

An Evaluative Comparison of Google OR-Tools and Machine Learning Algorithms for Global Path Planning in Unmanned Ground Vehicles

Alexandre Benoit, and Pedram Asef,

Abstract—This work provides a comprehensive analysis of the Global Path Planning (GPP) subsystem for unmanned ground vehicles (UGV). An autonomous mining sampling robot, defined as ROMIE, used for geochemical mining sampling processes. GPP is pivotal to ROMIE's efficient functioning, as it necessitates resolving the well-known graph theory problem, the Traveling Salesman Problem (TSP). An NP-hard problem has profound implications for ROMIE's operations as it is employed to identify the most efficient way to cover all sampling points in a prospective mining operation field. The application of TSP not only ensures high cost and time efficiency but also enhances competitiveness to current human labour. This paper aims to identify, develop, and validate the most cost-effective solution to TSP for UGVs in mining applications. Performance analysis of newly developed real-time optimization solver coupled with Google OR-Tools and reinforcement learning (RL) methods, i.e., Q-Learning and Double Q-Learning algorithms. Through comprehensive analysis, our research reveals that Q-Learning consistently outperforms other techniques, firmly positioning itself as the premier optimization strategy across all evaluated datasets. While the performance can vary with different datasets, on average, Q-Learning's solutions deviate by merely 1.2% from the optimal solutions within the set, underscoring its superior efficiency. Overall, across all datasets gathered, Q-Learning-based algorithms proved to be the most performant, indicating their potential in providing cost-effective and efficient solutions in real-world applications enhancing autonomous operations in the mining industry such as ROMIE.

Index Terms—Travelling Salesman Problem, Global Path Planning, Google OR-Tools, Reinforcement Learning, Q-learning algorithm, Double Q-learning algorithm.

I. INTRODUCTION

ROMIE (Robotic Ore Mineral Identification and Exploration) is a pioneering robot specializing in tackling the field of exploration and geochemical sampling within the mining industry. Undertaken before mining activities, the geochemical sampling process entails the collection and analysis of soil samples to identify potential mineral deposits. Traditionally relying on manual labor, the existing methods of sampling pose significant issues in terms of time, cost, working conditions, and sustainability. The process is labor-intensive and demands a substantial workforce, placing a burden on related companies.

To date, the process of sampling kilometers of fields in the mining industry has been performed manually becoming a

laborious task. [17]. In response to this challenge, the ROMIE software and hardware have been investigated and developed as an innovative autonomous global path planning system to optimize the navigation of mobile robots in a variety of environments. The system utilizes advanced algorithms and state-of-the-art hardware to generate efficient and reliable paths, taking into account obstacles, constraints, and objectives. This is performed by a state-of-the-art object detection algorithm supported by two stereo cameras as well as a detailed mapping executed by a LIDAR. ROMIE traverses the field to go to specific point having mineral of interest, drill and analyse the soil with an on-board earth auger and X-ray fluorescence (XRF).

Solving the Global Path Planning (GPP) problem, is analogous to solving the Travelling Salesman Problem (TSP) [33]. The TSP is a well known NP-hard problem [23] in combinatorial problems, important in theoretical computer science [1] and optional research [5]. It has become common practise to use optimization algorithms which are broadly classified as deterministic, yielding consistent results for the same inputs, and stochastic, which use randomness to explore multiple solution paths [8]. Stochastic algorithms, subdivided into heuristic and meta-heuristic types, are particularly useful for challenging optimization problems due to their exploratory methods and practicality. The TSP is a prime example where heuristic approaches often deliver near-optimal solutions efficiently, making them suitable for NP-hard problems. The choice of algorithm, however, depends significantly on the application's specifics.

Numerous renowned methods, e.g., Genetic Algorithm [31], Tabu Search [39], Particle Swarm Optimization [11], Simulated Annealing [4], and Ant Colony Optimization [15], have been applied to approximate the optimal solution to the TSP.

More recently, machine learning (ML) methods have emerged as viable alternatives, offering solutions that are both computationally efficient and closer to the exact solution. Specifically, reinforcement learning (RL) has demonstrated its utility as a straightforward and accessible technique to address the TSP [9]. RL is a branch of ML where an agent maximizes rewards through trial-and-error interactions with its environment. Essential this is the Markov Decision Process, where decisions rely on the present state, embodying all relevant past information, thus simplifying the learning process [40].

This paper offers a new software, for autonomous sampling robots with a unique comparative analysis of real-time

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optimization algorithms, namely, Google OR-Tools and RL algorithms, such as Q-Learning and Double Q-Learning. The main objective is to identify the optimal TSP tour among the sampling points in real-time operations. The software prioritizes the minimization of travel distance over computational time efficiency because a shorter distance translates into less prospecting time, resulting in huge cost savings. For the same size of land, ROMIE is **6** times faster and **1.5** cheaper than human labour [13]. Finally, the developed ROMIE software is physically developed and experimentally validated.

This paper unfolds as follows: Section II reviews previous studies that have applied Google OR-Tools and reinforcement learning to the TSP. Section III provides a comprehensive overview of the various optimization techniques for TSP resolution. Section IV outlines our experimental design and methodology as well as the analyze of our experimental findings, alongside a depiction of the software interface where GPP is implemented. Lastly, Section V encapsulates the conclusions drawn from our research and hints at prospective directions for future studies.

II. RELATED WORKS

For TSPs, numerous methods have been employed to identify the most cost-efficient route. Comparative studies often benchmark their outcomes against exact algorithms, while computationally expensive to deliver optimal TSP tours. Prominently, the Concorde solver [2] has been acclaimed as the most efficient, solving a TSP with 85,900 Euclidean cities [3]. Given the substantial computational time exact algorithms require, alternative methods have been sought to approximate the exact solution with a tolerable deviation percentage. These algorithms, often classified as heuristics or metaheuristics, are favored for their computational efficiency and precision.

The Christofides Algorithm, developed in 1976 [49], has proven to be a significant contribution to TSP solutions over the past decades. Empirical evaluations typically indicate that it achieves an accuracy of roughly 90% when compared to an optimal TSP solution, thus providing a minimum performance guarantee of 50%. The refinement of this algorithm through recent studies has led to an improved performance assurance, estimated to be $1.5 - 10^{-36}$ [12].

Various Heuristic methods have been compared over the years. A. Hanig Halim, et al., they compared six meta-heuristic algorithms (Nearest Neighbor, Genetic Algorithm, Simulated Annealing, Tabu Search, Ant Colony Optimization, Tree Physiology Optimization) on their ability to solve the TSP. Nearest Neighbor, Tree Physiology Optimization, and Genetic Algorithm were the fastest in computation, while Tabu Search and Ant Colony Optimization required more computation time. Tabu Search, Tree Physiology Optimization, and Genetic Algorithm achieved results nearest to the optimal solution, but the accuracy of Tabu Search declined with larger problems. The study highlights the need for more research in real-world contexts to better understand these algorithms' strengths and weaknesses.

A pertinent study aligned with ROMIE's application was conducted by Jie Chen, et al., where they addressed the TSP

for unmanned aerial vehicles (UAVs) utilizing two parallel optimization algorithms. The study focuses on the application of two parallel optimization algorithms, Iterative Genetic Algorithm (IGA) and Particle Swarm Optimization - Ant Colony Optimization (PSO-ACO), for effective UAV path planning. It establishes a suitable TSP model for UAV path planning and demonstrates that these proposed algorithms provide more reasonable and effective solutions than comparative approaches, thereby optimizing UAV path planning [21].

Regrettably, as far as our research indicates, a comprehensive comparison among Google OR-Tools algorithms is absent from the literature. A single study [24] was found that made a comparison between heuristic methods from Google OR-Tools, ML, and Exact algorithms. Nevertheless, gaining insights into the behaviors of the aforementioned heuristics contributes to our understanding of the performance of Google OR-Tools, especially considering the similarity of some of their methods to the algorithms presented in the study by A. Hanig Halim, et al.

In the recent years, ML methods particularly Deep Neural Networks (DNNs) and RL, are revolutionizing the algorithmic landscape. Over recent years, Neural Networks (NN) have showcased potential in learning combinatorial algorithms, including both supervised and RL techniques.

The TSP exemplifies a practical application where ML can yield effective solutions. TSP, a combinatorial optimization problem, involves finding optimal solutions from numerous potential ones. The trade-off between optimality and computational efficiency is traditionally addressed using exact methods or heuristics [27].

DNNs can potentially enhance these heuristic methods. Training these networks might lead to more efficient algorithms for combinatorial optimization problems, posing the question if Deep Learning can supersede traditional heuristic algorithms [28].

The study [7] proposes UTSP (Unsupervised Traveling Salesman Problem), a state-of-the-art unsupervised learning approach for solving the TSP using Graph Neural Networks (GNNs) with a surrogate loss function to overcome challenges faced by RL. The approach involves building a heatmap from GNN output and feeding it into a search algorithm and emphasizes the importance of expressive GNNs for efficiency. UTSP outperforms or competes with other learning-based TSP heuristics in terms of solution quality, running speed, and reduced training requirements.

Numerous academic papers have scrutinized the continuously evolving state-of-the-art ML solutions for the TSP, including [26] and [29]. These works provide extensive comparisons of ML models tested on varying numbers of cities (e.g., TSP20, TSP50, TSP100, TSP200, TSP500, and TSP1000). Notable algorithms like Concorde, Gurobi, and LKH3 generate optimal TSP solutions, with others applying ML to simple TSP heuristics or optimized problems. The analysis involves metrics, such as tour length, optimality gap, evaluation time, and the type of NN used by different deep learning approaches, including heuristic (H), supervised learning (SL), reinforcement learning (RL), sampling (S), greedy (G), beam search (BS), beam search and shortest tour heuristic (BS*), and 2OPT

local search.

A RL-driven Iterated Greedy Algorithm (RLIGA) for the TSP is proposed in [30]. A key component is the incorporation of a Q-Learning which guides the algorithm to select the optimal parameters for iterative searches based on historical experience with a unique aspect of this approach being the introduction of a 'damage size d ' selection mechanism informed by Q-learning. This mechanism is designed to prevent the algorithm from engaging in blind searches, increasing its ability to find optimal solutions.

Studies from Jiazhaio Liang [45] suggest that the potential use of the Q-learning and RL method to solve the TSP. The approach, while currently based on a simple model focused on distance, demonstrates successful convergence. The study indicates the need for future work to incorporate more complex factors for a better reality model. The authors demonstrate the proposed Q-learning method in the case of optimal path finding among warehouses, underscoring the relevance of RL in addressing combinatorial optimization problems.

Wang, et al. [18] applied three RL algorithms, such as Q-Learning, Sarsa, and Double Q-Learning, to solve the TSP. They found the Double Q-Learning algorithm to be the most effective, when using a reward function of $R1 = 1/d_{ij}$.

Despite the increasing application of optimization algorithms like Google OR-Tools and Q-Learning methods in solving complex problems, such as TSP, comprehensive performance analysis and comparison between them is conspicuously absent in the existing literature. This lack of comparative analysis limits our understanding of the strengths and weaknesses of these approaches when applied to practical, real-world problems. In response to this knowledge gap, this study provides an unprecedented comparative study between Google OR-Tools optimization algorithms and Q-Learning methods, aiming to evaluate their effectiveness in optimizing the TSP solutions.

III. OPTIMIZATION METHODS FOR SOLVING THE TRAVELING SALESMAN PROBLEM

A. Traveling Salesman Problem

The TSP is a renowned NP-hard problem [47] in the field of combinatorial optimization, which poses a significant query: "Given n cities and the distances between each pair, what is the most efficient route that visits each city once and returns to the starting city?" [46]. This problem, pivotal in industrial sectors, such as transportation becomes exponentially complex when the number of nodes or cities increases, thereby underscoring the necessity for effective resolutions. A mathematical interpretation of the problem would involve grasping the vast number of potential solutions that route optimization can generate. There are $(n - 1)!/2$ possible combinations. [43] Additionally, the researchers can depict the problem through different methods to present the optimal path. Assuming P_n as the total permutations of the set $1, 2, \dots, n$ and n equals x points. The TSP involves the quest for $\pi = (\pi(1), \pi(2), \dots, \pi(n))$ in P_n so that the function in Eq.1 is minimized. Here, C signifies the cost or distance between two cities and is known as the tour length:

$$C_{\pi(n)}\pi(1) + \sum_{i=1}^{n-1} C_{\pi(i)\pi(i+1)} \quad (1)$$

Because the exact solution is expensive to find, a plethora of meta-heuristic algorithms, each embodying unique strategies of intensification and diversification, have been put forward in scholarly literature to seek gratifying solutions for the TSP [43]. ML [4] offers an alternative route to crafting solutions manually, which could be costly or demand a high level of specialized knowledge. Recent strides in graph neural network techniques stand out as they inherently function on the graph structure of such problems. Thus, the creation and evaluation of algorithms for resolving the TSP remains a vibrant area of investigation in the research community.

B. Google OR-Tools

Google's OR-Tools are an open-source suite designed for optimization, providing a plethora of algorithms for routing, flow, graph, linear programming, and constraint problems. In the context of the TSP, OR-Tools offer a robust and adaptable platform for constructing and solving TSP instances [44]. The library includes a variety of first solution and local search algorithms, forming a comprehensive online toolset for addressing the TSP. Table. I below highlights these useful algorithms provided by the OR-Tools API.

A myriad of renowned algorithms for resolving the TSP, as shown in Table.I, have been explored. Google OR-Tools provides a swift avenue to experiment with TSP solutions, offering pre-implemented optimization algorithms through its API. However, it should be noted that this approach restricts full control over their implementation details.

There are some high level explanations of the theory behind some of the deterministic and stochastic algorithms [41].

- **Nearest Neighbour (Path_Cheapest_Arc)**

The Nearest Neighbor (NN) algorithm is a commonly used heuristic for the TSP. It starts from a random city and sequentially visits the nearest unvisited city until all cities are visited. Despite its non-optimal nature [32], the NN algorithm is efficient and practical with a runtime complexity of $O(n^2)$, where n is the number of cities. It offers rapid solutions for small to moderately large problem instances [35].

- **Minimum Spanning Tree (Similar to Local_Cheapest_Insertion)**

The Minimum Spanning Tree (MST) Heuristic, used in solving the TSP, finds the minimum total edge cost that connects all nodes in a graph. The TSP constraints are met by adding more edges to the MST, ensuring each node is visited once without backtracking [34].

The proposed MST implementation employs Prim's Algorithm, which has a time complexity of $O(|E| + |n|\log(|n|))$, where $|E|$ represents the edge number and $|n|$ the vertex number. In TSP's complete graph context, $|E|$ simplifies to $|n|(|n| - 1)$ [35]. The MST heuristic builds a foundation for TSP heuristic algorithms by creating an MST, doubling each edge, finding an Eulerian

TABLE I
VARIOUS LOCAL SEARCH AND META-HEURISTIC METHODS USED IN OR-TOOLS.

Category	Method	Algorithm
First Local Search	PATH_CHEAPEST_ARC (PCA)	Greedy
	LOCAL_CHEAPEST_INSERTION (LCI)	Local insertion heuristic
	LOCAL_CHEAPEST_ARC (LCA)	Localised Greedy
	CHRISTOFIDES (C)	Graph Theory heuristic
Meta-heuristic	GREEDY_DESCENT (GD) (Hill Climbing)	Greedy heuristic
	GUIDED_LOCAL_SEARCH (GLS)	Penalty-based
	SIMULATED_ANNEALING (SA)	Probabilistic
	GENERIC_TABU_SEARCH (GTS)	Memory-based

circuit in the doubled tree, and finally shortcutting the circuit into a Hamiltonian cycle to form a TSP tour. While not always yielding the optimal TSP solution, the MST heuristic offers a valuable starting point or approximation method, especially when combined with Prim's or Kruskal's Algorithm. The MST's cost will always be less than the TSP's cost, offering a lower bound for the TSP and a means to conceptualize the optimal TSP tour by creating a "1-tree" through edge removal.

- **Christofides:**

The Christofides' Algorithm, a renowned method in theoretical Computer Science, for TSP. This multistep algorithm guarantees a solution within 1.5 times the optimal. This algorithm encompasses several steps:

- 1) **MST:** Develop a Minimum Spanning Tree.
- 2) **Odd degrees:** Identify vertices with odd degrees in MST, ensuring no node repetition. This step, with a time complexity of $O(|n|)$, relies on available degree count for each vertex.
- 3) **Matching:** Find smallest distance matchings, potentially using algorithms like Blossom V. This step, the most computationally expensive, has a runtime complexity of at least $O(|n|^4)$ [22].
- 4) **Euler:** Combine matching and MST to form an Eulerian multigraph [25], ensuring even node degrees. This multigraph underlies the Eulerian tour, computed using Fleury's algorithm [20].
- 5) **Hamiltonian:** Verify that each city has been visited; if yes, proceed to the next city.

- **Hill Climbing**

Hill Climbing begins with a random sequence of cities and iteratively improves it. The algorithm considers all successor states, each obtained by swapping two adjacent cities, and chooses the best one as the new state. This process continues until an adequate solution is found. Although Hill Climbing is simple and efficient, it's limited by its short-sighted approach, often stopping at local optima and potentially missing the global optimum [19].

- **Simulated Annealing**

The Simulated Annealing algorithm is a probabilistic search technique that aims to find an optimal solution by exploring the solution space, allowing for the acceptance of worse solutions to escape local optima. It uses a temperature (T) parameter that gradually decreases over time, controlling the probability of accepting worse solutions. The cooling schedule, represented by the parameter α ,

determines how the temperature decreases.

A mathematical representation of the probability of choosing changes in a solution (move) is calculated as:

$$P(\text{move } A \rightarrow B) = e^{-\Delta/T} \quad (2)$$

where $\delta = f(A) - f(B)$ and T is the current temperature. The temperature is set to decrease by [10]:

$$T = \alpha T_0 \quad (3)$$

where α is a number in (0,1) acting as a decreasing factor. The following figure describes the probabilistic acceptance that the SA method follows for exploring the solution space and helping in avoiding getting trapped in local optima [14]. Decreasing the temperature helps with this transition. In the final stages of the search, the minimum number of bad solutions is to be chosen to improve the accuracy of the solution as much as possible. This is called exploitation. The only mechanism to balance exploitation and exploration in the Simulated Annealing algorithm is the temperature.

- **Tabu Search**

Tabu Search, is a global optimization algorithm. Starting from an initial feasible solution, it explores the solution's neighborhood to identify candidate solutions. If a candidate fulfills predetermined rules, its tabu status is bypassed, it is deemed as the current solution and potentially the global optimum, and added to the tabu list. If not, the optimal solution among the non-taboo ones is selected as the current optimal solution, also added to the tabu list. This process continues until a termination condition is met [37] [16].

C. RL: Reinforcement Learning

Geochemical sampling processes may require more detailed prospection over time, necessitating scalability and responsiveness to customer needs in the field. In the event that the customer wishes to sample a specific area following promising results on the sampling heat maps, the use of ML algorithms, such as UTSP may provide a viable solution. However, the application of such algorithms can be complex and present certain limitations, including an internal lack of current knowledge of Graph Neural Networks (GNN) for optimal implementation. Nevertheless, some state-of-the-art solutions offer a reasonable degree of optimality and free resources are available, although scalability for high point

counts remains an issue. Q-learning, a type of RL, is selected for its straightforward implementation and comprehensibility. It offers a practical and intuitive approach to solving problems, making it accessible and easy to understand. The alternative description of this method was not provided in the preceding section.

1) Q-Learning: The Q-Learning is a model-free RL algorithm (Algorithm 1) [50]. In this work, the Q-learning is applied to a 2D map. The Q-learning algorithm learns a policy to determine the optimal actions, i.e., moving from one point to another. The objective is to maximize the cumulative reward, i.e., minimizing the overall distances of the tour. The most important mathematical feature is the Q-Table which maps states and actions to Q-values. It estimates the quality of taking certain action in a given state. The Q-Values are represented as a state-action function (s, a) . This pair is a cumulative reward starting in state ' s ' (sampling points) taking an action ' a ' (choice of the next sampling points). It uses the Bellman equation as a simple value iteration update, using the weighted average of the current value and the new information [51].

$$Q^n(s_t, a_t) = (1 - \alpha)Q(s_t, a_t) + \alpha \left(r_t + \gamma \max_a Q(s_{t+1}, a) \right) \quad (4)$$

where r_t is the reward received when moving from state s_t to state s_{t+1} , and α is the learning rate ($0 < \alpha \leq 1$). $Q^{new}(s_t, a_t)$ is the updated Q value and it is composed of three factors:

- $(1 - \alpha)Q(s_t, a_t)$: the current value (weighted by one minus the learning rate)
- αr_t : the reward r_t to obtain if action a_t is taken when in state s_t (weighted by the learning rate)
- $\alpha \gamma \max_a Q(s_{t+1}, a)$: the maximum reward that can be obtained from state s_{t+1} (weighted by the learning rate and discount factor γ)

The learning process in Q-learning (Eq. 1) involves iteratively solving the TSP on the same set of points. After each step, the agent updates the Q-table by incorporating the received reward and estimating the Q-values of the next state.

Algorithm 1 Q-Learning

```

Initialize Q-Table with zeros
Initialize  $\epsilon$ ,  $\epsilon_{\min}$ ,  $\epsilon_{\text{decay}}$ ,  $\gamma$ ,  $\alpha$ 
for each episode do
  Initialize state  $s$ 
  while episode is not done do
    Choose action  $a$  from state  $s$  using policy derived from
    Q-Table (e.g.,  $\epsilon$ -greedy)
    Take action  $a$ , observe reward  $r$ , next state  $s'$ 
     $Q[s, a] \leftarrow Q[s, a] + \alpha \cdot (r + \gamma \cdot \max_a Q[s', a] - Q[s, a])$ 
    if  $\epsilon > \epsilon_{\min}$  then
       $\epsilon \leftarrow \epsilon \cdot \epsilon_{\text{decay}}$ 
    end if
     $s \leftarrow s'$ 
  end while
end for

```

2) Double Q-Learning: The double Q-Learning (Algorithm 2) is an enhancement over Q-learning, designed to mitigate the

overestimation issues in action value approximations inherent in Q-learning. It does this by using two separate Q-tables instead of one, each updated using information from the other, providing a more robust and unbiased estimate of action values which is different from the vanilla Q-Learning only taking the maximum Q-value for the updates. However, it can occasionally lead to underestimations and requires more computational resources due to the maintenance of two Q-tables. It is particularly beneficial in environments with high variance in the reward structure [52].

$$Q_{t+1}^A(s, a) = Q_t^A(s, a) + \alpha \left[r(s, a) + \gamma \max_{a'} Q^B(s', a') - Q_t^A(s, a) \right] \quad (5)$$

$$Q_{t+1}^B(s, a) = Q_t^B(s, a) + \alpha \left[r(s, a) + \gamma \max_{a'} Q^A(s', a') - Q_t^B(s, a) \right] \quad (6)$$

The equation above is similar to the vanilla Q-Learning but the difference lies in how these tables are updated in this study. When the algorithm is about to update Q-TableA (QA), the best action (the action with the highest Q-value) is selected based on QA, but it uses the Q-value for that action from Q-TableA (QB) to perform the update. Similarly, when updating QB, the proposed algorithm selects the best action based on QB, but uses the Q-value from QA to perform the update.

Algorithm 2 Double Q-Learning

```

Initialize Q-TableA and Q-TableB with zeros
Initialize  $\epsilon$ ,  $\epsilon_{\min}$ ,  $\epsilon_{\text{decay}}$ ,  $\gamma$ ,  $\alpha$ 
for each episode do
  Initialize state  $s$ 
  while episode is not done do
    Choose action  $a$  from state  $s$  using policy derived from
    Q-Table (e.g.,  $\epsilon$ -greedy)
    Take action  $a$ , observe reward  $r$ , next state  $s'$ 
    if  $\text{rand}() < 0.5$  then
       $a' \leftarrow \arg \max_a QA[s', a]$ 
      Update QA with eq.2
    else
       $a' \leftarrow \arg \max_a QB[s', a]$ 
      Update QB with eq.3
    end if
    if  $\epsilon > \epsilon_{\min}$  then
       $\epsilon \leftarrow \epsilon \cdot \epsilon_{\text{decay}}$ 
    end if
     $s \leftarrow s'$ 
  end while
end for

```

3) Epsilon greedy:

The epsilon-greedy strategy is a popular method for handling the exploration-exploitation dilemma in RL. The dilemma arises because, to find the optimal policy, an agent

needs to explore its environment to learn the reward associated with various actions in different states. But, the agent also needs to exploit its current knowledge to maximize the rewards it receives. If it only exploits its current knowledge, it may never discover better policies. If it only explores, it will fail to maximize the rewards that it could have received based on its current knowledge.

The epsilon-greedy strategy is a simple method for balancing exploration and exploitation. It works as follows:

- 1) With probability $1 - \epsilon$, the agent selects the action that it believes has the maximum expected reward. This is the exploitation part.
- 2) With probability ϵ , the agent selects an action randomly. This is the exploration part.

The parameter epsilon can be any value between 0 and 1, and it controls the balance between exploration and exploitation. A higher value means more exploration and less exploitation, while a lower value means more exploitation and less exploration [53] [54].

The epsilon-greedy strategy can be formalized as:

$$\begin{cases} \max_a Q_t(s, a), & \text{with probability } 1 - \epsilon, \\ \text{random}, & \text{from all actions with probability } \epsilon \end{cases}$$

4) Model Definition

- 1) **State:** In our context of TSP, a state is represented by the current position of the agent and the set of unvisited sampling points.
- 2) **Action:** An action corresponds to moving from the current sampling point position to an unvisited city.
- 3) **Reward:** The reward function in TSP could be designed to encourage shorter tours. A common approach is to use the negative of the travel cost as the reward. For example, if the agent moves from sampling point 1 to sampling point 2 and the cost of travel is 10 units, then the reward would be -10. The goal of the agent (Algorithm 1) would be to maximize the total reward, which would equate to finding the shortest possible tour.

IV. RESULTS DISCUSSION: THE DIFFERENCE BETWEEN LOCAL AND GLOBAL OPTIMUM

As elucidated in part III, certain metaheuristics have a propensity to get ensnared in local optima, engendering fluctuations in the cost function's behavior. A useful approach to comprehending the results of various Google OR-Tools algorithms is to anticipate the performance each algorithm might deliver.

Commonly utilized algorithms like Hill Climbing (HC) and Simulated Annealing (SA) elucidate the issue of an agent becoming stuck or attempting to evade local optima.

We have simulated a solitary peak landscape (eq 7) and a multiple peak landscape (eq 8), each represented as a 3D surface. The intent is to comprehend the agent's behavior in each scenario by assigning identical initial positions in both algorithms.

In the case of the singular peak, the agent commences at $\text{initial}=[0, 1]$ and for the multiple peaks, the agent initiates from position 1 $\text{initial}=[0.8, -0.5]$ and position 2 $\text{initial}=[0,$

$-1]$). We simulated a 3D depiction where the reward function aims to reach the peak, thus concentrating on a maximization problem. The agent is capable of moving (taking steps) across the grid-based environment such as North, South, East, West, as well as NE, NW, SE, SW, with a constant step size of $\text{stepSize} = [0.05, 0.05]$. This ensures consistent initial decision-making.

$$o = - \sum_{i=1}^n x_i^2 \quad (7)$$

$$o = -1 (0.2 + x_1^2 + x_2^2 - 0.1 \cos(6\pi x_1) - 0.1 \cos(6\pi x_2)) \quad (8)$$

For the SA algorithm, the optimal temperature and alpha values were determined to be 1 and 0.99, respectively, established through experimentation and tuning.

In equation 7's simulation, both methods reach a cost of 0, indicating they have achieved the peak.

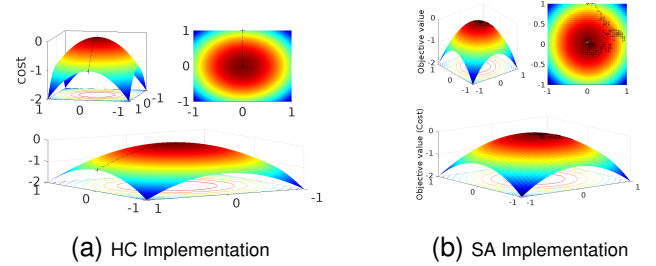


Fig. 1. Comparison between HC and SA by implementing Eq. 7.

The cost graph reveals that for a simple problem like this, HC is more suitable, as SA tends to overexploit the space, resulting in a less smooth cost vs iteration graph. This is also an initial insight into understanding the probability and temperature behaviors in SA implementation:

In the process's early stages, when the temperature is high, the algorithm is more likely to accept worse solutions. This allows the algorithm to explore the search space and avoid getting stuck in local minima. As the temperature decreases, the agent approaches the goal (global minimum), and the probability of accepting worse solutions decreases. This steers the algorithm towards the best solution.

The temperature parameter in SA governs the equilibrium between exploration and exploitation. At the search's onset, the high temperature allows the algorithm to explore a vast range of solutions, both better and worse. As the search progresses and the agent approaches the goal, the temperature is gradually decreased. This 'cooling schedule' lessens the likelihood of accepting worse solutions and makes the search more concentrated.

In scenarios where the search space incorporates multiple peaks, the HC agent faces challenges in identifying the global optimum, as depicted in Fig. 3a. Conversely, the SA agent would be capable of pinpointing the global optimum. The following illustration (Eq. 8) showcases the agent's strategy of evading local optima to determine the most significant optimum within its region. Fig. 3b delineate the navigational route the agent undertakes to ascend to the hill's apex. These

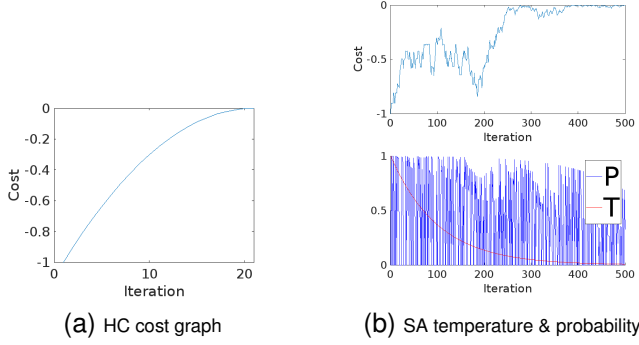


Fig. 2. Cost graph results for HC and SA using Eq. 7.

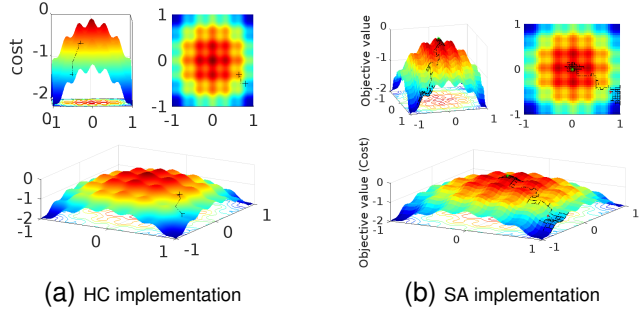


Fig. 3. Heat map comparison between HC and SA by Implementing Eq. 8 by starting on position 1.

figures also represent the evolution of its travel cost function alongside the diminishing temperature and probability management, as shown in Fig. 4b. The resulting graph clearly demonstrates the behavior described above, illustrating the impact of temperature and the exploration-exploitation trade-off in the simulated annealing algorithm and its attempts to break free from local optima, occasionally opting for inferior decisions even when better alternatives were previously available by accepting varying probabilities. It is critical to mention that achieving such a satisfactory solution required multiple simulation runs, leading to an optimal solution in approximately 1 out of 5 simulations. This behavior can be attributed to several factors, such as the Temperature Schedule decreasing too swiftly, thereby inhibiting the exploration of more regions. Additionally, the initial temperature may be too low, leading to a premature convergence. Other contributing factors include multiple simulation runs and the complexity of the Neighborhood Function. It is also observed that the agent tends to favor horizontal (North, South, East, West) rather than vertical movements, possibly to avoid the increased complexity of the chosen route.

Hill Climbing starts from an arbitrary solution and iteratively makes small changes to improve it. However, it often gets stuck in local optima because it lacks a mechanism to explore other regions of the solution space once it reaches a peak. As a result, the algorithm may exhibit significant variations as it explores different solutions, leading to a more erratic cost curve. The improvement is more gradual and less erratic when a higher number of sampling points is considered.

On the other hand, Simulated Annealing utilizes a temperature-controlled mechanism, which allows it to escape

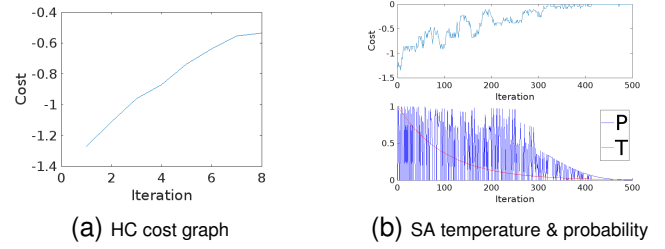


Fig. 4. Cost graph results for HC and SA using Eq. 8 in position 1.

local optima and increase the likelihood of finding the global optimum. This is achieved by occasionally accepting "bad" moves (moves that lead to worse solutions) to explore more of the solution space. As the "temperature" decreases over time, the algorithm becomes less likely to accept bad moves and more inclined to exploit the regions of the solution space that have been identified as good, with the hope of converging on the global optimum.

V. EXPERIMENTAL RESULTS AND VALIDATION

A. Experiment Set-up

1) *Hardware Set-up*: The GPP testing and implementation is conducted using a Jetson Orion development kit and a Lenovo ThinkPad P53 equipped with an Intel Core i7-9750H CPU running at 2.6GHz (12 cores) and an Nvidia Corporation TU177GLM GPU. The ThinkPad P53 also has 16GiB of memory. The prototyped ROMIE is presented in Fig. 14.

2) *Software language*: Python and MATLAB are effective language for solving the Traveling Salesman Problem (TSP) due to their advanced numerical abilities, user-friendly syntax, and high-level problem-solving abstractions. Both have valuable features: Python includes packages like Numpy and Panda for array management, and MATLAB has a robust mathematical toolbox.

While JavaScript is used for web application backend functionalities, compiled languages like Java or C++ might be chosen when faster execution is needed. However, these languages typically require more coding.

Despite the potential speed advantages of Java and C++, the report primarily focuses on Python and MATLAB because of their popularity among engineers and the abundance of open-source TSP models they offer.

3) *Webframe work*: Django was chosen as the web framework for the project due to its numerous advantages linked with Python capabilities. Its major qualities include its robustness, scalability, and extensive built-in features that facilitate rapid development.

4) *Data set*: The report utilizes six distinct datasets, featuring 20, 100, 200, and 1000 sampling points respectively (some graphs will show Tabu Search (TS) which was already computed). Each dataset comprises longitude and latitude coordinates, illustrating the region of Endeavour Mining's Ity Mine, located in Ivory Coast. The selection of this specific location facilitates practical application, enabling ROMIE to work with real datasets that mining engineers and geologists might select to carry out their prospecting efforts.

TABLE II

OVERALL TSP FOR RUNNING THE PRE-DEFINED DATASET USING GOOGLE OR-TOOLS, Q-LEARNING, AND DOUBLE Q-LEARNING ON A PROSPECTING LAND OF 11KM² 1/2. TOUR LENGTH (TL) IN M, GAP TO BEST (GB) IN % AND T (TIME) IN SECONDS ARE THE THREE PARAMETERS STUDIED.

	TSP20			TSP100			TSP500			TSP1000		
Method	TL (m)	GB (%)	T (s)	TL (m)	GB (%)	T (s)	TL (m)	GB (%)	T (s)	TL (m)	GB (%)	T (s)
PMCA	12951	17	0.01	26094	6.1	0.2	38048	5.8	0.9	80404	1.0	60
LCI	12949	17	0.01	26133	6.3	0.1	38345	6.6	0.5	82144	3.2	49
LCA	12951	17	0.01	25931	5.5	0.2	38532	7.1	0.9	81873	2.9	52
C	12951	17	0.01	26211	6.6	0.2	37739	4.9	0.7	79704	0.1	49
GD	13306	20	0.01	26233	6.7	0.1	38486	7.0	0.6	80003	0.5	50
GLS	13306	20	0.01	25662	4.4	33.4	36862	2.5	108	79919	0.4	92
SA	13306	20	0.01	26088	6.1	0.1	38486	7.0	0.4	79607	0	53
GTS	13306	20	0.01	26088	6.1	0.1	38486	7.0	0.6	79607	0	49.8
Q-L	11096	0	0.02	24588	0	0.3	35962	0	13	84589	6.3	120
D Q-L	11096	0	0.05	24687	0	0.4	35985	0.1	16.3	84647	6	120

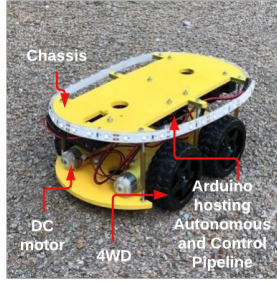


Fig. 5. Miniature size of ROMIE for testing the pipeline and mechanical properties.

B. Google OR-Tools VS RL comparative table

The results have been computed for three different methods: OR-Tools, Q-Learning and Double Q-Learning. Tables II and III provide a summary of the results obtained during the simulation. The analysis of the results is provided in the subsequent sections to evaluate the performance of TSP model.

C. ORTOOLS Implementation and Results

Google OR-Tools algorithms are partitioned into two categories: First Solution (FS) Strategies and Local Search (LS) Meta-heuristics. Broadly speaking, the performance of first solution strategies aligns closely with certain Local Search Metaheuristics. The following Table IV provides a comparative evaluation that we generated of Google OR-Tools' performance in terms of first solution and local search strategies relative to the optimal performance from the dataset.

TABLE III
AVERAGE GOOGLE OR-TOOLS RESULTS ACROSS THE DATASET FOR EACH SOLUTION STRATEGY

	TSP20	TSP50	TSP100	TSP200	TSP500	TSP1000
FS	16.7%	2.7%	8.1%	6.9%	2.2%	2.4%
LS	19.9%	2.7%	5.9%	6.0%	1.1%	0.2%

The performance of each meta-heuristic set can be visualized through a graph depicting the amount of time required to compute the best TSP tour by taking the running average to obtain smoother results. The red circle shows the cost effective

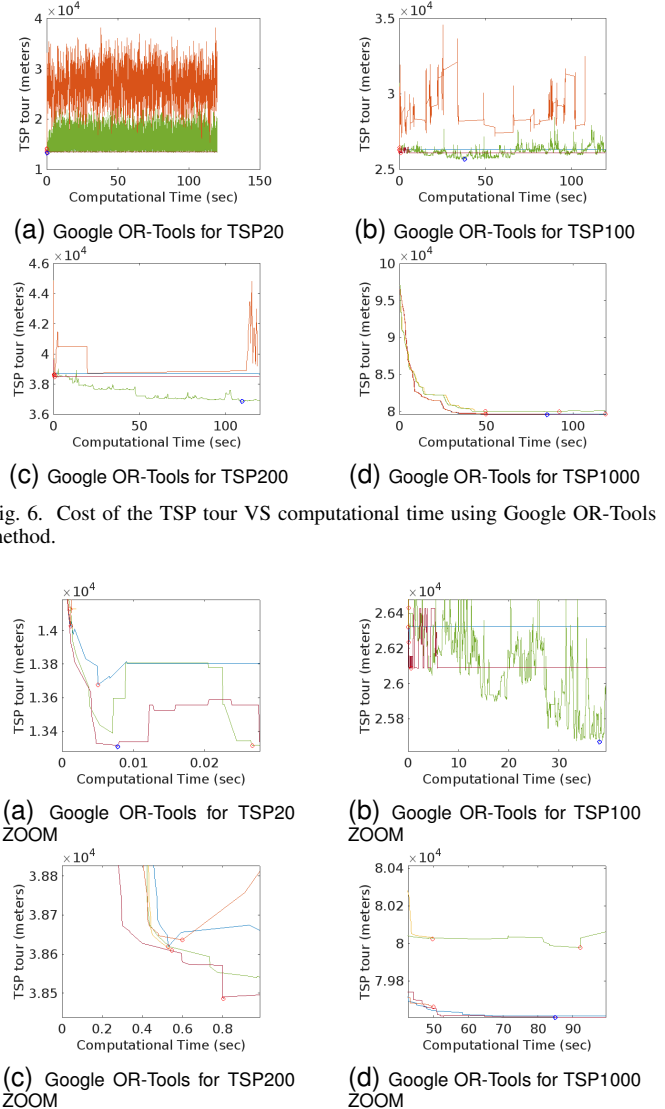


Fig. 6. Cost of the TSP tour VS computational time using Google OR-Tools method.

Fig. 7. Cost Of The TSP Tour VS Computational Time Using Google OR-Tools Method ZOOM

distance results for each methods and the blue circle shows the overall cost effective distance result of the data set.

These charts elucidate the stability and variability of each method for a specified quantity of sampling points aimed at determining the optimal TSP tour. As verified earlier in

our comparison between Simulated Annealing (SA) and Hill Climbing (HC), it is projected that with an increased count of sampling points, the agent tends to break free from local optima more swiftly than HC, even if the disparity is marginal for larger data sets (0.3%). Nonetheless, the promptness of a method's convergence to the objective function underscores its stability and precision, thus ensuring a reliable forecast for subsequent computations of the best TSP tour. For a smaller set of sampling points, the meta-heuristic method seems to over-complicate the problem, resulting in a highly variable pattern as it gets trapped in local optima.

The Gradient Descent (GD) method typically halts its computation first, often finding itself ensnared in local optima (refer to Fig. 7 for the yellow line).

The following figures represents different OR-Tools algorithms: in cyan (GTS), in yellow (GD), in lime (GLS), in red (SA) and in orange (TS).

- 1) **TSP20:** Results demonstrate a highly inconsistent and fluctuating Generalized Local Search (GLS) and Tabu Search (TS), in contrast to Guided Tabu Search (GTS) and SA, which rapidly locate the optimal solution and evade local optima.
- 2) **TSP100:** As we increase the number of sampling points, we see marginal improvements in the behavior of GLS and TS. GTS quickly avoids local optima, but SA seems to have more difficulty escaping local optima, exhibiting slight volatility in the initial 5 seconds. SA and GLS showcase a more binary behavior in the zoomed version (Fig.7b). GLS achieves the best solution for this round.
- 3) **TSP200:** A clear differentiation is seen here with SA, TS, and GTS struggling to avoid local optima, and an unusual exploration-exploitation tradeoff in TS. Nonetheless, GLS consistently seems to escape local optima as the problem's complexity (i.e., more sampling points) increases, resulting in the best outcome within this dataset.
- 4) **TSP1000:** This is the sole instance where all methods exhibit smooth behavior, demonstrating precision in terms of exploration and exploitation. However, GTS outperforms the others, closely followed by the remaining methods. GTS's success can be attributed to its strategy of maintaining a "tabu list" of recent solutions and forbidding or penalizing moves that revert to these solutions. This approach is often more effective at finding global optima in complex solution spaces, which may explain its superior performance here.

Here is one of the results of a tsp tour for Google OR-Tools:

D. RL Implementation and Results

The results from both Q-Learning and Double Q-Learning are highly pleasing, achieved through diligent hyperparameter adjustments as elaborated earlier. Tables II and III showcase the consistent supremacy of both Q-Learning and Double Q-Learning for problems ranging from TSP20 to TSP200, with an average difference of approximately 8.2% to FS and LS. However, performance deteriorates as the number of sampling points escalates, with a noticeable performance gap for

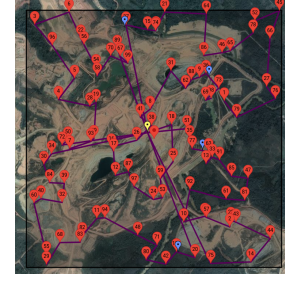


Fig. 8. Google OR-Tools TSP100.

TSP500 at 1.36% for QL and 3% for Double QL. Evidently, TSP1000 encounters difficulty in escaping local optima, standing as the least efficient of the dataset, diverging more than 6% from the optimal solution. Despite various attempts at hyperparameter tuning, this deviation can be attributed to the high computational complexity and the extensive Q-table, which results in incomplete learning and subsequently, subpar solutions. This could also be due to the balance between exploration and exploitation, which might have been skewed towards lower exploration. This is experienced by the rapid epsilon decay applied in the model. The swift convergence is due to a limited number of episodes.

The graphical representation of the iterations against the reward, distance (negative reward), and the decay in epsilon value for the TSP500 dataset can be seen in Fig. 9. All training graphs across the dataset exhibit a similar pattern. The triangular pattern observed can primarily be attributed to the alternation between exploration and exploitation phases in the Q-learning algorithm. The algorithm maximizes the current best action during the exploitation phase, leading to an increase in reward. Subsequently, during the exploration phase, it opts for a random action to accumulate new knowledge. However, as this action might not be the most optimal one, the reward may decrease, causing a downward slope in the graph and forming the characteristic triangular pattern.

The learning rate, denoted as alpha, plays a pivotal role. This parameter dictates how much the new information will replace the old. A non-optimal learning rate could result in the algorithm 'forgetting' beneficial strategies, causing the reward to drop after reaching a peak. This phenomenon contributes to the triangular shape of the graph. Therefore, to address this, one may need to fine-tune the hyperparameters or provide a stable environment during the training phase to ensure a consistent increase in reward over iterations.

Fig. 10 depicts the physical route that ROMIE will follow for different datasets, as generated by the Q-Learning algorithm

E. ROMIE GPP Application of The TSP

This study was carried out in conjunction with the development of the developed GPP software, which resulted in a user-friendly web interface. The interface, built on Django, offers efficient communication between JavaScript and Python files, utilizing the Fetch API for HTTP requests and integrating a security feature called CSRF token.

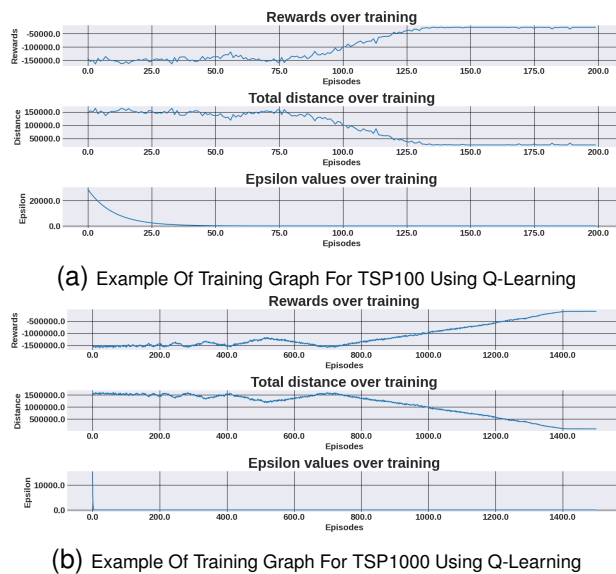
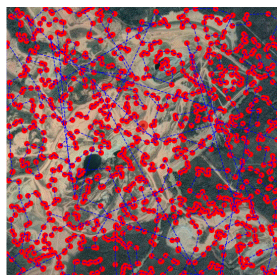


Fig. 9. Results For Q-Learning and Double Q-Learning Implementation.



(a) TSP graph for TSP 1000

Fig. 10. Results of ROMIE Waypoints route generated by Q-learning.

The primary objective was to create an interface that is intuitive for mining engineers, geologists, and researchers. This led to the incorporation of an interactive Google Map. Users can interact with this map in a number of ways, including selecting the mining site's location, delineating the site by dragging on the map (which simultaneously generates the site's coordinates), and choosing sampling points.

The users have two options for selecting sampling points: they can either use a grid-based method, or manually click on the map to specify the points where the robot needs to conduct prospecting. Upon the selection of sampling points, the system generates the most efficient TSP route and displays the path that ROMIE will take.

The interface is organized to have the interactive map on one side, and the user input fields on the other. This layout enhances the user experience by providing a clear and convenient workspace.

In essence, this study demonstrates a practical application of solving the TSP problem in the mining industry, bridging the gap between computational solutions and real-world challenges.

VI. CONCLUSION

In this study, a new unmanned ground vehicle (UGV) was developed using innovative software, e.g., Q-learning-

based Google methods, and hardware tools. Also, a comparative analysis of the performance of Google OR-Tools algorithms and reinforcement learning (RL) algorithms, such as Q-Learning and Double Q-Learning. This work offers a comprehensive analysis of the Global Path Planning (GPP) subsystem for UGVs employed as autonomous mining sampling vehicles. A newly developed robotic ore mineral identification and exploration (ROMIE) vehicle is designed and used for geochemical mining sampling processes. GPP is pivotal to ROMIE's efficient functioning, as it necessitates resolving the well-known graph theory problem, the Traveling Salesman Problem (TSP). The simulation tests were conducted on a data set ranging from 20 to 1,000 sampling points. The achieved results were experimentally validated in this report.

The findings indicate that, on average, the Local Search algorithm in Google OR-Tools deviates from the optimal solution by 5.7%, compared to the First Solution strategy's deviation of 6.3%. For sampling points less than 500, Q-Learning exhibits superior performance compared to Double Q-Learning and Google OR-Tools. However, for problems with 500 sampling points, the Guided Local Search algorithm shows superior performance, and for 1,000 points, the Generic Tabu Search algorithm is more effective.

The researchers see a need for future research to explore the application of Deep Neural Networks. Investigating the current state of the art, such as UTSP, or implementing Deep Q-Learning algorithms could potentially yield even better results and seems to be the logical next step in advancing this field.

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