Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 2

Created by Prof. Diego Maldonado and Prof. Virginia Naibo

Name: ____

Due date: Thursday, February 7th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises

1. Heidi spent 200 dollars buying sheep, goats, and hogs, to the number of 80. The sheep cost her 1 dollar a-piece; the goats, 2 dollars; and the hogs 3 dollars. If the number of goats is twice the number of sheep; how many sheep, goats, and hogs did she get?(Show your work)

Answer:

sheep	goats	hogs

2. A health food store mixes granola that costs them 4 dollars per pound and raisins that cost them 2 dollars per pound together to make 25 pounds of raisin granola. How many pounds of raisins should they include if they want the mixture to cost them a total of 80 dollars? (Show your work)

Answer:

granola	raisins

3. A math test consisting of 40 problems is worth 100 points. The test has two kinds of problems: the easy ones and the hard ones. If the hard ones are worth 6 points and the easy ones 2 points. How many problems of each value are there on the test? (Show your work)

Answer:

hard problems	easy problems

4. Solve the system (S) below **by hand**. Indicate the size, the matrix of coefficients, the augmented matrix, and consistency or inconsistency of the system (S). If consistent, indicate the number of free variables and express the solution in terms of the free variables.

$$(S)\left\{\begin{array}{rcrcrcr} 2x_1 & + & 2x_2 & + & 3x_3 & = & 4 \\ 2x_1 & & & + & x_3 & = & 2 \\ 4x_1 & + & 2x_2 & + & 4x_3 & = & 6 \end{array}\right.$$

5. A system (S) has the following augmented matrix

```
M =

     2     1     3     1     2    -1
     3     4     5     1     1     2
     4     5     6     7     0     4
```

What is the size of (S)? What is the matrix of coefficients of (S)? Is (S) consistent or inconsistent? If consistent, find its solution(s). If there are free variables, express the solution in terms of the free variables.

6. Do there exist real numbers x_1, x_2, and x_3 such that

$$x_1 \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}? \tag{1}$$

If so, what are they?

Hint: Think of the equality (1) as the vector form of a linear system with unknowns x_1, x_2, and x_3. Make that system explicit and determine its consistency or inconsistency.

7. A system (S) has the following augmented matrix

```
M =

     2     1     3     1     2
     3     4     5     1     1
     4     5     6     7     0
```

What is the size of (S)? What is the matrix of coefficients of (S)? Is (S) consistent or inconsistent? If consistent, find its solution(s). If there are free variables, express the solution in terms of the free variables.

8. Do there exist real numbers x_1, x_2, x_3, and x_4 such that

$$x_1 \begin{bmatrix} 2 \\ 1 \\ 4 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 1 \\ 4 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 1 \\ 2 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 6 \\ 2 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 2 \end{bmatrix}? \tag{2}$$

If so, what are they?

Hint: Think of the equality (2) as the vector form of a linear system with unknowns x_1, x_2, x_3, and x_4. Make that system explicit and determine its consistency or inconsistency.

9. True or False - **Circle the right one** (One point each)

T or **F**. We always have that $rank(M) = rank(rref(M))$.

T or **F**. There is an $m \times n$ system that has exactly 11 solutions.

T or **F**. If $Ax = b$ has a solution, then $Ay = c$ has a solution for any other right-hand side vector c.

T or **F**. Given an $m \times n$ matrix B, the system

$$Bx = 0,$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad 0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (m \text{ zero entries}),$$

has *at least* one solution. In other words, any $m \times n$ homogeneous system always has at least one solution.

T or **F**. If A and B are 3×3 matrices with $rank(A) = rank(B) = 3$, then $rank(A+B) = 3$.

Points obtained in this assignment (out of 16): ______________