Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 6

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Name: ______________________________

Due date: Thursday March 7th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. Find a 2×2 lower-triangular matrix L such that

$$L \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

2. Graphs and websites. The following 8×8 binary matrix A describes the connectivity of 8 websites, that is, $A_{ij} = 1$ if there is a link from website i to website j, and $A_{ij} = 0$ otherwise.

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A =

     1     0     1     0     0     1     0     0
     1     0     0     0     0     0     0     1
     0     0     1     1     0     1     0     0
     1     0     0     1     1     1     0     1
     0     0     0     0     1     1     1     1
     0     1     0     0     0     1     0     1
     0     0     1     1     1     1     0     0
     0     1     1     0     1     1     0     0
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What is the least number of clicks required to get from website 1 to website 7? What website can be most accessed by clicking twice?

3. A video store has two locations: Downtown (D) and Suburbs (S). The company has 2000 videos that can be rented and returned to either location. Any videos that are rented are returned the next day without fail. A video rented at D has a 90% chance of being returned to location D the morning after. A video rented at S has an 80% chance of being returned to location S the morning after. Find the step matrix for the discrete dynamical system that models the distribution of videos from day to day. Assuming that the videos are split equally between the locations on day 0, find the distribution of videos between the locations after 4, 8, and 10 days. Find the stationary distribution vector $u = [u_1, u_2]'$ that describes the distribution of videos in the long run.

4. Determine the cubic function $y = a_0 + a_1x + a_2x^2 + a_3x^3$ that passes through the points $(-1, 1)$, $(0, 2)$, $(1, 1)$, and $(2, 4)$.

5. Find bases for the column space, the row space, and the null space of the matrix

$$N = \begin{bmatrix} 2 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}.$$

Indicate the corresponding dimensions of those subspaces.

6. Consider the vectors

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \\ -2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 1 \\ -2 \\ 3 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 4 \end{bmatrix} \quad \text{and} \quad v_5 = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

Answer the following questions (Justify and show your work in each case)

(i) Is v_3 a linear combination of v_1, v_2, and v_4?

(ii) Is the set $\{v_2, v_3, v_4, v_5\}$ linearly independent?

(iii) Does v_4 belong to $span(v_1, v_3, v_5)$?

(iv) Does $v_1 + v_3$ belong to $span(v_2 + v_1, v_3)$?

(v) Which scalars x_1, x_2, x_3, x_4 satisfy the equation

$$x_1 v_3 + x_2 v_2 + x_3 v_5 + x_4 v_1 = v_4?$$

7. Consider an economy with three industries: electricity, oil, and pipeline. For each dollar of electricity generated there are charges of 20 cents for electricity to run auxiliary equipment, 40 cents for oil to power the generators, and 10 cents in pipeline usage. For each dollar of oil produced there are 10 cents spent on electricity and 40 cents for oil to produce steam that is pumped into the well. Finally, for each dollar in pipeline usage there are 30 cents spent on electricity and 20 cents is spent on oil to heat the pipeline. In addition, suppose that there is an outside demand for \$12600 worth of pipeline usage, \$4200 worth of electricity, and \$84000 worth of oil. Is this an open or a closed economy model? Identify the consumption matrix and find the production schedule.

8. True or False - **Circle the right one** (1 point each)

T or **F**. There exist real numbers x_1, x_2, x_3, and x_4 such that

$$x_1 \begin{bmatrix} 2 \\ 1 \\ 4 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 1 \\ 4 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 1 \\ 2 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 6 \\ 2 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

T or **F**. If $Ax = b$ then the vector b is a linear combination of the columns of A.

T or **F**. Given an $m \times n$ matrix B, the system

$$Bx = 0,$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad 0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (m \text{ zero entries}),$$

has *at least* one solution. In other words, any $m \times n$ homogeneous system always has at least one solution.

T or **F**. If P and Q are 3×3 matrices with $rank(P) = rank(Q) = 3$, then $rank(P+Q) = 3$.

T or **F**. If the system $Mx = 0$ has exactly one solution, the columns of M are linearly independent.

Points obtained in this assignment (out of 16): ____________