Math 321

Q's

Knight → always tells truth
Knave → always lies

55 A says "At least one of us is a knave"

B says nothing.

a) if A was a knight what he says is true.

$\boxed{\text{A: Knight} \quad \text{B: Knave}}$

b) if A was a knave what he says is false

"At least one of us is a knave" = F

yet for a knave ↑ is true!

56 focus on B if B is a knave → A is a knight
if B is a knight → A is a knave ←

A says "we are both knights"

but by ⟷ we know they are not the same.

So A is a knave ∧ B is a knight.

23c Given $\boxed{p \to q}$ (implication)

converse $q \to p$

inverse $\neg p \to \neg q$

contrapos $\neg q \to \neg p$

pos. int. is prime only if only 1 and itself divides it.

Note: $\square \to \triangle$

$\square$	$\triangle$
if	only if
suff.	then
when	nes.
whenever	follows

prime $\to$ 1 & itself are only divisors

Converse: (prime) $\leftarrow$ (1 & itself are only divisors)

it is suff. for a number to have only 1 and itself a divisors for it to be prime.

etc.

1.2 Sameness of compound propositions

Idea: two compound propositions are the "same" if the have the same truth values when the individual proposition truth values match.

Def: (1) a compound proposition that is always true (no matter the truth values of the propositions in it) is a tautology.

(2) Always false? contradiction

(3) Sometimes True. Sometimes false.

Contingency.

ex

$\neg p$	p	q	$p \to q$	$p \wedge \neg p$	$p \vee \neg p$
F	T	T	T	F	T
F	T	F	F	F	T
T	F	T	T	F	T
T	F	F	T	F	T
			Contingency	Contradiction	tautology

ex $p \to q$ vs $\neg p \vee q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Def: If $p \leftrightarrow q$ is a tautology then $p \equiv q$ are logically equivalent.

ex show $(p \to q) \equiv (\neg p \vee q)$

means show $(p \to q) \leftrightarrow (\neg p \vee q)$ is a tautology

p	q	$p \to q$	$\neg p$	$\neg p \vee q$	$(p \to q) \leftrightarrow (\neg p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

tautology

so $(p \to q) \equiv (\neg p \vee q)$

Logical Equivalence to Know

$\boxed{p. 24}$ table 6, table 7 (1^{st} two), table 8

ex $\neg(p \wedge q) \equiv (\neg p \vee \neg q)$

De Morgan's law

$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$

Using Logical Equivalences.

ex $(p \wedge q) \to p$ $\qquad \square \to \Delta$

$\equiv \neg(p \wedge q) \vee p$ $\qquad \equiv \neg\square \vee \Delta$

$\equiv \neg p \vee \neg q \vee p$

$\equiv \neg p \vee p \vee \neg q$

$\equiv (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T$