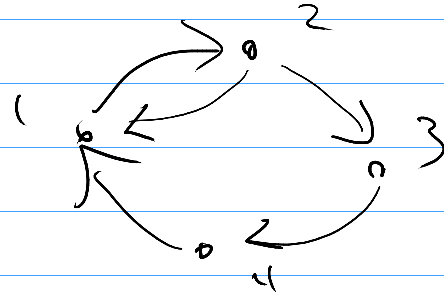
Math 322

~~Q5~~ $M_{R^*} = M_R \vee M_R^{[2]} \vee \cdots \vee M_R^{[|A|]}$

9.5 (25a) $A = \{1,2,3,4\}$

$R = \{(1,2),(2,1),(2,3),(3,4),(4,1)\}$

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$



Use: $M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee M_R^{[4]}$

$$M_R^{[2]} = M_R \odot M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} =$$

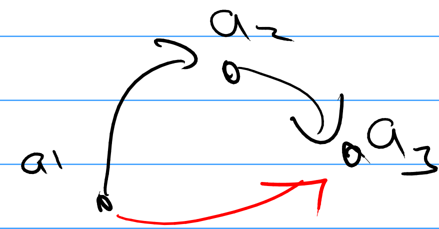
$$M_R^{[2]} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_R^{[3]} = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{M_R^{[2]}} \underbrace{\odot}_{\odot} \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{M_R} = \text{etc.}$$

Alg. 1 $\quad M_{R^*} = M_R \vee M_R^{[2]} \vee \cdots \vee M_R^{[|A|]}$

<u>Warshall's</u>

Ex $\quad W_0 = M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & \cancel{0} & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 1 & \cancel{0} & 0 & 0 \end{array}$



(2,2)

(2,1) (1,2)

(4,1)

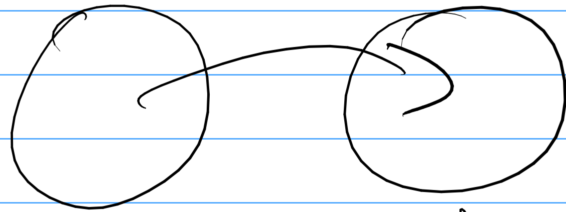
(4,2)

$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \cancel{0} & 1 & \cancel{0} & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 1 & 1 & \cancel{0} & 0 \end{array}$

$W_2 = \begin{bmatrix} 1 & 1 & 1 & \cancel{0} \\ 1 & 1 & 1 & \cancel{0} \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & \cancel{0} \end{bmatrix} \qquad W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

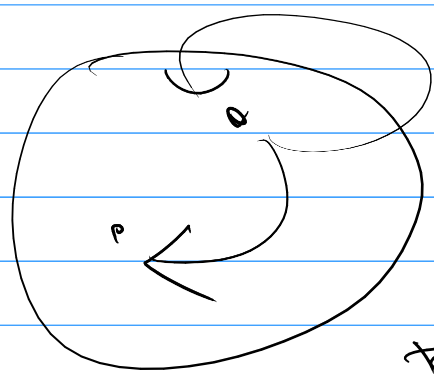
$W_4 = M_{R^*} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

8.5 Relations .. set of (a,b)



Domain   Codomain

R on a set A.



what rules on R are needed for R to act as the question..

A

"Are a and b the same?"

what does R need?

1. Reflexive
2. Symmetric
3. transitive

If R a relation on the set A

is
1. reflexive
2. symmetric
3. transitive

then R is called an <u>equivalence relation</u>

<u>Notation</u>: if R <u>is</u> an equivalence relation and $(a,b) \in R$

aRb rather use $a \sim b$

"a is related to b" "a is equivalent to b"

ex's p. 563 (3)

functions from $\mathbb{Z}$ to $\mathbb{Z}$

$f(z) : \mathbb{Z} \to \mathbb{Z}$

<u>ex</u> $f(z) = 3z + 2$

R relate some $f_1(z)$ to some $f_2(z)$

A

(5a) $\boxed{f R g \text{ when } f(1) = g(1)}$ ✓

ex $f(z) = \boxed{z+3}$ ✓ $f(1) = 4$ ✓

$g(z) = z^2 + 3$ $g(1) = 4$

1) is R reflexive?: $\forall f\,(f R f)$

$\forall f\,(f(1) = f(1))$ $\boxed{\text{true.}}$

2) is R symmetric?:

$\forall f \forall g\,(f R g \rightarrow g R f)$

$\forall f \forall g\,(f(1) = g(1) \rightarrow g(1) = f(1))$ $\boxed{\text{true}}$

3) is R transitive?:

$\forall f \forall g \forall h\,(f R g \wedge g R h \rightarrow f R h)$

$\forall f \forall g \forall h\,(f(1) = g(1) \wedge g(1) = h(1) \rightarrow f(1) = h(1))$

$\boxed{\text{true}}$!

R is an equivalence relation.

Equivalence Classes.

set of all elements related to an element a.

$$[a]_R = \{ s \mid aRs \}$$

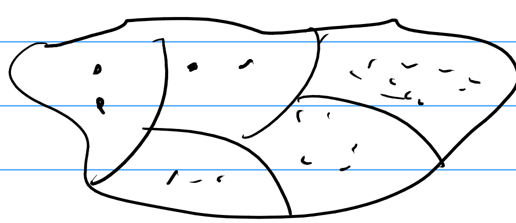
Thm: the three statements are equivalent

① aRb

② $[a]_R = [b]_R$

③ $[a]_R \cap [b]_R \neq \emptyset$.

Partitions. A partition of a set is a collection of disjoint non-empty subsets that have the set as their union.



S

A_i is a partition of S iff

① $A_i \neq \emptyset$

② $A_i \cap A_j = \emptyset$ if $i \neq j$

③ $\bigcup_i A_i = S$

Thm: the equivalence classes of R an equivalence relation on S form a partition.

ex5 $f R g$ when $f' = g'$

reflexive? $\forall h\,(hRh)$

$\forall h\,(h' = h')$ $\boxed{\text{true}}$

symmetric? $\forall h \forall g\,(hRg \to gRh)$

$\forall h \forall g\,(h' = g' \to g' = h')$

$\boxed{\text{true}}$

transitive?

$\forall h \forall f \forall g\ (h' = f' \wedge f' = g' \rightarrow h' = g')$

$\boxed{\text{true}}$

Yes. R is an equi. relation.

$[f(x) = x^2]_R = \{ s \mid x^2 R s \}$

$= \{ s \mid 2x = s' \}$

$s' = 2x$

$s = \int 2x\, dx = x^2 + C$

$[x^2]_R = \{ x^2 + C \text{ for } C \in \mathbb{R} \}$

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thm: Given a partition of a set S there is an equivalence relation on S that has as its equivalence classes the partition.

on $S = \{\text{all people}\}$

1. $R = \{(a,b) \mid a, b \text{ are the same age}\}$

a) ref?: $\forall a$ (a, a are the same age) true.

b) sym?: $\forall a \forall b$ (a, b are same age

$\rightarrow b, a$ are same age) true.

c) tran?: $\forall a \forall b \forall c$ (

a, b same age $\wedge$ b, c same age $\rightarrow$ a, c same age) true.

Yes

2. $\{(a,b) \mid a \text{ met } b\}$

a) ref?: $\forall a$ (a met a) true

b) sym?: $\forall a \forall b$ (a met $b \rightarrow b$ met a) true

c) trans?:

$\forall a \forall b \forall c$ (a met $b \wedge b$ met c $\rightarrow$ a met c)

No: Mark met (student)
$\wedge$ (student) met (student parent)
but Mark ~~met~~ (student parent)