Math 321

Q's $S = \{a, a+1, a+2, \ldots, a+(n-1)\}$

$s_i \in S \qquad s_i \bmod n \in \{0, 1, 2, \ldots, n-1\}$

$$a \bmod n = a \bmod n + 0$$
$$(a+1) \bmod n = a \bmod n + 1$$
$$(a+2) \bmod n = a \bmod n + 2$$
$$\vdots$$
$$(a+(n-1)) \bmod n = a \bmod n + (n-1)$$

$|S| = n \qquad |\text{remainders}| = n$

b/c $\frac{n}{n} = 1$ all remainders occur exactly once.

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

$$(x+y)^n = \sum_{j=0}^{n} \frac{n!}{(n-j)!\,j!} x^{n-j} y^j$$

Ex $$(x-3)^4 = \frac{4!}{4!\,0!} x^4 + \frac{4!}{3!\,1!} x^3(-3)^1 + \frac{4!}{2!\,2!} x^2(-3)^2 + \frac{4!}{1!\,3!} x(-3)^3 + \frac{4!}{0!\,4!}(-3)^4$$

$$= x^4 + 4x^3(-3) + 6x^2(-3)^2 + 4 \cdot x \cdot (-3)^3 + (-3)^4$$

$$= x^4 - 12x^3 + 54x^2 - 108x + 81$$

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

ex 1 let $x=1$ $y=1$

$$\boxed{2^n = \sum_{j=0}^{n} \binom{n}{j}}$$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = \boxed{2^n}$$

ex 2 let $x=+1$ $y=-1$

$$0 = \sum_{j=0}^{n} \binom{n}{j} \boxed{(+1)^{n-j}} (-1)^j$$

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots$$

$$\binom{n}{0} + \binom{n}{2} + \cdots = \binom{n}{1} + \binom{n}{3} + \cdots$$

ex 3 let $x=1$ $y=2$

$$3^n = \sum_{j=0}^{n} \binom{n}{j} 1^{n-j} 2^j = \sum_{j=0}^{n} \binom{n}{j} 2^j$$

$$3^n = 1 + n\cdot 2^1 + \binom{n}{2}\cdot 2^2 + \binom{n}{3}2^3 + \cdots + \binom{n}{n}2^n$$

Pascal's Triangle (coeff. $(x+y)^n$)

$n=0$ — 1

$n=1$ — $1 \quad 1$

$n=2$ — $1 \quad 2 \quad 1$

$n=3$ — $1 \quad 3 \quad 3 \quad 1$

$n=4$ — $1 \quad 4 \quad 6 \quad 4 \quad 1$

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{3}{1} = \binom{2}{1} + \binom{2}{0}$$

Pascal's Identity

$$\boxed{\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}}$$

goal: $\dfrac{(n+1)!}{k!(n-k+1)!}$

Pf:

$$\binom{n}{k} + \binom{n}{k-1} = \frac{(n-k+1)n!}{\underbrace{(n-k+1)k!(n-k)!}_{(n-k+1)!}} + \frac{k\cdot n!}{\underbrace{k\cdot(k-1)!}_{k!}(n-k+1)!}$$

$$= \frac{(n-k+1)\cdot n! + k\cdot n!}{k!(n+1-k)!} = \frac{[(n-\cancel{k}+1)+\cancel{k}]n!}{k!(n+1-k)!}$$

$$= \frac{(n+1)\cdot n!}{k!(n+1-k)!} = \frac{(n+1)!}{k!(n+1-k)!} = \binom{n+1}{k}$$

Pf

idea:

people = {Mark, ... }

|people| = 15

choose 5 people

$$\binom{15}{5} = \binom{\text{Set of 5}}{\text{with Mark}} + \underbrace{\binom{\text{Set of 5}}{\text{without Mark}}}$$

$$1 \cdot \binom{14}{4} + \binom{14}{5}$$

$$\binom{15}{5} = \binom{14}{4} + \binom{14}{5}$$

$T = \{a, \dots\}$ $\quad |T| = n+1$

choose k.

$$\binom{n+1}{k} = \binom{\text{has}}{a} + \binom{\text{doesn't have}}{a}$$

$$\binom{n+1}{k} = 1 \cdot \binom{n}{k-1} \qquad \binom{n}{k}$$

$$\boxed{\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}}$$

n	2^n	
0	1	$= 1$
1	2	$= 1+1$
2	4	$= 1+2+1$
3	8	$= 1+3+3+1$
4	16	$= 1+4+6+4+1$
5	32	$= 1+5+10+10+5+1$
6	64	$= 1+6+15+20+15+6+1$

1, 1, 2, 3, 5, 8, 13

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots \binom{n}{n} = 2^n$$

$$\binom{0}{0} = f_1 = 1$$

$$\binom{1}{0} = f_2 = 1$$

$$\binom{2}{0} + \binom{1}{1} = f_3 = 2$$

$$\binom{3}{0} + \binom{2}{1} = f_4 = 3$$

$$\binom{4}{0} + \binom{3}{1} + \binom{2}{2} = f_5 = 5$$

$$\binom{5}{0} + \binom{4}{1} + \binom{3}{2} = f_6 = 8$$