CIS 450
Computer Architecture and Organization

Lecture 17: Cache Memories

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Topics

- Caching in the memory hierarchy
- Generic cache memory organization
- Direct mapped caches
- Set associative caches
- Impact of caches on performance
- The memory mountain

Memory Hierarchies

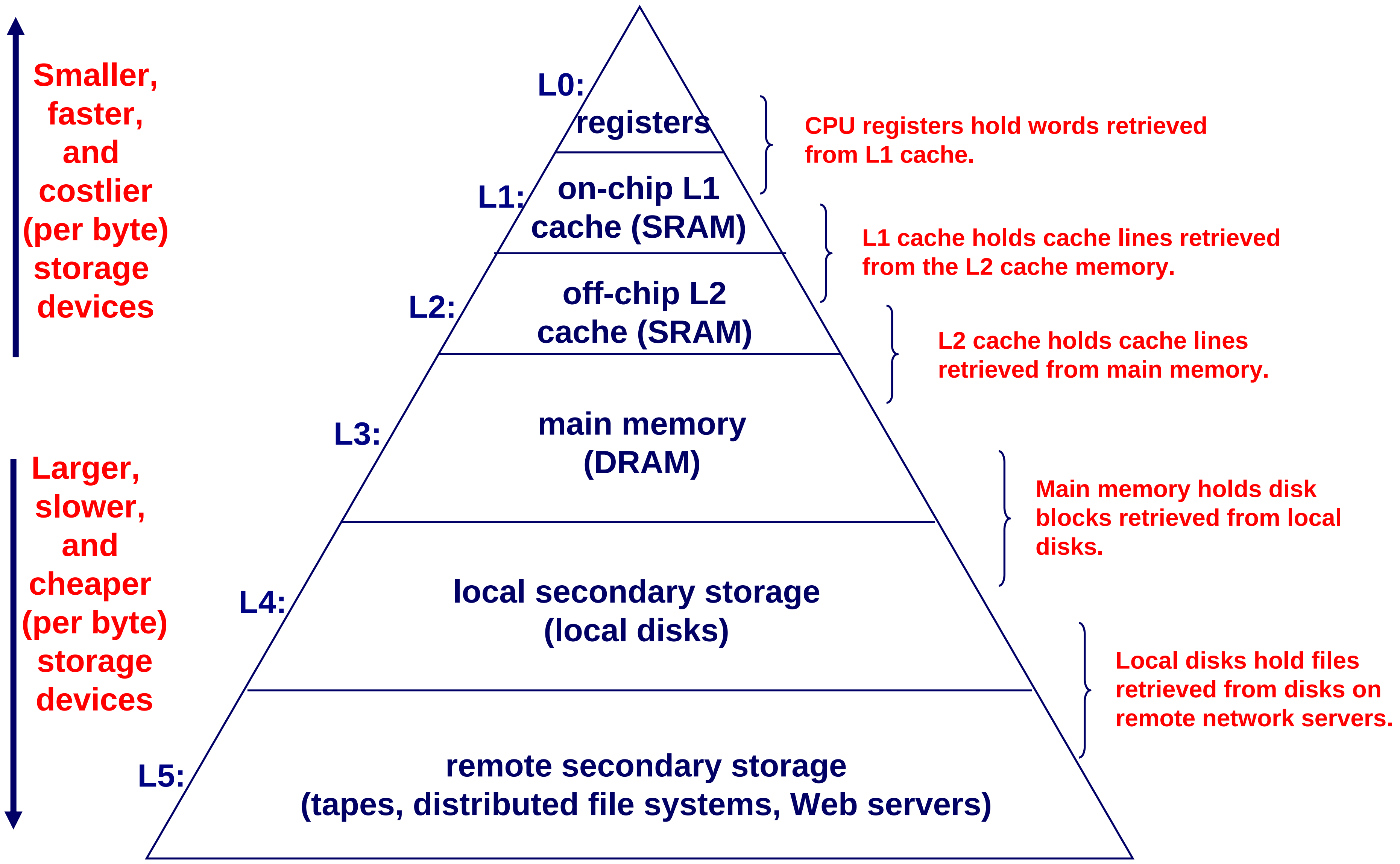
Some fundamental and enduring properties of hardware and software:

- Fast storage technologies cost more per byte, have less capacity, and require more power (heat!).
- The gap between CPU and main memory speed is widening.
- Well-written programs tend to exhibit good locality.

These fundamental properties complement each other beautifully.

They suggest an approach for organizing memory and storage systems known as a memory hierarchy.

An Example Memory Hierarchy



Smaller, faster, and costlier (per byte) storage devices
L0:
registers
CPU registers hold words retrieved from L1 cache.
L1:
on-chip L1 cache (SRAM)
L1 cache holds cache lines retrieved from the L2 cache memory.
L2:
off-chip L2 cache (SRAM)
L2 cache holds cache lines retrieved from main memory.
L3:
main memory (DRAM)
Main memory holds disk blocks retrieved from local disks.
Larger, slower, and cheaper (per byte) storage devices
L4:
local secondary storage (local disks)
Local disks hold files retrieved from disks on remote network servers.
L5:
remote secondary storage (tapes, distributed file systems, Web servers)

Caches

Cache: A smaller, faster storage device that acts as a staging area for a subset of the data in a larger, slower device.

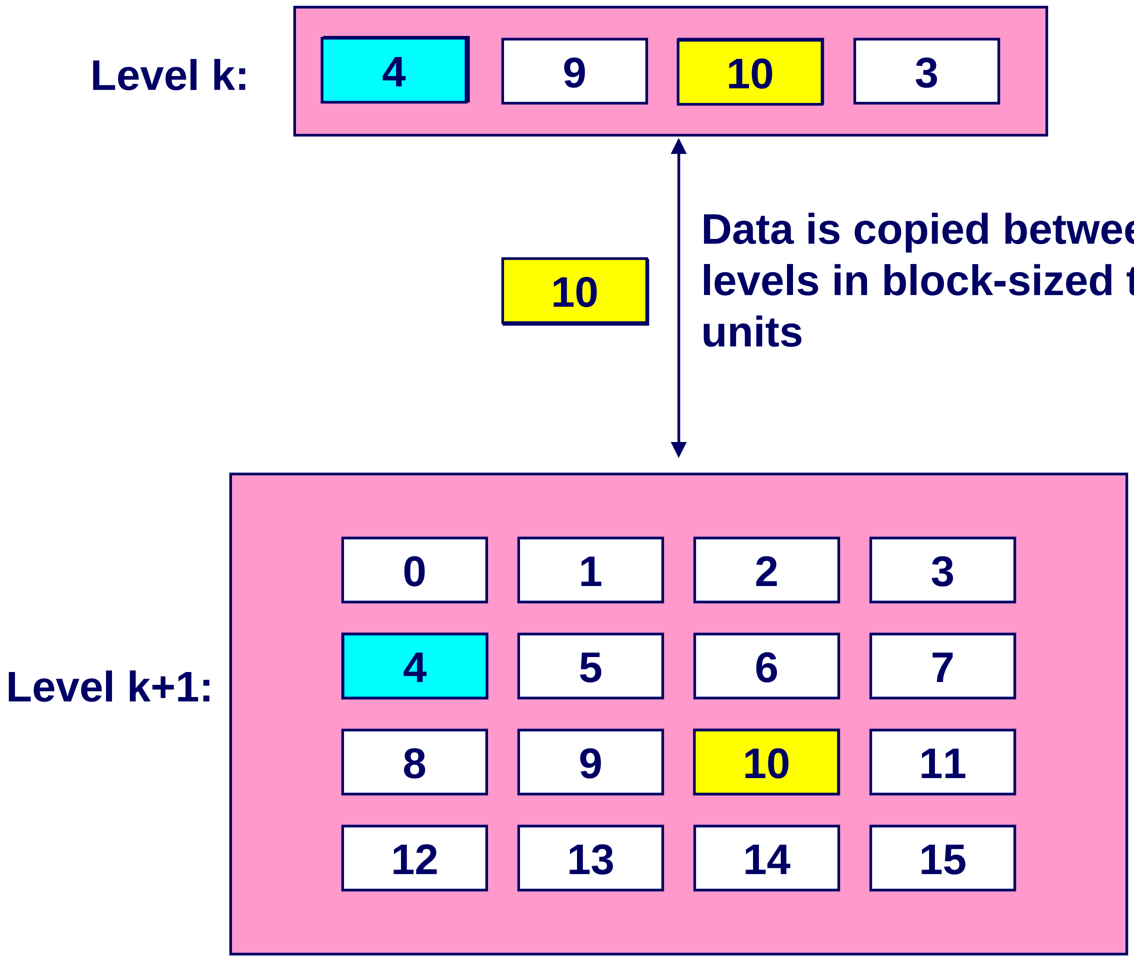
Fundamental idea of a memory hierarchy:

- For each k, the faster, smaller device at level k serves as a cache for the larger, slower device at level k+1.

Why do memory hierarchies work?

- Programs tend to access the data at level k more often than they access the data at level k+1.
- Thus, the storage at level k+1 can be slower, and thus larger and cheaper per bit.
- Net effect: A large pool of memory that costs as much as the cheap storage near the bottom, but that serves data to programs at the rate of the fast storage near the top.

Caching in a Memory Hierarchy

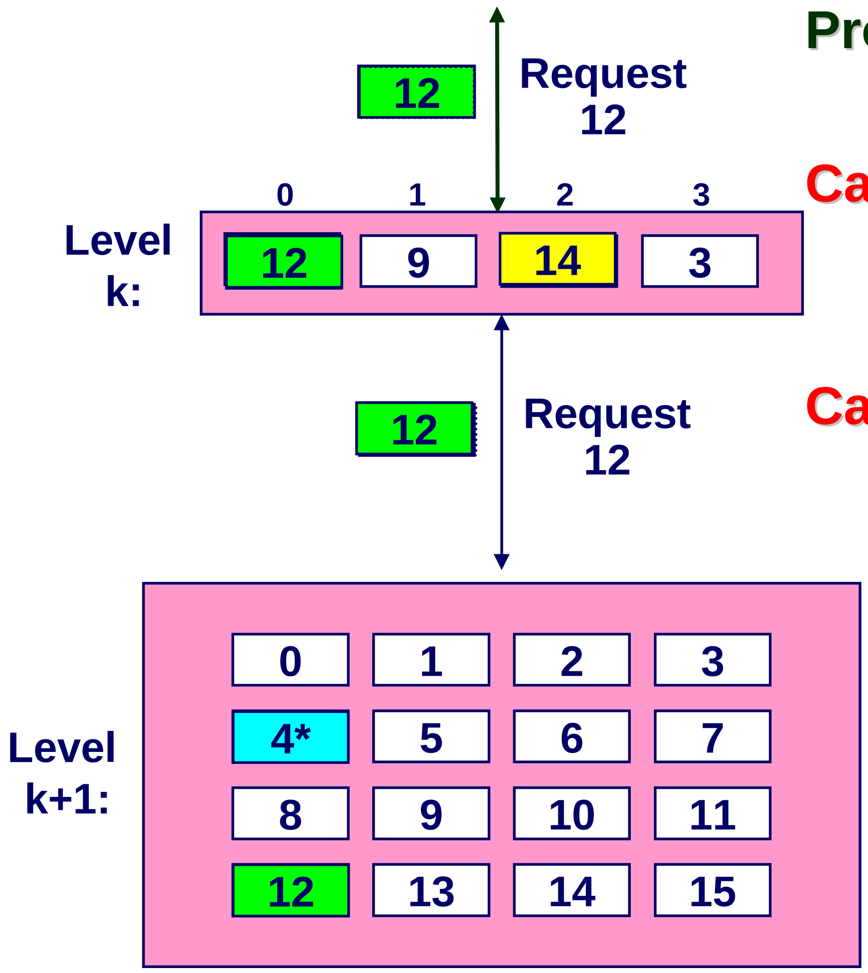


Smaller, faster, more expensive device at level k caches a subset of the blocks from level k+1

Data is copied between levels in block-sized transfer units

Larger, slower, cheaper storage device at level k+1 is partitioned into blocks.

General Caching Concepts



Program needs object d, which is stored in some block b.

Cache hit

- Program finds b in the cache at level k. E.g., block 14.

Cache miss

- b is not at level k, so level k cache must fetch it from level k+1. E.g., block 12.
- If level k cache is full, then some current block must be replaced (evicted). Which one is the "victim"?
  - Placement policy: where can the new block go? E.g., b mod 4
  - Replacement policy: which block should be evicted? E.g., LRU

General Caching Concepts

Types of cache misses:

- Cold (compulsory) miss
  - Cold misses occur because the cache is empty.
- Conflict miss
  - Most caches limit blocks at level k+1 to a small subset (sometimes a singleton) of the block positions at level k.
  - E.g. Block i at level k+1 must be placed in block (i mod 4) at level k+1.
  - Conflict misses occur when the level k cache is large enough, but multiple data objects all map to the same level k block.
  - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.
- Capacity miss
  - Occurs when the set of active cache blocks (working set) is larger than the cache.

Examples of Caching in the Hierarchy

Cache Type	What is Cached?	Where is it Cached?	Latency (cycles)	Managed By
Registers	4-byte words	CPU core	0	Compiler
TLB	Address translations	On-Chip TLB	0	Hardware
L1 cache	64-bytes block	On-Chip L1	1	Hardware
L2 cache	64-bytes block	Off-Chip L2	10	Hardware
Virtual Memory	4-KB page	Main memory	100	Hardware+ OS
Buffer cache	Parts of files	Main memory	100	OS
Network buffer cache	Parts of files	Local disk	10,000,000	AFS/NFS client
Browser cache	Web pages	Local disk	10,000,000	Web browser
Web cache	Web pages	Remote server disks	1,000,000,000	Web proxy server

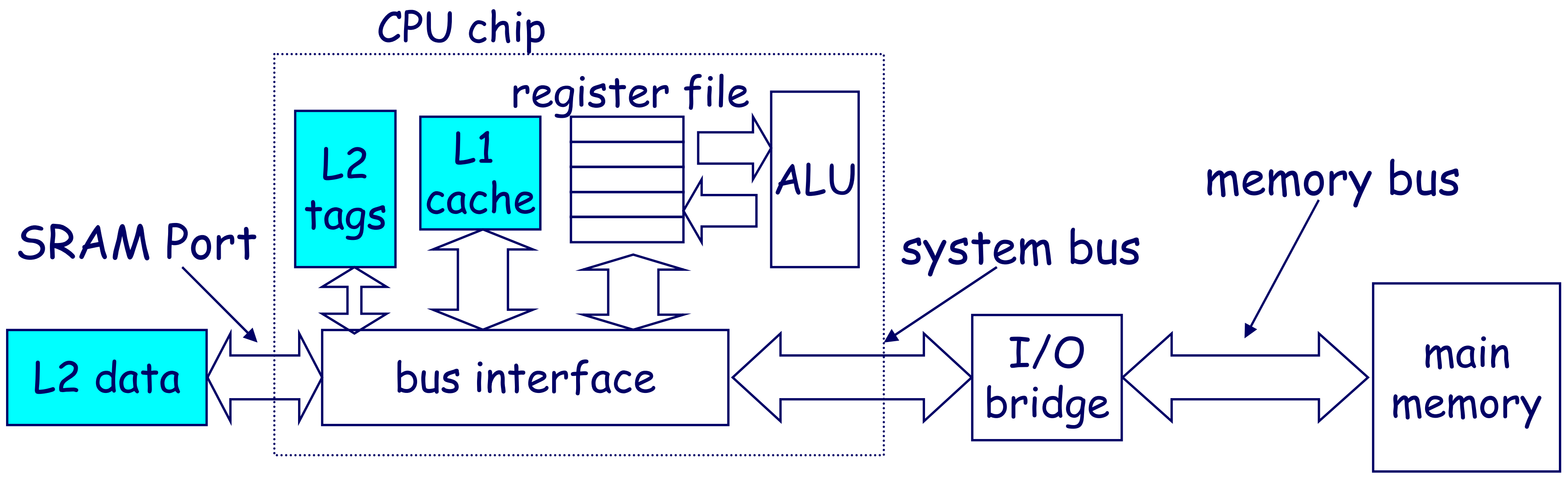
Cache Memories

Cache memories are small, fast SRAM-based memories managed automatically in hardware.

- Hold frequently accessed blocks of main memory

CPU looks first for data in L1, then in L2, then in main memory.

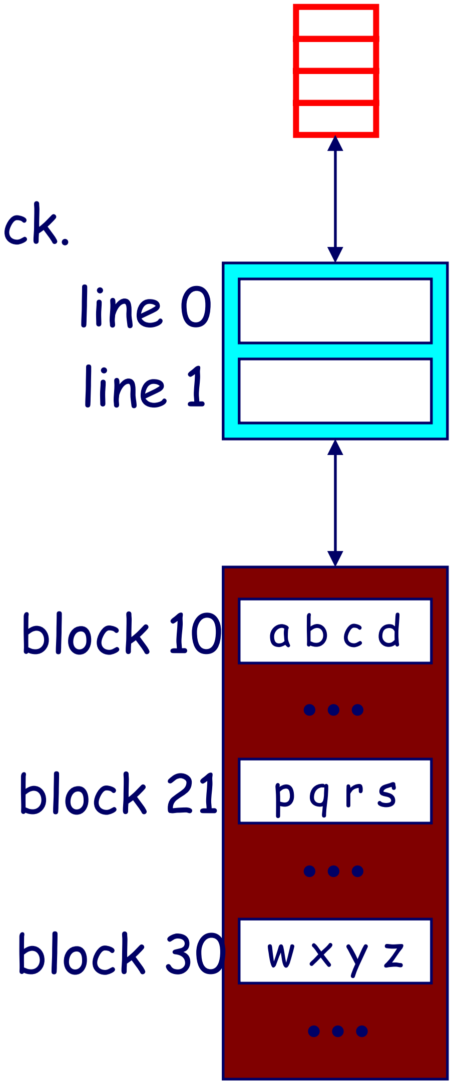
Typical system structure:



Inserting an L1 Cache Between the CPU and Main Memory

The transfer unit between the CPU register file and the cache is a 4-byte block.

The tiny, very fast CPU register file has room for four 4-byte words.



The small fast L1 cache has room for two 4-word blocks.

The transfer unit between the cache and main memory is a 4-word block (16 bytes).

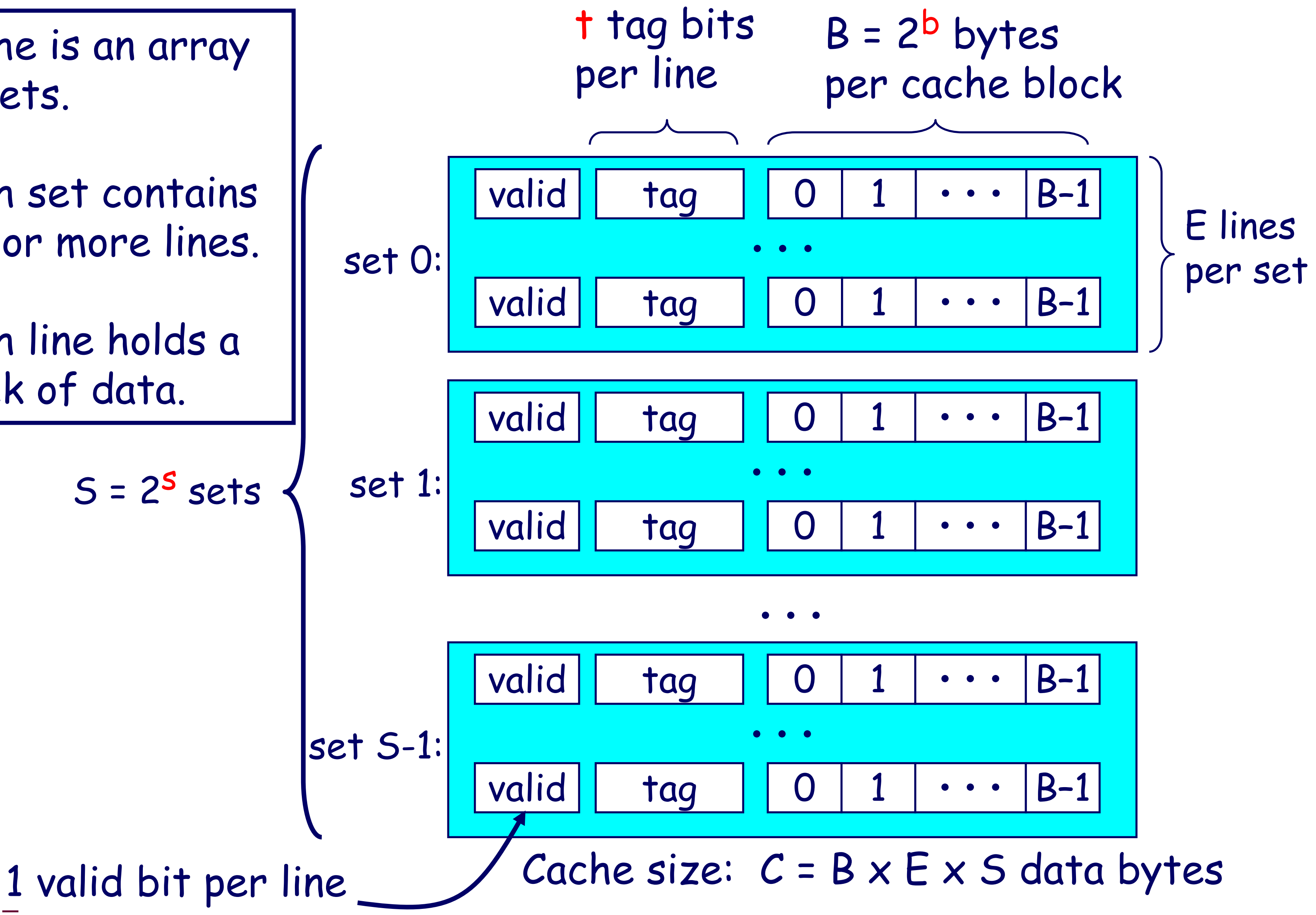
The big slow main memory has room for many 4-word blocks.

General Organization of a Cache

Cache is an array of sets.

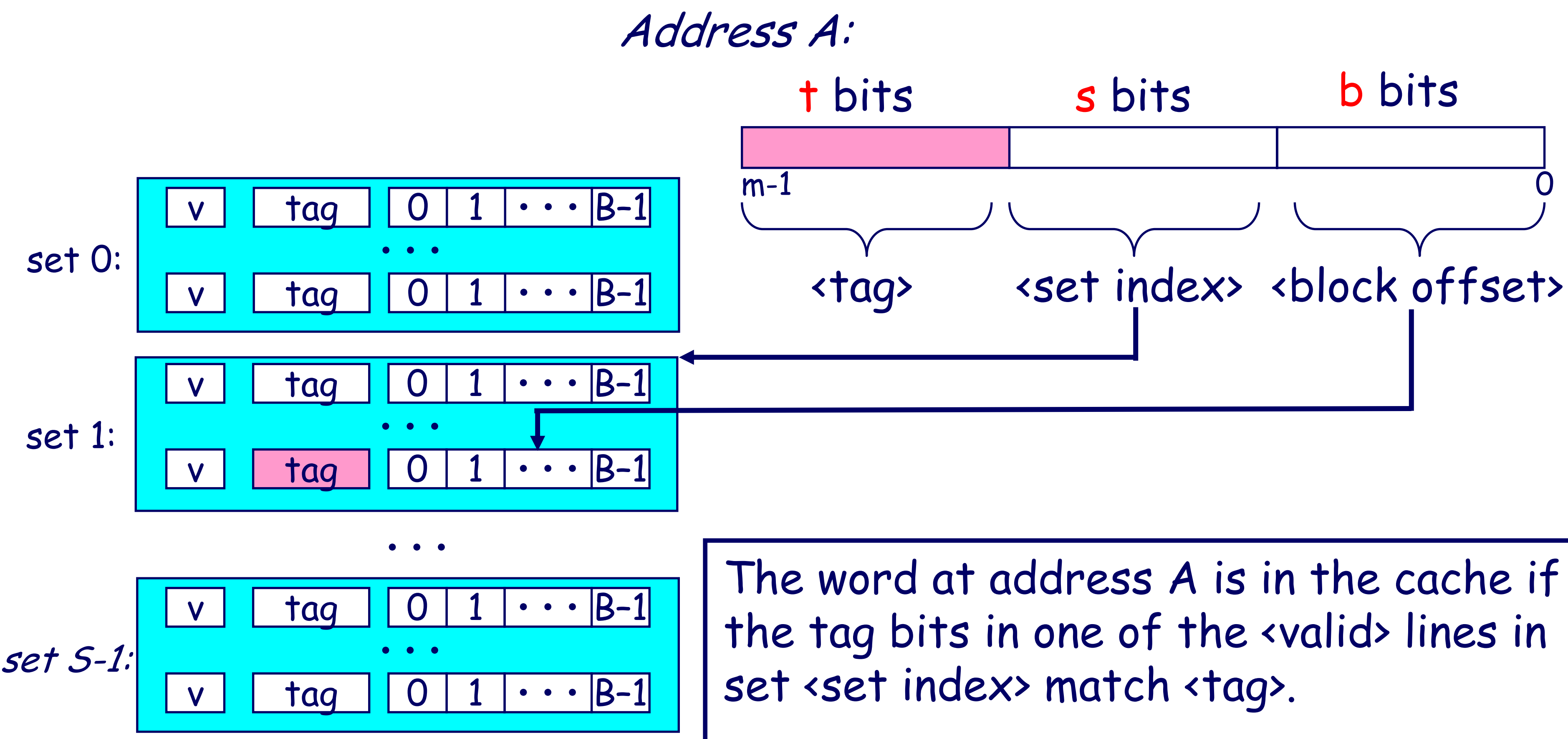
Each set contains one or more lines.

Each line holds a block of data.



Cache size: C = B x E x S data bytes

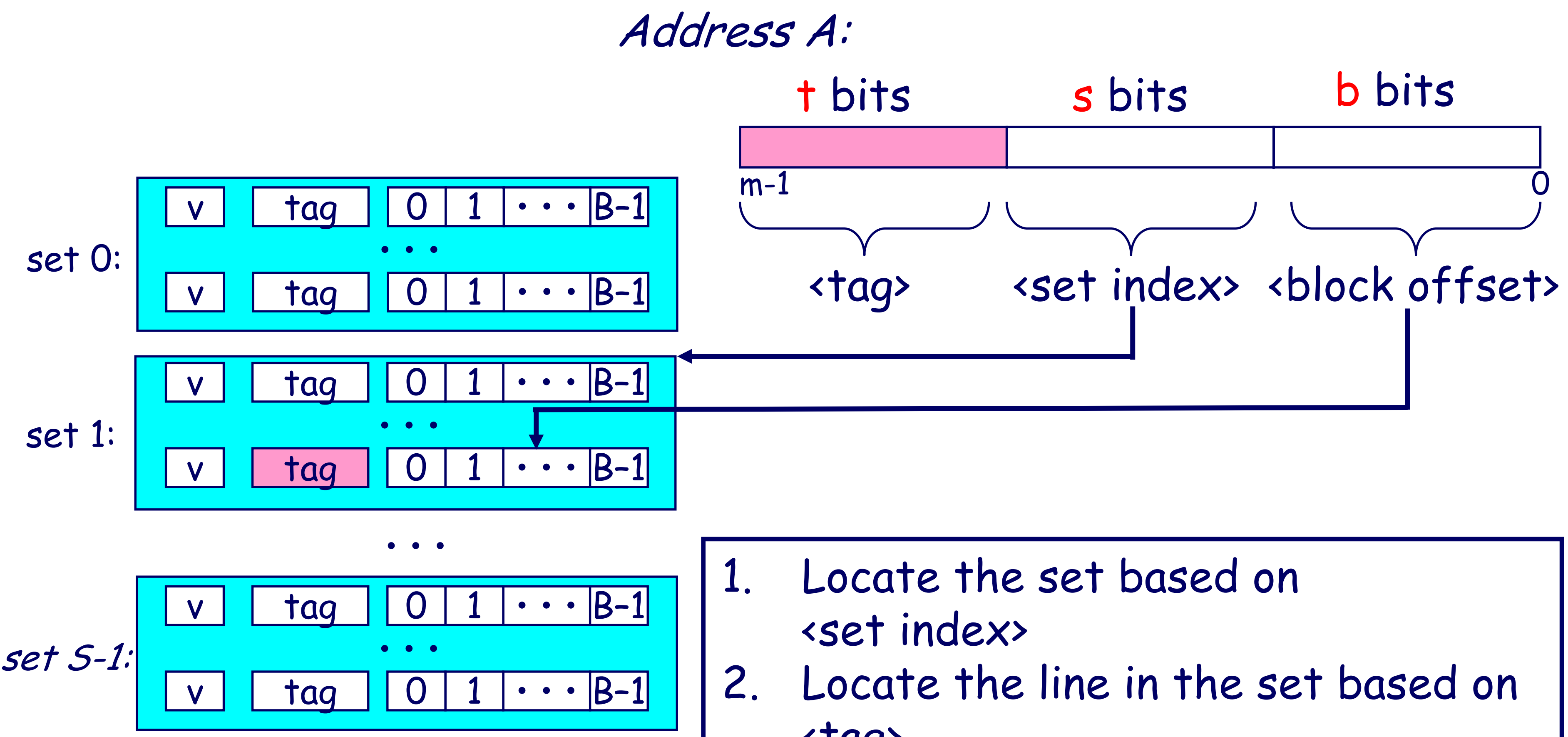
Addressing Caches



The word at address A is in the cache if the tag bits in one of the <valid> lines in set <set index> match <tag>.

The word contents begin at offset <block offset> bytes from the beginning of the block.

Addressing Caches

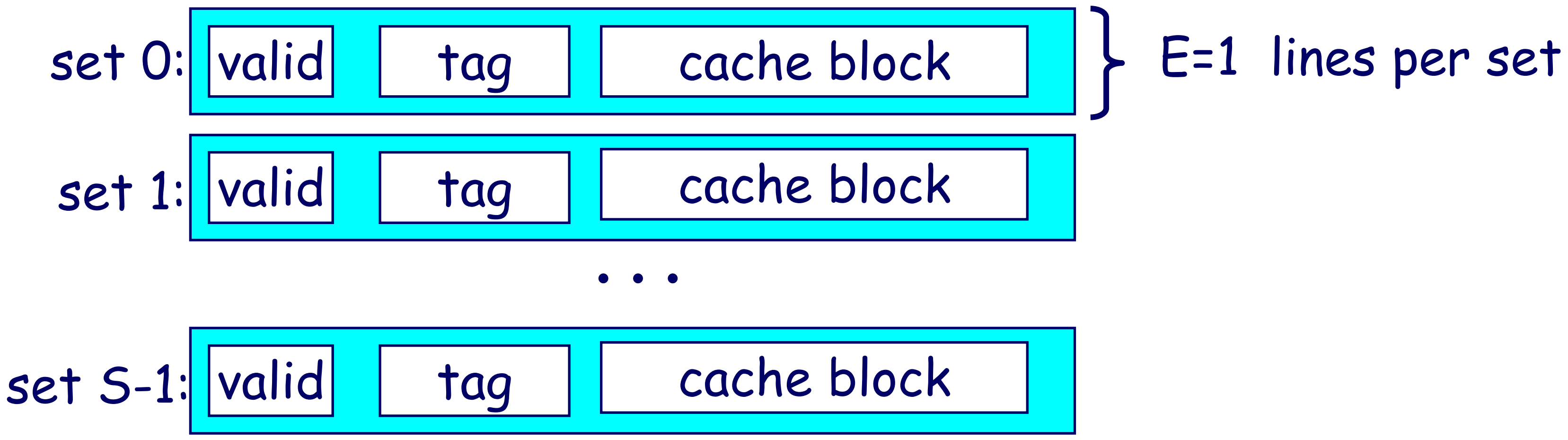


1. Locate the set based on <set index>
2. Locate the line in the set based on <tag>
3. Check that the line is valid
4. Locate the data in the line based on <block offset>

Direct-Mapped Cache

Simplest kind of cache, easy to build
(only 1 tag compare required per access)

Characterized by exactly one line per set.

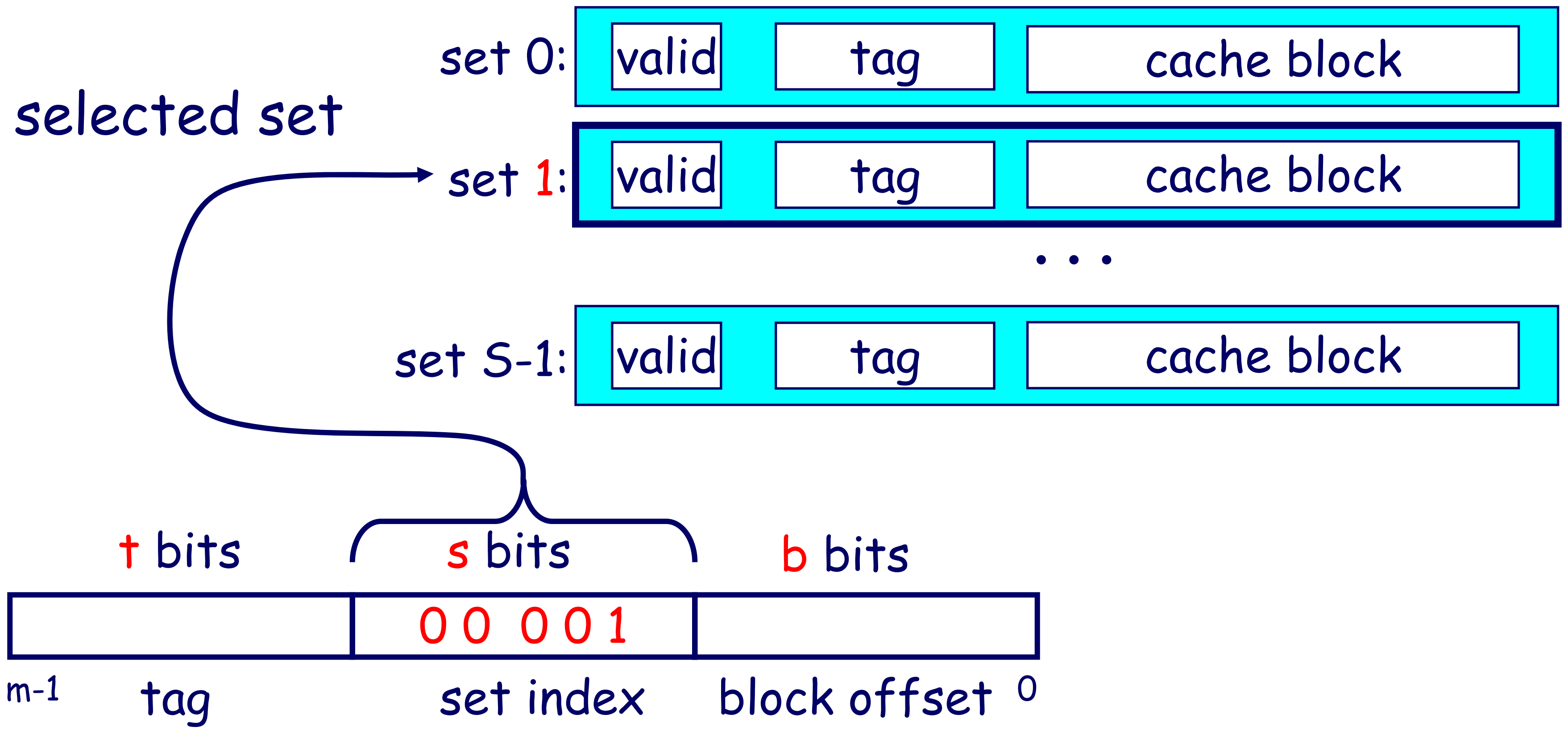


Cache size: C = B x S data bytes

Accessing Direct-Mapped Caches

Set selection

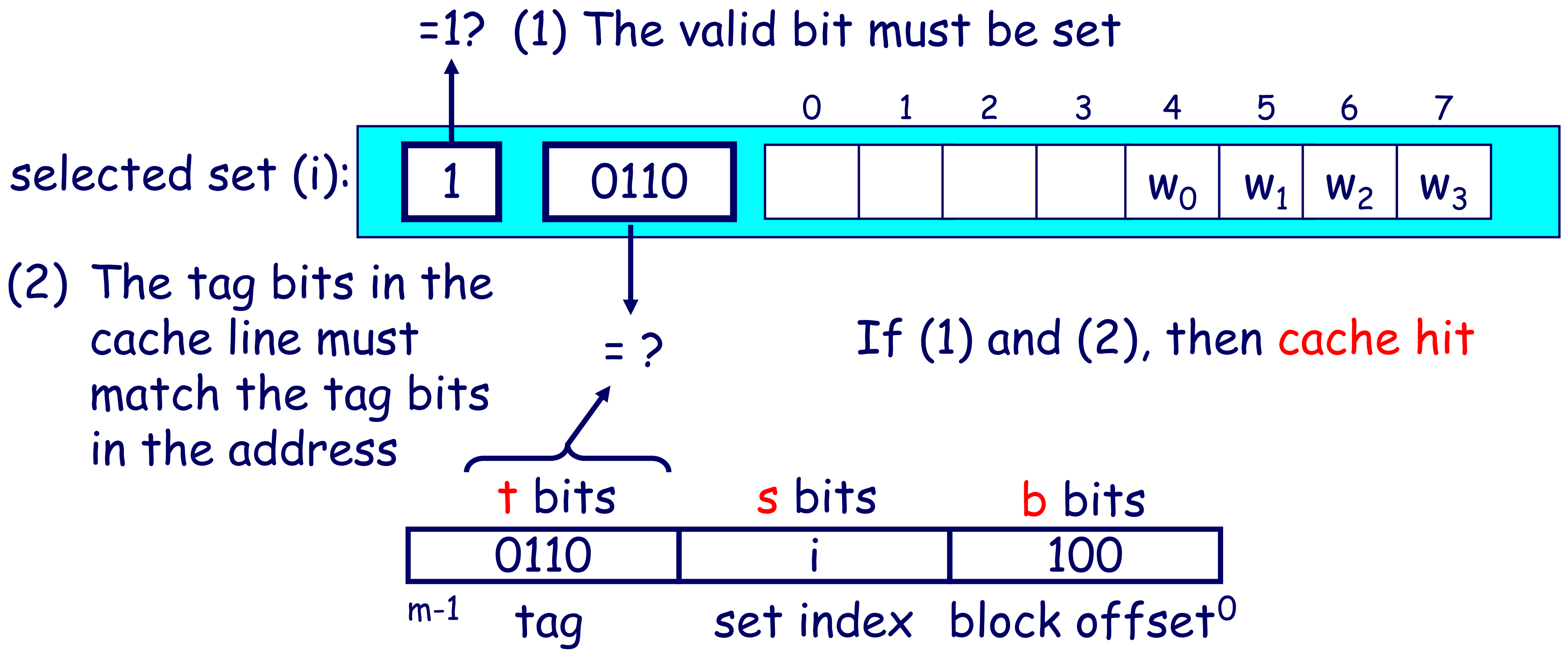
- Use the set index bits to determine the set of interest.



Accessing Direct-Mapped Caches

Line matching and word selection

- **Line matching: Find a valid line in the selected set with a matching tag**
- **Word selection: Then extract the word**

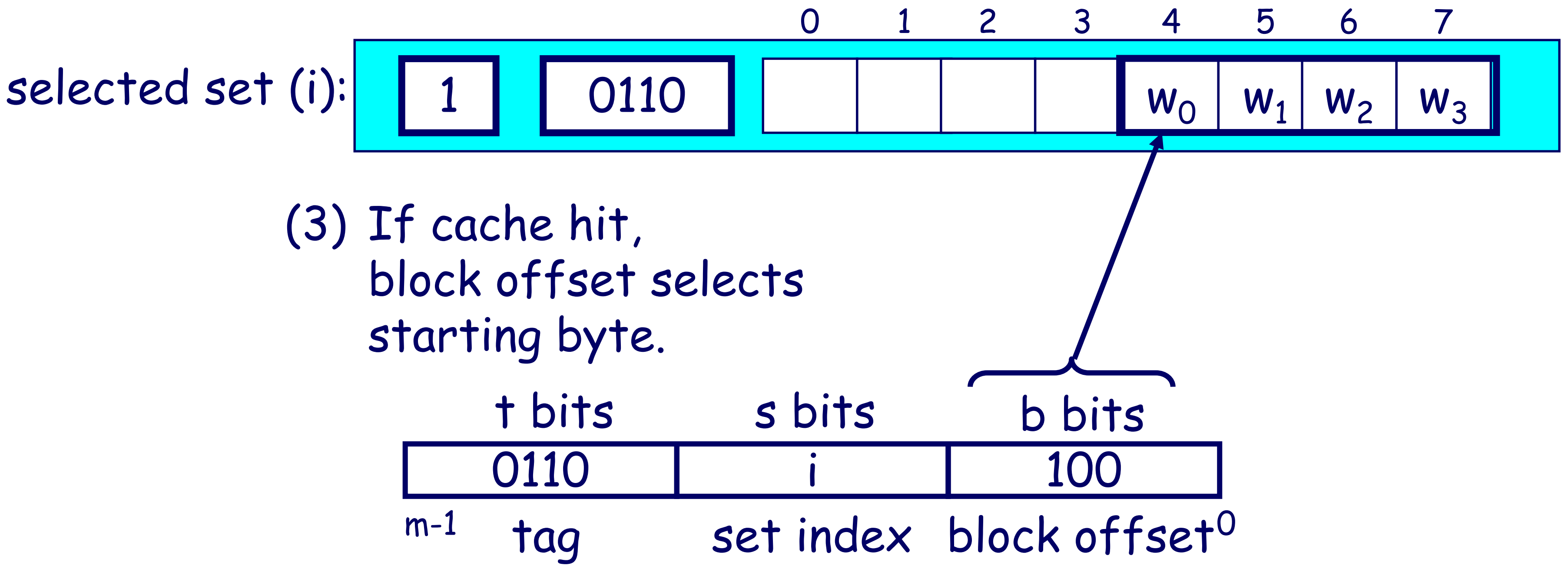


=1?
(1) The valid bit must be set
0
1
2
3
4
5
6
7
selected set (i):
1
0110
w_0
w_1
w_2
w_3
(2) The tag bits in the cache line must match the tag bits in the address
= ?
If (1) and (2), then cache hit
t bits
s bits
b bits
0110
i
100
m-1
tag
set index
block offset
0

Accessing Direct-Mapped Caches

Line matching and word selection

- **Line matching: Find a valid line in the selected set with a matching tag**
- **Word selection: Then extract the word**



Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 byte addresses, B=2 bytes/block,
S=4 sets, E=1 entry/set

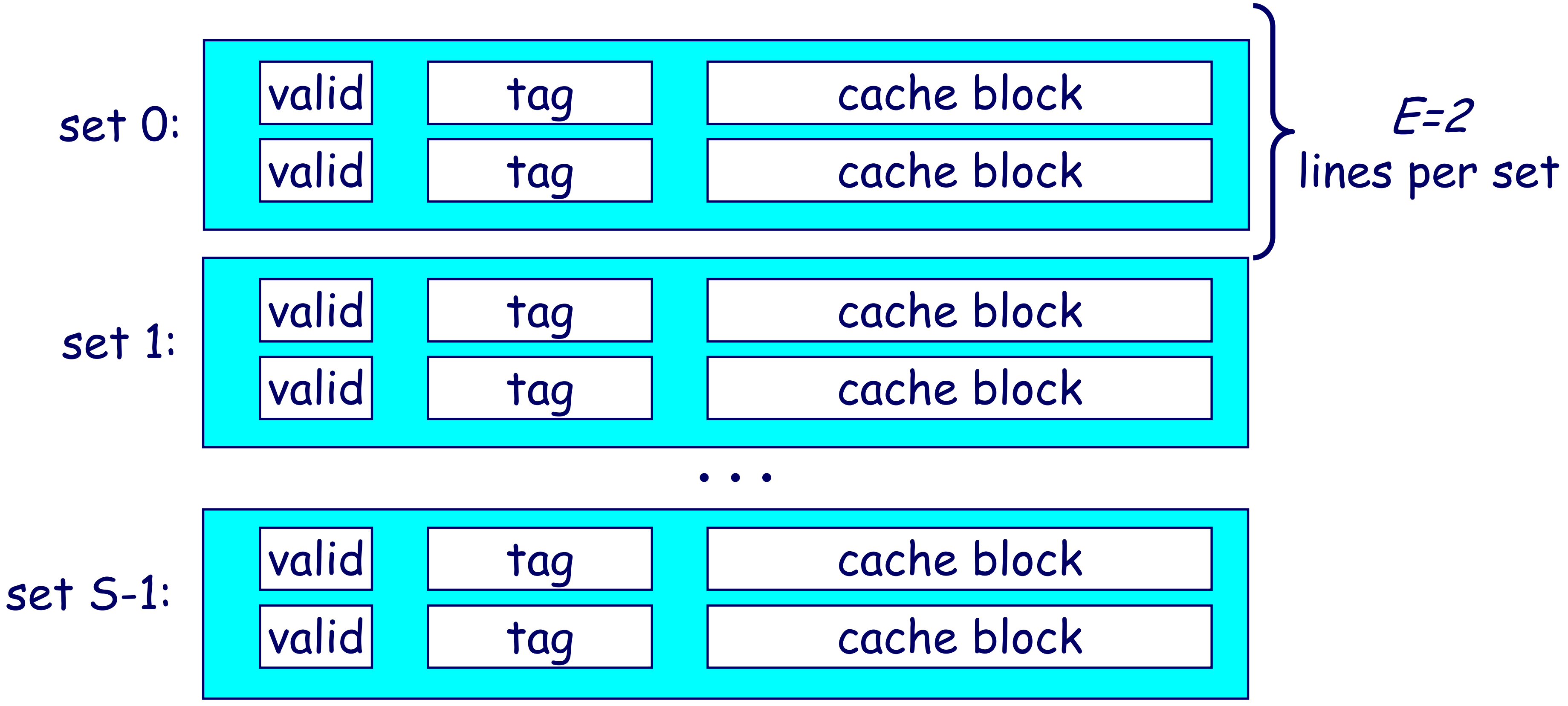
Address trace (reads):

0	**[0000_2],**	miss
1	**[0001_2],**	hit
7	**[0111_2],**	miss
8	**[1000_2],**	miss
0	**[0000_2]**	miss

v	tag	data
1	0	M[0-1]
1	0	M[6-7]

Set Associative Caches

Characterized by more than one line per set

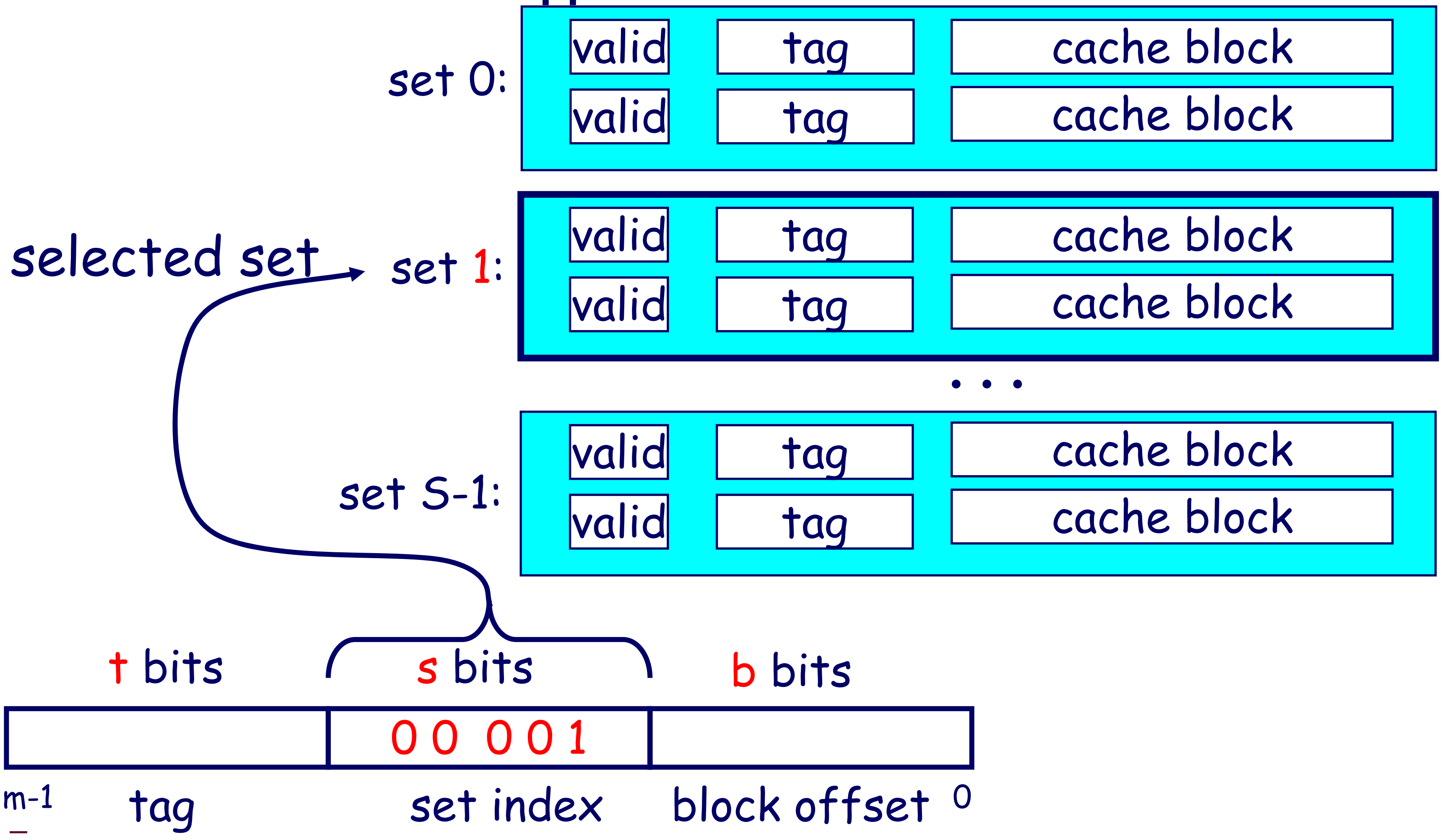


E-way associative cache

Accessing Set Associative Caches

Set selection

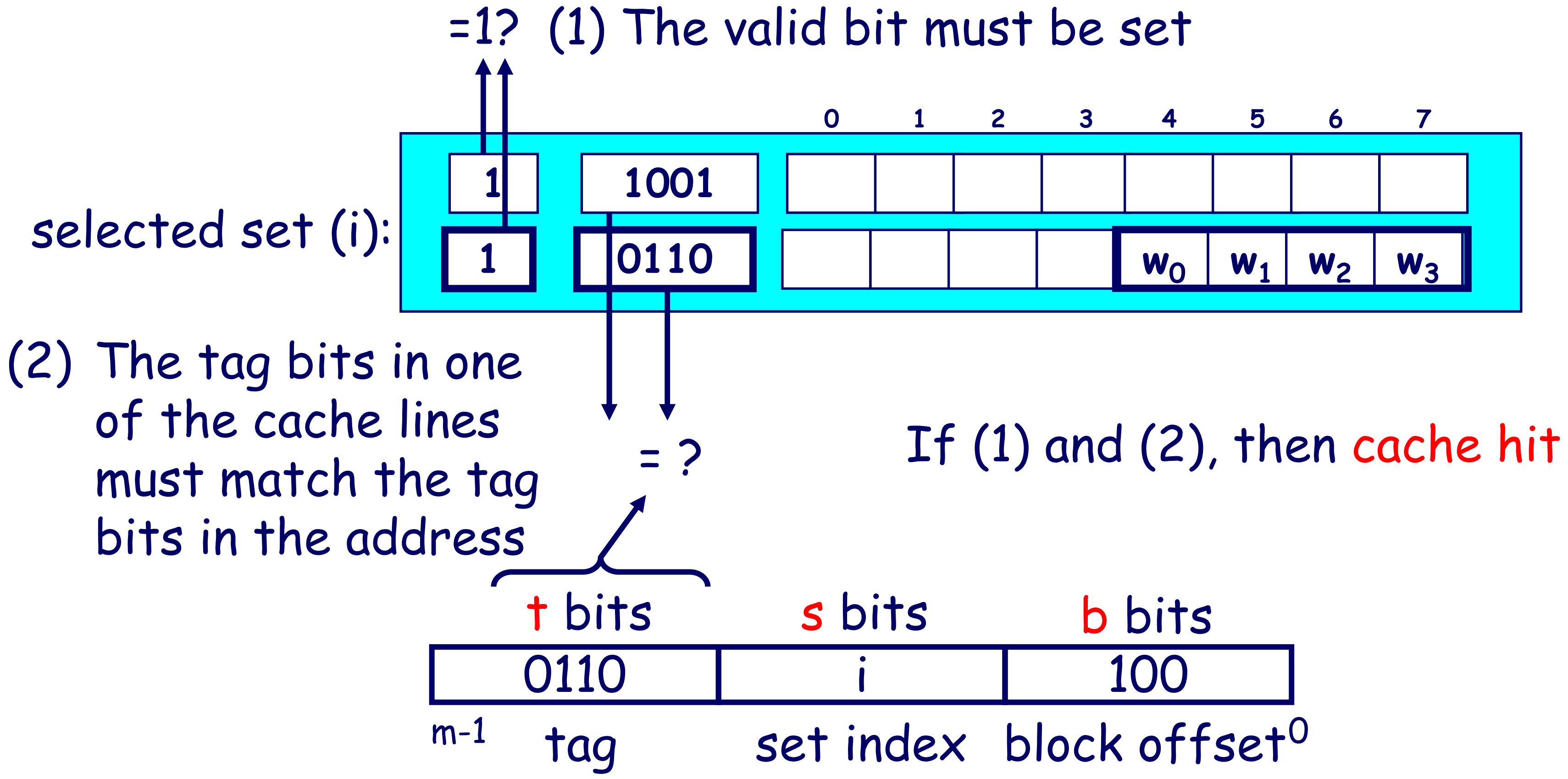
- identical to direct-mapped cache



Accessing Set Associative Caches

Line matching and word selection

- must compare the tag in each valid line in the selected set.

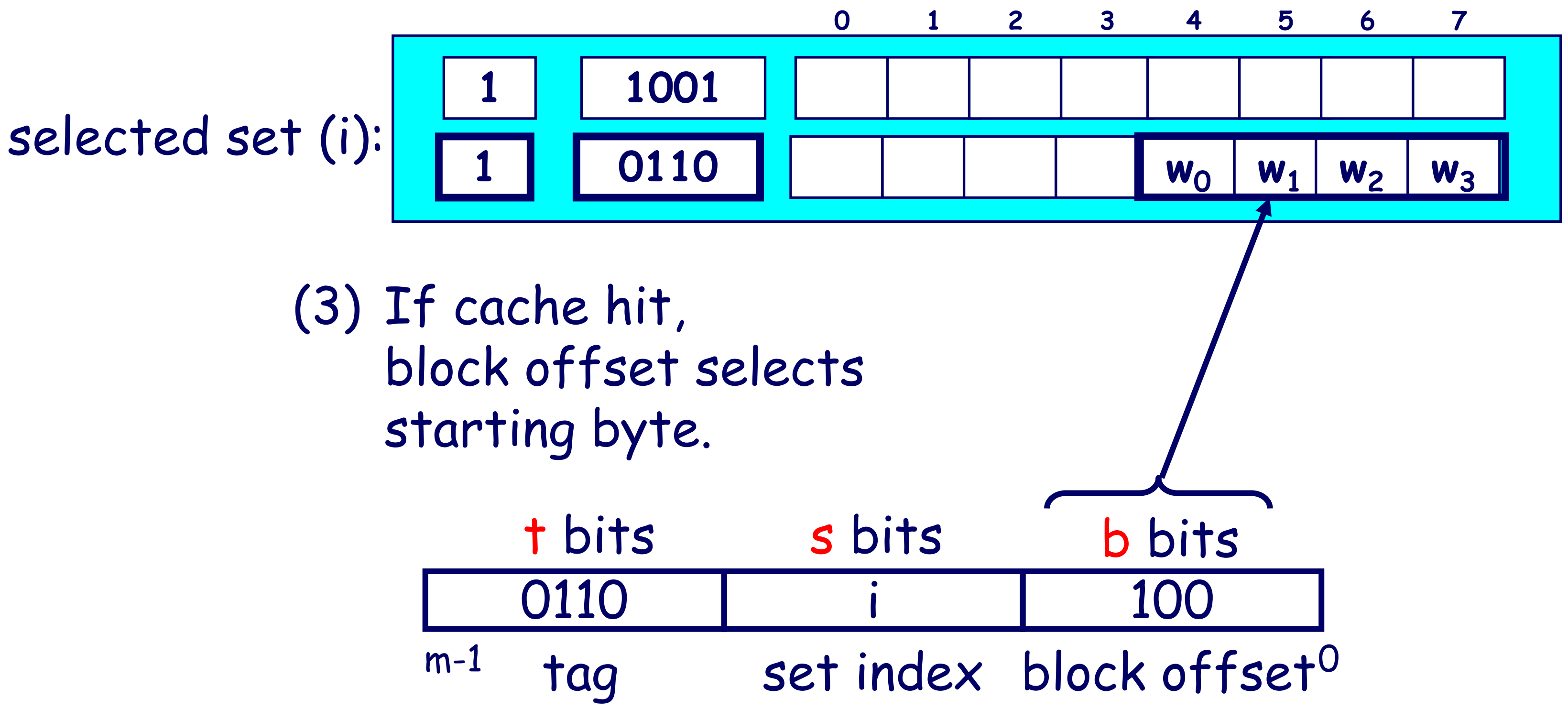


=1?
(1) The valid bit must be set
0
1
2
3
4
5
6
7
1
1001
selected set (i):
1
0110
w_0
w_1
w_2
w_3
(2) The tag bits in one of the cache lines must match the tag bits in the address
= ?
If (1) and (2), then cache hit
t bits
s bits
b bits
0110
i
100
m-1
tag
set index
block offset
0

Accessing Set Associative Caches

Line matching and word selection

- Word selection is the same as in a direct mapped cache



2-Way Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

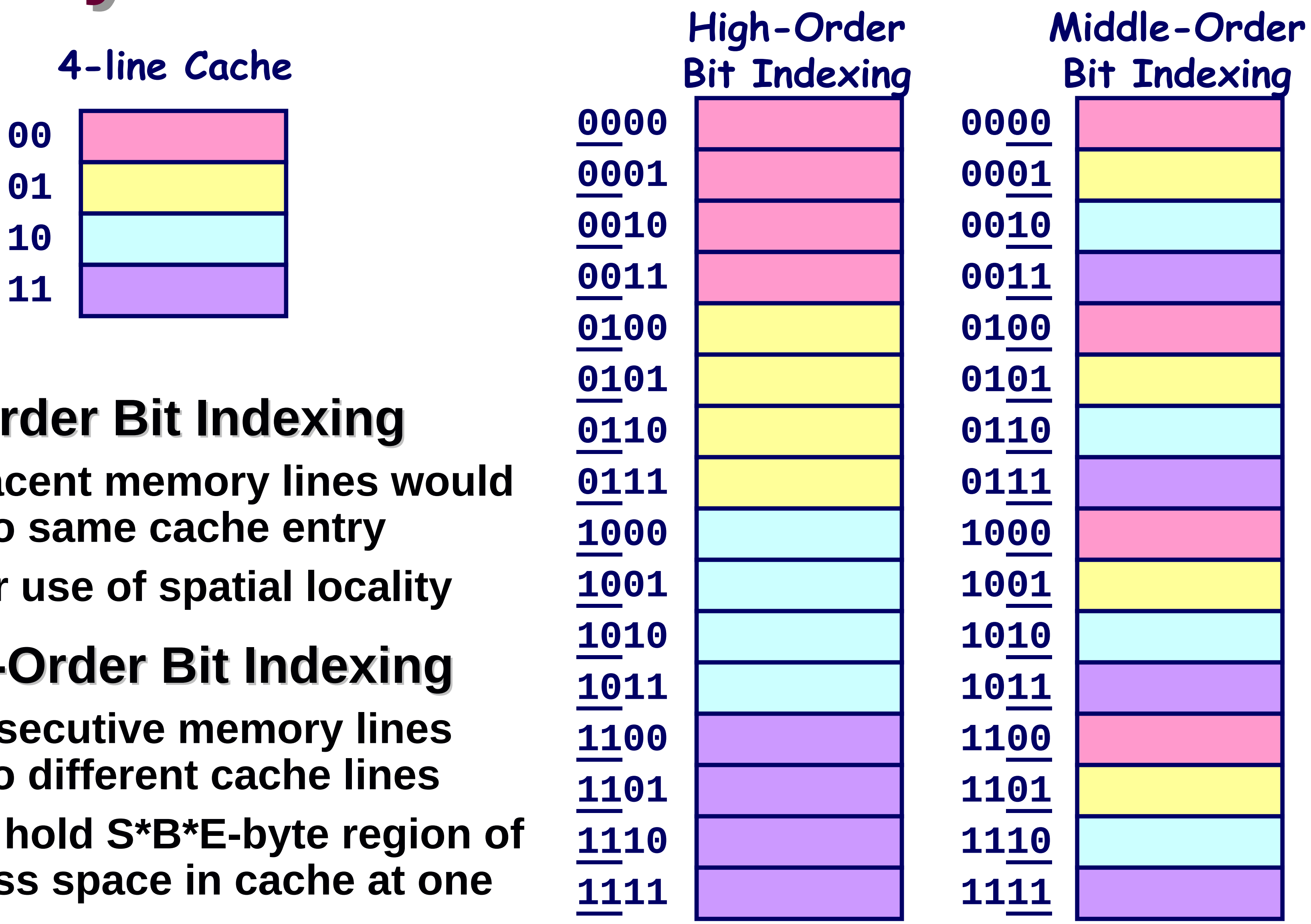
M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 entry/set

Address trace (reads):

0	$[0000_2]$,	miss
1	$[0001_2]$,	hit
7	$[0111_2]$,	miss
8	$[1000_2]$,	miss
0	$[0000_2]$	hit

v	tag	data
1	00	M[0-1]
1	10	M[8-9]
1	01	M[6-7]
0		

Why Use Middle Bits as Index?



High-Order Bit Indexing

- Adjacent memory lines would map to same cache entry
- Poor use of spatial locality

Middle-Order Bit Indexing

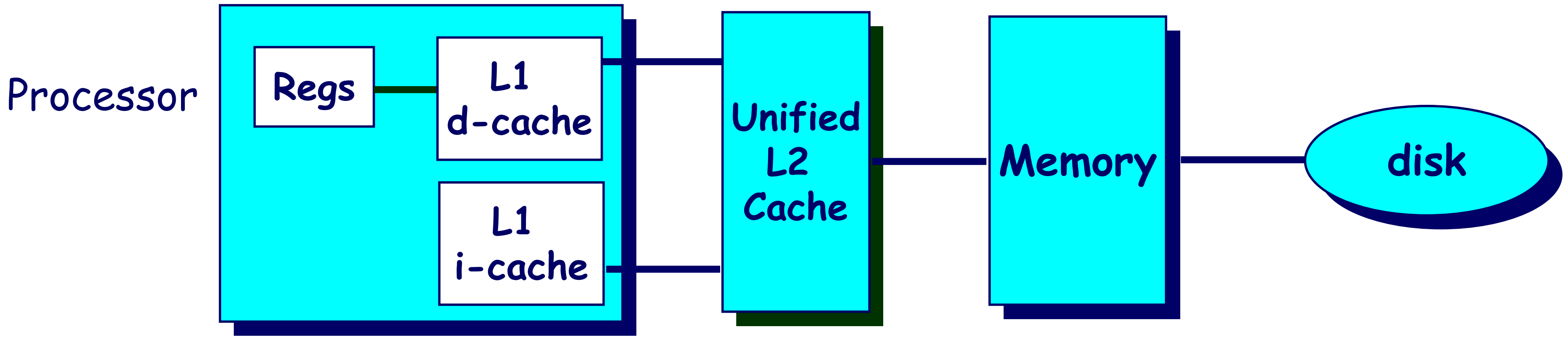
- Consecutive memory lines map to different cache lines
- Can hold S*B*E-byte region of address space in cache at one time

Maintaining a Set-Associate Cache

- How to decide which cache line to use in a set?
  - Least Recently Used (LRU), Requires $\lceil lg_2(E!) \rceil$ extra bits
  - Not recently Used (NRU)
  - Random
- Virtual vs. Physical addresses:
  - The memory system works with physical addresses, but it takes time to translate a virtual to a physical address. So most L1 caches are virtually indexed, but physically tagged.

Multi-Level Caches

Options: separate data and instruction caches, or a unified cache



size:	200 B	8-64 KB	1-4MB SRAM	128 MB DRAM	30 GB
speed:	3 ns	3 ns	6 ns	60 ns	8 ms
$/Mbyte:			$100/MB	$1.50/MB	$0.05/MB
line size:	8 B	32 B	32 B	8 KB	

larger, slower, cheaper →

What about writes?

Multiple copies of data exist:

- L1
- L2
- Main Memory
- Disk

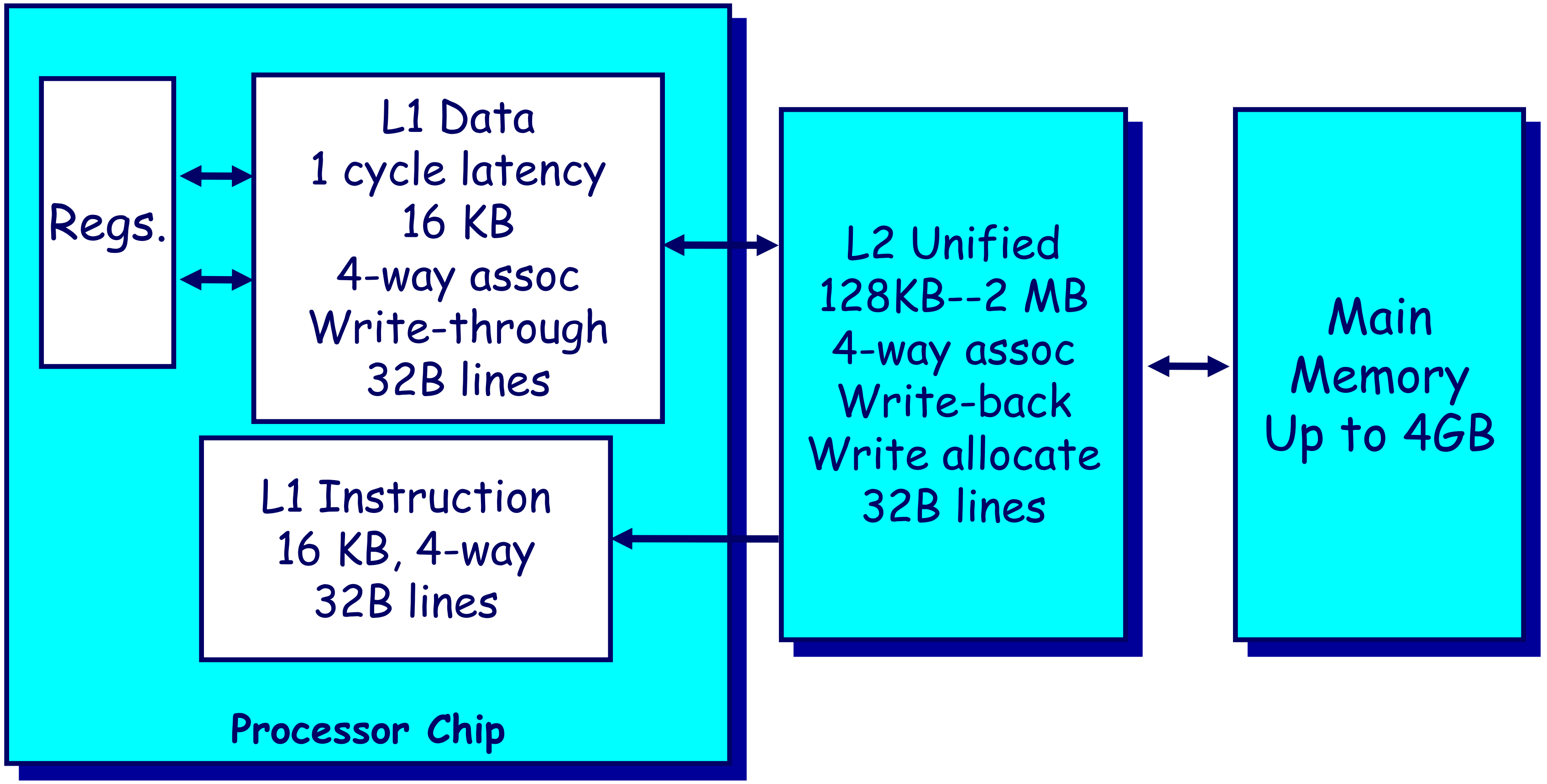
What to do when we write?

- Write-through
- Write-back
  - need a dirty bit
  - What to do on a write-miss?

What to do on a replacement?

- Depends on whether it is write through or write back

Intel Pentium III Cache Hierarchy



Cache Performance Metrics

Miss Rate

- Fraction of memory references not found in cache (misses / references)
- Typical numbers:
  - 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Hit Time

- Time to deliver a line in the cache to the processor (includes time to determine whether the line is in the cache)
- Typical numbers:
  - 1-2 clock cycle for L1
  - 5-20 clock cycles for L2

Aside for architects:
-Increasing cache size?
-Increasing block size?
-Increasing associativity?

Miss Penalty

- Additional time required because of a miss
  - Typically 50-200 cycles for main memory (Trend: increasing!)

Writing Cache Friendly Code

- Repeated references to variables are good (temporal locality)
- Stride-1 reference patterns are good (spatial locality)
- Examples:
  - cold cache, 4-byte words, 4-word cache blocks

```
int sum_array_rows(int a[M][N])
{
  int i, j, sum = 0;

  for (i = 0; i < M; i++)
     for (j = 0; j < N; j++)
        sum += a[i][j];
  return sum;
}
```

Miss rate = 1/4 = 25%

```
int sum_array_cols(int a[M][N])
{
  int i, j, sum = 0;

  for (j = 0; j < N; j++)
     for (i = 0; i < M; i++)
        sum += a[i][j];
  return sum;
}
```

Miss rate = 100%

The Memory Mountain

Read throughput (read bandwidth)

- Number of bytes read from memory per second (MB/s)

Memory mountain

- Measured read throughput as a function of spatial and temporal locality.
- Compact way to characterize memory system performance.

Memory Mountain Test Function

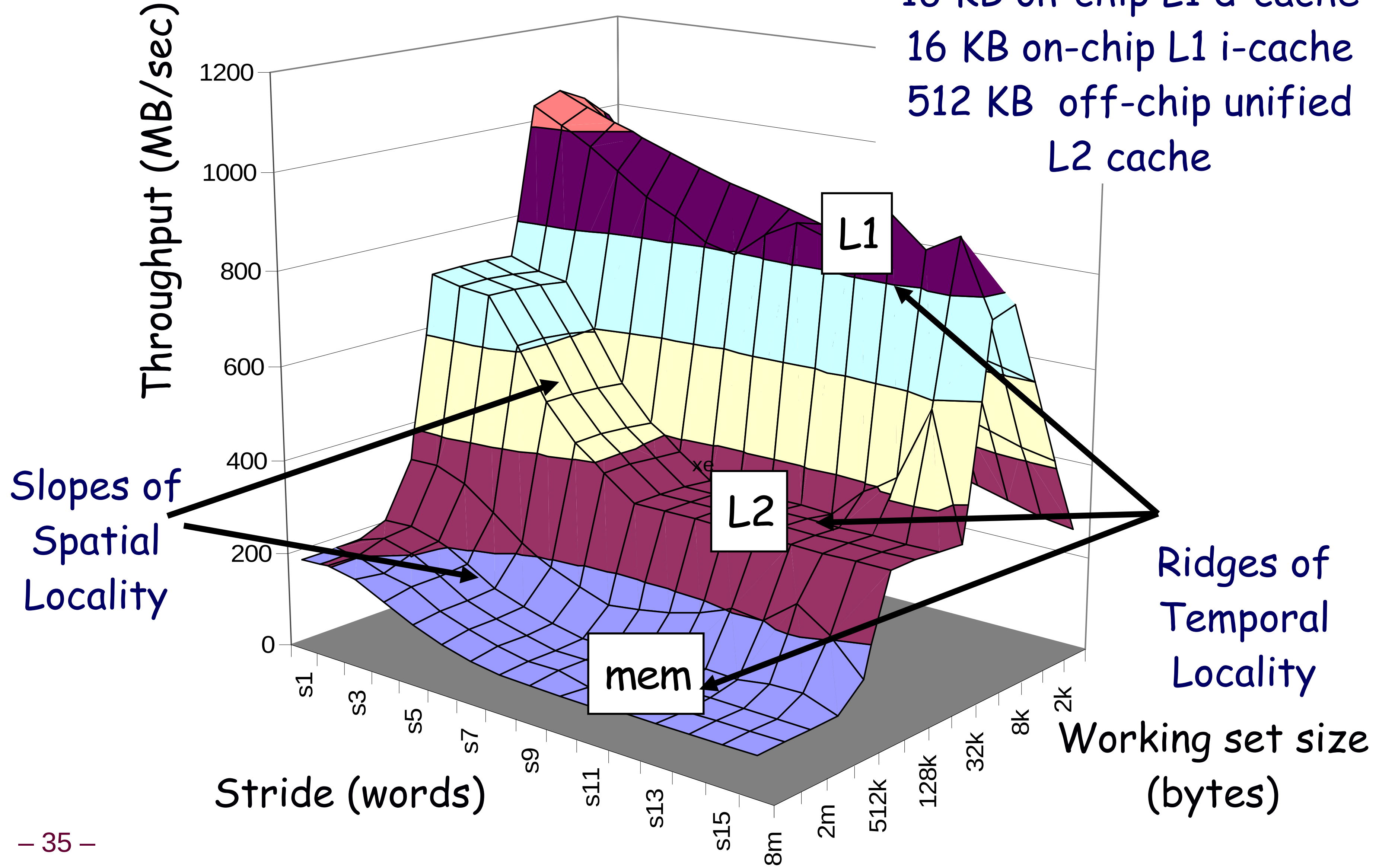
```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride);                      /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0);  /* call test(elems,stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

Memory Mountain Main Routine

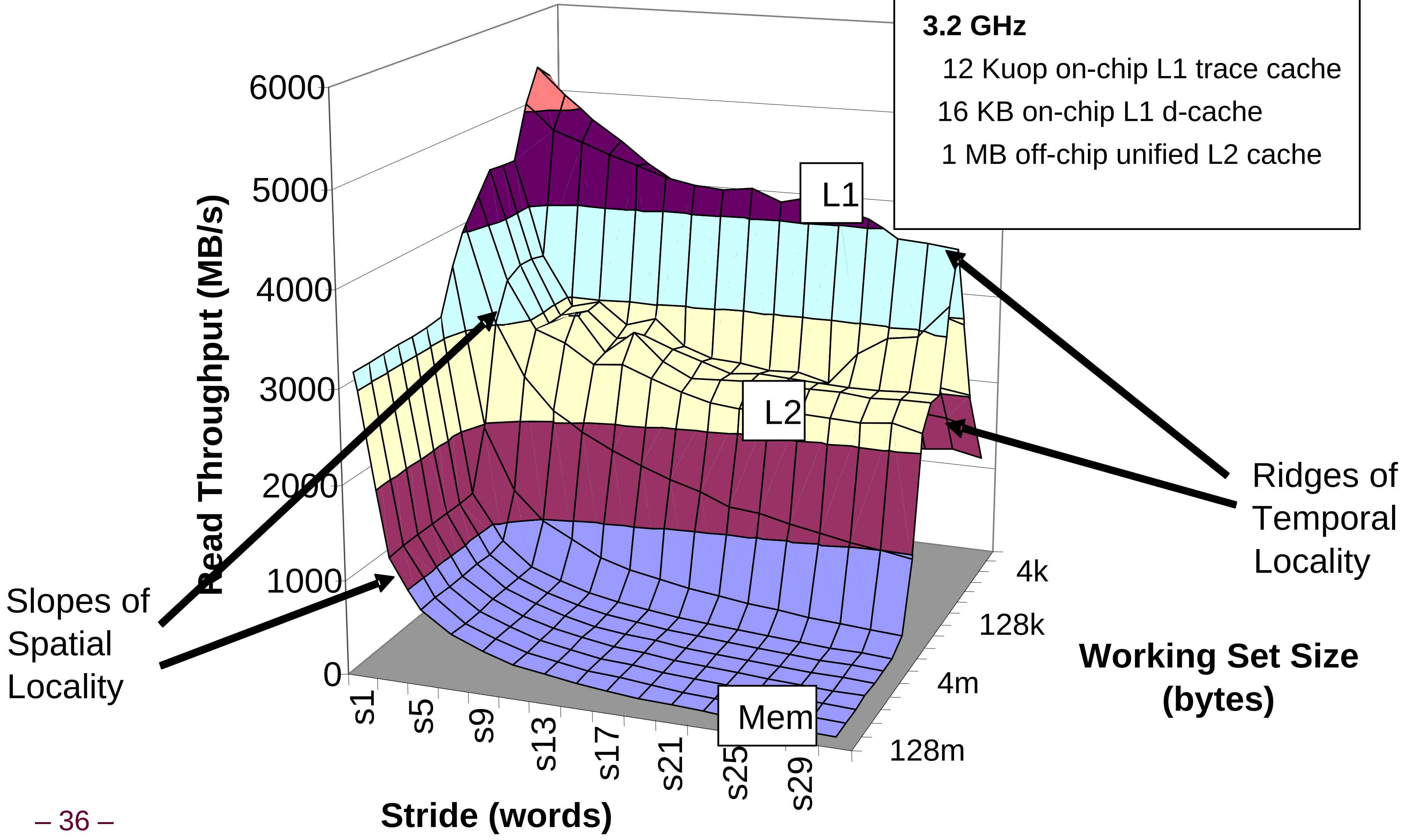
```
/* mountain.c - Generate the memory mountain. */
#define MINBYTES (1 << 10)  /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23)  /* ... up to 8 MB */
#define MAXSTRIDE 16        /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS];         /* The array we'll be traversing */

int main()
{
    int size;        /* Working set size (in bytes) */
    int stride;      /* Stride (in array elements) */
    double Mhz;      /* Clock frequency */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    Mhz = mhz(0);              /* Estimate the clock frequency */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride, Mhz));
        printf("\n");
    }
    exit(0);
}
```

The Memory Mountain

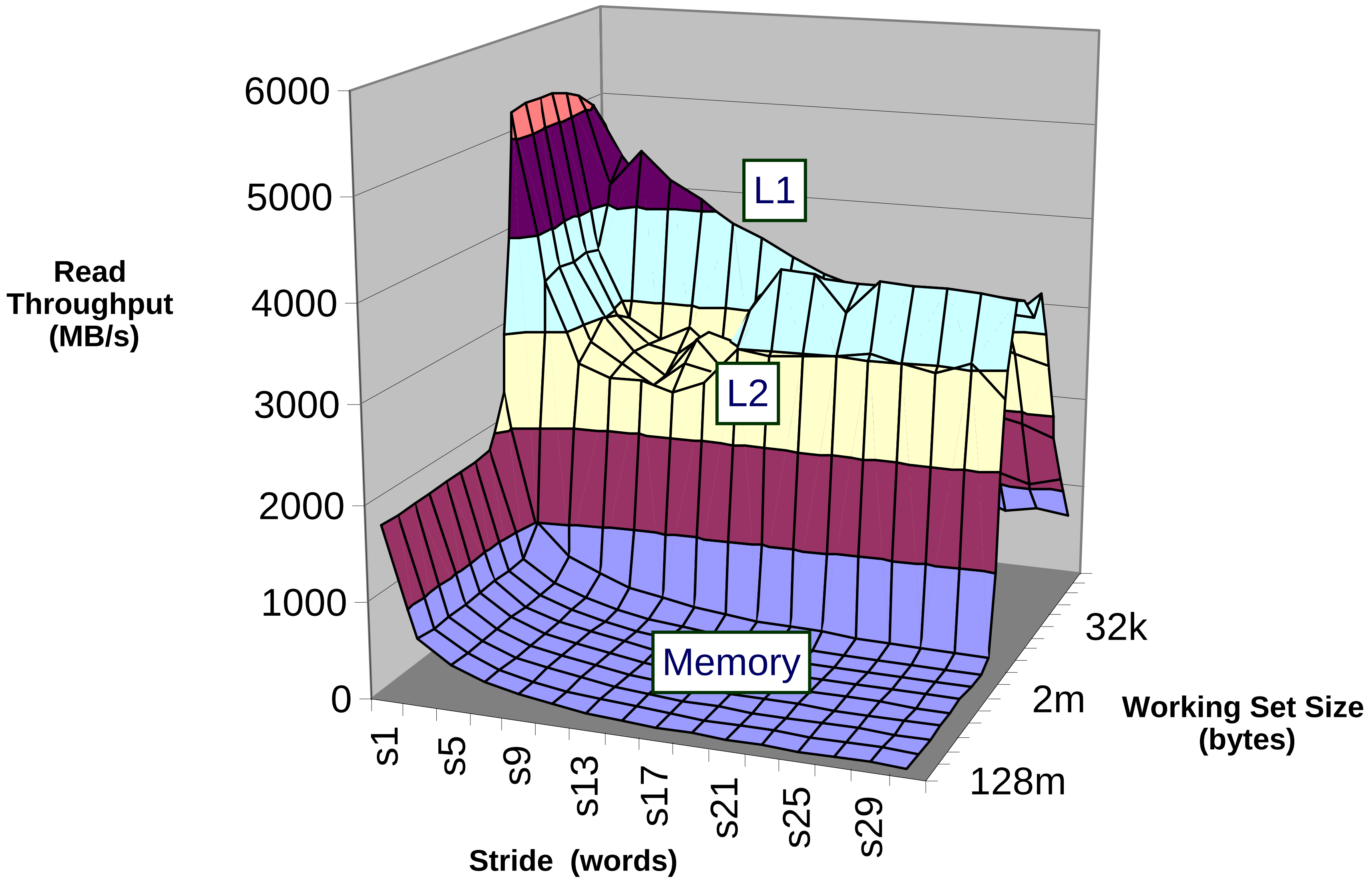
Pentium III
550 MHz
16 KB on-chip L1 d-cache
16 KB on-chip L1 i-cache
512 KB off-chip unified
L2 cache



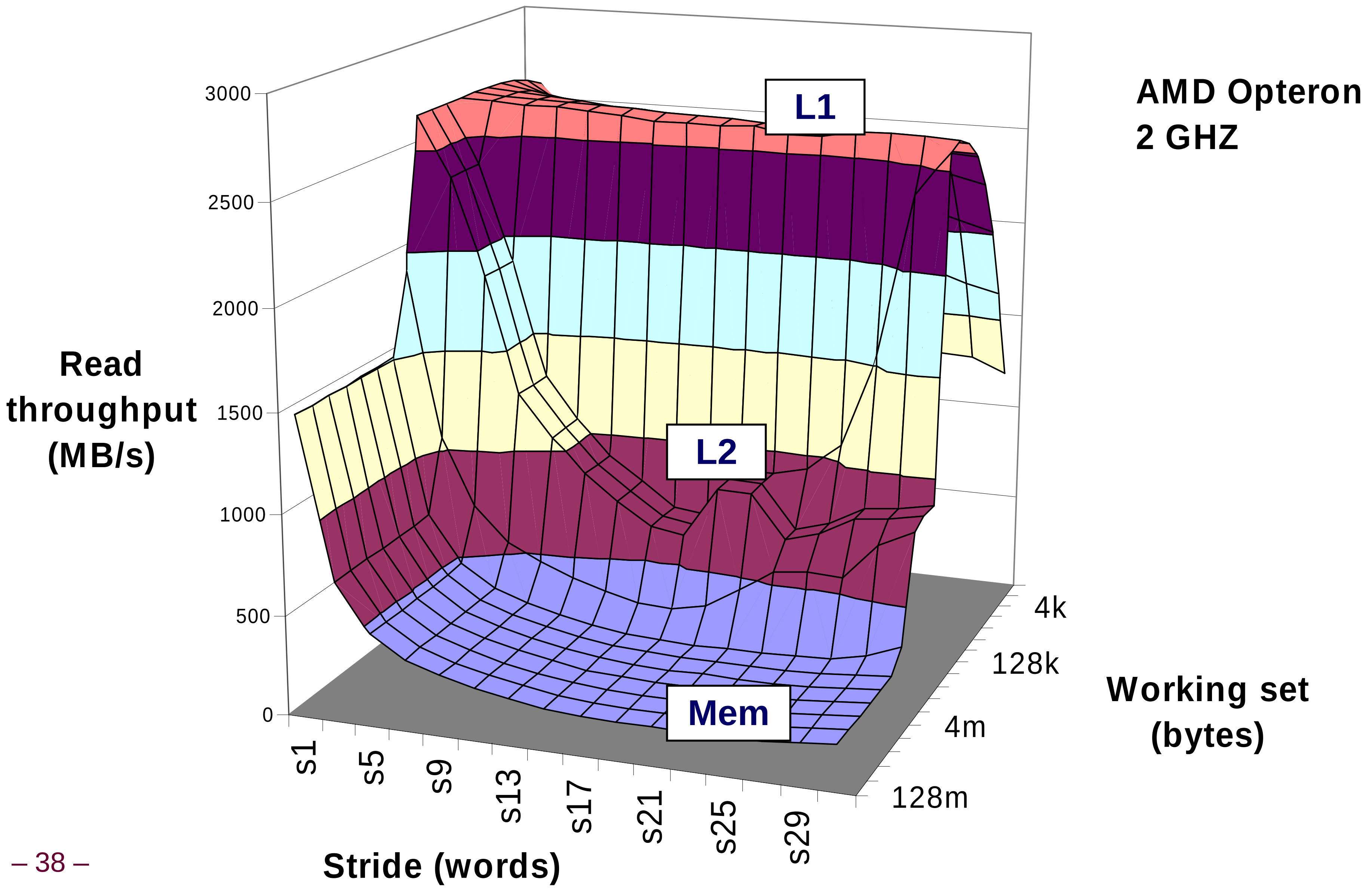
X86-64 Memory Mountain



Camaro.cis.ksu.edu Memory Mountain
(Intel(R) Xeon(TM) CPU 2.66GHz)



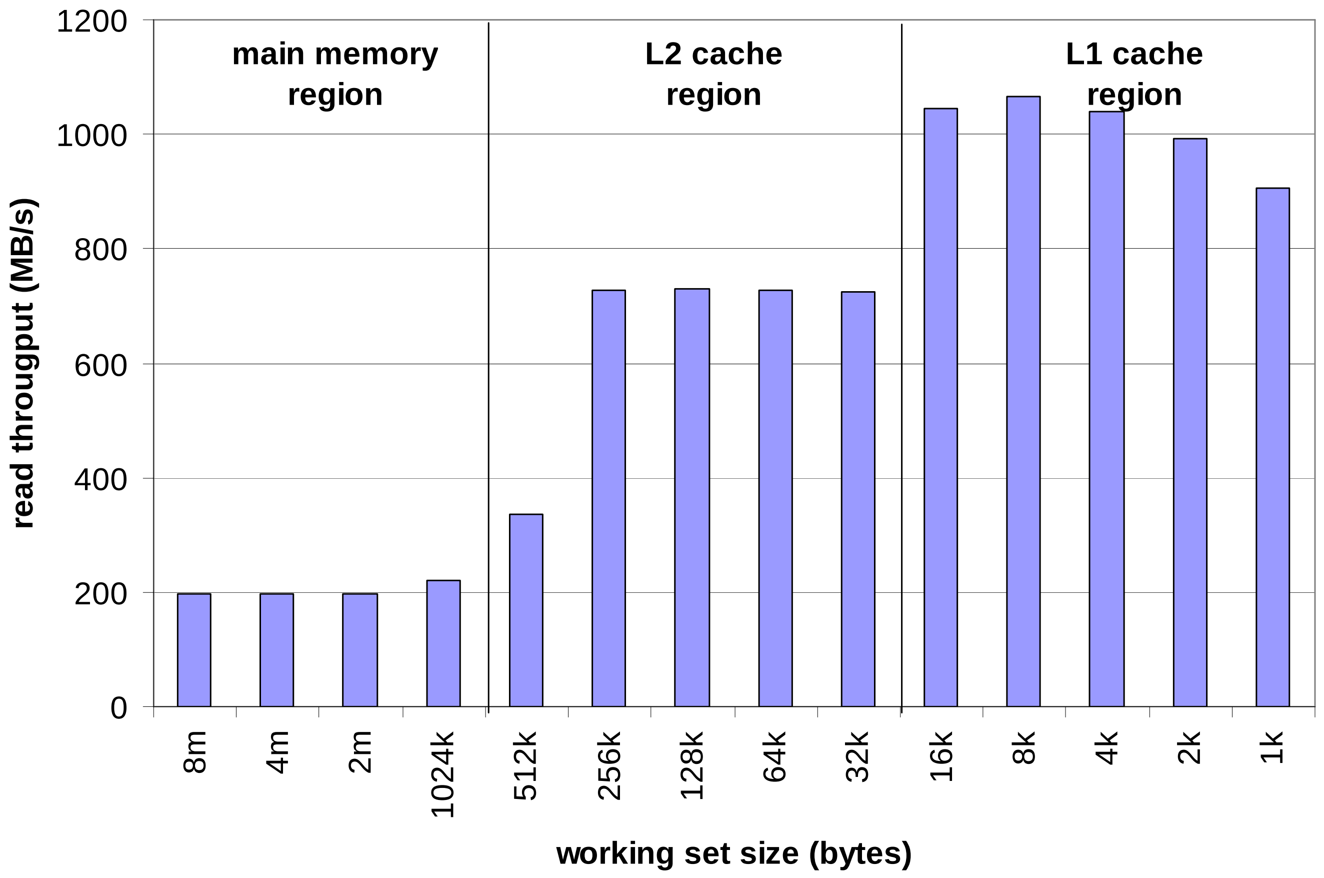
Opteron Memory Mountain



Ridges of Temporal Locality

Slice through the memory mountain with stride=1

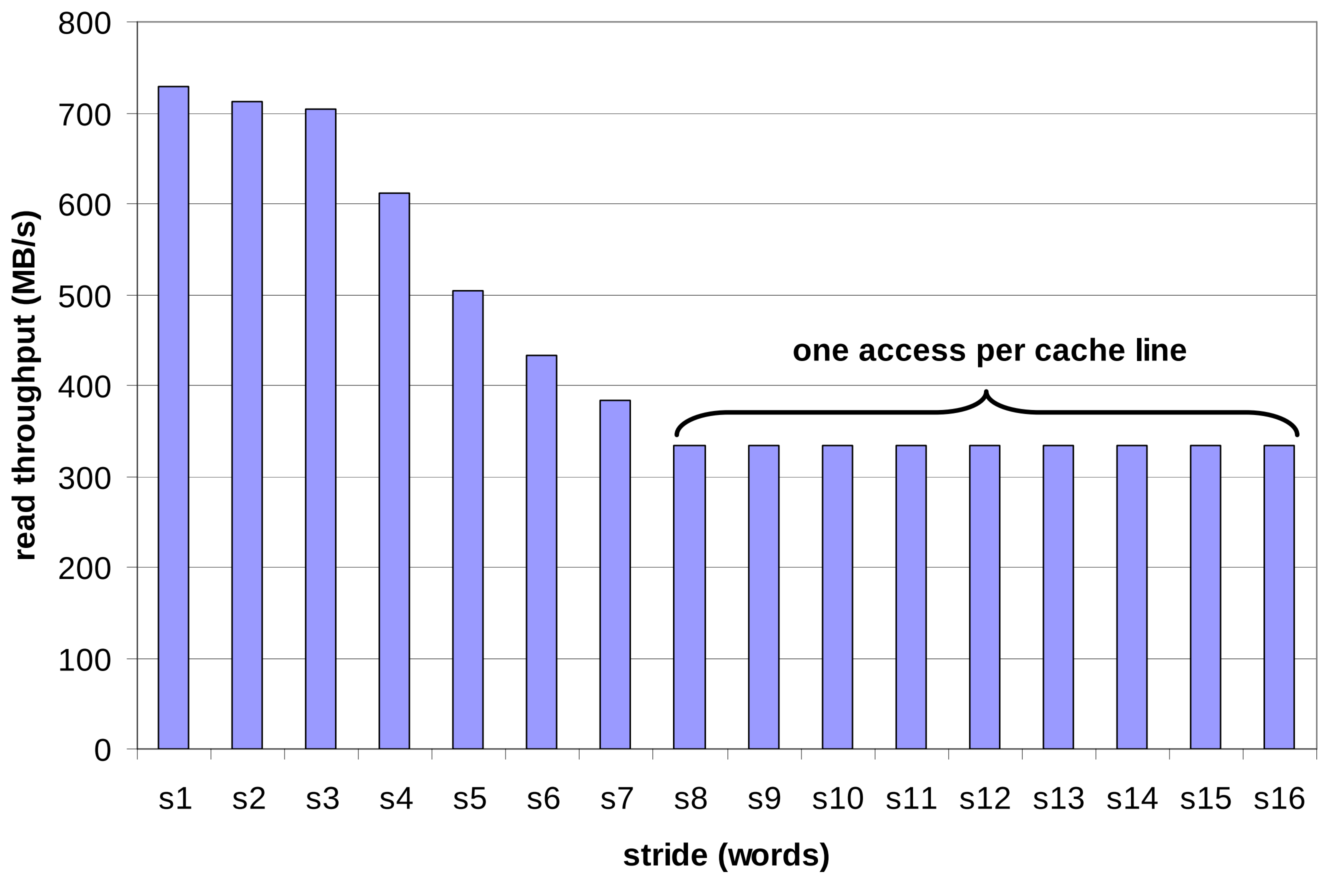
- illuminates read throughputs of different caches and memory



A Slope of Spatial Locality

Slice through memory mountain with size=256KB

- shows cache block size.



Matrix Multiplication Example

Major Cache Effects to Consider

- Total cache size
  - Exploit temporal locality and keep the working set small (e.g., use blocking)
- Block size
  - Exploit spatial locality

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable **sum** *held in register*

Description:

- Multiply N x N matrices
- O(N3) total operations
- Accesses
  - N reads per source element
  - N values summed per destination
    - » but may be able to hold in register

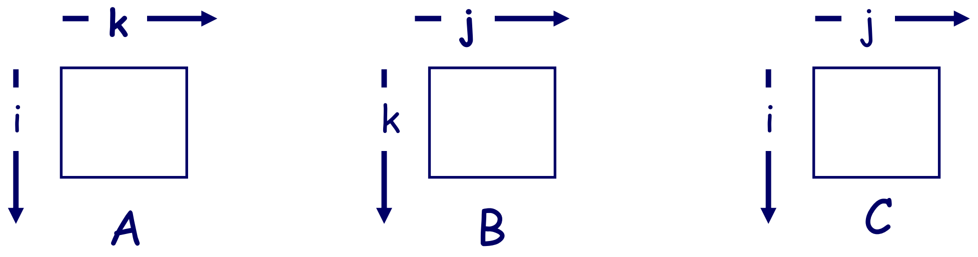
Miss Rate Analysis for Matrix Multiply

Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
  - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

- Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

C arrays allocated in row-major order

- each row in contiguous memory locations

Stepping through columns in one row:

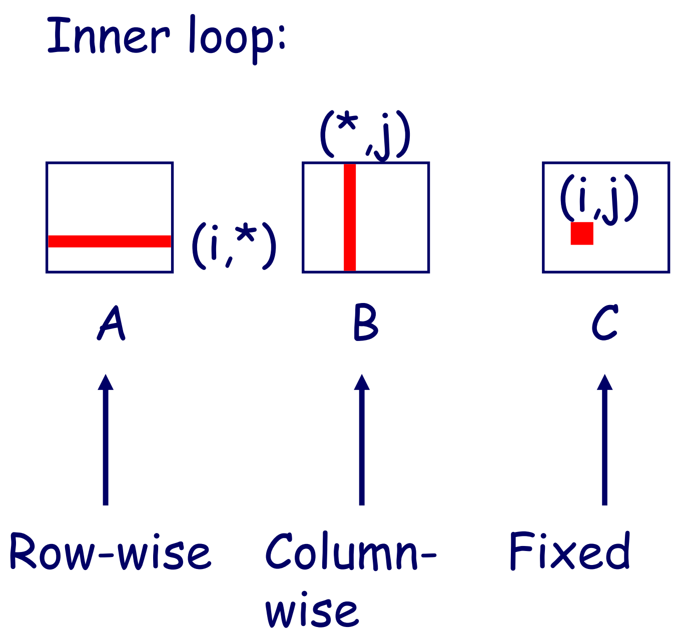
-
  ```
  for (i = 0; i < N; i++)
    sum += a[0][i];
  ```
- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
  - compulsory miss rate = 4 bytes / B

Stepping through rows in one column:

-
  ```
  for (i = 0; i < n; i++)
    sum += a[i][0];
  ```
- accesses distant elements
- no spatial locality!
  - compulsory miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```



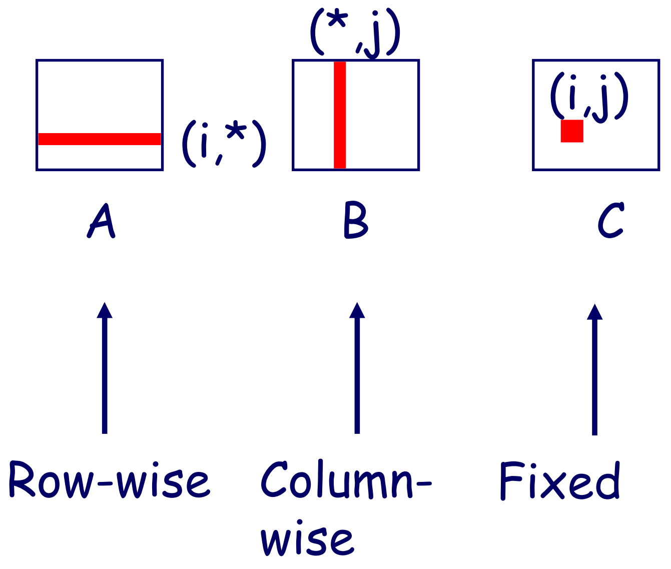
Misses per Inner Loop Iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

Inner loop:

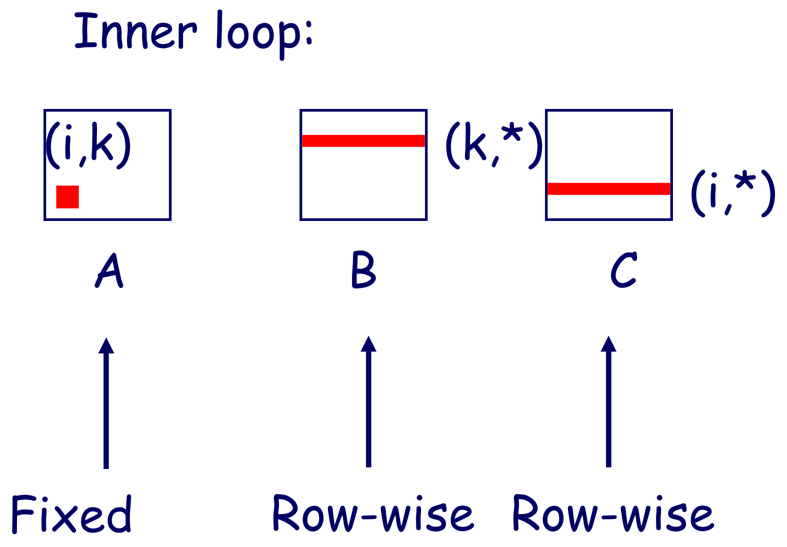


Misses per Inner Loop Iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

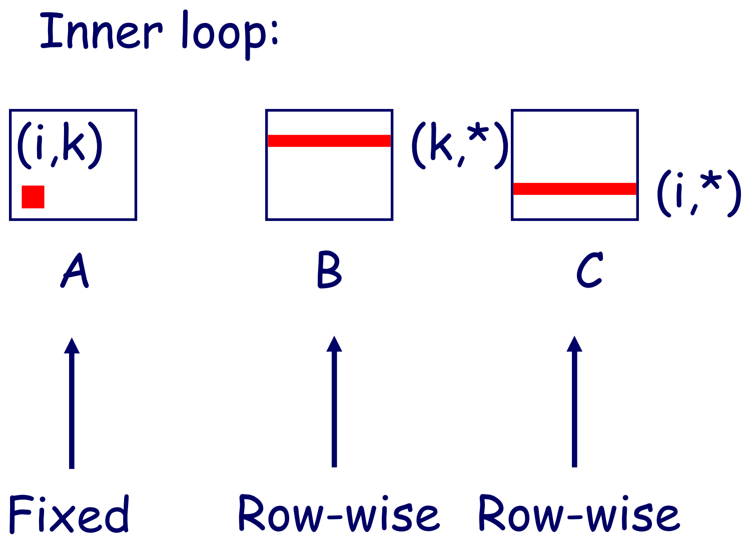


Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

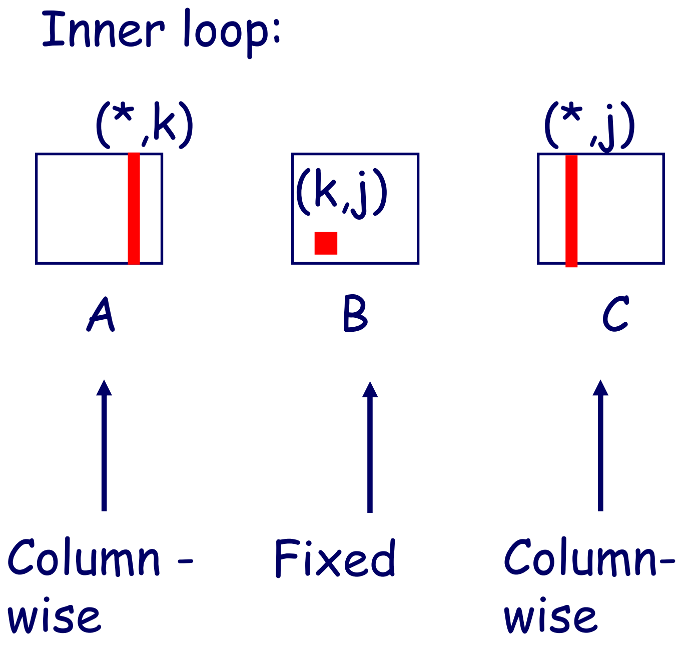


Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

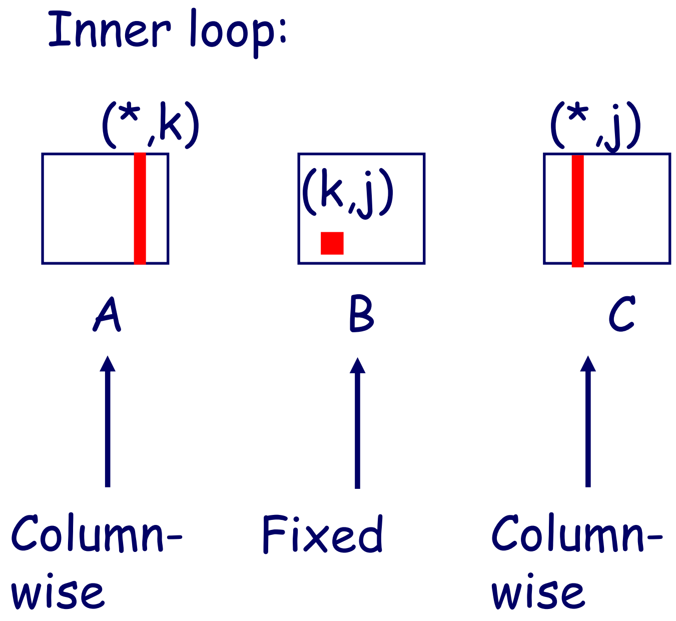


Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
  for (i=0; i<n; i++) {
   for (j=0; j<n; j++) {
     sum = 0.0;
     for (k=0; k<n; k++)
       sum += a[i][k] * b[k][j];
     c[i][j] = sum;
  }
 }
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

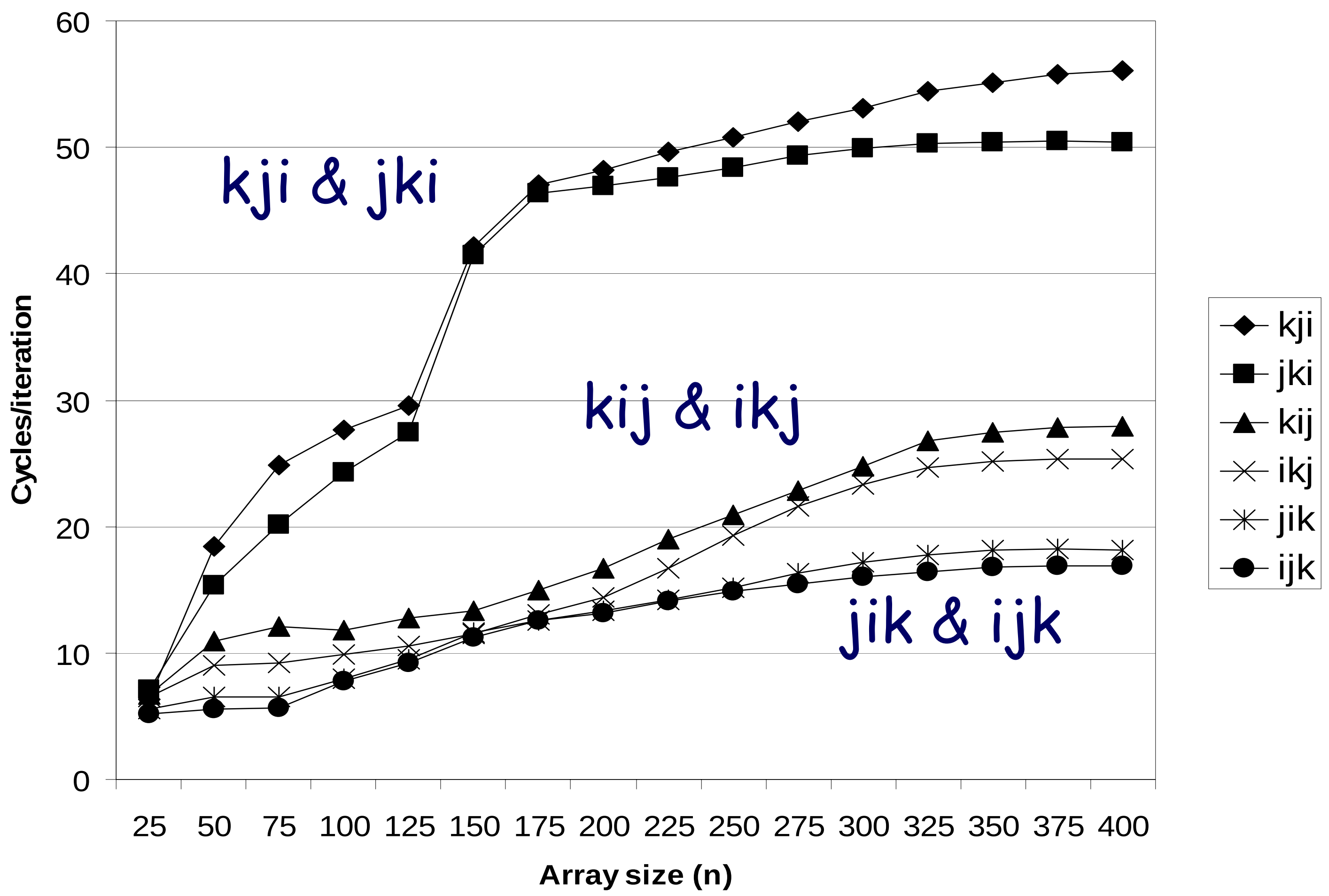
```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

Pentium Matrix Multiply Performance

Miss rates are helpful but not perfect predictors.

- Code scheduling matters, too.



Improving Temporal Locality by Blocking

Example: Blocked matrix multiplication

- "block" (in this context) does not mean "cache block".
- Instead, it mean a sub-block within the matrix.
- Example: N = 8; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., $\mathbf{A}_{xy}$) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Blocked Matrix Multiply (bijk)

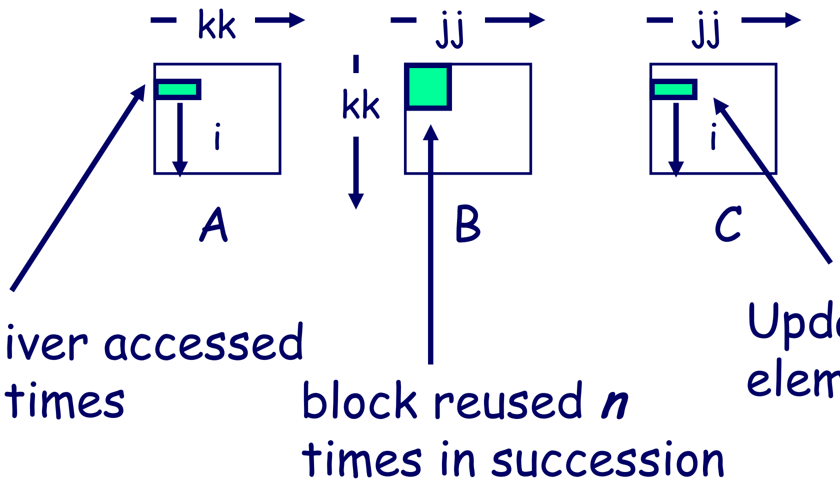
```
for (jj=0; jj<n; jj+=bsize) {

  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;

  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

Blocked Matrix Multiply Analysis

- Innermost loop pair multiplies a *1 X bsize* sliver of *A* by a *bsize X bsize* block of *B* and accumulates into *1 X bsize* sliver of *C*
- Loop over *i* steps through *n* row slivers of *A* & *C*, using same *B*

```
for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
```

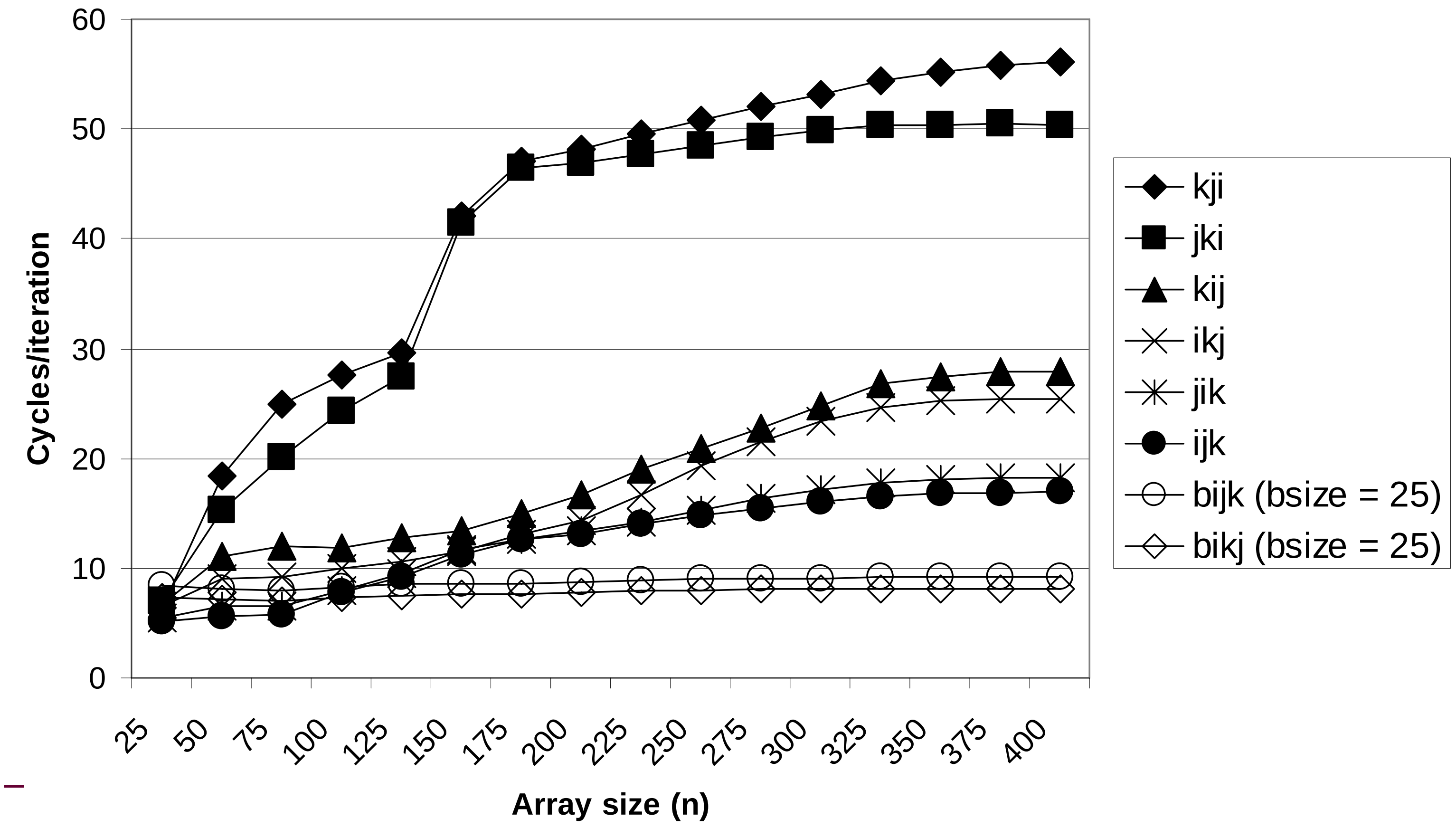
Innermost Loop Pair


kk
jj
jj
kk
i
i
A
B
C
row sliver accessed *bsize* times
block reused *n* times in succession
Update successive elements of sliver

Pentium Blocked Matrix Multiply Performance

Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)

- **relatively insensitive to array size.**



Concluding Observations

Programmer can optimize for cache performance

- How data structures are organized
- How data are accessed
  - Nested loop structure
  - Blocking is a general technique

All systems favor “cache friendly code”

- Getting absolute optimum performance is very platform specific
  - Cache sizes, line sizes, associativities, etc.
- Can get most of the advantage with generic code
  - Keep working set reasonably small (temporal locality)
  - Use small strides (spatial locality)