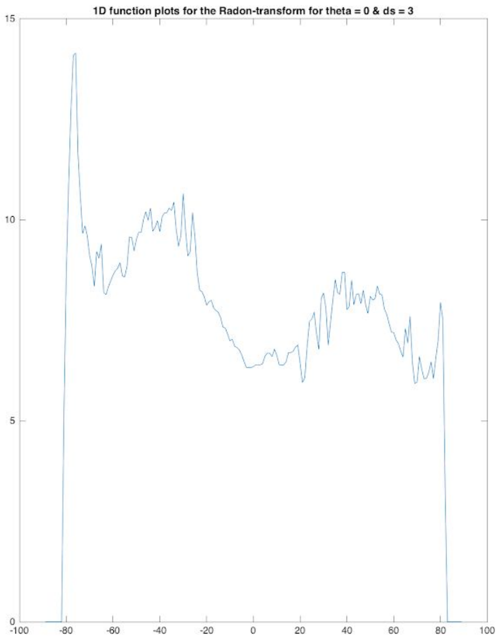
(Δs = 0.5 pixel-width) (Δs = 1 pixel-width) (Δs = 3 pixel-width)

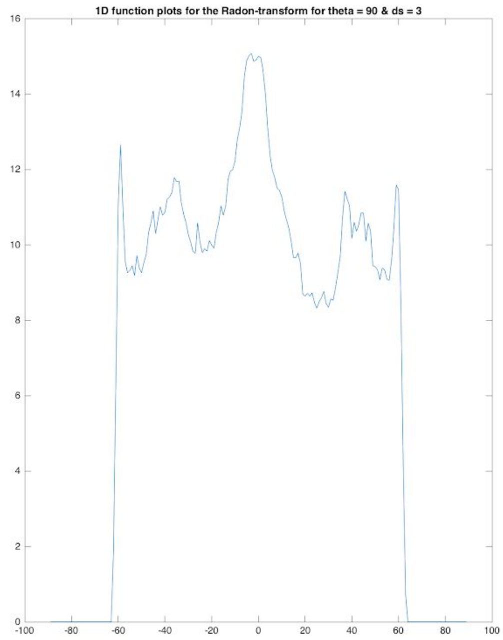
It is clearly visible that the image corresponding to Δs = 0.5 pixel-width is the smoothest. This is because we are considering more number of points (closer points) while evaluating the line integrals and hence they will have more continuity/smoothness.









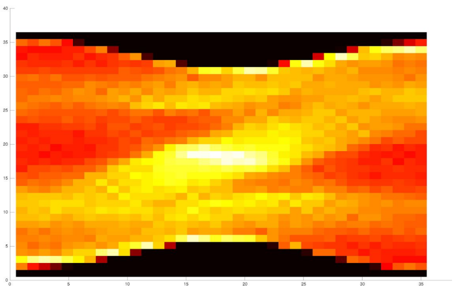


Here is is clearly visible that for Δs = 0.5 pixel-width the curve for θ = 0, 90 is the smoothest. This is again due to the fact that we are considering more number of points while evaluating the integral and hence this imparts a continuous/smooth nature to the curve.

d) If I were designing a CT scanner the best parameter settings for Δt and $\Delta\theta$ would be 1 (assuming the reconstructed image is 180x180 and t ranges from [-90:90], θ ranges from [0:180]). This is due to the following reasons -

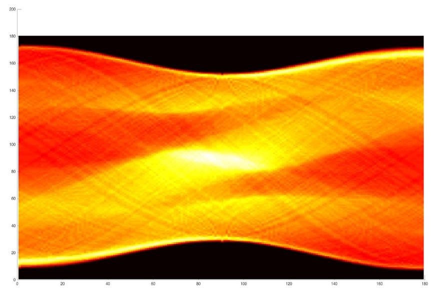
i) Due to the discrete nature of the image. If we make Δt, $\Delta\theta$ any smaller, many of our integrals will correspond to the same pixels. This means that two different lines in (x,y) domain may pass through same set of pixels and hence give out same integrals.

ii) Making it any smaller would sharply increase the time required for calculation of the radon transform and at the same time, may not add substantial value to the the plot.



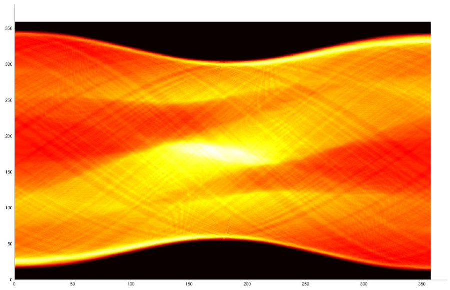
($\Delta t = \Delta\theta = 5$)

0.967567 seconds.



($\Delta t = \Delta\theta = 1$)

22.919310 seconds.



($\Delta t = \Delta\theta = 0.5$)

97.334063 seconds.

From these plots we can see that the radon transform for ($\Delta t = \Delta\theta = 5$) is very discrete and hence the backprojection will not be an accurate reconstructed image. Hence we desire a small vale of Δt, $\Delta\theta$. But on decreasing the value from 1 to 0.5 the change in sharpness of image is not very significant and hence no substantial value is added to the reconstructed image. But the difference is the computation time is very high and hence the scanner will become in efficient.

e) For choosing the number of pixels in the grid we would rely on the of values of Δt and $\Delta\theta$ and the size of the final image. The number of pixels in the grid should be = size of image / Δt. This would ensure that we make complete use of CT data (integrals). For Δs, we must choose it to be close to $\sqrt{2}$ pixel-width. This will ensure that we don't skip any pixels while evaluating the line integrals.

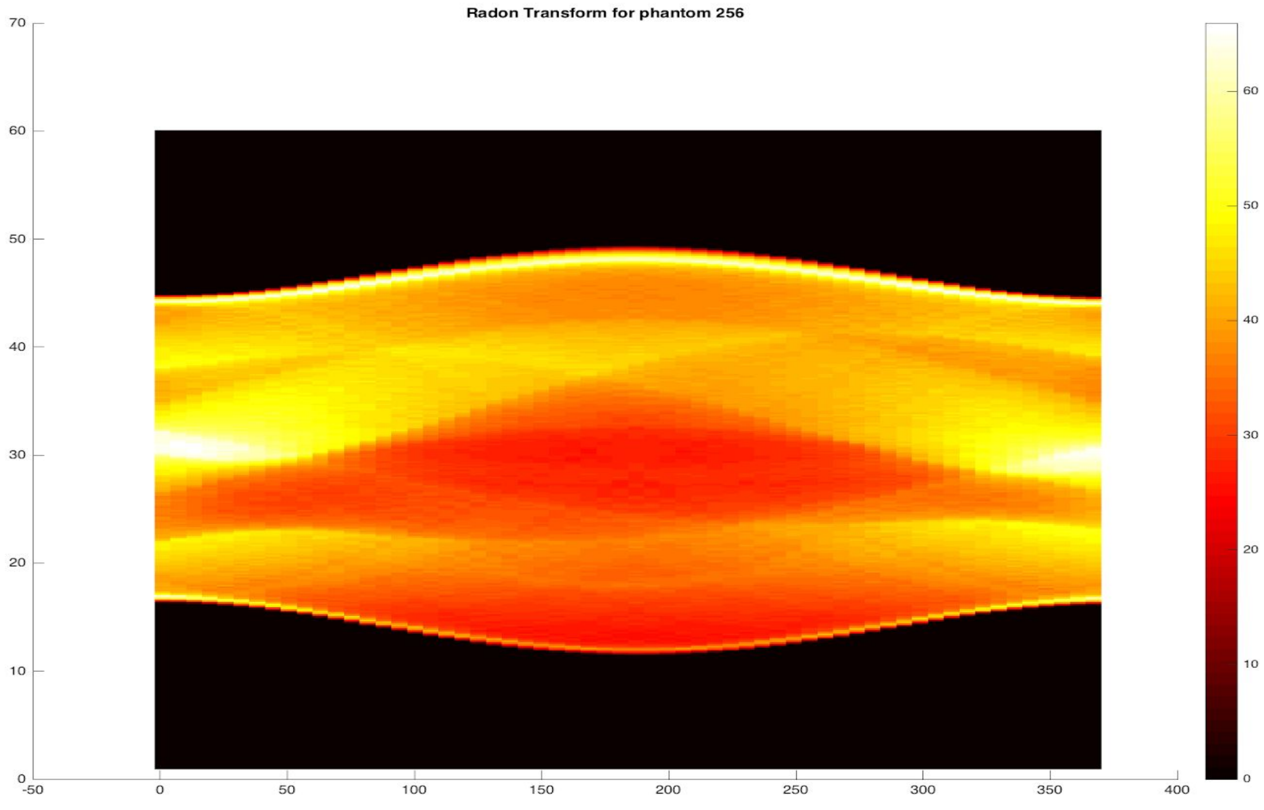
Effect of choosing $\Delta s >> 1$: This will lead to skipping of pixel-intensities along the line and hence may not give the exact line integral.

Effect of choosing $\Delta s << 1$: This will lead to unnecessary accounting of the same pixels while evaluating the line integrals which is redundant.

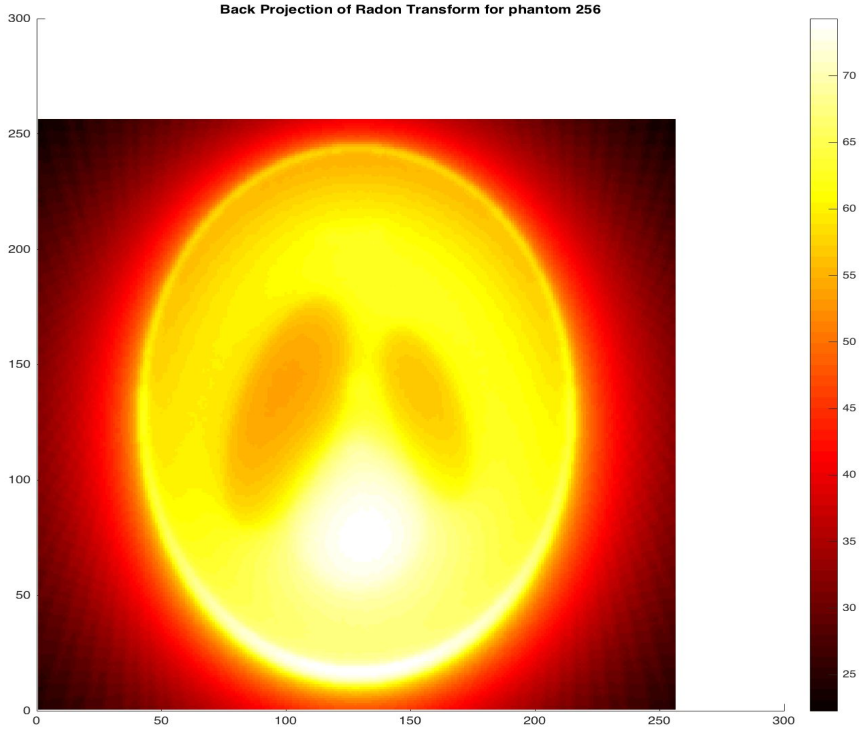
Question 2

a) Radon Transform and Filtered back Projection with different filtering functions.
The fourier transform of the Radon transform is a representation of image signal in the frequency domain. In the filtered back projection of radon transform we dampen amplitudes of the lower frequencies and amplify the amplitudes of higher frequencies till a certain point as frequencies beyond that belong to noise and will get amplified too.
So in the back projection we notice that the filtered backprojection image for Cosine is better than the Shepp-Logan which is better than Ram-Lak filtering. The image obtained from Ram-Lak has some bright pixel signals which came because the Ram-Lak filter amplifies the amplitudes of higher frequencies too(till the rectangle limit).
Also by reducing the rectangle with from wmax to wmax/2 increases the blurriness in the obtained image as we are removing the higher frequency components. To obtain the best image we need to manually tune the parameter L.

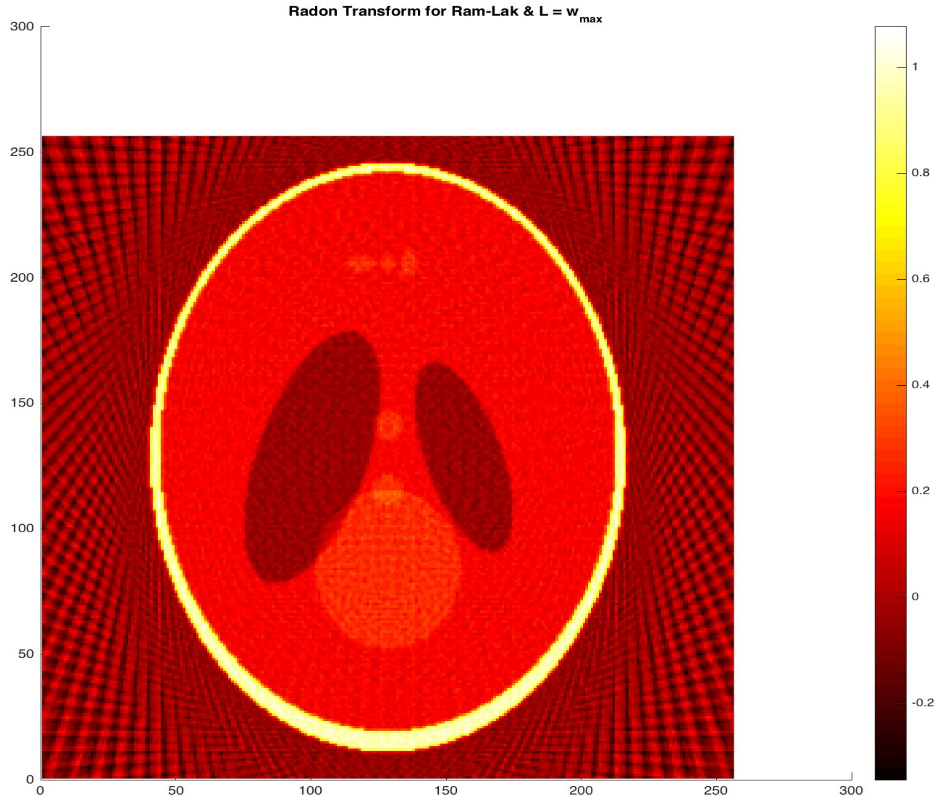
i) Radon Transform of the Phantom 256 data



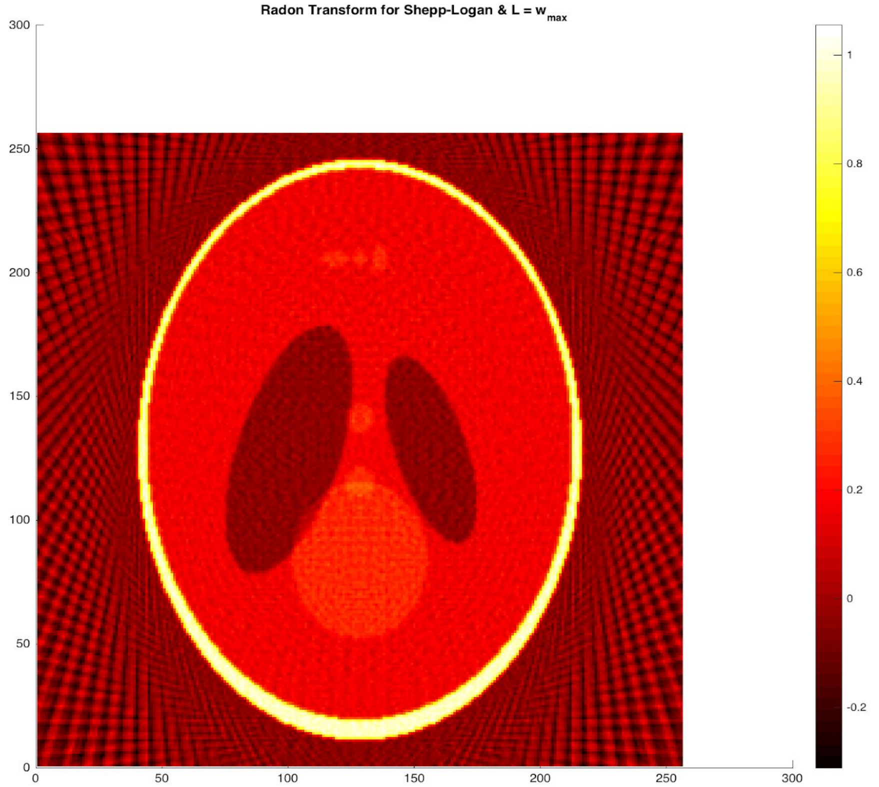
ii) Direct Back projection of the Radon Transform



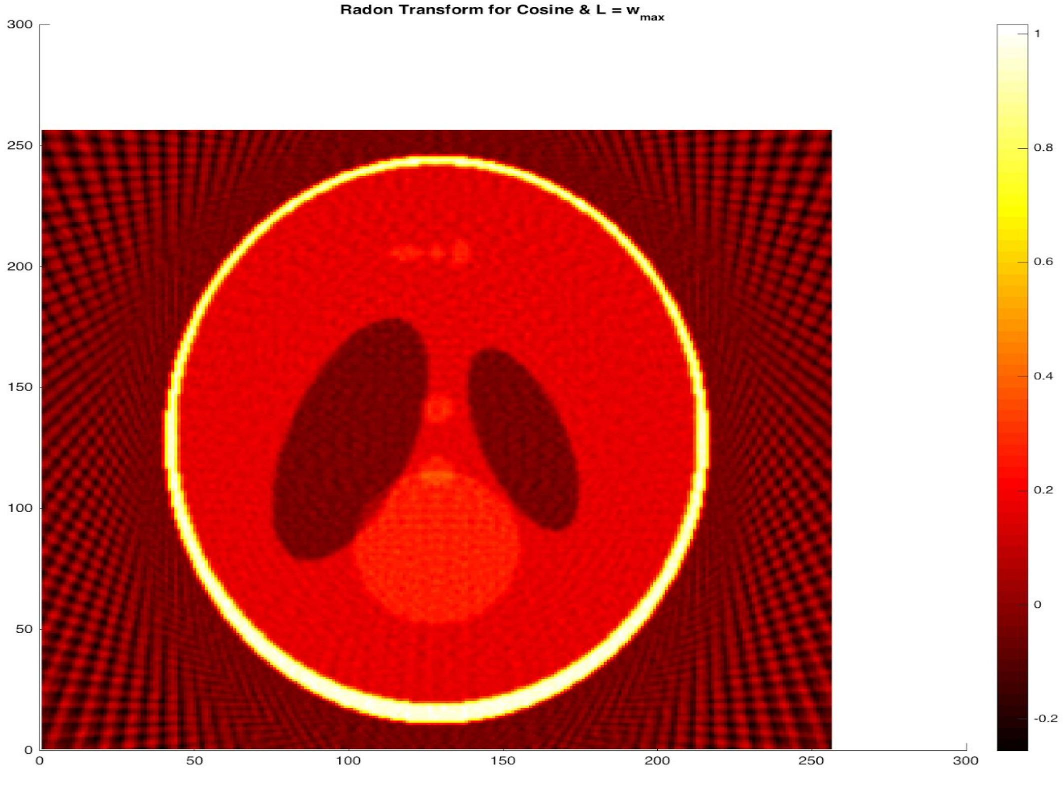
iii) Filtered Back Projection using Ram-Lak filter with L = w_max



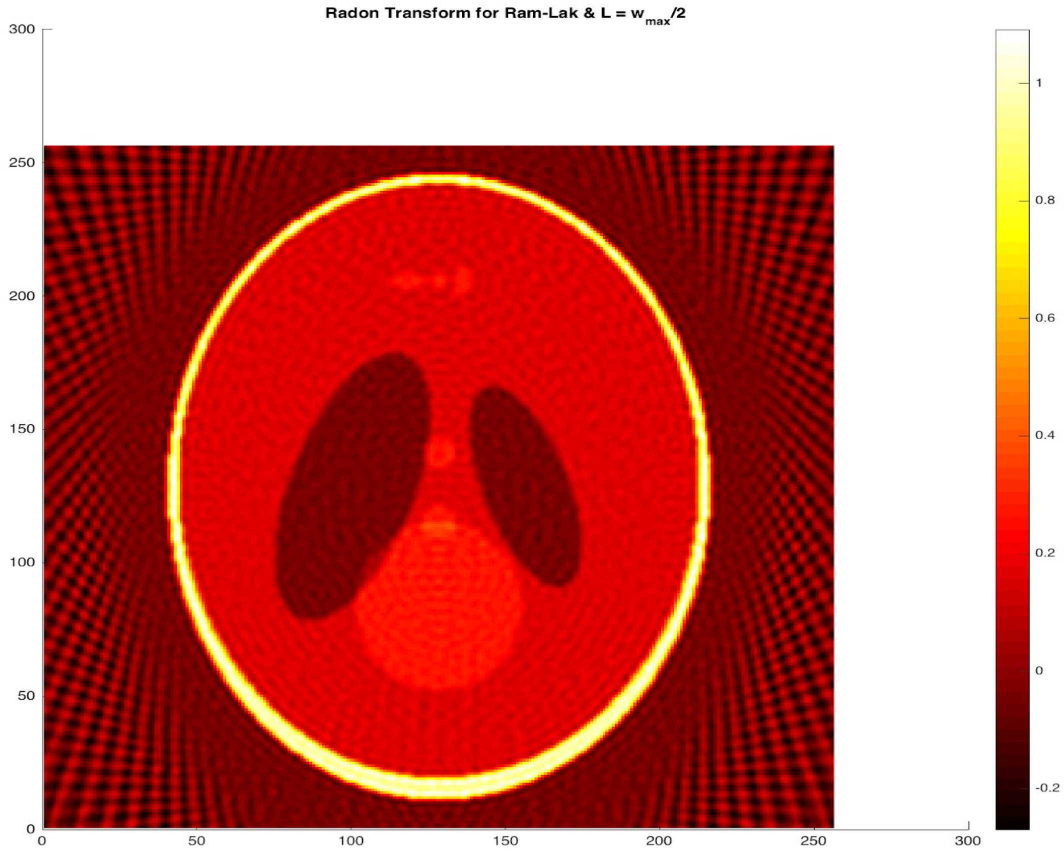
iv) Filtered Back Projection using Shepp-Logan filter with L = w_max



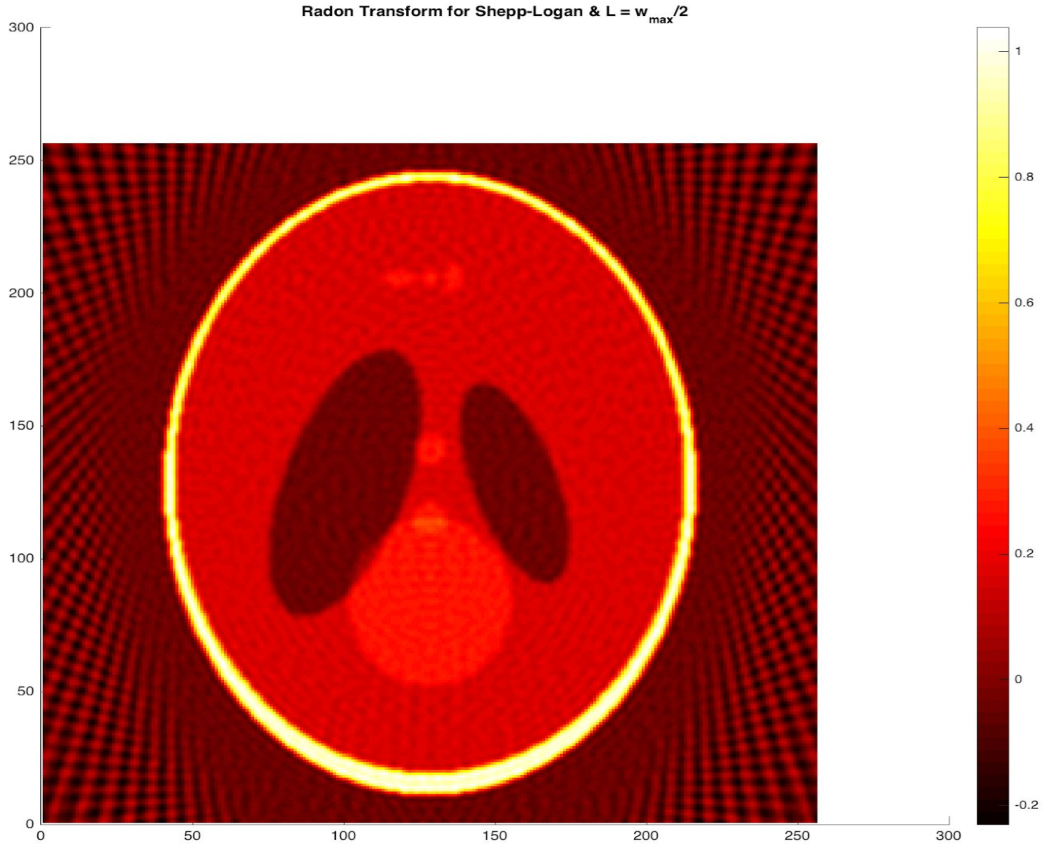
v) Filtered Back Projection using Cosine filter with L = w_max



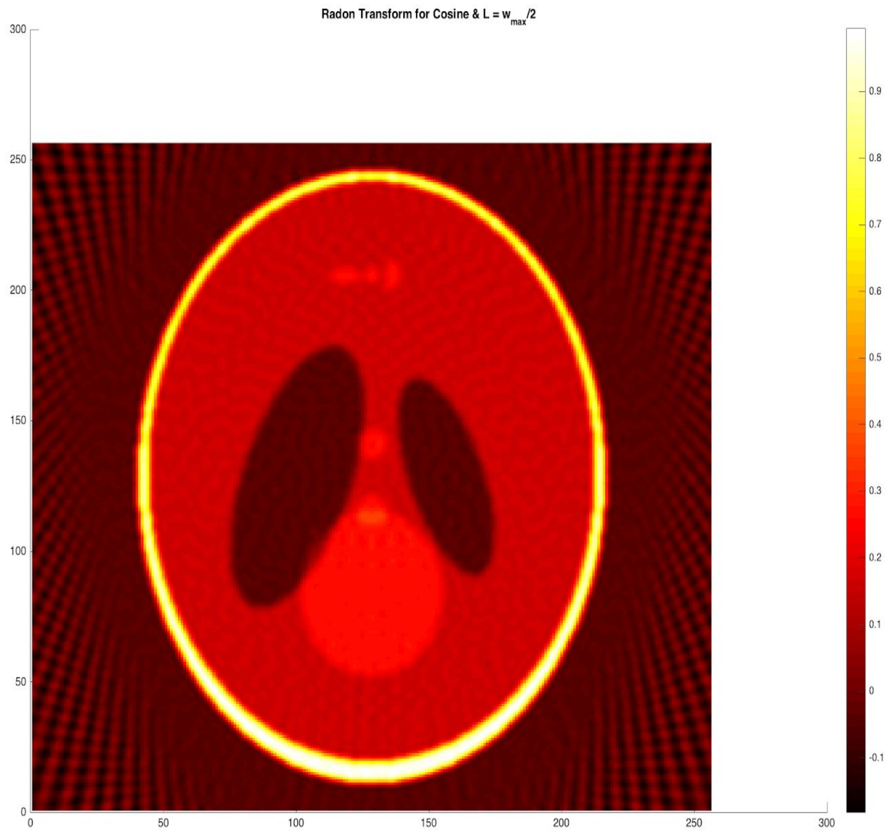
vi) Filtered Back Projection using Ram-Lak filter with L = w_max/2



vii) Filtered Back Projection using Shepp-Logan filter with L = w_max/2



viii) Filtered Back Projection using Cosine filter with L = w_max/2



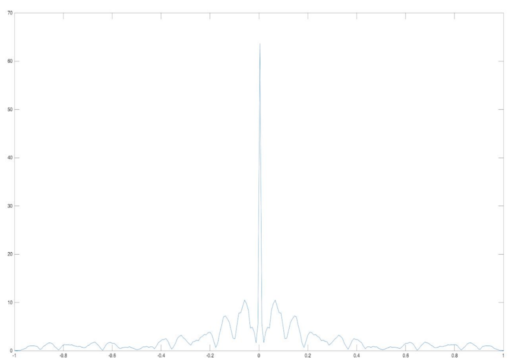
b) Analysis of Gaussian Blurring added image and Filtered Back Projection

Higher the blurring for the image the lesser is the RRMSE for it's filtered back projected image with the original image. This is because in the blurred image the components of the higher frequencies will not be present and so the filtered back projection will be very similar to the original image. Where as for the sharp image the filtered back projection will remove the higher frquency components and the RRMSE of the obtained image with the original image will be higher as can be seen from the data.

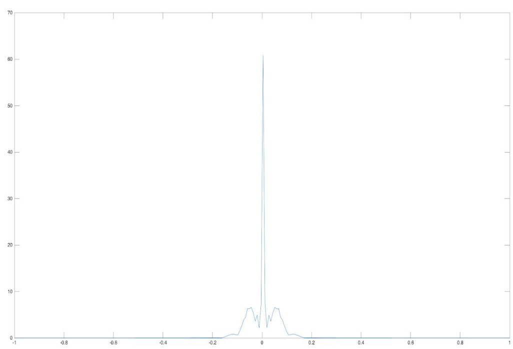
RRMSE for original image and it's back projection = 0.3269

RRMSE for mask 1 blurred image and it's back projection = 0.2065

RRMSE for mask 5 blurred image and it's back projection = 0.2039

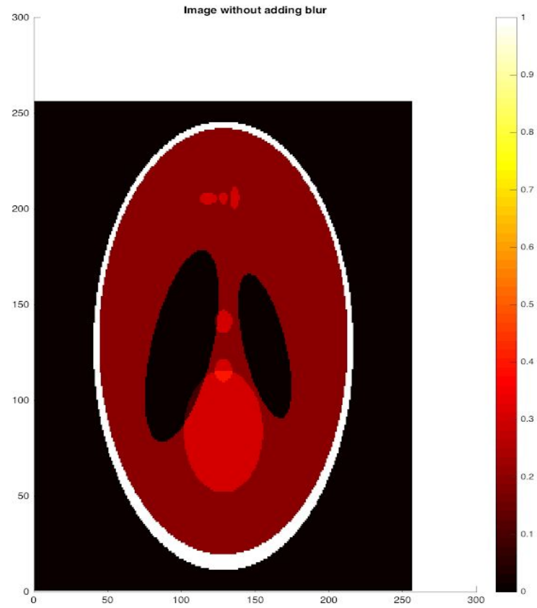


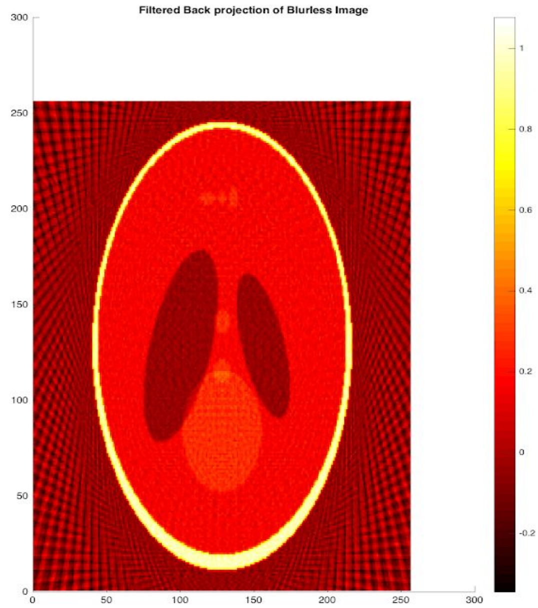
(Fourier Transform for Original Sharp Image)



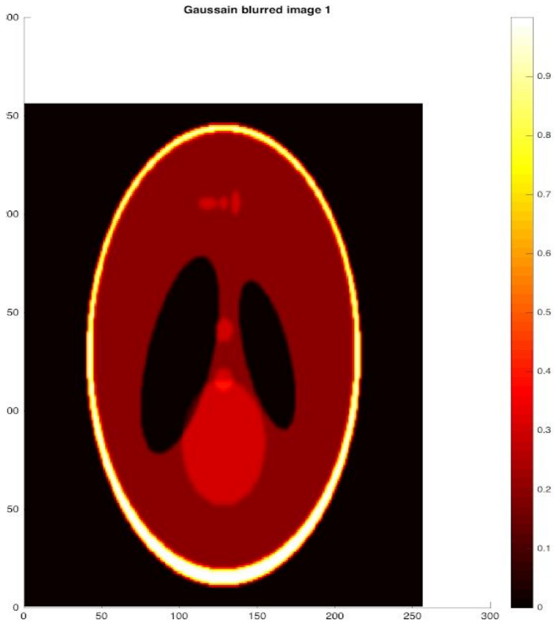
(Fourier Transform for Blurred Image)

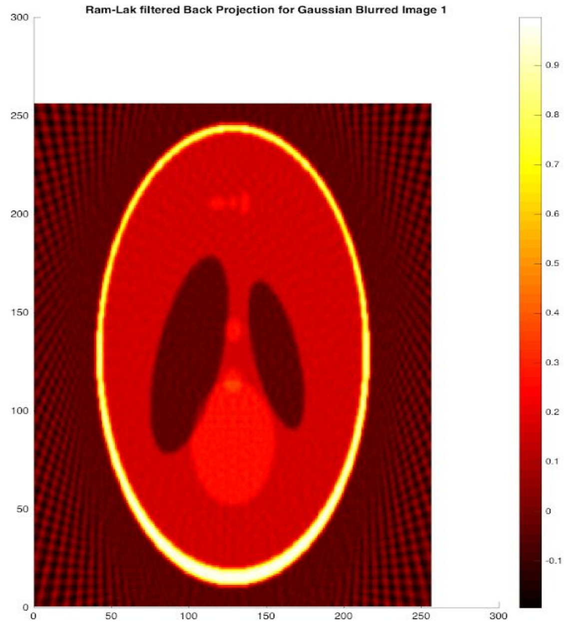
i) The Original Image and it's back project



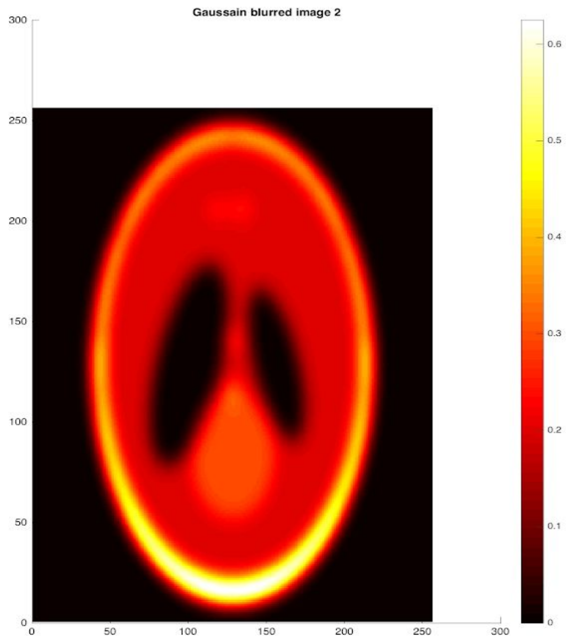


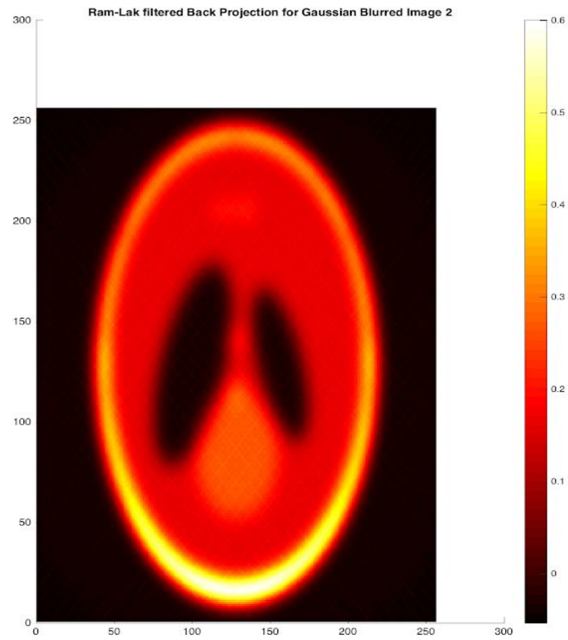
ii) Gaussian blur with mask = fspecial ('gaussian', 11, 1) and it's filtered back projection :





iii) Gaussian blurr with mask = fspecial ('gaussian', 51, 5) and it's filtered back projecti

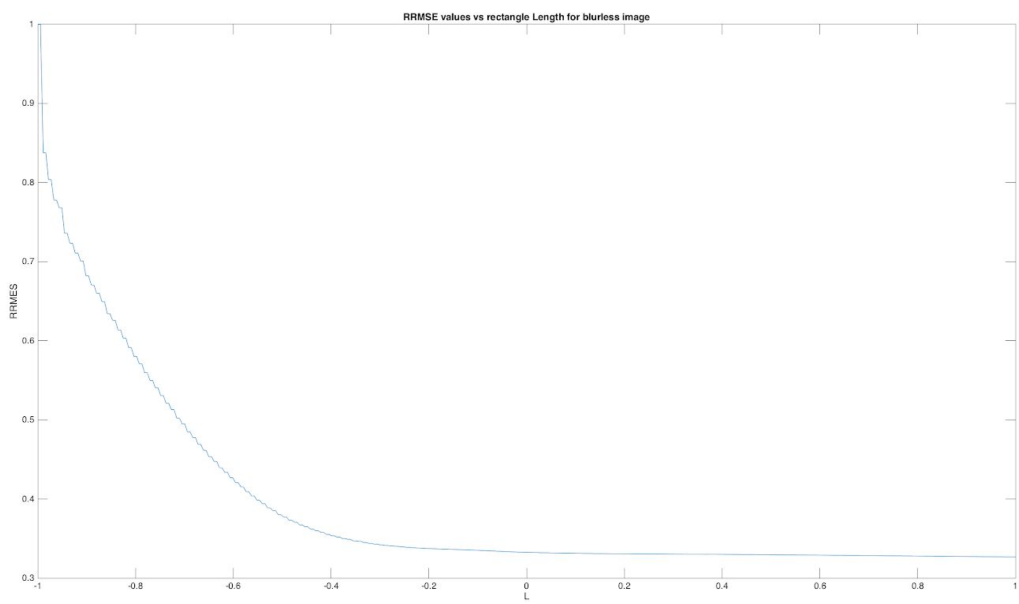




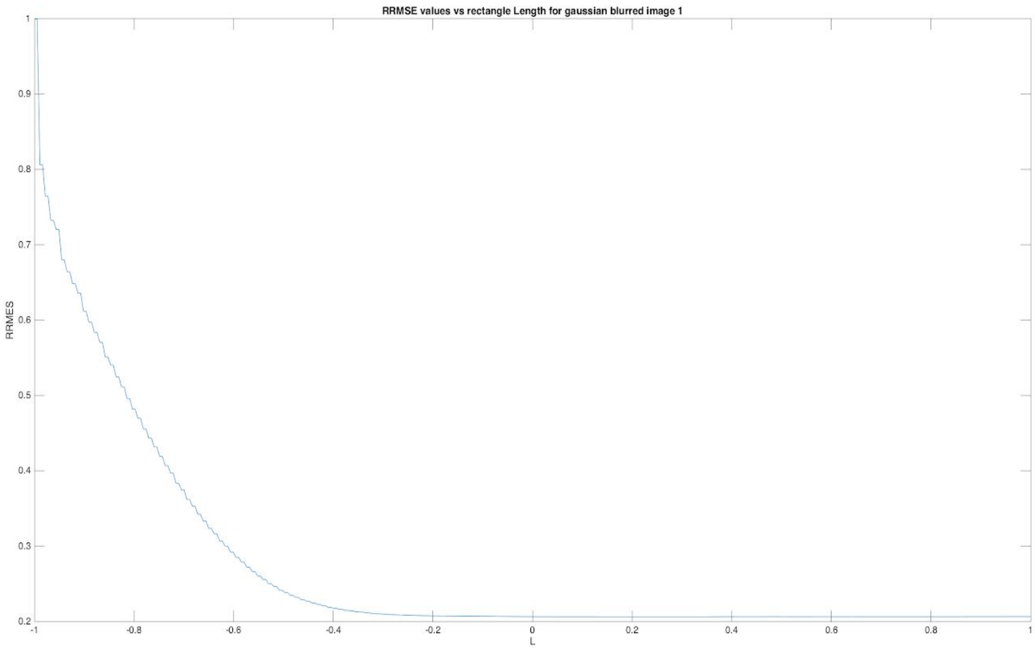
c) Analysis for Rectangle length parameter :

The RRMSE value will be decreasing as the width of the rectangle increases. The frequency components after the rectangle width are removed in the filtered back projection. So when the rectangle width is lower most of the frequency components will be removed also lower frequency components are damped. This will result in larde RRMSE value. So as rectangle width increases RRMSE decreases. But for width equal to max frequency also the RRMSE will be finite as the lower frequency components are altered. And as the width increases most of the frequency component will be included already. This explains the asymptotic nature of the RRMS graph.

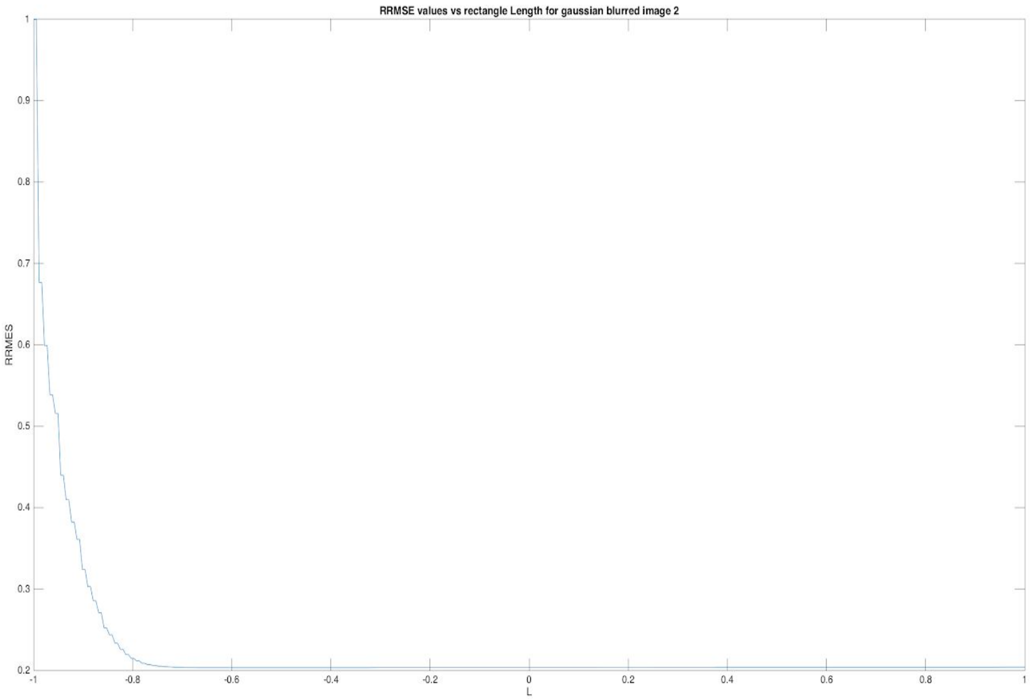
i) RRMSE vs rectangle length for blureless image



ii) RRMSE vs rectangle length for blured image 1

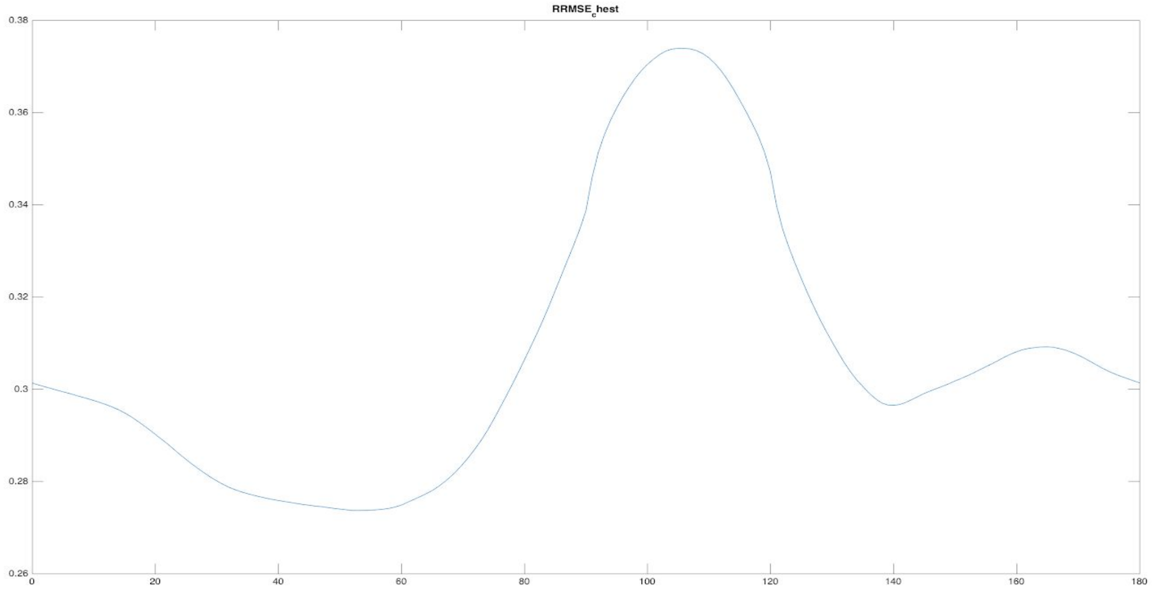


iii) RRMSE vs rectangle length for blured image 2

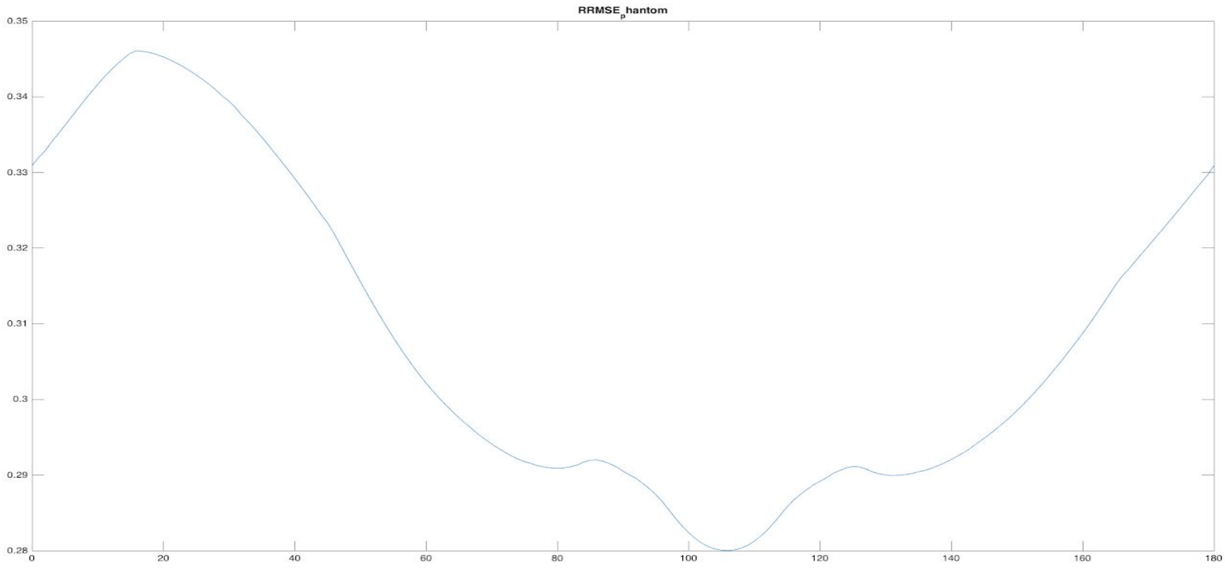


Question 3

a) RRMSE values between ground-thruth image and reconstructed image are as show in figure:
    i) For Chest: minimum RRMSE for theta = [54 : 203]



ii) For phantom: minimum RRMSE for theta = [107 : 256]



b) Best Reconstructed images on the basis of RRMSE values to the ground thruth image.