Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 7

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Name: ____

Due date: Thursday March 14th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. Find a 2×2 matrix A that represents a linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ such that

$$T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -9 \end{bmatrix}.$$

2. Find bases for the kernel and the range of the linear transformation $T : \mathbf{R}^4 \to \mathbf{R}^3$ represented by the matrix

$$A = \begin{bmatrix} -3 & 0 & -1 & 7 \\ 1 & 6 & 4 & 0 \\ 1 & 2 & 2 & 5 \end{bmatrix}.$$

Is T onto? Is T one-to-one?

3. Determine all the numbers x_1, x_2 and x_3 such that the vectors

$$u_1 = \begin{bmatrix} -3 \\ 1 \\ x_1 \\ 3 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 5 \\ 1 \\ 1 \\ x_2 \end{bmatrix}, \quad \text{and} \quad u_3 = \begin{bmatrix} x_3 \\ 3 \\ 2 \\ 4 \end{bmatrix}$$

are all mutually orthogonal.

4. Find a matrix A that represents a linear operator T in $\mathbf{R}^2$ such that T maps the point $(4, 5)$ to $(-1, 2)$ and the point $(-1, -1)$ to $(3, 2)$.

5. Use a *for loop* to write a Matlab function that takes an arbitrary $m \times n$ matrix A and an $m \times 1$ column vector v and returns the sum of all the distances from v to the columns of A.

6. For the vectors

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \\ -2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 1 \\ -2 \\ 3 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 4 \end{bmatrix} \quad \text{and} \quad v_5 = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

compute the following:

(i) $\|v_1\| + v_2 \cdot v_3 =$

(ii) $\|v_1 - v_5\| =$

(iii) $d(v_3, v_2) =$

(iv) Angle between v_4 and $v_3 =$

(v) $\mathrm{proj}_{v_2}(v_4) =$

(vi) $\mathrm{comp}_{v_1}(v_3) =$

(vii) $\mathrm{refl}_{v_3}(v_5) =$

(viii) $\mathrm{comp}_{v_2}(v_3) =$

(ix) $d(v_1 + v_4, v_2 - v_3) =$

(x) $\mathrm{proj}_{v_3}(v_1) =$

7. Consider the vectors

$$w_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \\ -2 \\ 1 \\ 5 \end{bmatrix} \quad \text{and} \quad w_2 = \begin{bmatrix} 4 \\ 1 \\ 7 \\ 1 \\ 2 \\ -8 \end{bmatrix}.$$

Find a basis for the subspace of all the vectors in $\mathbf{R}^6$ which are orthogonal to w_1 and w_2.

8. True or False - **Circle the right one** (1 point each)

T or **F**. The linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ represented by the matrix

$$B = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 5 & 0 \\ 4 & 2 & -1 \end{bmatrix}$$

is onto.

T or **F**. If the non-zero vectors v_1, v_2, and v_3 are mutually orthogonal, then they are linearly independent.

T or **F**. If $null(A) = \{0\}$, then the linear transformation represented by A is one-to-one.

T or **F**. An $n \times n$ matrix P is orthogonal if and only if its columns are mutually orthogonal vectors.

T or **F**. The linear transformation represented by the matrix

$$D = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & -1 \end{bmatrix}$$

is one-to-one.

Points obtained in this assignment (out of 16): ______________