Math 243

Q's / Exam on Ch 8 #12

$$f(x) = \frac{\ln(2x)}{2x}, a = \frac{1}{2}, n = 3$$

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f\left(\tfrac{1}{2}\right) = \frac{\ln\left(2\cdot\frac{1}{2}\right)}{2\cdot\frac{1}{2}} = 0$$

$$f'(x) = \frac{2 - 2\ln(2x)}{4x^2}$$

$$f'\left(\tfrac{1}{2}\right) = 2$$

etc

Q's /

$$r = a\sin\theta + b\cos\theta \qquad (ab \neq 0)$$

'Looked' like a circle.

$$r^2 = a(r\sin\theta) + b(r\cos\theta) \qquad (r \neq 0)$$

$$x^2 + y^2 = ay + bx$$

$$x^2 - bx + \left(\tfrac{b}{2}\right)^2 + y^2 - ay + \left(\tfrac{a}{2}\right)^2 = 0 + \left(\tfrac{b}{2}\right)^2 + \left(\tfrac{a}{2}\right)^2$$

$$\left(x - \tfrac{b}{2}\right)^2 + \left(y - \tfrac{a}{2}\right)^2 = \frac{b^2 + a^2}{4}$$

circle:

$$\left(x - \frac{b}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \frac{b^2 + a^2}{4}$$

center $\left(\frac{b}{2}, \frac{a}{2}\right)$

$$r = \frac{1}{2}\sqrt{b^2 + a^2}$$

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horz. / vert. tangents

$$r = a\sin\theta + b\cos\theta \qquad (ab \neq 0)$$

horz. $\frac{dy}{dx} = 0$ $\qquad$ vertical $\frac{dy}{dx}$ d.n.e.

$$\frac{dy}{dx} = \frac{\frac{d}{d\theta}[y(\theta)]}{\frac{d}{d\theta}[x(\theta)]} = \frac{\frac{d}{d\theta}[r \cdot \sin\theta]}{\frac{d}{d\theta}[r \cdot \cos\theta]}$$

$$= \frac{\frac{d}{d\theta}\left[(a\sin\theta + b\cos\theta) \cdot \sin\theta\right]}{\frac{d}{d\theta}\left[(a\sin\theta + b\cos\theta) \cdot \cos\theta\right]}$$

$$= \frac{(a\cos\theta - b\sin\theta)\sin\theta + (a\sin\theta + b\cos\theta)\cos\theta}{(a\cos\theta - b\sin\theta)\cos\theta - (a\sin\theta + b\cos\theta)\sin\theta}$$

$$= \frac{2a\cos\theta\sin\theta - b\sin^2\theta + b\cos^2\theta}{-2b\cos\theta\sin\theta + a\cos^2\theta - a\sin^2\theta}$$

$$= \frac{a\sin 2\theta + b\cos 2\theta}{-b\sin 2\theta + a\cos 2\theta}$$

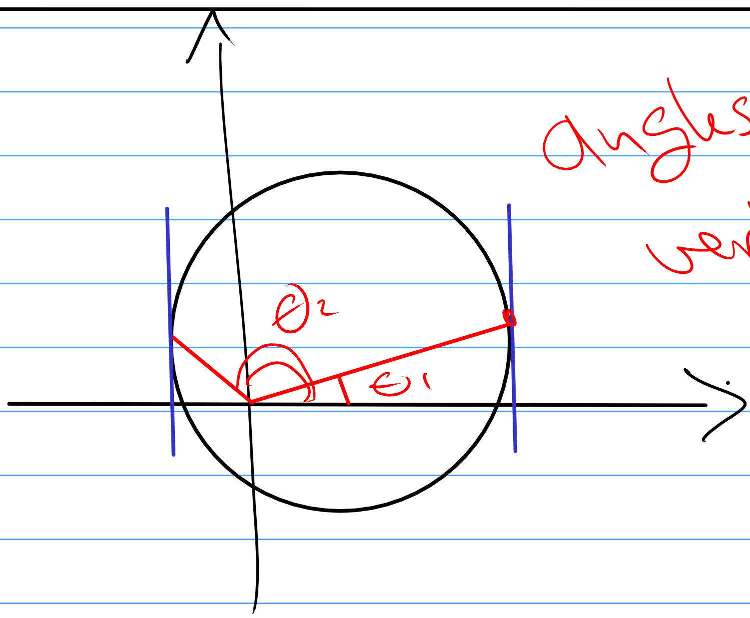
Vert. Tangent: $-b\sin 2\theta + a\cos 2\theta = 0$

$$b\sin 2\theta = a\cos 2\theta$$

$$\tan 2\theta = a/b$$

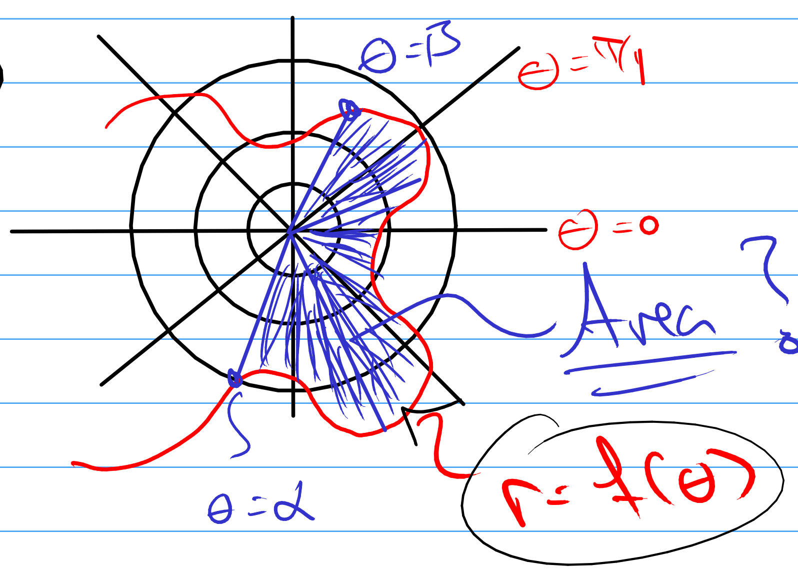
horz. tangent: $a\sin 2\theta + b\cos 2\theta = 0$

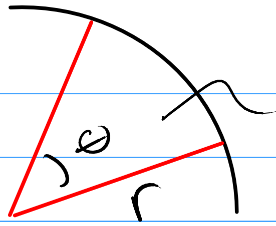
$$\tan 2\theta = -b/a$$



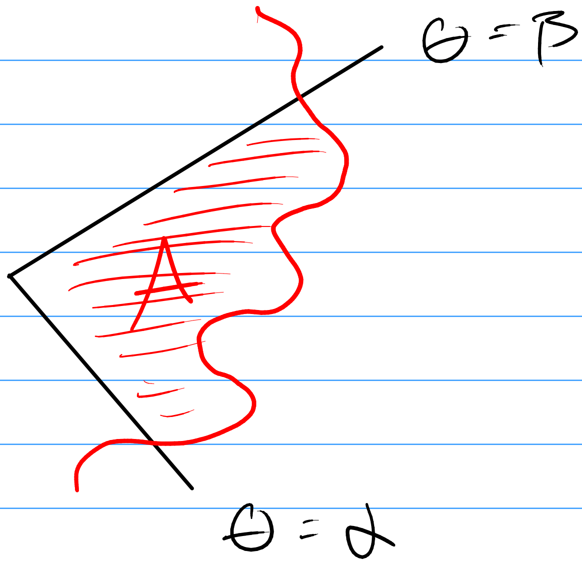
angles that form vert. tangents on $r = f(\theta)$

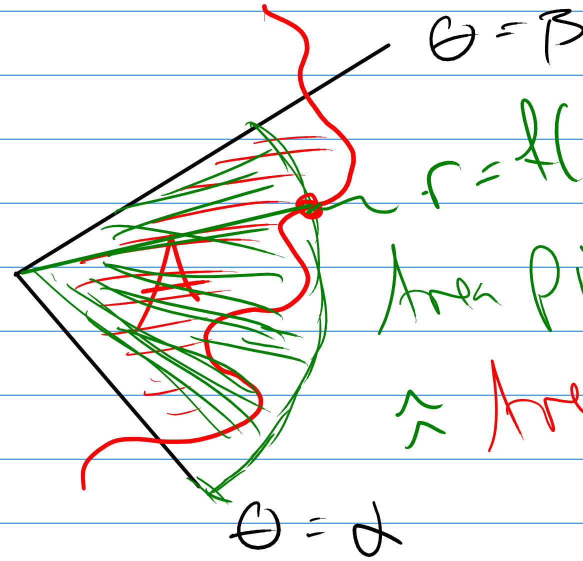
9.4





$$\text{Area} = \frac{1}{2}\theta r^2$$

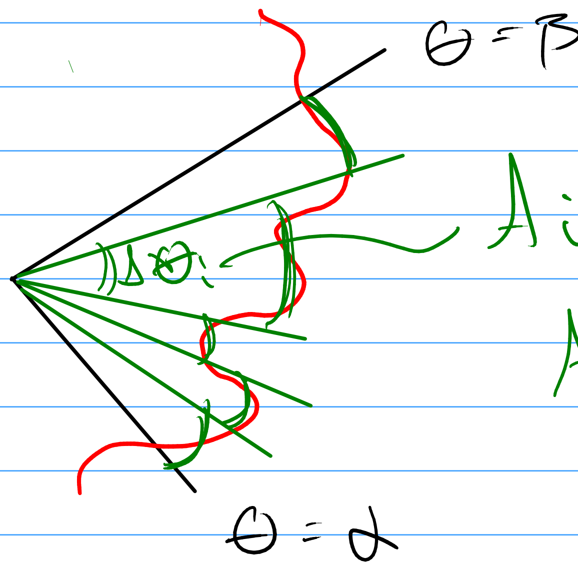




$r = f(\theta^*)$

Area pie $\approx$ Area

where θ^* is between α and β.



$$A_i = \frac{1}{2}\Delta\theta_i \, r_i^2$$

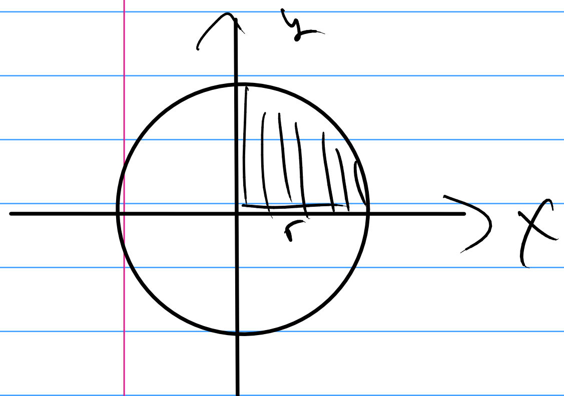
$$A_i = \frac{1}{2}(\Delta\theta_i) f(\theta_i^*)^2$$

$$\rightarrow \quad A \approx \sum_{i=1}^{n} \frac{1}{2} f(\theta_i^*)^2 \Delta\theta_i$$

$$\rightarrow \quad A = \lim_{\max \Delta\theta_i \to 0} \sum_{i=1}^{n} \frac{1}{2} f(\theta_i^*)^2 \Delta\theta_i$$

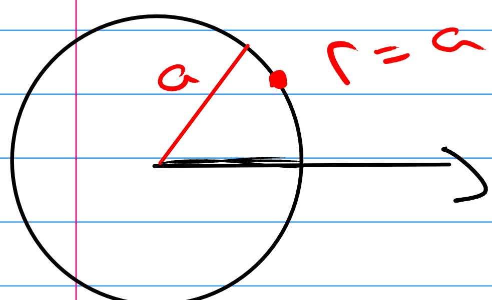
$$A = \int_{\alpha}^{\beta} \frac{1}{2} f(\theta)^2 \, d\theta$$ (if there is no question of $r = f(\theta)$)

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$



$$A = 4\int_0^r \sqrt{r^2 - x^2} \, dx$$

use trig sub to finish.



$$A = \int_0^{2\pi} \frac{1}{2} a^2 \, d\theta$$

$$A = \frac{1}{2} a^2 \theta \Big|_0^{2\pi} = \boxed{\pi a^2}$$

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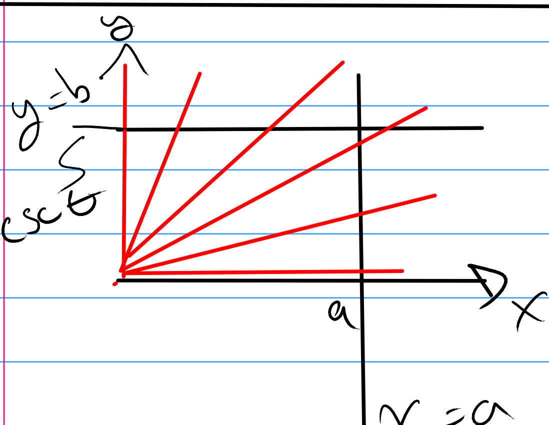
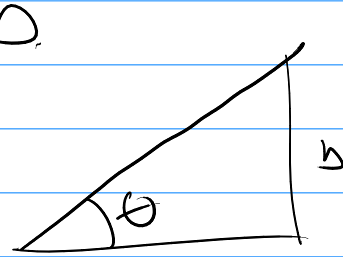
ex



$r = b\csc\theta$

Area $= a \cdot b$



$x = a$

$r\cos\theta = a \to r = a\sec\theta$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

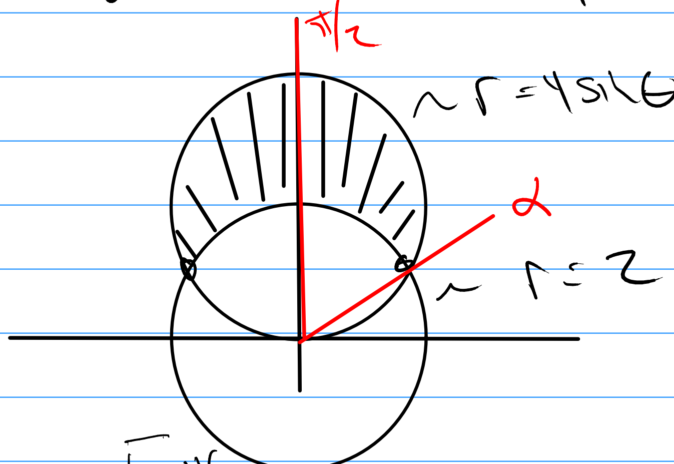
$$A = \int_0^{\tan^{-1}(b/a)} \frac{1}{2}(a\sec\theta)^2\,d\theta + \int_{\tan^{-1}(b/a)}^{\pi/2} \frac{1}{2}(b\csc\theta)^2\,d\theta$$

$$= \frac{a^2}{2}\tan\theta\Big|_0^{\tan^{-1}(\frac{b}{a})} - \frac{b^2}{2}\cot\theta\Big|_{\tan^{-1}(\frac{b}{a})}^{\pi/2}$$

$$= \left(\frac{a^{\cancel{2}}}{2}\cdot\frac{b}{\cancel{a}} - 0\right) - \left(0 - \frac{b^{\cancel{2}}}{2}\cdot\frac{a}{\cancel{b}}\right)$$

$$= \frac{1}{2}ab + \frac{1}{2}ab = \boxed{ab}$$

Area between $r = 4\sin\theta$ $\quad r = 2$



$$\text{Area} = 2\left[\int_\alpha^{\pi/2} \frac{1}{2}(4\sin\theta)^2\,d\theta - \int_\alpha^{\pi/2} \frac{1}{2}(2)^2\,d\theta\right]$$

$$= 16\int_\alpha^{\pi/2} \sin^2\theta\,d\theta - 4\int_\alpha^{\pi/2} d\theta$$

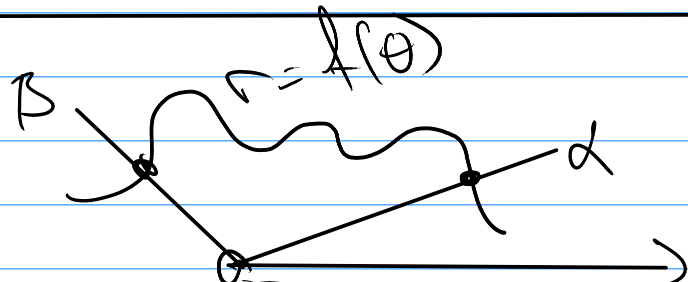
What is α?

$r = 4\sin\theta \qquad r = 2$

$$4\sin\theta = 2$$
$$\sin\theta = 1/2$$
$$\theta = \pi/6$$

$$A = 16\int_{\pi/6}^{\pi/2} \sin^2\theta \, d\theta - 4\int_{\pi/6}^{\pi/2} d\theta = \text{Finish!}$$

Arclength



$x = f(\theta)\cos\theta \qquad y = f(\theta)\sin\theta$

$$\theta \in [\alpha, \beta]$$

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$\frac{dx}{d\theta} = f'(\theta)\cos\theta - f(\theta)\sin\theta$$

$$\frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta$$

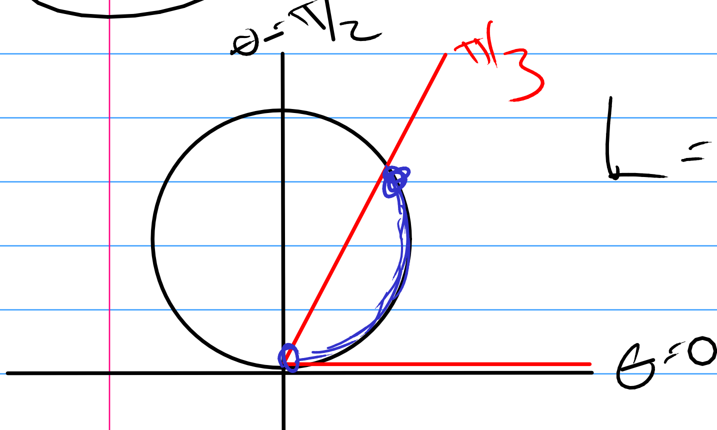
$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \boxed{f'(\theta)^2\cos^2\theta + f'(\theta)^2\sin^2\theta} \cancel{- 2f'(\theta)f(\theta)\cos\theta\sin\theta} + f(\theta)^2\sin^2\theta \cancel{+ 2f'(\theta)f(\theta)\cos\theta\sin\theta} + f(\theta)^2\cos^2\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = (f'(\theta))^2 + (f(\theta))^2$$

$$\therefore \quad \boxed{L = \int_\alpha^\beta \sqrt{(r')^2 + (r)^2}\, d\theta}$$

Arc Length

Ex5 $\quad r = 3\sin\theta \qquad \theta \in [0, \pi/3]$



$$L = \int_0^{\pi/3} \sqrt{\underbrace{(3\cos\theta)^2 + (3\sin\theta)^2}_{9}}\, d\theta$$

$$L = \int_0^{\pi/3} 3\, d\theta$$

$$L = 3\theta \Big|_0^{\pi/3} = \boxed{\pi}$$

Ex6 $\quad r = e^{2\theta} \qquad \theta \in [0, 2\pi]$

$$L = \int_0^{2\pi} \sqrt{(2e^{2\theta})^2 + (e^{2\theta})^2}\, d\theta$$

$$L = \int_0^{2\pi} e^{2\theta}\sqrt{5}\, d\theta = \frac{\sqrt{5}}{2} e^{2\theta} \Big|_0^{2\pi}$$

$$= \frac{\sqrt{5}}{2} e^{4\pi} - \frac{\sqrt{5}}{2} = \boxed{\frac{\sqrt{5}}{2}\left(e^{4\pi} - 1\right)}$$

ex $r = 3\sin 2\theta$ one petal $\theta \in [0, \pi/2]$

$0 = 3\sin 2\theta$

$0 = \sin 2\theta$

$2\theta = 0 + n\pi$

$\theta = 0 + n\pi/2$

$$L = 4\int_0^{\pi/2} \sqrt{(6\cos 2\theta)^2 + (3\sin 2\theta)^2}\, d\theta$$

$$L = 4\int_0^{\pi/2} \sqrt{36\cos^2 2\theta + 9\sin^2 2\theta}\, d\theta$$

$L \approx$ (Simpsons?, taylor?, midpt?, Maxima?, wlms?)