Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 11

Created by Prof. Diego Maldonado and Prof. Virginia Naibo

Name: ____

Due date: Thursday, April 18th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. Find a 2×2 matrix A that represents a linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ such that

$$T\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix} \quad \text{and} \quad T\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \end{bmatrix}.$$

2. Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be a linear transformation and $\beta = \{u_1, u_2, u_3\}$ be a basis of $\mathbf{R}^3$ such that

$$u_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 4 \\ 4 \\ -1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

and

$$T(u_1) = \begin{bmatrix} -7 \\ 9 \\ -1 \\ 1 \end{bmatrix}, \quad T(u_2) = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad T(u_3) = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix}.$$

Find $T(u)$ where

$$u = \begin{bmatrix} -5 \\ 0 \\ 4 \end{bmatrix}.$$

Find a basis for $range(T)$. Is T onto? Is T one-to-one? (Justify).

3. Find a decomposition of the vector $v = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ of the form $v = \bar{v} + w$ such that $\bar{v}$ belongs to the subspace $\mathcal{U} = span\{u_1, u_2\}$, where $u_1 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, and $w \in \mathcal{U}^\perp$. Determine the distance from v to $\mathcal{U}$.

4. Consider the vectors

$$w_1 = \begin{bmatrix} 4 \\ -1 \\ 4 \\ 1 \end{bmatrix} \quad \text{and} \quad w_2 = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}.$$

Find an **orthonormal** basis for the subspace of all the vectors in $\mathbf{R}^4$ which are orthogonal to w_1 and w_2.

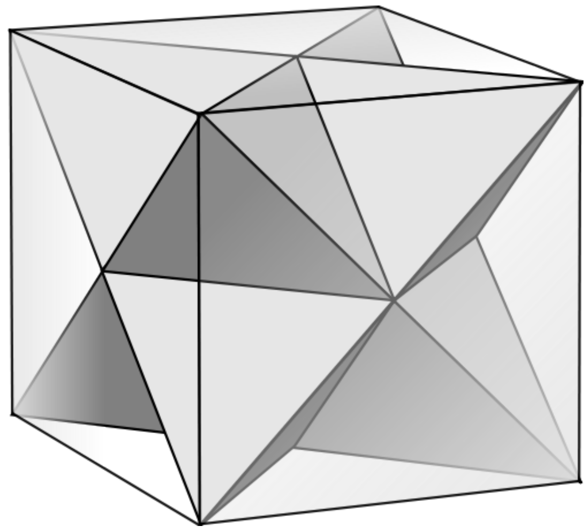
5. Use two *for loops* and *branching* to write a Matlab function that takes an arbitrary $n \times n$ matrix A and returns a matrix B whose (i, j)-entry is the dot product between the i-th row and j-th column of A. If the entered matrix A is not a square matrix, the code must display a message indicating so.

6. Let W be the 2×5 matrix

$$W = \begin{bmatrix} -1 & 3 & 1 & 3 & 2 \\ 1 & -3 & 2 & -1 & 1 \end{bmatrix}$$

Find an **orthonormal** basis for $null(W)$ by first finding any basis and then applying the Gram-Schmidt process to it.

7. Use the cross product to find the volume of the intersection of the two tetrahedra shown in the picture. Assume that the inscribing cube has side length equal to 1 (unit of length).



8. True or False - **Circle the right one**.

T or **F**. If $T : \mathbf{R}^n \to \mathbf{R}^n$ is onto, then it is one-to-one.

T or **F**. If the columns of the $n \times n$ matrix A are linearly independent, then given any $b \in \mathbf{R}^n$ the system $Ax = b$ has exactly one solution.

T or **F**. If the vector b is a linear combination of the columns of a matrix A, then the system $Ax = b$ has at least one solution.

T or **F**. Given vectors u_1, u_2, and u_3 with $u_1 \in span\{u_2, u_3\}$, then dim$(span\{u_1, u_2, u_3\}) = 2$.

T or **F**. If $Ax = Ay$, for a matrix A and some vectors x and y, then $x = y$.

Points obtained in this assignment (out of 16): ______________