ASPECTS OF CONFINING STRINGS

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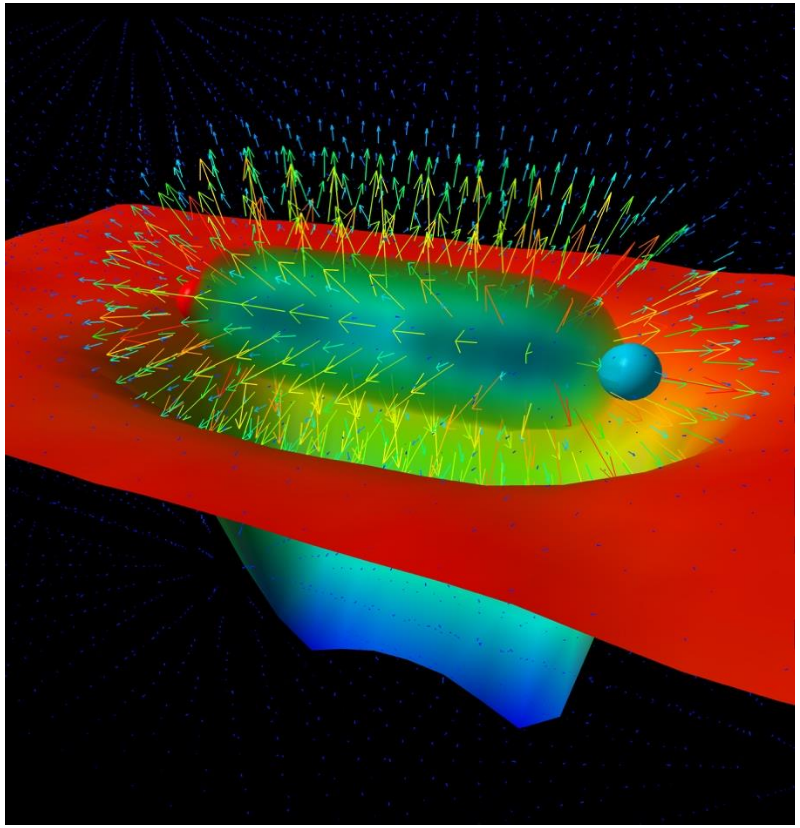
BASED ON WORK WITH THOMAS DUMITRESCU AND YANYAN LI

CONFINING STRINGS



Leinweber

- Absence of free quarks and gluons
- Examples : Yang Mills (YM), QCD
- Characteristic feature : chromoelectric flux tubes
- Numerical lattice evidence
- Study properties of confining strings

UNIVERSAL ASPECTS

- Is the vacuum in a confining phase ?
- Closely related – Do we have finite tension strings / flux tubes ? Is there a one-form symmetry that guarantees the existence / stability ?
- Given such a string, there are two Nambu Goldstone bosons, associated with the broken translational invariance.

UNIVERSAL AND NON-UNIVERSAL ASPECTS

- Is the vacuum in a confining phase ?

  What is the bulk mass spectrum ?

- Closely related – Do we have finite tension strings / flux tubes ? Is there a one-form symmetry that guarantees the existence / stability ?

  Do fundamental strings attract / repel ? Do they form bound states ?

- Given such a string, there are two Nambu Goldstone bosons, associated with the broken translational invariance.

  What is the spectrum of massive excitations ?

FLUX TUBES IN PURE YANG MILLS

- Is the vacuum in a confining phase ? ✓

  What is the bulk mass spectrum / taxonomy of light modes ?

  [Athenodorou, Teper '21, ….]

- $\mathbb{Z}_N^{(1)}$ one-form symmetry

  Believe that confining strings in YM attract and form bound states

- 2 NGBs and a Pseudoscalar Axion as the lightest massive excitation

[Dubovsky, Flauger, Gorbenko' 12 ,Athenodorou, Dubovsky, Luo, Teper ' 24, ….]

FLUX TUBES IN PURE YANG MILLS

Axion is the lightest massive excitation in a setting where fundamental confining strings attract !

These are important data points, essentially numerical since pure YM is strongly coupled. We do not have an analytic understanding of answers to these minimally non universal questions.

SUPERCONDUCTING AND CONFINING STRINGS

- Stable, finite tension extended excitations also arise in simple weakly coupled tractable physical systems in 3+1 dimensions :

  - Abrikosov strings in Ginzburg Landau effective theory of superconductivity
  - Closely related Nielsen-Oleson strings in Abelian Higgs Models (AHMs)

  Abrikosov-Nielsen-Oleson (ANO) superconducting vortex strings (magnetic flux tubes)

- Simple Abelian models are good laboratories for studying properties of strings

- ‘t Hooft - Mandelstam **DUAL SUPERCONDUCTIVITY**: Confining strings (electric) EM dual to superconducting strings (magnetic)

- Dual superconductivity is made explicit in **Deformed Seiberg Witten (SW) theory** – a **dual Abelian Higgs model** with **Confining Electric** flux tubes

OUTLINE OF TODAY'S TALK

Properties of Strings

[Dumitrescu, AG ‘25]

Fluctuations of Strings

[Dumitrescu, AG, Li ‘25]

Deformed SW theory

[Dumitrescu, AG ’25]

Superconducting Vortex strings (Magnetic flux tubes)

Confining Strings (Electric flux tubes)

SUPERCONDUCTING VORTEX STRINGS

PROPERTIES OF STRINGS

String Tension, Phases of giant strings

Do strings attract or repel ?

Do strings form bound states ?

Forthcoming [Dumitrescu, AG ‘25]

UNIVERSAL FEATURES OF AHM VARIANTS

$$\mathcal{L} = -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} - |(\partial_\mu - i a_\mu)\phi|^2 - V(|\phi|)$$ Keep general

- $U(1)$ gauge theory, single complex scalar of unit charge
- Dimensionless gauge coupling e
- $V(|\phi|)$ admits one Higgs vacuum requirement
- $U(1)_m^{(1)}$ one-form magnetic flux symmetry

$$\Phi_B = \int_{xy-\mathrm{plane}} dS\, B_z$$

CHOICES OF POTENTIAL

Well studied !

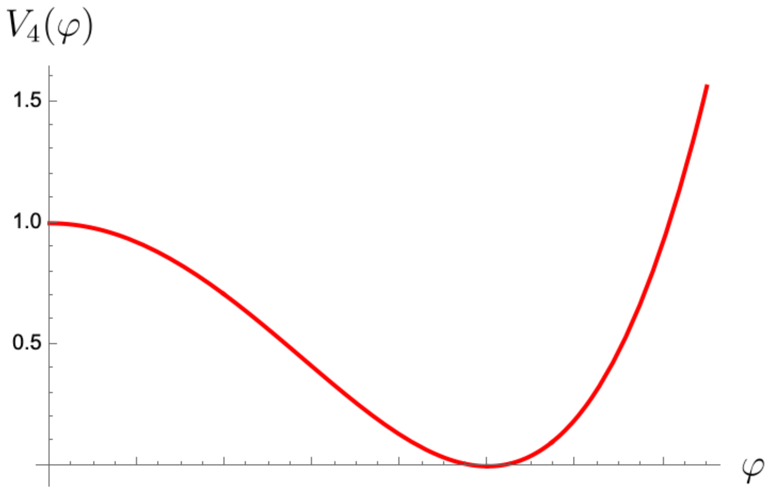
CONVENTIONAL MODEL

$$V_4(|\phi|) = \frac{\lambda}{2}(|\phi|^2 - v^2)^2$$

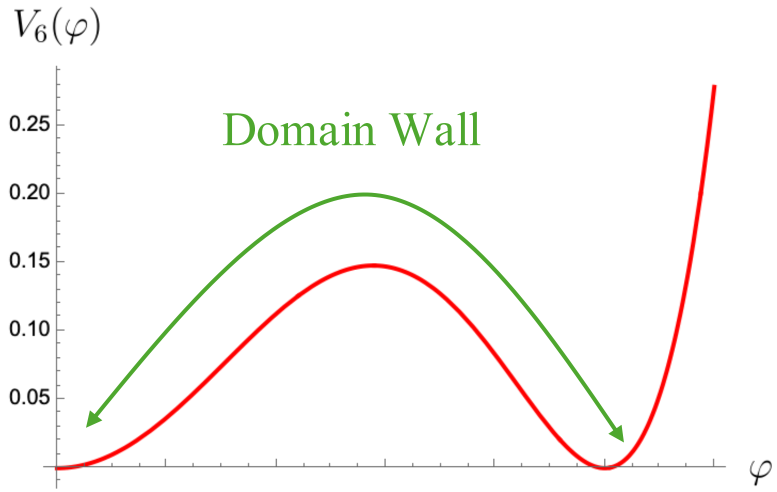
$\lambda > 0$

DEGENERATE MODEL

$$V_6(|\phi|) = \frac{\lambda}{2}|\phi|^2(|\phi|^2 - v^2)^2$$



The Higgs vacuum is **trivial** and **gapped** – massive Higgs boson m_H and vector boson m_V



Unique Higgs vacuum

Higgs vacuum + Coulomb vacuum

TYPE - I AND TYPE - II CLASSIFICATION

$\beta < 1$ Type - I $m_H < m_V$

$\beta > 1$ Type – II $m_H > m_V$

$$\beta = \frac{m_H^2}{m_V^2} \begin{array}{ll} = \lambda/e^2 & \text{Conventional} \\ = \lambda v^2/e^2 & \text{Degenerate} \end{array}$$

$\beta = 1$ conventional AHM - BPS embedding into $\mathcal{N} = 1$ supersymmetry

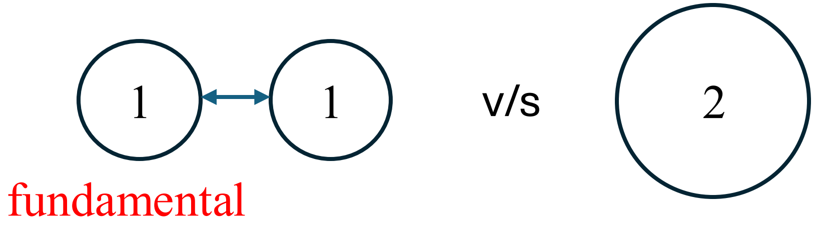
Ratio controls many, potentially correlated, qualitative properties.

Purely classical analysis

ABRIKOSOV-NIELSEN-OLESON (ANO) STRINGS

- **Magnetic** flux tubes. To exist as finite tension excitations, study the Higgs vacuum in which the one form flux symmetry is unbroken. Charged objects : **ANO strings**.

- For any flux n what is the **lowest energy** n **- string ?**

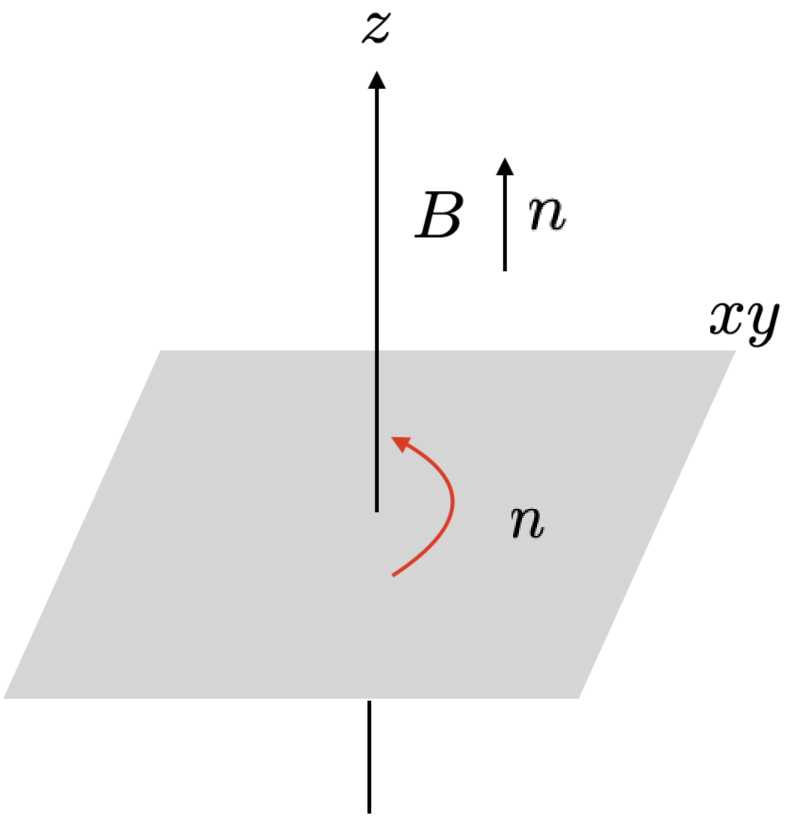


- There can be many strings but in any sector, there would be a **ground state** or lowest energy string expected to be the most stable one.

- Write down non linear equations of motion of Lagrangian and solve them subject to flux constraint. Instead solve in **analytically tractable regime.**

ROTATIONALLY SYMMETRIC STRINGS

ROTATIONALLY SYMMETRIC STRINGS



Tension OR Energy per unit length.

T_n

- Simplifying assumption

- Also Static and translationally invariant

$$\phi(x) = v\varphi(r)e^{in\theta}$$
$$a_\theta(x) = n(1 - a(r))$$

Finite energy requirement forces the winding to be equal to the flux

- These might **NOT** be the minimum energy string configurations

STRING EQUATIONS

ODEs instead of PDEs

$$\varphi''(u) + \frac{1}{u}\varphi'(u) = \frac{n^2}{u^2}\varphi^2(u)a^2(u) + \frac{1}{2}\tilde{V}'(\varphi,\beta) \rightarrow$$

Rescaled version of the potential

$$a''(u) - \frac{1}{u}a'(u) = 2a(u)\varphi^2(u)$$

Regularity and finite tension requirements for the boundary conditions

$$\underline{\varphi(0) = 0 \qquad a(0) = 1} \qquad \underline{\varphi(\infty) = 1 \qquad a(\infty) = 0}$$

(vacuum in degenerate model)

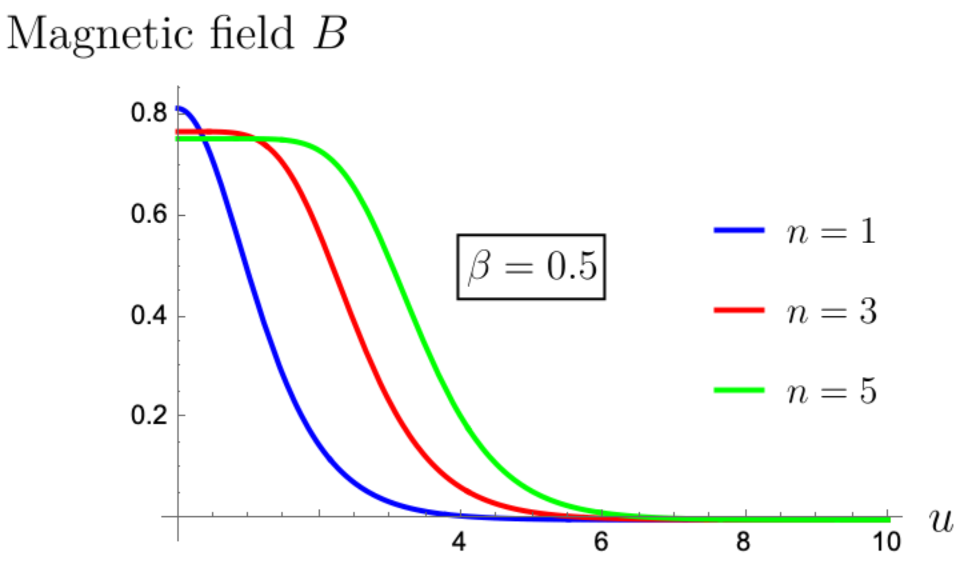
Higgs vacuum

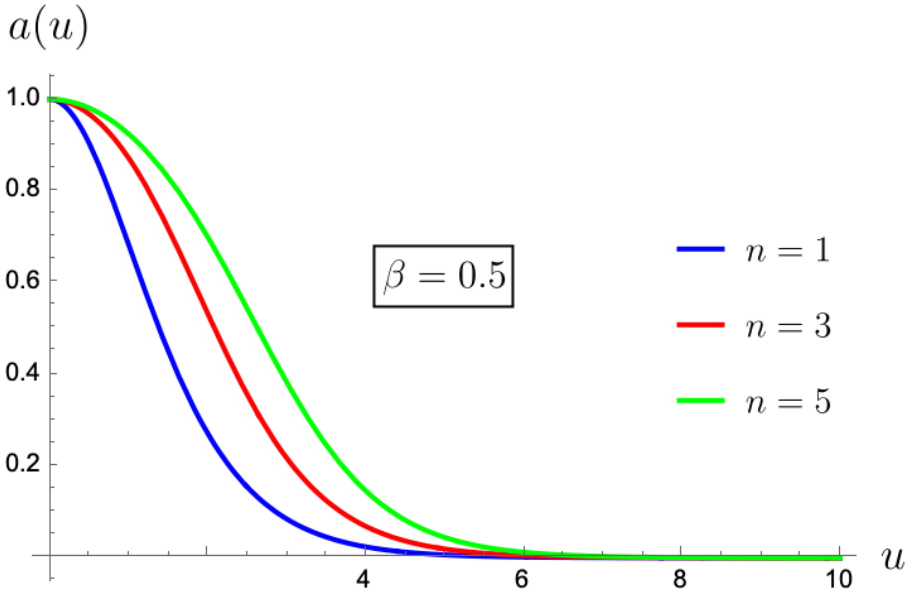
- Cannot be solved analytically, but can be numerically – profiles only depend on β and n
- Analytic solution in large flux limit

$$\phi(x) = v\varphi(r)e^{in\theta}$$

$$a_\theta(x) = n(1 - a(r))$$

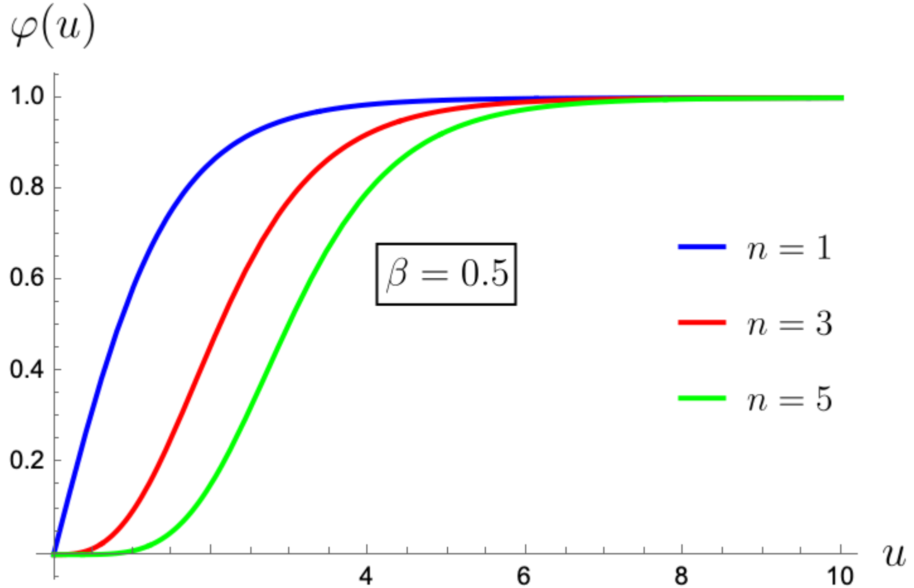
VARIATION WITH n

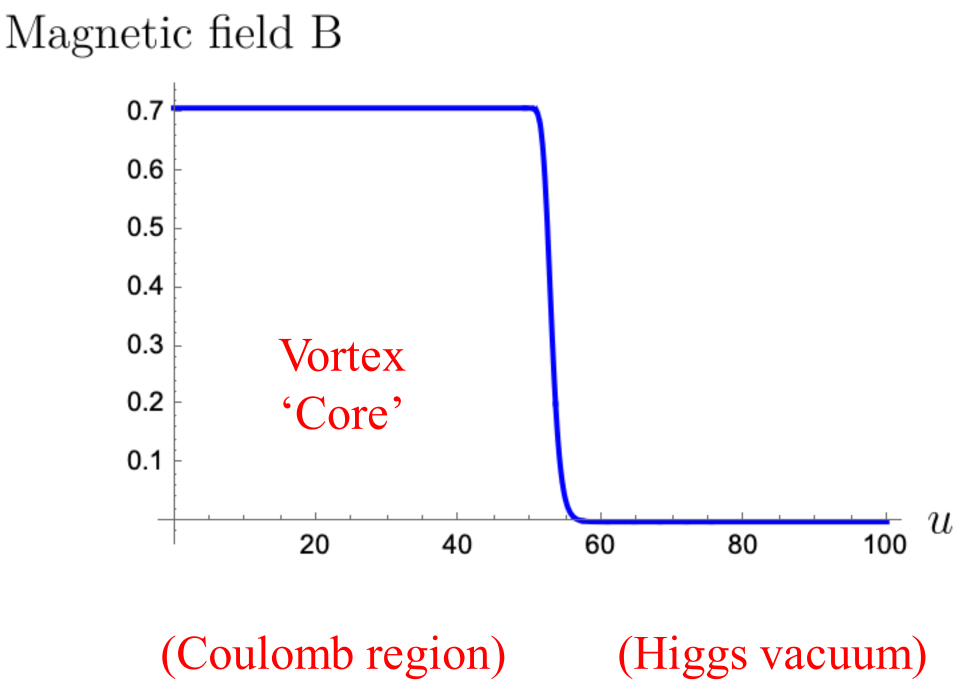




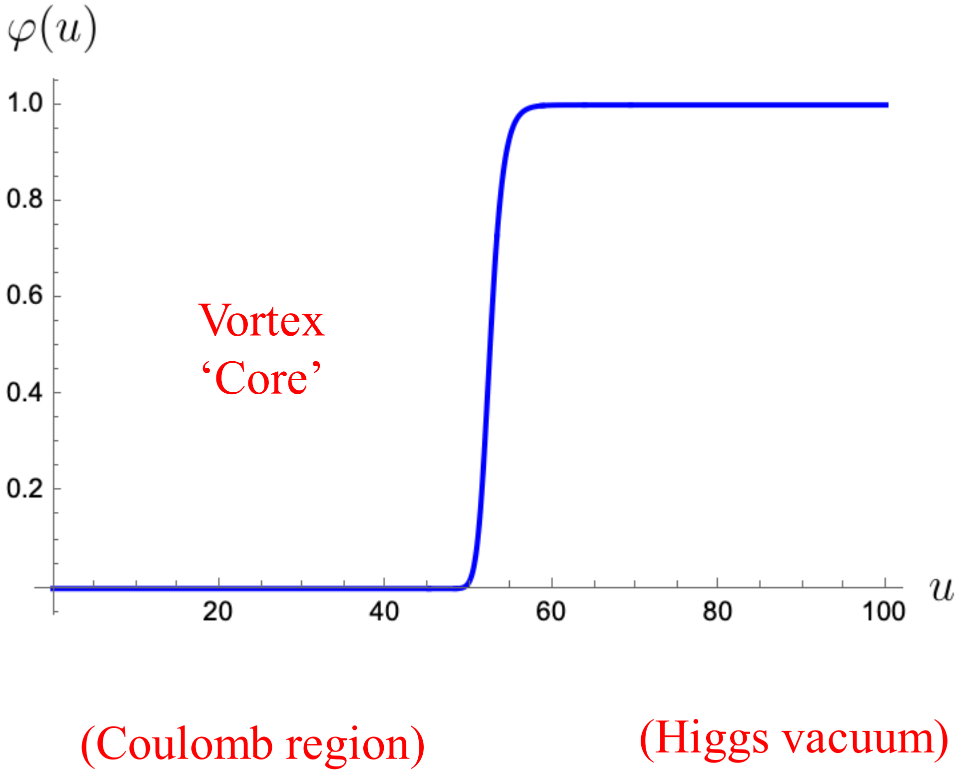
$$B = -\frac{n}{u}a'(u)$$

Conventional AHM





$n = 1000 \qquad \beta = 0.5$



Conventional AHM

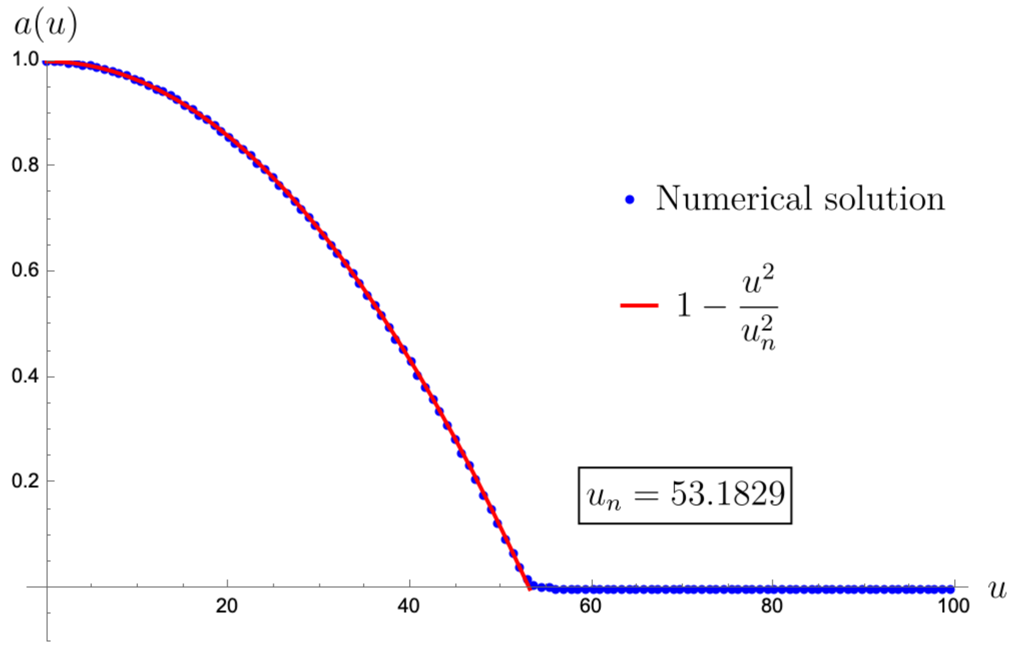
BUT THE TREND IS TRUE
EVEN FOR THE DEGENERATE MODEL !

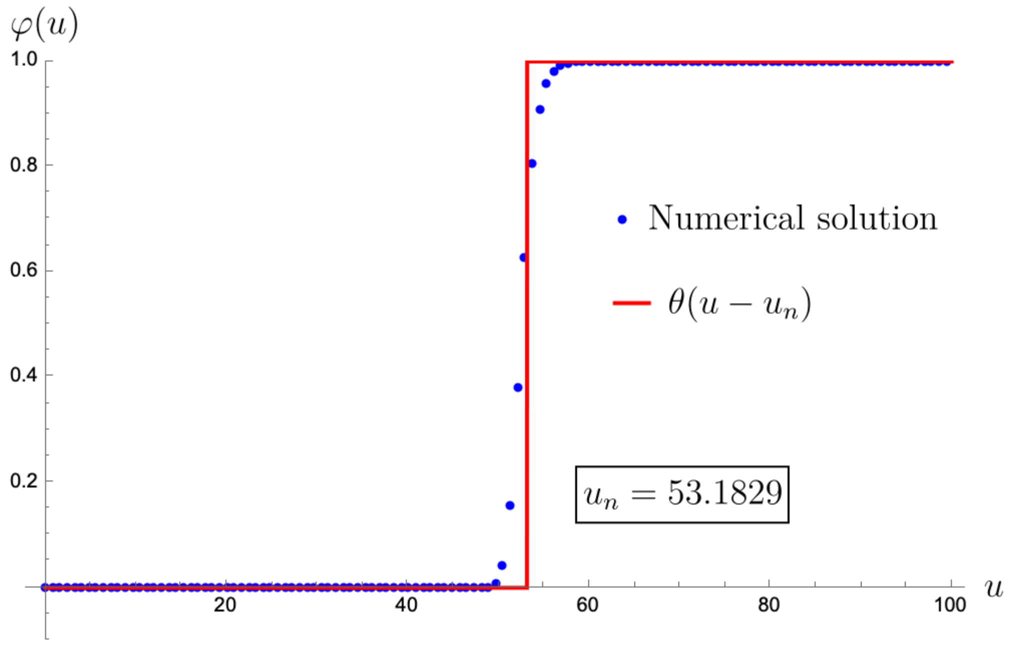
GIANT STRINGS : CONVENTIONAL AHM

$$a(u) = \begin{cases} 1 - \frac{u^2}{u_n^2} & u \leq u_n \\ 0 & u > u_n \end{cases}$$

$$\varphi(u) = \begin{cases} 0 & u \leq u_n \\ 1 & u > u_n \end{cases}$$

JUST COULOMB REGION AND HIGGS VACUUM

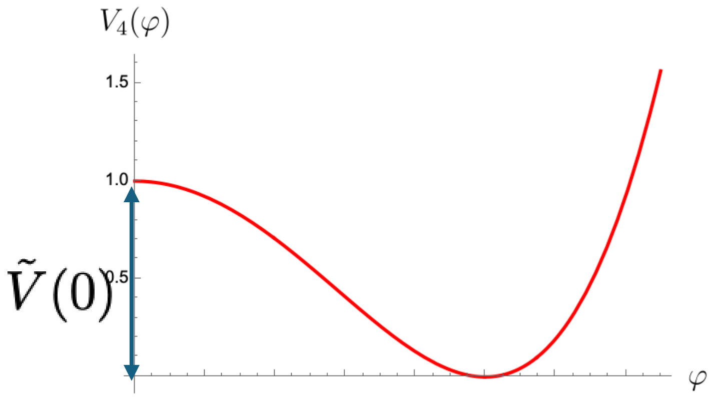




Quadratic fall off for gauge field translates to constant magnetic field !

$$B = -\frac{n}{u} a'(u)$$

$$n = 1000 \qquad \beta = 0.5$$



STRING TENSION GUESSTIMATE

Constant energy density proportional to the area of the string

$$T_n \sim \int_0^{u_n} u du \left(B^2 + \tilde{V}(0)\right) = \frac{\sqrt{2}n^2}{u_n^2} + \frac{\beta}{2\sqrt{2}} u_n^2$$

Not a vacuum ! Hence, there's a penalty

Flux wants to spread out

Variational minimization sets optimal radius and string tension

$$u_n^2 = \frac{2n}{\sqrt{\beta}} \qquad \frac{T_n}{2\pi} = \sqrt{2\beta n}$$

LARGE FLUX SOLUTION

- We **<u>need to</u>** account for physics from the transition region
- By producing large-n solution everywhere

Surface tension

$$\frac{T_n}{2\pi} = \underbrace{\sqrt{2\beta n}}_{\text{Core}} + \underbrace{\sigma u_n}_{\text{Boundary}}$$

SUBLEADING TO CORE !

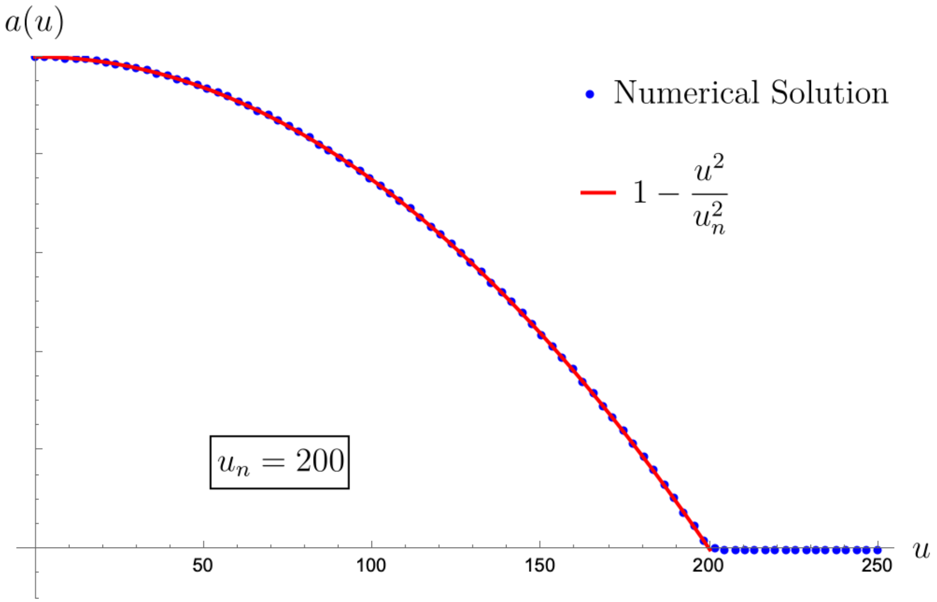
$$\sigma > 0 \leftrightarrow \beta < 1, \quad \sigma < 0 \leftrightarrow \beta > 1, \quad \sigma = 0 \leftrightarrow \beta = 1$$

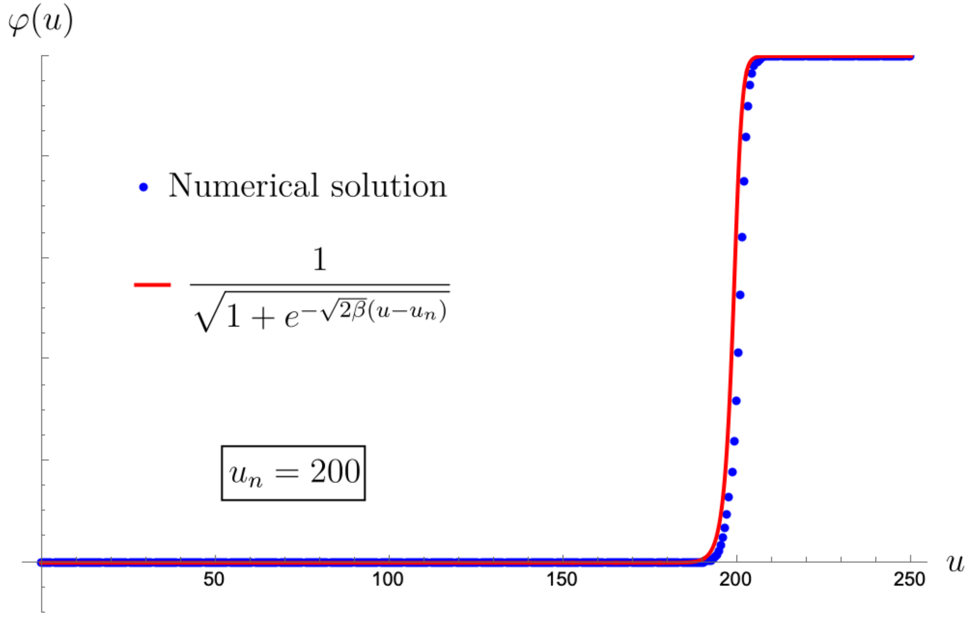
GIANT STRINGS: DEGENERATE MODEL

$$a(u) = \begin{cases} 1 - \frac{u^2}{u_n^2} & u \leq u_n \\ 0 & u > u_n \end{cases}$$

Domain wall solution connecting the Coulomb and Higgs vacuum

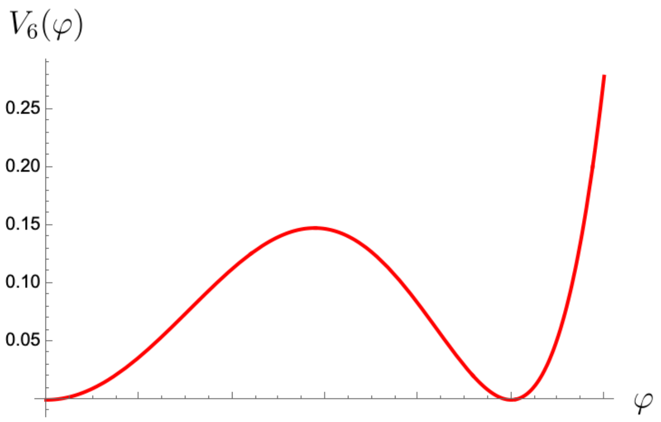
$$\varphi_{\mathrm{DW}}(u) = \frac{1}{\sqrt{1+e^{-\sqrt{2\beta}(u-u_n)}}}$$





Note the difference in size for the same values of parameters

$$n = 1000 \qquad \beta = 0.5$$



STRING TENSION

Domain wall tension – Difference in competition, no urgency for the scalar field to leave the Coulomb vacuum !

$$\frac{T_n}{2\pi} = \frac{\sqrt{2}n^2}{u_n^2} + \sigma_{\mathrm{DW}} u_n$$

Flux wants to spread out

Variational minimization sets optimal radius and string tension

$$\frac{T_n}{2\pi} = \frac{3}{2^{7/6}}\beta^{1/3} n^{2/3} \qquad u_n = 2^{5/6}\frac{n^{2/3}}{\beta^{1/6}}$$

PHASES OF GIANT STRINGS

CONVENTIONAL AHM

- BULK PHASE
- Core radius $O(\sqrt{n})$
- String tension $O(n)$
- Energy density $\beta\sqrt{2}$

Infinite volume region – Large Strings have phases !

$$\frac{T_n}{\pi u_n^2}$$

DEGENERATE MODEL

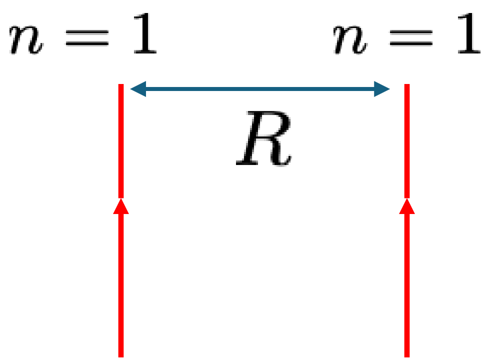
- DOMAIN WALL PHASE
- Core radius $O(n^{2/3})$
- String tension $O(n^{2/3})$
- Energy density 0

[Penin, Weller ’21 , …]
[Bolognesi, Gudnason ’05, …]

BREAK ROTATIONAL SYMMETRY

FORCES BETWEEN SEPARATED STRINGS

$$V_{\text{int}}(R) = -A^2 \sqrt{\frac{\pi}{2m_H}} \frac{e^{-m_H R}}{\sqrt{R}} + B^2 \sqrt{\frac{\pi}{2m_V}} \frac{e^{-m_V R}}{\sqrt{R}}$$

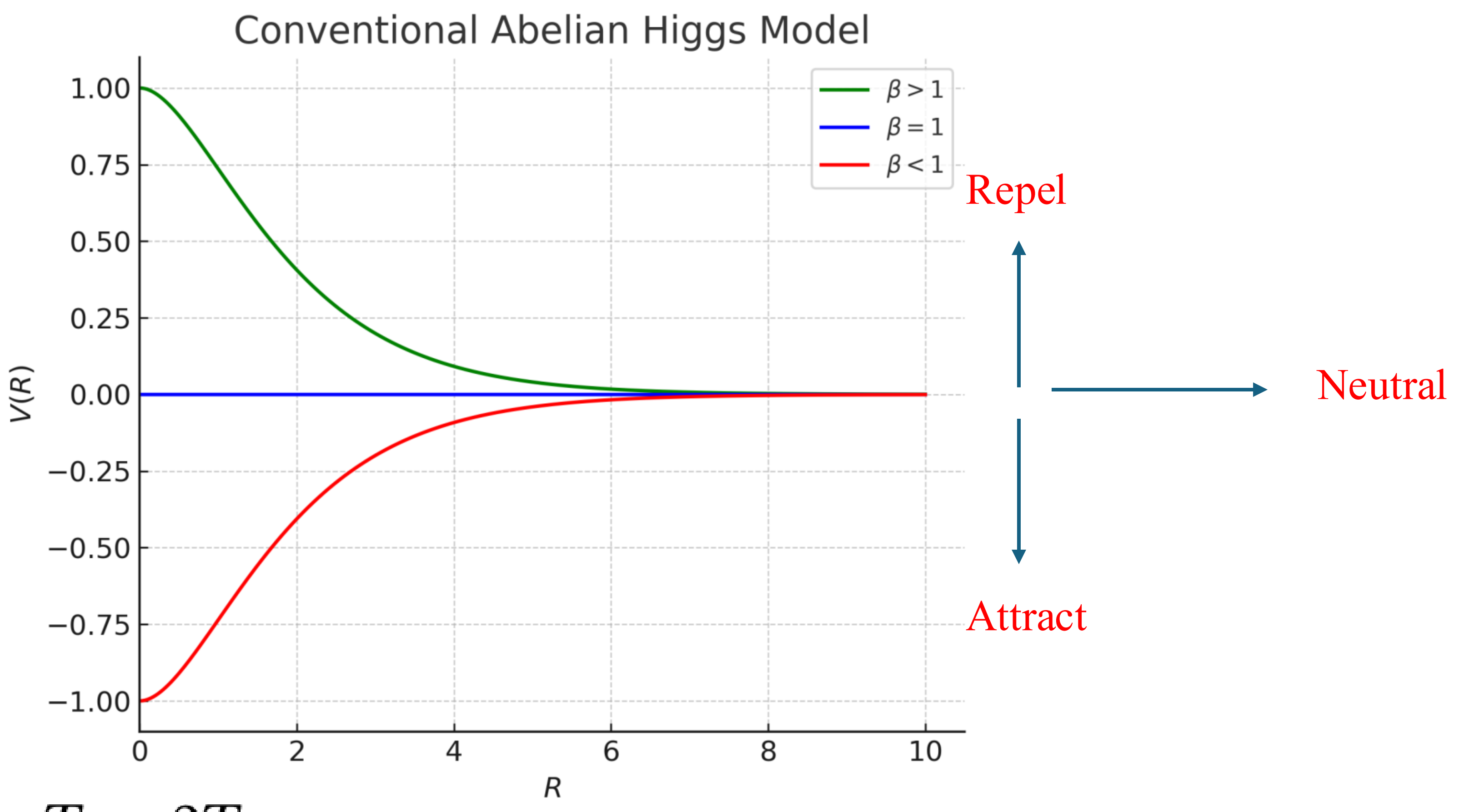


- Force between two $n = 1$ strings separated by a distance R

- **ATTRACT** in Type – I ; **REPEL** in Type – II

- Just the **Mass Spectrum** β determines the answer to this question

INTERACTION POTENTIAL $V(R)$ AS A FUNCTION OF R



$T_2 - 2T_1$

Binding energy of a 2-string

Interaction energy of superconducting vortices [Jacobs, Rebbi '79, …]

Just knowing the spectrum tells us something non trivial about the strings themselves. This is generically not true – **DEGENERATE MODEL**

YM BELIEF : Strings attract at infinity and form stable bound states

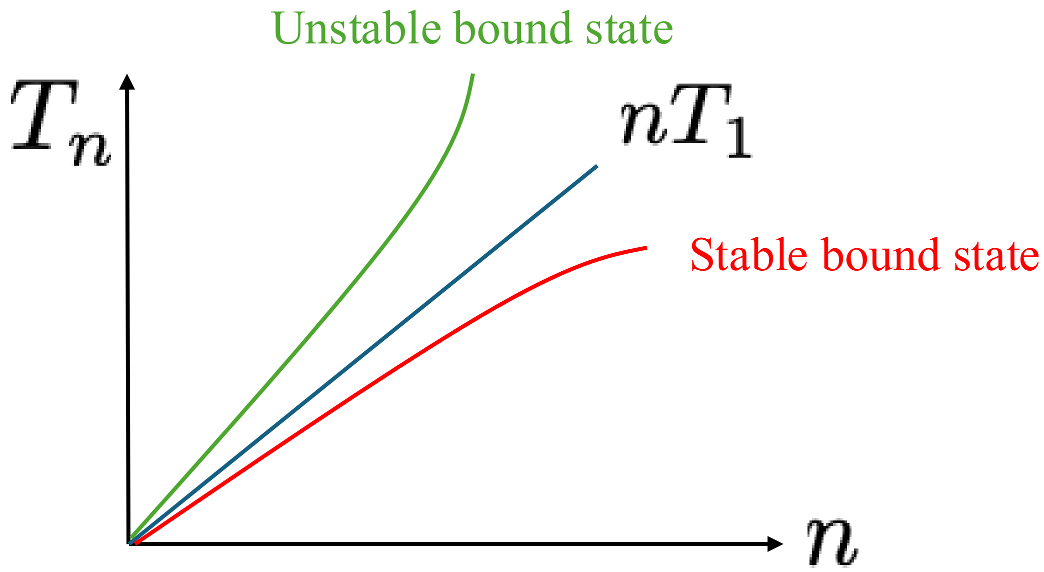
ROTATIONALLY SYMMETRIC BOUND STATES

Compare T_n with nT_1

- Numerics at small flux

- Stability of giant strings - convexity of tension at large n

$$\frac{d^2 T_n}{dn^2}$$



STABILITY OF GIANT BOUND STATES

CONVENTIONAL AHM

- $\frac{d^2 T_n}{dn^2} = -\frac{\sigma}{4n^{3/2}} \begin{cases} < 0 & \beta < 1 \text{ Stable} \\ = 0 & \beta = 1 \text{ Neutral} \\ > 0 & \beta > 1 \text{ Unstable} \end{cases}$

$$\frac{T_n}{2\pi} = \sqrt{2\beta}n + \sigma u_n$$

- Stability directly correlated w/ β
- Stability of 2-string ; same as above

DEGENERATE MODEL

- $$\frac{d^2 T_n}{dn^2} < 0$$

$$\frac{T_n}{2\pi} = \sqrt{2}n^{2/3}\beta^{1/3}$$

- Stable for all values of β
- Stability of 2-string – not as straightforward

FEATURES

- **Phases of Giant strings** : Scaling of size and tension with n
- **Interacting forces** between separated fundamental strings
- Stable versus unstable symmetric **bound states**
- These are important data points gathered from simple AHMs

STRING FLUCTUATIONS IN CONVENTIONAL AHM

Forthcoming [Dumitrescu, AG, Li ‘25]

FLUCTUATION PROBLEM AND ZERO MODES

- String is an object in 2D space ; **Two** NGBs – Broken translational symmetry

$$X^{i=1,2}(t,z)$$

- Additional moduli for BPS at higher flux

For this talk focus on fluctuations of the **fundamental** string

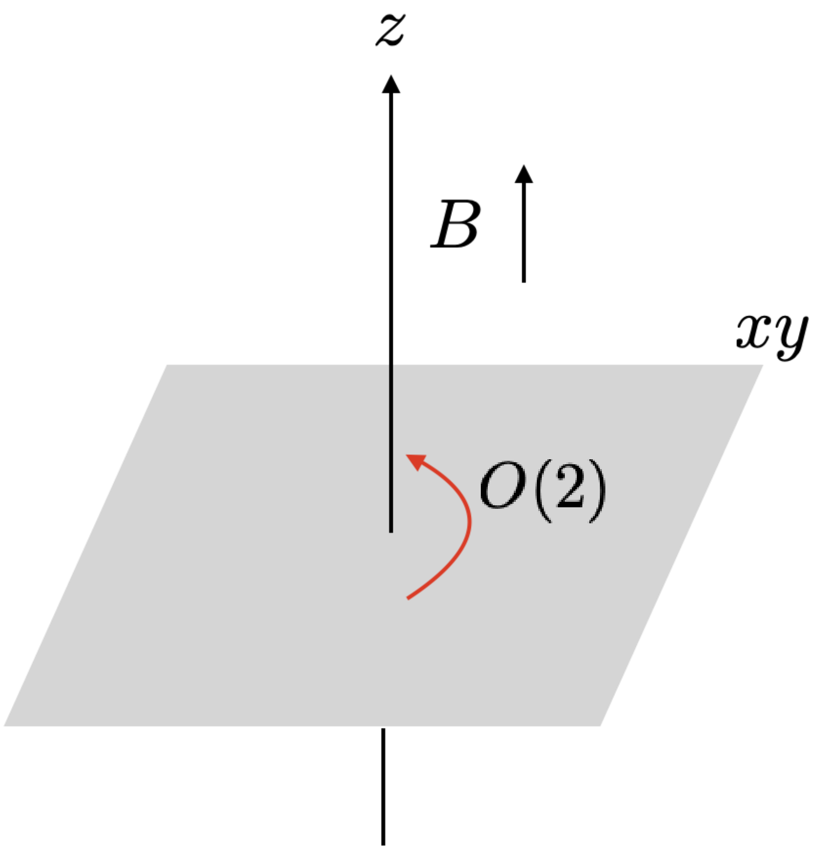
- Other small fluctuation modes are **gapped** and extremely rich

- Linearize equations and solve linearized problem

MASSIVE MODES

Continuum of scattering states

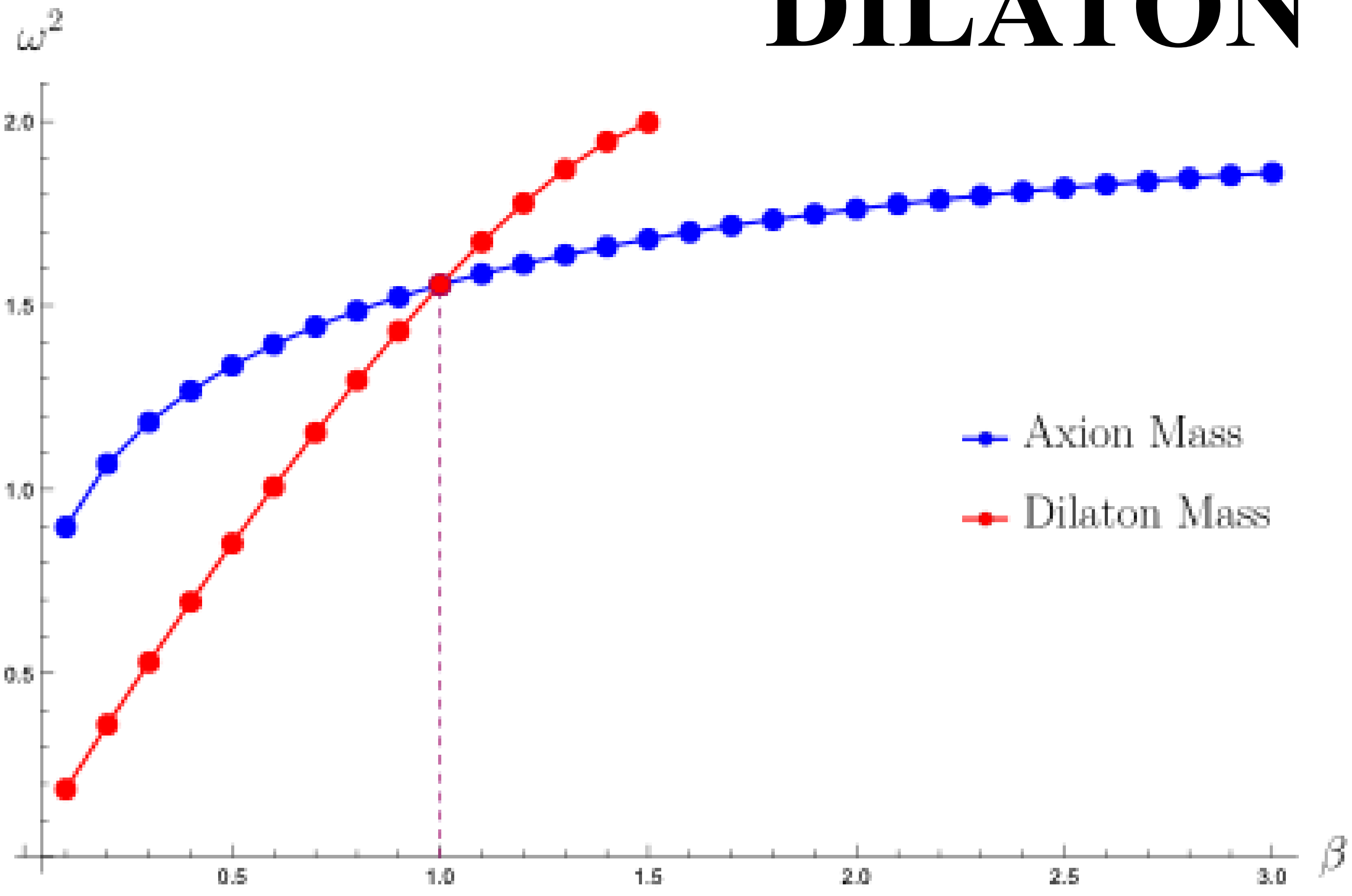
Bound state excitations



- $O(2)$ transverse spin – rotates the GBs.
- Use this symmetry to organize states
- Transverse Parity : GBs transform as a vector
- Focus on spin – 0 : Axion and Dilaton

Pseudoscalar Scalar

MASS HIERARCHY : AXION AND DILATON



Type – I

Type - II

Axion is the lightest massive excitation in the type-II regime - a setting where fundamental strings **<u>REPEL</u>** !

YM CONFINING FLUX TUBES

- Pseudoscalar Axion is the lightest non-trivial fluctuation

  [Dubovsky, Flauger, Gorbenko' 13, Athenodorou, Dubovsky, Luo, Teper ' 24, ….]

- Believe that confining strings in YM attract and form bound states

  [Athenodorou, Teper '21, ….]

Axion is the lightest massive excitation in a setting where fundamental confining strings **ATTRACT** !

EFFECTIVE ACTION OF LONG STRINGS

- Calculate three point couplings of GBs to massive modes

$$aXX \longrightarrow \text{(Suppressed indices)}$$

- Integrate to calculate backreaction on the GBs. At leading order,

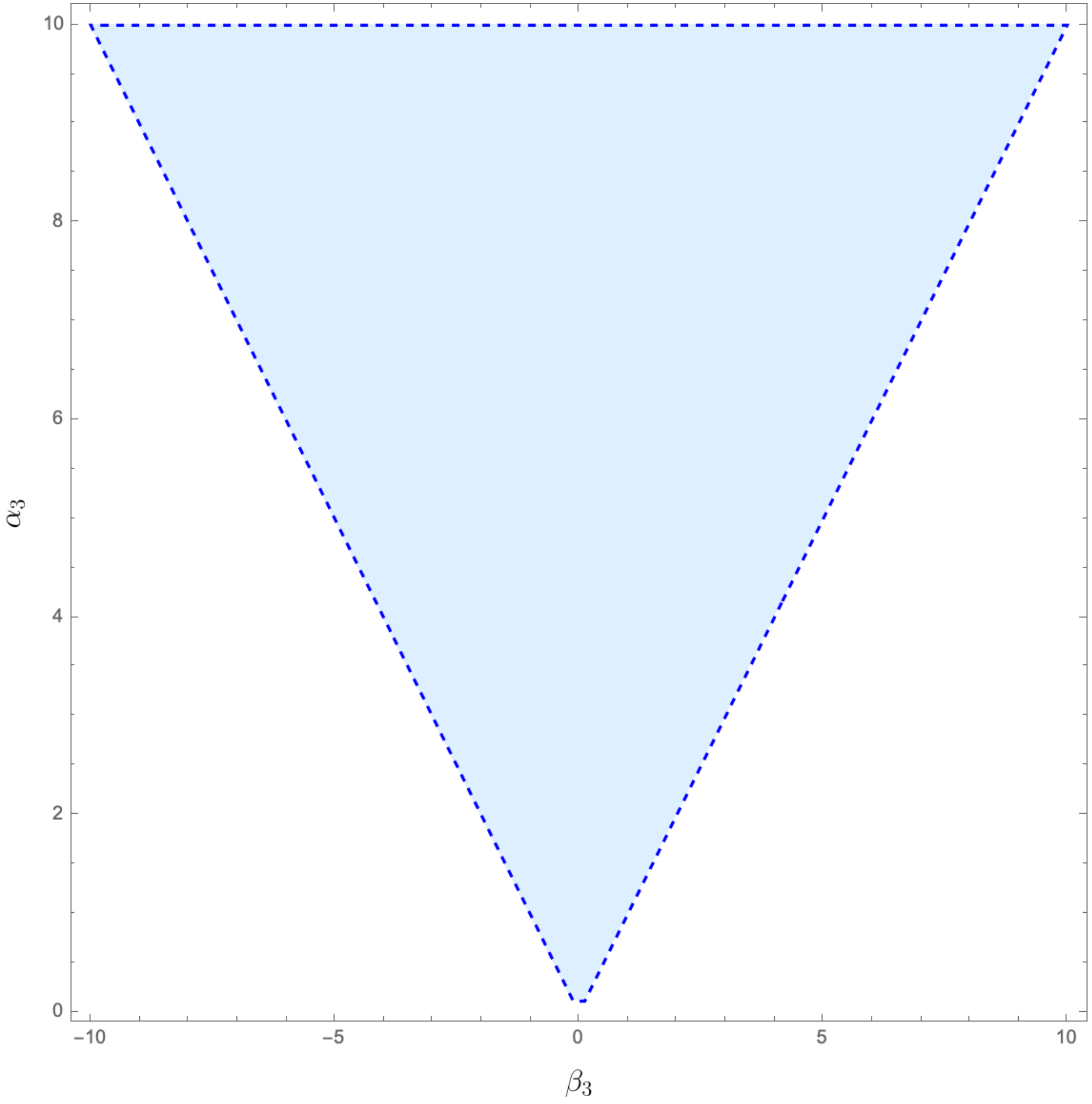
[Dubovsky, Flauger, Gorbenko '12, Aharony, Komargodski '13, ….]

Interesting characteristic numbers of the string

$$\mathcal{L} \sim T\sqrt{-h}\left(1 + \frac{\tilde{\alpha}_3}{T^2}K^4 + \frac{\tilde{\beta}_3}{T^2}R^2 + \dots\right)$$

Extrinsic curvature

Ricci scalar



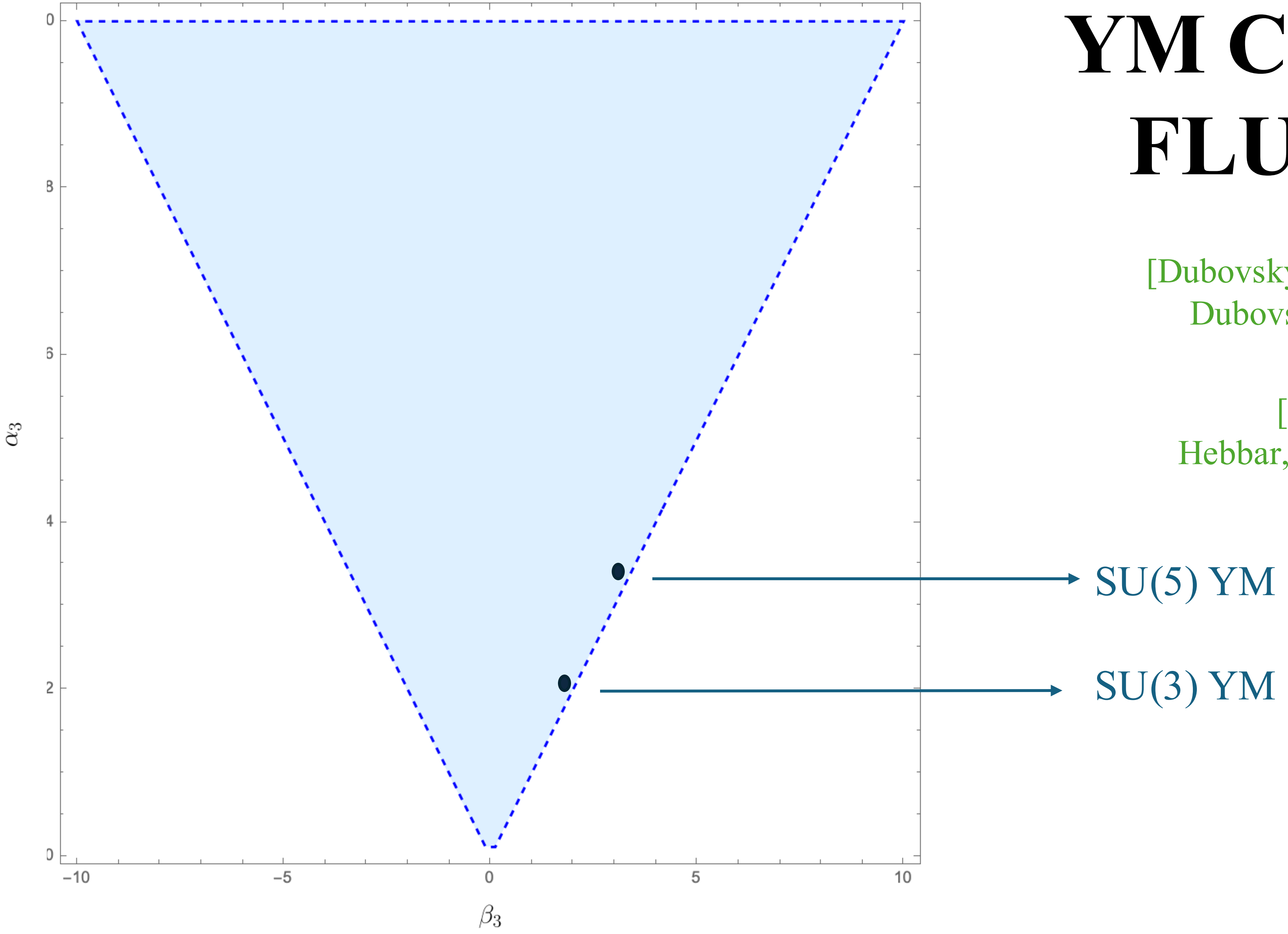
BOOTSTRAP BOUNDS

Requiring a consistent UV completion of the branon S-matrix, put bounds on its low energy expansion and bound the EFT parameters

[Miro, Guerrieri, Hebbar, Penedones, Vieira '19]

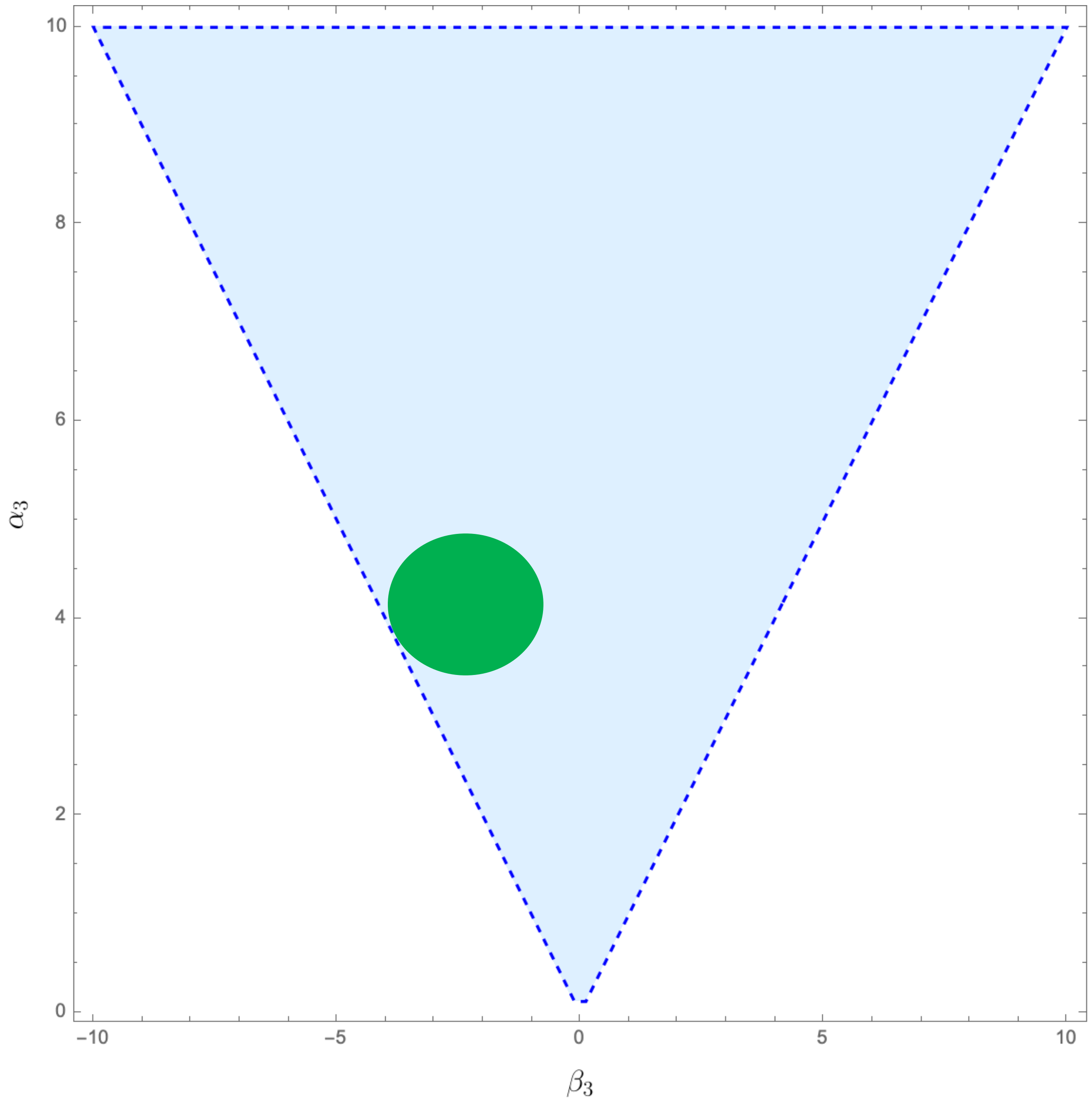
[Miro, Guerrieri '21]

YM CONFINING FLUX TUBES



[Dubovsky, Flauger, Gorbenko ’14, Dubovsky, Gorbenko ’15,…]

[Miro, Guerrieri, Hebbar, Penedones, Vieira ’19]

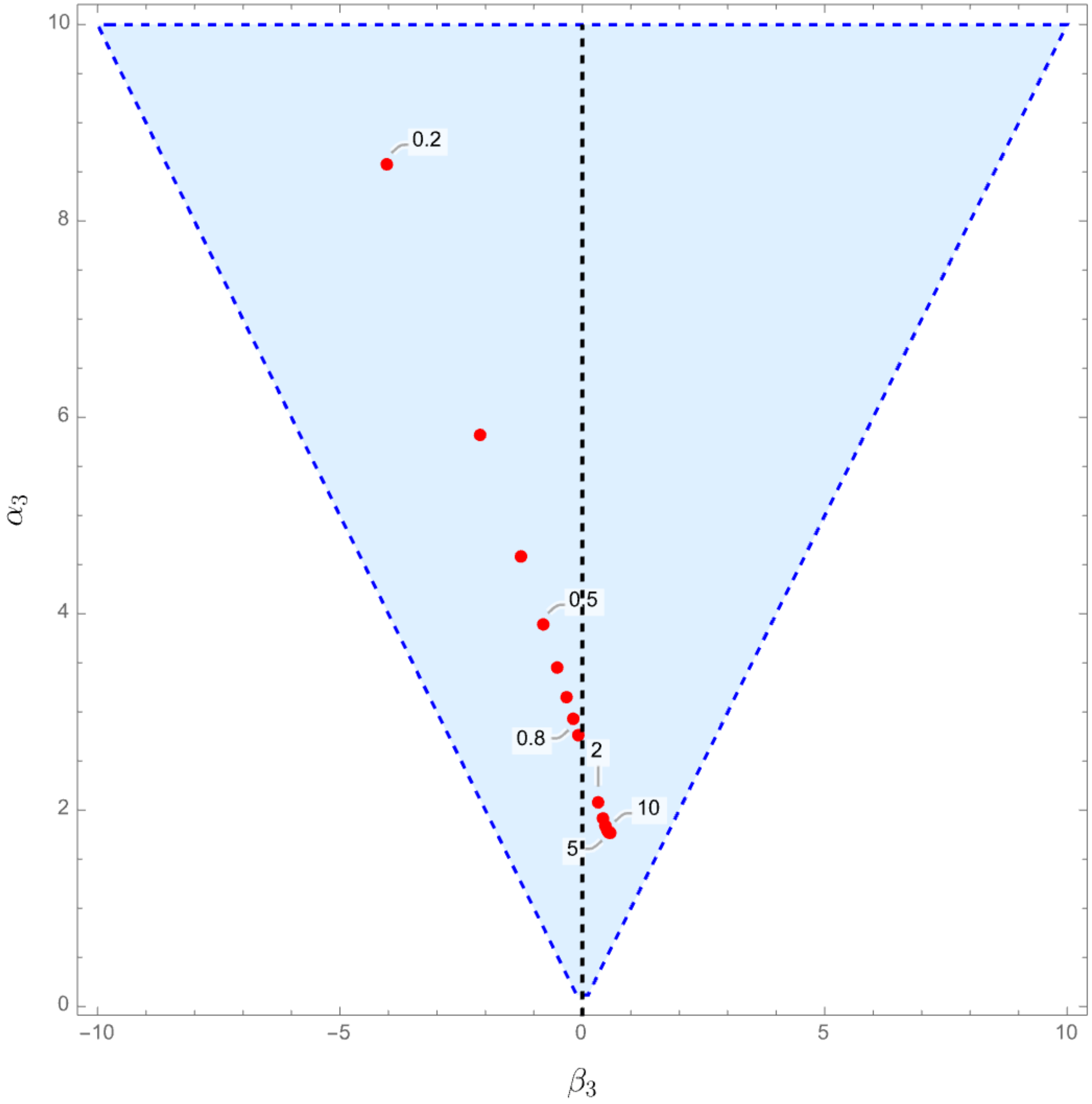


HOLOGRAPHIC CONFINING GAUGE THEORIES

Expect flux tubes in holographic confining gauge theories to have a **light dilaton** fluctuation mode

[Polyakov '98]

[Aharony, Karzbrun'09,..]



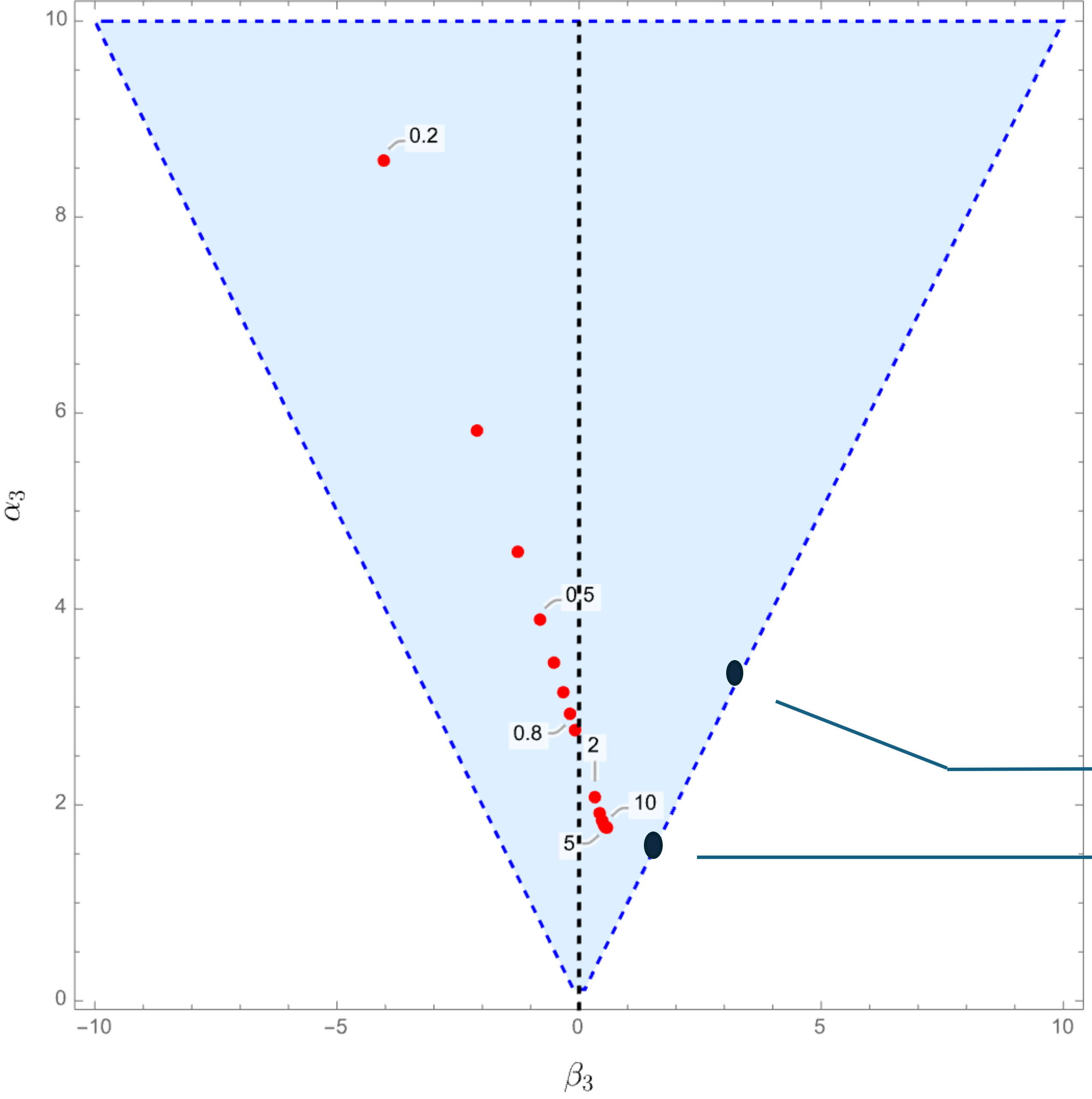
TYPE- I AHM

TYPE-II AHM

CONVENTIONAL AHM

In red, we plot the conventional AHM for different values of β

$$\alpha_3,\ \beta_3 = \frac{f(\beta)}{e^4}$$



THE YM CONFINING FLUX TUBES LIE IN THE REGION WHERE TYPE-II SUPERCONDUCTING STRINGS RESIDE.

This simulates the similar tension as before !

SU(5) YM

SU(3) YM

TYPE- I
AHM

TYPE-II
AHM

MINIMAL AHM IS **NOT** A COMPELLING DUAL DESCRIPTION FOR YM CONFINING STRINGS !

NATURAL NEXT VARIANT !

- **Deformed SW theory**: Abelian model of confinement, **MANY** more fields : scalars and fermions

- It is known from condensed matter literature that if one has multiple Higgs fields and you play w/ potentials, one can dramatically change the way in which strings attract / repel

- **NEXT** : Study properties of **CONFINING STRINGS** in deformed Seiberg Witten theory.

CONFINING STRINGS IN DEFORMED SEIBERG WITTEN (SW) THEORY

Forthcoming [Dumitrescu, AG ‘25]

WHY SEIBERG WITTEN THEORY?

$\mathcal{N}=2$ SYM

$\mathcal{N}=1$ SYM

Pure YM

Today – weakly deform SW theory and restrict our analysis to $\mathcal{N}=1$ theories put box here

Explicit realization of dual superconductivity (dual Abelian Higgs model) with electric flux tubes instead !

SEIBERG WITTEN THEORY

- $\mathcal{N}=2$ SYM with gauge group $SU(2)$ -- asymptotically free
- Matter content in the UV $(\phi^a, \lambda_\alpha^{ia}, f_{\mu\nu}^a)$
- $\mathbb{Z}_2^{(1)}$ conserved one-form electric (center) symmetry

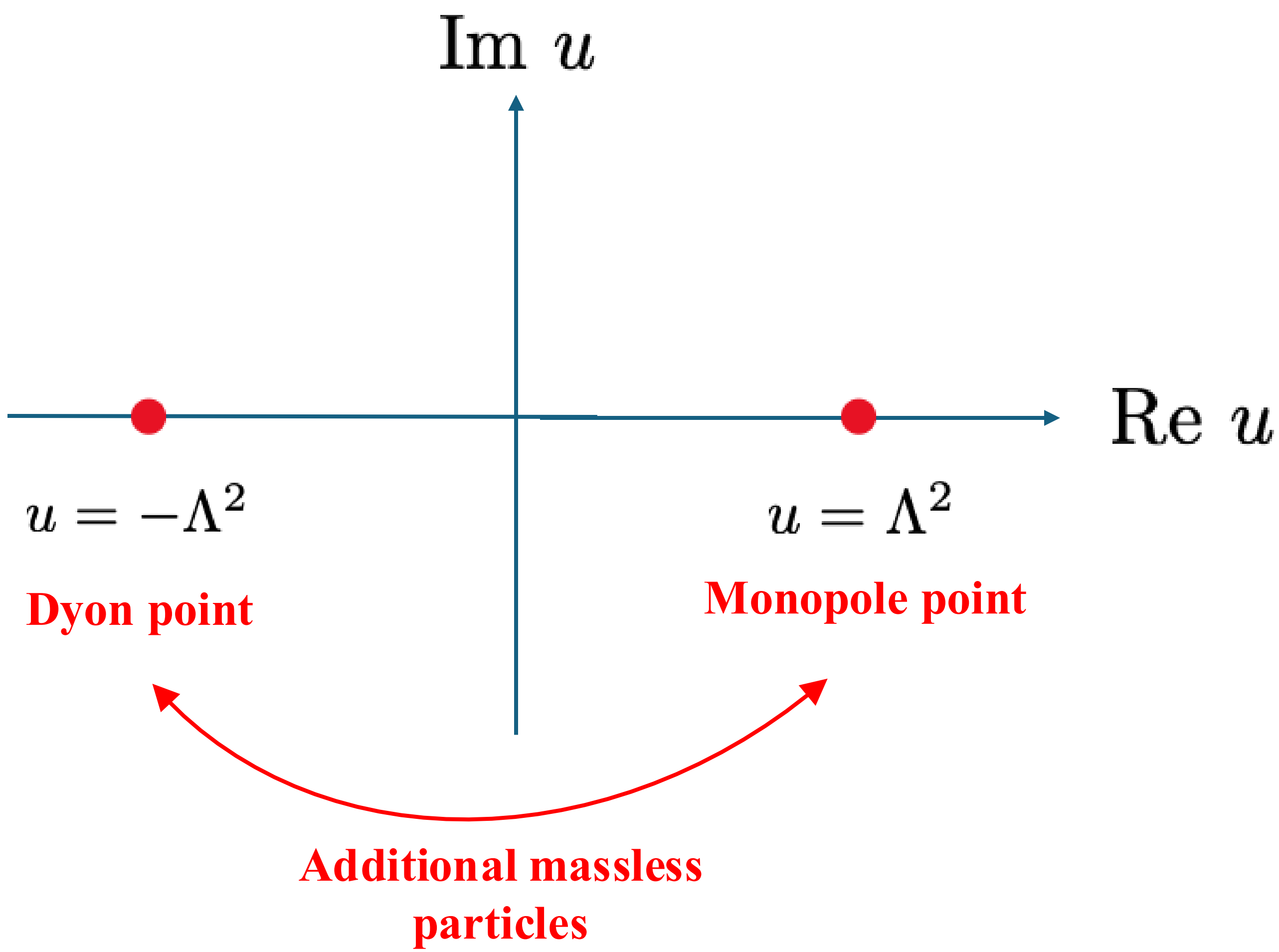
Scalar potential $V(\phi) \sim \mathrm{Tr}[\phi, \phi^\dagger]^2$

$$SU(2) \to U(1) \qquad \phi \sim \sigma^3$$

MODULI SPACE OF VACUA

Gauge-invariant quantity parametrizing the space of vacua $\leftarrow$ $u = \mathrm{Tr}\,\phi^2$

[Seiberg, Witten ’94]



Λ Strong coupling scale

Naively, expect confining theory at low energies – IR is instead described by Coulomb phase !

Single $\mathcal{N} = 2$ $U(1)$ Vector multiplet at generic u

THEORY AT THE MONOPOLE POINT

$$L = \int d^4\theta \Big[\tfrac{1}{e^2} \overline{A}_D A_D + M e^{-2V} \overline{M} + \widetilde{M} e^{2V} \overline{\widetilde{M}} \Big]$$

$U(1)$ gauge coupling

$$+ \int d^2\theta \Big[\tfrac{1}{4e^2} W^\alpha W_\alpha + \sqrt{2} A_D M \widetilde{M} \Big] + (\text{h.c})$$

$\mathcal{N} = 2$

SQED

Weakly coupled theory of monopoles and photons

$\mathcal{N} = 2$

Vector multiplet

$\mathcal{N} = 1$

Chiral multiplets

$\mathcal{N} = 2$

Hypermultiplet (Monopole fields)

$0 \quad A_D = (a_D, \dots)$

$V = (A_\mu, \dots)$

$M = (m, \dots) \quad +1$

$\widetilde{M} = (\widetilde{m}, \dots) \quad -1$

$\mathcal{N} = 1$ Vector multiplet

SOFT SUSY BREAKING

$$\mathcal{N}=2\to\mathcal{N}=1$$

- UV deformation : mass to the chiral, vector multiplet massless.

$$\Delta\mathcal{W}=m\mathrm{Tr}\Phi^2$$

- $m\to\infty\ :\ \mathcal{N}=1\ SU(2)$ SYM

The theory is in a confining phase for all values of m

SOFT SUSY BREAKING

$$\mathcal{N}=2 \rightarrow \mathcal{N}=1$$

Unique Confining vacuum at the monopole point

- UV deformation manifested at the monopole point as,

$$\Delta\mathcal{W} = mU(A_D)$$

Magnetic Higgs mechanism

- Vacuum: $A_D = 0$ $\quad M = \tilde{M} \neq 0$

- Vacuum structure protected by **holomorphy and non-renormalization** theorems for all values of m

Close to the monopole point

$$u(a_D) = 4i\Lambda a_D - \frac{a_D^2}{4} - \frac{i a_D^3}{64\Lambda} + \ldots$$

[Seiberg, Witten ’94, D’Hoker, Phong ’97]

SUPERPOTENTIAL

Truncated superpotential

$$\mathcal{W} = \underbrace{\sqrt{2}A_D M\widetilde{M}}_{\text{Undeformed}} + \underbrace{\xi A_D + \alpha_2 A_D^2 + \alpha_3 A_D^3}_{\text{Deformation}}$$

Consider only renormalizable terms.

We gain more control by staying close to the monopole point and hence consider a small m expansion. This is also required so as to not go beyond the confines of our IR effective theory.

A small m expansions allows us to perform a Taylor expansion of $U(A_D)$ and justifies considering a truncated model. For our purposes, we consider only renormalizable terms in the superpotential.

Consider superpotential for **all values** of α_2, α_3 and later scale back to SW solution.

$\xi, \alpha_2, \alpha_3 \in \mathbb{R}$ $\qquad$ $\xi \sim m\Lambda = 1$ $\qquad$ $\alpha_2 \sim m$ $\qquad$ $\alpha_3 \sim m^2$

ENHANCED SUSY $\mathcal{N}=2$

$$\{\bar{Q}^i_{\dot\alpha}, Q^j_\alpha\} \sim \sigma^\mu_{\alpha\dot\alpha}(\epsilon^{ij}P_\mu + Z^{ij}_\mu)$$

$$\mathcal{W} = \sqrt{2}A_D M\widetilde{M} + \underline{A_D}$$

$\mathcal{N}=2$

Fayet-Iliopoulos Term

- Accidental $\mathcal{N}=2$ supersymmetry – deformation is an FI term

Same String tension as the conventional non-susy AHM !

- Theory admits ½-BPS strings obeying

$$\frac{T_n}{2\pi} = \sqrt{2}n$$

BULK PHASE !

[Douglas, Shenker '95]

Discusses strings in the deformed SW theory specifically at the BPS point – sine law !

FEATURES OF DUAL AHM

$$L = -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} - |(\partial_\mu - i a_\mu) m|^2 - |(\partial_\mu + i a_\mu)\tilde{m}|^2 - \frac{|\partial_\mu a_D|^2}{e^2}$$

$$\underbrace{-\frac{e^2}{2}(|m|^2 - |\tilde{m}|^2)^2}_{\text{D-terms}} \quad \underbrace{-2|a_D|^2(|m|^2 + |\tilde{m}|^2) \quad -e^2|\sqrt{2} m\tilde{m} + 1 + 2\alpha_2 a_D + 3\alpha_3 a_D^2|^2}_{\text{F-terms}}$$

- Kinetic terms : consider **two-derivative and renormalizable** terms.
- $U(1)^{(1)}_{\text{electric}}$ one-form electric flux symmetry
- Classical analysis : turn fermions off
- $U(1)$ gauge theory with complex scalars

$$\mathbb{Z}_2^{(1)} \xrightarrow{\text{ENHANCEMENT IN THE IR}} U(1)^{(1)}_{\text{electric}}$$

(Broken in UV -- heavy W-bosons)

VACUUM STRUCTURE

CONFINING (DUAL HIGGS) VACUUM

H

$$A_D = 0 \qquad \sqrt{2} M \widetilde{M} + 1 = 0$$

TWO COULOMB VACUA

$C_{\pm}$

$$M = \widetilde{M} = 0 \qquad 1 + 2\alpha_2 A_D + 3\alpha_3 A_D^2 = 0$$

Note the presence of extra Coulomb vacua in the truncated superpotential – markedly different from deformed SW theory !

Only at the BPS point, we have a unique confining vacuum !

CONFINING VACUUM

Unique and trivially gapped

$$M_V^2 = 2\sqrt{2}e^2$$

Massive vector multiplet

$$M_L^2 = 2e^2\left(\sqrt{2} + e^2\alpha_2^2 - \sqrt{2\sqrt{2}e^2\alpha_2^2 + e^4\alpha_2^4}\right)$$

$$M_H^2 = 2e^2\left(\sqrt{2} + e^2\alpha_2^2 + \sqrt{2\sqrt{2}e^2\alpha_2^2 + e^4\alpha_2^4}\right)$$

Massive chiral multiplets

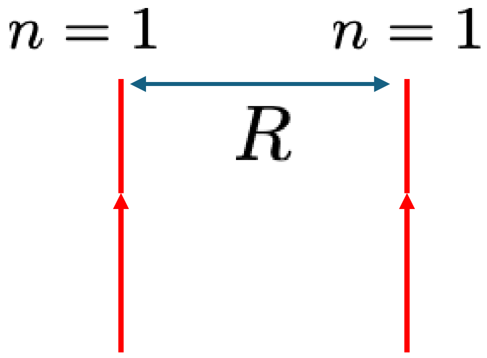
For all values of parameters, in the confining vacuum, there is a light scalar mode !

$$\boxed{M_L < M_V < M_H}$$

FORCES BETWEEN SEPARATED CONFINING STRINGS

$$V_{\text{int}}(R) = -A^2 \sqrt{\frac{\pi}{2m_H}} \frac{e^{-m_H R}}{\sqrt{R}} + B^2 \sqrt{\frac{\pi}{2m_V}} \frac{e^{-m_V R}}{\sqrt{R}}$$

$$-C^2 \sqrt{\frac{\pi}{2m_L}} \frac{e^{-m_L R}}{\sqrt{R}} \qquad M_L < M_V < M_H$$



- Force between two $n = 1$ strings separated by a distance R
- **ATTRACT** for all values of α_2, α_3
- Separated fundamental strings attract at large separations

CONFINING STRINGS

Unlike vacuum, strings generically **NOT** protected by SUSY.

Study properties of confining strings,

1. Do they form bound states ?
2. **Phases of Giant Confining Flux Tubes**

[Douglas, Shenker '95]

[Hanany, Strassler, Zaffaroni' 98]

[Vainshtein, Yung '01, Hou' 01]

[Klebanov, Herzog '02]

ROTATIONALLY SYMMETRIC STRINGS

ROTATIONALLY SYMMETRIC STRINGS

$$m(x) = \frac{i}{2^{1/4}}\varphi(r)e^{in\theta} \qquad \tilde{m}(x) = \frac{i}{2^{1/4}}\varphi(r)e^{-in\theta}$$

$$a_\theta = n(1 - A(r)) \qquad a_D(x) = a_D(r)$$

$$\varphi, a_D \in \mathbb{C}$$

- System of **five** coupled non-linear ODES– profiles depend on n, α_2, α_3
- Need to solved numerically; analytic solution in the large flux limit

$$\underbrace{\varphi(0) = 0 \quad A(0) = 1}_{\text{Regularity near origin}} \quad \underbrace{\varphi(\infty) = 1 \quad A(\infty) = 0 \quad a_D(\infty) = 0}_{\text{Confining vacuum (finite tension)}}$$

$a_D(0)$ needs to be determined numerically

BPS STRINGS IN $\mathcal{N} = 2$

$$\mathcal{W} = \sqrt{2} A_D M \tilde{M} + \underline{A_D}$$

$\mathcal{N} = 2$
Fayet-Iliopoulos Term

- Accidental $\mathcal{N} = 2$ supersymmetry – deformation is an FI term

- Theory admits ½-BPS strings obeying

$$\frac{T_n}{2\pi} = \sqrt{2} n$$

Giant Strings in the
Bulk Phase

Same field profiles and string tension as the BPS point in the conventional AHM from before !

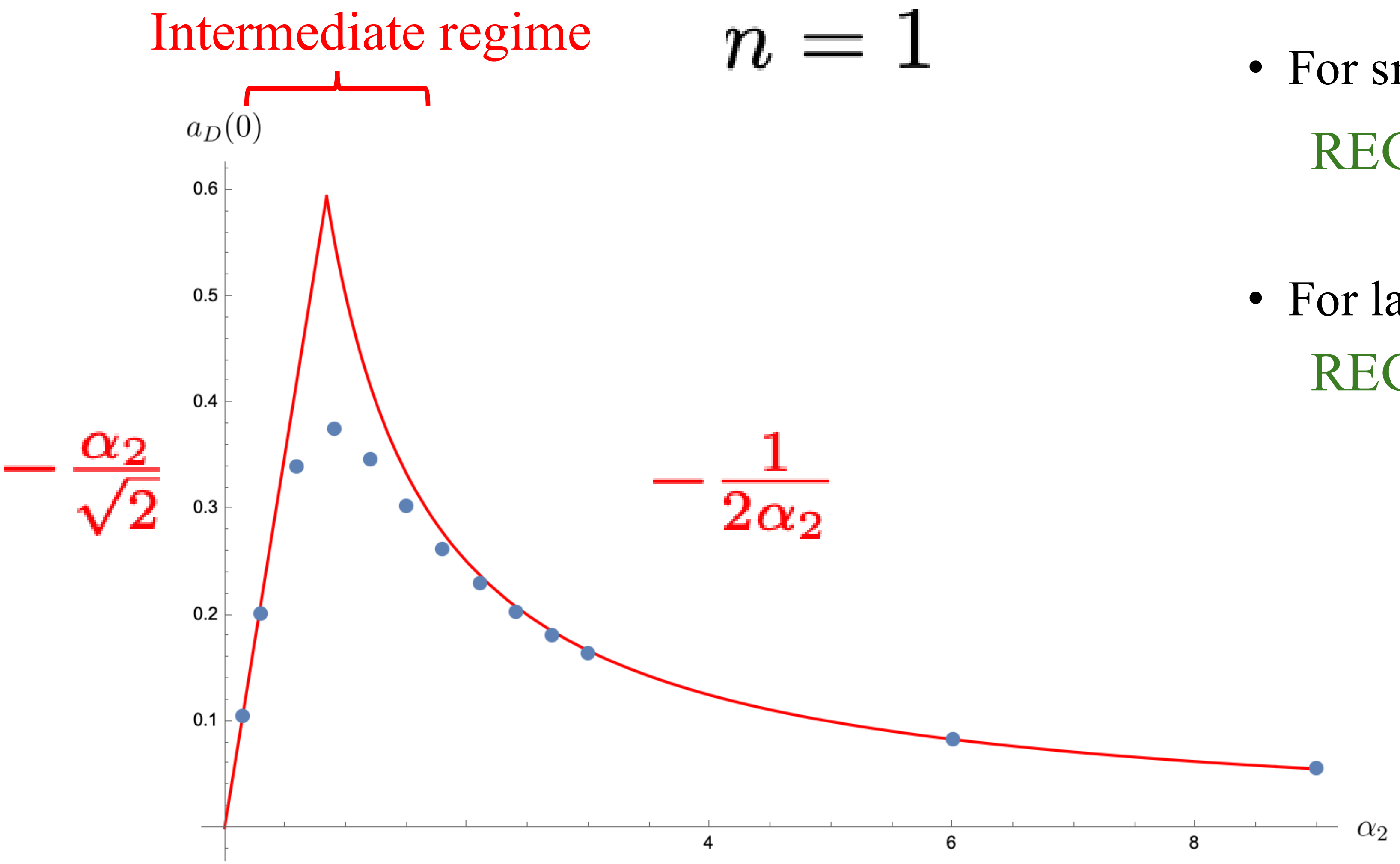
$$\varphi(u) \in \mathbb{R} \qquad a_D = 0$$

DIAGNOSIS NEAR THE ORIGIN

$$\sqrt{2}A_D M\tilde{M} + A_D + \alpha_2 A_D^2$$

For any given finite flux n , the following statements are true,

$$n = 1$$



- For small α_2 $\quad a_D(0) = -\frac{n\alpha_2}{\sqrt{2}}$

  REGIME 1

- For large α_2 $\quad a_D(0) = -\frac{1}{2\alpha_2}$

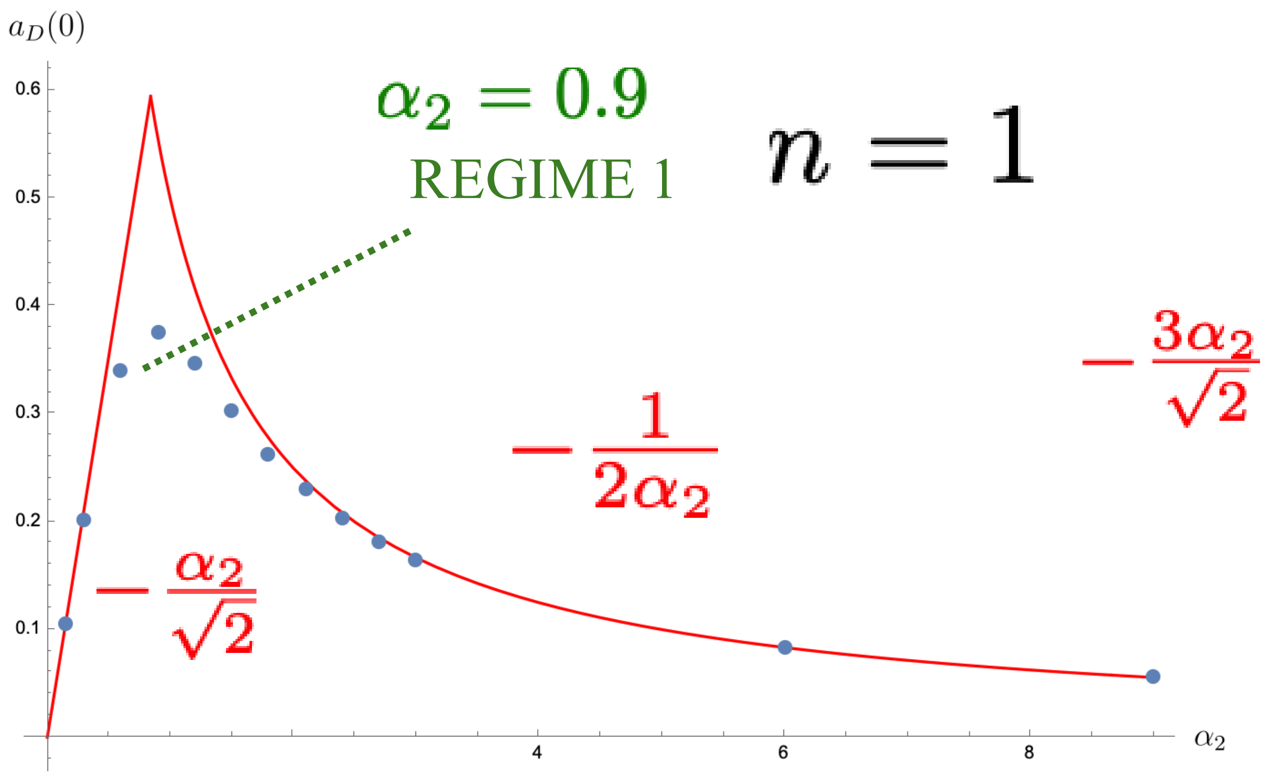
  REGIME 2

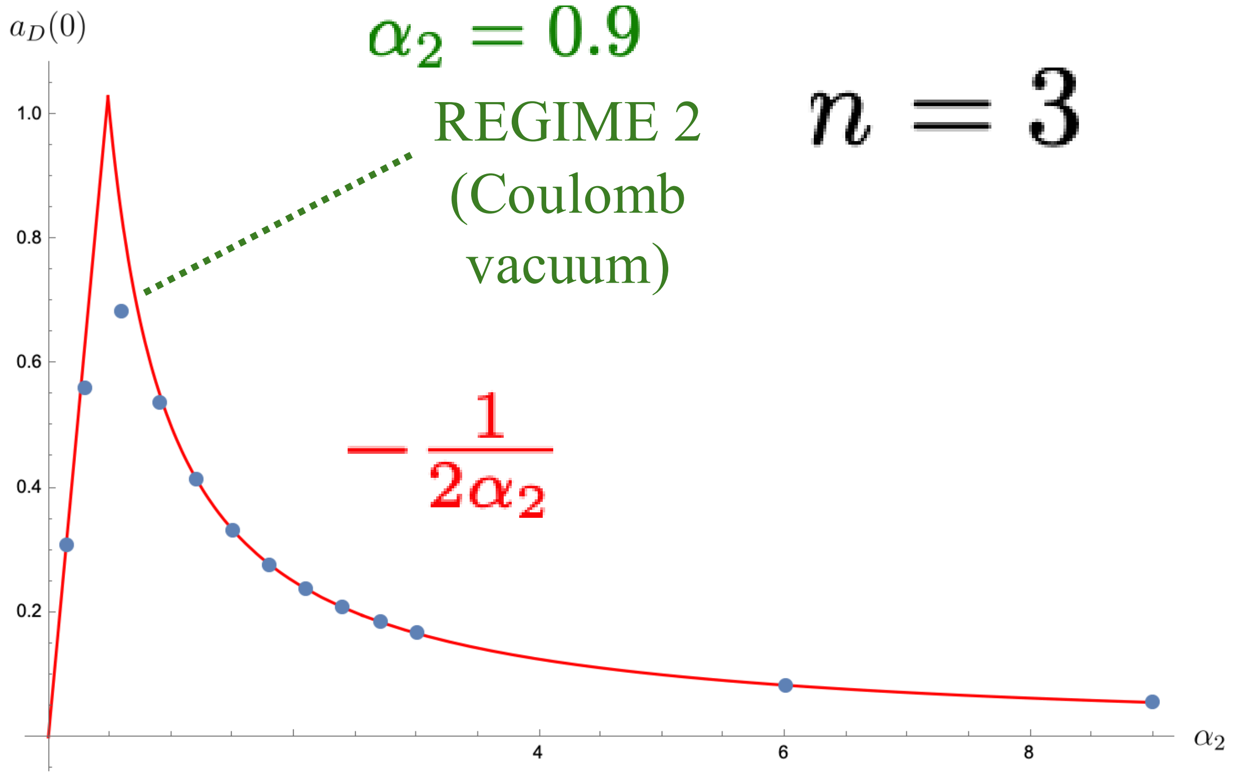
Coulomb vacuum at the origin !

$$1 + 2\alpha_2 A_D = 0$$

Coulomb vacuum condition

VARIATION WITH FLUX





Numerically we find that the crossover takes place at smaller values of α_2 with increasing flux

With increase in flux, range of validity for

REGIME 1 ↓ **REGIME 2** ↑

SUMMARY OF NUMERICAL FINDINGS

- **REGIME 1 :** Small values of $\alpha_2, \ \alpha_3$

  It's **range of validity decreases with increase in flux** !

CAN BE EXPLAINED IN PERTURBATION THEORY !

- **REGIME 2 :** Larger values of $\alpha_2, \ \alpha_3$

  With increasing flux, the crossover to this regime happens at smaller $\alpha_2, \ \alpha_3$

  DOMAIN WALL STRINGS !

STRINGS IN PERTURBATION THEORY

PERTURBATION THEORY SETUP

$$\varphi(u) = \varphi_{\rm BPS}(u) + \alpha_2 \varphi_1(u) \qquad A(u) = A_{\rm BPS}(u) + \alpha_2 A_1(u)$$

$$a_D(u) = \alpha_2 g_1(u)$$

Needs to be solved numerically

$$A_1 = \varphi_1 = 0 \qquad a_D \in \mathbb{R}$$

Perturbation theory around the BPS point : small α_2 as illustration

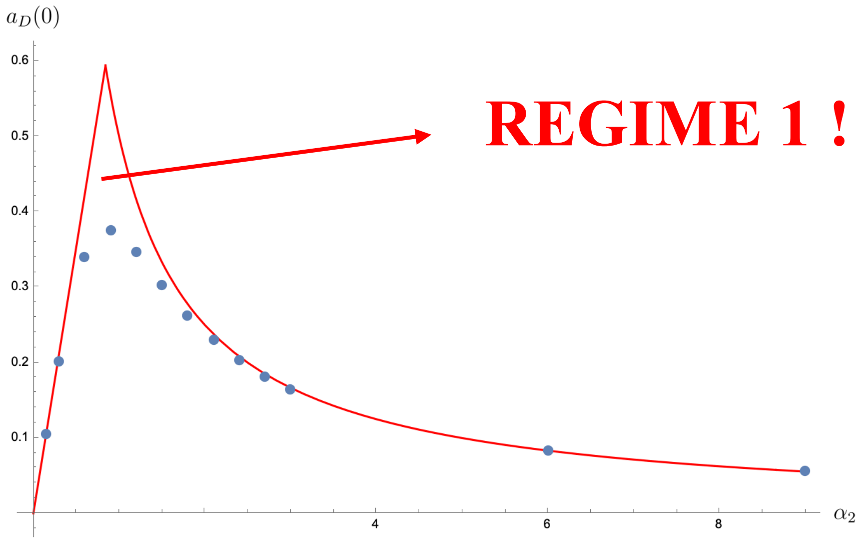
GIANT STRINGS : PERTURBATION THEORY

$$A(u) = \begin{cases} 1 - \frac{u^2}{2n} & u \leq \sqrt{2n} \\ 0 & u > \sqrt{2n} \end{cases} \qquad \phi(u) = \begin{cases} 0 & u \leq \sqrt{2n} \\ 1 & u > \sqrt{2n} \end{cases} \qquad u_n = \sqrt{2n}$$

$$a_D(u) = \alpha_2 g_1(u) \qquad g_1(u) = \begin{cases} -\frac{n}{\sqrt{2}} + \frac{u^2}{2\sqrt{2}} & u \leq \sqrt{2n} \\ 0 & u > \sqrt{2n} \end{cases}$$

Core radius same as the conventional AHM at the BPS point ! Consistent w/ the fact that

$$A_1 = \varphi_1 = 0$$



REGIME 1 !

$$a_D(0) = -\frac{n\alpha_2}{\sqrt{2}}$$

In Perturbation theory the strings **DO NOT** realize the Coulomb vacuum !

STRING TENSION IN PERTURBATION THEORY

$$u_n = \sqrt{2n}$$

$$\frac{T_n}{2\pi} = \underline{\sqrt{2n}} - \frac{1}{2}n^2\alpha_2^2$$

BPS String tension

BULK-LIKE STRINGS!

REGIME OF VALIDITY OF PERTURBATION THEORY

$$\alpha_2 \sim \frac{1}{\sqrt{n}}$$

With increasing flux, the range over which PT is valid decreases !

This was also visible via numerical findings from before !

$$\frac{d^2T_n}{dn^2} = -\alpha_2^2 \quad < 0$$

STABLE **GIANT ROTATIONALLY SYMMETRIC** STRINGS !

SUMMARY OF PERTURBATIVE ANALYSIS

$$a_D(u) = \alpha_2\left(-\frac{n}{\sqrt{2}} + \frac{u^2}{2\sqrt{2}}\right) + \alpha_2\alpha_3\left(\frac{9}{8}n^2 - \frac{3}{4}n^2u^2 + \frac{3}{32}u^4\right)$$

$$\frac{T_n}{2\pi} = \sqrt{2}n - \frac{1}{2}n^2\alpha_2^2 + \frac{1}{\sqrt{2}}n^3\alpha_2^2\alpha_3$$

REGIME OF VALIDITY OF PERTURBATION THEORY

$$|\alpha_2| \sim \frac{1}{\sqrt{n}} \qquad |\alpha_3| \sim \frac{1}{n}$$

1. BULK-LIKE STRINGS

2. STABLE ROTATIONALLY SYMMETRIC

3. SHRINKING RANGE OF VALIDITY WITH INCREASING FLUX

GO BEYOND PT !

DOMAIN WALLS IN $\mathcal{N} = 1$

$$\mathcal{W} = \sqrt{2} A_D M \widetilde{M} + A_D + \alpha_2 A_D^2 + \alpha_3 A_D^3$$

TAXONOMY OF BPS DOMAIN WALLS

$$\{Q_\alpha, Q_\beta\} \sim \sigma_{\alpha\beta}^{[\mu\nu]} Z_{[\mu\nu]}$$

$$C_+ \leftrightarrow C_-$$

Generically
Always present

$$C_+ \leftrightarrow H \qquad C_- \leftrightarrow H$$

Pertinent for Domain Wall Strings

$$1 + 2\alpha_2 A_D + 3\alpha_3 A_D^2 = 0$$

$$\frac{-\alpha_2 \pm \sqrt{D}}{3\alpha_3}$$

$$D \equiv \alpha_2^2 - 3\alpha_3$$

$$D > 0$$

Real roots

$$D = 0$$

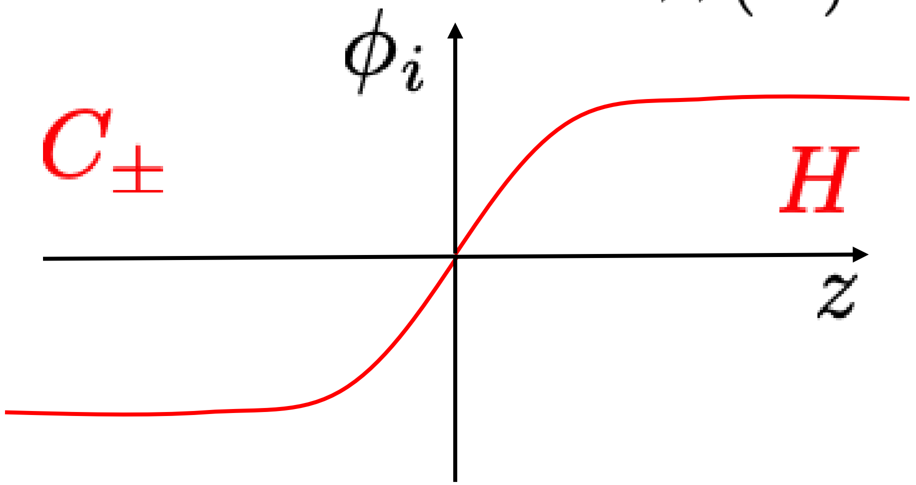
Coincident Coulomb vacua

$$D < 0$$

Imaginary roots

BPS EQUATIONS

$$\mathcal{W}(H) = 0$$



$$\phi_i = (m, \tilde{m}, a_D)$$

$$\partial_z \overline{\phi_i} = e^{i\eta} \frac{\partial \mathcal{W}}{\partial \phi_i}$$

$$|m| = |\tilde{m}|$$

$$\sigma_\pm = 2|\mathcal{W}(C_\pm)|$$

BPS Domain Wall tension if it exists

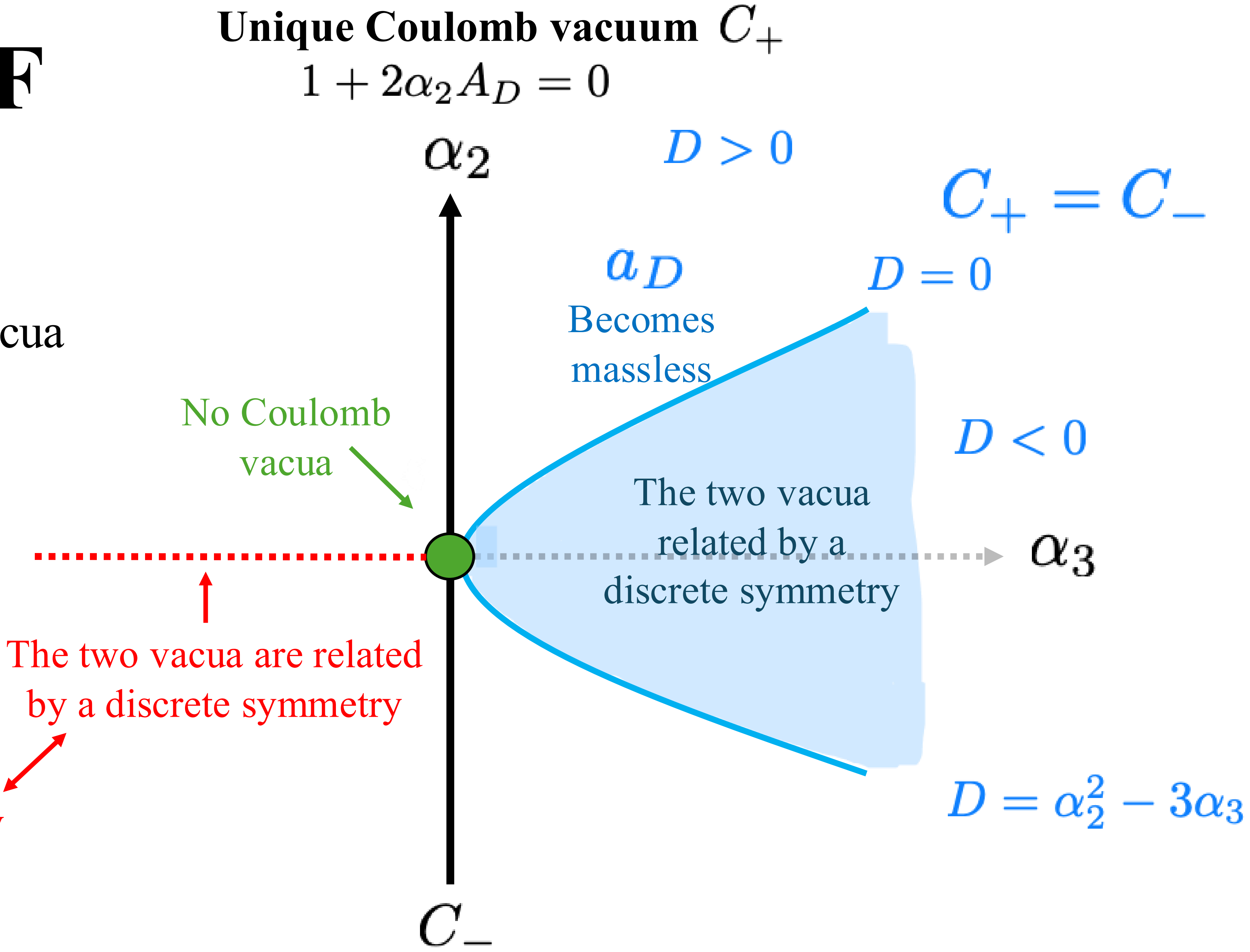
BPS EQUATIONS ARE OVER CONSTRAINED AND MAY OR MAY NOT HAVE SOLUTIONS !

SPACE OF VACUA

Generically, three vacua

$H \quad C_\pm$

Always Present !



Enhanced $\mathbb{Z}_2$ symmetry
on the α_3 axis

WALL CROSSING

[Cecotti, Vafa,…]

[Cecotti, Fendley, Intriligator, Vafa,…]

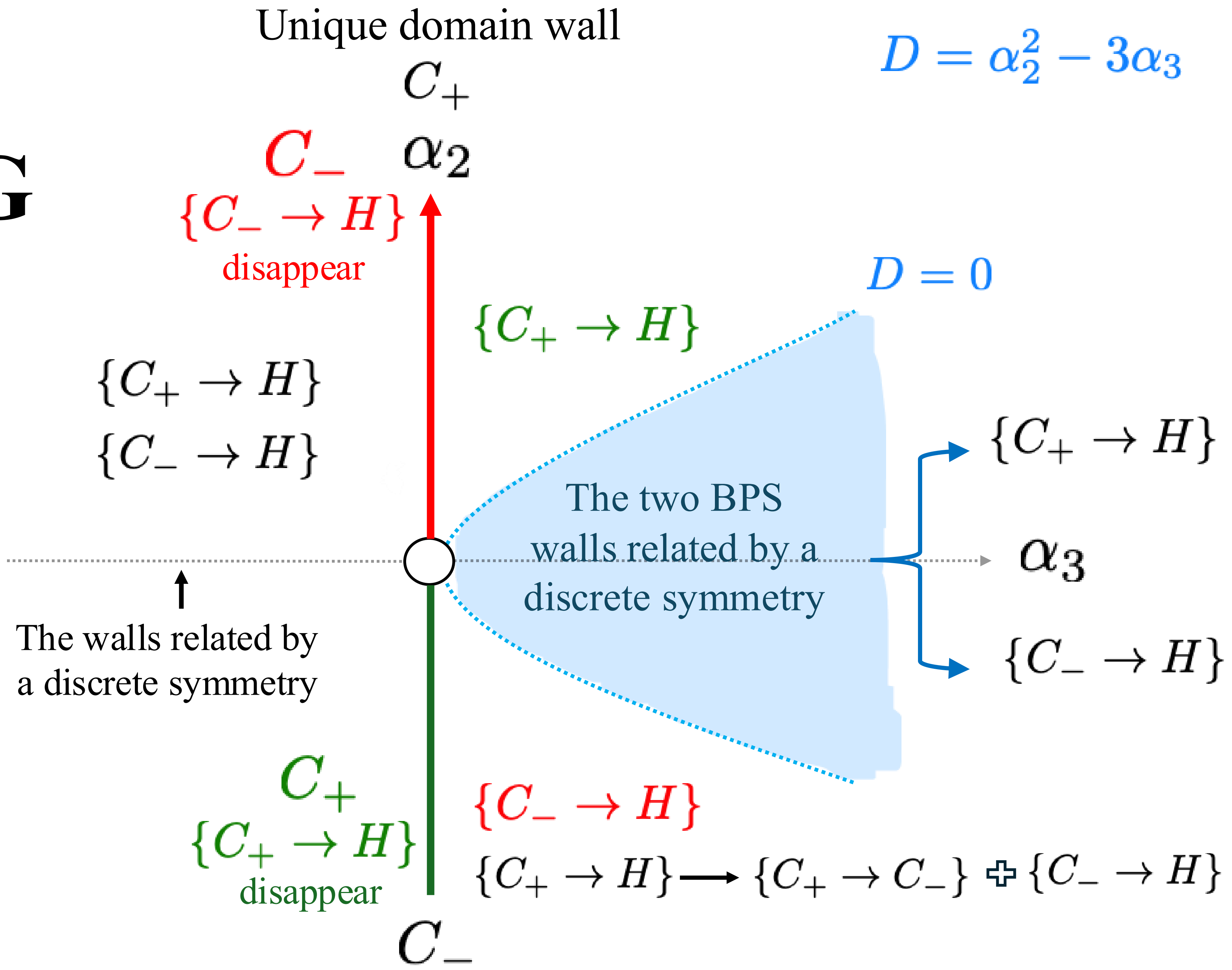
$$C_+ \leftrightarrow C_-$$

Generically
Always present

Here we map BPS domain walls

$\{C_+ \to H\}$

$\{C_- \to H\}$

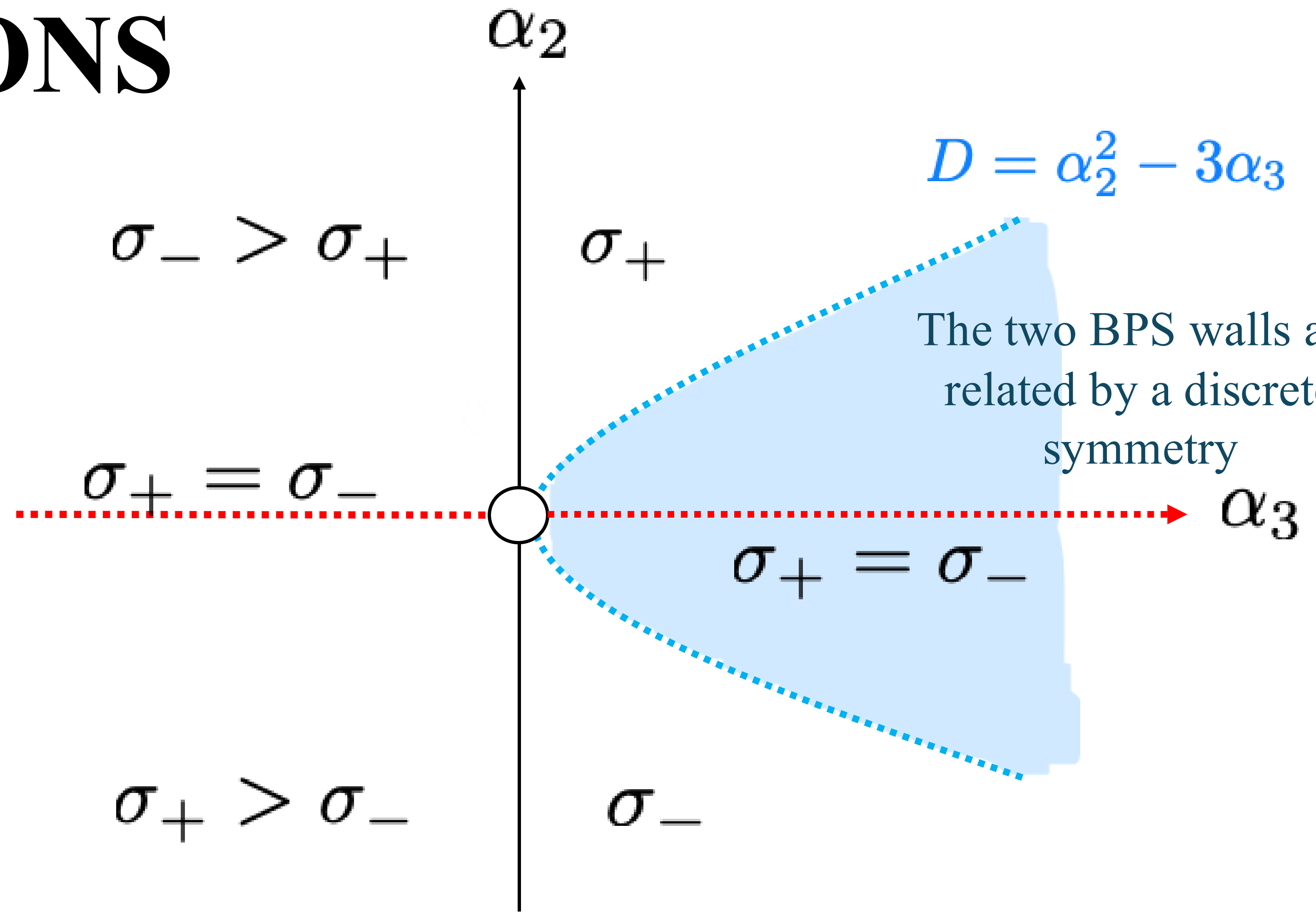


BPS WALL TENSIONS

$\{C_+ \to H\}$ σ_+

$\{C_- \to H\}$ σ_-

Walls are related by a discrete symmetry



HIERARCHY OF BPS DOMAIN WALL TENSIONS WHEN THEY EXIST

STRINGS BEYOND PERTURBATION THEORY

$$\mathcal{W} = \sqrt{2} A_D M \widetilde{M} + A_D + \alpha_2 A_D^2 + \alpha_3 A_D^3$$

FROM DOMAIN WALLS TO STRINGS

REGIME 1 (PERTURBATION THEORY)

$$|\alpha_2| \lesssim \frac{1}{\sqrt{n}} \quad |\alpha_3| \lesssim \frac{1}{n}$$

Small values of

$\alpha_2, \ \alpha_3$

Taxonomy of BPS domain walls governs the phases of giant non-BPS strings !

REGIME 2 (DOMAIN WALL REGIME)

Strings realize the Coulomb vacuum at the origin.

In the Infinite flux limit, giant strings realize the Domain Wall phase

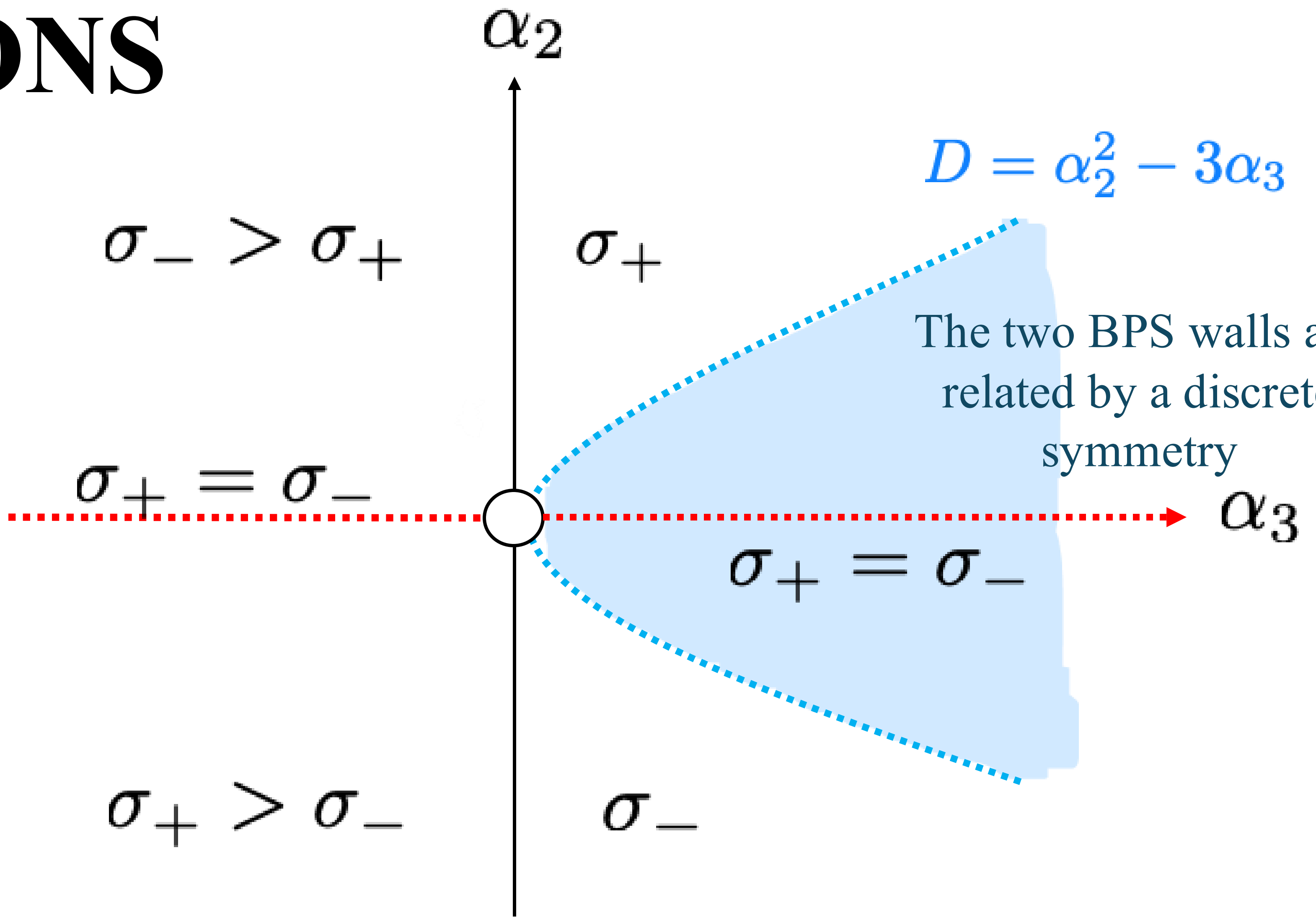
Interested in the **GROUND STATE / LOWEST ENERGY** string. $T_n \sim (n\sigma_\pm)^{2/3}$

BPS WALL TENSIONS

$\{C_+ \to H\}$ σ_+

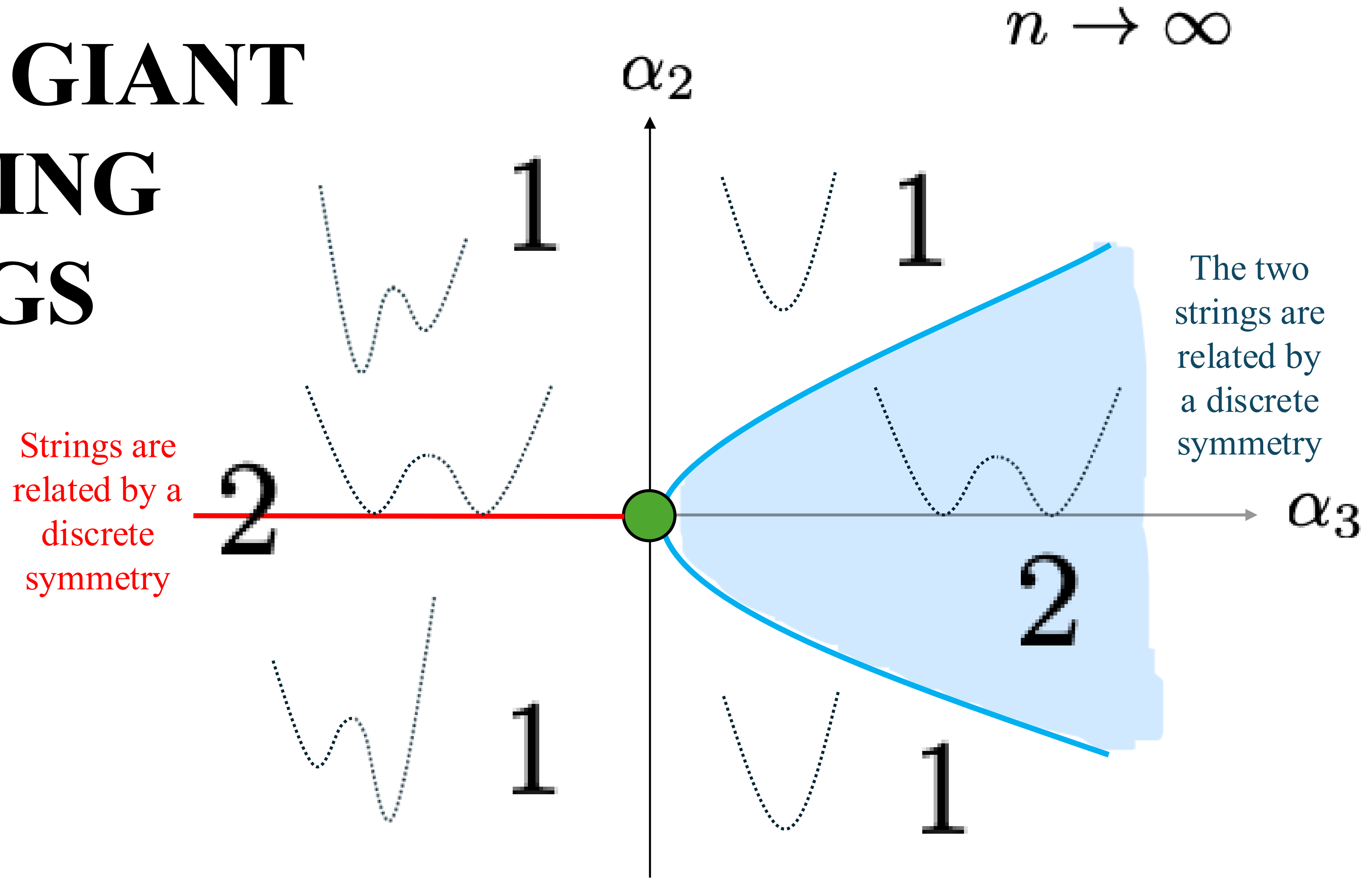
$\{C_- \to H\}$ σ_-

Walls are related by a discrete symmetry



HIERARCHY OF BPS DOMAIN WALL TENSIONS WHEN THEY EXIST

PHASES OF GIANT CONFINING STRINGS



MULTICRITICAL POINT

SECOND ORDER LINE

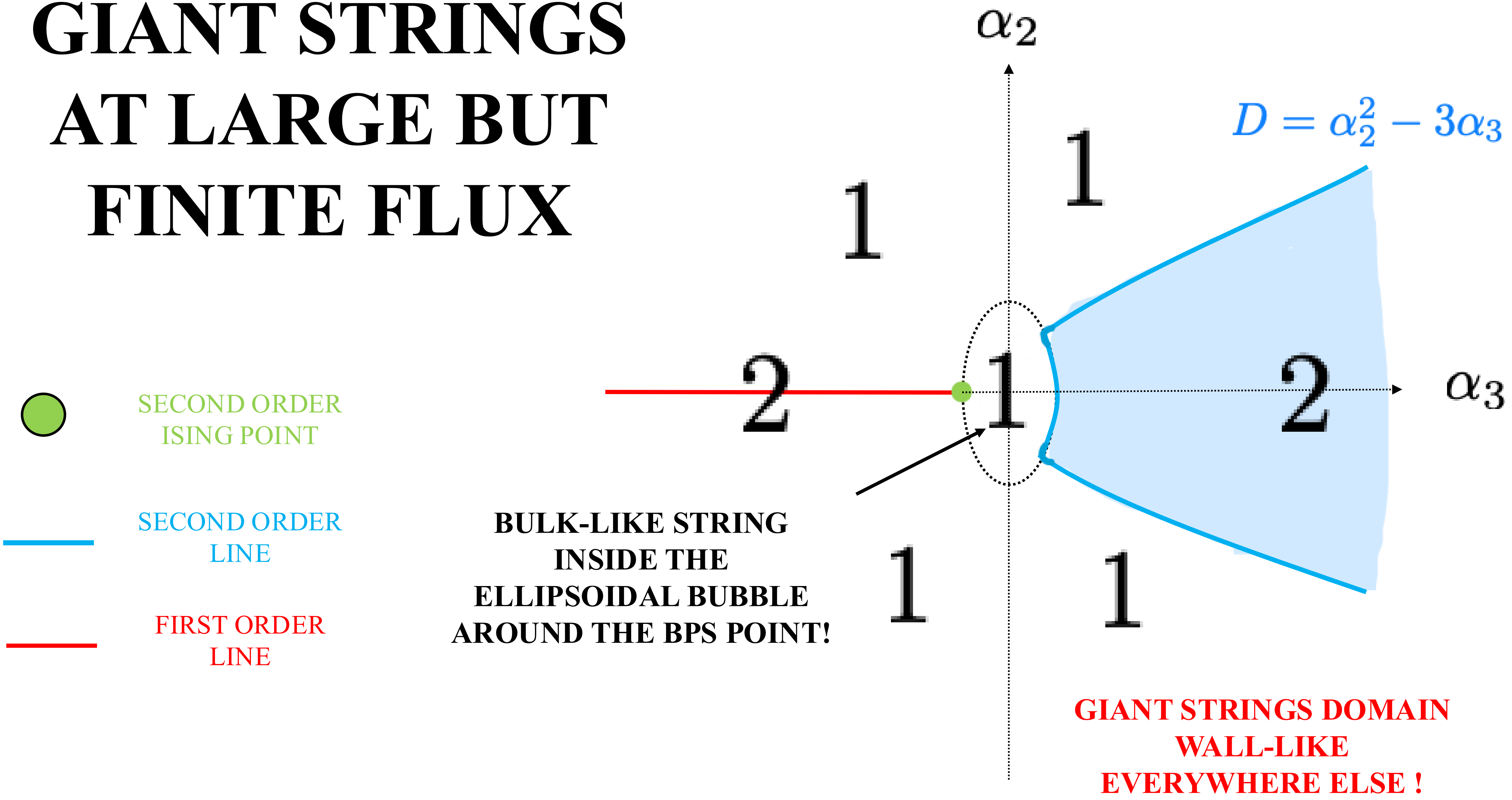
FIRST ORDER LINE

BULK PHASE ONLY AT THE BPS POINT – MULTICRITICAL POINT !

$$T_n \sim (n\sigma_\pm)^{2/3}$$

GIANT STRINGS IN THE DOMAIN WALL PHASE EVERYWHERE ELSE !

GIANT STRINGS AT LARGE BUT FINITE FLUX



ROTATIONALLY SYMMETRIC BOUND STATES

Compare T_n with nT_1

STABLE FOR ALL PARAMETER VALUES !

Numerics at small flux -- **stable** for all values of α_2, α_3

Stability of giant strings - convexity of tension at large n

$$T_n \sim (n\sigma_\pm)^{2/3}$$

Domain Wall phase

$$\frac{T_n}{2\pi} = \sqrt{2}n - \frac{1}{2}n^2\alpha_2^2 + \frac{1}{\sqrt{2}}n^3\alpha_2^2\alpha_3$$

Perturbation Theory : Bulk phase

$$\boxed{\frac{d^2T_n}{dn^2} < 0}$$

SUMMARY

1. **Minimal** Abelian Higgs models (AHMs) **do not** serve as compelling dual descriptions of YM flux tubes !

What is the lightest fluctuation mode around a string background ?

Do separated strings attract or repel ?

2. Deformed SW provides a setting where dual superconductivity can be made precise. The minimal AHMs serve as effective toy models to study strings in this richer setting.

3. The large flux limit is a tractable limit and the physics of the giant strings gets simplified.

Domain Wall Phase – In this phase the behavior of non-BPS strings mimics the behavior of BPS Domain Walls !

Bulk phase

NEXT STEPS !

Study **fluctuations around the confining strings** backgrounds in the SW dual AHM

Is the axion lighter or the dilaton – study this question as a function of the deformation parameter m ?

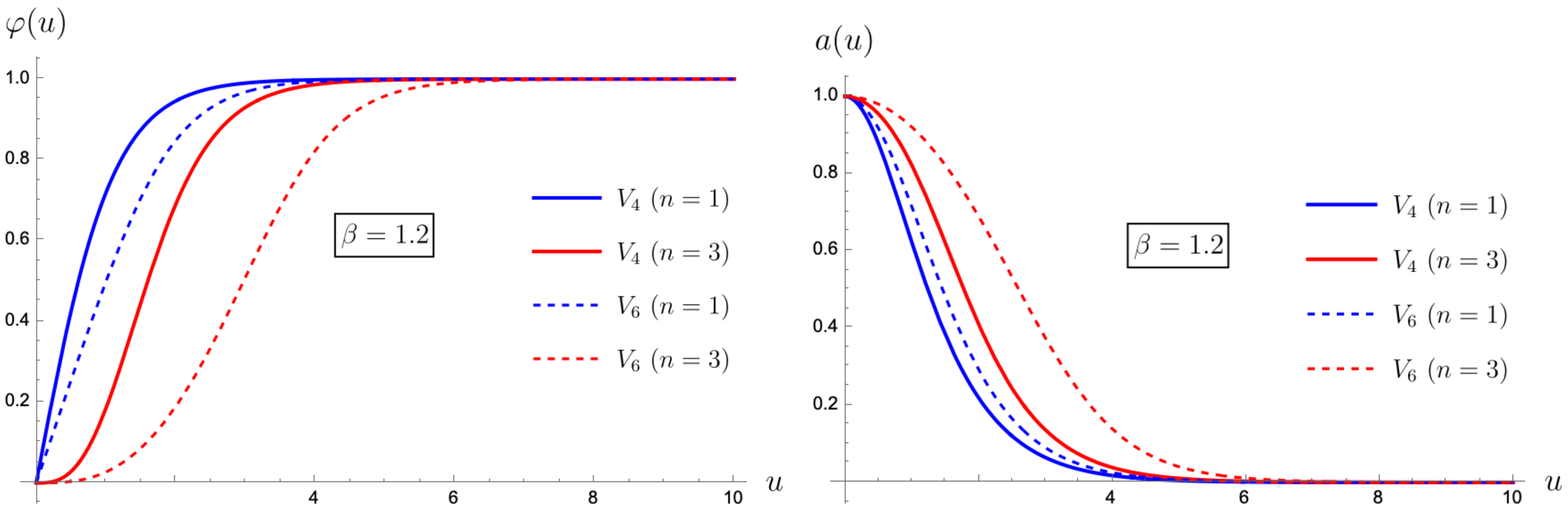
What conclusions can we draw about strings in Seiberg-Witten theory ?

Study fermionic fluctuations around the string background ? Is a fermion the lightest mode ?

Adjoint QCD

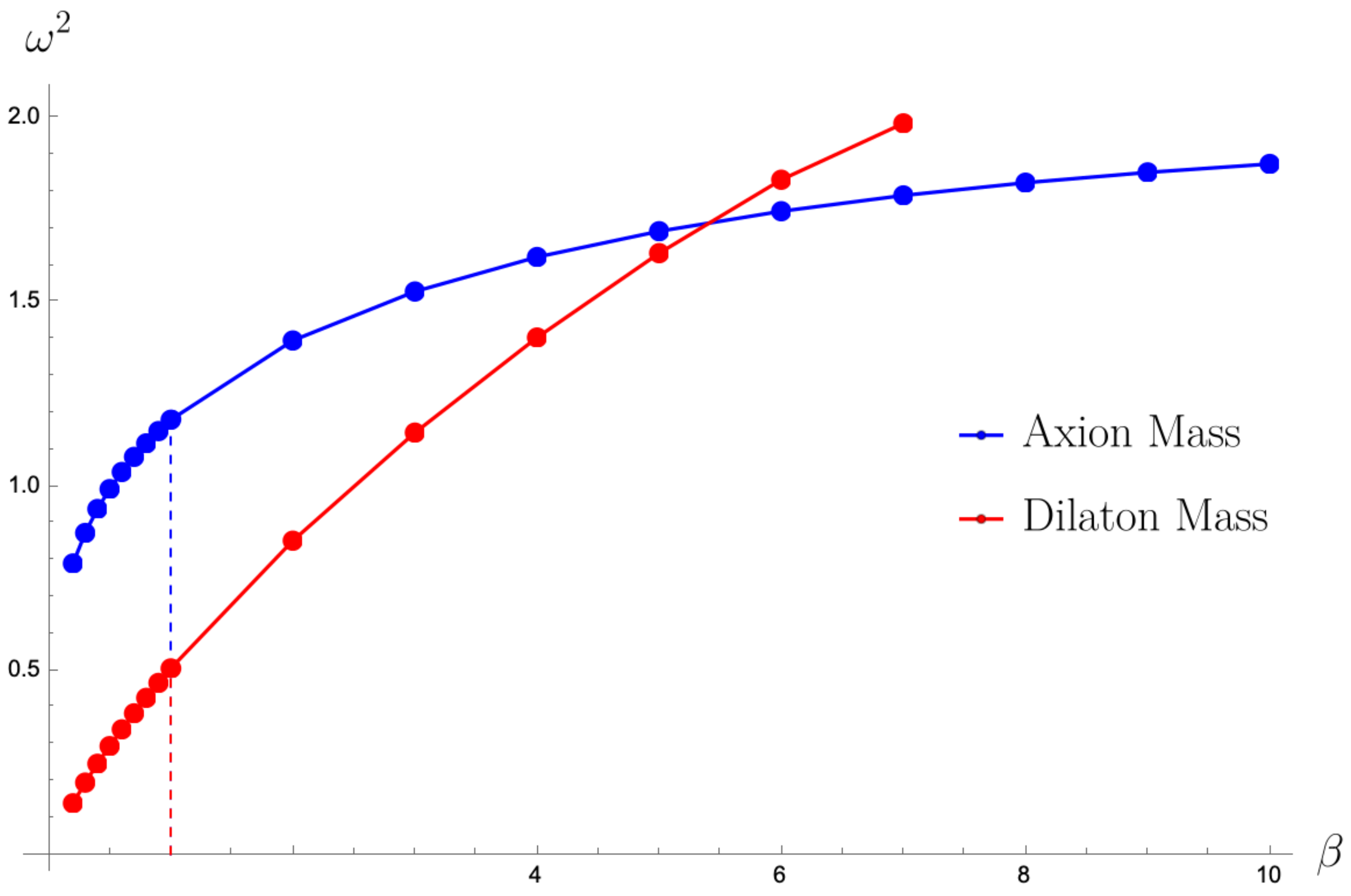
THANK YOU !

Comparing the conventional AHM and Degenerate models qualitatively



The degenerate model is more dilute for same value of β

MASS HIERARCHY : DEGENERATE MODEL



Type – I

Type - II

Axion is <u>again</u> the lightest massive excitation in the type-II regime, where fundamental strings repel !

LINDE WEINBERG BOUND

- If λ gets very small, then there are quantum effects that become important and completely destroy the Higgs phase. There is a lower bound on β that comes from quantum corrections.

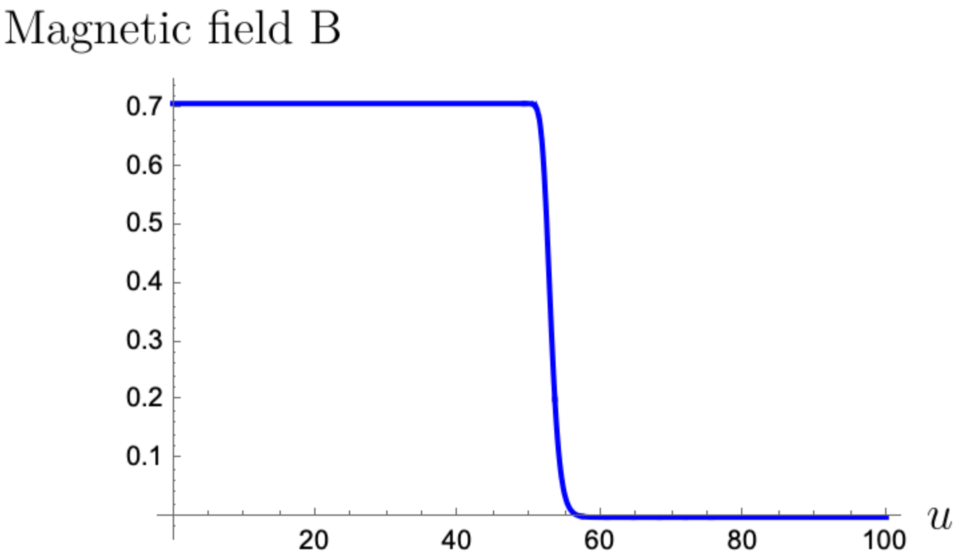
- There is no problem in cutting off the analysis at the point where the quantum corrections become important

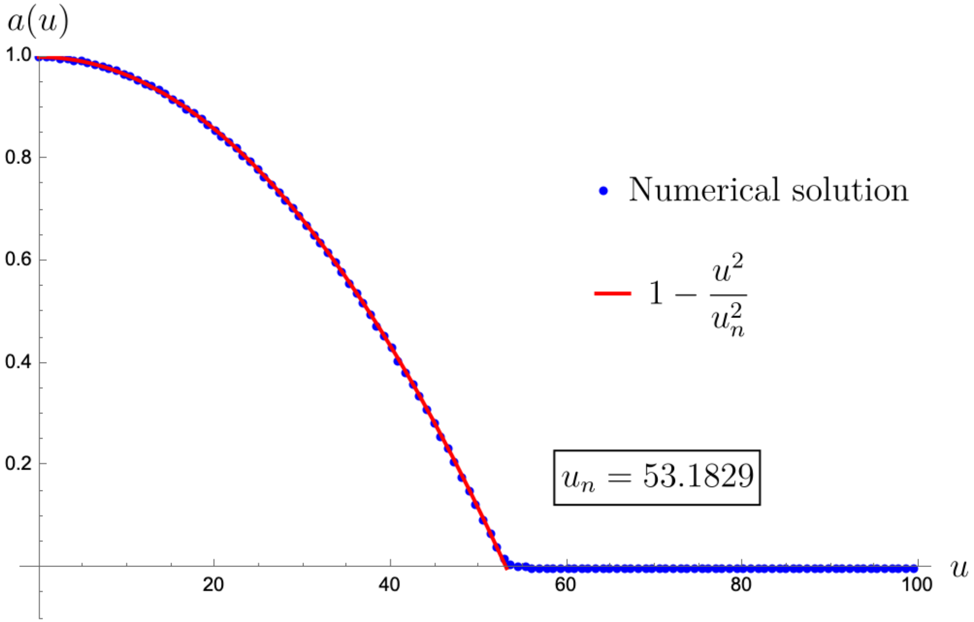
$$\lambda \gtrsim e^4 \qquad\qquad \beta \gtrsim e^2$$

WHY IS THE DILATON LIGHTER ?

- The region that opens up in the middles is a Coulomb region with a nearly massless photon and the whole model has a magnetic flux symmetry. The photon that becomes massless in this large bucket is the GB for that symmetry, Precisely the mode of wiggling the magnetic field is the Dilaton – it preserves all the symmetries of the string and it is natural that the mode which is the Goldstone mode of that system should be the lightest modes in this regime.
- Free Maxwell has 2 GBs. In the full AHM, there's only the mag symmetry. The thing that breaks the degeneracy b/w them is the interactions w/ the charged Higgs. If one had to guess, which one survived or which one is lighter, it is maybe more natural to guess the pseudo GB. This is a-posteriori reasoning !

UNIFORM MAGNETIC FIELD INSIDE CORE





STRING TENSION (CONTINUED)

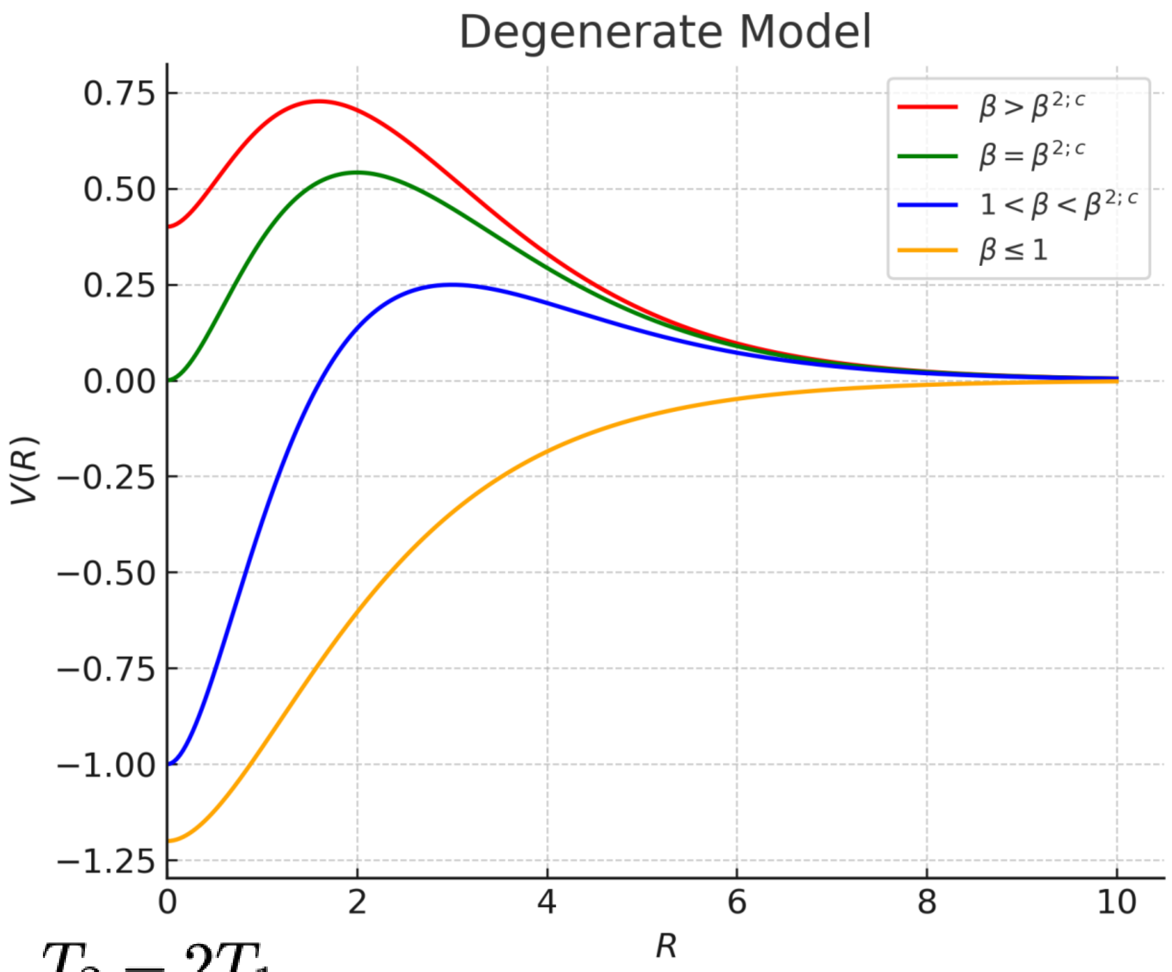
- Fact for all n : BPS point $\frac{T_n}{2\pi} = \sqrt{2}n$
- To leading order in n marginal BPS-like behavior – we did not account for physics from the boundary
- Break marginality by producing large n solution everywhere

Surface tension

$$\frac{T_n}{2\pi} = \underbrace{\sqrt{2\beta}n}_{\text{Core}} + \underbrace{\sigma u_n}_{\text{Boundary}}$$

$$\sigma > 0 \leftrightarrow \beta < 1, \quad \sigma < 0 \leftrightarrow \beta > 1, \quad \sigma = 0 \leftrightarrow \beta = 1$$

INTERACTION POTENTIAL $V(R)$ AS A FUNCTION OF R



$T_2 - 2T_1$

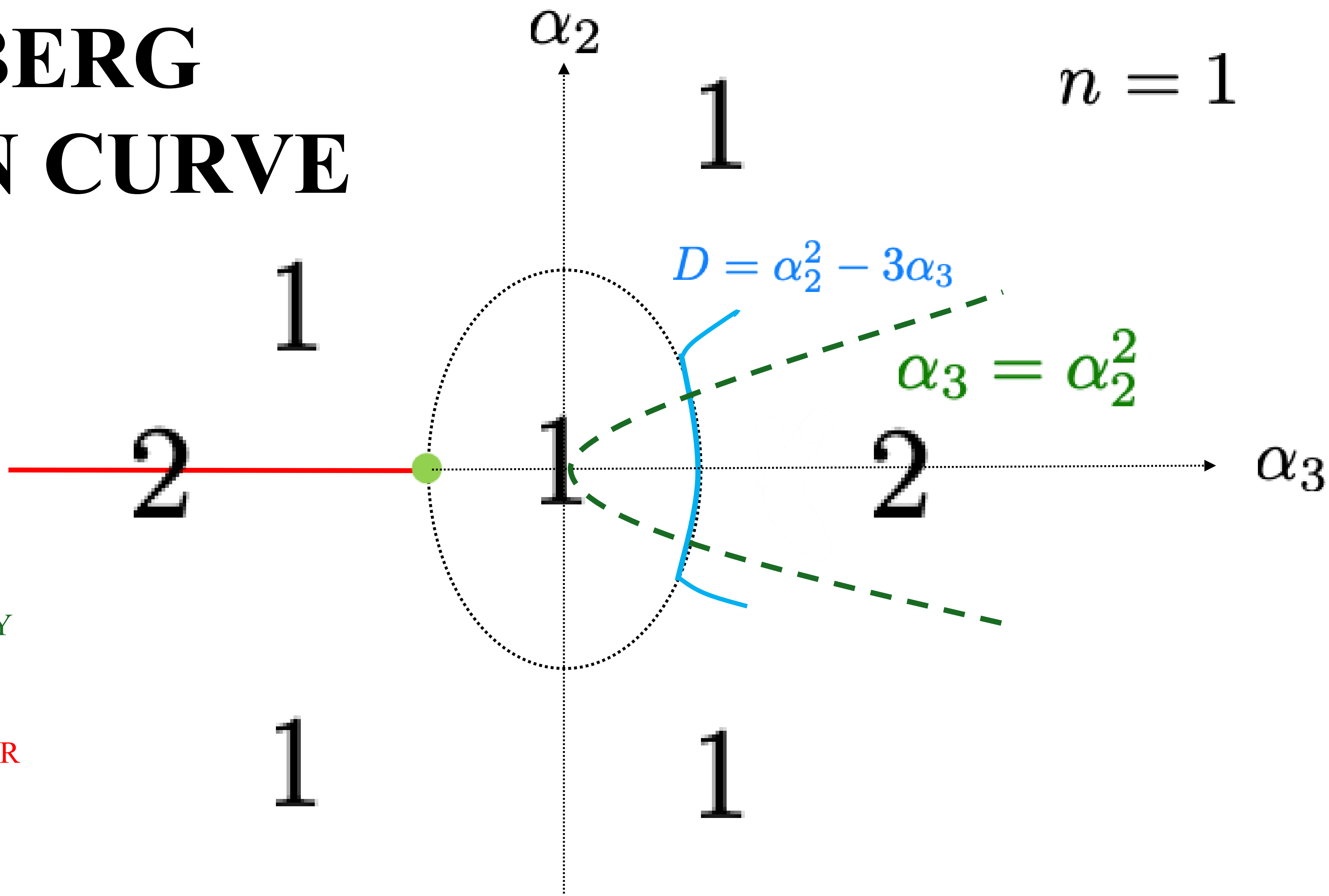
SCHEMATIC CARTOON PREDICTION

Strings repel at infinity but can form rotationally symmetric bound states.

SEIBERG WITTEN CURVE

$\alpha_2 \sim m$

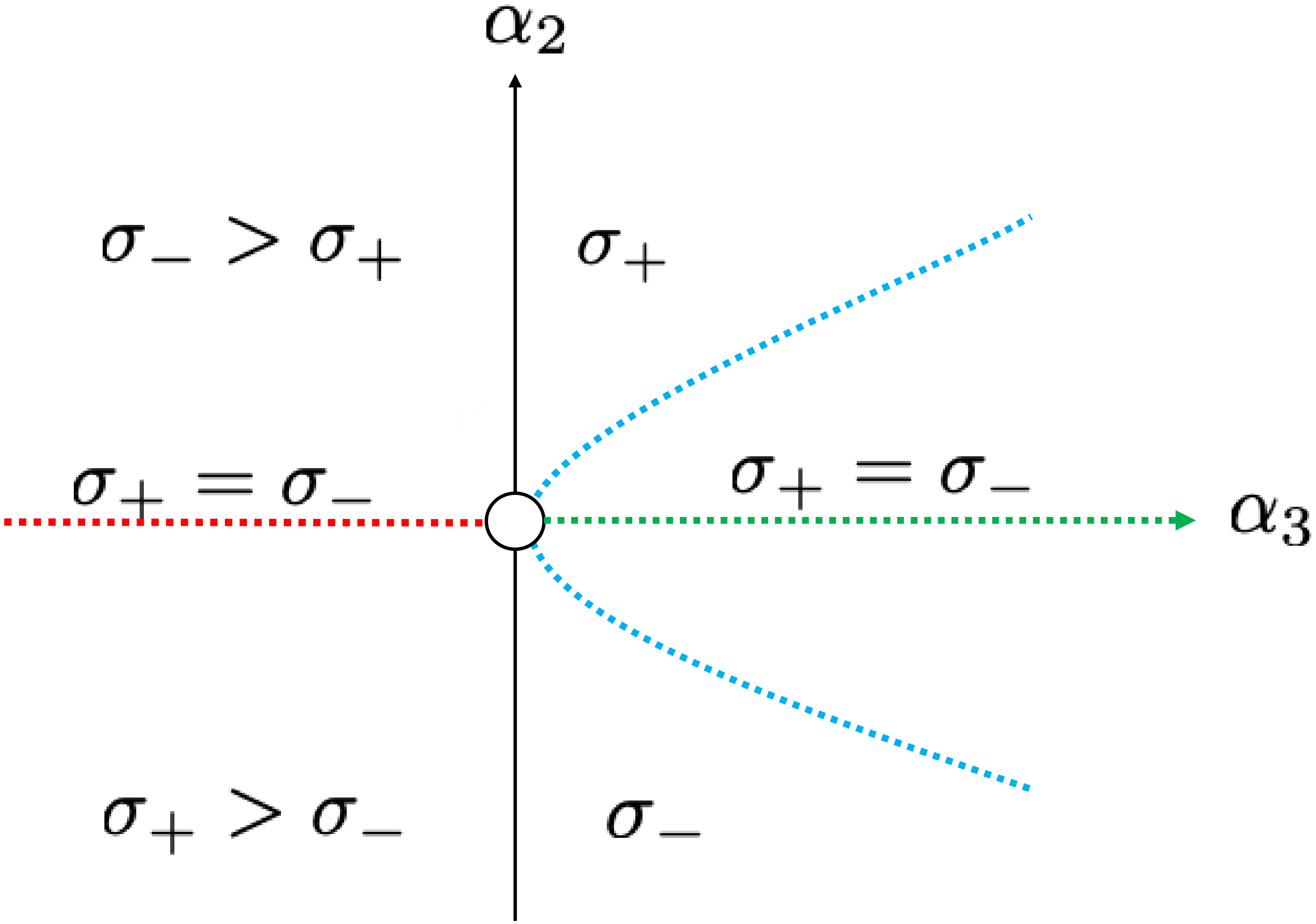
$\alpha_3 \sim m^2$



SEIBERG WITTEN THEORY CURVE

SECOND ORDER LINE

FIRST ORDER LINE



α_2
$\sigma_- > \sigma_+$
σ_+
$\sigma_+ = \sigma_-$
$\sigma_+ = \sigma_-$
α_3
$\sigma_+ > \sigma_-$
σ_-

TRASH SLIDES

COUNTING VACUA

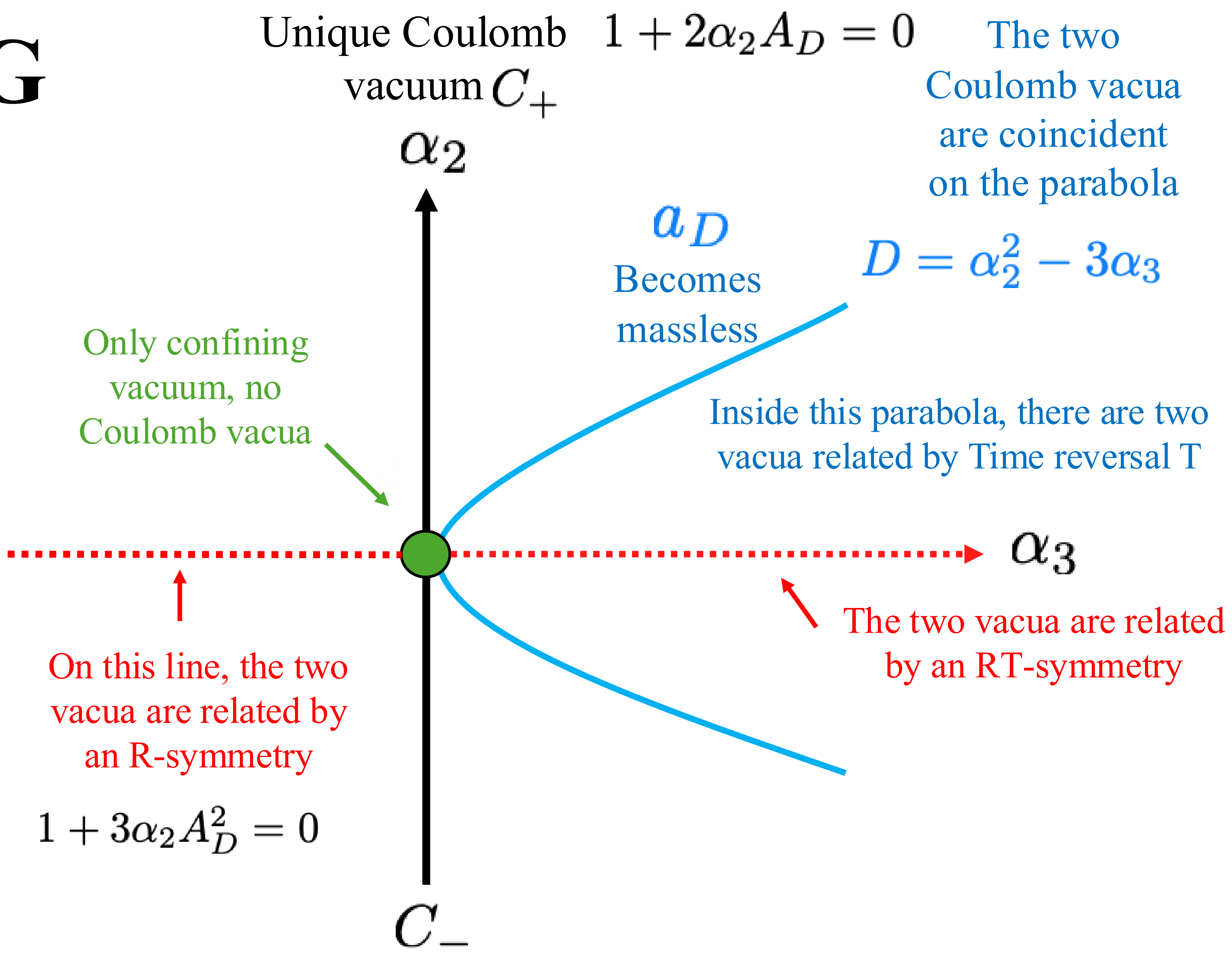
$$T : A_D \to A_D$$

$$R : A_D \to -A_D$$

Enhanced $\mathbb{Z}_2$ R-symmetry on the locus

$$\alpha_2 = 0 \neq \alpha_3$$

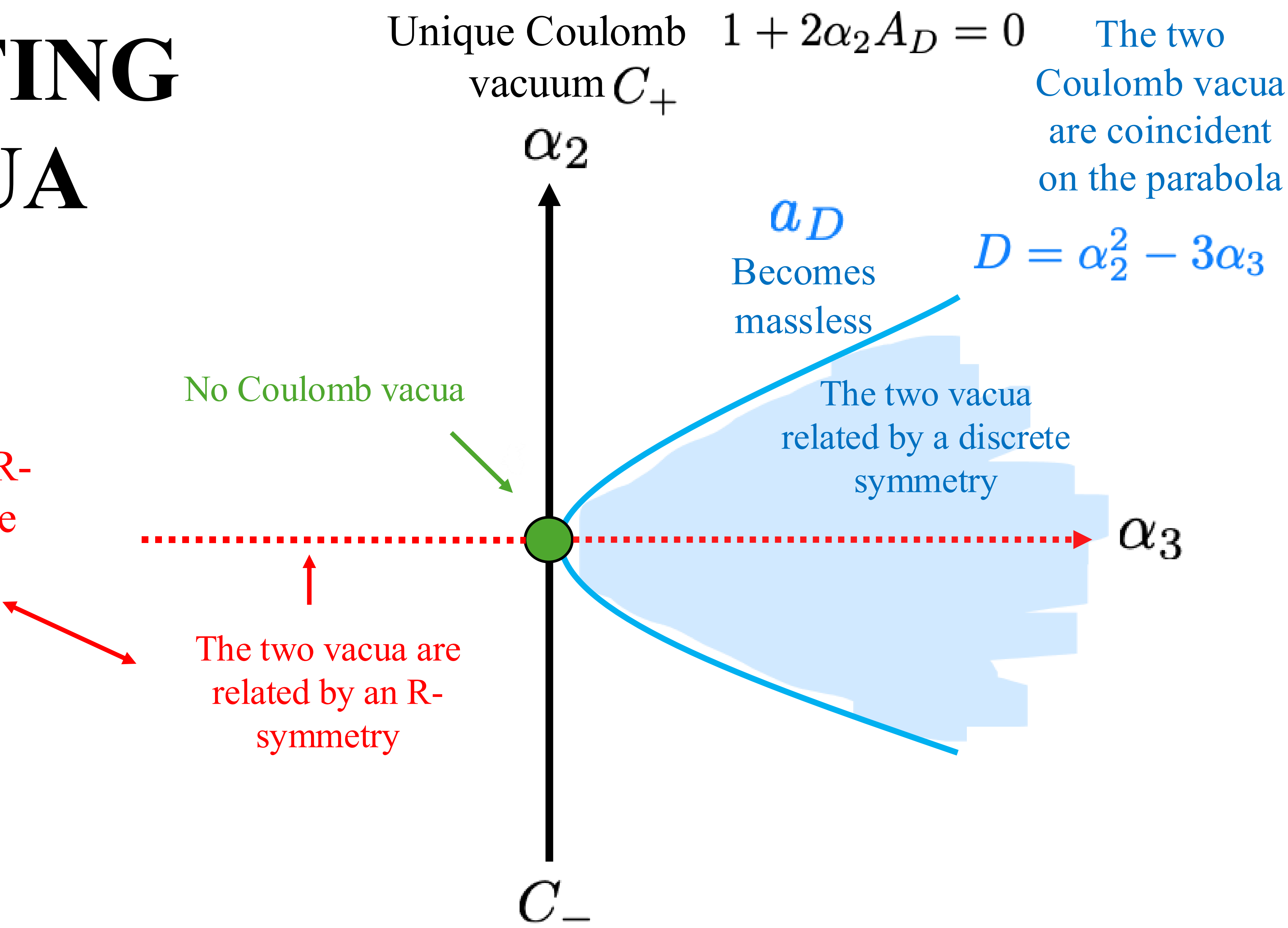
The confining vacuum is always present !



COUNTING VACUA

Enhanced $\mathbb{Z}_2$ R-symmetry on the α_3 axis

H is always present !



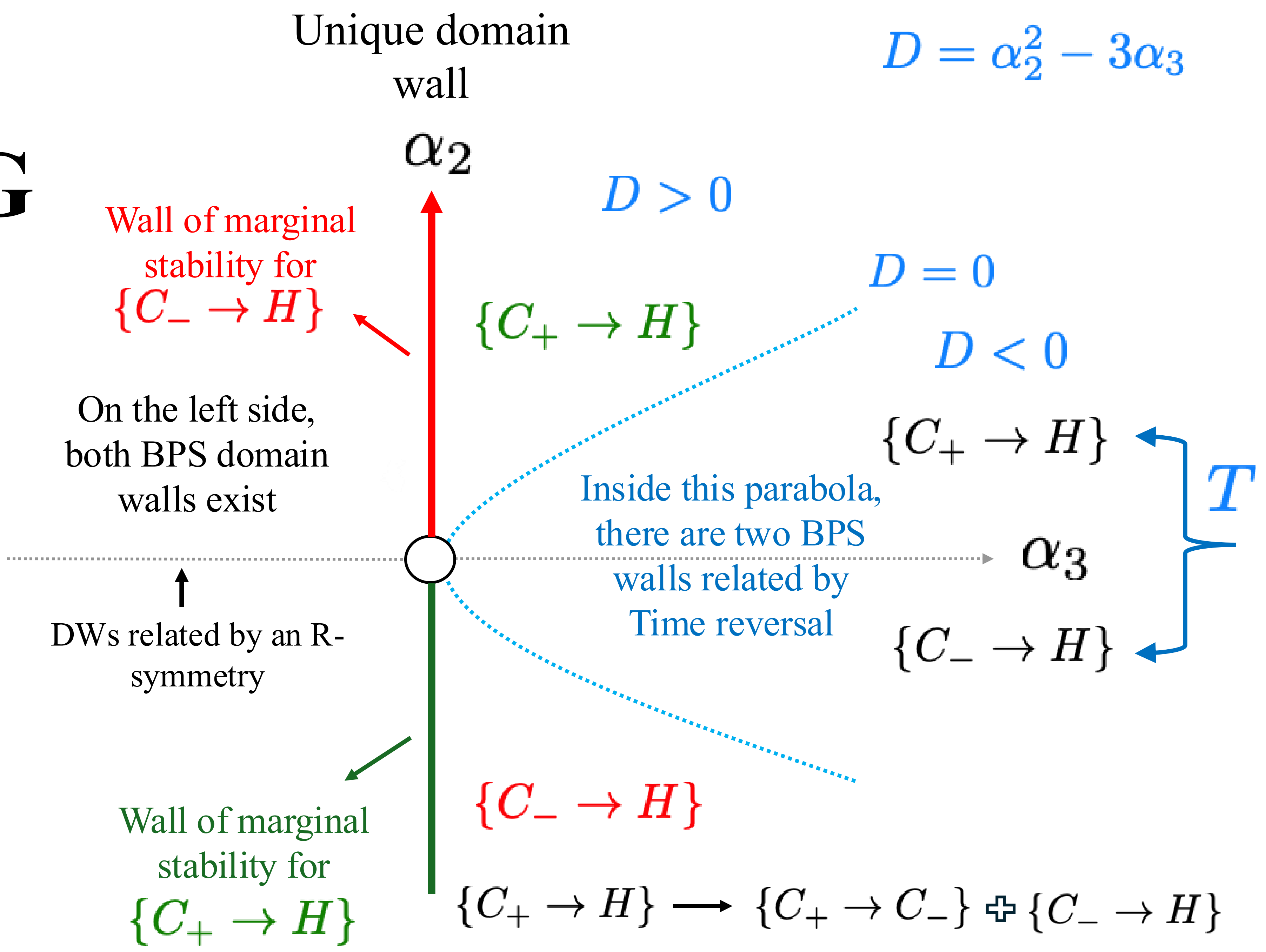
WALL CROSSING

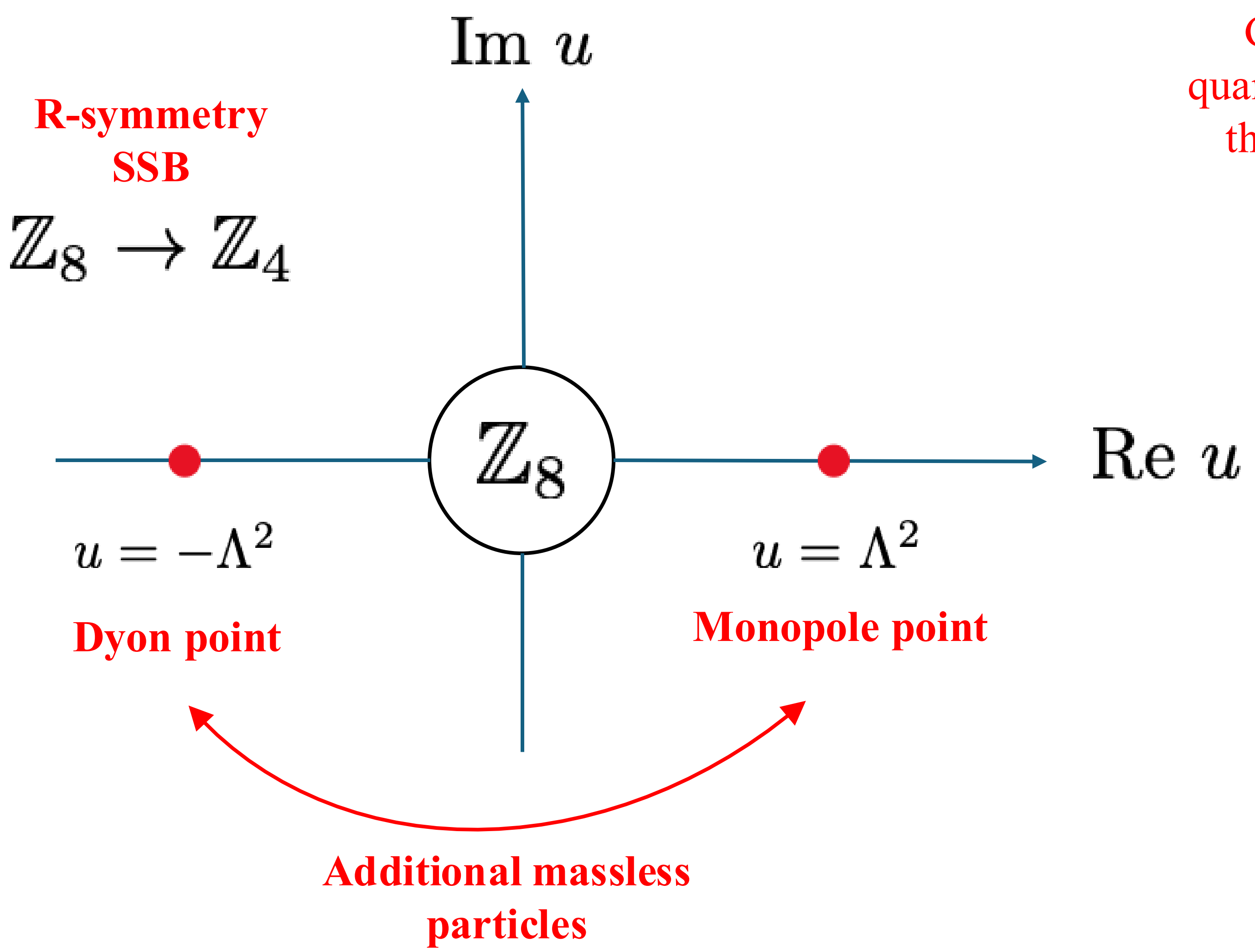
[Cecotti, Fendley, Intriligator, Vafa,…]

$\{C_+ \to H\}$

$\{C_- \to H\}$

Walls do not break T spontaneously in the $D > 0$ regime





Gauge-invariant quantity parametrizing the space of vacua ← $u = \mathrm{Tr}\ \phi^2$

$u \sim a^2$

[Seiberg, Witten '94]

As we vary u the low energy physics changes smoothly but undergoes electric magnetic duality transformations

Naively, expect confining theory at low energies – IR is instead described by Coulomb phase !

Single $\mathcal{N} = 2$ $U(1)$ Vector multiplet at generic u

THEORY AT THE MONOPOLE POINT

$$L = \int d^4\theta \left[\frac{1}{e^2} \overline{A}_D A_D + M e^{-2V} \overline{M} + \widetilde{M} e^{2V} \overline{\widetilde{M}} \right]$$

$e \sim 1/g$ Low energy $U(1)$ gauge coupling

$$+ \int d^2\theta \left[\frac{1}{4e^2} W^\alpha W_\alpha + \sqrt{2} A_D M \widetilde{M} \right] + (\text{h.c})$$

$\mathcal{N} = 2$

SQED

Weakly coupled theory of monopoles and photons

$\mathcal{N} = 2$

Vector multiplet

$\mathcal{N} = 1$

Chiral multiplets

$\mathcal{N} = 2$

Hypermultiplet (Monopole fields)

0 $\quad A_D = (a_D, \psi_\alpha, F)$

$V = (a_\mu, \lambda_\alpha, D)$

$\widetilde{M} = (\widetilde{m}, \psi_-, F_-)$ $\quad +1$

$M = (m, \psi_+, F_+)$ $\quad -1$

$\mathcal{N} = 1$ Vector multiplet

SUPERPOTENTIAL

Truncated superpotential

$$\mathcal{W} = \underbrace{\sqrt{2}A_D M \widetilde{M}}_{\text{Undeformed}} + \underbrace{\xi A_D + \alpha_2 A_D^2 + \alpha_3 A_D^3}_{\text{Deformation}}$$

Consider only renormalizable terms.

We gain more control by staying close to the monopole point and hence consider a small m expansion. This is also required so as to not go beyond the confines of our IR effective theory.

A small m expansions allows us to perform a Taylor expansion of $U(A_D)$ and justifies considering a truncated model. For our purposes, we consider only renormalizable terms in the superpotential.

Consider superpotential for **all values** of α_2 , α_3 and later scale back to SW solution.

$$\xi = 4im\Lambda \qquad \alpha_2 = -\frac{m}{4} \qquad \alpha_3 = -i\frac{m}{64\Lambda}$$

SCALING LIMIT

$$\mathcal{W} = \sqrt{2} A_D M \widetilde{M} + A_D + \alpha_2 A_D^2 + \alpha_3 A_D^3$$

Time Reversal symmetry

$$\xi, \alpha_2, \alpha_3 \in \mathbb{R}$$

Set units

$$\xi = 1$$

Compare with SW coefficients

$$m\Lambda = \frac{1}{4} \qquad \alpha_2 = \frac{m}{4} \qquad \alpha_3 = \frac{m^2}{16}$$

Scaling Limit

$$\Lambda \to \infty \qquad m \to 0$$

To stay close to the monopole point and IR effective theory does not break down

PROBES OF DUALITY

- Interested in studying simple tractable models to study if these could be **viable dual descriptions** for Yang Mills confining strings

- Ask minimal non-universal questions to probe this duality

  - Do fundamental strings attract or repel ?

  - Beyond NGBs what is the lightest massive fluctuation around string backgrounds ?

- Compare results to what is known about YM flux tubes