
QUIZ 6

Name:

Time: March 22 , 2016

Instructions: Please fill in the solutions in the space provided for the questions highlighted in red.

Consider the language $L_{01} = \{0^k 1^k \mid k \geq 0\}$. Consider the following grammar $G_{01} = (V, \Sigma, R, S)$, where $V = \{S\}$, $\Sigma = \{0, 1\}$, the rules $R = \{S \rightarrow \epsilon \mid 00S11 \mid 000S111\}$.

We will prove that all the words derived from the grammar belong to the language L_{01} . We will prove that by showing that the statement $S(i)$ below holds for all i .

$S(i)$: If $S \Rightarrow^* w$ in i steps, then $w \in L_{01}$.

If $S(i)$ holds for all i , then note that we have shown that all words derived by the grammar are in L_{01} , because if a word is derivable in the grammar then it is derivable in some i steps, hence, by $S(i)$, $w \in L_{01}$.

We will show that $S(i)$ holds for all i by induction on i .

1. Base Case $i = 1$: **What is $S(1)$?**

Show that $S(1)$ holds. (Hint: What ws are derivable in 1 step? Do they belong to L_{01} ?)

2. Induction step: **Prove that if $S(i)$ holds for $1 \leq i \leq n$, then $S(n+1)$ holds.**

(Hint: Let $S \Rightarrow^* w$ in $n+1$ steps. What are the different ways in which you can split the derivation $S \Rightarrow^* w$ into sub-derivations of length less than or equal to n ? Note that since n is at least 1, $S \Rightarrow \epsilon$ is not a derivation of $n+1$ steps.)