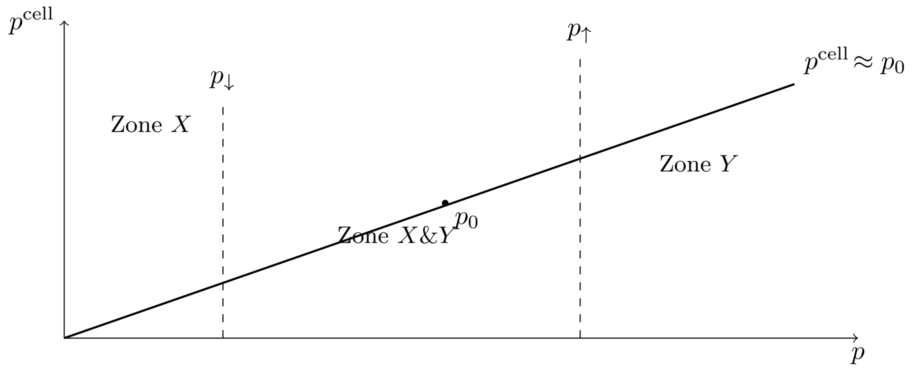
CURVE STABLECOIN: INTUITION OF THE LIQUIDATION AMM (LLAMMA)

We consider two assets: Y (e.g. ETH) and X (USD). Let the market price be p (measured in units of X/Y). We denote by p_0 the current oracle/market price and fix two thresholds $p_\downarrow < p_\uparrow$ that determine the liquidation band. We also use the notations $[USD] = X$, $[ETH] = Y$, and a portfolio $P = [USD, ETH] = [X, Y]$.

Idea and Zones



The key idea is to use a special AMM for liquidations. This makes liquidations smoother than discrete jumps of a traditional trigger. The inventory inside the AMM cell ("bucket") depends on the price p_0 relative to the band. We partition the p-line into three zones:

- **Zone** Y: the cell holds only Y. If the true price were in the X-region, one could buy Y on the market at price $\lesssim p_{\text{sell}}$ and sell into the cell at $\gtrsim p_{\text{cell}}$.
- **Zone** X: the cell holds only X. If the true price were in the Y-region, one could buy in the cell at $\lesssim p_{\text{cell}}$ and sell on the market at $\gtrsim p_0$.
- **Zone** $X\&Y$: the cell holds a mix of X and Y; trading happens inside.

Intuitively, when the external price p_0 is high (to the right), the cell should contain mostly Y (collateral); when p_0 is low (to the left), the cell should contain mostly X (stablecoin received in liquidation).

Making the Band Depend on p_0

To achieve the above behavior we let the cell's internal "buy" and "sell" thresholds depend on the external price p_0. Introduce two functions

$$p_\uparrow^{\text{cell}}(p_0), \qquad p_\downarrow^{\text{cell}}(p_0),$$

which play the role of the cell's upper/lower effective prices. We require the following monotonicity around the band edges $p_\uparrow$ and $p_\downarrow$:

$$\begin{aligned} p_\uparrow^{\text{cell}}(p_0) &< p_0 \quad \text{when } p_0 < p_\downarrow, \\ p_\uparrow^{\text{cell}}(p_0) &= p_0 \quad \text{when } p_0 = p_\downarrow, \\ p_\uparrow^{\text{cell}}(p_0) &> p_0 \quad \text{when } p_0 > p_\downarrow; \end{aligned}$$

and symmetrically

$$\begin{aligned} p_\downarrow^{\text{cell}}(p_0) &< p_0 \quad \text{when } p_0 < p_\uparrow, \\ p_\downarrow^{\text{cell}}(p_0) &= p_0 \quad \text{when } p_0 = p_\uparrow, \\ p_\downarrow^{\text{cell}}(p_0) &> p_0 \quad \text{when } p_0 > p_\uparrow. \end{aligned}$$

Invariant (schematic form)

Let X and Y denote the current reserves inside the cell. The liquidation AMM maintains an invariant of the form

$$\big(X + \alpha(p_0)\big)\,\big(Y + \beta(p_0)\big) \equiv I, \tag{1}$$

where $I > 0$ is constant along a trade and the shifts α, β depend on p_0 in such a way that the cell gradually moves between the pure-X and pure-Y states as p_0 crosses the band.

Functions $\alpha(p_0)$ and $\beta(p_0)$ can be any functions that satisfy conditions above. For simplicity, we assume that $p_\uparrow^{\text{cell}} = p_0^3/p_\downarrow^2$ and $p_\downarrow^{\text{cell}} = p_0^3/p_\uparrow^2$.

Our invariant becomes:

$$\left(X + \frac{\sqrt{I\,p_0^3}}{p_\uparrow}\right)\left(Y + \sqrt{\frac{I}{p_0^3}}\,p_\downarrow\right) = I \tag{2}$$

Then let's assume that $I = p_0 * (A\,y_0)^2$, where y0 is some function of p_0 and is measured in [Y]:

$$\left(X + \frac{p_0^2}{p_\uparrow}\,A\,y_0\right)\left(Y + \frac{p_\downarrow}{p_0}\,A\,y_0\right) = p_0\,A^2\,y_0^2 \tag{3}$$

Let's break cells following way:

$$\frac{p_\downarrow}{p_\uparrow} = \frac{A-1}{A}, \qquad A = \text{const (for example 100)} \tag{4}$$

Then:

$$\left(X + \frac{p_0^2}{p_\uparrow}\,A\,y_0\right)\left(Y + \frac{p_\uparrow}{p_0}\,\frac{A-1}{A}\,A\,y_0\right) = p_0\,A^2\,y_0^2 \tag{5}$$

Let's expand the brackets:

$$\begin{aligned}
&\left(X + \frac{p_0^2}{p_\uparrow}Ay_0\right)\left(Y + \frac{p_\uparrow}{p_0}(A-1)y_0\right) = p_0A^2y_0^2 \\
&\Rightarrow XY + X\left(\frac{p_\uparrow}{p_0}(A-1)y_0\right) + Y\left(\frac{p_0^2}{p_\uparrow}Ay_0\right) + \left(\frac{p_0^2}{p_\uparrow}Ay_0\right)\left(\frac{p_\uparrow}{p_0}(A-1)y_0\right) = p_0A^2y_0^2 \\
&\Rightarrow XY + X\,p_\uparrow\frac{A-1}{p_0}y_0 + Y\,\frac{p_0^2A}{p_\uparrow}y_0 + p_0A(A-1)y_0^2 = p_0A^2y_0^2.
\end{aligned}$$

And we get the original quadratic equation for y_0:

$$y_0^2\,(p_0\,A) - y_0\left(p_\uparrow\frac{A-1}{p_0}X + \frac{p_0^2A}{p_\uparrow}Y\right) - XY = 0 \tag{6}$$