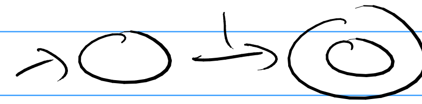
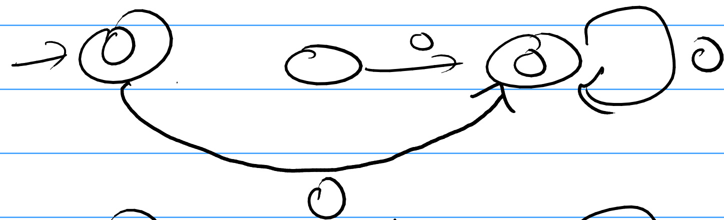
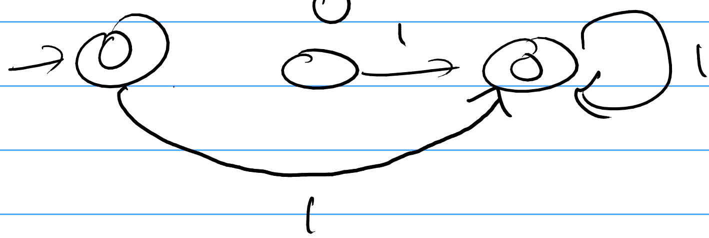
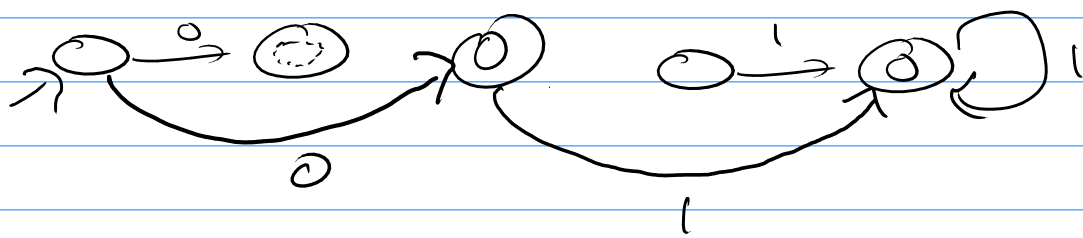
Math 322

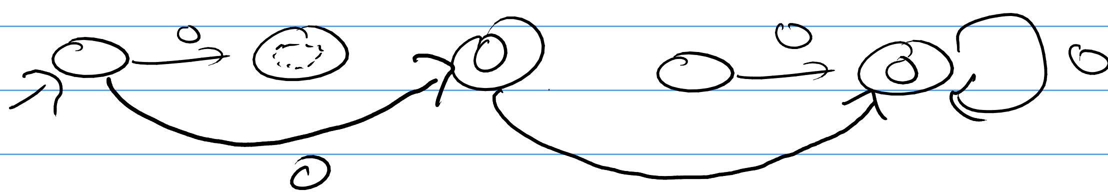
~~Q's~~ 12.4 (13c) $01^* \cup 00^* 1$

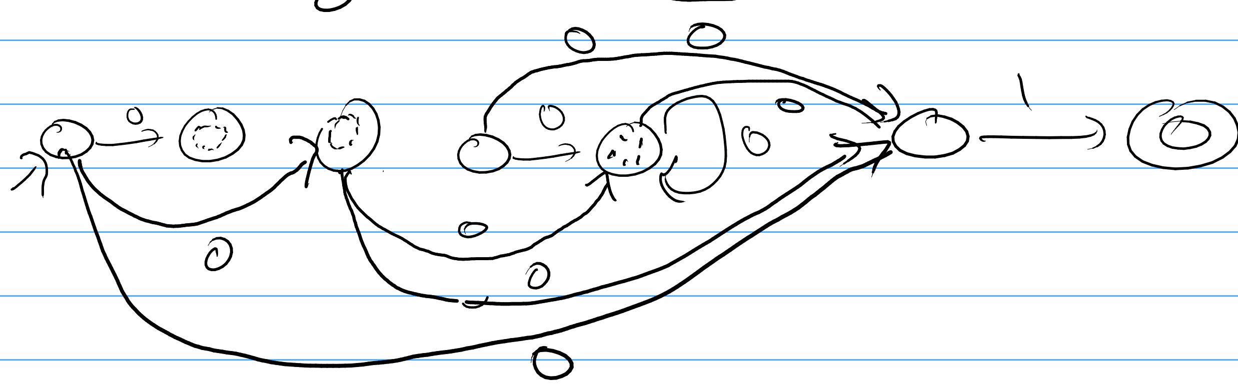
M_0 :  M_1 : 

M_{0^*} : 

M_{1^*} : 

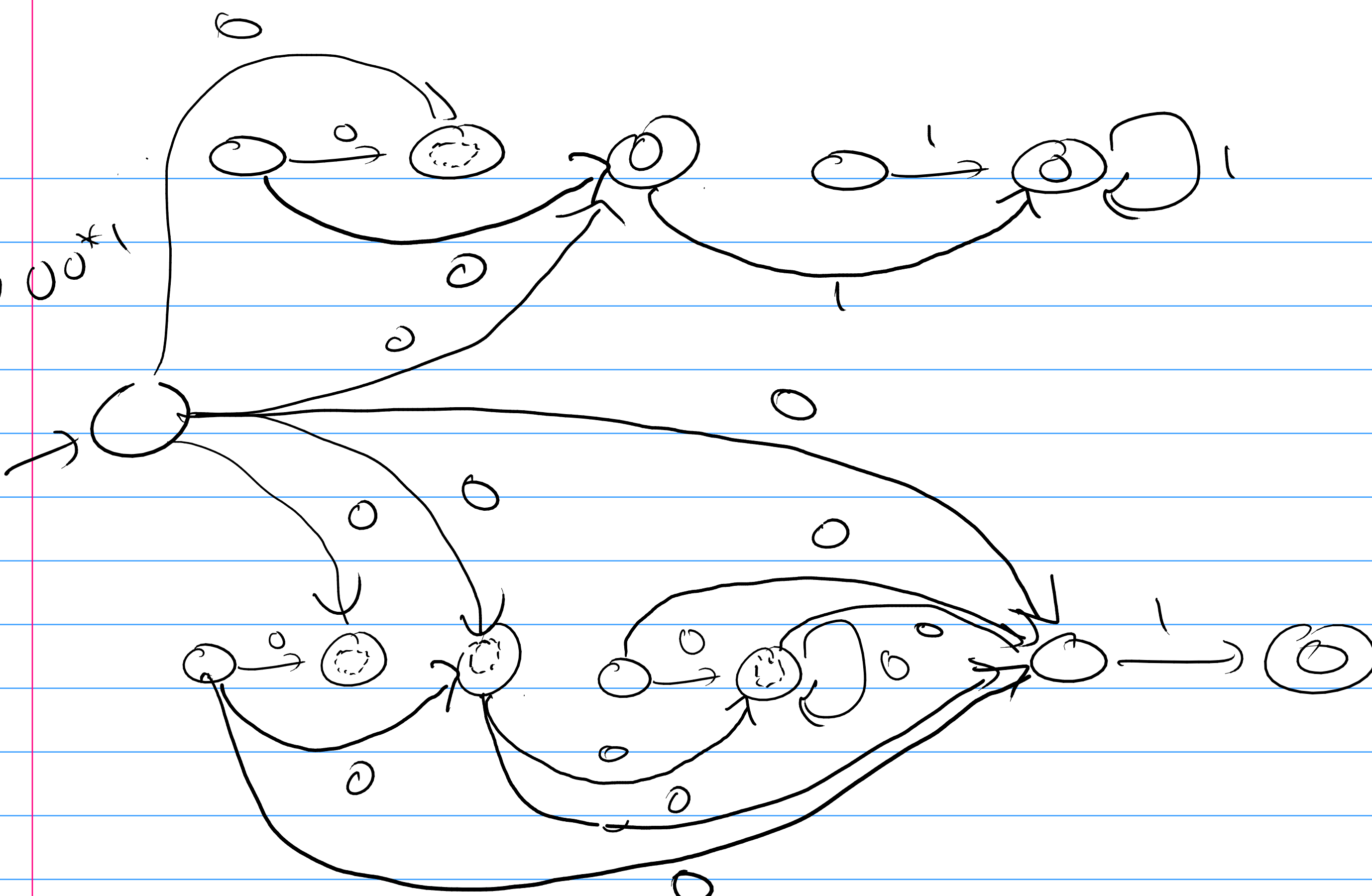
M_{01^*} : 

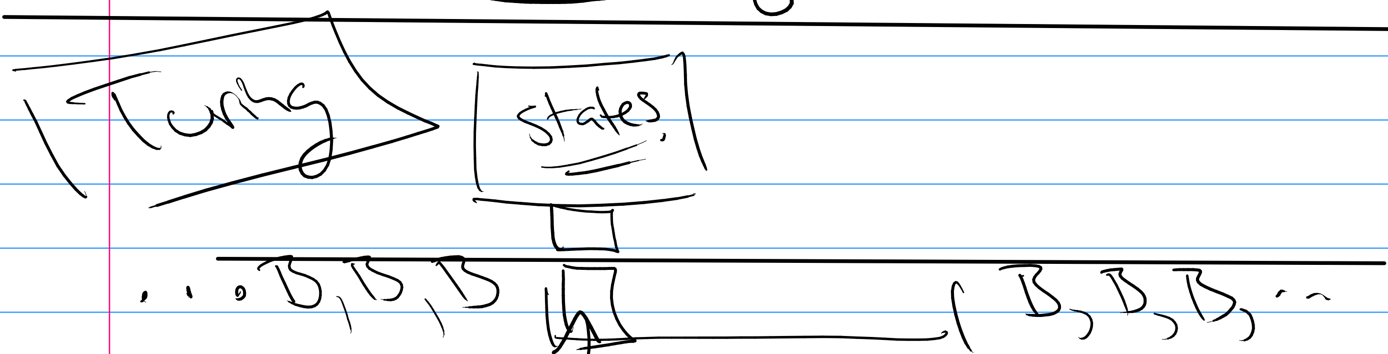
M_{00^*} : 

M_{00^*1} : 

M

$01^* \cup 00^*1$





x is an input string.

1st non-blank symbol.

$f \leftarrow$ set of 5-tuples (partial function)

$(s, i, s', i', M) \in f$

(ex) $(S_0, 1, S_1, B, R) \in$
$(S_1, *, S_3, B, R)$
$(S_1, 1, S_2, B, R)$
$(S_2, 1, S_2, 1, R)$
$(S_2, *, S_3, 1, R)$

S_0

B, B, 1, 1, 1, *, 1, 1, B, B, ...

S_1

B, B, B, 1, 1, *, 1, 1, B, B, ...

S_2

B, B, B, B, 1, *, 1, 1, B, B, ...

S_2

B, B, B, B, 1, *, 1, 1, B, B, ...

S_3

B, B, B, B, 1, 1, 1, 1, B, B, ...

Halts

$V \subseteq I$, T recognizes $x \in V^*$

iff: when x is on the tape w/ T in starting position, T halts in a final state.

any state that is not listed in front of a 5-tuple.

ex

$(S_0, 1, S_1, B, R)$
$(S_1, *, S_3, B, R)$
$(S_1, 1, S_2, B, R)$
$(S_2, 1, S_2, 1, R)$
$(S_2, *, S_3, 1, R)$

$S = \{S_0, S_1, S_2, S_3\}$

final state

Number Theoretic Function: $\mathbb{N} = \{0, 1, 2, \ldots\}$

$$f : \underbrace{\mathbb{N} \times \mathbb{N} \times \ldots \times \mathbb{N}}_{k\text{-tuples}} \to \mathbb{N}$$

example of $f(n_1, n_2) = n_1 + n_2$

unary rep. of $\mathbb{N} = \{\underset{0}{1}, \underset{1}{11}, \underset{2}{111}, \underset{3}{1111}, \ldots\}$

$f(2,1) = 2+1 = 3$

1111

$BB\boxed{111 * 11}BB$

6 wide

$(S_0, 1, S_1, B, R)$
$(S_1, *, S_3, B, R)$
$(S_1, 1, S_2, B, R)$
$(S_2, 1, S_2, 1, R)$
$(S_2, *, S_3, 1, R)$

addition machine

Power?

Church-Turing Thesis

If a problem can be solved by an effective algorithm

$\rightarrow$ a Turing Machine can solve it.

$P(n)$ – Yes n is prime
no n is not prime.

$2 | n$

yes: not

no: $3 | n$

y: not

$5 | n$

y: not

"effective" algorithm

T exists to solve it.

$(\text{prime} < \sqrt{n}) | n$

yes: not

prime

Can we add power to Turing machines?

- → deterministic vs non-deterministic
- → multihead (+) multi tape?
- → plane memory
- → inf. in one direction

<u>all</u> are of same <u>power</u>.

Power means if one can solve a problem <u>any</u> other Turing machine can as well.

(maybe easier to use one or another)

<u>Universal Turing Machine</u>

↑ set of 5-tuples that will do <u>all</u> problems that <u>any</u> T can do.

<u>Halting Problem</u>

given T and x its input

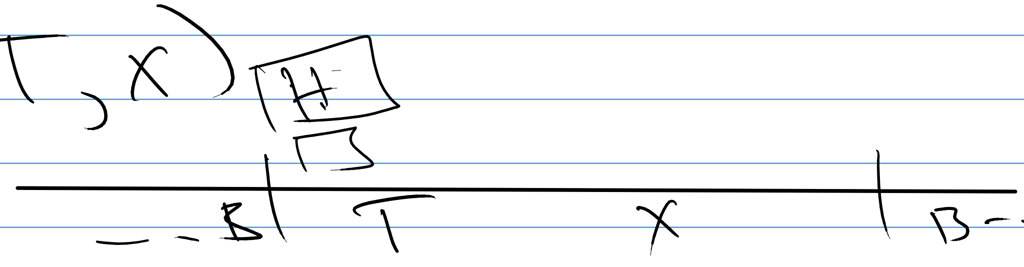
will T halt? or loop?

$\neg$ halt

(yes/no problem)

$\boxed{\text{assume}}$ a turing machine H exists to solve this.

H(T, x)



Consider ① $H(T, T)$

encode T into symbols

feed them into T

$T(T)$

② Flipper $F(F) = \neg F(F)$

$T(T) = \text{halts} \rightarrow F(T) = \text{loop}$

$T(T) = \text{loops} \rightarrow F(T) = \text{halt}.$

Question:

$$H(T, x) = T(x) \begin{cases} \text{halt} \\ \neg\text{halt} \end{cases}$$

$H(F, F)$

Ans. 1 $H(F, F) = \text{halt}$

$\rightarrow F(F) = \text{halt}.$

but! $F(F) = \neg F(F) = \text{loop}$

ans 2 $H(F, F) = \text{loops}$

$\rightarrow F(F) = \neg F(F) = \text{halt}$

contradiction

Computability

① if T can compute a function
→ the function is computable.

② if no T exists → uncomputable.

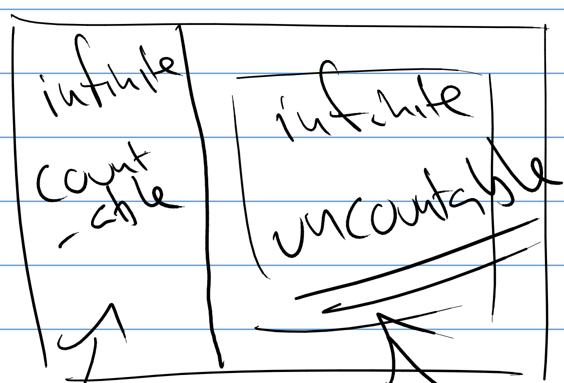
is $\mathbb{R}$ countable.

part of the idea

$$r_1 = 0.d_{11}d_{12}d_{13}\ldots$$
$$r_2 = 0.d_{21}d_{22}d_{23}\ldots$$
$$\vdots$$

$$\left.\begin{array}{l} f_{r_1}(n) = d_{1n} \\ f_{r_2}(n) = d_{2n} \\ \vdots \end{array}\right]$$ uncountable number of functions.

all functions

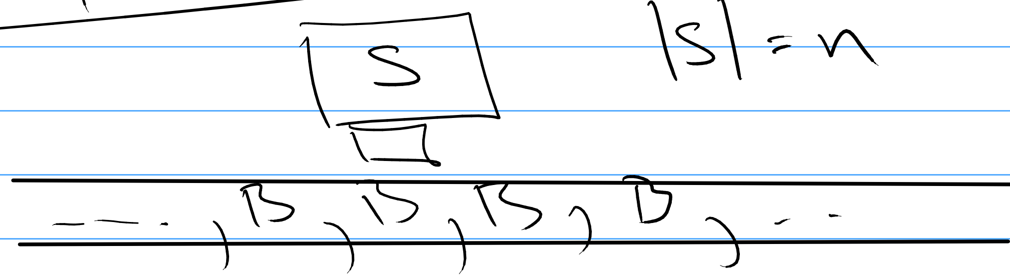


(infinite + uncountable)

computable

uncomputable

Busy Beaver T w/ $A = \{1, B\}$



$|S| = n$

for all possible Turing machines (that halt) with $|S| = n$ what is the greatest number of 1's that is written before halting?

$B(2) = 4$, $B(3) = 6$, $B(4) = 13$

$B(5) \geq 4098$ $B(6) \geq 1.29 \times 10^{865}$

P vs NP

$T, |x| = n, p(n)$ $T, |x| = n, p(n)$

number of transitions before halt.

Deterministic non-deterministic

$P \subseteq NP$

open is $P \overset{?}{=} NP$