Quiz 5

1. Let us define an equivalence relation $\equiv$ on strings with respect to a language L as follows: $x \equiv_L y$ if and only if $\text{suffix}(L, x) = \text{suffix}(L, y)$. Let L be the set of all strings with odd number of $1s$. Which of the following is true?

   (A) $0 \equiv_L 1$
   (B) $00 \equiv_L 10$
   (C) $01 \equiv_L 00$
   (D) $01 \equiv_L 10$

   Correct answer is (D).

   (A) False, $\epsilon \notin \text{suffix}(L, 0)$, $\epsilon \in \text{suffix}(L, 1)$
   (B) False, $\epsilon \notin \text{suffix}(L, 00)$, $\epsilon \in \text{suffix}(L, 10)$
   (C) False, $\epsilon \notin \text{suffix}(L, 00)$, $\epsilon \in \text{suffix}(L, 01)$
   (D) True, $\text{suffix}(L, 01) = \text{suffix}(L, 10) =$ set of all strings with even number of 1s.

2. Let $\equiv_L$ be as defined in Problem 1. We define an equivalence class $[x]_{\equiv_L}$ of a string x with respect to the relation $\equiv_L$ as follows: $[x]_{\equiv_L} = \{y \mid x \equiv_L y\}$. Let $L = \mathbf{L}((0 \cup 1)^* 11 (0 \cup 1)^*)$. Then, $[1]_{\equiv_L}$ is the set

   (A) $\mathbf{L}((0 \cup 1)^* 1 (0 \cup 1)^*)$
   (B) $\mathbf{L}((0 \cup 1)^* 1)$
   (C) $\mathbf{L}((0 \cup \epsilon)(10 \cup 0)^* 1)$
   (D) $\mathbf{L}((01 \cup 0)^* 1)$

   Correct answer is (C).

   L is the set of all strings with the substring 11.

   (A) $10 \in \mathbf{L}((0 \cup 1)^* 1 (0 \cup 1)^*)$. $10 \notin [1]_{\equiv_L}$, because $11 \in L$ but $101 \notin L$.
   (B) $011 \in \mathbf{L}((0 \cup 1)^* 1)$. $011 \notin [1]_{\equiv_L}$, because $0110 \in L$ but $10 \notin L$.
   (C) Let $\mathbf{L}((0 \cup \epsilon)(10 \cup 0)^* 1) = L'$. No element in L' has substring 11 and all substring in L' end with a 1. Note in order for $1z \in L$, z either start with 1, or z contain 11 as a substring. This holds true for all strings in L'.
   (D) $011 \in \mathbf{L}((01 \cup 0)^* 1)$, and the rest is the same as (B).

3. Let $\mathcal{C}_{\text{equiv}}(L) = \{[x]_{\equiv_L} | x \in \Sigma^*\}$, it is the set of all equivalence classes of $\equiv_L$. Let L_{odd} be the set of all strings with odd number of 1s, and L_{even} the set of all strings with even number of 1s. Then $\mathcal{C}_{\text{equiv}}(L_{odd})$ is:

   (A) $\{L_{odd}\}$
   (B) $\{L_{odd}, L_{even}\}$
   (C) $\{L_{even}\}$
   (D) $\{\}$

Correct answer is (B).

All strings that have odd number of 1s are equivalent, because their suffix language is the set of all strings with even number of 1s. Similarly, all strings that have even number of 1s are equivalent, because their suffix language is the set of all strings with odd number of 1s.

4. Recall $\mathcal{C}_{\text{suf}}(L)$ is the set of all suffix languages of L. Let $|S|$ denote the number of elements in S. Which of the following is true?

   (A) $|\mathcal{C}_{\text{equiv}}(L)| \geq |\mathcal{C}_{\text{suf}}(L)|$

   (B) $|\mathcal{C}_{\text{equiv}}(L)| = |\mathcal{C}_{\text{suf}}(L)|$

   (C) $|\mathcal{C}_{\text{equiv}}(L)| \leq |\mathcal{C}_{\text{suf}}(L)|$

   (D) $|\mathcal{C}_{\text{equiv}}(L)| \geq 2^{|\mathcal{C}_{\text{suf}}(L)|}$

   Correct answer is (B).

   Note that all strings in an equivalence class $[x]_{\equiv_L}$ have the same suffix language (since, by definition two strings are equivalent when their suffix languages are the same). And two strings whose suffix languages are not the same are not in the same equivalence class. Therefore equivalence classes and suffix languages are in one-to-one correspondence.

5. Which of the following is false?

   (A) For every regular language L, $\mathcal{C}_{\text{equiv}}(L)$ is finite.

   (B) If $\mathcal{C}_{\text{equiv}}(L)$ is finite, then L is regular.

   (C) $\mathcal{C}_{\text{equiv}}(L)$ is finite for some non-regular languages.

   (D) $\mathcal{C}_{\text{equiv}}(L)$ is infinite for all non-regular languages.

   The correct answer is (C). Note that since there is a one-to-one correspondence between $\mathcal{C}_{\text{equiv}}(L)$ and $\mathcal{C}_{\text{suf}}(L)$, we can essentially replace $\mathcal{C}_{\text{equiv}}(L)$ in the above definition with $\mathcal{C}_{\text{suf}}(L)$. Hence, all statements except C are true.