

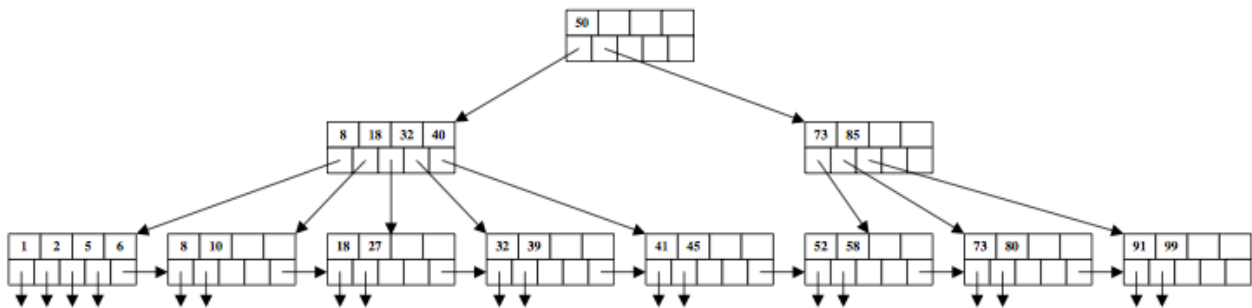
Homework Assignment 8 –part 1 [15 points] – due November 8th at 11:59PM

Note 1: Please remember that you are allowed to discuss the assigned exercises, but you should write your own solution. Identical solutions will receive 0 points.

Note 2: For full credit, show your work (not only the final answers).

Exercise I (B+ trees) [12 points]

Consider the following B-tree of degree $d = 2$ (i.e., each index node can hold at least $d = 2$ keys and at most $2d = 4$ keys):



- (a) What is the maximum and minimum number of records that the tree can hold (without increasing its height)? Explain your answer.

Minimum: The root needs to have at least 2 pointers. Thus, there will be at least two nodes on the second level, each having at least two keys and three pointers. That means each node on the second level points to 3 leaf nodes and each leaf should have at least two keys and pointers to two records. Therefore, we have at least $2 \times 3 \times 2 = 12$ records in a tree of height 2.

Maximum: The root can have up to 5 pointers to 5 nodes on level 2. Each of the nodes on level 2 can also have up to 5 pointers to leaf nodes. Each leaf node can have up to 4 pointers to records. Therefore, the maximum number of records that the tree can hold is $5 \times 5 \times 4 = 100$.

- (b) Show the steps in executing the following operation: Lookup the record 40. [See example 14.14 in the textbook]

$40 < 50 \rightarrow$ follow 1st pointer

$40 \geq 40 \rightarrow$ follow 5th pointer

First record is 41

$41 > 40$, so we conclude that 40 does not belong to the tree.

- (c) Show the steps in executing the following operation: Lookup all records in the range from 10 to 58 (including 10 and 58). [See example 14.15 in the textbook]

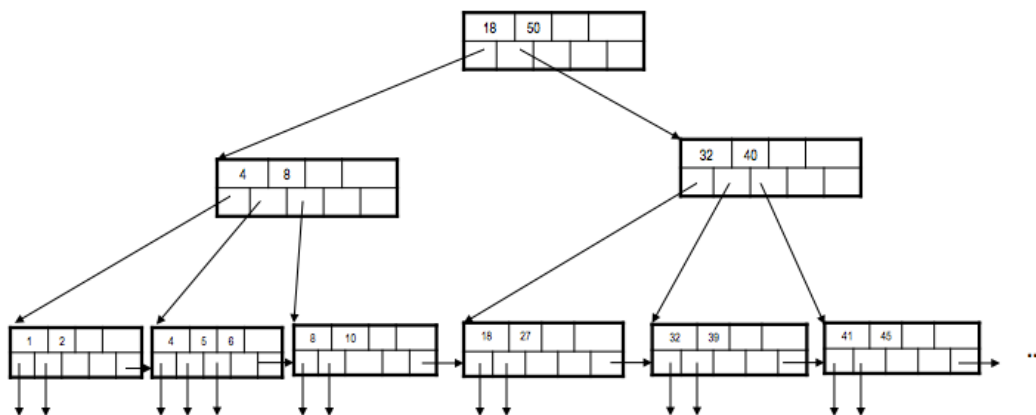
$10 < 50 \rightarrow$ follow 1st pointer

$8 < 10 < 18 \rightarrow$ follow 2nd pointer

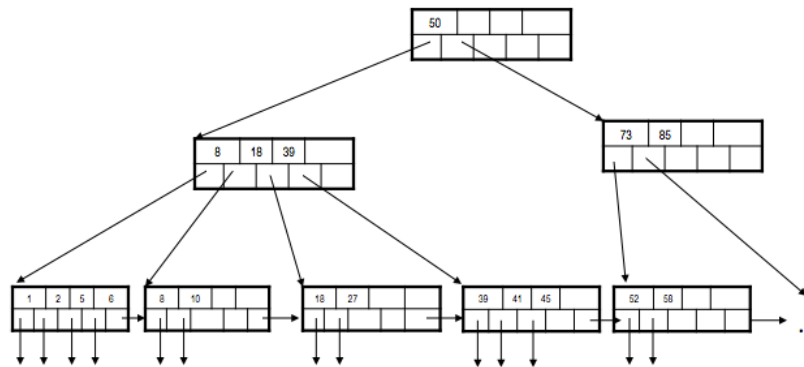
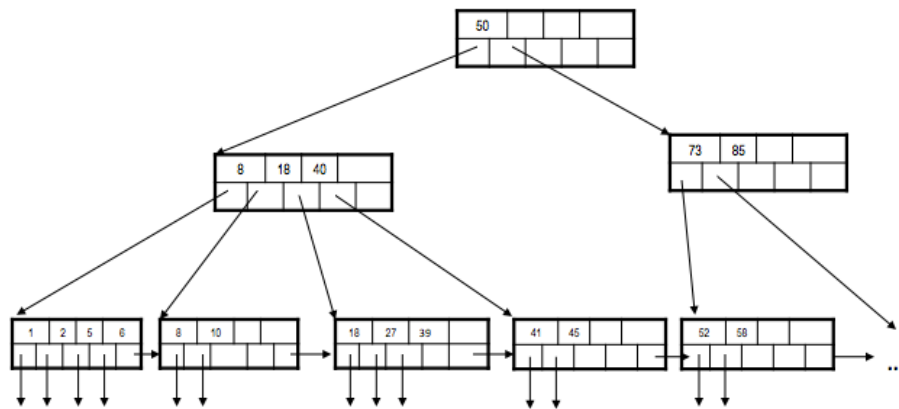
find 10 in the 2nd key

get records with keys 10, 18, 27, 32, 39, 41, 45, 52, 58 by following chains.

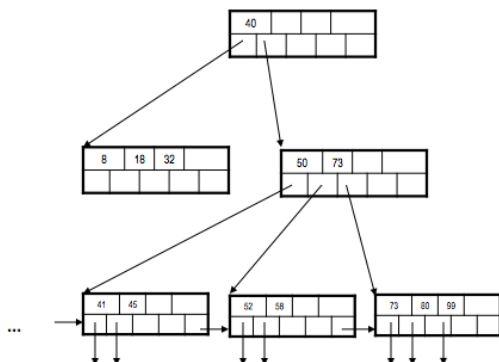
- (d) Show the B+ tree that would result from inserting the data entry with key 4 in the original tree. [It's ok to redraw only the part of the tree where changes occur.]



- (e) Show the B+ tree that would result from deleting the data entry with key 32 from the original tree. [It's ok to redraw only the part of the tree where changes occur.]



- (f) Show the B+ tree that would result from deleting the data entry with key 91 from the original tree. [It's ok to redraw only the part of the tree where changes occur.]



Exercise II (Sparse versus dense indexes) [3 points]

Suppose a block holds either 20 records, or 50 key-pointer pairs. If a data file has 100,000 records, how many blocks do we need to hold this data file and:

(a) A dense index?

We need $100,000/20 = 5,000$ blocks to hold the data file.

We need $100,000/50 = 2,000$ blocks to hold a dense index for the data file (one key-pointer pair for each data record.)

That means, together we need 7,000 blocks for the data file and a dense index.

(b) A sparse index?

As before, we need $100,000/20 = 5,000$ blocks to hold the data file.

We need $5,000/50 = 100$ blocks to hold a sparse index for the data file (one key-pointer pair for each data block).

That means, together we need 5,100 blocks for the data file and a sparse index.