Math 321

Induction

Basis: Show $P(1)$ is true — for whatever the first part is

weak | Strong

Inductive Step

show $\boxed{P(k) \to P(k+1)}$

show $\boxed{[P(1) \wedge \dots \wedge P(k)] \to P(k+1)}$

by: assume $P(k)$ is true
show $P(k+1)$ is true

by: assume $P(1) \wedge P(2) \wedge \dots \wedge P(k)$ True
show $P(k+1)$ true.

Note:

$$< \frac{p^5 - p^4 - 3}{p^6} \leq \frac{p^5}{p^6} = \frac{1}{p}$$

show $\forall n$ $\quad \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n} \qquad n \geq 2$

a) $P(2)$: $1 + \frac{1}{4} < 2 - \frac{1}{2}$

b) is $P(2)$ true? $\quad \frac{5}{4} < \frac{3}{2}$

$5\left(\frac{1}{4}\right) < 6\left(\frac{1}{4}\right)$ **yes.**

If you would prove the statement

$$\forall n \geq 2 \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n} \right) \text{ is true}$$

Pf: **Basis:** $P(2)$ is true (See above)

Inductive: (show $P(k) \to P(k+1)$)

assume $P(k)$: $1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} < 2 - \frac{1}{k}$

have to **show** $P(k+1)$: $1 + \frac{1}{4} + \cdots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k+1}$

given $1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} < 2 - \frac{1}{k}$ is true.

$$1 + \cdots + \frac{1}{(k+1)^2} < 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$$

want to get here!

we are here!

$$1 + \cdots + \frac{1}{(k+1)^2} < 2 - \left(\frac{1}{k} - \frac{1}{(k+1)^2}\right) = 2 - \left[\frac{(k+1)^2 - k}{k(k+1)^2}\right]$$

$$'' < 2 - \left[\frac{k^2 + k + 1}{k(k+1)^2}\right]$$

$$< 2 - \left[\frac{k^2 + k}{k(k+1)^2}\right] = 2 - \frac{1}{k+1}$$

$\boxed{\text{true.}}$

want

$$\frac{1}{k+1} = \frac{k(k+1)}{k(k+1)^2} = \frac{k^2 + k}{k(k+1)^2}$$

Nim: you take stuff. last player to take something loses.

4.2 #11 rule take at most 3 sticks.

one pile of n sticks.. $n = \{1, 2, 3, 4, \ldots\}$

Base:

$\boxed{n=1}$ player one loses $\quad n \bmod 4 = 1$ player 1 loses

$n=2$ player two loses

$n=3$ player two loses

$n=4$ player two loses $\quad n \bmod 4 = 0, 2, 3$ player 2 loses

$\boxed{n=5}$ player one loses

$n=6$ player two loses.

⋮

$n = 4j+0$, $n = 4j+2$, $n = 4j+3$ player 2 loses

$n = 4j+1$ player one loses

Inductive: assume the pattern is true for $n = 1, 2, 3, \ldots, K$

show pattern works for $K+1$

$K+1 \bmod 4 = 0, 1, 2$ or 3

Case 1

$K+1 \bmod 4 = 1$

remove 1 $\quad K \bmod 4 = 0$

2 $\quad K-1 \bmod 4 = 3$

3 $\quad K-2 \bmod 4 = 2$

Case 2

$K+1 \bmod 4 = 0, 2, 3$

$K+1 \bmod 4 = 0 \xrightarrow{\text{take } 3} K-2 \bmod 4 = 1$

$K+1 \bmod 4 = 2 \xrightarrow{\text{take } 1} K \bmod 4 = 1$

$K+1 \bmod 4 = 3 \xrightarrow{\text{take } 2} K-1 \bmod 4 = 1$

4.2 #7 $2 and $5

$2, 2\cdot 2=4, 5, 3\cdot 2=6, 5+2=7, 4\cdot 2=8, 2\cdot 2+5=9, 2\cdot 5=10,$

$+\$5$

Basis: $2\cdot 2=\boxed{4}, \boxed{5}, 3\cdot 2=\boxed{6}, 5+2=\boxed{7}, 4\cdot 2=\boxed{8}$

true.

Inductive:

assume $\$4, \$5, \$6, \$7, \$8, \ldots, \$K$

take: $\underbrace{\$(K-4)}_{\text{this is formed by } \$2 \text{ and } \$5} + 1\cdot\$5 = \$(K+1)$

true.