# Math 321

Q's of 5.3 #26   $n = 13$

(a) $\binom{13}{10} = \frac{13!}{10!\,3!}$

(b) $P(13,10) = \frac{13!}{3!}$

(c) 10 men  3 women

$$|\text{all}| = |0 \text{ women}| + \underbrace{|\text{exactly } 1| + |\text{exactly } 2| + |\text{exactly } 3|}_{\text{at least one women}}$$

$$\binom{13}{10} - \binom{10}{10}$$

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Variation: 16 guys 3 girls.
(Same question as above)

$$\binom{19}{10} - \binom{16}{10}$$

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## 7.1 Recurrence Relation

$a_n$ = "expression of $a_{n-1}$, $a_{n-2}$, etc"

Given $a_0, a_1$ (start values) (Initial Conditions)

$\boxed{\text{Ex!}}$ Initial Value: $a_0 = 2 + 1 = 3$

Recurrence Relation: $a_n = 2 \cdot a_{n-1} + 1$

$$a_n = 2 \cdot \underbrace{\boxed{a_{n-1}}}_{= 2a_{n-2}+1} + 1$$

$$a_n = 2(2a_{n-2} + 1) + 1$$

$$a_n = 2^2 a_{n-2} + 2 + 1$$

$$a_n = 2^2(2a_{n-3} + 1) + 2 + 1$$

$$a_n = 2^3 a_{n-3} + 2^2 + 2 + 1$$

$$a_n = 2^n \underbrace{a_0}_{= 3} + 2^{n-1} + \cdots + 2^2 + 2 + 1$$

$$a_n = (2+1)2^n + 2^{n-1} + \cdots + 2^2 + 2 + 1$$

$$a_n = 2^{n+1} + 2^n + 2^{n-1} + \cdots + 2^2 + 2 + 1$$

$$\boxed{a_n = 2^{n+2} - 1}$$ soln.

Initial Value Prob.

$$\begin{cases} a_0 = 2 + 1 = 3 \\ a_n = 2 \cdot a_{n-1} + 1 \end{cases}$$

check.

$a_0 = 2^2 - 1 = 3$ ✓

$2^{n+2} - 1 = 2(2^{n+1} - 1) + 1$

$2^{n+2} - 1 = 2^{n+2} - 2 + 1$

$2^{n+2} - 1 = 2^{n+2} - 1$ ✓

31st day $a_{30} = 2^{32} - 1$

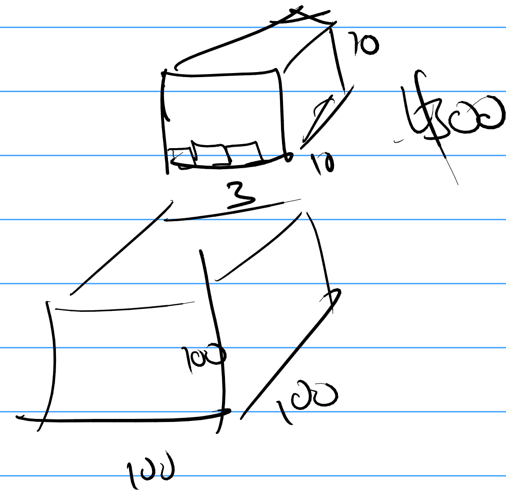
$\approx 2^{32}$

$2^{32} = \boxed{4 \cdot GB}$

$\approx 4,000,000,000$

$\approx 1,000,000,000$

$\approx 10\ m^3$

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(ex) Tower of Hanoi

$a_n = 2^n - 1$

$a_{64} = 2^{64} - 1$

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$a_0$ $\quad a_n = f(a_{n-1}) \quad a_i \in \mathbb{C}$

$a_n = a_{n-1}^2 + a_0$

$a_1 = a_0^2 + a_0$

$a_2 = a_1^2 + a_0 = (a_0^2 + a_0)^2 + a_0$

$a_2 = a_0^4 + 2a_0^3 + a_0^2 + a_0$

$a_3 = a_2^2 + a_0$

$a_n = C_0 \cdot a_0 + C_1 \cdot a_0^2 + C_2 \cdot a_0^3 + \cdots$

$C_0 = 1 \qquad C_1 = 1$

$C_n = C_0 C_{n-1} + C_1 C_{n-2} + \cdots + C_{n-1} C_0$

$$\begin{array}{cc} 1 & 1 \\ 1 & 1 \\ \hline \end{array} \qquad \begin{array}{ccc} 1 & 1 & 2 \\ 2 & 1 & 1 \\ \hline \end{array}$$

$1 + 1 = 2 \qquad 2 + 1 + 2 = 5$

$$\begin{array}{cccc} 1 & 1 & 2 & 5 \\ 5 & 2 & 1 & 1 \\ \hline \end{array}$$

$5 + 2 + 2 + 5 = 14$

Initial Values: $C_0 = 1 \quad C_1 = 1$

Recurrence Relation:

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-k-1}$$

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

## Catalan Numbers:

$C_0 = 1 \quad C_1 = 1 \qquad C_n = \sum_{k=0}^{n-1} C_k C_{(n-1)-k}$

p. 456

$n_0 \times n_1 \times n_2 \times \ldots \times n_n$

$n+1$ numbers
$n$ products

## Degrees & Recurrence Relations

ex

$a_n = 2a_{n-1} + 1$ Degree 1

$a_n = a_{n-1} + a_{n-2}$ Degree 2

$\vdots$

$a_n = a_{n-7} - n \cdot a_{n-11}$ Degree 11