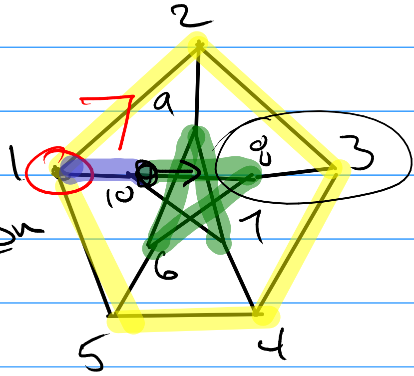
Math 322

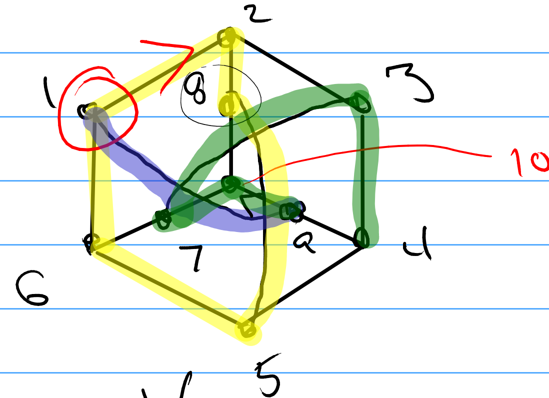
Q's

Isomorphism

u	v
1	1
2	2
3	8
4	5
5	6
10	9
8	10
6	7
9	3
7	4



U



V

Invariants: ① number of vertices: same

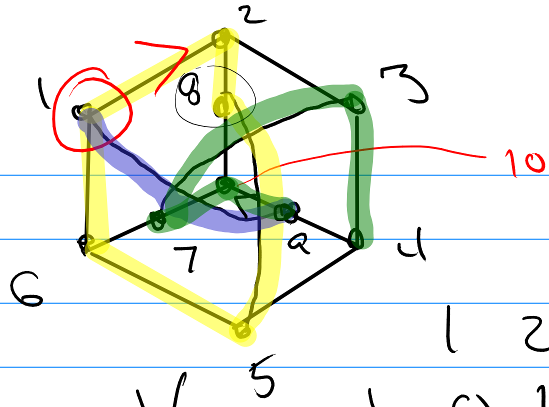
② number of edges: same

③ degree: all are $\deg(v)=3$

Maybe isomorphic!

$$A_u = \begin{bmatrix} 0&1&0&0&1&0&0&0&0&1 \\ 1&0&1&0&0&0&0&0&1&0 \\ 0&1&0&1&0&0&0&1&0&0 \\ 0&0&1&0&1&0&1&0&0&0 \\ 1&0&0&1&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&1&1&0 \\ 0&0&0&1&0&0&0&0&1&1 \\ 0&0&1&0&0&1&0&0&0&1 \\ 0&1&0&0&0&1&1&0&0&0 \\ 1&0&0&0&0&0&1&1&0&0 \end{bmatrix}$$

u	v
1 →	1
2 →	2
3 →	8
4 →	5
5 →	6
6 →	7
7 →	4
8 →	10
9 →	3
10 →	9



V

A_V

	1	2	8	5	6	7	4	10	3	9
1	0	1	0	0	1	0	0	0	0	1
2										
8										
5										
6										
7										
4										
10										
3										
9										

etc.

9.4 Paths and Connectedness

Paths: seq of edges from a to b.

$(a, x_1), (x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, b)$

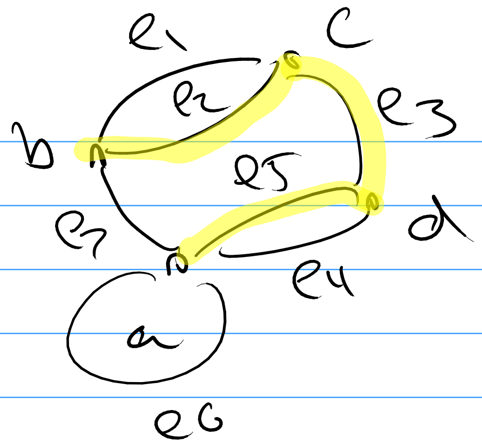
$a = x_0$, $b = x_n$

each (x_i, x_i) or $\{x_i, x_i\}$ are in E.

length is the number of edges (n)

if you can, without confusion, only list the vertices.

$a = x_0, x_1, x_2, \ldots, x_n = b$



Yellow path...

$\boxed{e_5, e_3, e_2}$

$\underbrace{a, d}, c, b$

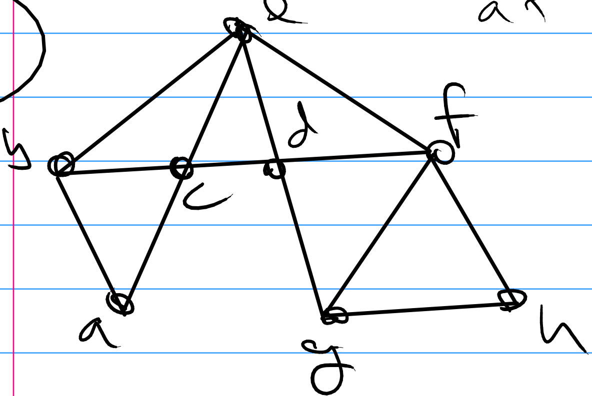
$\{a, d\} \leftarrow$ which?

Special Paths:

① Simple Path: an edge is only used once.

② Circuit: Starts & ends at same vertex $(a = b)$.

ex



ex's

1) not a path

a, d

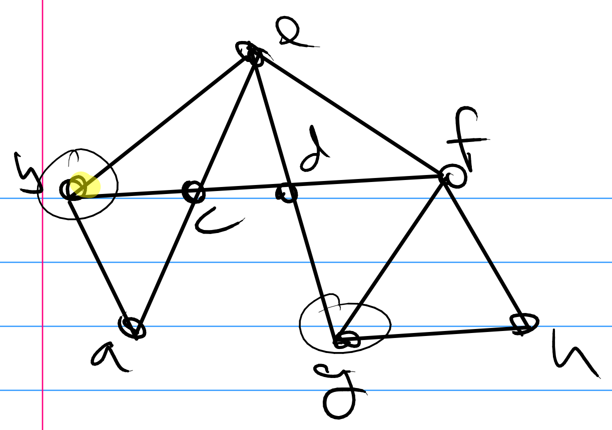
$a, b, \underbrace{c, g}_{\text{not an edge}}, f, h$

2) path of length 6

b, c, e, d, f, h, g (simple path)

3) path of length 4, not simple path

$a, \underbrace{\underbrace{b, e}, \underbrace{e, b}}_{\text{same edge}}, c$



Simple circuit visiting each vertex

g, h, f, e, b, a, c, d, g

simple circuit that crosses every edge. (no)

Simple path that crosses every edge

b, e, c, b, a, c, d, e, f, h, g, f, d, g

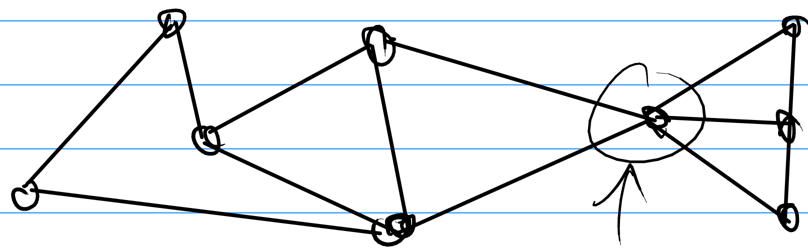
Connectedness

Def: G, an undirected graph, is connected if there is a path between all pairs of distinct vertices.

- If G is not connected it is made of two or more connected components.

- If you remove an edge from G and inc. the number of connected components it is a cut edge.

- If you remove a vertex with all edges that are assoc. with it and it inc the number of connected comp. it is a cut vertex



cut vertex

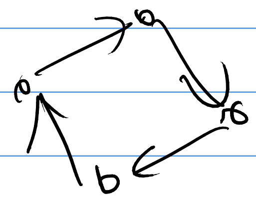
Directed Graphs:

① if there is a path to and from every distinct pair → $\boxed{\text{Strongly Connected}}$

② if you ignore direction and consider the graph as being undirected if it connected → $\boxed{\text{Weakly Connected}}$

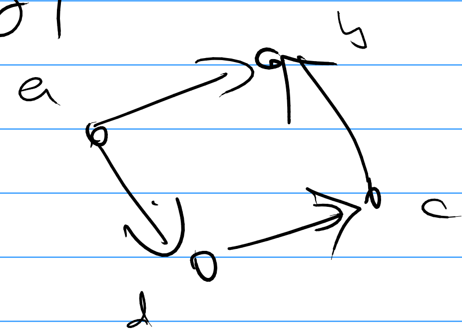
1. Strongly and weakly connected

ex



2. not strongly and is weakly connected

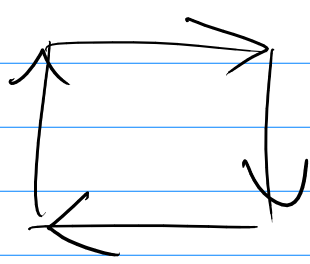
ex



3. Strongly and not weakly connected

Impossible.

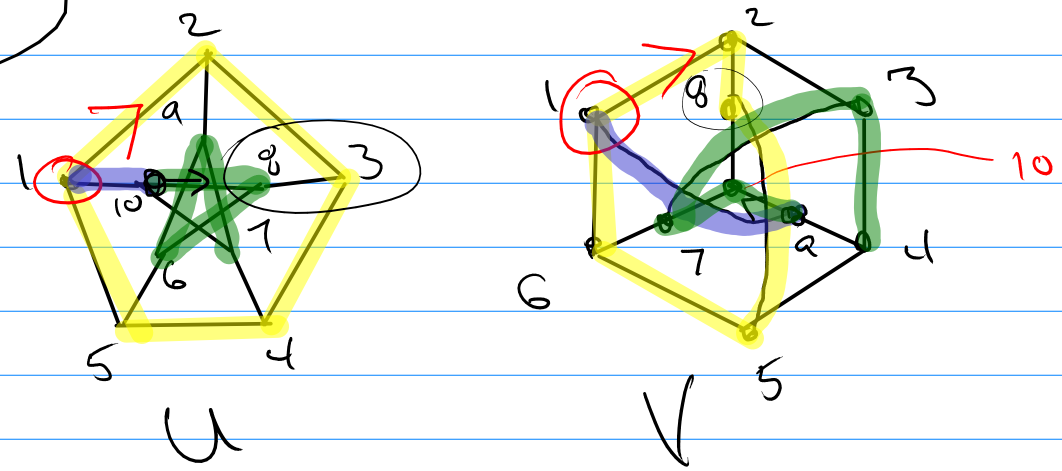
4. not strong ∧ not weak




Invariants and Isomorphisms.

<u>New Invariant</u>: <u>paths are invariant</u>

ex



Count possible paths:

A_G is G's Adj. Matrix.

$A_G^r = [c_{ij}]$

c_{ij} is the number of paths from v_i to v_j of length r.
