Math 321

Q's ~~f~~ Useful Math Software

1. Numerics
   - → Scilab
   - → octave
2. Symbolic
   - → Maxima
   - → yacas
3. Stats
   - → r-project

Show $a \cdot b = \gcd(a,b) \cdot \operatorname{lcm}(a,b)$

$$\gcd(a,b) = p_1^{\min(a_1,b_1)} \cdot p_2^{\min(a_2,b_2)} \cdot \cdots \cdot p_n^{\min(a_n,b_n)}$$

$$\operatorname{lcm}(a,b) = p_1^{\max(a_1,b_1)} \cdot p_2^{\max(a_2,b_2)} \cdot \cdots \cdot p_n^{\max(a_n,b_n)}$$

$$\gcd(a,b) \cdot \operatorname{lcm}(a,b) =$$

$$\underbrace{p_1^{\min(a_1,b_1)} \cdot p_1^{\max(a_1,b_1)}} \cdot \underbrace{p_2^{\min(a_2,b_2)} \cdot p_2^{\max(a_2,b_2)}} \cdot \cdots \cdot \underbrace{p_n^{\min(a_n,b_n)} \cdot p_n^{\max(a_n,b_n)}}$$

$$= p_1^{(\min(a_1,b_1) + \max(a_1,b_1))} \cdot \cdots \cdot p_n^{\min(a_n,b_n) + \max(a_n,b_n)}$$

$$= p_1^{a_1+b_1} \cdot p_2^{a_2+b_2} \cdot \cdots \cdot p_n^{a_n+b_n}$$

$$= p_1^{a_1} p_1^{b_1} p_2^{a_2} p_2^{b_2} \cdots p_n^{a_n} p_n^{b_n}$$

$$= \left(p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n}\right) \cdot \left(p_1^{b_1} p_2^{b_2} \cdots p_n^{b_n}\right)$$

$$= a \cdot b$$

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36 $n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_2 b^2 + a_1 b^1 + a_0 b^0$

base b expansion of n

(base 1) 7 → |||||||

|||||||||||||||
|||||||||| → 25 =

Money.

5¢

10¢

25¢

50¢

$1

$2(10\text{'s}) + 5(1\text{'s})$

343

17 21 11

positional numbers

$$n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_2 b^2 + a_1 b^1 + a_0 b^0$$

$b = \{2, 3, \dots\}$
$n = \{1, 2, 3, \dots\}$
$K = \{0, 1, 2, \dots\}$
$a_i \in \{0, 1, 2, \dots, b-1\}$
$a_k \neq 0$

base 10

$$\underline{7} \cdot 10^4 + \underline{0} \cdot 10^3 + \underline{3} \cdot 10^2 + \underline{1} \cdot 10^1 + \underline{6} \cdot 10^0$$

$$(7, 0, 3, 1, 6)_{10}$$

ex

$$(1, 0, 0, 1, 1, 1, 0)_2 = 64 + 8 + 4 + 2 = (7, 8)_{10}$$

$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$$(23, 0, 4, 59)_{60} = 23 \cdot 60^3 + 0 \cdot 60^2 + 4 \cdot 60 + 59$$

$60^3 \quad 60^2 \quad 60^1 \quad 60^0 = 1$

$= ?$

$$\overset{5\ 11}{1,261} = (\ ?\ )_4 = 1 \cdot 4^5 + 237$$

1024   192

$$= (1, 0, 3, 2, 3, 1)_4 \qquad = 1 \cdot 4^5 + 0 \cdot 4^4 + 3 \cdot 4^3 + 45$$

1024

$4^5 \quad 4^4 \quad 4^3 \quad 4^2 \quad 4^1 \quad 4^0 = 1$

$$= 1 \cdot 4^5 + 0 \cdot 4^4 + 3 \cdot 4^3 + 2 \cdot 4^2 + 13$$

1024   256   64   16   4

Operations:

$$\begin{array}{r} 1 \\ (2,1,3)_4 \\ +\ (2,0,3)_4 \\ \hline (1,0,2,2)_4 \end{array}$$

$6 = (1,2)_4$

$$\begin{array}{r} 12 \\ *\ 61 \\ \hline 12 \\ 720 \\ \hline 732 \end{array}$$

$(60+1)(10+2)$

$$\begin{array}{r} (2,0,1)_3 \\ *\ (1,2)_3 \\ \hline (1,1,0,2)_3 \\ +\ (2,0,1,0)_3 \\ \hline (1,0,1,1,2)_3 \end{array}$$

div. vs mod

$a = b \cdot q + r$

q = a div b r = a mod b

by Sub.

$23 = a \quad 4 = b$

$23 = 4 \cdot 0 + 23$ -4

$23 = 4 \cdot 1 + 19$ -4

$23 = 4 \cdot 2 + 15$ -4

$23 = 4 \cdot 3 + 11$ -4

$23 = 4 \cdot 4 + 7$ -4

$23 = 4 \cdot \boxed{5} + \boxed{3}$

$\frac{23}{4} = 5.$ something.

$23 \text{ div. } 4 = \lfloor 23/4 \rfloor$

$23 \bmod 4 = 23 - 4 \cdot \lfloor 23/4 \rfloor$

Euclidean Alg.

if $a = bq + r \rightarrow \gcd(a,b) = \gcd(b,r)$

$\gcd(24, 14) = 2$

$24 = 14 \cdot 1 + 10$

$14 = 10 \cdot 1 + 4$

$10 = 4 \cdot 2 + \boxed{2}$ ← gcd

$4 = 2 \cdot 2 + \underline{0}$