Introduction to AI and ML

Matrix Project

Binaya Kumar Sahoo(EE17BTECH11048),
Hrithik Pawar(EE17BTECH11006)

February 14, 2019

Geometry Question:

The point

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

is translated parallel to the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4 \tag{1}$$

by $2\sqrt{3}$ units. If the new point $\mathbf{B}$ lies in the third quadrant, then find the equation of the line passing through $\mathbf{B}$ and perpendicular to L.

Approach to Question:

The required point which is at a distance of $2\sqrt{3}$ units can be found by adding the given point with $2\sqrt{3}$ times the direction vector of the given line. Then the equation of the required line is calculated by using slope of given line.

Solution in Matrix form:

Given point:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2}$$

Given line:

$$L_1 : \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4 \tag{3}$$

Normal vector of given line:

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4}$$

Direction vector of given line:

$$\mathbf{m}_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{n}_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{5}$$

Solution in Matrix form:(contd)

Required point:

$$\mathbf{B} = \mathbf{A} + 2\sqrt{3} * \mathbf{m}_1/\sqrt{2} \tag{6}$$

or

$$\mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 2\sqrt{3} * \begin{pmatrix} -1 \\ -1 \end{pmatrix} /\sqrt{2} \tag{7}$$

or

$$\mathbf{B} = \begin{pmatrix} 2 - \sqrt{6} \\ 1 - \sqrt{6} \end{pmatrix} \tag{8}$$

Solution in Matrix form:(contd)

Slope of reqd. line:

$$\mathbf{m}_2 = \mathbf{n}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{9}$$

Normal vector of reqd. line:

$$\mathbf{n}_2 = \mathbf{m}_1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{10}$$

Therefore, equation of reqd. line:

$$L_2 : \mathbf{n}_2^T (\mathbf{x} - \mathbf{B}) = 0 \tag{11}$$

or

$$\begin{pmatrix} -1 & -1 \end{pmatrix} \mathbf{x} = -\begin{pmatrix} -1 & -1 \end{pmatrix} \begin{pmatrix} 2 - \sqrt{6} \\ 1 - \sqrt{6} \end{pmatrix} \tag{12}$$

or

$$L_2 : \begin{pmatrix} -1 & -1 \end{pmatrix} \mathbf{x} = 3 - 2\sqrt{6} \tag{13}$$

Solution Figure: