CIS 770: Formal Language Theory

Pavithra Prabhakar

Kansas State University

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Administrivia

Instructional Staff

- Instructor:
  - Pavithra Prabhakar (`pprabhakar`)
- Teaching Assistant:
  - Hongmin Li (`hongminli`)
- Meeting time: Tuesdays and Thursdays at 8:05-9:20 am in Durland Hall 1029
- Office Hours:
  - Pavithra: Tuesdays 10-11:30am or by appointment at 2176 Engineering Hall
  - Hongmin: Wed 8-10am or by appointment at Graduate Student Area

Canvas

- Main source of information/communication about the class
- Check Canvas regularly
- All announcements will be made there
- Lecture notes will be posted after the class
- Homeworks will be uploaded on Canvas, and also the grades
- Quizzes may also be assigned on Canvas.
- Check for exam schedules, class cancellations, etc.

Textbooks

- **Prerequisites:** Students should preferably have taken MATH 510 (Discrete Mathematics), or an equivalent course. Specifically, students are expected to have the following skills:
  - Comprehend and write mathematical definitions.
  - Comprehend and write mathematical proofs.
- **Text-book:** Introduction to the Theory of Computation by Michael Sipser
- **Lecture Notes:** Available on Canvas
- **Additional References**
  - Introduction to Automata Theory, Languages, and Computation: Hopcroft, and Ullman
  - Introduction to Automata Theory, Languages, and Computation: Hopcroft, Motwani, and Ullman
  - Elements of the Theory of Computation: Lewis, and Papadimitriou

Total Grade and Weight

- Homeworks: 20%
- Quizzes: 10%
- Midterms: 40% (2×20)
- Finals: 30%

Grades are curved, but as a rule of thumb, expect that it requires 80 % to earn an A, and 60 % to earn a B.

Homeworks

- One homework every other week: Assigned on Tuesday, either on Canvas or in class, and due the following Tuesday (at 23:59pm).
- You can either submit electronic copy to Canvas (pdf file) or submit paper copy in class or the homework tray in CIS office. Please make sure you include your name, the course number, and if you submit to the CIS office, also the timestamp (day and time).
- No LATE homework. Lowest homework score will be dropped.
- Read the statements regarding Grievances, Academic integrity and expectation for classroom conduct on course website.
- Statement regarding students with disabilities

Quizzes

- One every week. The lowest quiz score will be dropped when calculating the average.
- Will be in class (and maybe online on Compass)
- Goal: To help you ascertain your understanding of the material

Examinations

- First Midterm: March 8 or 10 in class
- Second Midterm: April 7 or 12 in class
- Midterms will only test material since the previous exam
- Final Exam will test all the course material

Three Tales of Computation . . .

The three Tales

- What is the nature of infinity?
- How do we learn language?
- How does the human brain work?

These diverse threads of scientific enquiry led to an understanding of computation.

The first story

Understanding Infinity
One machine to solve them all . . .

Infinitum actu non datur

There are no *actual* infinities; only *potential* infinities.

> *"I protest against the use of infinite magnitude as something completed, which is never permissible in mathematics. Infinity is merely a way of speaking, the true meaning being a limit which certain ratios approach indefinitely close, while others are permitted to increase without restriction." : Gauss*

> *"There are more primes than in any given collection of prime numbers" : Euclid*
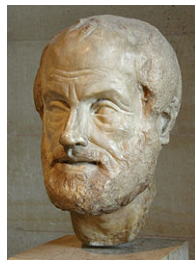
Aristotle

- Laid the foundations of the theory of infinite sets
- Developed the theory of infinite ordinals (numbers)
- Showed how the size of (infinite) sets could be measured
  - Showed there were more real numbers than natural numbers
  - Presaged ideas that would later show that very few problems can actually be solved computationally


Georg Cantor

Devil or Messiah?

- Cantor's work received widespread opposition during his time
  - Theologians saw Cantor's work as a challenge to the uniqueness of absolute infinity in the nature of God
  - Poincaré called it the "great disease" infecting mathematics
  - Kronecker called Cantor a "charlatan", a "renegade", and a "corrupter of youth"!
- But, it also had some supporters

  "No one shall expel us from the Paradise created by Cantor": Hilbert

Crisis in Set Theory
Bertrand Russell (1872–1970)

- Mathematician, philosopher, writer, and political activist who won the Nobel prize in literature!
- Discovered disturbing paradoxes in Cantor's theory
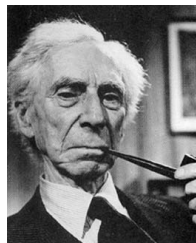
Bertrand Russell

Russell's Paradox

- Riddle: "In a small village, the barber shaves all the men who do not shave themselves (and only them). Does the barber shave himself?"

- Russell: "Consider the set A of all sets that are not members of themselves. Is A a member of itself?"

- Solution to the crises: Formalism that avoids the paradoxes
  - Define concepts precisely
  - Define axioms and rules of inference that can be used to write down formal proofs

David Hilbert

- Euclid's Elements sets out
  - Axioms (or postulates), which are *self evident truths*, and
  - Proves all results in geometry from these truths formally

Euclid of Alexandria

Euclid's Postulates

A1 A straight line can be drawn from any point to any point.

A2 A finite line segment can be extended to an infinite straight line.

A3 A circle can be drawn with any point as center and any given radius.

A4 All right angles are equal.

A5 If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will meet on that side on which the angles are less than two right angles.
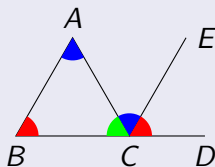
Example of a Formal Proof

Proposition

The interior angles of a triangle sum to two right angles.

Proof.

1. Extend one side (say) BC to D [A2]
2. Draw a line parallel to AB through point C; call it CE [P31]
3. Since AB is parallel to CE, BAC = ACE and ABC = ECD [P29]
4. Thus, the sum of the interior angles = ACB + ACE + ECD = 180° □

Consistency and Completeness

Proof System

Precise definition of what constitutes a proof — each line is an axiom, or is derived from previous lines by rule of inference.

- Correctness of proof reduced to checking if it follows the rules of the system; no ambiguity!

Consistency

A proof system is consistent if it only allows true statements to be proved; no false conclusions.

Completeness

A proof system is complete if every true statement has a proof that adheres to its rules.

- Suggested 23 open problems to be investigated in the 20th century; some remain open to this day!
- One was to obtain a consistent and complete proof system for mathematics — axioms and rules that will allow all (and only) mathematical truths to be proved

- In 1930, at the annual meeting of the Society of German Scientists and Physicians, Hilbert said of his program, "We must know. We will know."
- At one of the satellite conferences of the same meeting, Kurt Gödel pronounced that Hilbert's program was fated to fail!
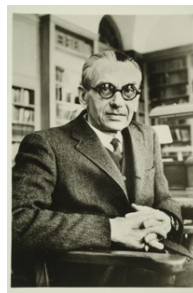
Kurt Gödel

Incompleteness Theorem: Gödel showed that given any consistent proof system for number theory, one can construct a statement about numbers that is true but cannot be proved!

- Relied on the Liar's Paradox which says "This statement is false."
- To get incompleteness, Gödel constructed the statement "This statement is unprovable" inside number theory!
- Proof is a hacker's dream!

Mechanized Computation

- The heart of the axiomatic method, is that proof correctness reduces to mechanical checking.
- But what is mechanical checking?
- What are limits of mechanical computation?

The Computer Revolution
Alonzo Church, Emil Post, and Alan Turing (1936)

Alonzo Church   Emil Post   Alan Turing

- Church (λ-calculus), Post (Post's machine), Turing (Turing machine) independently come up with formal definition of mechanical computation that are equivalent
- Discovered problems that cannot be solved computationally

Berry's Paradox: Russell

> *"The smallest natural number that cannot be defined in less than ninety characters"*

- The above sentence describes a number as there are only finitely many strings with less than 90 characters
- The sentence itself has only 81 characters, and describes a number that cannot be described by less than 90 characters!

Busy Beaver Function

- Simple Program: A program (in say C) that takes no inputs, computes, prints some number and stops.
- Busy Beaver Function: $BB(n)$ is the largest number printed by a simple program of less than n bytes

An uncomputable function!

Theorem

There is no C program that given number n computes BB(n).

- Suppose (for contradiction) there is a C function compBB to compute $BB(n)$. Let compBB be of size m bytes.

- Consider the following program:

```
int compBB (int n) {
    ......
}
void main() {
    printf(''%d'',compBB(5000+2*m)+1);
}
```

- Size of above program $\leq m + 5000 + \log m$ but prints a number $> BB(5000 + 2 * m)$! Contradiction!

Spectacular Failure of Hilbert's Program

Uncomputable functions provide more examples of incompleteness

- Suppose there is a consistent proof system that for every value of n proves that $BB(n) = k$ for some k.
- The function compBB(n) simply goes through all strings one by one, till it comes across a string that is a correct proof of $BB(\text{n}) = \text{k}$.

The second story

Understanding Language
One grammar to generate it all . . .

The Problem of Language Acquisition

- Language is an important human cognitive process that allows us to share information, thoughts and subjective experiences
- Though language is complex, it is acquired and used skillfully by children
- What is the mechanism behind its acquisition and use?

Behaviorism

- All things that organisms do — actions, thinking, feeling — are behaviors, in response to sensory input
- Behaviors are the only measurable things
- Scientific description of such behavior should not rely on internal physiological events or hypothetical constructs such as the mind

- The child's mind is a blank slate, and language is learned
- The learning process is a gradual change based on sensory input provided to the organism
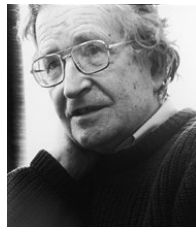- Thinking is a form of "verbal behavior"
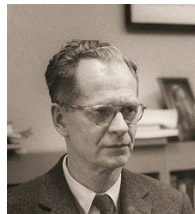
B.F. Skinner

Critique of Verbal Behavior
Noam Chomsky (1928–)

Behaviorist account is flawed because the underpinnings of natural language are highly abstract principles, and children acquire language without explicit instruction or environment clues to these principles.

Noam Chomsky

"A person capable of producing sentences with up to 20 words, can deal with at least 10^{20} sentences. At the rate of 5 seconds per sentence, she would need 100 trillion years (with no time for eating or sleeping) to memorize them.": Pinker

Humboldt: "infinite use of finite media"
Recursive build-up

- The longest English sentence in the Guinness Book of World Records is a 1,300 word sentence in Faulkner's Absalom, Absalom! that begins "They both bore it as though ..."

- Steven Pinker thought of submitting the following record breaker: Faulkner wrote, "They both bore it as though ..."

- But that could be easily broken by: Pinker wrote that Faulkner wrote, "They both ..."

Grammatical Correctness independent of Cognition
Ungrammatical Comprehensible Sentences

- The child seems sleeping.
- It's flying finches, they are.
- Sally poured the glass with water.
- Who did a book about impress you?
- This sentence no verb.

- Chomsky: "Colorless green ideas sleep furiously."
- Edward Lear: "It's a fact the whole word knows, That Pobbles are happier without their toes."

The key to language acquisition is learning a Generative Grammar for the language that describes

- Word categories (like nouns, verbs, etc.)
- Rules determining how categories are put together

Chomsky's theory: A core Universal Grammar is innate to all humans.

Chomsky Hierarchy

Chomsky found grammars of different complexity conveniently describe various aspects of languages. Echoed in compiler design

- Lexical tokens described using "regular expressions"
- Language syntax described using "context-free grammars"

The third story

Understanding the Brain
One machine to think it all ...

The Human Brain

- Central organ in the body that controls and regulates all human activity
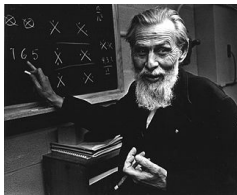- Believed the seat of "higher mental activity": thought, reason and abstraction
- How does it work?

- Neurons are the primary functional unit of the brain and the central nervous system
- Neurons receive information at dendrites and transmit via axons
- They communicate with each other through junctions called synapses
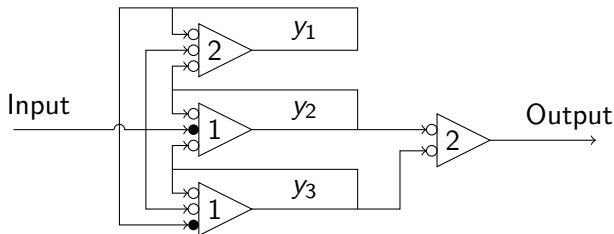
Santiago Ramón y Cajal

Warren Sturgis McCullough

Walter Pitts

- Came up with a mathematical model of a neuron
- Motivation to compare the computational power of networks of neurons with Turing Machines

Simple Neural Net

A Simple Neural Net

- Neurons have excitatory (circles) or inhibitory (dots) synapses
- Neuron produces 1 if the number of excitatory synapses with 1-input exceeds the number of inhibitory synapses with 1-input by at least the threshold of the neuron (number inside triangle)

Finite State Automata

- Finite automata model introduced by Huffman (1954), Moore (1956) and Mealy (1955) to model sequential circuits
- Kleene, 1956: Neural Nets of McCullough-Pitts the same as Finite Automata

Michael Rabin

Dana Scott

- Introduced the notion of Nondeterministic Finite Automata
- Understanding the power of nondeterministic computation has remained a fundamental problem since then

Three Tales of Computation

- What is the nature of infinity?
- How do we learn language?
- How does the human brain work?

Computation underlies all these diverse scientific enquiries and they laid the foundations of a theory that we will explore.

Understand the nature of computation in a manner that is independent of our understanding of physical laws (or of the laws themselves)

- Its a fundamental scientific question
- Provides the foundation for the science of computationally solving problems

Problems through the Computational Lens

Mathematical problems look fundamentally different when viewed through the computational lens

- Not all problems equally easy to solve — some will take longer or use more memory, no matter how clever you are
- Not all problems can be solved!
- The "complexity" of the problem influences the nature of the solution
  - May explore alternate notions of "solving" like approximate solutions, "probabilistically correct" solutions, partial solutions, etc.

Course Overview

The three main computational models/problem classes in the course

Computational Model	Applications
Finite State Machines/ Regular Expressions	text processing, lexical analysis, protocol verification
Pushdown Automata/ Context-free Grammars	compiler parsing, software modeling, natural language processing
Turing machines	undecidability, computational complexity, cryptography