Q's: 2.4 (11)

$a_n = n^{\text{th}}$ pos int. that is <u>not</u> a perfect square

$$a_n = n + \{\sqrt{n}\} = n + \boxed{[\sqrt{n}]} \quad \text{round.}$$

$\sqrt{n}$: $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3, \sqrt{10}, \dots, \sqrt{15}, 4, \dots$

$\boxed{\text{Pos. Int.}}$: $\boxed{1}, 2, 3, \boxed{4}, 5, 6, 7, 8, \boxed{9}, 10, 11, 12, 13, \dots$

$\{a_n\} = 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, \dots$

$\quad\quad\quad \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow$

$\quad\quad\quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad \dots$

2nd Rule: $\{n + \text{round}(\sqrt{n})\} \quad n = 1, 2, 3, 4, \dots$

$\quad\quad\quad\quad\quad [\sqrt{n}]$

$= 1 + [\sqrt{1}], \quad 2 + [\sqrt{2}], \quad 3 + [\sqrt{3}], \quad 4 + [\sqrt{4}],$

$\quad 5 + [\sqrt{5}], \dots, 8 + \boxed{[\sqrt{8}]}, 9 + [\sqrt{9}], 10 + [\sqrt{10}], \dots$

$= 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, \dots$

$+3 \quad\quad\quad +5$

n : $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots$

$\sqrt{n}$: $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3, \sqrt{10}, \sqrt{11}, \dots$

$[\sqrt{n}]$: $1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 3, \dots$

21) $\sum\limits_{k=1}^{n} k^2 - (k-1)^2 = \sum\limits_{k=1}^{n} (2k-1)$ used to find $\sum k$

$\sum\limits_{k=1}^{n} k^1$

telescoping.

$(1^2 - 0^2) + (2^2 - 1^2) + \cdots + (n^2 - (n-1)^2)$

$n^2 = \sum\limits_{k=1}^{n}(2k-1) = 2\left[\sum\limits_{k=1}^{n}k\right] - \sum\limits_{k=1}^{n}1$

$\sum\limits_{k=1}^{n} k^2$ 0 $\sum\limits_{k=1}^{n} k^3 - (k-1)^3 =$

$\sum\limits_{k=1}^{n} k^3$ 0 $\sum\limits_{k=1}^{n} k^4 - (k-1)^4 =$

$\sum\limits_{k=1}^{n} k^4$ 0 $\sum\limits_{k=1}^{n} k^5 - (k-1)^5 =$
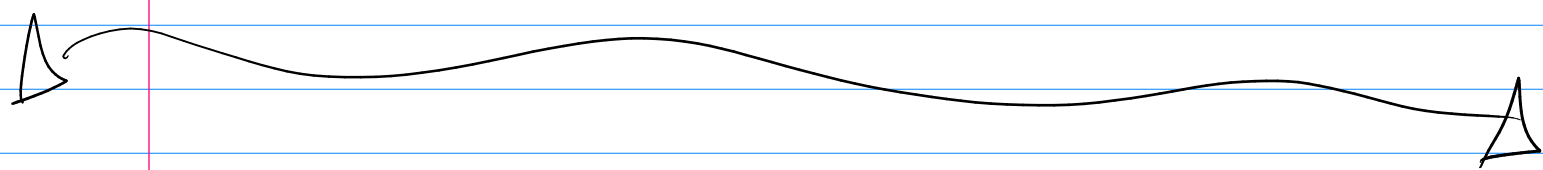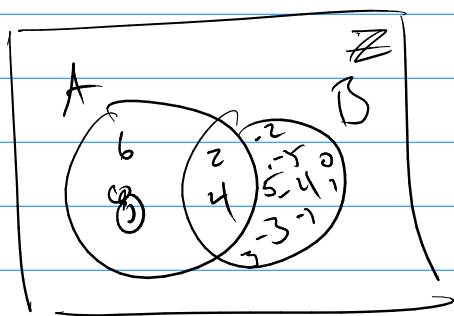
Exam 2   10 probs (+) 1 extra credit.

Sets (3 probs)

① Representing Sets

    a) Venn
    b) Set-builder
    c) lists.

$$A = \{a \mid a \text{ is even} \wedge 2 \leq a \leq 8\} = \{2,4,6,8\}$$

$$B = \{b \mid b \in \mathbb{Z} \wedge -6 < b < 6\} = \{-5,-4,\dots,5\}$$

② Set ops using lists.

given A, B list. $A \cap B, A \cup B, \bar{A}, A-B,$
                                   etc

③ Set ops. using membership tables,

    Venn Diagrams.

Functions - Seq - Series   (5 probs)

① given $f: A \to B$ discuss is it onto?
one-to-one? , operations?
                              ↖ composition

② You give a function that satisfies
asked properties.

③ Prove: $\mathbb{Q}$ is countable.

④ evaluate a telescoping series.

ex $\displaystyle\sum_{k=1}^{n} 2^{k+3} - 2^{k}$

$\displaystyle\sum_{k=3}^{n} \sqrt{k+2} - \sqrt{k}$

⑤ Series (non-telescoping) p. 157

$\displaystyle\sum_{k=1}^{100} \left(k^2 + 3 \cdot \left(\frac{1}{2}\right)^k \right)$

Matrices (2 probs)

① number matrix ops.

② zero-one matrix ops.

Extra Credit

Find $\displaystyle\sum_{k=1}^{n} k^q$ using a telescoping series technique.