Math 321

Q's / 2.3(21) $f:\mathbb{R}\to\mathbb{R}$

f is strictly dec

Note: $f(x_1) < f(x_2)$ when $x_1 < x_2$ strictly Inc.

$f(x_1) > f(x_2)$ when $x_1 < x_2$ strictly Dec

$\frac{1}{f(x)}$ is strictly inc.

Pf: f is strictly dec

iff $f(x_1) > f(x_2)$ when $x_1 < x_2$

iff $\frac{1}{f(x_1)} < \frac{1}{f(x_2)}$ when $x_1 < x_2$

iff $\frac{1}{f(x)}$ is strictly inc.

2.3 Function $f: A \to B$

is a rule that assigns to every $a \in A$ exactly one $b \in B$.

$f: \mathbb{N} \to S$

$f: \mathbb{Z} \to S$

a subset of $\mathbb{Z}$

$f: \mathbb{R} \to \mathbb{Z}$

$\lceil x \rceil = n \qquad n-1 < x \leq n$

$\lfloor x \rfloor = n \qquad n \leq x < n+1$

$n! = n(n-1)\cdots(2)(1)$

<u>def:</u> $0! = 1$

2.4 Seq's & Sums

<u>Sequence:</u> A function that maps a subset of $\mathbb{Z}$ to a set S.

$$f: \underset{\mathbb{Z}}{\overset{\text{subset}}{\text{of}}} \to S$$

ex $\quad f: \{1, 2, 3, \ldots\} \to S$

Normal function notation $f(n) =$ rule involving "n"

ex $f(n) = n^2+1$ $n = -1, 0, 1, 2, \ldots$

$f(n) = \frac{1}{n}$ $n = 1, 2, \ldots$

$f(n) = n!$ $n = 0, 1, 2, \ldots$

Seq. Notation

$\{n^2+1\}$ $n = -1, 0, 1, 2, \ldots$

$\{1/n\}$ $n = 1, 2, 3, \ldots$

$\{n!\}$ $n = 0, 1, 2, \ldots$

In general

$\{a_n\}$ $n = \ldots$

ex $\{n^2+1\}$ $n = -1, 0, 1, 2, \ldots$

$\boxed{\text{seq.}} \rightarrow 2, 1, 2, 5, 10, \ldots$

$\{a_n\}$ $n = 0, 1, 2, \ldots$ ← index of seq

$\boxed{\text{seq}} \rightarrow a_0, a_1, a_2, a_3, \ldots$ ← terms of seq.

ex

$\{n\}$ $n=1,2,3,\ldots$

$a_1=1,\ a_2=2,\ a_3=3,\ a_4=4,\ \ldots$

$\{n^2\}$ $n=0,1,2,3,\ldots$

$a_0=0,\ a_1=1,\ a_2=4,\ a_3=9,\ \ldots$

harmonic seq.

$\{1/n\}$ $n=1,2,3,\ldots$

$a_1=1,\ a_2=1/2,\ a_3=1/3,\ \ldots$

arithmetic seq

$\{a+d\cdot n\}$ $n=0,1,2,\ldots$

$a,\ a+d,\ a+2d,\ a+3d,\ \ldots$

geometric seq

$\{a\cdot r^n\}$ $n=0,1,2,\ldots$

$a,\ ar,\ ar^2,\ \ldots$

Seq $\longrightarrow$ rule?

rule $\longrightarrow$ seq?

ex $a_0 = 0$, $a_1 = 1$

$$a_n = a_{n-1} + a_{n-2} \quad n = 2, 3, \ldots$$

$a_0 = 0$, $a_1 = 1$, $a_2 = 1$, $a_3 = 2$, ...

Sums add up some of a seq's terms.

$$\sum_{i=1}^{4} \left(\frac{1}{i}\right) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

upper limit $\rightarrow$ n

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + \cdots + a_n$$

a_k $\leftarrow$ seq.

k: index

m: lower limit

ex's

$$\sum_{k=1}^{3} k = 1 + 2 + 3 = 6$$

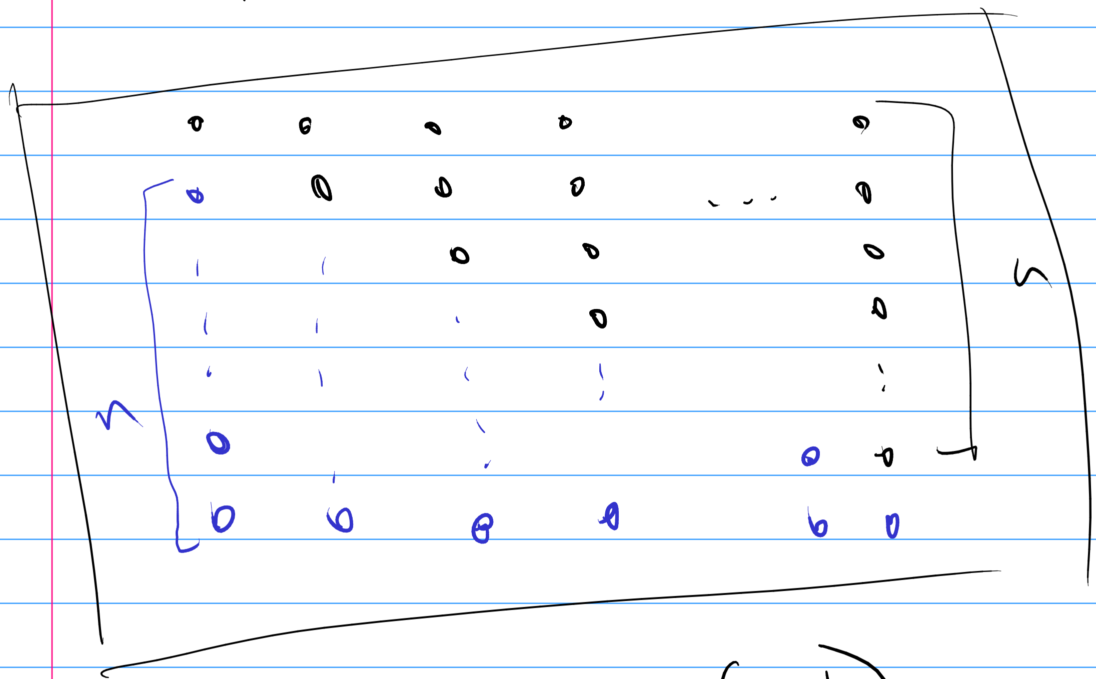
$$\sum_{k=1}^{100} k = 1 + 2 + 3 + \cdots + 99 + 100$$

$$100 + 99 + 98 + \cdots + 2 + 1$$

$$101 + 101 + 101 + \cdots + 101 + 101 = 100(101)$$

$$\sum_{k=1}^{100} k = \frac{100(101)}{2} = \boxed{5050}$$

$$\sum_{x=1}^{n} x = \frac{n(n+1)}{2}$$



$$\text{area} = n(n+1)$$

$$\text{sum} = \boxed{\frac{n(n+1)}{2}}$$

$$\sum_{k=1}^{n} k^2 = 1 + 4 + 9 + 16 + \dots + n^2 = ?$$

$\boxed{\text{telescoping sum}}$

$$\sum_{k=1}^{5} \left(a_k^{+1} - a_{k-1}\right)$$

$$= (\cancel{a_1} - a_0) + (\cancel{a_2} - \cancel{a_1}) + (\cancel{a_3} - \cancel{a_2}) +$$

$$+ (\cancel{a_4} - \cancel{a_3}) + (a_5 - \cancel{a_4})$$

$$= a_5 - a_0$$

$$\sum_{k=1}^{n} k^2$$