# Math 321

~~Q's~~ Showing $p \equiv q$ **means** $\boxed{(p \leftrightarrow q) \equiv T}$

**How?** ① Truth Table

② Use "older" known logical equivalences.

③ $(p \leftrightarrow q) \equiv (p \land q) \lor (\neg p \land \neg q)$

**want:** $(p \land q) \lor (\neg p \land \neg q) \equiv T$

$(p \land q) \equiv T$ or $(\neg p \land \neg q) \equiv T$

show $p$ and $q$ are true for same truth values

or

show $p$ and $q$ are false for same truth values.

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ex #21 $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$

Ver 1

| $p$ | $q$ | $p \leftrightarrow q$ | $\neg(p \leftrightarrow q)$ | $\neg p \leftrightarrow q$ | $\neg(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | F | T | T | T |
| F | T | F | T | T | T |
| F | F | T | F | F | T |

Ver 2

$$\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$$

Use $\square \leftrightarrow \Delta \equiv (\square \wedge \Delta) \vee (\neg\square \wedge \neg\Delta)$

$$\neg[(p \wedge q) \vee (\neg p \wedge \neg q)] \equiv (\neg p \wedge q) \vee (p \wedge \neg q)$$

$$\neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q) \equiv (\neg p \wedge q) \vee (p \wedge \neg q)$$

$$(\boxed{\neg p \vee \neg q}) \wedge (p \vee q) \equiv (\neg p \wedge q) \vee (p \wedge \neg q)$$

$$[(\neg p \vee \neg q) \wedge p] \vee [(\neg p \vee \neg q) \wedge q] \equiv \text{ditto}$$

$$[\underbrace{(\neg p \wedge p)}_{F} \vee (\neg q \wedge p)] \vee [(\neg p \wedge q) \vee \underbrace{(\neg q \wedge q)}_{F}] \equiv \text{ditto}$$

$$(\neg q \wedge p) \vee (\neg p \wedge q) \equiv \text{ditto}$$

Same

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$$\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$$

Ver 3

Listen to lecture!

1.3 **Propositions:** declaritive sentence that is T xor F.

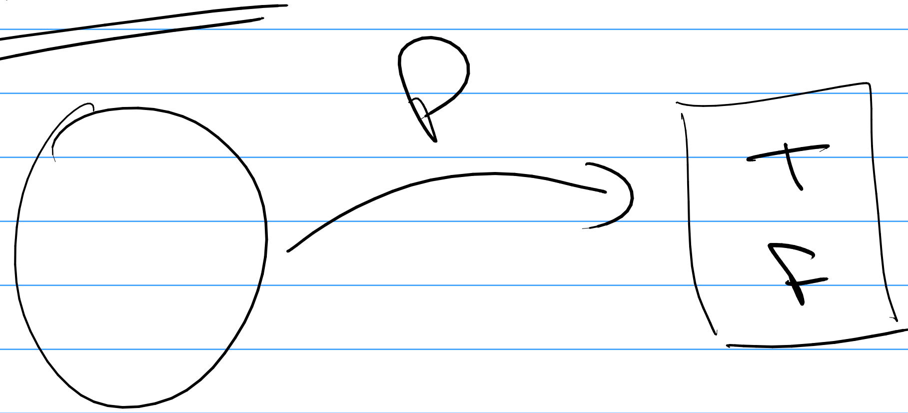
ex: l: "Lions like cheese"

m: "mice like cheese"

H: "lions are happy"

So propositions have things you are talking about (variable) and the property they should have (predicate)

**Propositional Functions:**




Domain

(universe of discourse)

$P(x)$: "$\underbrace{x}_{\text{element from the domain}}$ has $\underbrace{\text{property } P}_{\text{predicate}}$"

$P(x)$ is <u>not</u> a proposition.

How to create a proposition from $P(x)$?

$C(x)$: "x likes cheese"

(1) Evaluation (plug in an element from domain)

$C(\text{Mark})$ or $C(\text{Iron})$

(2) <u>Quantification</u>

(a) Universal Quantification

$\forall x\, P(x)$: "$P(x)$ for all values of $x$ in domain"

(b) Existential Quantification

$\exists x\, P(x)$: "there is an $x$ in the domain such that $P(x)$"

if domain $x_1, x_2, \dots, x_n$

$\forall x P(x) \equiv P(x_1) \land P(x_2) \land \dots \land P(x_n)$

$\exists x P(x) \equiv P(x_1) \lor P(x_2) \lor \dots \lor P(x_n)$

| | true? | False? |
|---|---|---|
| $\forall x P(x)$ | $P(x)$ is true for <u>all</u> $x$ | $P(x)$ is false for some $x$ (counter example) |
| $\exists x P(x)$ | $P(x)$ is true for one or more $x$. | $P(x)$ is false for <u>all</u> $x$. |

<u>Negation:</u> $\neg(\forall x P(x)) \equiv \exists x \neg P(x)$

$\neg(\exists x P(x)) \equiv \forall x \neg P(x)$

ex $\neg(\forall x (P(x) \land Q(x)))$

$\equiv \exists x \neg(P(x) \land Q(x))$

$\equiv \exists x (\neg P(x) \lor \neg Q(x))$

(New)

Order of operations

1. Parentheses
2. Quantification
3. $\neg$
4. $\land$, $\lor$, $\oplus$
5. $\rightarrow$, $\leftrightarrow$

ex

$C(x)$ : "x is a comedian"

$F(x)$ : "x is funny"

$\forall x ( C(x) \rightarrow F(x))$