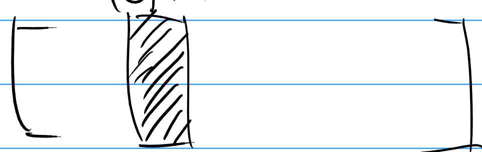
Math 243

Exam on Ch 10

Tues 14	Wed 15	Thurs 16

36 hrs

90 min

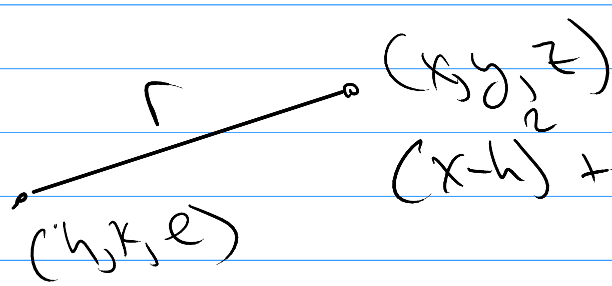


open — Tues 10^{30}

Due — Thurs 10^{30} am

10.5 / 10.6 Equations in 3 space

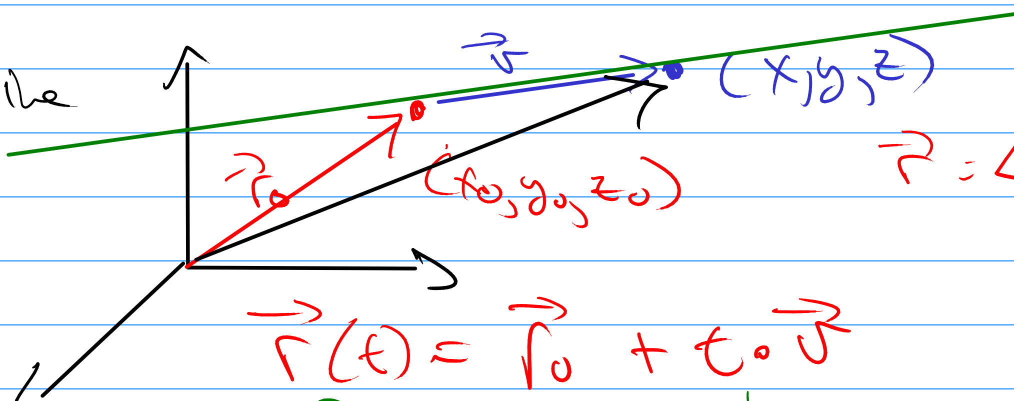
①



$$(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$$

<u>sphere</u>

② line



$\vec{r} = \langle x_0, y_0, z_0 \rangle$

$$\vec{r}(t) = \vec{r}_0 + t \cdot \vec{v}$$

r traces out the green line

<u>lines:</u> vector eqn $\vec{r} = \vec{r}_0 + t\,\vec{v}$

<u>Need:</u> ① point (x_0, y_0, z_0) on line

$\to \vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

② $\vec{v} = \langle a, b, c \rangle$ any vector in the $\underbrace{\text{same direction}}_{\text{parallel}}$ as the line.

(or) eqn as components (parametric equations)

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

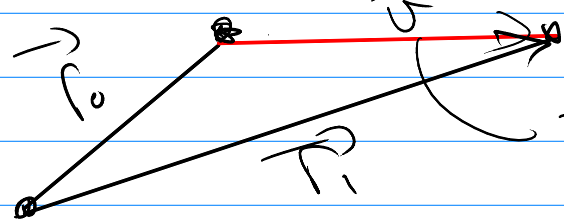
$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

(or) Symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

③ Line Segment

$$\vec{r} = \vec{r}_0 + t(\vec{r}_1 - \vec{r}_0)$$



$\vec{r}_0 + \vec{v} = \vec{r}_1$

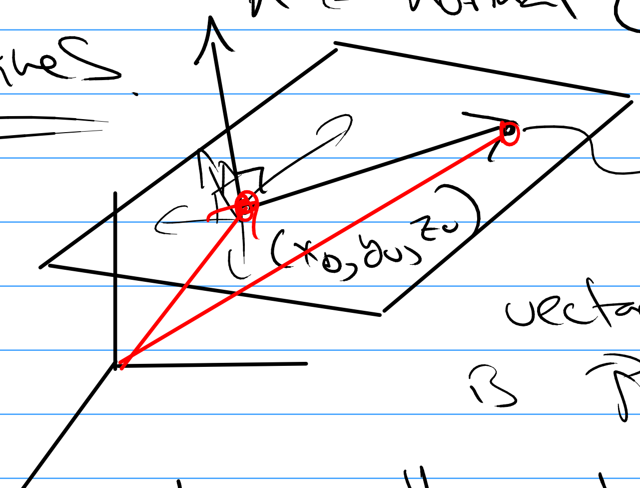
$\vec{v} = \vec{r}_1 - \vec{r}_0$

$$\vec{r} = \vec{r}_0 + t\vec{r}_1 - t\vec{r}_0$$

$$\boxed{\vec{r} = (1-t)\vec{r}_0 + t\,\vec{r}_1 \qquad t \in [0,1]}$$

④ Planes

$\vec{n} \leftarrow$ normal ($\perp$ to plane)



vector in plane is $\vec{r} - \vec{r}_0$

b/c $\vec{n}$ is $\perp$ to all vectors in plane

vector eqn of a plane

$$\boxed{\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0}$$

$$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r}_0$$

$$ax + by + cz = ax_0 + by_0 + cz_0 \leftarrow \text{Scalar}$$

"d"

Scalar Equation of plane

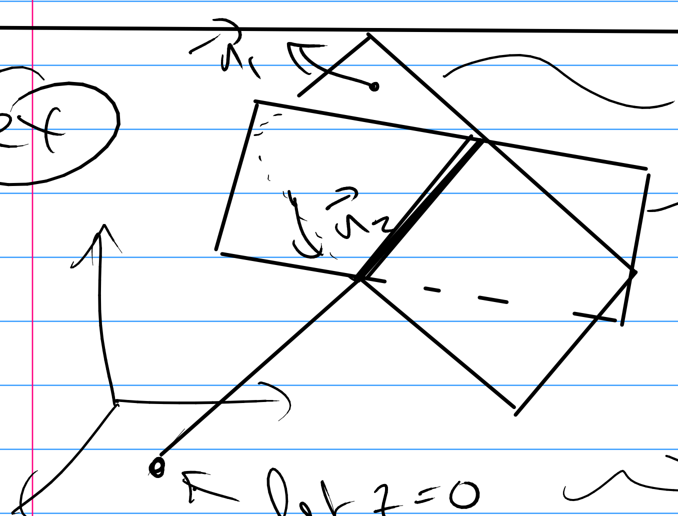
$$ax + by + cz + d = 0$$

Review: ① to find sphere
Need: pt, and radius

② line → Need: pt and parallel vector

③ plane → Need: pt and perpendicular vector

ex



$ax + by + cz + d = 0$

$ex + fy + gz + h = 0$

let $z = 0$ → plug it into eqn's
solve: (x_0, y_0, z_0)

b/c line is in the planes (both)

$\perp$ to $\vec{n}_1$ and $\vec{n}_2$

$\rightarrow$ parallel to $(\vec{n_1} \times \vec{n_2}) = \vec{v}$

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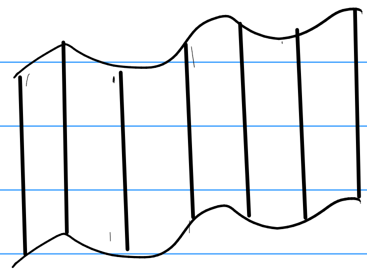
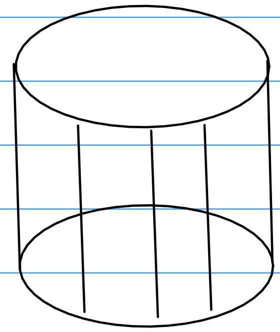
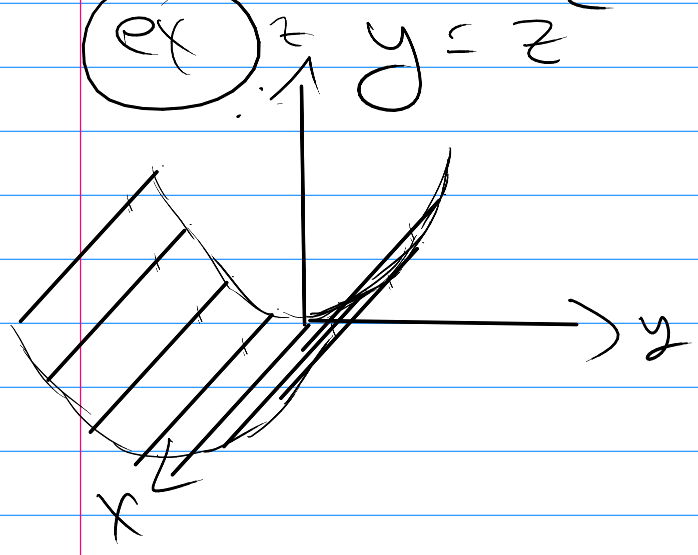
⑤ <u>Cylinders</u>





Given a planer curve, a cylinder is the collection of parallel lines (to a given line) that pass through the planer curve.

ex $y = z^2$



Quadric Surfaces

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

by rotation and translation

$$Ax^2 + By^2 + Cz^2 + J = 0$$

or

$$Ax^2 + By^2 + Iz = 0$$

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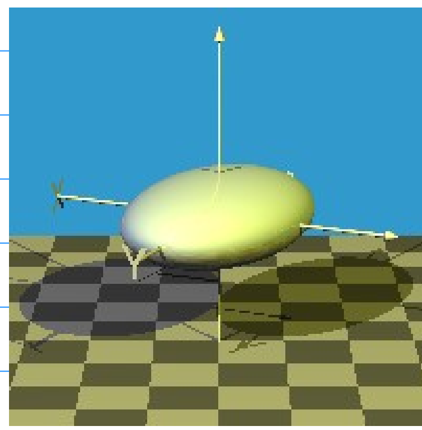
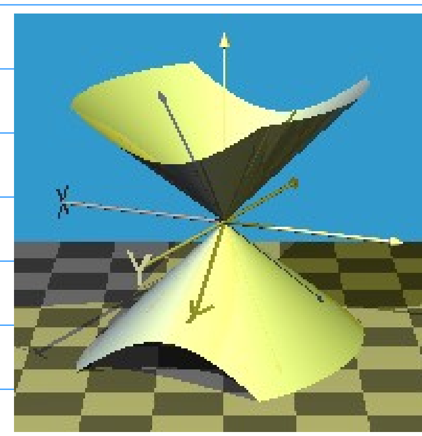
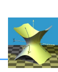
ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

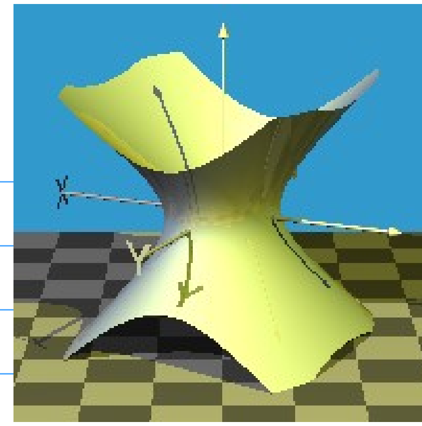


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$ Cone



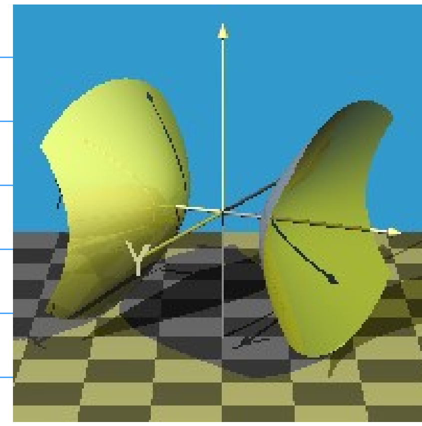


hyperboloid of one sheet



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

hyperboloid of two sheets



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Elliptic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

hyperbolic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$