
CIS 721 - Real-Time Systems

Lecture 31: Advanced UPPAAL Models

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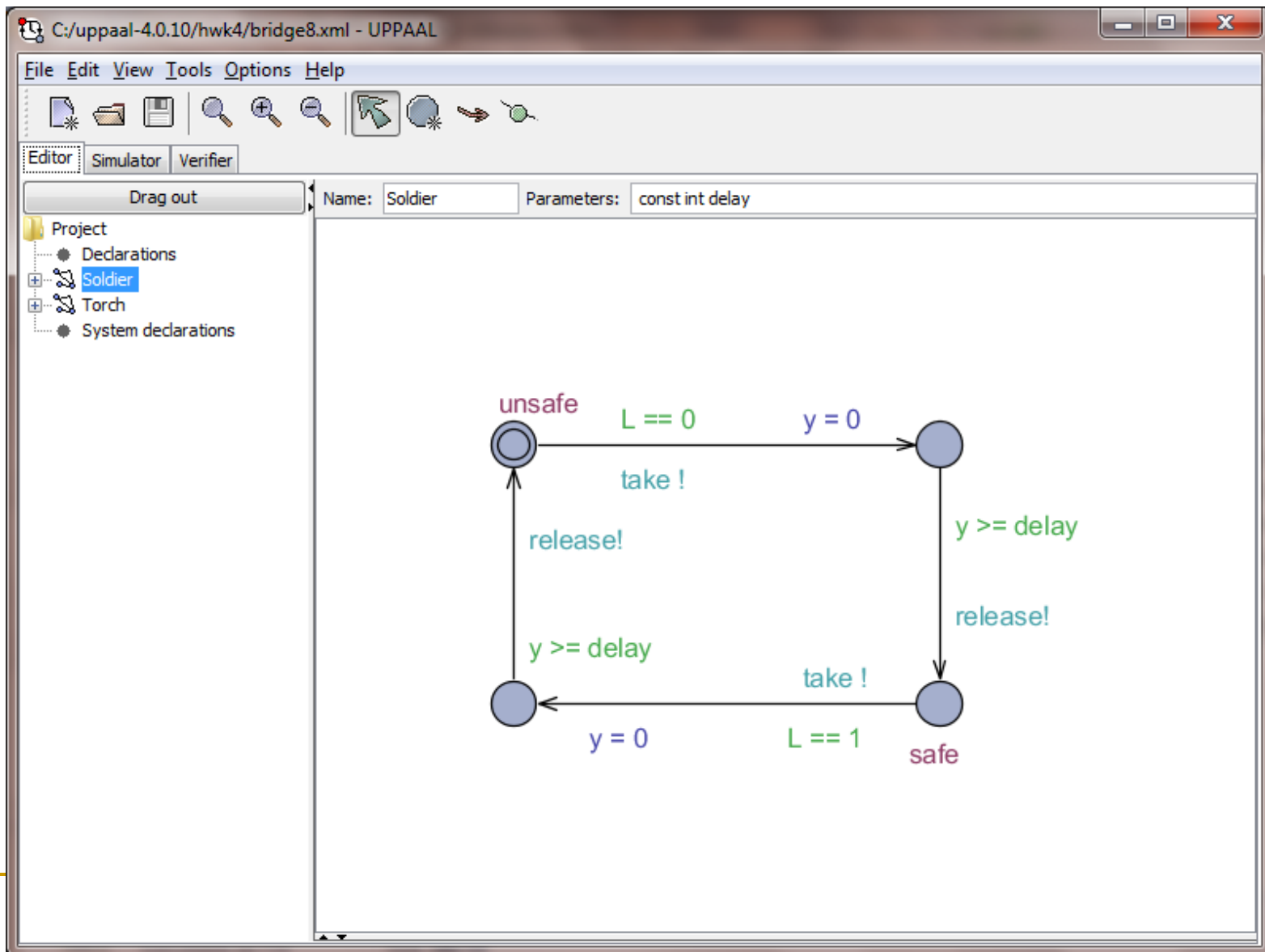
Uppaal Models

- Vikings' Bridge Problem
- Gossiping Girls' Problem
- Prioritized Token-Based Mutual Exclusion Algorithms

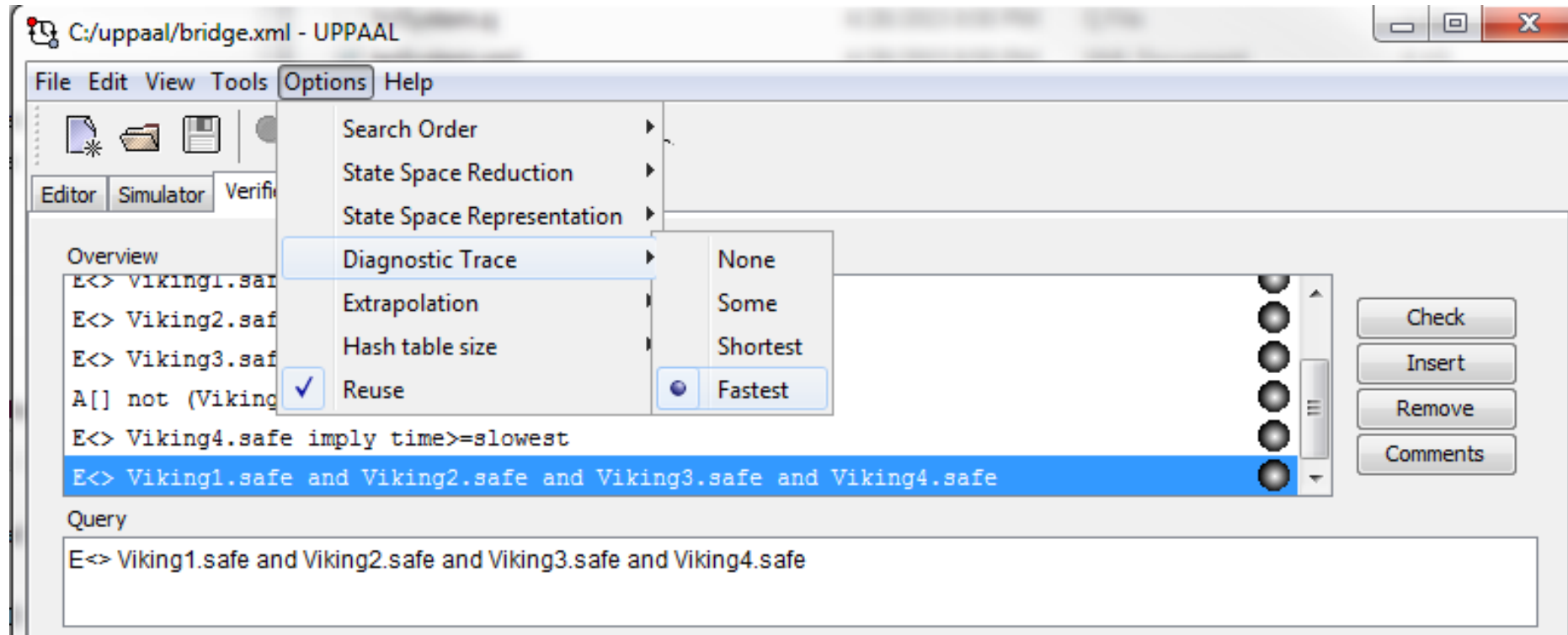
Vikings' Bridge Problem

- Adjust the solution to the Vikings' Problem given in bridge.xml to handle four vikings with crossing times of 3, 6, 12, and 15. Determine the minimum time required for all four vikings to cross the bridge.
- Repeat the problem, but vary the time required for the third viking (currently 12); e.g., try 3, 6, 9, and 15, etc.
- What range of values require the same amount of time as the first run with crossing times of 3, 6, 12, and 15?
- Finally, develop a general solution (on paper) for 4 vikings with crossing times of k , $2*k$, $4*k$, and $5*k$, for any fixed $k > 0$.

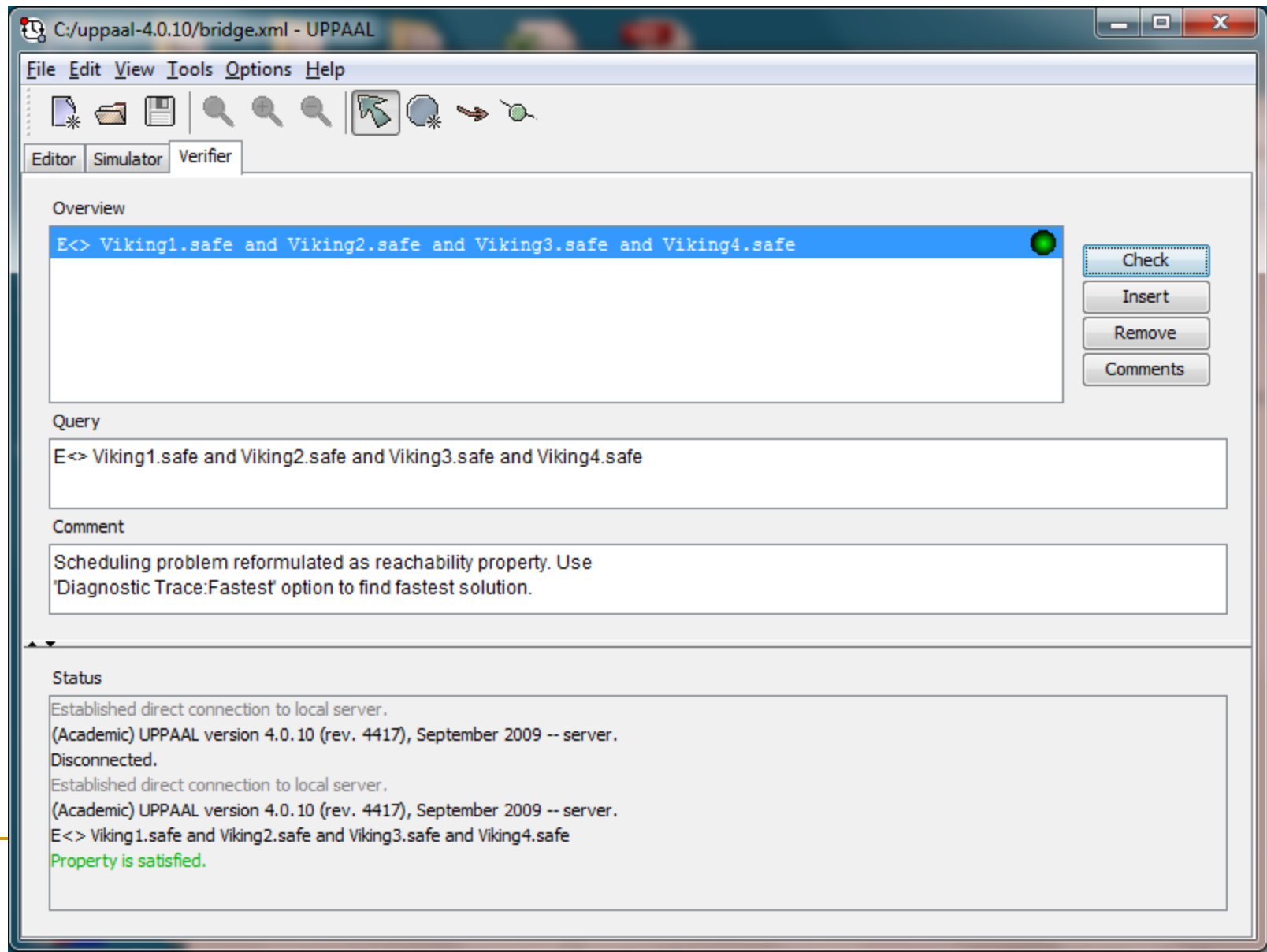
Vikings' Bridge Problem



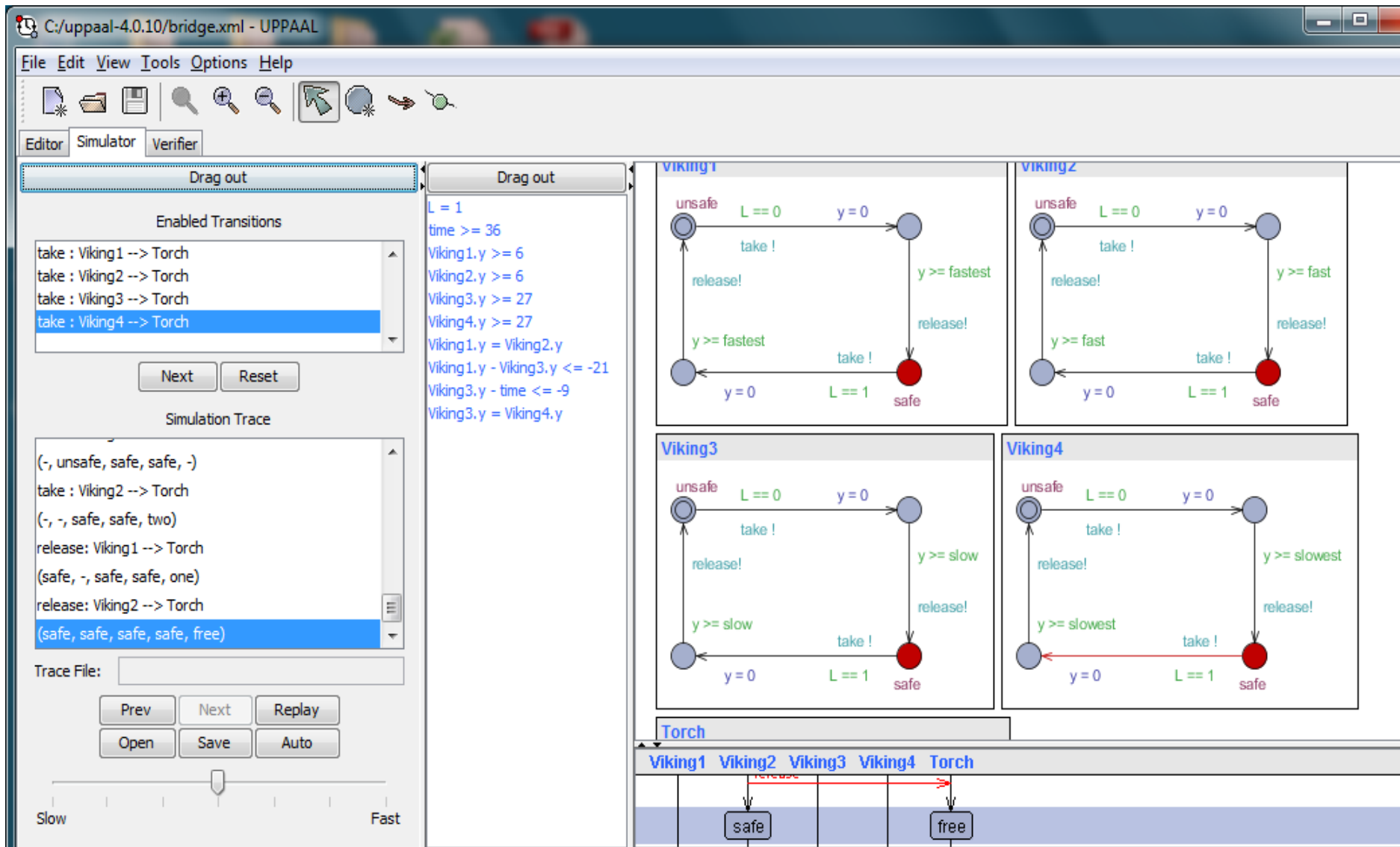
Diagnostic Trace + Fastest



Vikings' Bridge Problem



Vikings' Bridge Problem



Vikings' Bridge Problem

The image displays two windows of the UPPAAL software interface, showing the Vikings' Bridge Problem model.

Top Window (Editor): The file is `C:/uppaal-4.0.10/bridge.xml - UPPAAL`. The **Editor** tab is active. The code defines variables and constants for the bridge problem:

```
chan take, release;           // Take and release torch
int[0,1] L;                    // The side the torch is on
clock time;                     // Global time

const int fastest = 3;
const int fast = 6;
const int slow = 8;
const int slowest = 15;
```

Bottom Window (Simulator): The file is `C:/uppaal-4.0.10/bridge.xml - UPPAAL`. The **Simulator** tab is active. It shows the state transition diagram for the model, divided into two parts: **Enabled Transitions** and **Drag out**.

Enabled Transitions: A list of transitions that are currently enabled:

- take : Viking1 --> Torch
- take : Viking2 --> Torch
- take : Viking3 --> Torch
- take : Viking4 --> Torch

Drag out: A list of conditions that are currently true in the state:

- `L == 1`
- `time >= 35` (highlighted with a red circle)
- `Viking1.y >= 15`
- `Viking2.y >= 35`
- `Viking3.y >= 26`
- `Viking4.y >= 15`
- `Viking1.y - Viking3.y <= -11`
- `Viking1.y = Viking4.y`
- `Viking2.v <= time`

State Transition Diagram: The diagram shows the state of the system. It includes two parts: **Viking1** and **Viking2**. The **Viking1** part shows a state where `unsafe` is true, `L == 0`, and `y = 0`. The **Viking2** part shows a state where `unsafe` is true, `L == 0`, and `y >= fast`. The diagram illustrates the sequence of events: `take !`, `release!`, `y >= fastest`, and `take !`.

Vikings' Bridge Problem

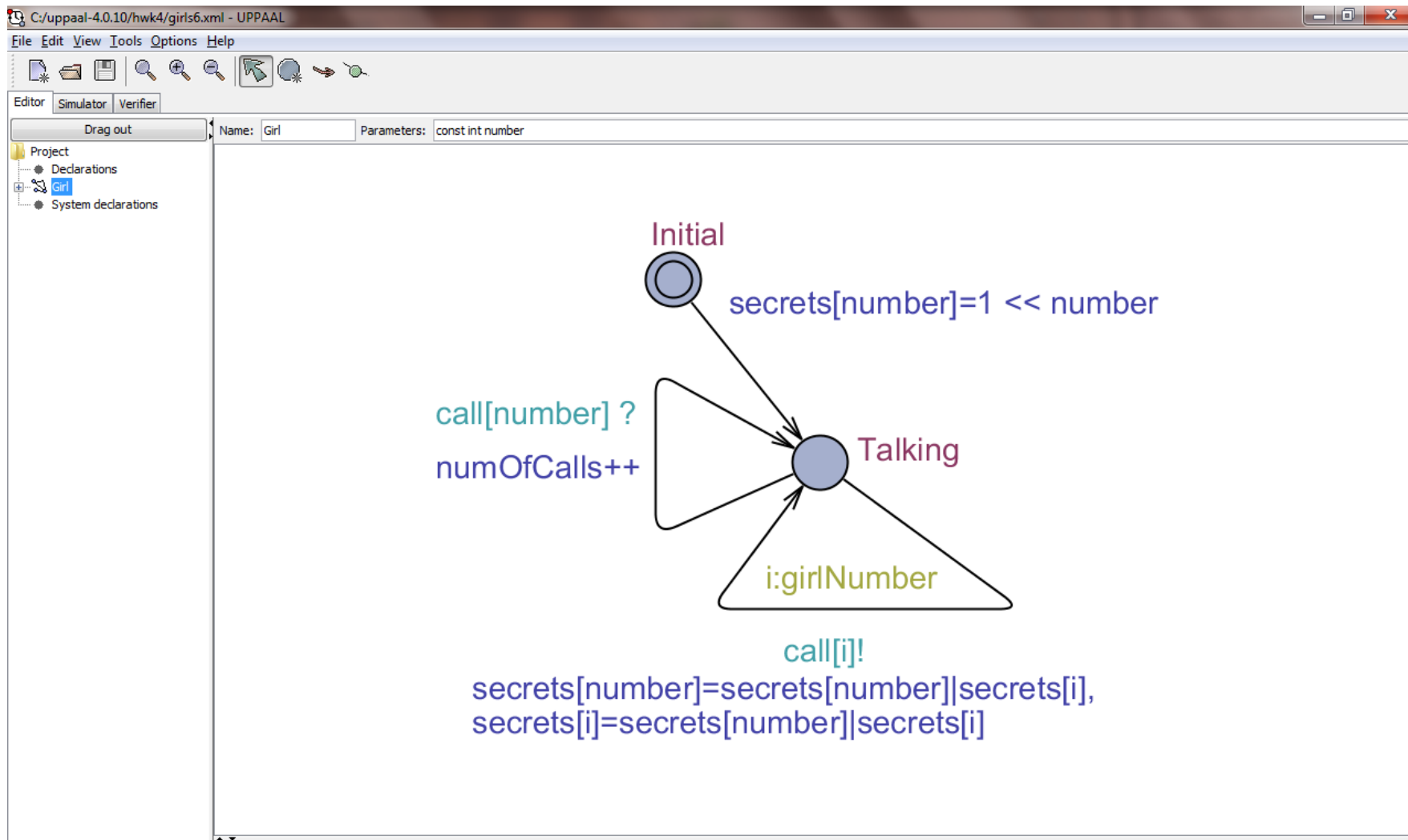
- Values from 9 to 15 for the third viking require the same time, 36, for all four to cross.
- Generalize:
 - $k=3$: 3, 6, 12, 15 \Rightarrow 36 crossing time
 - $k=4$: 4, 8, 16, 20 \Rightarrow 48 crossing time
 - $k=5$: 5, 10, 20, 25 \Rightarrow 60 crossing time

So, total time = $12*k$, for crossing times of k , $2*k$, $4*k$, $5*k$

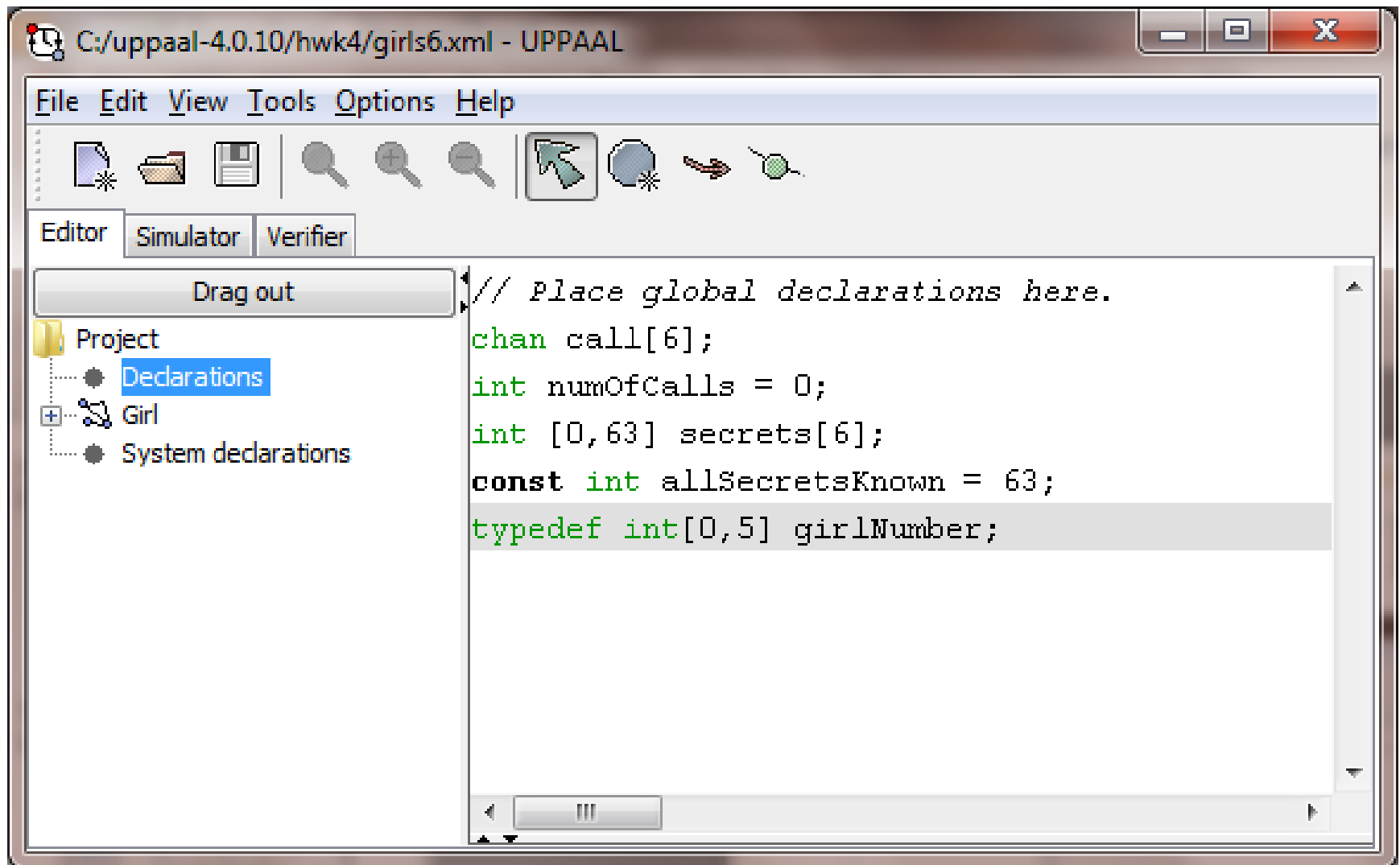
Gossiping Girls' Problem

- Model and analyze the following gossiping girls problem in UPPAAL. A number of girls initially know one distinct secret each. Each pair of girls has access to a phone line that can be used to share their secrets. Each time two girls talk to each other they always exchange all secrets that they currently know with each other (thus after the phone call they both know all secrets they knew together before the phone call). Only binary communication (between two girls in each phone call) is possible.
- (a) Use UPPAAL to find the minimal number of phone calls needed for **two**, **three**, or **four** girls to know all secrets.
- (b) Analytically determine how many phone calls are needed to solve the gossiping girls' problem for n girls. Hint: Use a recursive or inductive approach. Consider the solution for $n = 2$, $n = 3$, etc. Then, write a proof by induction.

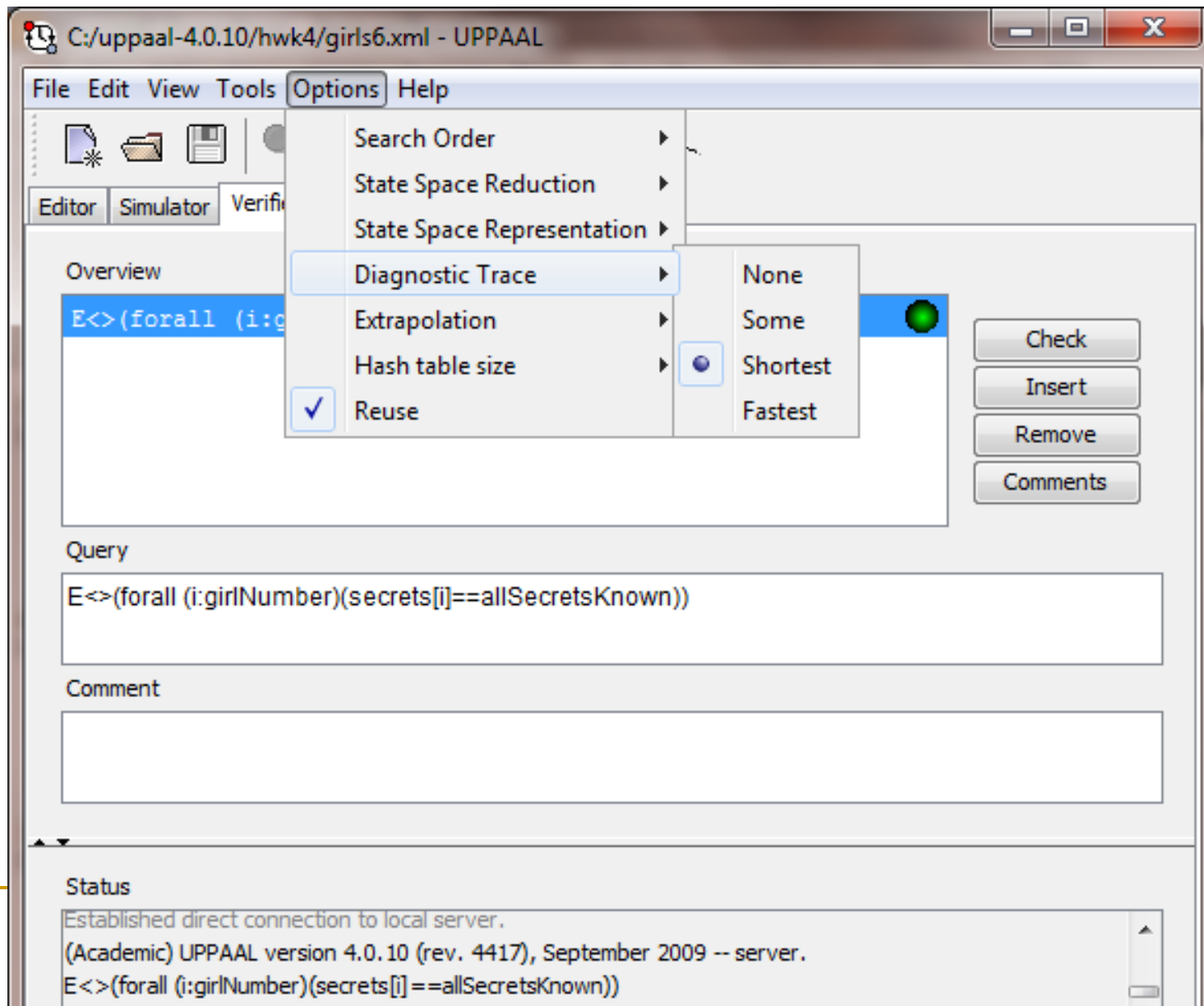
Gossiping Girls' Problem



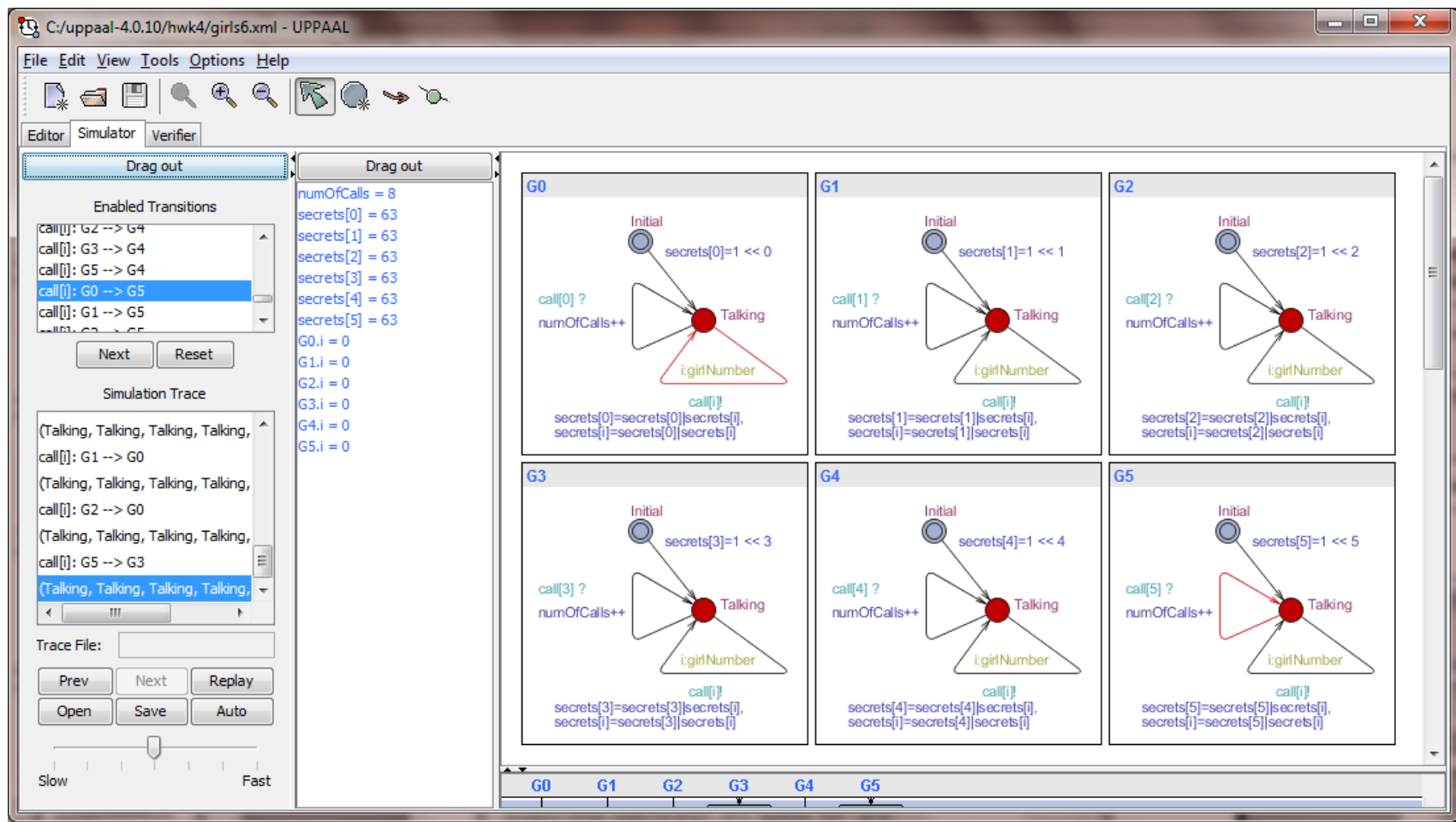
Gossiping Girls' Problem



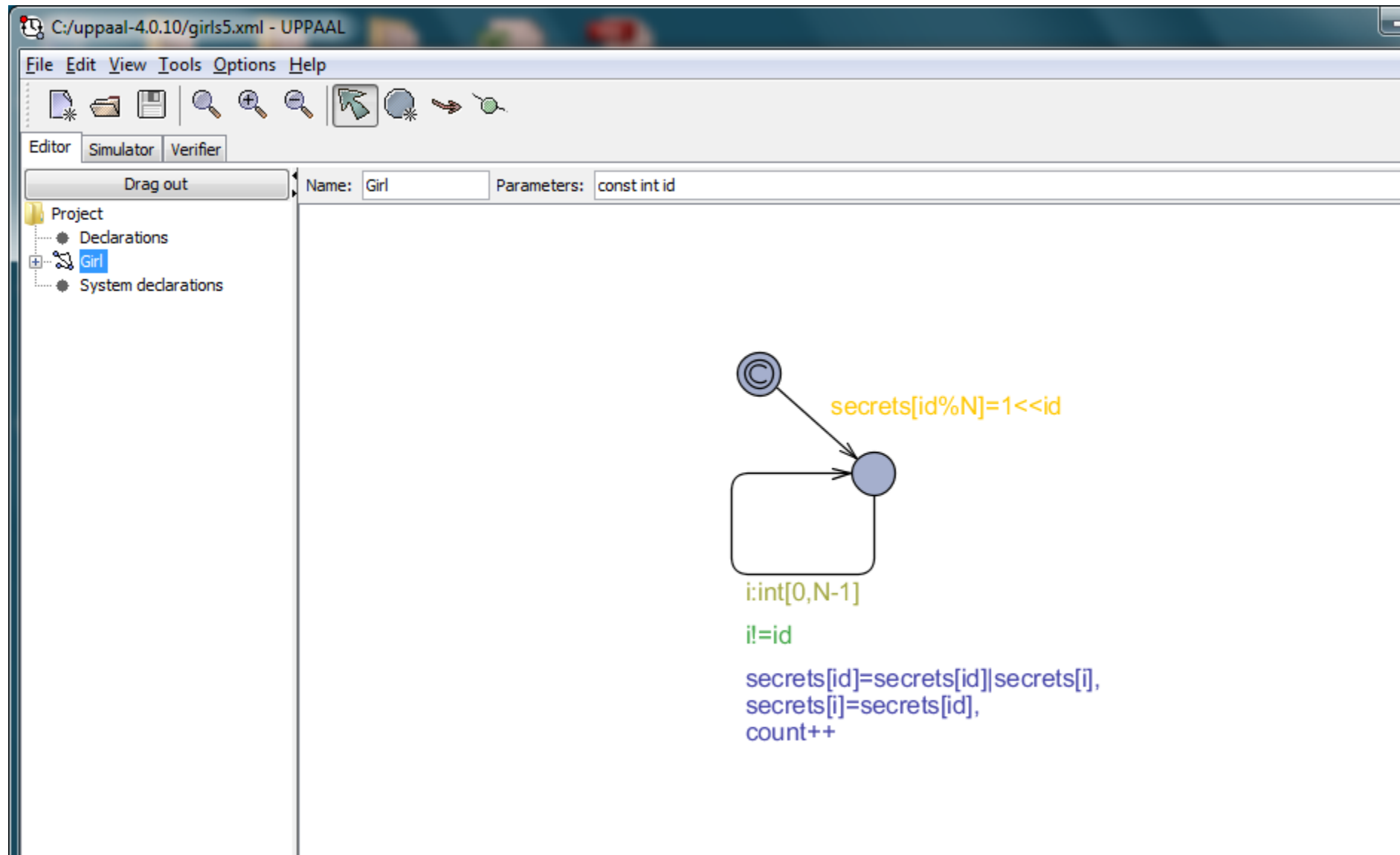
Gossiping Girls' Problem



Gossiping Girls' Problem



Gossiping Girls' Problem



Gossiping Girls' Problem

- Special case, $n = 4$: 4 calls required.
- Other cases:
 - $n = 2 \Rightarrow 1$ call
 - $n = 3 \Rightarrow 3$ calls
 - $n = 4 \Rightarrow 4$ calls
 - $n = 5 \Rightarrow 6$ calls
 - $n = 6 \Rightarrow 8$ calls
 - $n = 7 \Rightarrow 10$ calls
 - $n = 8 \Rightarrow 12$ calls
 - For $(n \geq 4)$, $2*n - 4$ calls

Distributed Mutual Exclusion Algorithms

Permission based

Lamport
Ricart, et.al.
Maekawa
Sanders
Agrawal, et al.
Singhal
Raynal

Token based

Logical structure

No logical structure

Suzuki, et al.
Ricart, et al.
Singhal
Helary, et al.

Dynamic

(path reversal)

Trehel, et al.
Bernabeu-Auban, et al.
Ginat, et al.

Static

(edge reversal)

Raymond
van de Snepscheut
Neilsen, et al.

**Generalized
Algorithm**

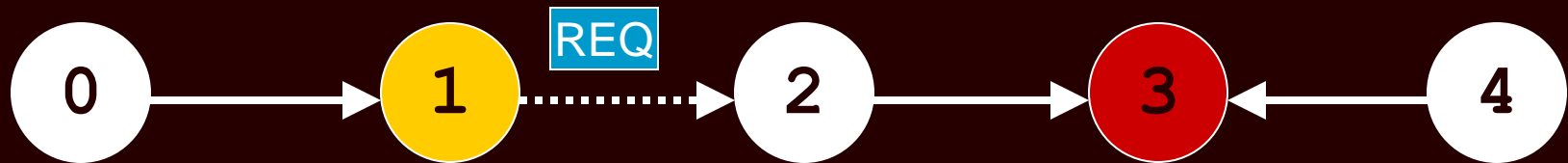
Dynamic Logical Structure – Path Reversal



 = token holder

 = requesting token

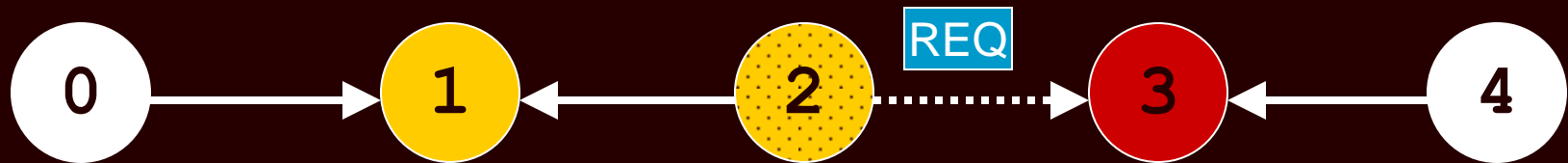
Dynamic Logical Structure – Path Reversal



 = token holder

 = requesting token

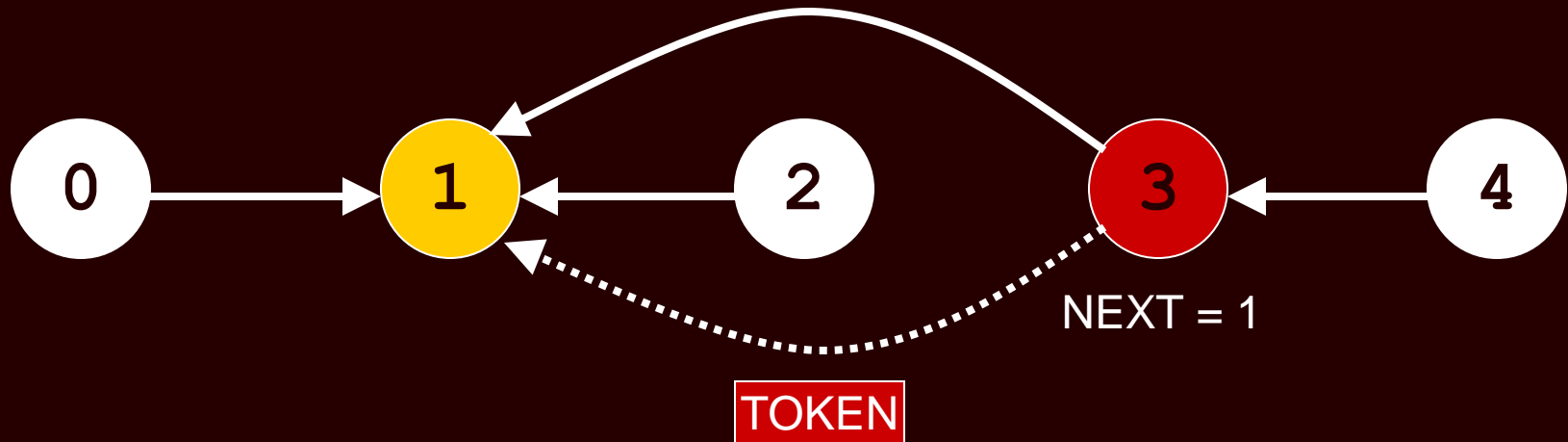
Dynamic Logical Structure – Path Reversal



 = token holder

 = requesting token

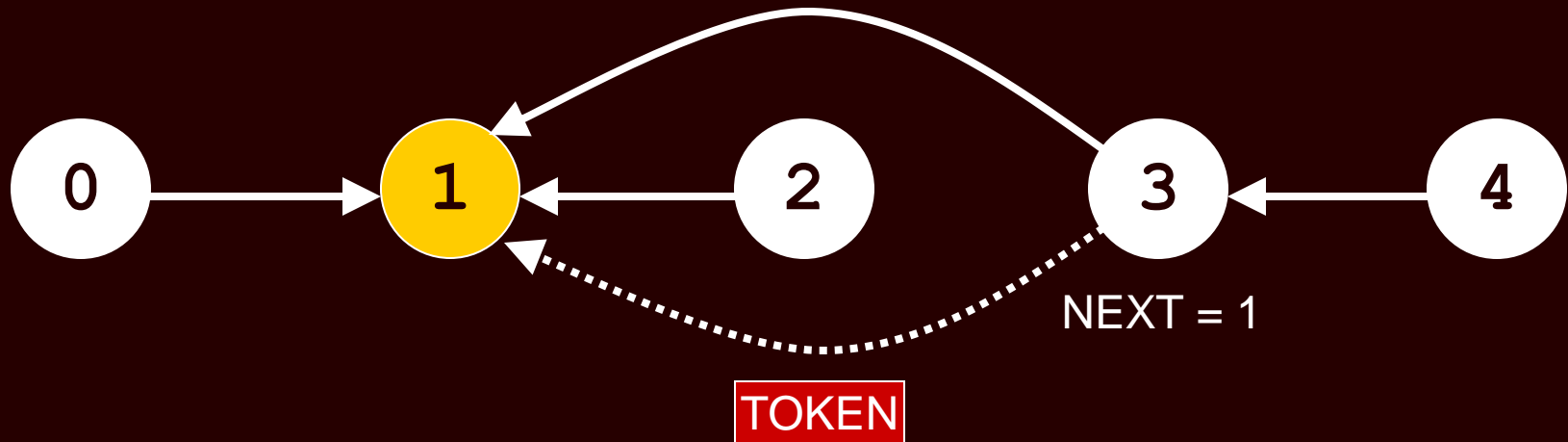
Dynamic Logical Structure – Path Reversal



 = token holder

 = requesting token

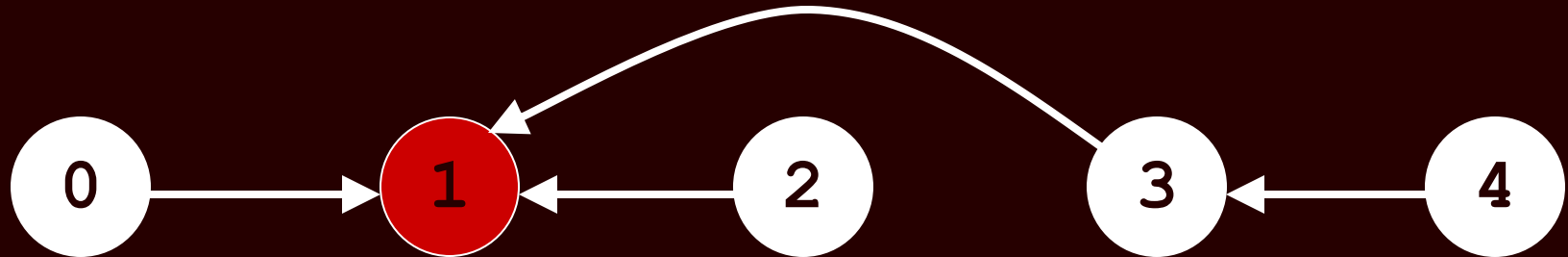
Dynamic Logical Structure – Path Reversal



 = token holder

 = requesting token

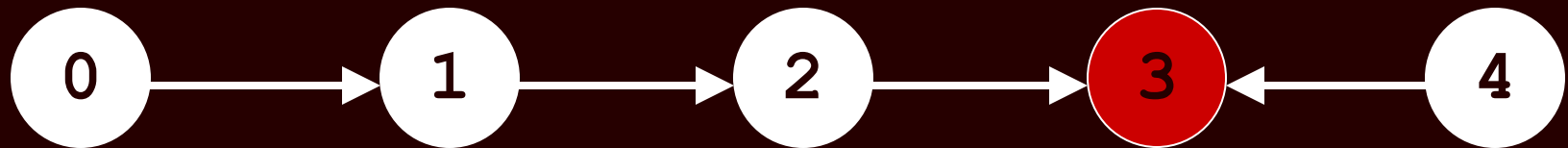
Dynamic Logical Structure – Path Reversal



 = token holder

 = requesting token

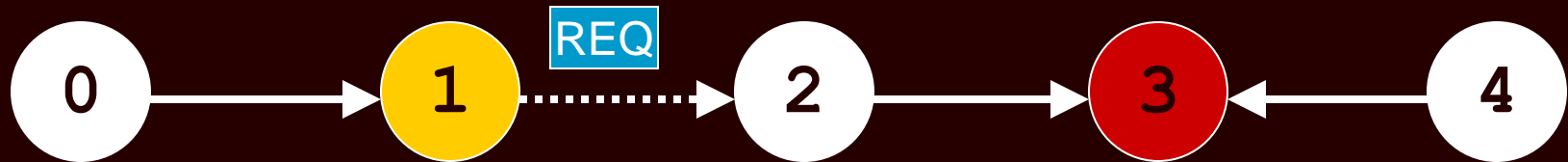
Static Logical Structure – Edge Reversal



 = token holder

 = requesting token

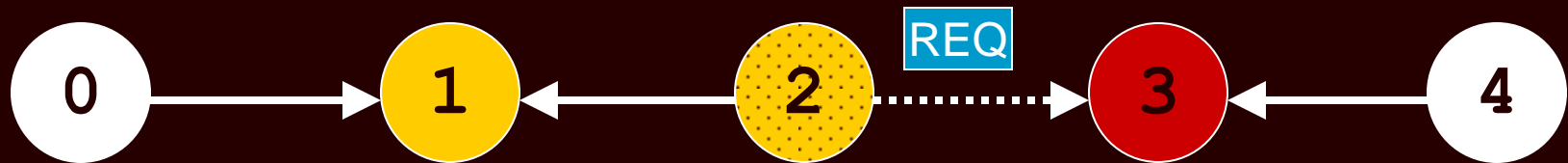
Static Logical Structure – Edge Reversal



 = token holder

 = requesting token

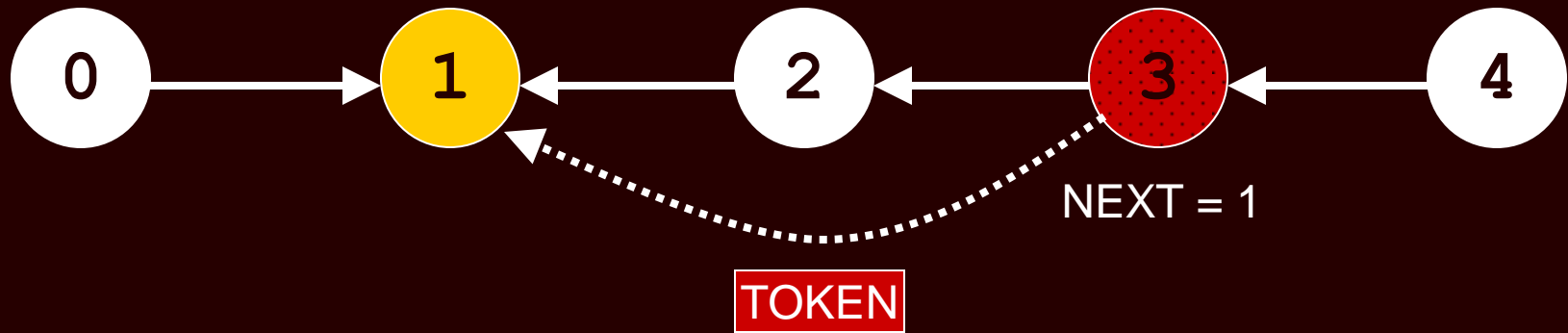
Static Logical Structure – Edge Reversal



 = token holder

 = requesting token

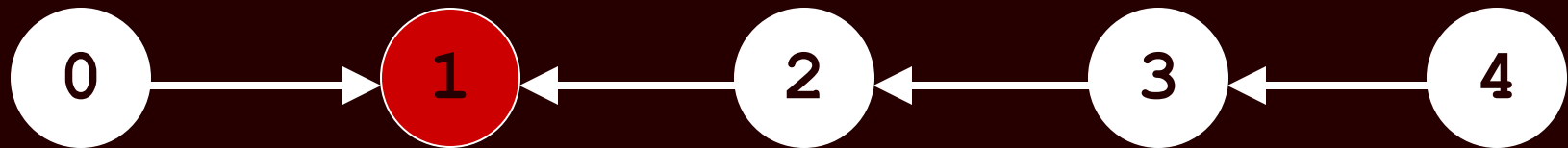
Static Logical Structure – Edge Reversal



 = token holder

 = requesting token

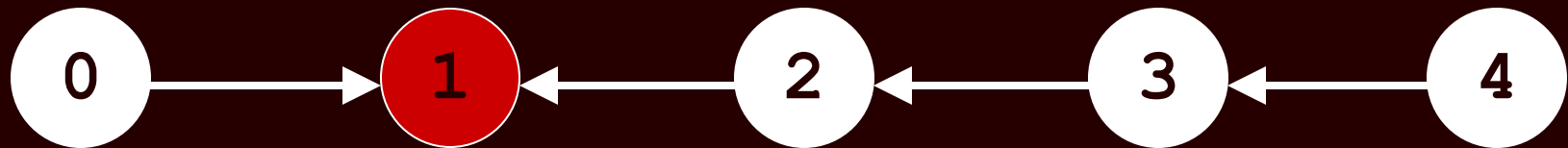
Static Logical Structure – Edge Reversal



 = token holder

 = requesting token

Static Logical Structure – Edge Reversal



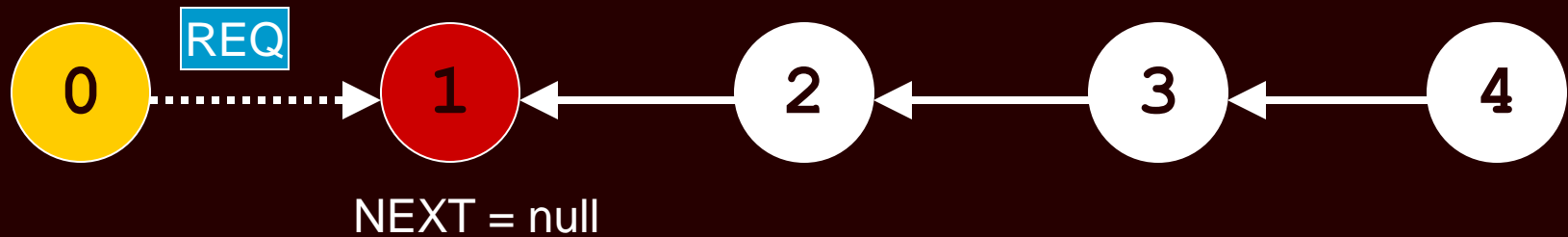
 = token holder

 = requesting token

Logical Wait Queue



Static Logical Structure – Edge Reversal



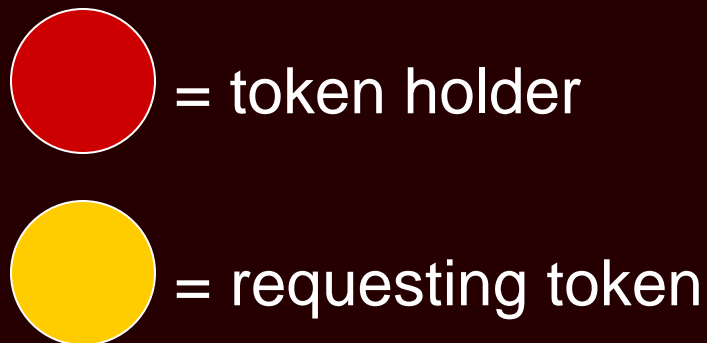
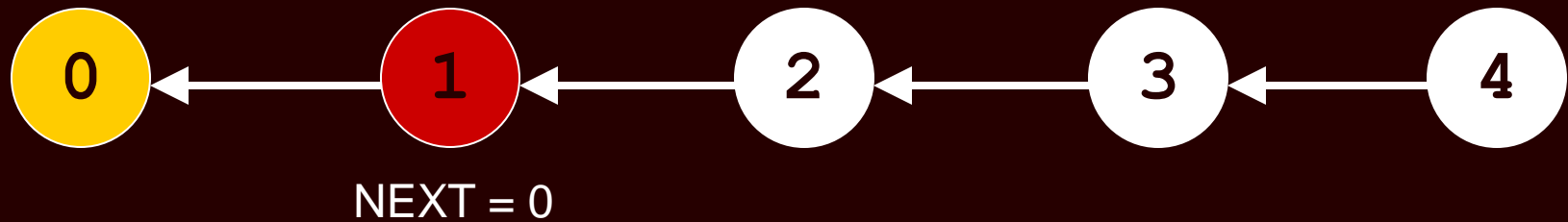
 = token holder

 = requesting token

Logical Wait Queue



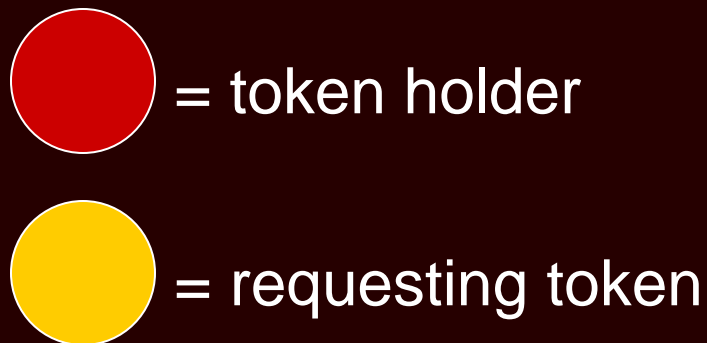
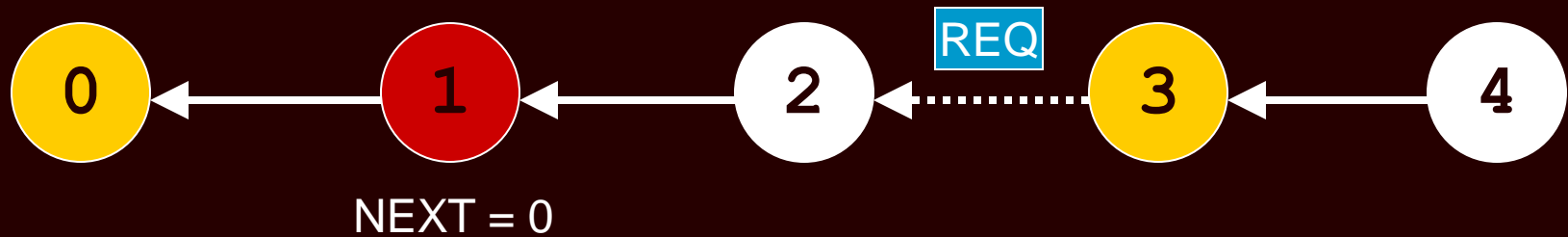
Static Logical Structure – Edge Reversal



Logical Wait Queue



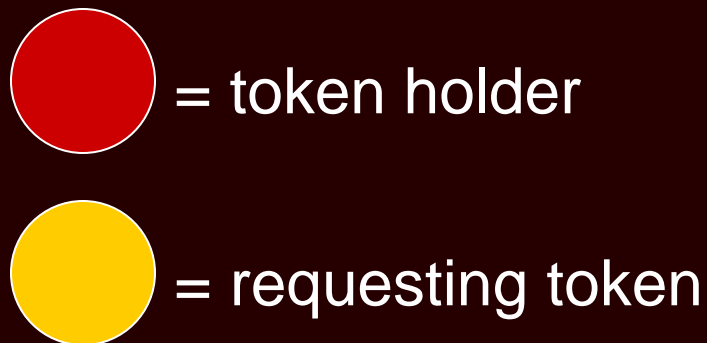
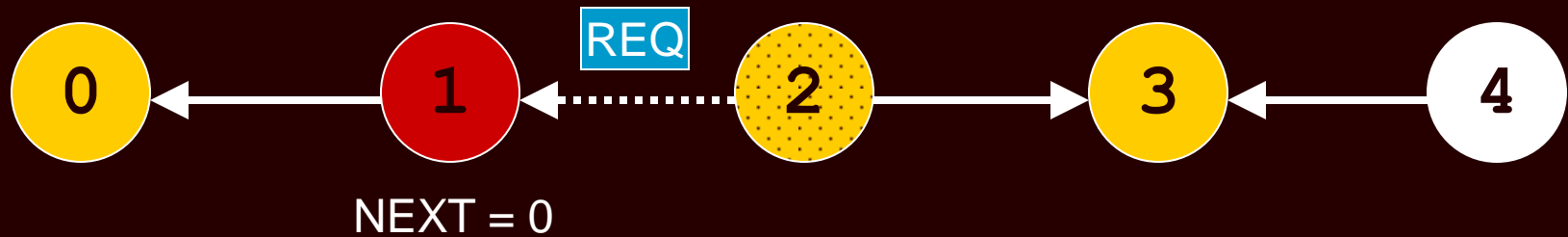
Static Logical Structure – Edge Reversal



Logical Wait Queue



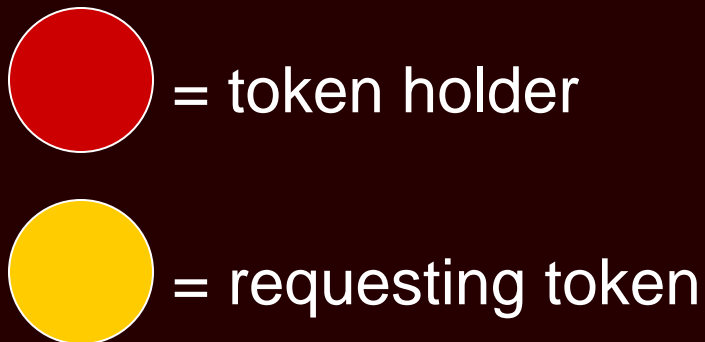
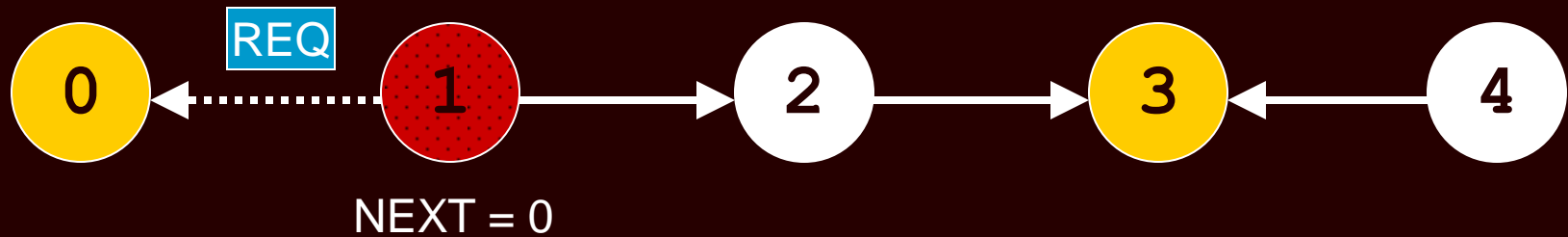
Static Logical Structure – Edge Reversal



Logical Wait Queue



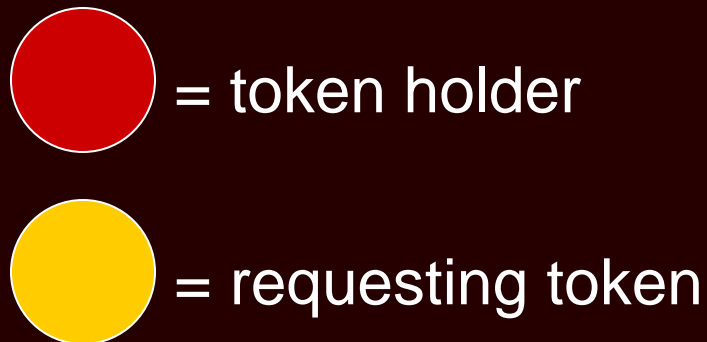
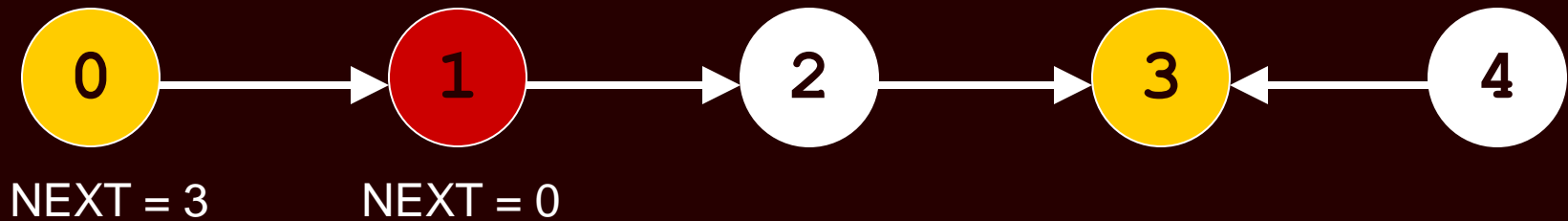
Static Logical Structure – Edge Reversal



Logical Wait Queue



Static Logical Structure – Edge Reversal

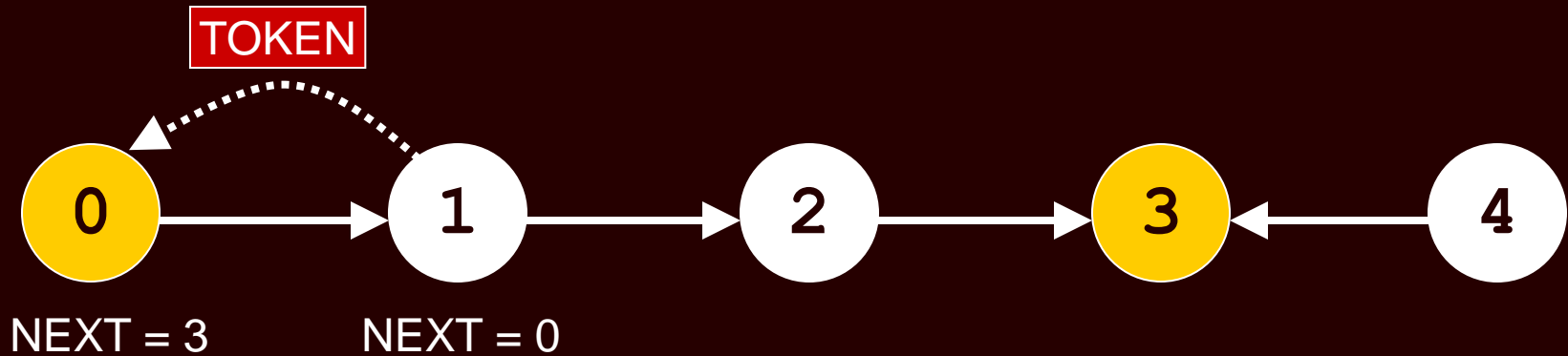


Logical Wait Queue

0, 3



Static Logical Structure – Edge Reversal



 = token holder

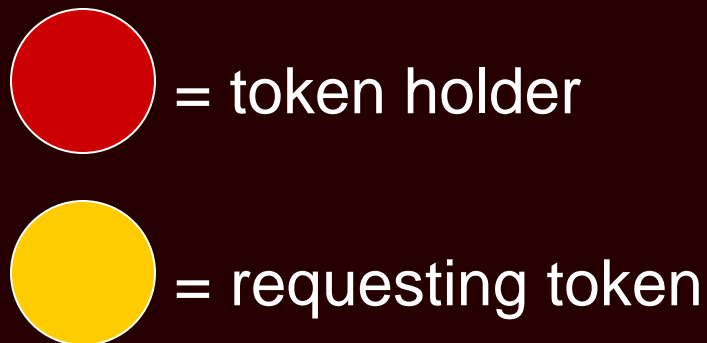
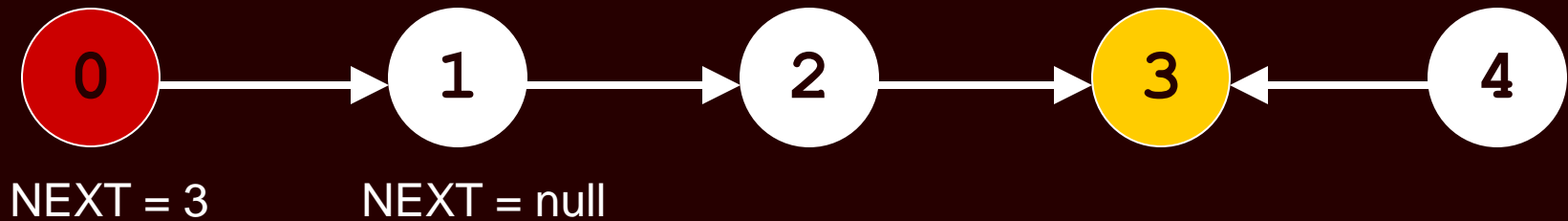
 = requesting token

Logical Wait Queue

0, 3



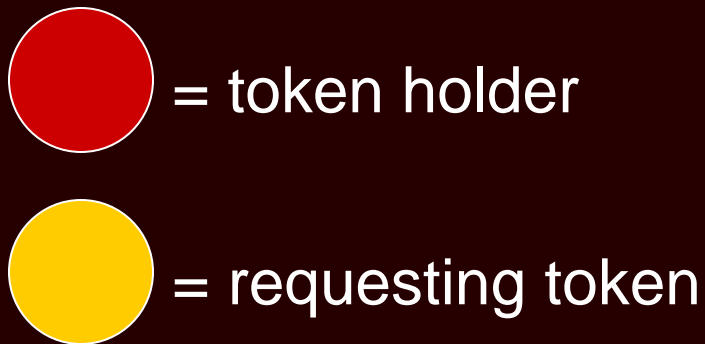
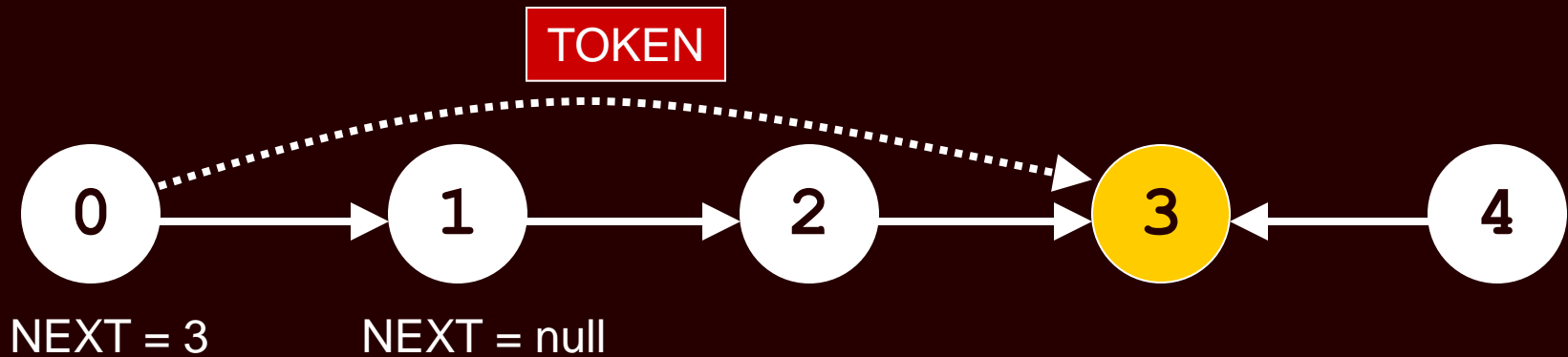
Static Logical Structure – Edge Reversal



Logical Wait Queue



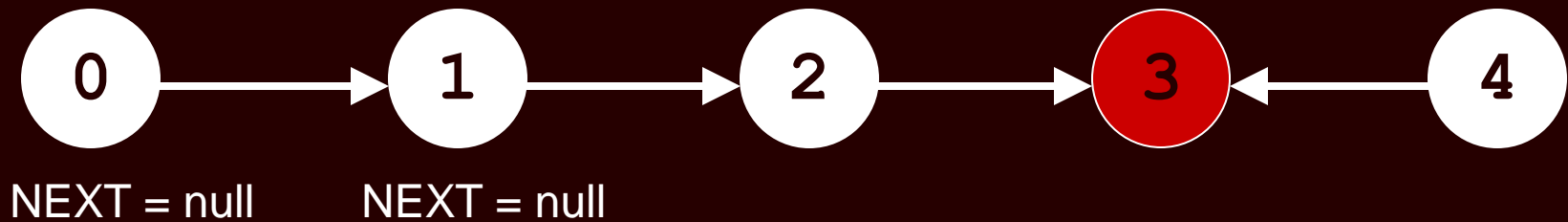
Static Logical Structure – Edge Reversal



Logical Wait Queue



Static Logical Structure – Edge Reversal



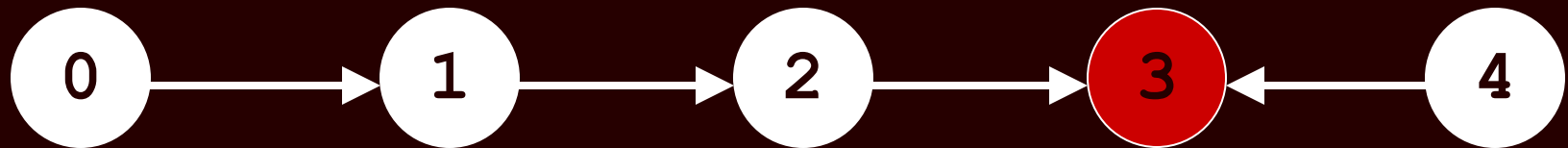
 = token holder

 = requesting token

Logical Wait Queue



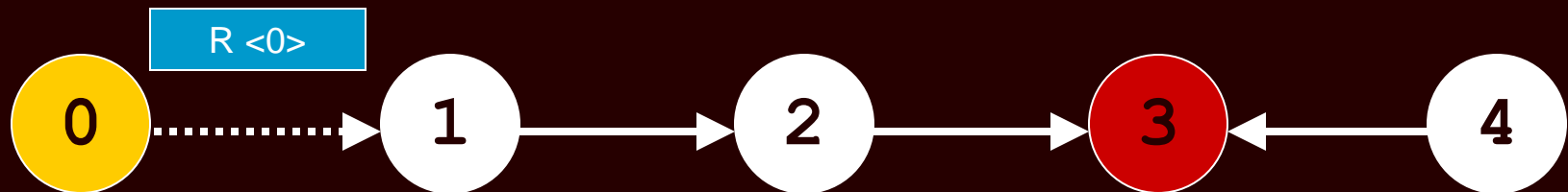
Generalized Algorithm: Edge+Path Reversal



 = token holder

 = requesting token

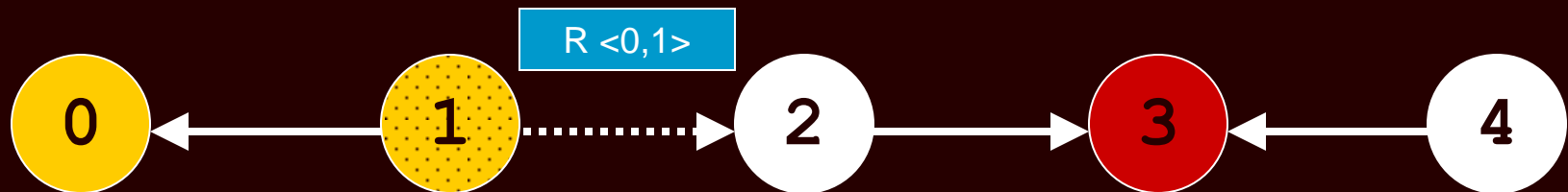
Generalized Algorithm: Edge+Path Reversal



 = token holder

 = requesting token

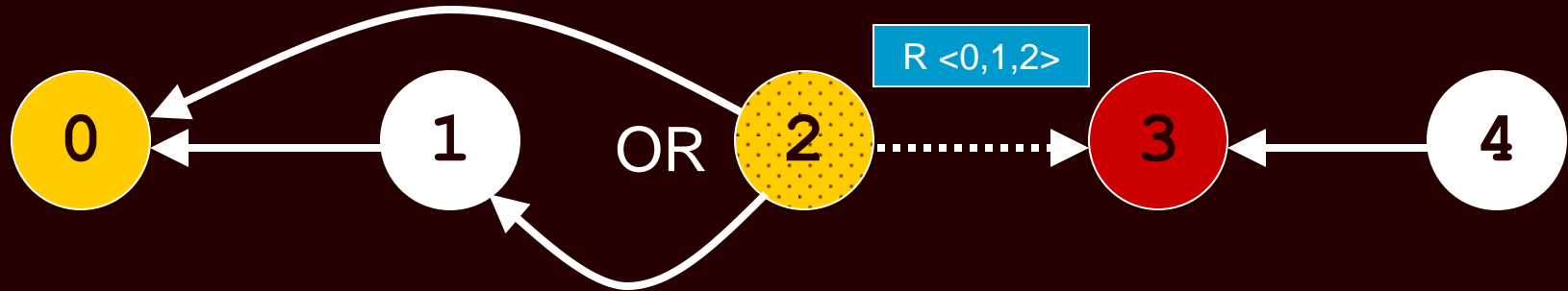
Generalized Algorithm: Edge+Path Reversal



 = token holder

 = requesting token

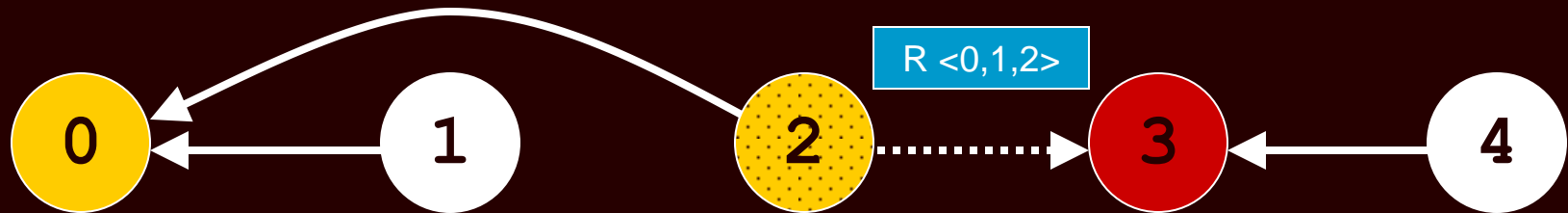
Generalized Algorithm: Edge+Path Reversal



 = token holder

 = requesting token

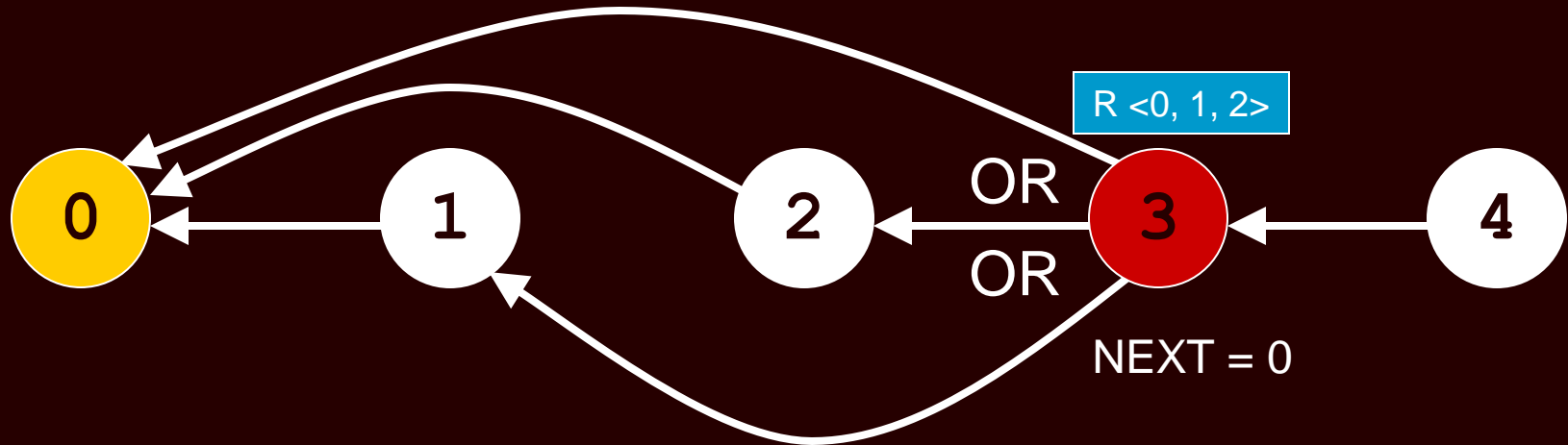
Generalized Algorithm: Edge+Path Reversal



 = token holder

 = requesting token

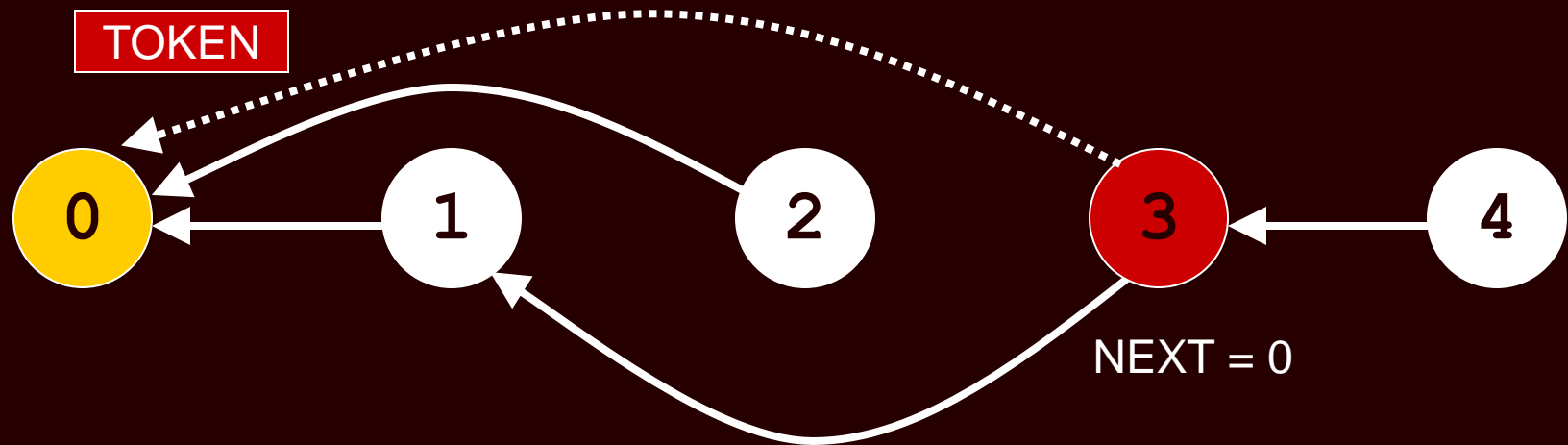
Generalized Algorithm: Edge+Path Reversal



 = token holder

 = requesting token

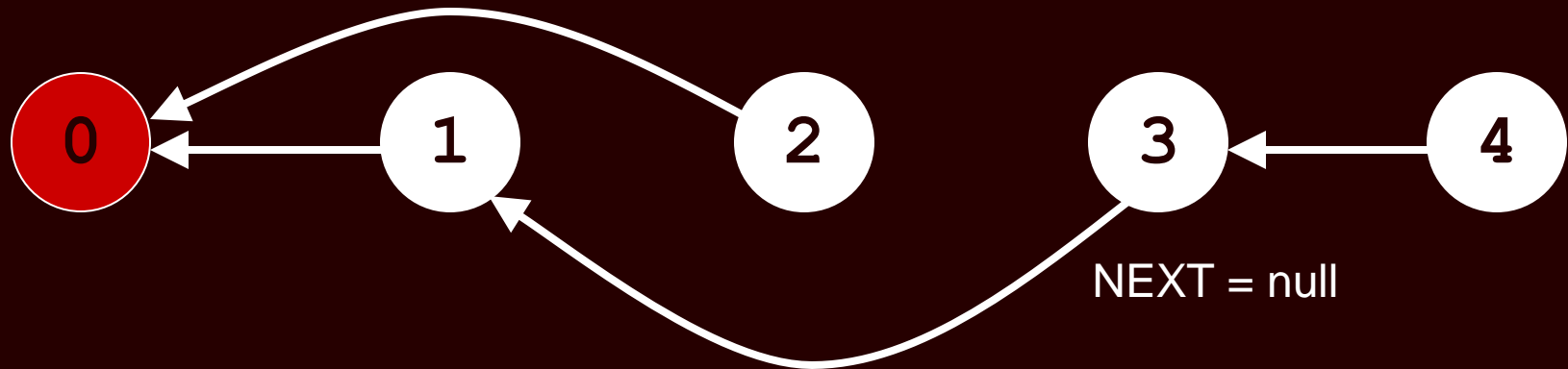
Generalized Algorithm: Edge+Path Reversal



 = token holder

 = requesting token

Generalized Algorithm: Edge+Path Reversal

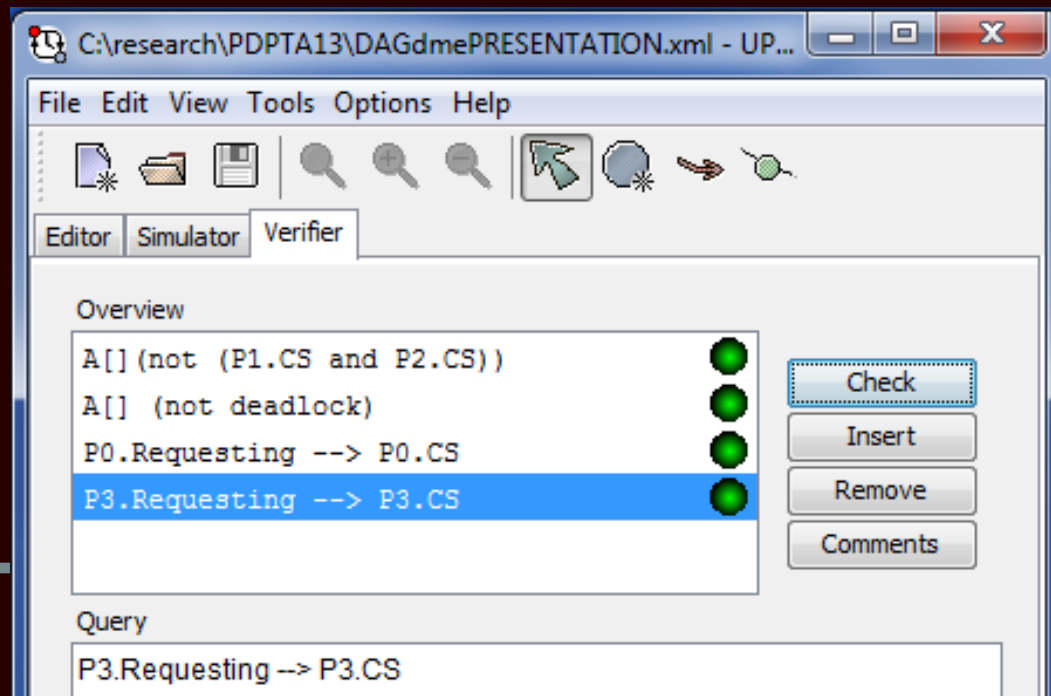


 = token holder

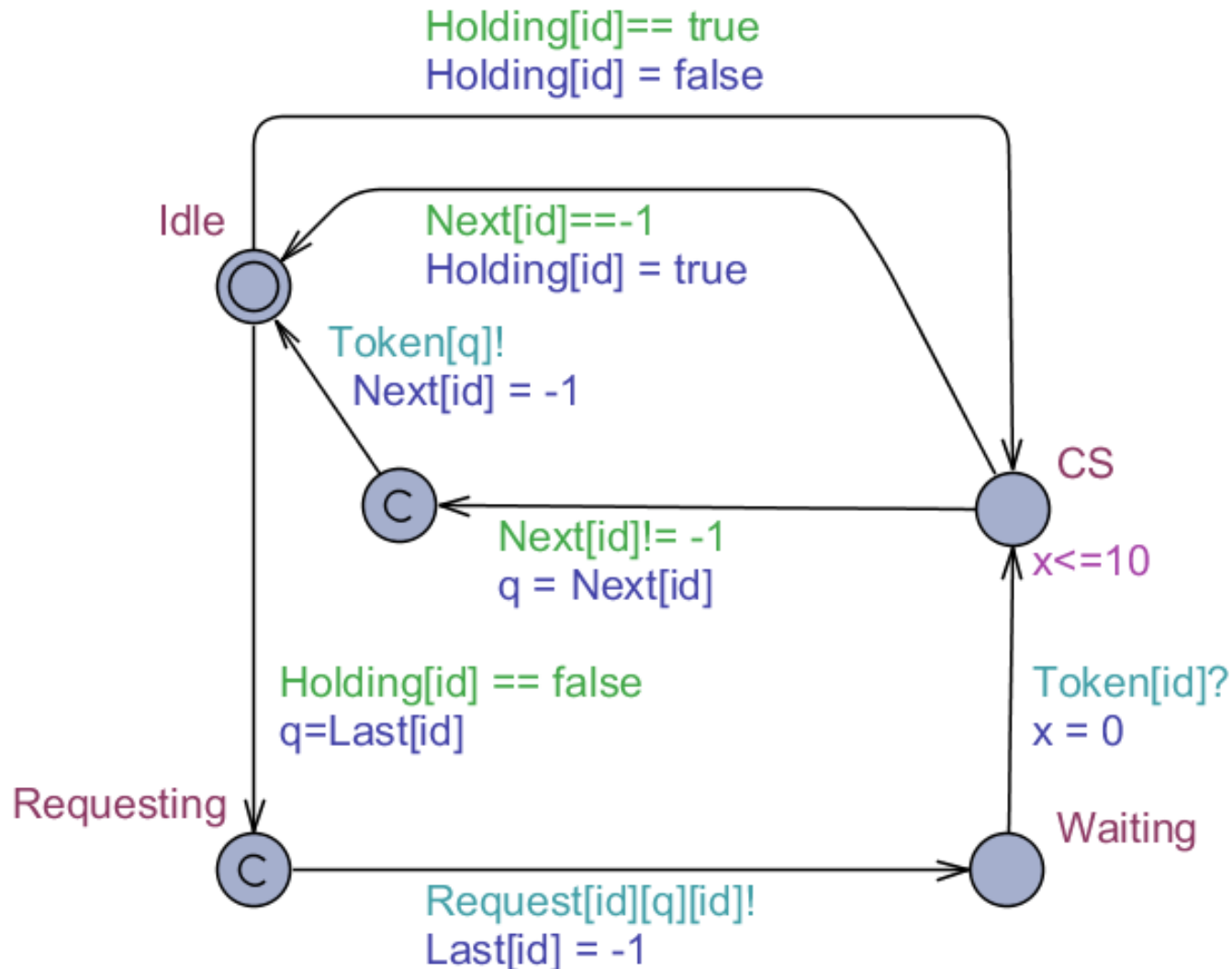
 = requesting token

Properties to Verify

- Mutual exclusion is guaranteed.
- The algorithm is deadlock free.
- The algorithm is starvation free.



UPPAAL Model – ProcessWork(id)

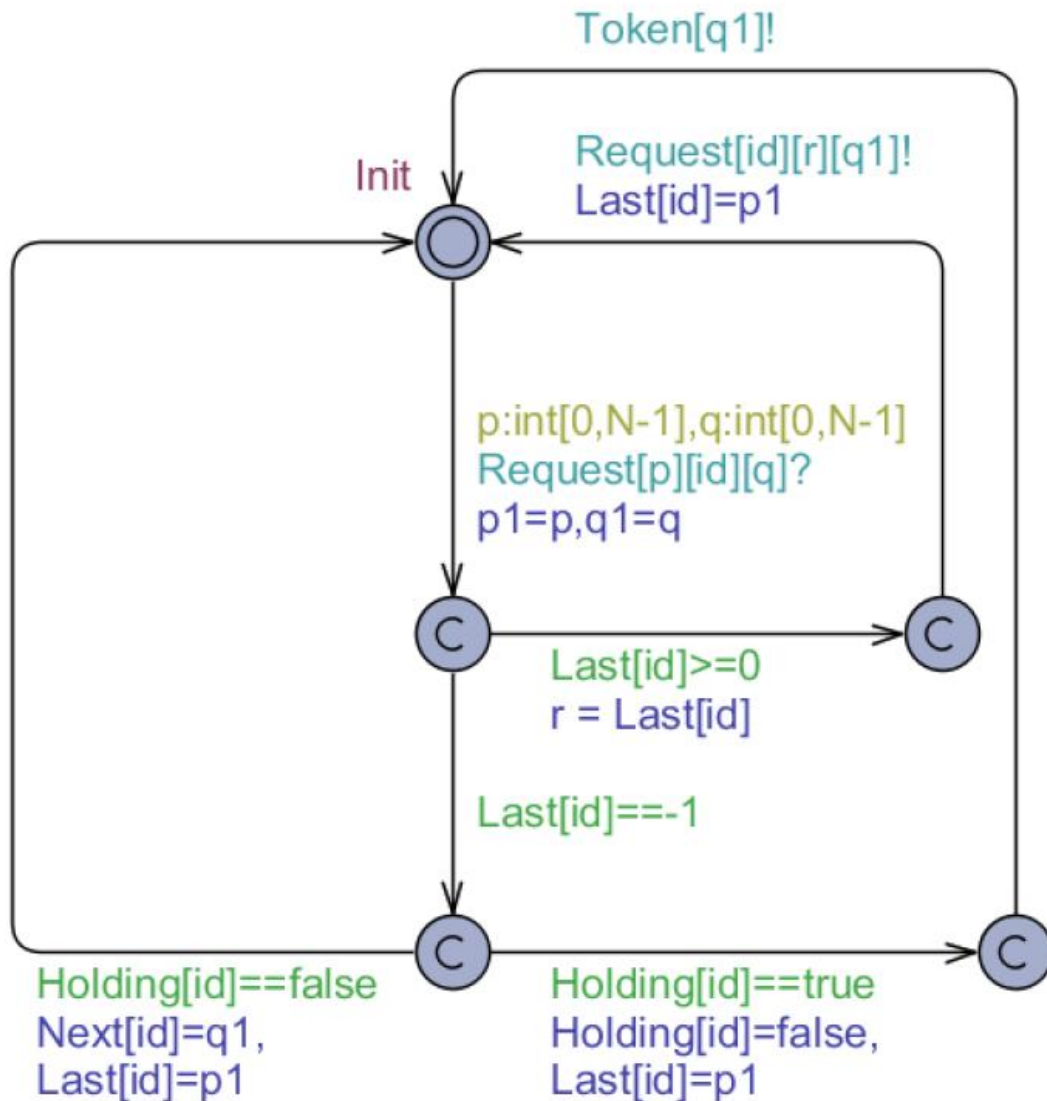


Holding – holding, but not using the token

Next – next node to receive token

Last – next node to send request to (node on path to last request)

UPPAAL Model – ProcessRequest(id)



Holding – holding, but not using the token

Next – next node to receive token

Last – next node to send request to (node on path to last request)

Drag out

Drag out

Enabled Transitions

Holding[0] = 1
Holding[1] = 0
Holding[2] = 0
Holding[3] = 0
Next[0] = -1
Next[1] = -1
Next[2] = -1
Next[3] = -1
Last[0] = -1
Last[1] = 0
Last[2] = 0
Last[3] = 0
P0.q = 0
P1.q = 0
P2.q = 0
P3.q = 0
R0.r = 0

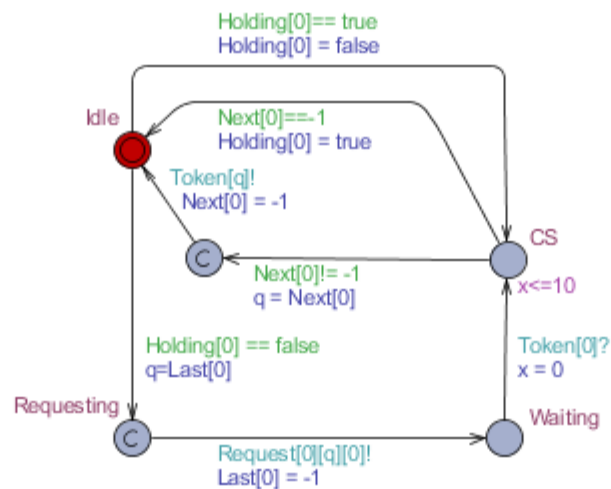
Next

Reset

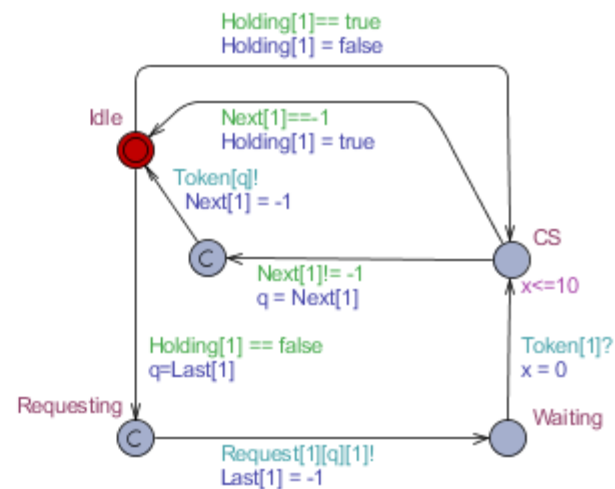
Simulation Trace

e, Idle, Idle, Init, Init, In

P0



P1

**Holding = true**



Drag out

Drag out

Enabled Transitions

P0
P1
P2
P3

Next Reset

Simulation Trace

(Idle, Idle, Idle, Idle, Init, Init, Init)

P0

(CS, Idle, Idle, Idle, Init, Init, Init)

P1

(CS, Requesting, Idle, Idle, Init, Init, Init)

Request[1][q][1]: P1 --> R0

(CS, Waiting, Idle, Idle, -, Init, Init, Init)

R0

(CS, Waiting, Idle, Idle, -, Init, Init, Init)

R0

(CS, Waiting, Idle, Idle, Init, Init, Init)

P2

(CS, Waiting, Requesting, Idle, Init, Init, Init)

Request[2][q][2]: P2 --> R0

P0.x = P1.x

P1.x = P2.x

P2.x = P3.x

P3.x = P0.x

Trace File:

Prev

Next

Replay

Holding[0] = 0

Holding[1] = 0

Holding[2] = 0

Holding[3] = 0

Next[0] = -1

Next[1] = -1

Next[2] = -1

Next[3] = -1

Last[0] = -1

Last[1] = 0

Last[2] = 0

Last[3] = 0

P0.q = 0

P1.q = 0

P2.q = 0

P3.q = 0

R0.r = 0

R0.p1 = 0

R0.q1 = 0

R1.r = 0

R1.p1 = 0

R1.q1 = 0

R2.r = 0

R2.p1 = 0

R2.q1 = 0

R3.r = 0

R3.p1 = 0

R3.q1 = 0

P0.x in [0, 10]

P1.x in [0, 10]

P2.x in [0, 10]

P3.x in [0, 10]

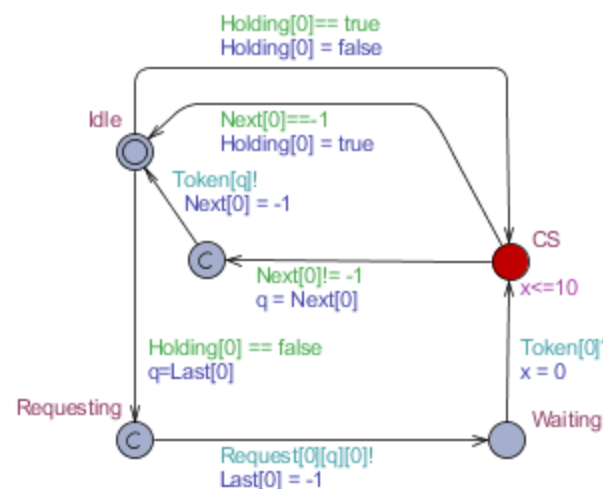
P0.x = P1.x

P1.x = P2.x

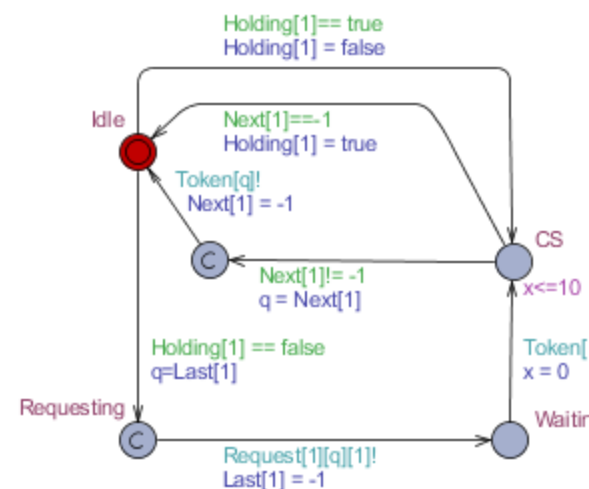
P2.x = P3.x

P3.x = P0.x

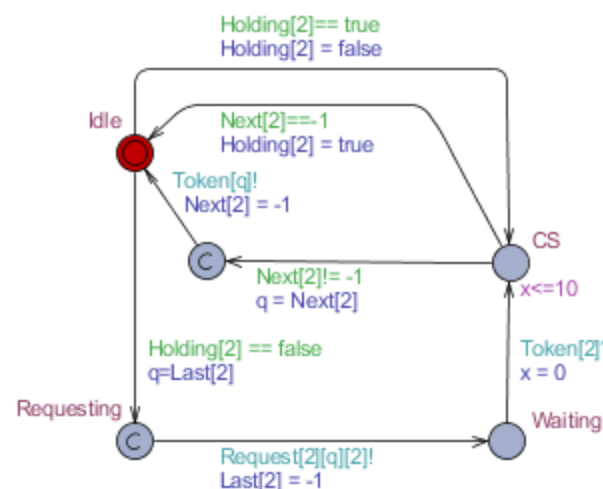
P0



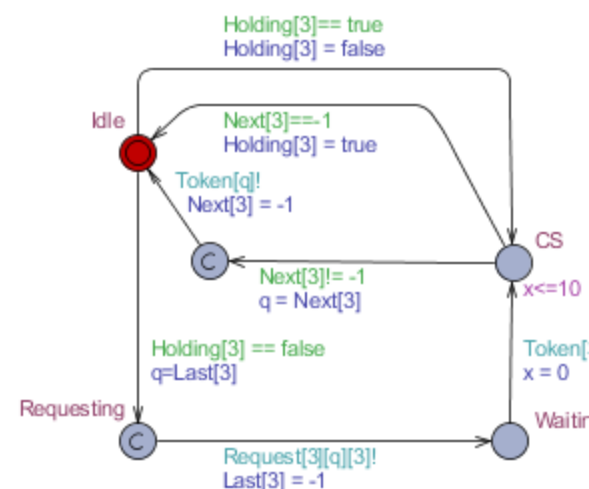
P1



P2



P3



P0

P1

P2

P3

R0

R1

R2

R3



Editor Simulator Verifier

Drag out

Drag out

Enabled Transitions

Request[1][q][1]: P1 --> R0

Holding[0] = 0
 Holding[1] = 0
 Holding[2] = 0
 Holding[3] = 0
 Next[0] = -1
 Next[1] = -1
 Next[2] = -1
 Next[3] = -1
 Last[0] = -1
 Last[1] = 0
 Last[2] = 0
 Last[3] = 0
 P0.q = 0
 P1.q = 0
 P2.q = 0
 P3.q = 0
 R0.r = 0
 R0.p1 = 0
 R0.q1 = 0
 R1.r = 0
 R1.p1 = 0
 R1.q1 = 0
 R2.r = 0
 R2.p1 = 0
 R2.q1 = 0
 R3.r = 0
 R3.p1 = 0
 R3.q1 = 0
 P0.x in [0,10]
 P1.x in [0,10]
 P2.x in [0,10]
 P3.x in [0,10]
 P0.x = P1.x
 P1.x = P2.x
 P2.x = P3.x
 P3.x = P0.x

Next Reset

Simulation Trace

(Idle, Idle, Idle, Idle, Init, Init, Init, Init)
 P0
 (CS, Idle, Idle, Idle, Init, Init, Init, Init)
 P1
 (CS, Requesting, Idle, Idle, Init, Init, Init, Init)
 Request[1][q][1]: P1 --> R0
 (CS, Waiting, Idle, Idle, -, Init, Init, Init, Init)
 R0
 (CS, Waiting, Idle, Idle, -, Init, Init, Init, Init)
 R0
 (CS, Waiting, Idle, Idle, Init, Init, Init, Init)
 P2
 (CS, Waiting, Requesting, Idle, Init, Init, Init, Init)
 Request[2][q][2]: P2 --> R0

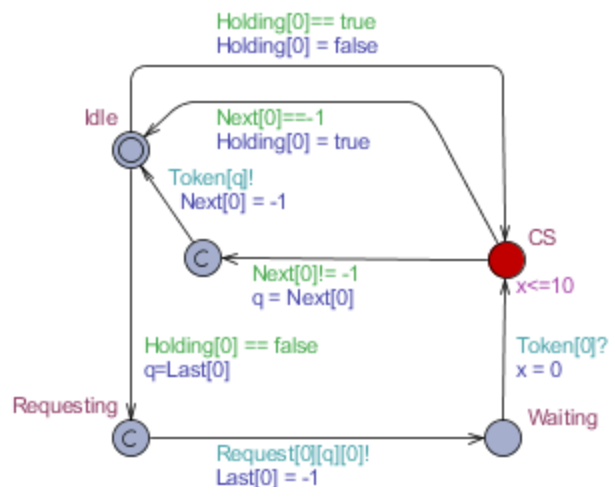
Trace File:

Prev

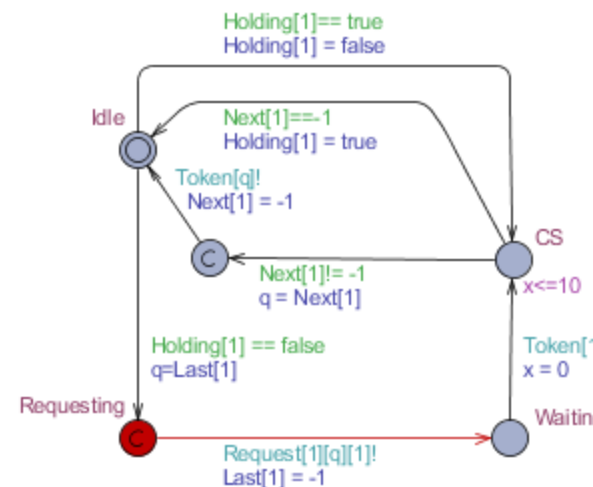
Next

Replay

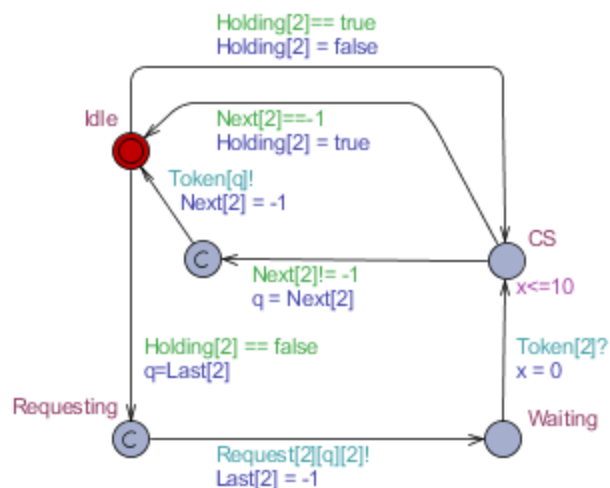
P0



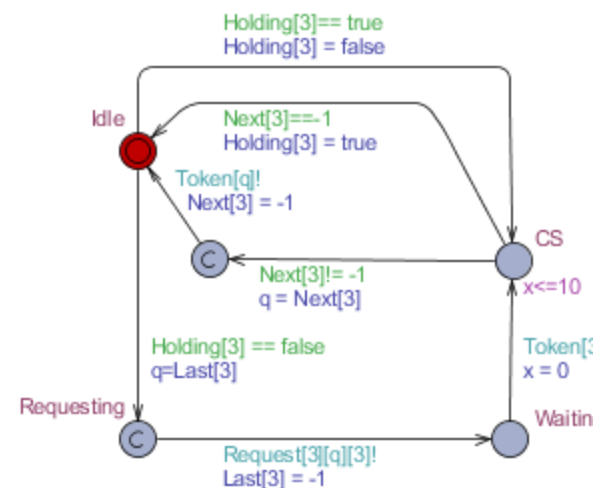
P1



P2



P3



P0

P1

P2

P3

R0

R1

R2

R3



Editor Simulator Verifier

Drag out

Drag out

Enabled Transitions

R0

Holding[0] = 0
 Holding[1] = 0
 Holding[2] = 0
 Holding[3] = 0
 Next[0] = 1
 Next[1] = -1
 Next[2] = -1
 Next[3] = -1
 Last[0] = 1
 Last[1] = -1
 Last[2] = -1
 Last[3] = 0
 P0.q = 0
 P1.q = 0
 P2.q = 0
 P3.q = 0
 R0.r = 0
 R0.p1 = 2
 R0.q1 = 2
 R1.r = 0
 R1.p1 = 0
 R1.q1 = 0
 R2.r = 0
 R2.p1 = 0
 R2.q1 = 0
 R3.r = 0
 R3.p1 = 0
 R3.q1 = 0
 P0.x in [0,10]
 P1.x in [0,10]
 P2.x in [0,10]
 P3.x in [0,10]
 P0.x = P1.x
 P1.x = P2.x
 P2.x = P3.x
 P3.x = P0.x

Next

Reset

Simulation Trace

P1

(CS, Requesting, Idle, Idle, Init, I
 Request[1][q][1]: P1 --> R0
 (CS, Waiting, Idle, Idle, -, Init, In
 R0
 (CS, Waiting, Idle, Idle, -, Init, In
 R0
 (CS, Waiting, Idle, Idle, Init, Init,
 P2
 (CS, Waiting, Requesting, Idle, In
 Request[2][q][2]: P2 --> R0
 (CS, Waiting, Waiting, Idle, -, Init
 R0
 (CS, Waiting, Waiting, Idle, -, Init

Trace File:

Prev

Next

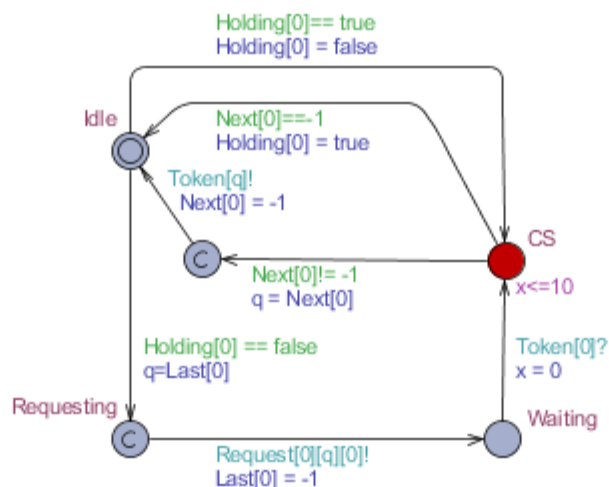
Replay

Open

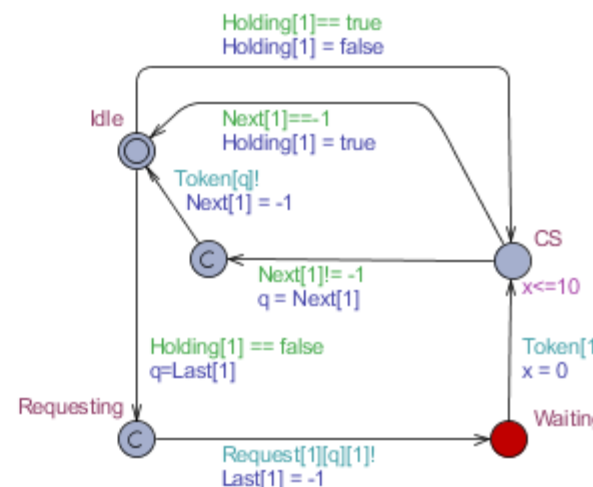
Save

Auto

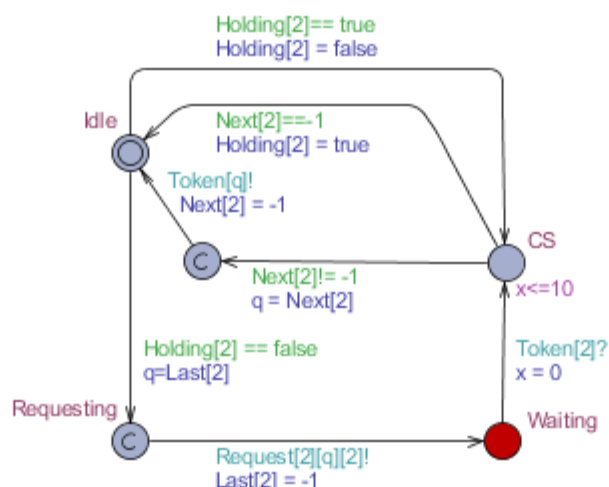
P0



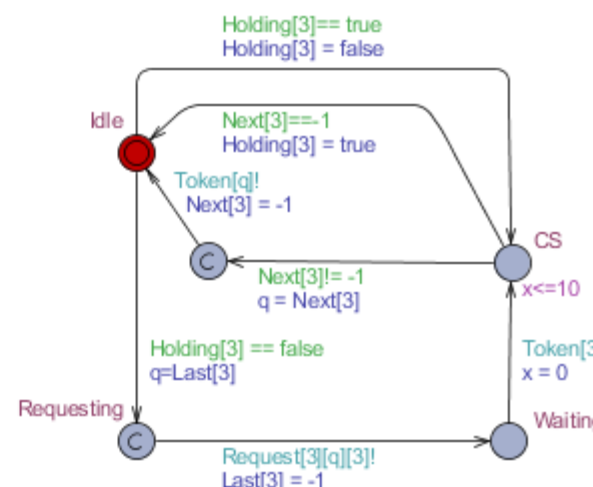
P1



P2



P3



P0

P1

P2

P3

R0

R1

R2

R3

Requesting



Editor Simulator Verifier

Drag out

Enabled Transitions

Token[q]: P0 --> P1

Next

Reset

Simulation Trace

(CS, Waiting, Waiting, Idle, -, Init, R0

(CS, Waiting, Waiting, Idle, -, Init, Request[0][r][q1]: R0 --> R1

(CS, Waiting, Waiting, Idle, Init, R1

(CS, Waiting, Waiting, Idle, Init, R1

(CS, Waiting, Waiting, Idle, Init, P0

(-, Waiting, Waiting, Idle, Init, Token[q]: P0 --> P1

(Idle, CS, Waiting, Idle, Init, P3

Trace File:

Prev

Next

Replay

Open

Save

Auto

Drag out

Holding[0] = 0

Holding[1] = 0

Holding[2] = 0

Holding[3] = 0

Next[0] = 1

Next[1] = 2

Next[2] = -1

Next[3] = -1

Last[0] = 2

Last[1] = 0

Last[2] = -1

Last[3] = 0

P0.q = 1

P1.q = 0

P2.q = 0

P3.q = 0

R0.r = 1

R0.p1 = 2

R0.q1 = 2

R1.r = 0

R1.p1 = 0

R1.q1 = 2

R2.r = 0

R2.p1 = 0

R2.q1 = 0

R3.r = 0

R3.p1 = 0

R3.q1 = 0

P0.x in [0,10]

P1.x in [0,10]

P2.x in [0,10]

P3.x in [0,10]

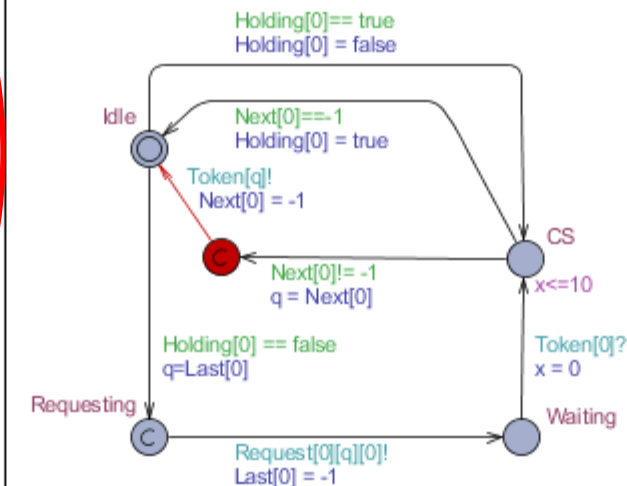
P0.x = P1.x

P1.x = P2.x

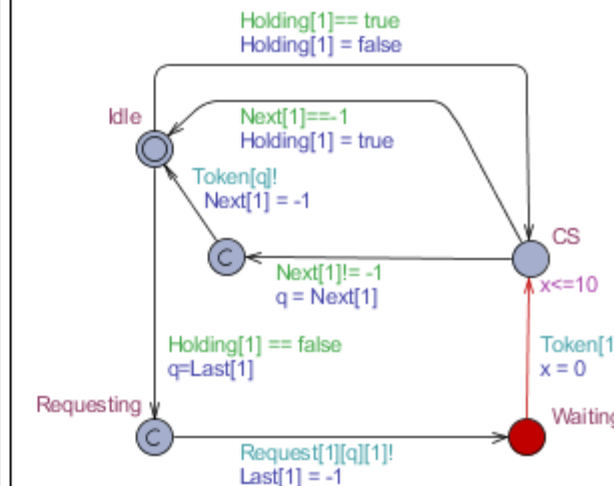
P2.x = P3.x

P3.x = P0.x

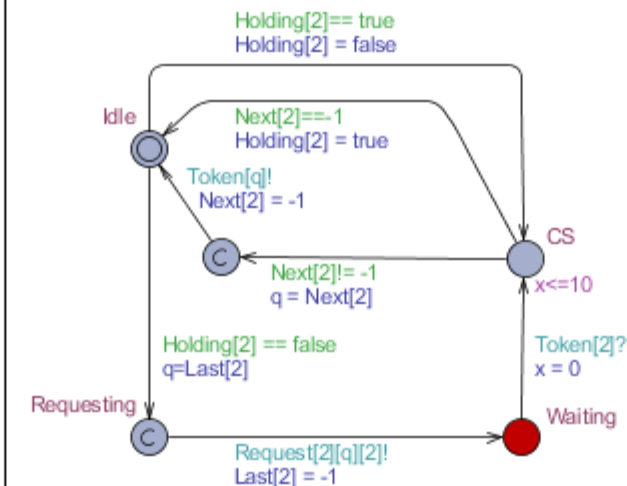
P0



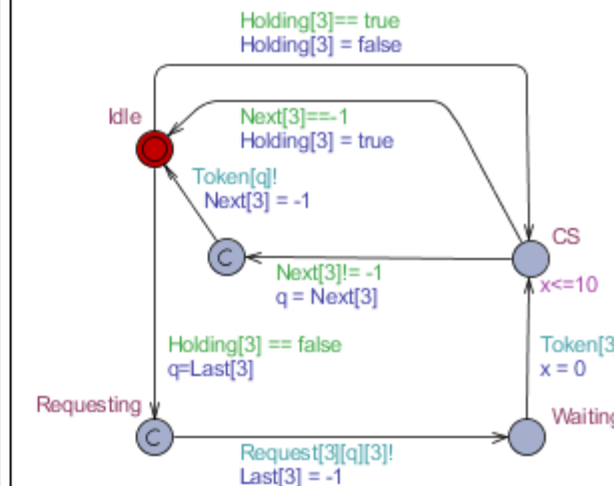
P1



P2



P3



P0

P1

P2

P3

R0

R1

R2

R3



Editor Simulator Verifier

Drag out

Drag out

Enabled Transitions

P0
P1
P3

Holding[0] = 0
Holding[1] = 0
Holding[2] = 0
Holding[3] = 0
Next[0] = -1
Next[1] = 2
Next[2] = -1
Next[3] = -1
Last[0] = 2
Last[1] = 0
Last[2] = -1
Last[3] = 0
P0.q = 1
P1.q = 0

Next

Reset

Simulation Trace

(CS, Waiting, Waiting, Idle, -, Init
R0
(CS, Waiting, Waiting, Idle, -, Init
Request[0][r][q1]: R0 --> R1
(CS, Waiting, Waiting, Idle, Init, -
R1
(CS, Waiting, Waiting, Idle, Init, -
R1
(CS, Waiting, Waiting, Idle, Init, I
P0
(-, Waiting, Waiting, Idle, Init, Ini
Token[q]: P0 --> P1
(Idle, CS, Waiting, Idle, Init, Init,
P3

Trace File:

Prev

Next

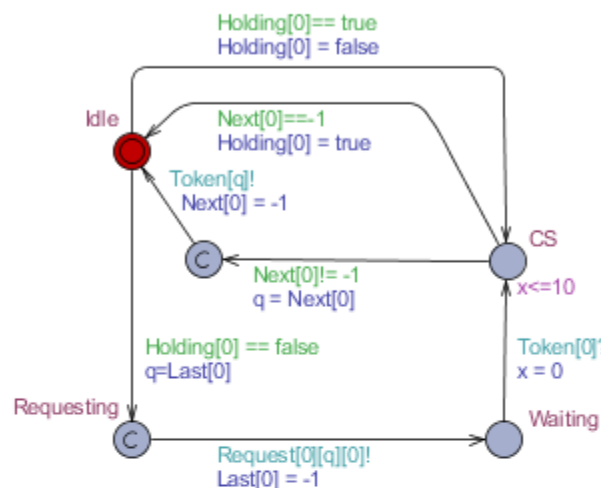
Replay

Open

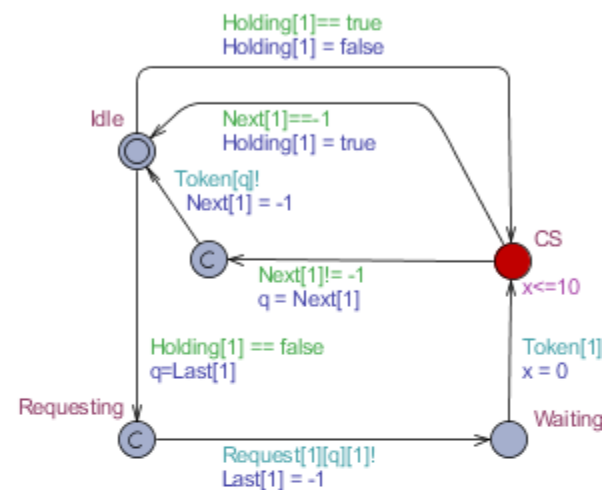
Save

Auto

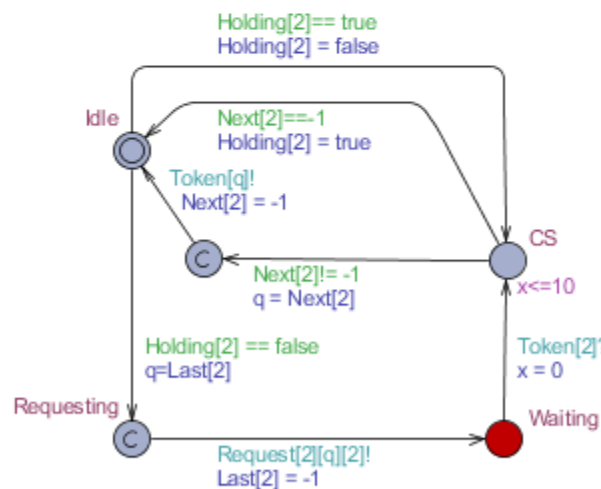
P0



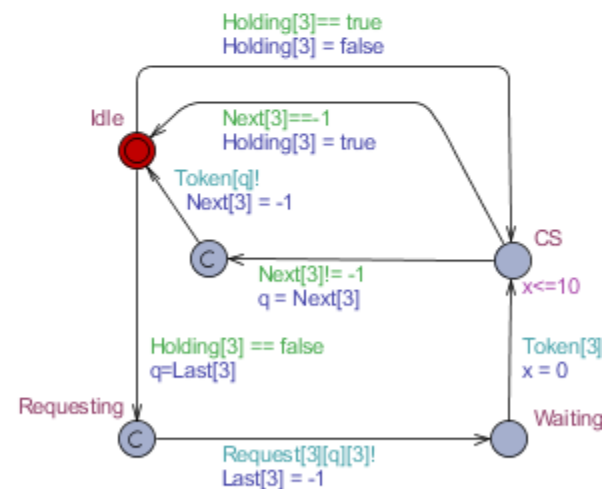
P1



P2



P3



P0

P1

P2

P3

R0

R1

R2

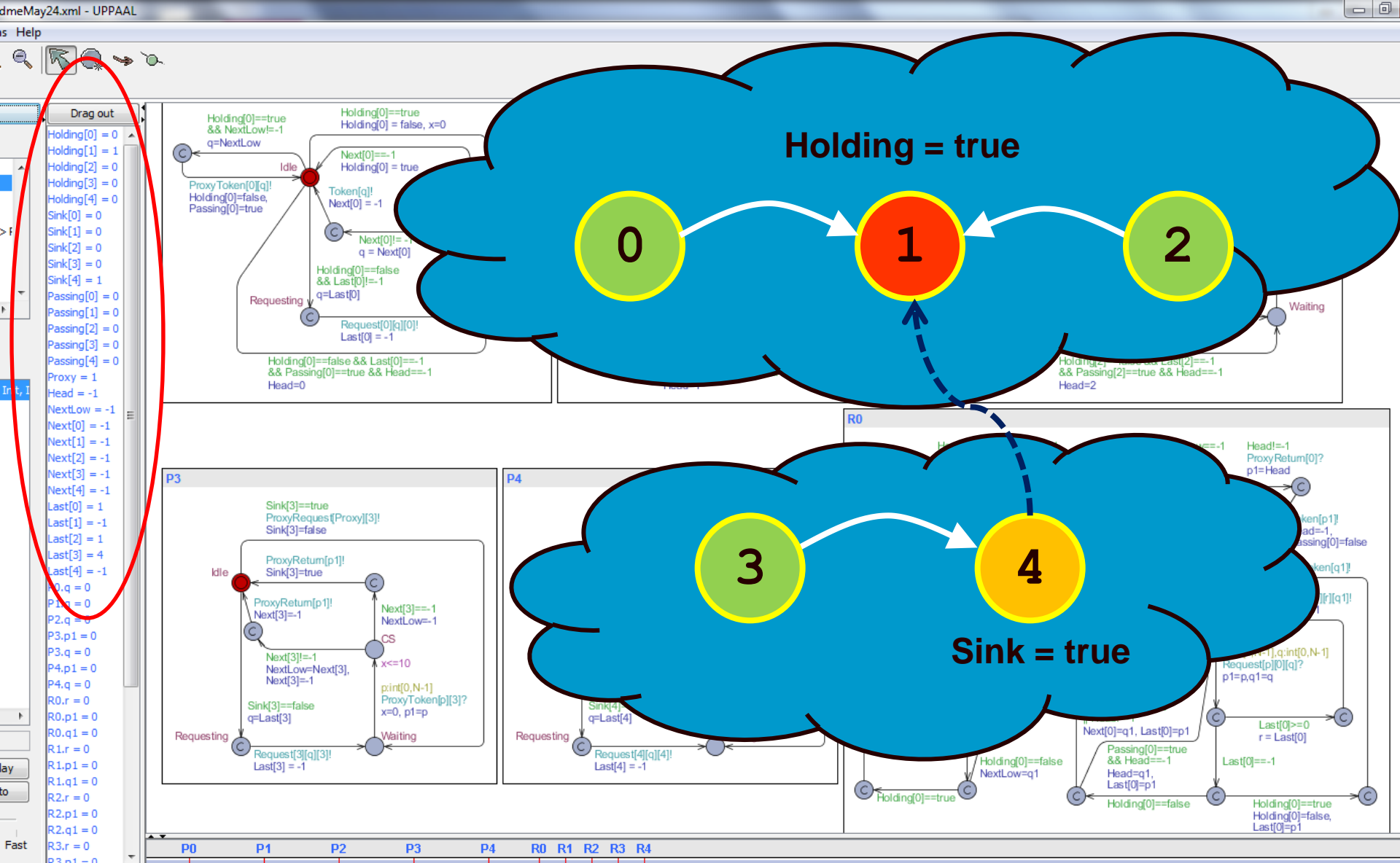
R3

Token[q]

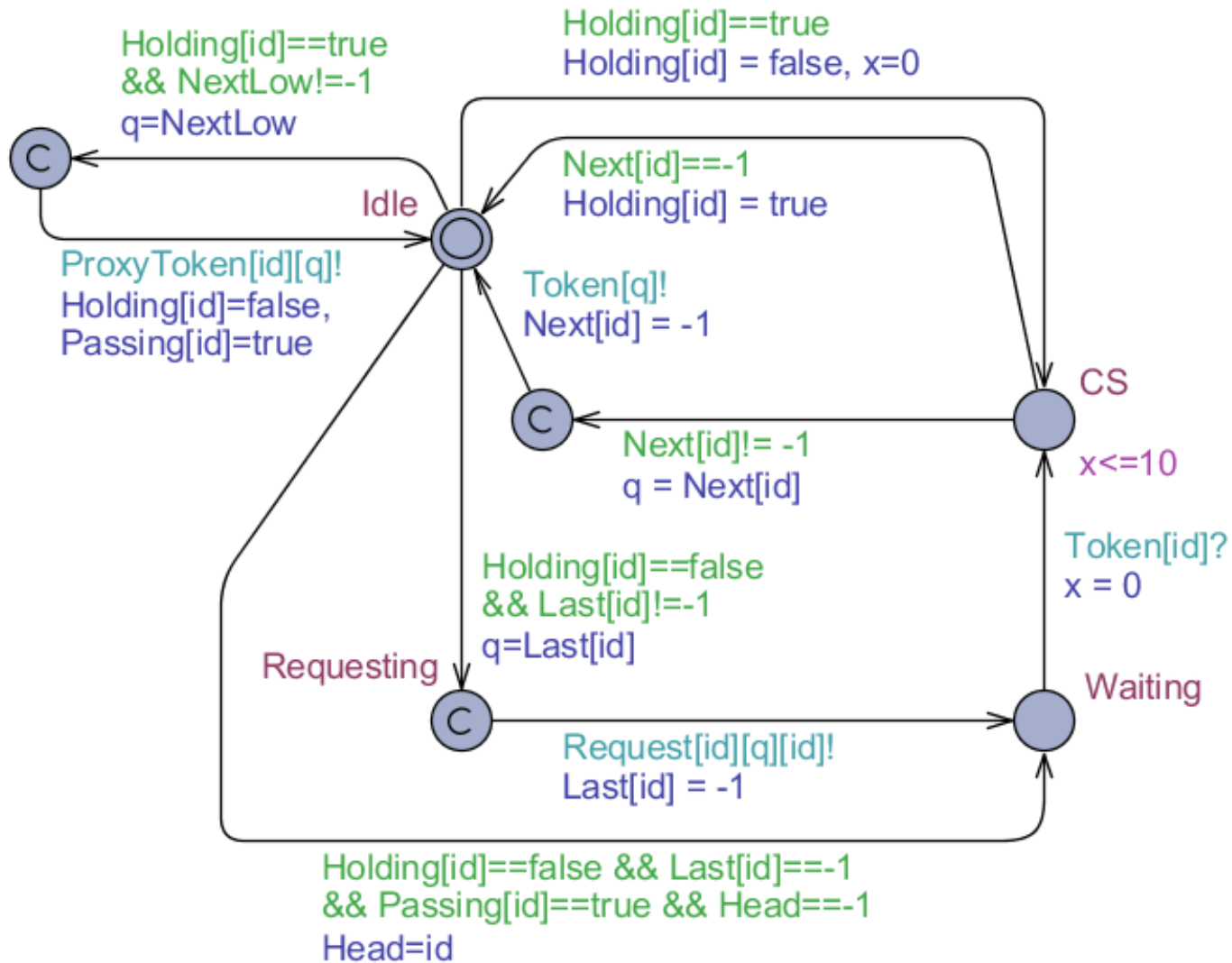
Prioritized Algorithms

- Modeled after real-time scheduling algorithm.
- Each priority level uses distributed algorithm for round-robin processing of requests at the same priority level.
- Proxy requests are passed up to reach a sink node at the highest priority level and to be enqueued in priority order in the token.
- Algorithm correctness is verified using same real-time verification tool, UPPAAL.
- Algorithm performance can also be determined using UPPAAL.

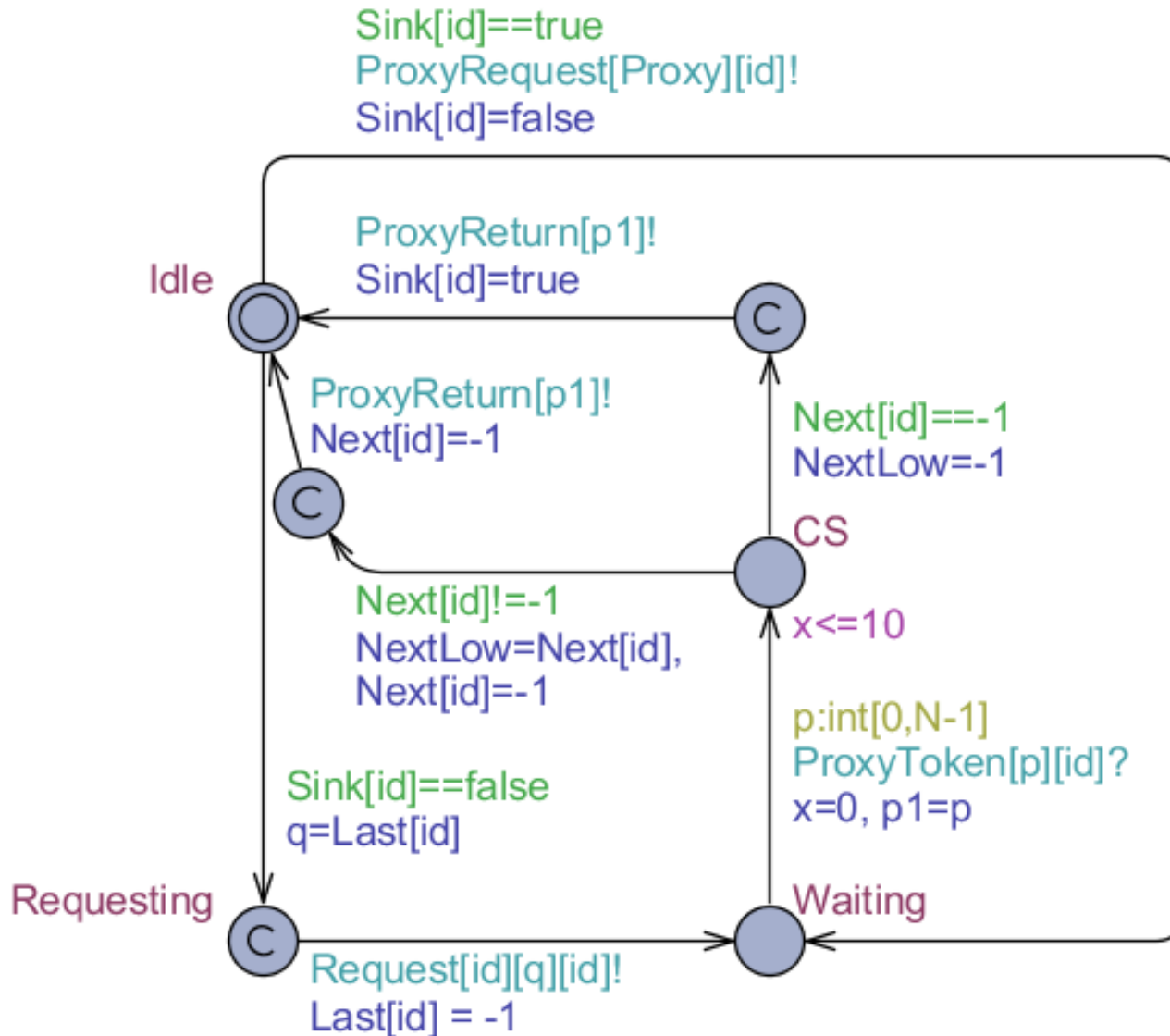
Example



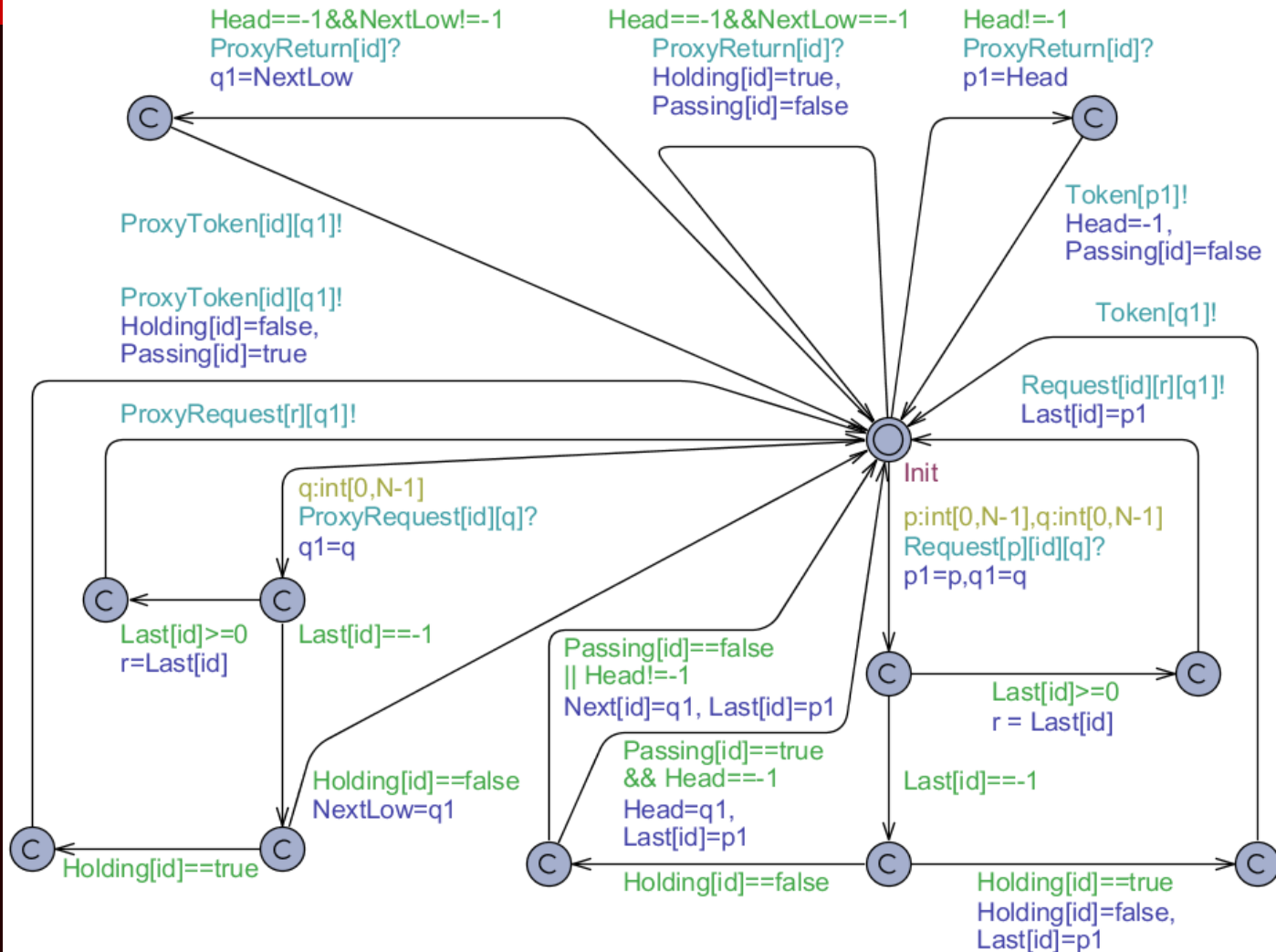
Prioritized ProcessWork(id)

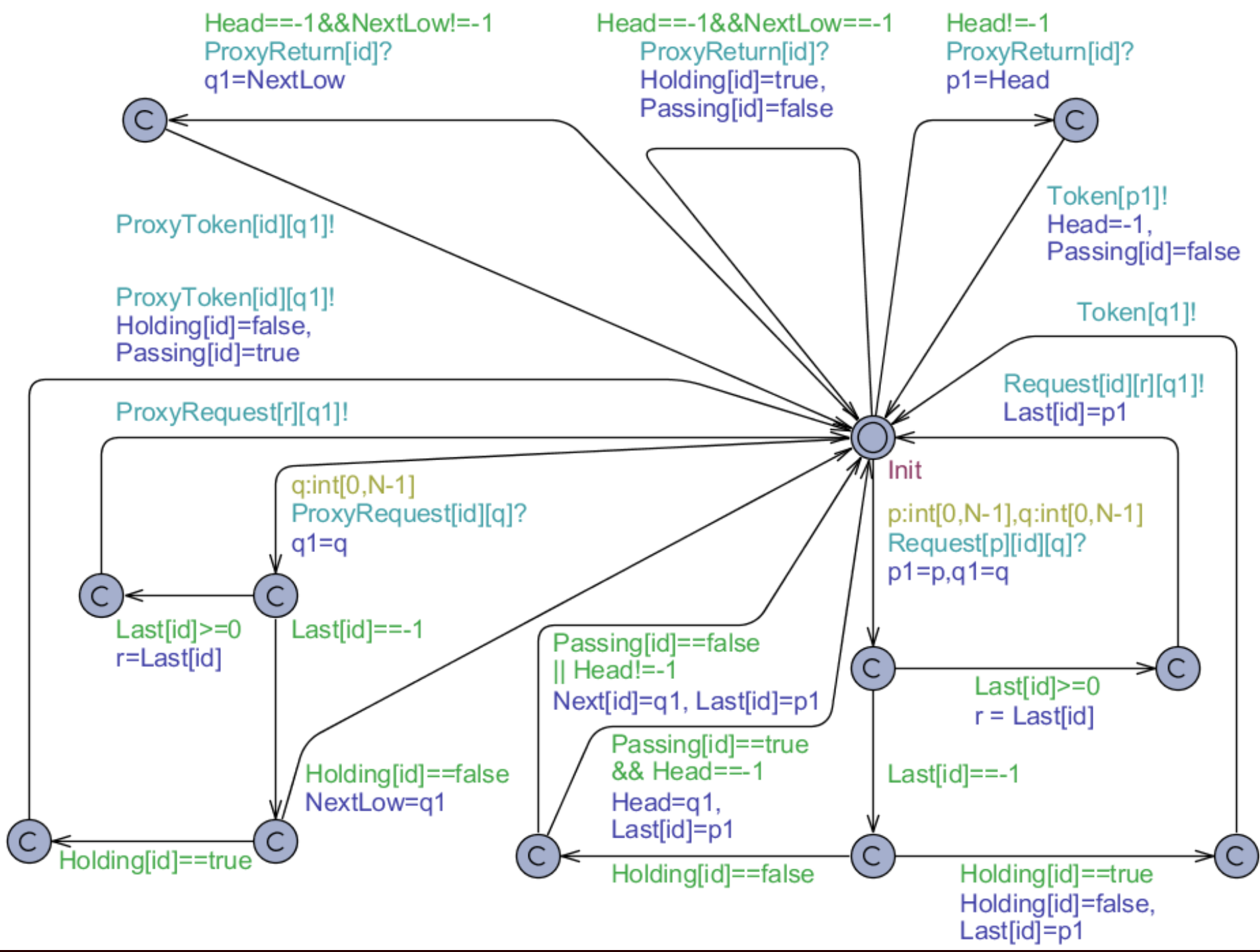


Prioritized ProcessWorkLow(id)

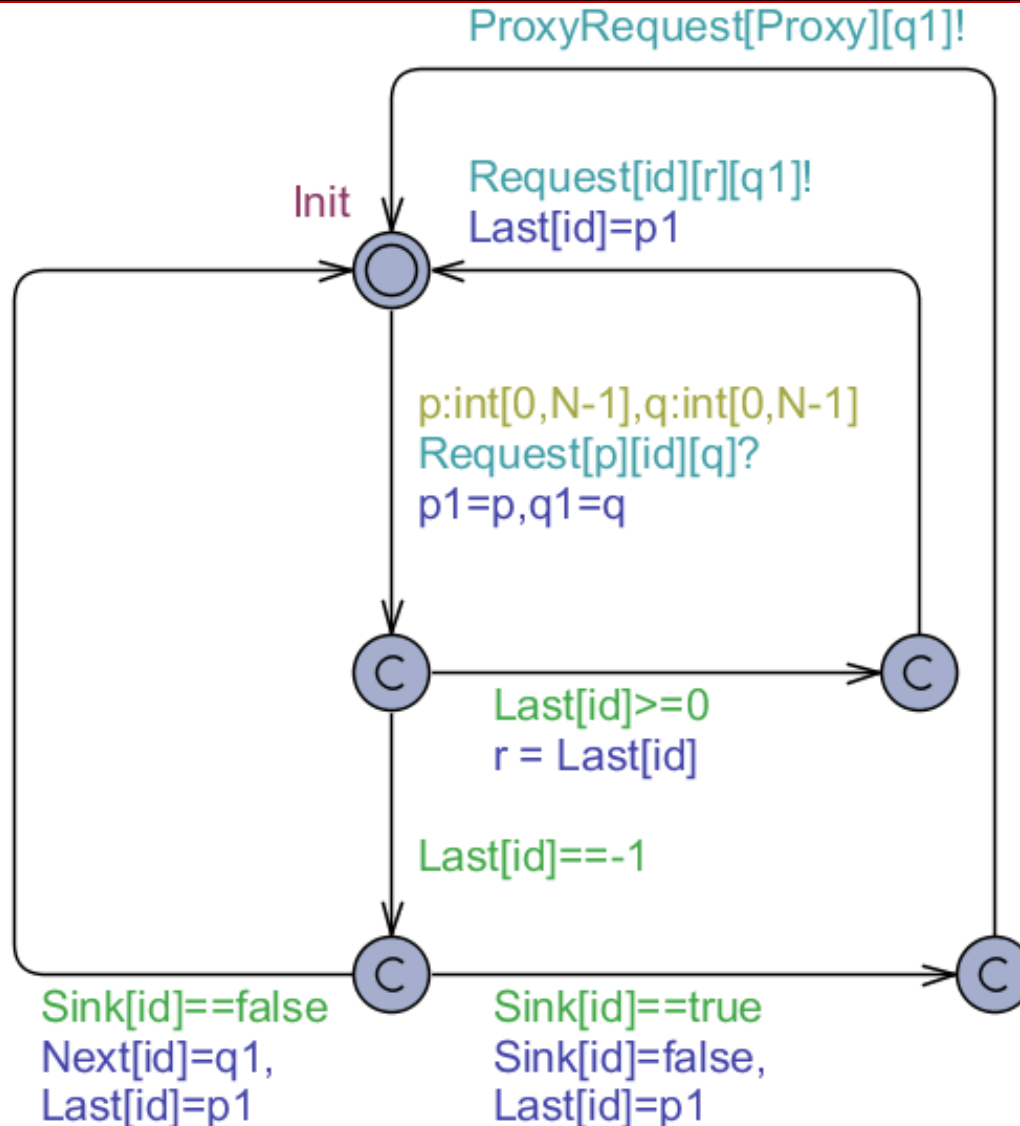


Prioritized ProcessRequest(id)





Prioritized ProcessRequestLow(id)



Example

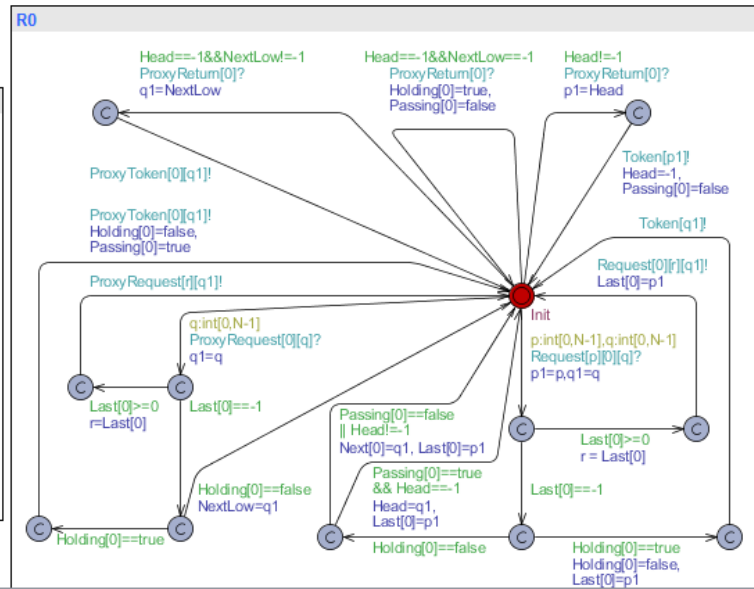
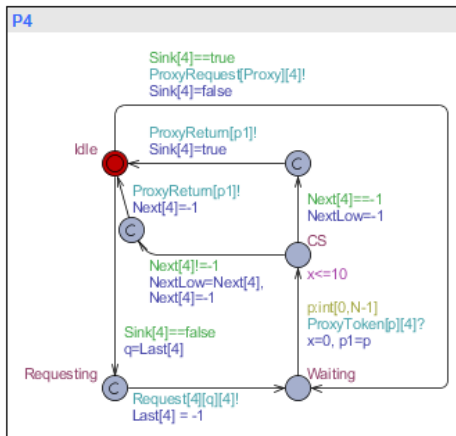
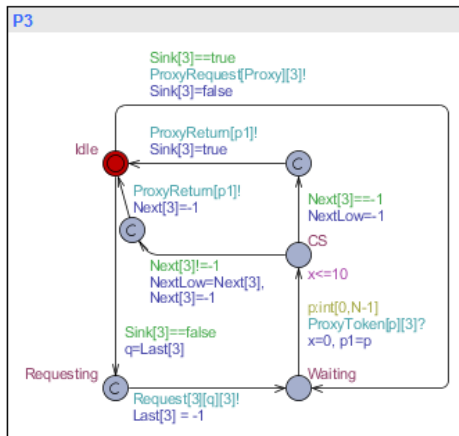
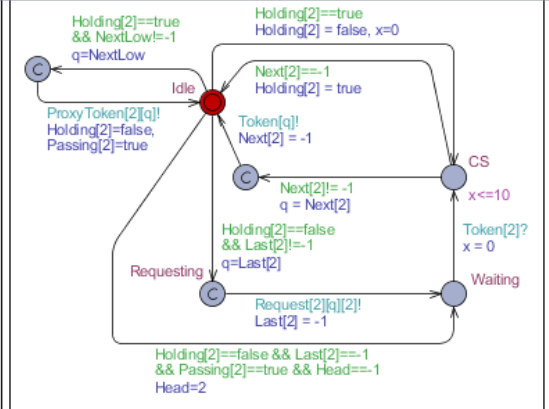
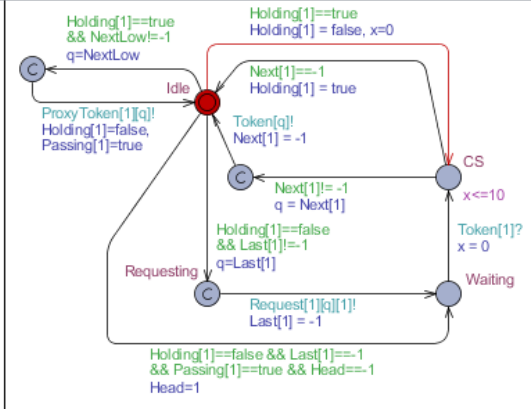
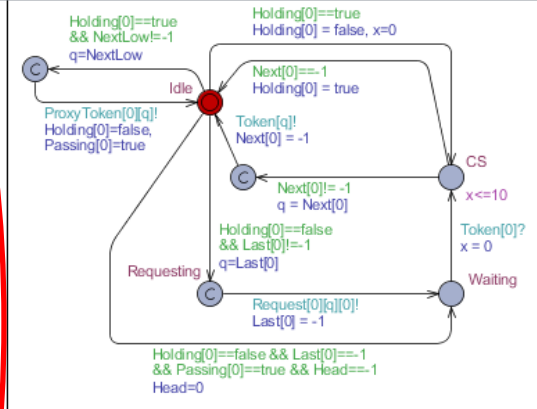
idmeMay24.xml - UPPAAL

ns Help

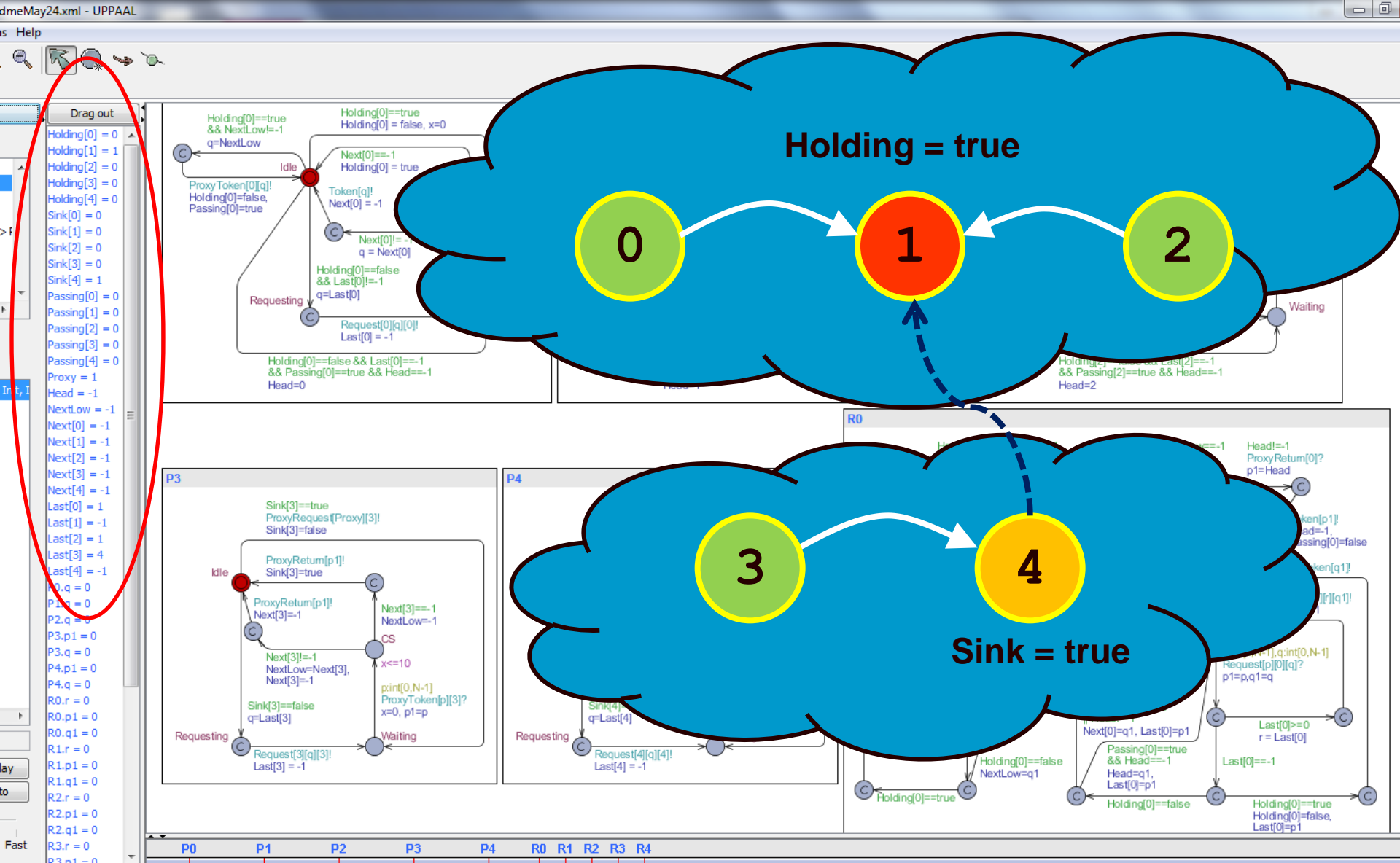


Drag out

Holding[0] = 0
Holding[1] = 1
Holding[2] = 0
Holding[3] = 0
Holding[4] = 0
Sink[0] = 0
Sink[1] = 0
Sink[2] = 0
Sink[3] = 0
Sink[4] = 1
Passing[0] = 0
Passing[1] = 0
Passing[2] = 0
Passing[3] = 0
Passing[4] = 0
Proxy = 1
Head = -1
NextLow = -1
Next[0] = -1
Next[1] = -1
Next[2] = -1
Next[3] = -1
Next[4] = -1
Last[0] = 1
Last[1] = -1
Last[2] = 1
Last[3] = 4
Last[4] = -1
P0.q = 0
P1.q = 0
P2.q = 0
P3.p1 = 0
P3.q = 0
P4.p1 = 0
P4.q = 0
R0.r = 0
R0.p1 = 0
R0.q1 = 0
R1.r = 0
R1.p1 = 0
R1.q1 = 0
R2.r = 0
R2.p1 = 0
R2.q1 = 0
R3.r = 0
R3.p1 = 0
R3.q1 = 0

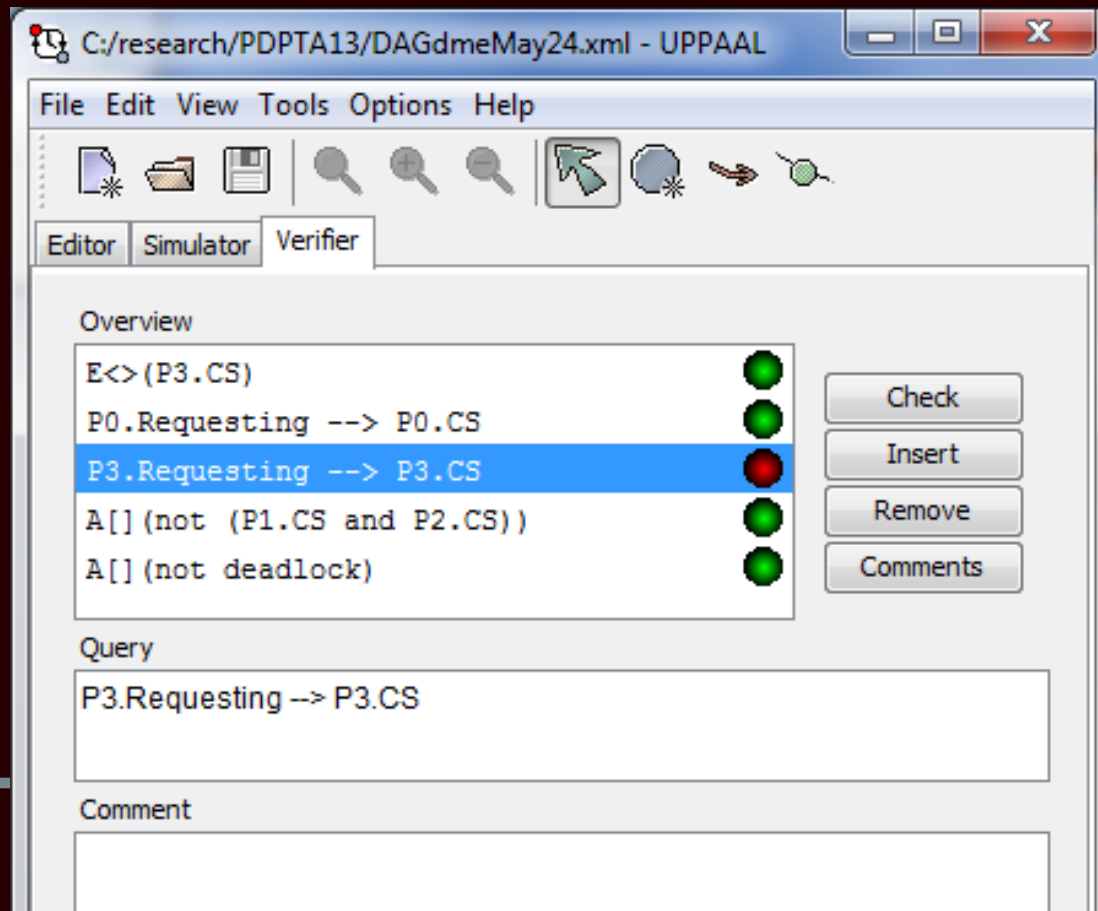


Example



Properties Checked

- Note: Starvation Freedom is not guaranteed for low-priority nodes.



Summary

- Generalized Algorithm generalizes all algorithms that impose a logical structure on the nodes.
- Using a star topology, the dag-based algorithm achieves performance that exceeds any known algorithm, even the centralized algorithm.
- Generalized Algorithm can be prioritized, and correctness is verified.
- Prioritized Algorithm is realized by passing token between priority levels and enqueueing low-priority requests in the token.

Summary

- Quiz #2 – this Friday, 4/25
- Final – take-home
- Project Presentations – start next week