

Math 321

Q's of Useful Math Software

(1) Numerics

→ Scilab

→ octave

(2) Symbolic

→ Maxima

→ yacas

(3) Stats

→ R-project

Show $a \cdot b = \gcd(a, b) \cdot \text{lcm}(a, b)$

$$\gcd(a, b) = p_1^{\min(a_1, b_1)} \cdot p_2^{\min(a_2, b_2)} \cdots p_n^{\min(a_n, b_n)}$$

$$\text{lcm}(a, b) = p_1^{\max(a_1, b_1)} \cdot p_2^{\max(a_2, b_2)} \cdots p_n^{\max(a_n, b_n)}$$

$$\gcd(a, b) \cdot \text{lcm}(a, b) =$$

$$\underbrace{p_1^{\min(a_1, b_1)} \cdot p_1^{\max(a_1, b_1)}}_{p_1^{\min(a_1, b_1) + \max(a_1, b_1)}} \cdot \underbrace{p_2^{\min(a_2, b_2)} \cdot p_2^{\max(a_2, b_2)}}_{p_2^{\min(a_2, b_2) + \max(a_2, b_2)}} \cdots \underbrace{p_n^{\min(a_n, b_n)} \cdot p_n^{\max(a_n, b_n)}}_{p_n^{\min(a_n, b_n) + \max(a_n, b_n)}}$$

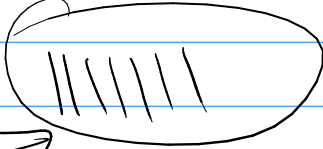
$$= p_1^{\min(a_1, b_1) + \max(a_1, b_1)} \cdots p_n^{\min(a_n, b_n) + \max(a_n, b_n)}$$

$$= p_1^{a_1 + b_1} \cdot p_2^{a_2 + b_2} \cdots p_n^{a_n + b_n}$$

$$\begin{aligned}
 &= a_1 b_1 a_2 b_2 \dots a_n b_n \\
 &= p_1 p_1 p_2 p_2 \dots p_n p_n \\
 &= (p_1^{a_1} p_2^{a_2} \dots p_n^{a_n}) (p_1^{b_1} p_2^{b_2} \dots p_n^{b_n}) \\
 &= a \cdot b
 \end{aligned}$$

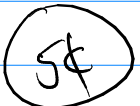



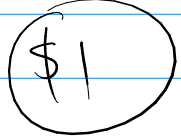
(36) $n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_2 b^2 + a_1 b^1 + a_0 b^0$

base b expansion of n

(base 1) $7 \rightarrow$ 

 $= 25$

Max.

$$2(10^5) + 5(1^5)$$

3 4 3

17

21

11

positional
number

$$n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_2 b^2 + a_1 b^1 + a_0 b^0$$

$$b = \{2, 3, \dots\}$$

$$n = \{1, 2, 3, \dots\}$$

$$k = \{0, 1, 2, \dots\}$$

$$a_i \in \{0, 1, 2, \dots, b-1\}$$

$$a_k \neq 0$$

base 10

$$7 \cdot 10^4 + 0 \cdot 10^3 + 3 \cdot 10^2 + 1 \cdot 10^1 + 6 \cdot 10^0$$

$$(7, 0, 3, 1, 6)_{10}$$

ex

$$(1, 0, 0, 1, 1, 1, 0)_2 = 64 + 8 + 4 + 2 = (78)_{10}$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$$(23, 0, 4, 59)_{60} = 23 \cdot 60^3 + 0 \cdot 60^2 + 4 \cdot 60 + 59$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $60^3 \quad 60^2 \quad 60^1 \quad 60^0$

= ?

$$1, 268 = (?)_4 = 1 \cdot 4^5 + 237$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $1024 \quad 256 \quad 64 \quad 16 \quad 4$

$$= (1, 0, 3, 2, 3, 1)_4 = 1 \cdot 4^5 + 0 \cdot 4^4 + 3 \cdot 4^3 + 2 \cdot 4^2 + 3 \cdot 4^1 + 1 \cdot 4^0$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $4^5 \quad 4^4 \quad 4^3 \quad 4^2 \quad 4^1 \quad 4^0$

$1024 \quad 256 \quad 64 \quad 16 \quad 4 \quad 1$

$= 1 \cdot 4^5 + 0 \cdot 4^4 + 3 \cdot 4^3 + 2 \cdot 4^2 + 3 \cdot 4^1 + 1 \cdot 4^0$

Operations.

$$\begin{array}{r}
 (2, 1, 3)_4 \\
 + (2, 0, 3)_4 \\
 \hline
 (1, 0, 2, 2)_4
 \end{array}$$

$$G = (1, 2)_4$$

$$\begin{array}{r}
 12 \\
 \times 6.1 \\
 \hline
 12 \\
 720 \\
 \hline
 732
 \end{array}$$

$$(60+1)(10+2)$$

$$\begin{array}{r}
 (2, 0, 1)_3 \\
 \times (1, 2)_3 \\
 \hline
 (1, 1, 0, 2)_3 \\
 + (2, 0, 1, 0)_3 \\
 \hline
 (1, 0, 1, 1, 2)_3
 \end{array}$$

div. (15) mod

$$a = b \cdot q + r$$

"
a div b

"
a mod b

by
Sub.

$$\begin{aligned} 23 &= a & 4 &= b \\ 23 &= 4 \cdot 0 + 23 & & -4 \\ 23 &= 4 \cdot 1 + 19 & & -4 \\ 23 &= 4 \cdot 2 + 15 & & -4 \\ 23 &= 4 \cdot 3 + 11 & & -4 \\ 23 &= 4 \cdot 4 + 7 & & -4 \\ 23 &= 4 \cdot 5 + 3 & & \end{aligned}$$

$$\frac{23}{4} = 5.75$$

$$23 \text{ div } 4 = \lfloor \frac{23}{4} \rfloor$$

$$23 \text{ mod } 4 = 23 - 4 \cdot \lfloor \frac{23}{4} \rfloor$$

Euclidean Alg.

$$\text{if } a = bq + r \rightarrow \gcd(a, b) = \gcd(b, r)$$

$$\gcd(24, 14) = 2$$

$$24 = 14 \cdot 1 + 10$$

$$14 = 10 \cdot 1 + 4$$

$$10 = 4 \cdot 2 + 2 \leftarrow \gcd$$

$$4 = 2 \cdot 2 + \underline{0}$$