CIS770 Homework 3

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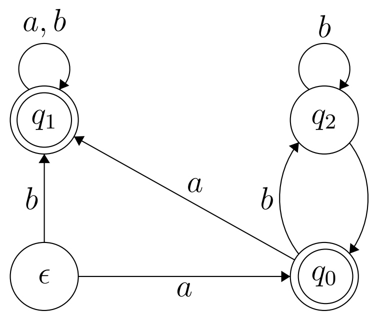
Problem1

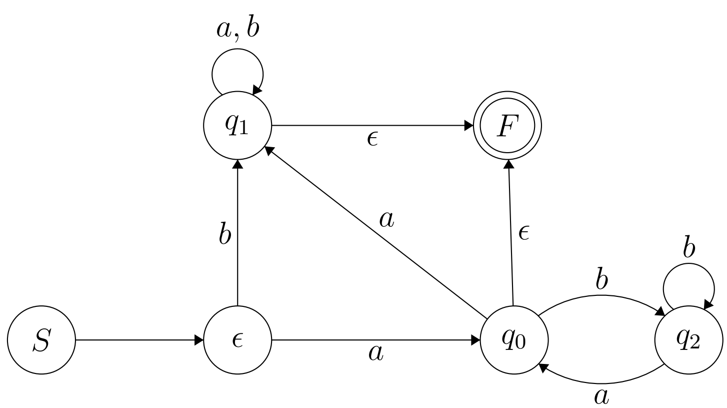
1.1.

This DFA was built directly. I first made a DFA to follow (abb*)* and then complemented it in order to fit the question. I have a state q_0 that remembers if it has seen the first a, and a state another state q_2 that remember if there was any number of b's following the first a as well as a dead state, q_1 that is gone to if it does not fit the language requirements. There is also an initial state q_ϵ, after this initial DFA was constructed I complemented it and the resulting DFA is below.

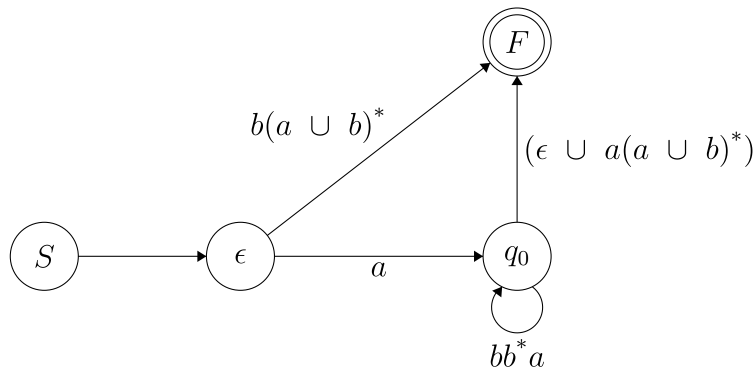


1.2.

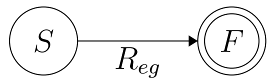
Step 1. Convert DFA to GNFA



Step 2. Remove q_2 and q_1



Step 3. Remove q_0 and ϵ



Now that all the states have been removed we can put together the pieces to make R_{eg} which is :

$$R_{eg} = b(a \cup b)^* \cup a(bb^*a)^*(\epsilon \cup a(a \cup b)^*)$$

Problem 2

2.1. left(A) = $\{\epsilon$, 10$\}$

2.2. left(A) = L(0^*1)

2.3. M = $(Q, \Sigma, \delta, q_0, F)$, M is the DFA that recognizes A, to determine if some word w belongs to left(A) we can simultaneously check the forward path at the initial state and backward path at the final state of w in our DFA M, the states of our new DFA are the same states as our old DFA M and the new initial state.

The new DFA M′= $(Q', \Sigma, \delta', q_0', F')$ where:

Q′= (Q x Q) $\cup$ $\{q_0'\}$

F′= $\{$(q,q) $|$ q $\in$ Q$\}$

$q_0' = \{q_0, F\}$: because we start at both ends of the original machine M

$$\delta'(q', a) = \begin{cases} \{q_0\} \text{ x } F & \text{if } q' = q_0' \text{ and } a = \epsilon \\ \{(q_1', q_2') | \delta(q_1, a) = q_1' and \delta(q_2, a) = q_2' & \text{if } q' = (q_1, q_2) \text{ and } a \in \Sigma \\ \emptyset & \text{else} \end{cases}$$

Problem 3

Assume C is regular, for contradiction, with p as the pumping lemma length. let w = $1^{p}01^{p}$ and w $\in$ C.

x, y, z such that $w = xyz$ and has the following properties:

$|y| > 0$

$|xy| \leq p$

Assume $x = 1^{i}$, $y = 1^{j}$, $z = 1^{k}01^{p}$, where i+j+k = p and j > 0.

Now w'= $xy^{0}z = 1^{i+k}01^{p}$. The number of ones preceding the first 0 is less than p and there are p ones following the first zero. Because there are p-1 number of ones preceding the first zero there must be p-1 number of ones following the first zero however there are p number of ones following so $w \notin$ C. C does not satisfy the pumping lemma and therefore is not regular.

Problem 4

4.1.

A = $F \cap L(ab^{*}c^{*}) = \{ab^{n}c^{n}|n \geq 0\}$, we can define a homomorphism h:$\{a,b,c\}^{*} \rightarrow \{0,1\}^{*}$ where h(a) = ϵ, h(b) = 0 and h(c) = 1. Now if we apply the homomorphism to our language A, h(A) = $\{0^{n}1^{n}|n \geq 0\} = L_{0^{n}1^{n}}$

Now consider h(A) to be regular, and let p be the pumping length for $L_{0^{n}1^{n}}$.
$w = 0^{p}1^{p}$
Since $|w|>p$ there are x, y, z such that $w = xyz$ where:

$|xy| \leq p$

$|y|>0$

and x = 0^{r}, y = 0^{s}, z = $0^{t}1^{p}$ where $|y|$ >0 and s >0

$$xy^{0}z = 0^{r}\epsilon 0^{t}1^{p} = 0^{r+t}1^{p}$$

since r+t < p, $xy^{0}z \notin L_{0^{n}1^{n}}$ this contradicts that h(A) is regular, thus proving F is also not regular.

4.2.

Let $p = 3$, where any $w \in$ F where $|w| \geq$ p.
Since i cannot be 0 and the word w is already divided for the case $i = 1$, we only need to show for cases when $i = 2$ and when $i \neq 2$, or when it is greater than 2.

If $i = 2$, divide w into x,y,z. $x =$ aa, y can be the next symbol and z makes up what is left of word w. This satifies the properties that $|xy| \leq$ p, and $|y| > 0$.

If $i \neq 2$, divide w into x,y,z. ...

Note: come back and finish 4.2