Math 321

~~Q's/~~ Exam

$f : A \to B$

4a $f(n) = \sqrt{n^2 - 2}$ $\qquad f : \mathbb{Z} \to \mathbb{R}$

$f(0) = \sqrt{-2} \notin \mathbb{R}$

~~$\sqrt{x}$ gives 2 ans.~~

4b

$\exists b$

$$\begin{tikzcd}
a \arrow[r, "g"] \arrow[rr, bend right, "f \circ g"'] & b \arrow[r, "f"] & c
\end{tikzcd}$$

$f(n) = n$ $\qquad f : \mathbb{Z} \to \mathbb{N}$

not a function.

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \;\ldots \\ \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

map pos. to $0, 2, 4, 6, \ldots$ $\qquad 2n$

map neg to $1, 3, 5, 7, \ldots$ $\qquad |2n+1|$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ -1 & -2 & -3 & -4 \;\ldots \end{array}$$

Th^m: primes are infinite

Pf: assume finite. $P = \{p_1, p_2, \dots, p_n\}$

let $Q = p_1 \cdot p_2 \cdots p_n + 1$

By Fund. th^m Q is prime or product of primes.

Case 1 Q is prime. Contradiction.

Case 2 Q is product of primes.

what prime is a factor (divides it)?

$$p^* \mid Q$$

but.. if any $p_i \in P$ $p_i \mid Q$

we also know $p_i \mid p_1 \cdot p_2 \cdots p_n$

$\rightarrow p_i \mid Q - p_1 \cdot p_2 \cdots p_n$

$\rightarrow p_i \mid 1$

so.. $p^* \notin P$ contradiction.

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we used:

Corr

$$a \mid b \wedge a \mid c$$

$$\rightarrow a \mid (m \cdot b + n \cdot c)$$

Density. **Def:** $\pi(x)$ = number of primes $\le x$ ex $\pi(10) = 4$

$\boxed{\text{Th}^{m}}$ $$\lim_{x\to\infty} \frac{\pi(x)}{x/\ln x} = 1$$

⇝ for large x $\quad \pi(x) \approx \dfrac{x}{\ln x}$

ex: $$\pi(10^{15}) \approx \frac{10^{15}}{\ln(10^{15})} = \frac{10^{15}}{\underset{\approx 10^{2}}{15}\cdot\ln(10)}$$

$$\pi(10^{15}) \approx \frac{10^{13}}{\ln(10)} \approx 5\times 10^{12}$$

1,000,000,000,000,000

% $\quad \dfrac{x/\ln x}{x} = \dfrac{1}{\ln x}$ % of primes below x.

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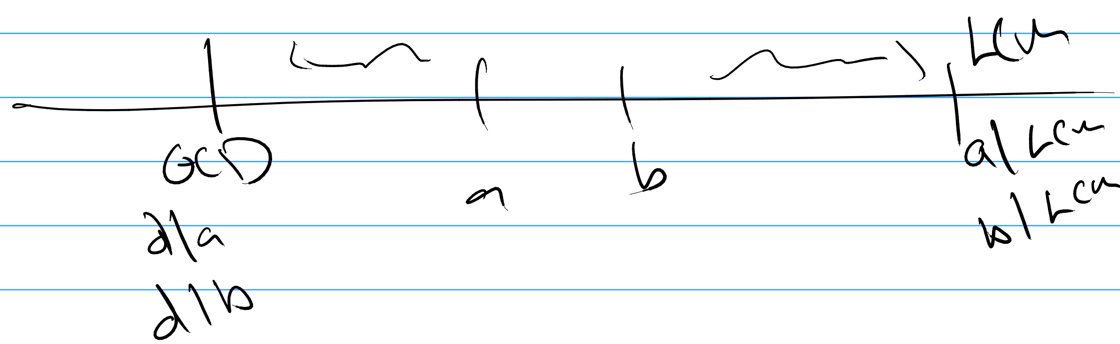
gcd(a,b) = greatest common divisor

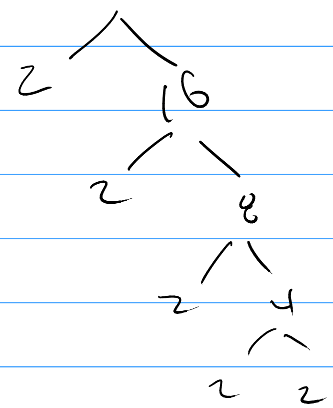
lcm(a,b) = least common multiple.

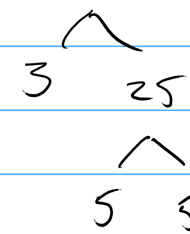


$$a = p_1^{a_1} \cdot p_2^{a_2} \cdot \ldots \cdot p_n^{a_n}$$
$$b = p_1^{b_1} \cdot p_2^{b_2} \cdot \ldots \cdot p_n^{b_n}$$

the unique prime factorization

$a = 32 = 2^5$ $\qquad$ $b = 75 = 3 \cdot 5^2$





$a = 2^5 \cdot 3^0 \cdot 5^0$ $\qquad$ $b = 2^0 \cdot 3^1 \cdot 5^2$

$$\gcd(a,b) = p_1^{\min(a_1,b_1)} \cdot \ldots \cdot p_n^{\min(a_n,b_n)}$$

$$\gcd(32,75) = 2^0 \cdot 3^0 \cdot 5^0 = \boxed{1}$$

$$\text{LCM}(a,b) = p_1^{\max(a_1,b_1)} \cdot \ldots \cdot p_n^{\max(a_n,b_n)}$$

$$\text{LCM}(32,75) = 2^5 \cdot 3^1 \cdot 5^2 = 75 \cdot (30+2)$$

$$2250 + 150$$

$$\boxed{2400}$$

Def: $\gcd(a,b)=1$ they are called relatively prime.

Note: $\mathbb{Q} = \{ \frac{a}{b} \mid a \in \mathbb{Z} \wedge b \in \mathbb{Z} \wedge b \neq 0 \wedge \gcd(a,b)=1 \}$

$n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_1 b^1 + a_0 b^0$

$b, n \in \mathbb{Z}^+ \wedge (b > 1) \wedge k = \{0, 1, 2, \ldots\}$

$\wedge (a_i < b) \wedge (a_k \neq 0)$

$1023 = 1 \cdot 10^3 + 0 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0$

$123 = 1 \cdot 60^2 + 2 \cdot 60^1 + 3 \cdot 60^0$

$123. = 1 \cdot 60^3 + 2 \cdot 60^2 + 3 \cdot 60 + 0 \cdot 60^0$