ASPECTS OF SUPERCONDUCTING STRINGS

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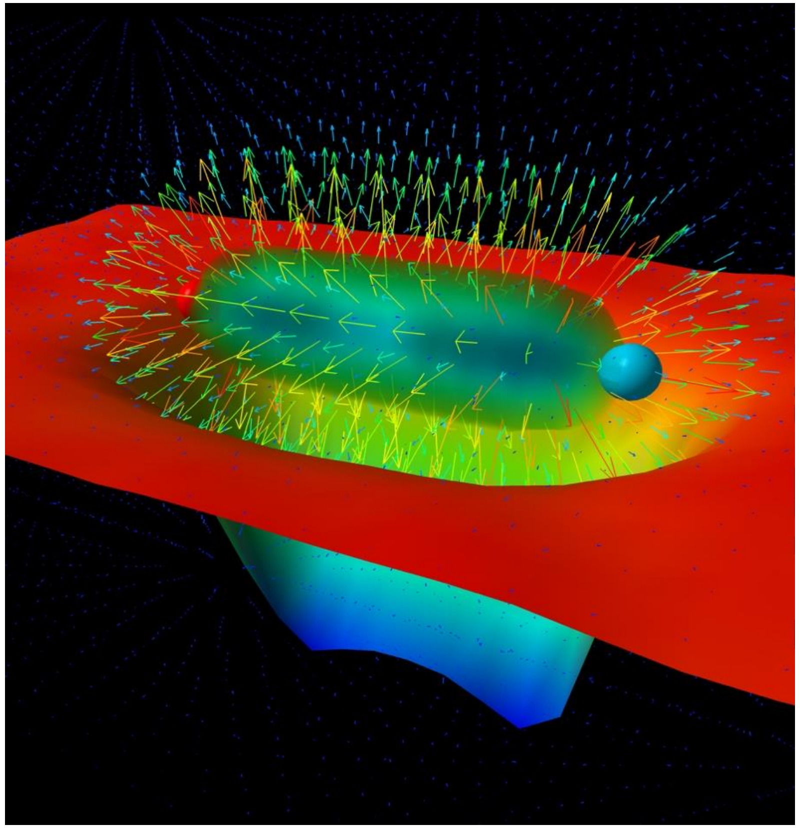
BASED ON WORK WITH THOMAS DUMITRESCU

CONFINING STRINGS



Leinweber

- Absence of free quarks and gluons
- Examples : Yang Mills (YM), QCD
- Characteristic feature : chromoelectric flux tubes
- Numerical lattice evidence
- Study properties of confining strings

UNIVERSAL ASPECTS

Is the vacuum in a
confining phase ?

Closely related – Do we have finite tension strings / flux tubes ? Is there a one-form symmetry that guarantees the existence / stability ?

Given such a string, there are two Nambu Goldstone bosons, associated with the broken translational invariance.

UNIVERSAL AND NON-UNIVERSAL ASPECTS

Is the vacuum in a confining phase ?

What is the bulk mass spectrum ?

Closely related – Do we have finite tension strings / flux tubes ? Is there a one-form symmetry that guarantees the existence / stability ?

Do fundamental strings attract / repel ? Do they form bound states ?

Given such a string, there are two Nambu Goldstone bosons, associated with the broken translational invariance.

What is the spectrum of massive excitations ?

FLUX TUBES IN PURE YANG MILLS

The vacuum is in a confining phase.

What is the bulk mass spectrum / taxonomy of light modes ?

[Athenodorou, Teper '21, ….]

$\mathbb{Z}_N^{(1)}$ one-form symmetry

Believe that confining strings in YM attract and form bound states

2 NGBs and a Pseudoscalar Axion as the lightest massive excitation

[Dubovsky, Flauger, Gorbenko' 12 ,Athenodorou, Dubovsky, Luo, Teper ' 24, ….]

FLUX TUBES IN PURE YANG MILLS

Axion is the lightest massive excitation in a setting where fundamental confining strings attract !

These are important data points, essentially numerical since pure YM is strongly coupled. We do not have an analytic understanding of answers to these minimally non universal questions.

SUPERCONDUCTING AND CONFINING STRINGS

- Stable, finite tension extended excitations also arise in simple weakly coupled tractable physical systems in 3+1 dimensions :

  - Abrikosov strings in Ginzburg Landau effective theory of superconductivity
  - Closely related Nielsen-Oleson strings in Abelian Higgs Models (AHMs)

  Abrikosov-Nielsen-Oleson (ANO) superconducting vortex strings (magnetic flux tubes)

- Simple Abelian models are good laboratories for studying properties of strings

- ‘t Hooft - Mandelstam **DUAL SUPERCONDUCTIVITY**: Confining strings (electric) EM dual to superconducting strings (magnetic)

- Dual superconductivity is made explicit in **Deformed Seiberg Witten (SW) theory** – a **dual Abelian Higgs model** with **<u>Confining Electric</u>** flux tubes

OUTLINE OF TODAY'S TALK

Properties of Strings

[Dumitrescu, AG '25]

Fluctuations of Strings

[Dumitrescu, AG, Li '25]

Superconducting Vortex strings

(Magnetic flux tubes)

SUPERCONDUCTING VORTEX STRINGS

PROPERTIES OF STRINGS

String Tension, Phases of giant strings

Do strings attract or repel ?

Do strings form bound states ?

Forthcoming [Dumitrescu, AG ‘25]

UNIVERSAL FEATURES OF AHM VARIANTS

$$\mathcal{L} = -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} - |(\partial_\mu - i a_\mu)\phi|^2 - V(|\phi|)$$ Keep general

- $U(1)$ gauge theory, single complex scalar of unit charge
- Dimensionless gauge coupling e
- $V(|\phi|)$ admits one Higgs vacuum requirement
- $U(1)_m^{(1)}$ one-form magnetic flux symmetry

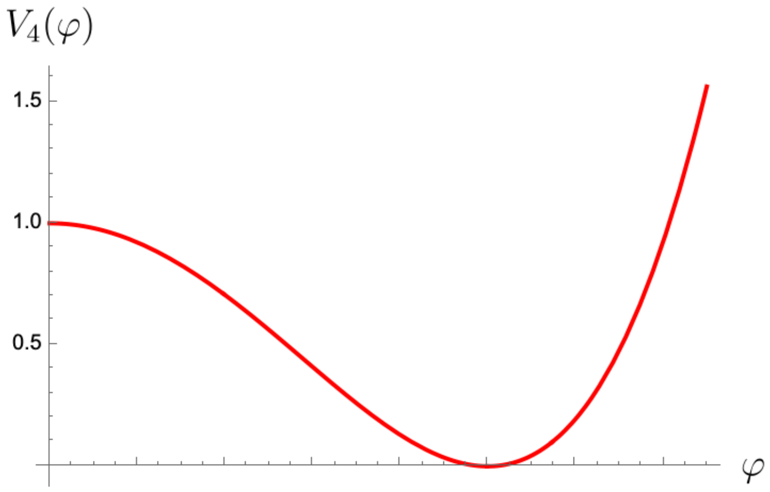
$$\Phi_B = \int_{xy-\text{plane}} dS \, B_z$$

CHOICES OF POTENTIAL

Well studied !

CONVENTIONAL MODEL

$$V_4(|\phi|) = \frac{\lambda}{2}(|\phi|^2 - v^2)^2$$



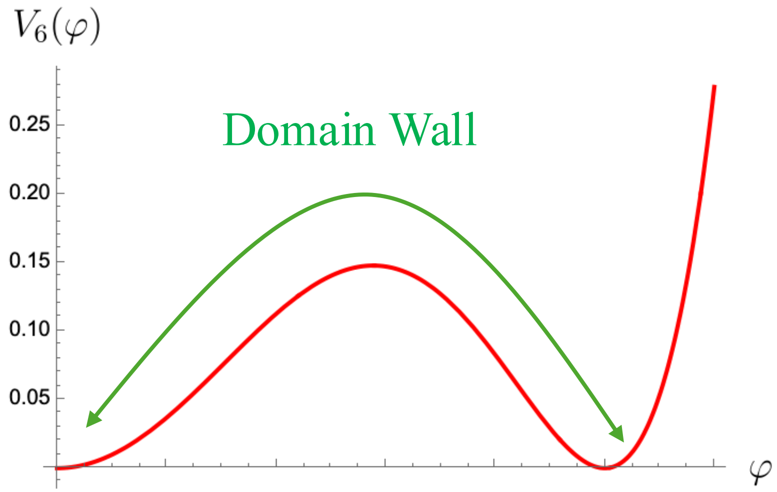
Unique Higgs vacuum

$$\lambda > 0$$

The Higgs vacuum is **trivial** and **gapped** – massive Higgs boson m_H and vector boson m_V

DEGENERATE MODEL

$$V_6(|\phi|) = \frac{\lambda}{2}|\phi|^2(|\phi|^2 - v^2)^2$$



Higgs vacuum + Coulomb vacuum

Finely tuned - disclaimer !

TYPE - I AND TYPE - II CLASSIFICATION

$\beta < 1$ Type - I $m_H < m_V$

$$\beta = \frac{m_H^2}{m_V^2} \begin{array}{l} = \lambda/e^2 \quad \text{Conventional} \\ = \lambda v^2/e^2 \quad \text{Degenerate} \end{array}$$

$\beta > 1$ Type – II $m_H > m_V$

$\beta = 1$ conventional AHM - BPS embedding into $\mathcal{N} = 1$ supersymmetry

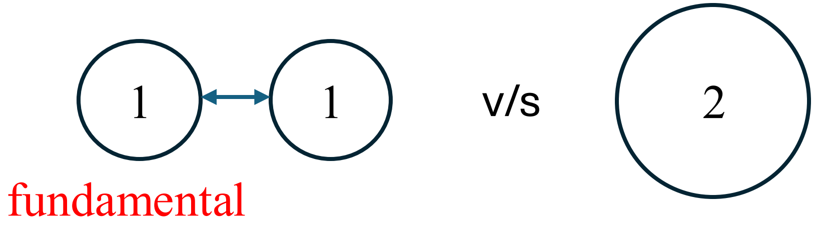
Ratio controls many, potentially correlated, qualitative properties.

Purely classical analysis

ABRIKOSOV-NIELSEN-OLESON (ANO) STRINGS

- **Magnetic** flux tubes. To exist as finite tension excitations, study the Higgs vacuum in which the one form flux symmetry is unbroken. Charged objects : **ANO strings**.

- For any flux n what is the **lowest energy** n **- string ?**

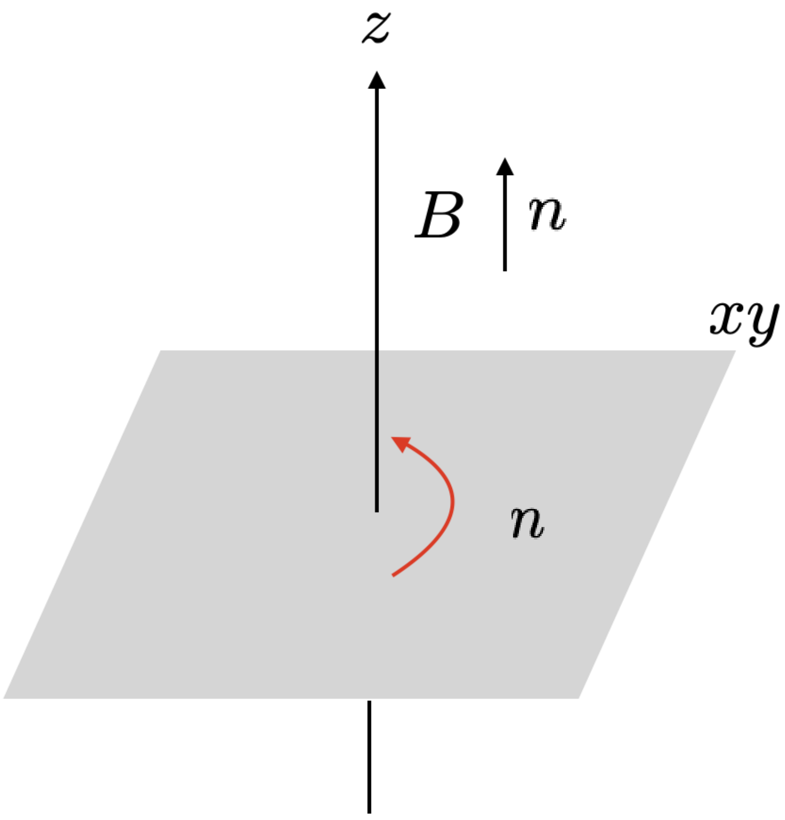


- There can be many strings but in any sector, there would be a **ground state** or lowest energy string expected to be the most stable one.

- Write down non linear equations of motion of Lagrangian and solve them subject to flux constraint. Instead solve in **analytically tractable regime.**

ROTATIONALLY SYMMETRIC STRINGS

ROTATIONALLY SYMMETRIC STRINGS



Tension OR Energy per unit length.

$$T_n$$

- Simplifying assumption

- Also Static and translationally invariant

$$\phi(x) = v\varphi(r)e^{in\theta}$$
$$a_\theta(x) = n(1 - a(r))$$

Finite energy requirement forces the winding to be equal to the flux

- These might **NOT** be the minimum energy string configurations

STRING EQUATIONS

ODEs instead of PDEs

$$\varphi''(u) + \frac{1}{u}\varphi'(u) = \frac{n^2}{u^2}\varphi^2(u)a^2(u) + \frac{1}{2}\tilde{V}'(\varphi, \beta) \rightarrow$$

Rescaled version of the potential

$$a''(u) - \frac{1}{u}a'(u) = 2a(u)\varphi^2(u)$$

Regularity and finite tension requirements for the boundary conditions

$$\underline{\varphi(0) = 0 \qquad a(0) = 1} \qquad \underline{\varphi(\infty) = 1 \qquad a(\infty) = 0}$$

(vacuum in degenerate model)

Higgs vacuum

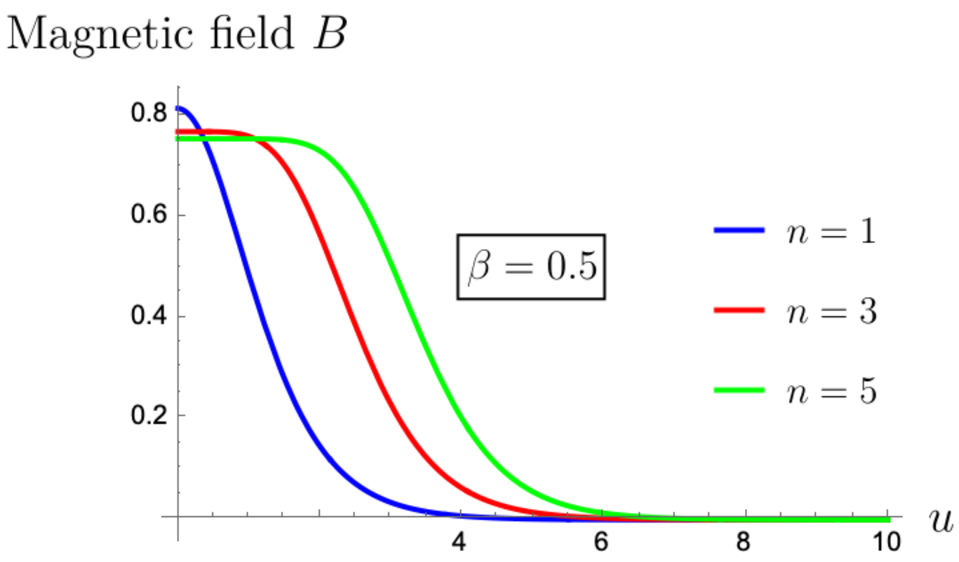
- Cannot be solved analytically, but can be numerically – profiles only depend on β and n

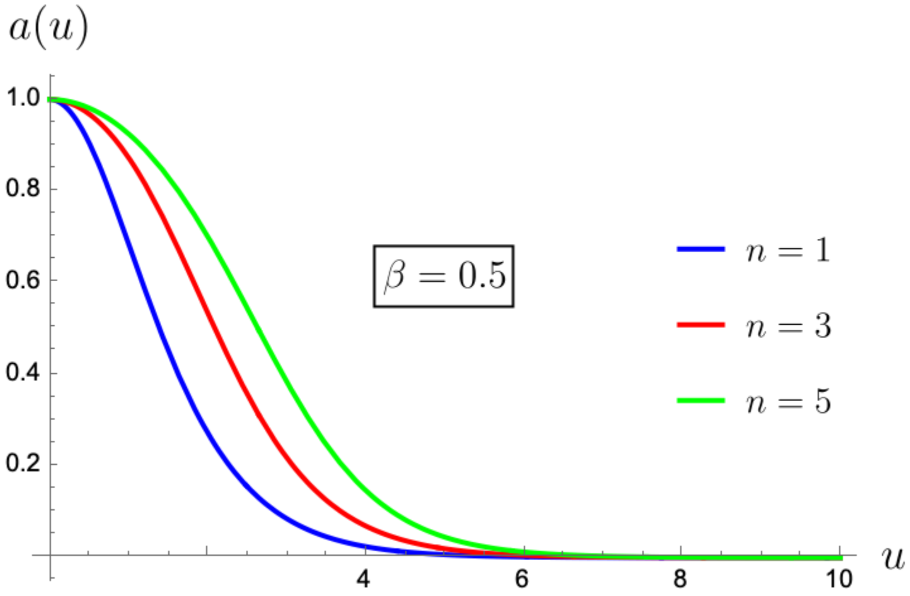
$$\phi(x) = v\varphi(r)e^{in\theta}$$

- Analytic solution in large flux limit

$$a_\theta(x) = n(1 - a(r))$$

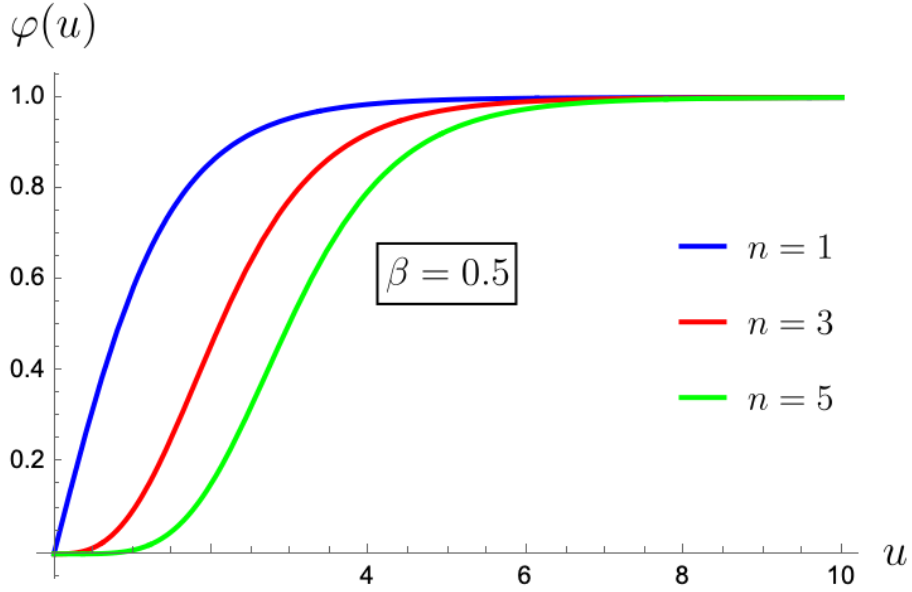
VARIATION WITH n



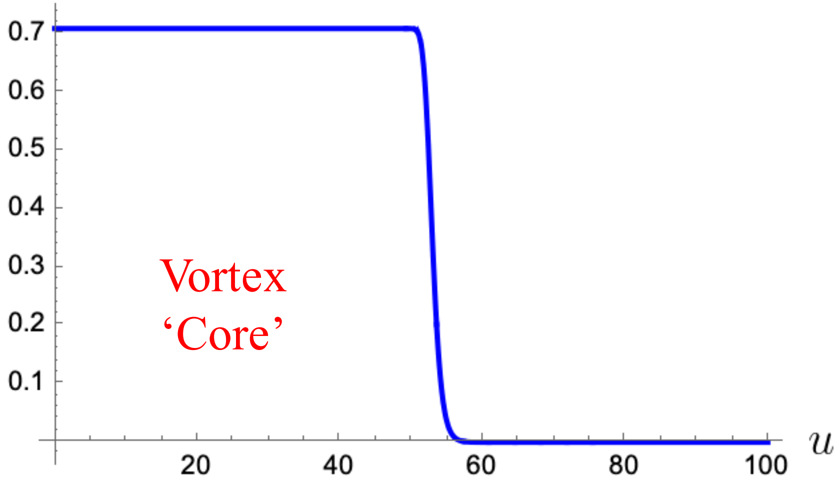


$$B = -\frac{n}{u} a'(u)$$

Conventional AHM

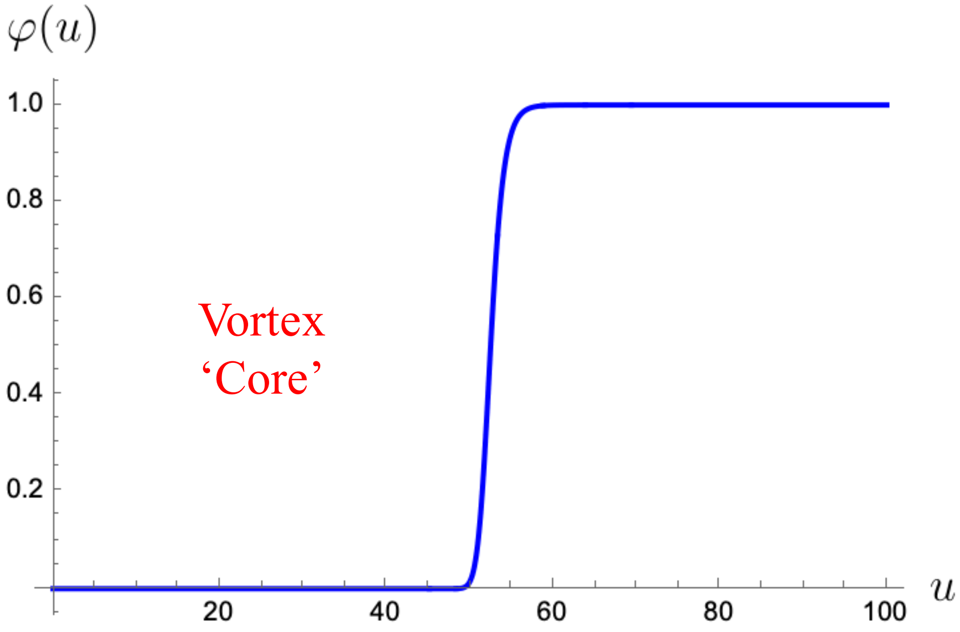


Magnetic field B



(Coulomb region) (Higgs vacuum)

$$n = 1000 \qquad \beta = 0.5$$



(Coulomb region) (Higgs vacuum)

Conventional AHM

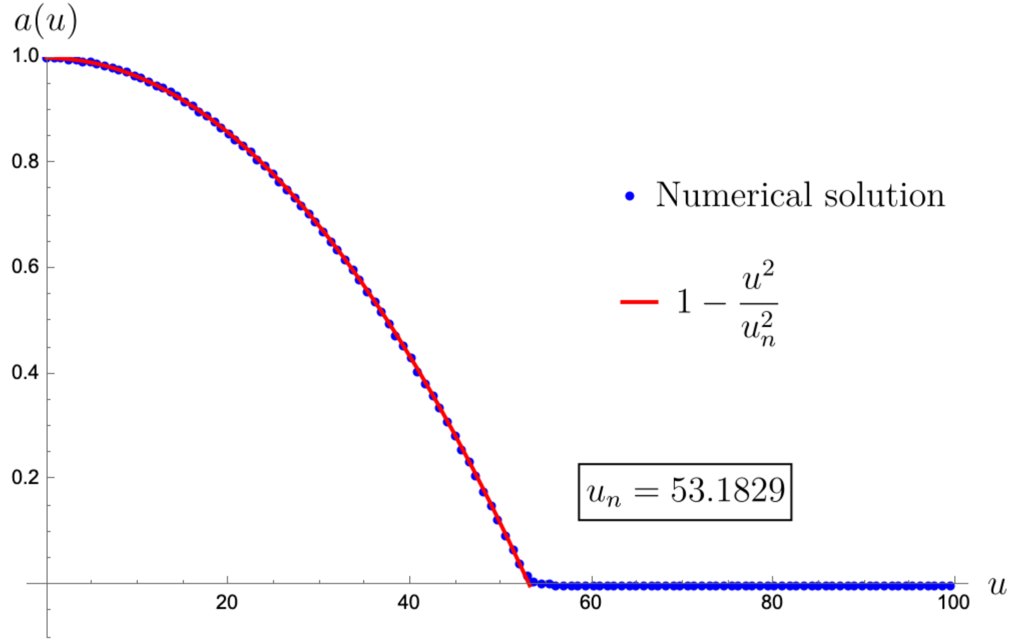
BUT THE TREND IS TRUE
EVEN FOR THE DEGENERATE MODEL !

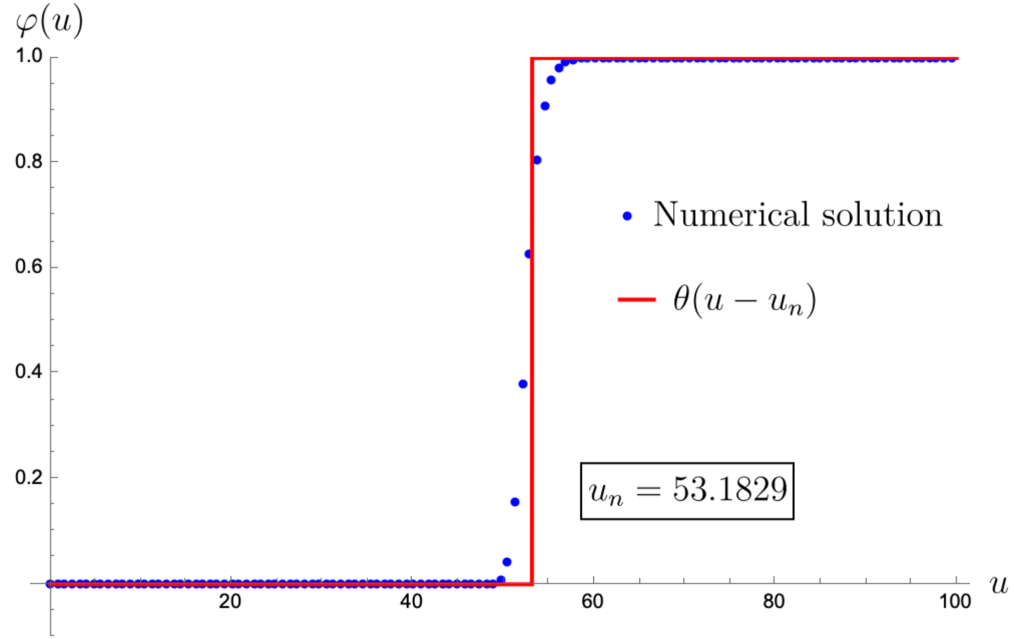
GIANT STRINGS : CONVENTIONAL AHM

$$a(u) = \begin{cases} 1 - \frac{u^2}{u_n^2} & u \leq u_n \\ 0 & u > u_n \end{cases}$$

$$\varphi(u) = \begin{cases} 0 & u \leq u_n \\ 1 & u > u_n \end{cases}$$

JUST COULOMB REGION AND HIGGS VACUUM

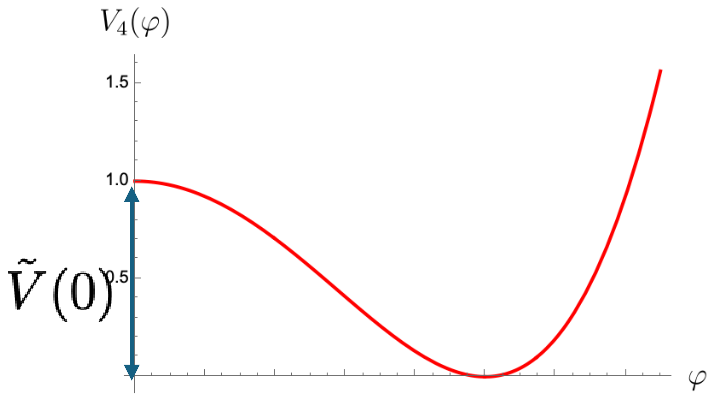




Quadratic fall off for gauge field translates to constant magnetic field !

$$B = -\frac{n}{u} a'(u)$$

$$n = 1000 \qquad \beta = 0.5$$



STRING TENSION GUESSTIMATE

Constant energy density proportional to the area of the string

$$T_n \sim \int_0^{u_n} u du \, (B^2 + \tilde{V}(0)) = \frac{\sqrt{2} n^2}{u_n^2} + \frac{\beta}{2\sqrt{2}} u_n^2$$

Not a vacuum ! Hence, there's a penalty

Flux wants to spread out

Variational minimization sets optimal radius and string tension

$$u_n^2 = \frac{2n}{\sqrt{\beta}} \qquad \frac{T_n}{2\pi} = \sqrt{2\beta n}$$

LARGE FLUX SOLUTION

- We **<u>need to</u>** account for physics from the transition region
- By producing large-n solution everywhere

Surface tension

$$\frac{T_n}{2\pi} = \underbrace{\sqrt{2\beta n}}_{\text{Core}} + \underbrace{\sigma u_n}_{\text{Boundary}}$$

SUBLEADING TO CORE !

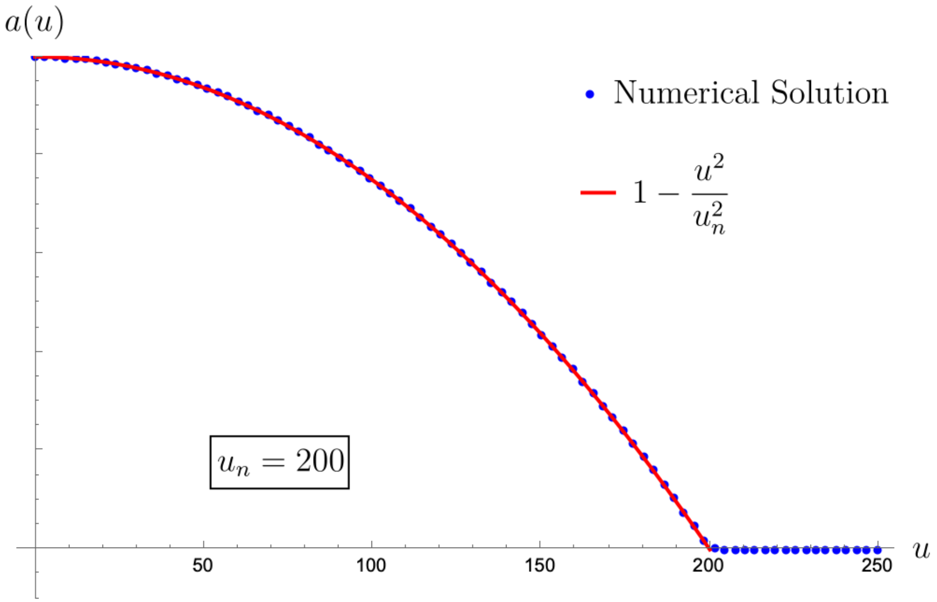
$$\sigma > 0 \leftrightarrow \beta < 1, \quad \sigma < 0 \leftrightarrow \beta > 1, \quad \sigma = 0 \leftrightarrow \beta = 1$$

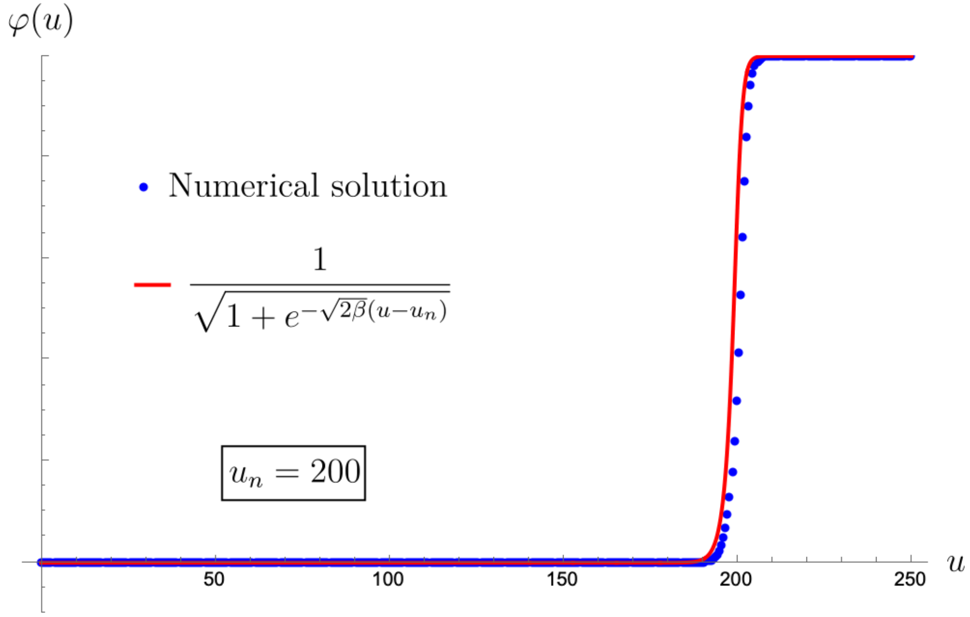
GIANT STRINGS: DEGENERATE MODEL

$$a(u) = \begin{cases} 1 - \frac{u^2}{u_n^2} & u \le u_n \\ 0 & u > u_n \end{cases}$$

Domain wall solution connecting the Coulomb and Higgs vacuum

$$\varphi_{\mathrm{DW}}(u) = \frac{1}{\sqrt{1+e^{-\sqrt{2\beta}(u-u_n)}}}$$

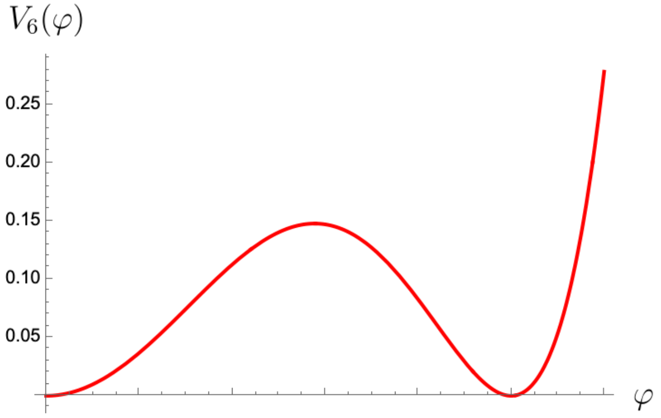




Note the difference in size for the same values of parameters

$$n = 1000 \qquad \beta = 0.5$$

STRING TENSION



$$\frac{T_n}{2\pi} = \frac{\sqrt{2}n^2}{u_n^2} + \sigma_{\mathrm{DW}} u_n$$

Domain wall tension – Difference in competition, no urgency for the scalar field to leave the Coulomb vacuum !

Flux wants to spread out

Variational minimization sets optimal radius and string tension

$$\frac{T_n}{2\pi} = \frac{3}{2^{7/6}} \beta^{1/3} n^{2/3} \qquad u_n = 2^{5/6} \frac{n^{2/3}}{\beta^{1/6}}$$

PHASES OF GIANT STRINGS

CONVENTIONAL AHM

- BULK PHASE
- Core radius $O(\sqrt{n})$
- String tension $O(n)$
- Energy density $\beta\sqrt{2}$

Infinite volume region – Large Strings have phases !

$$\frac{T_n}{\pi u_n^2}$$

DEGENERATE MODEL

- DOMAIN WALL PHASE
- Core radius $O(n^{2/3})$
- String tension $O(n^{2/3})$
- Energy density 0

[Bolognesi, Gudnason ’05, …]
[Penin, Weller ’21 , …]

FEATURES

The large flux limit is a tractable limit
and the physics of the giant strings
gets simplified.

PHASES OF GIANT STRINGS

Scaling of **size** and **tension** with flux.

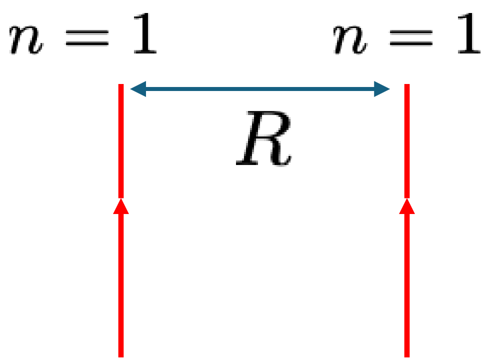
DOMAIN WALL PHASE

BULK PHASE

BREAK ROTATIONAL SYMMETRY

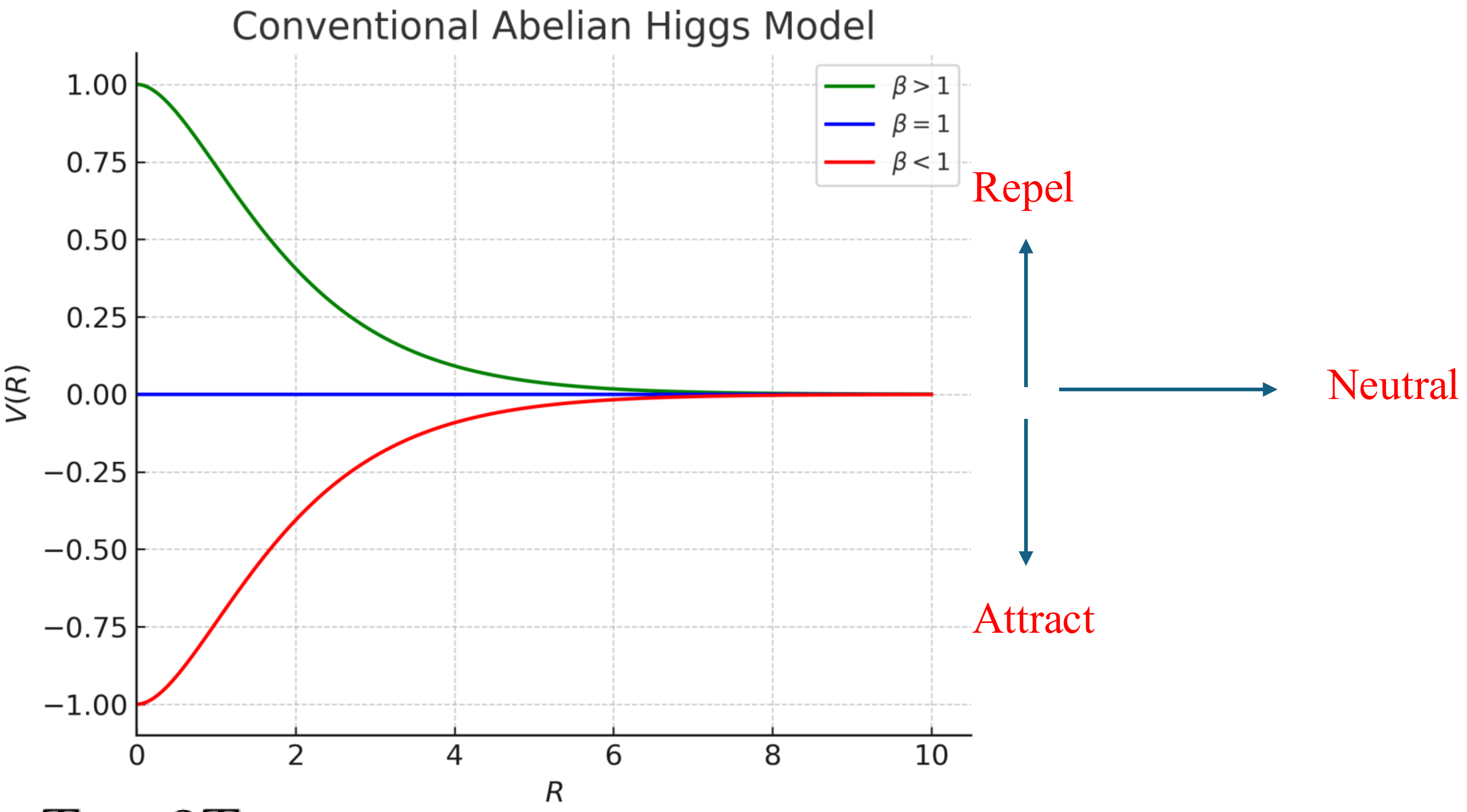
FORCES BETWEEN SEPARATED STRINGS

$$V_{\text{int}}(R) = -A^2 \sqrt{\frac{\pi}{2m_H}} \frac{e^{-m_H R}}{\sqrt{R}} + B^2 \sqrt{\frac{\pi}{2m_V}} \frac{e^{-m_V R}}{\sqrt{R}}$$



- Force between two $n = 1$ strings separated by a distance R
- **<u>ATTRACT</u>** in Type – I ; **<u>REPEL</u>** in Type – II
- Just the **<u>Mass Spectrum</u>** β determines the answer to this question

INTERACTION POTENTIAL $V(R)$ AS A FUNCTION OF R



$T_2 - 2T_1$

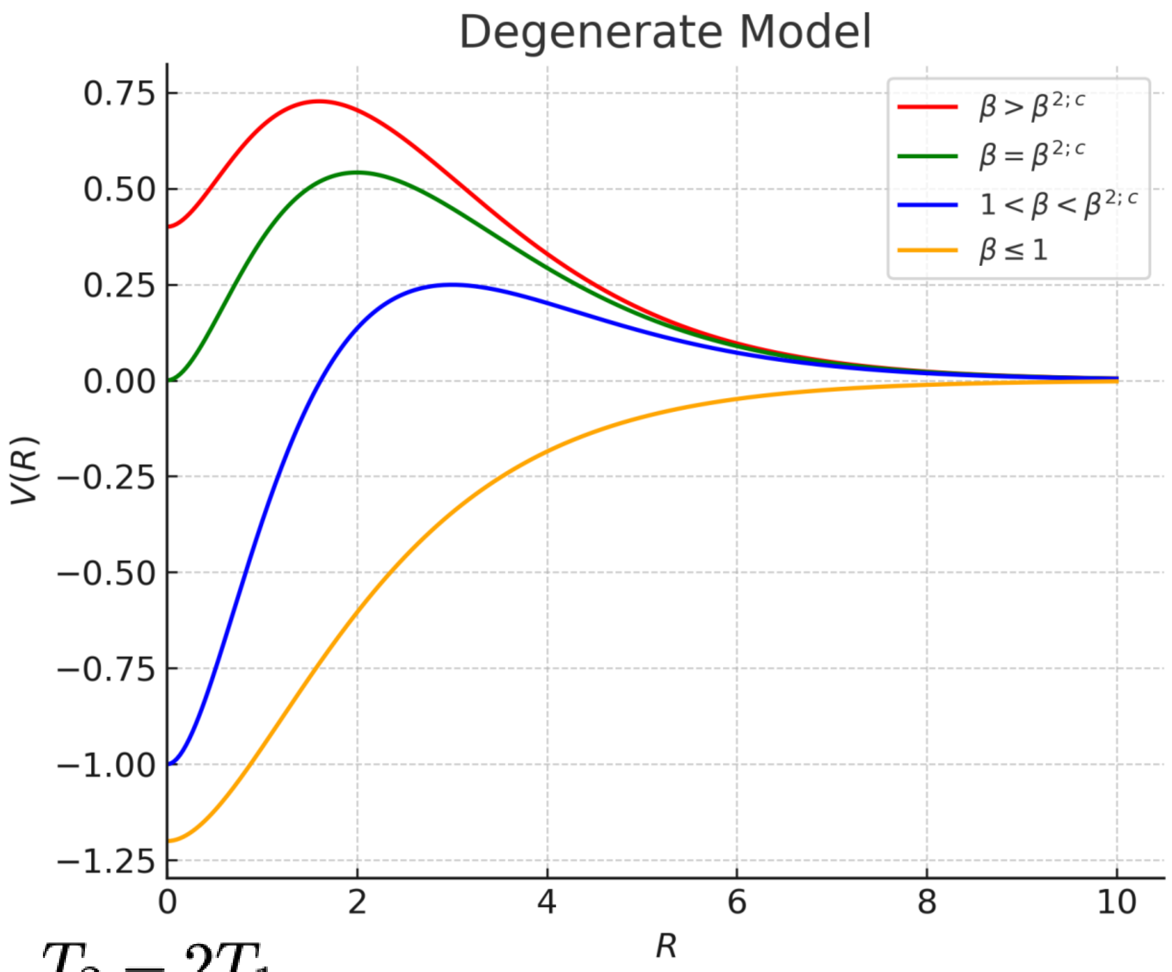
Binding energy of a 2-string

Interaction energy of superconducting vortices [Jacobs, Rebbi '79, …]

Just knowing the spectrum tells us something non trivial about the strings themselves. This is generically not true – **DEGENERATE MODEL**

YM BELIEF : Strings attract at infinity and form stable bound states

INTERACTION POTENTIAL $V(R)$ AS A FUNCTION OF R



$T_2 - 2T_1$

SCHEMATIC CARTOON PREDICTION

Strings repel at infinity but can form rotationally symmetric bound states.

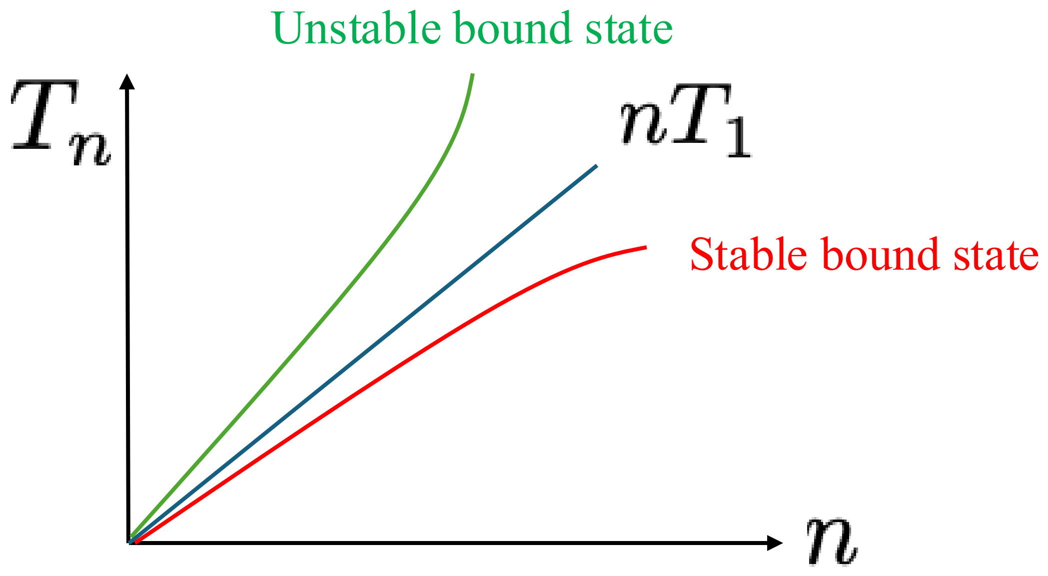
ROTATIONALLY SYMMETRIC BOUND STATES

Compare T_n with nT_1

- Numerics at small flux

- Stability of giant strings - convexity of tension at large n

$$\frac{d^2 T_n}{dn^2}$$



STABILITY OF GIANT BOUND STATES

CONVENTIONAL AHM

- $\frac{d^2T_n}{dn^2} = -\frac{\sigma}{4n^{3/2}} \begin{cases} < 0 & \beta < 1 \text{ Stable} \\ = 0 & \beta = 1 \text{ Neutral} \\ > 0 & \beta > 1 \text{ Unstable} \end{cases}$

$$\frac{T_n}{2\pi} = \sqrt{2\beta}n + \sigma u_n$$

- Stability directly correlated w/ β
- Stability of 2-string ; same as above

DEGENERATE MODEL

- $$\frac{d^2T_n}{dn^2} < 0$$

$$\frac{T_n}{2\pi} = \sqrt{2}n^{2/3}\beta^{1/3}$$

- Stable for all values of β
- Stability of 2-string – not as straightforward

FEATURES

Stable versus unstable symmetric
BOUND STATES

INTERACTING FORCES
between separated fundamental strings

PHASES OF GIANT STRINGS
Scaling of size and tension with flux

These are important data points gathered from simple AHMs

STRING FLUCTUATIONS IN CONVENTIONAL AHM

Forthcoming [Dumitrescu, AG, Li ‘25]

FLUCTUATION PROBLEM AND ZERO MODES

- String is an object in 2D space ; **Two** NGBs – Broken translational symmetry

$$X^{i=1,2}(t,z)$$

- Additional moduli for BPS at higher flux

For this talk focus on fluctuations of the **fundamental** string

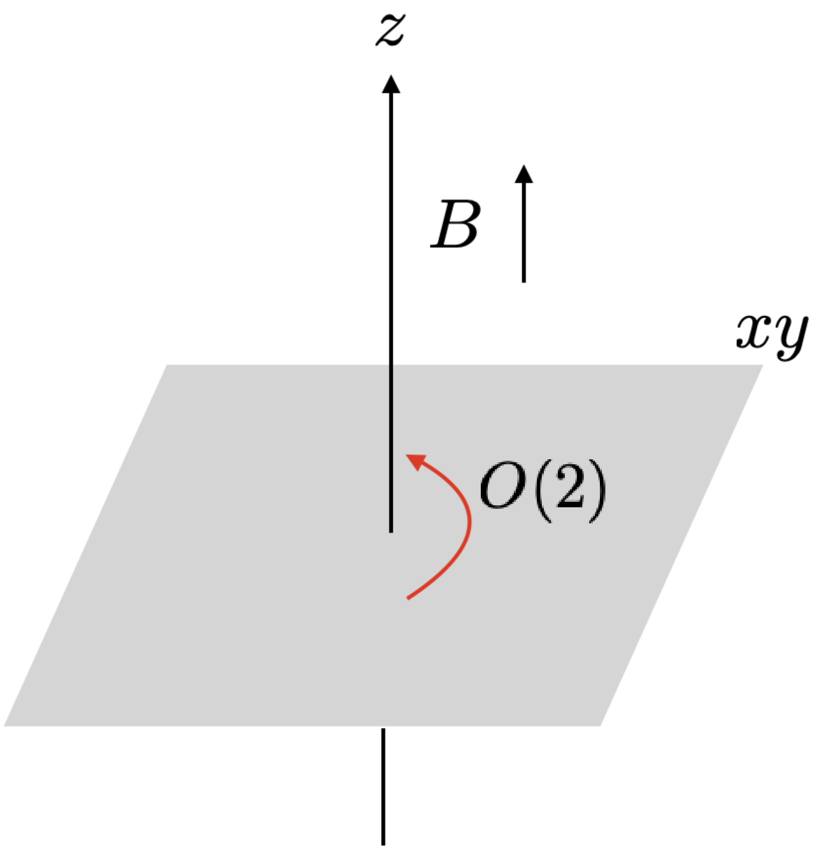
- Other small fluctuation modes are **gapped** and extremely rich

- Linearize equations and solve linearized problem

MASSIVE MODES

Continuum of scattering states

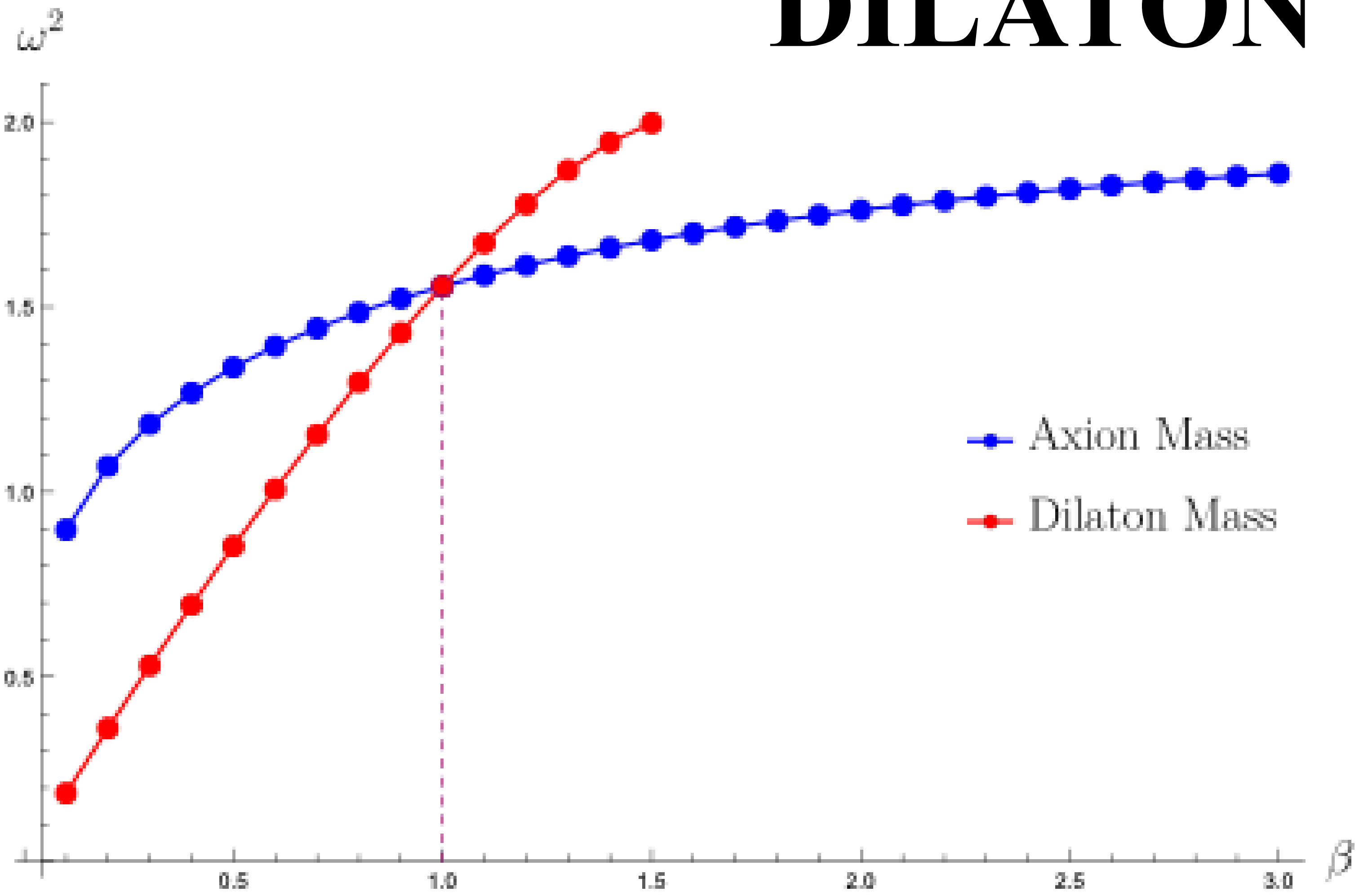
Bound state excitations



- $O(2)$ transverse spin – rotates the GBs.
- Use this symmetry to organize states
- Transverse Parity : GBs transform as a vector
- Focus on spin – 0 : Axion and Dilaton

Pseudoscalar Scalar

MASS HIERARCHY : AXION AND DILATON



Type – I

Type - II

Axion is the lightest massive excitation in the type-II regime - a setting where fundamental strings **<u>REPEL</u>** !

YM CONFINING FLUX TUBES

- Pseudoscalar Axion is the lightest non-trivial fluctuation

  [Dubovsky, Flauger, Gorbenko’ 13, Athenodorou, Dubovsky, Luo, Teper ’ 24, ….]

- Believe that confining strings in YM attract and form bound states

  [Athenodorou, Teper ’21, ….]

Axion is the lightest massive excitation in a setting where fundamental confining strings **ATTRACT** !

EFFECTIVE ACTION OF LONG STRINGS

- Calculate three point couplings of GBs to massive modes

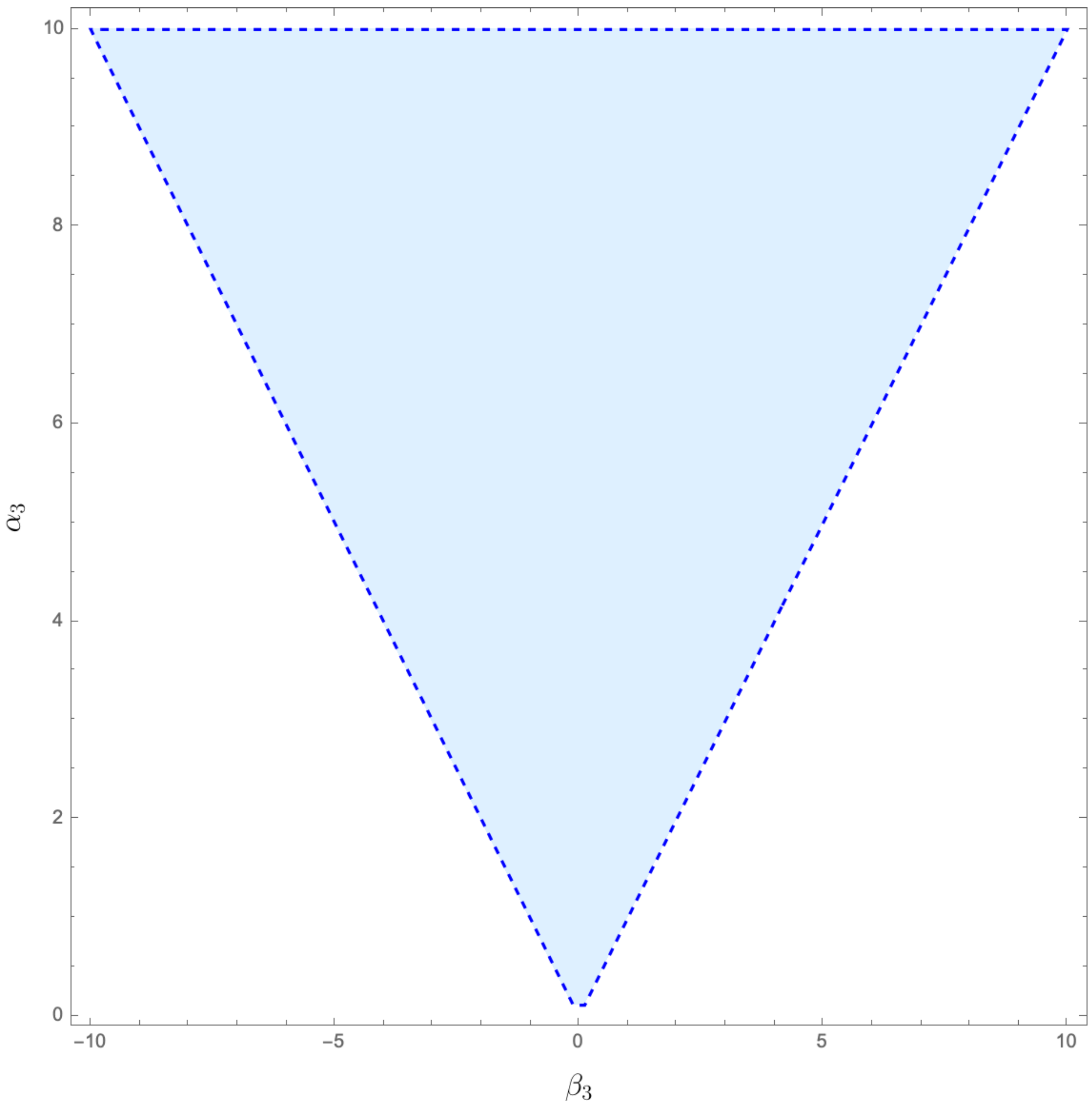
$$aXX \longrightarrow \text{(Suppressed indices)}$$

- Integrate to calculate backreaction on the GBs. At leading order,

[Dubovsky, Flauger, Gorbenko '12, Aharony, Komargodski '13, ….]

Interesting characteristic numbers of the string

$$\mathcal{L} \sim T\sqrt{-h}\left(1 + \frac{\tilde{\alpha}_3}{T^2}\underbrace{K^4}_{\text{Extrinsic curvature}} + \frac{\tilde{\beta}_3}{T^2}\underbrace{R^2}_{\text{Ricci scalar}} + \dots\right)$$



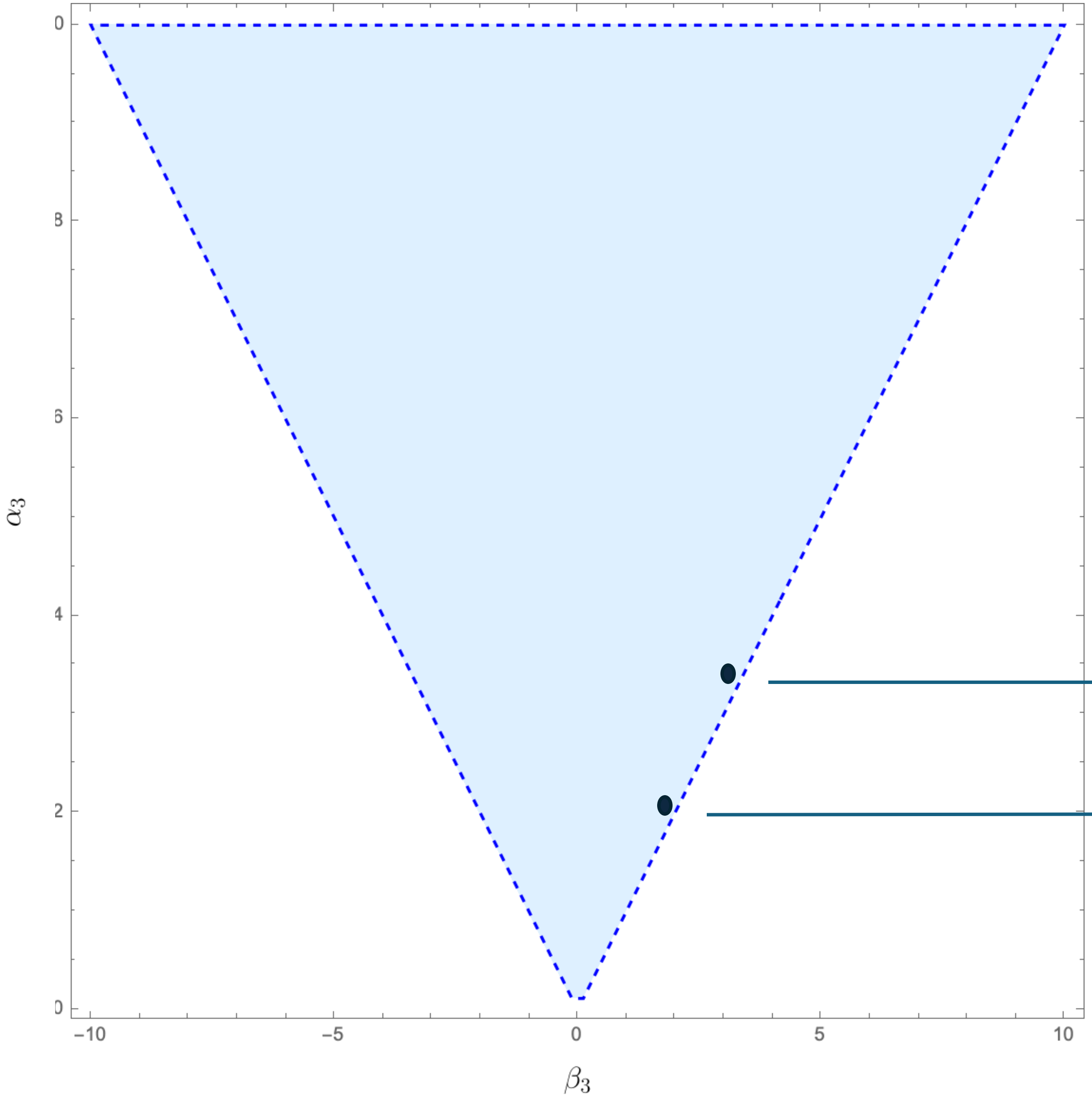
BOOTSTRAP BOUNDS

Requiring a consistent UV completion of the branon S-matrix, put bounds on its low energy expansion and bound the EFT parameters

[Miro, Guerrieri, Hebbar, Penedones, Vieira ’19]

[Miro, Guerrieri ’21]

YM CONFINING FLUX TUBES

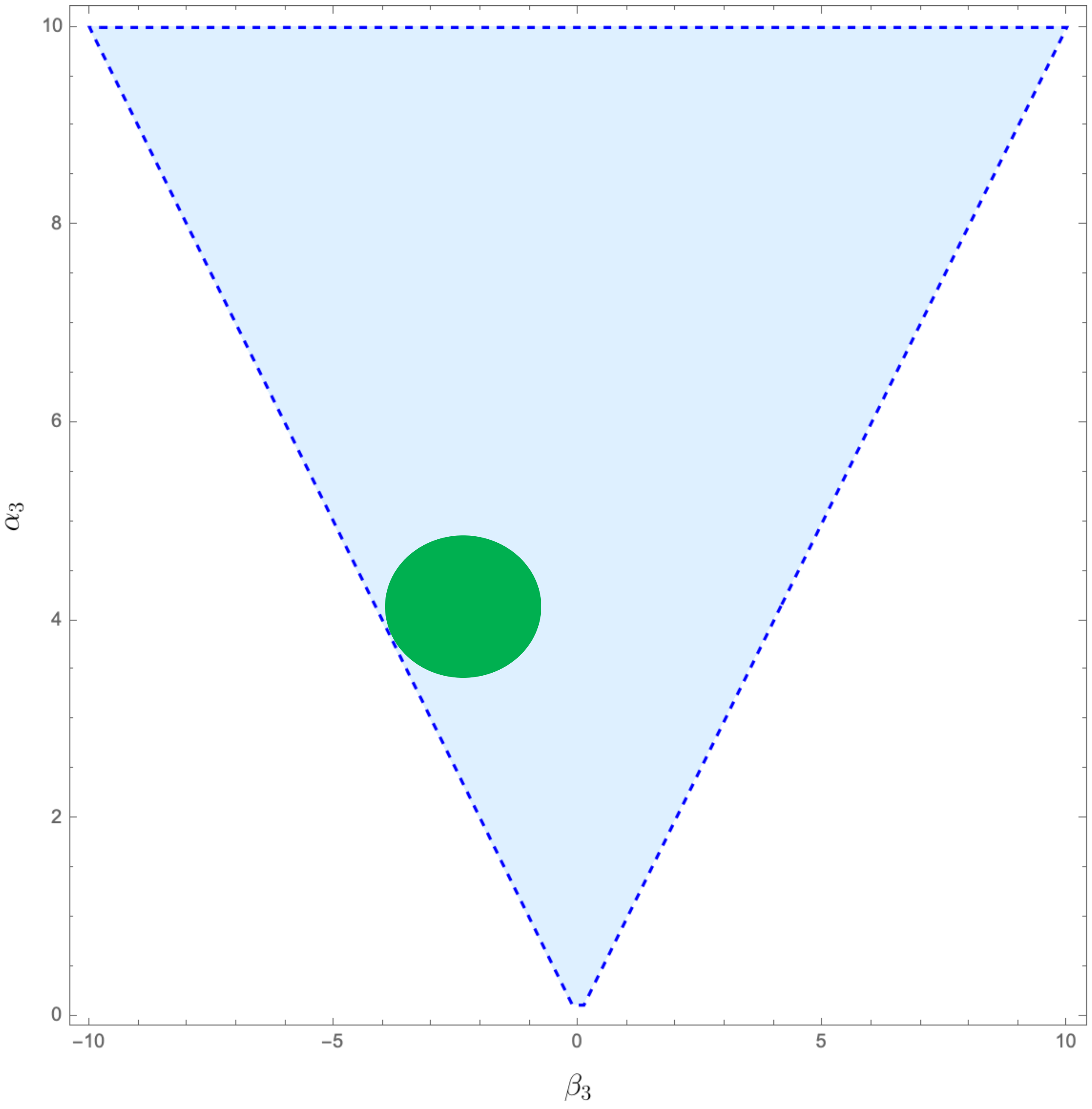


[Dubovsky, Flauger, Gorbenko ’14, Dubovsky, Gorbenko ’15,…]

[Miro, Guerrieri, Hebbar, Penedones, Vieira ’19]

SU(5) YM

SU(3) YM

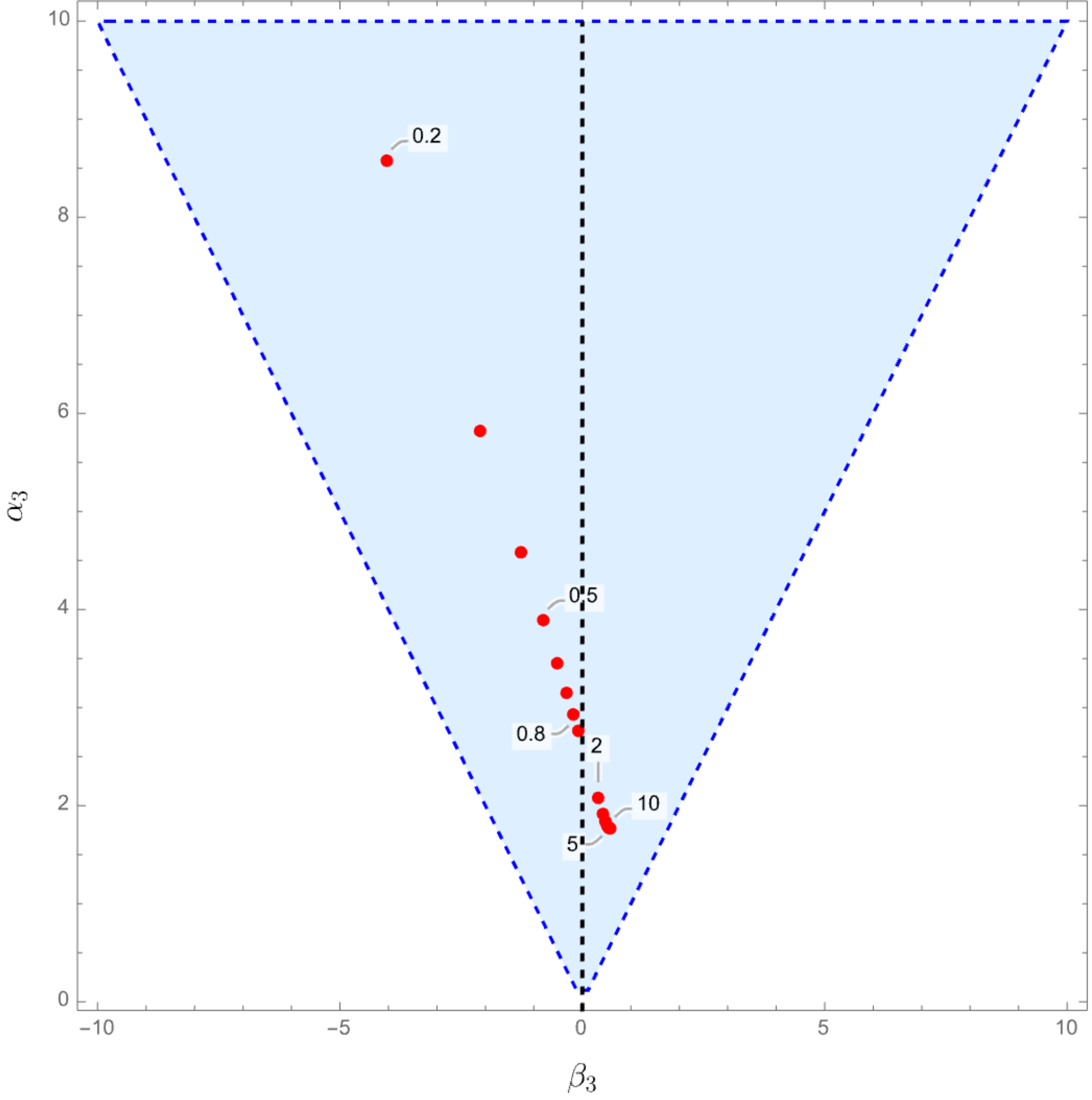


HOLOGRAPHIC CONFINING GAUGE THEORIES

Expect flux tubes in holographic confining gauge theories to have a **light dilaton** fluctuation mode

[Polyakov '98]

[Aharony, Karzbrun'09,..]



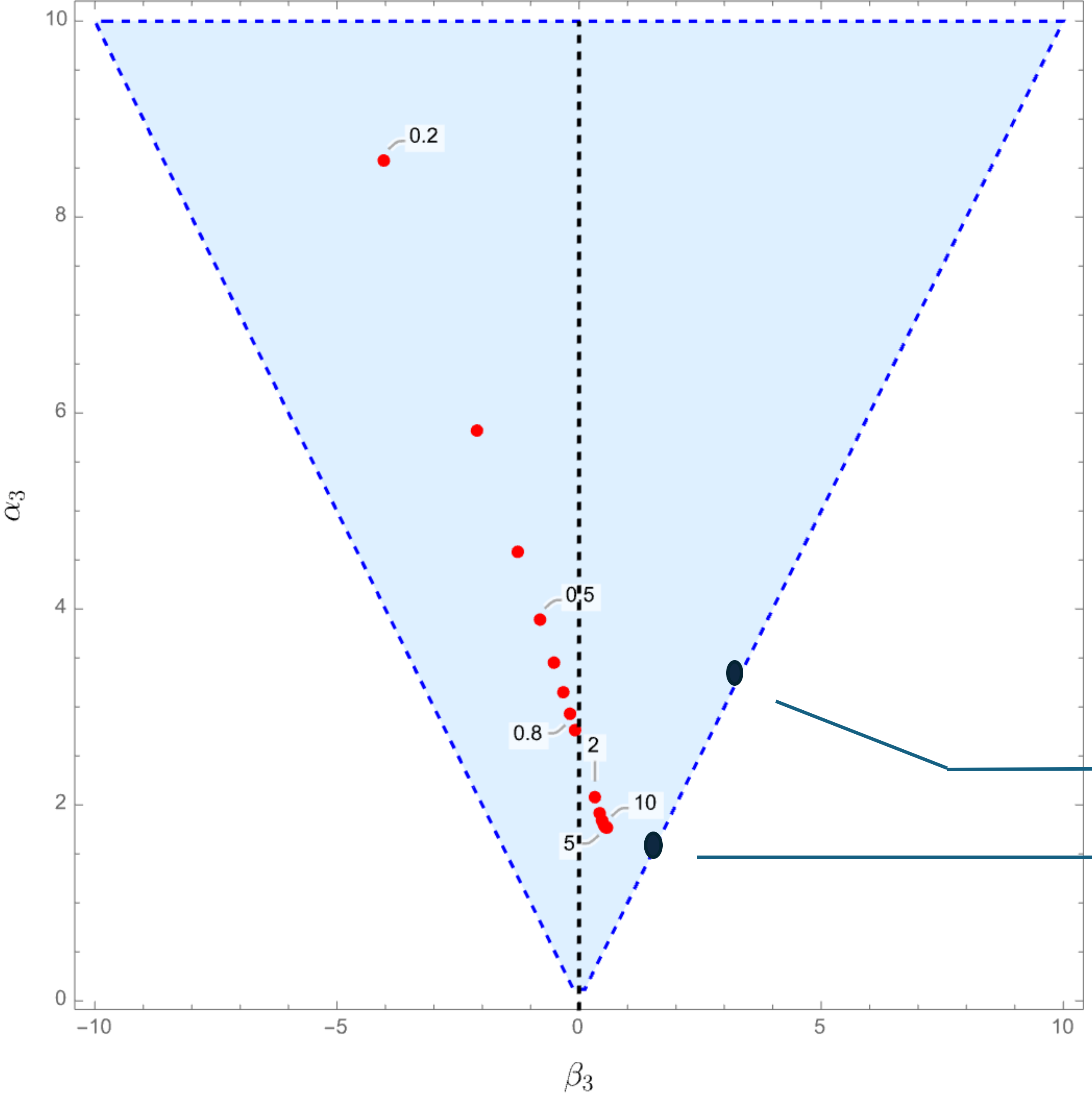
TYPE- I AHM

TYPE-II AHM

CONVENTIONAL AHM

In red, we plot the conventional AHM for different values of β

$$\alpha_3,\ \beta_3 = \frac{f(\beta)}{e^4}$$



THE YM CONFINING FLUX TUBES LIE IN THE REGION WHERE TYPE-II SUPERCONDUCTING STRINGS RESIDE.

This simulates the similar tension as before !

SU(5) YM

SU(3) YM

TYPE- I AHM

TYPE-II AHM

MINIMAL AHM IS **NOT** A COMPELLING DUAL DESCRIPTION FOR YM CONFINING STRINGS !

NATURAL NEXT VARIANT !

DEFORMED SEIBERG-WITTEN THEORY

Abelian model of confinement, **MANY** more fields : scalars and fermions

[Dumitrescu, AG ‘25]

It is known from condensed matter literature that if one has multiple Higgs fields and you play with potentials, one can dramatically change the way which strings attract or repel.

[Douglas, Shenker ‘95] [Hanany, Strassler, Zaffaroni’ 98] [Vainshtein, Yung ‘01, Hou’ 01]

PLETHORA OF LITERATURE !

[Klebanov, Herzog ’02] [Shifman, Yung ‘04] [Hanany, Tong ‘04]

[Auzzi, Bolognesi, Evslin, Konishi, Yung ‘03]

SUMMARY

1. **Minimal** Abelian Higgs models (AHMs) **do not** serve as compelling dual descriptions of YM flux tubes !

What is the lightest fluctuation mode around a string background ?

Do separated strings attract or repel ?

2. The large flux limit is a tractable limit and the physics of the giant strings gets simplified.

DOMAIN WALL PHASE

BULK PHASE

NEXT STEPS !

Study confining strings in deformed Seiberg-Witten theory
PHASES OF GIANT CONFINING STRINGS

[Dumitrescu, AG ‘25]

Study **fluctuations around the confining strings** backgrounds in the SW dual AHM

Study fermionic fluctuations around the string background ?
Is a fermion the lightest mode ?

THANK YOU !