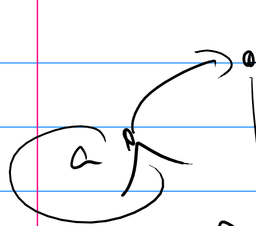
Math 322



$\boxed{\text{digraph}}$ $R = \{(a,b), (a,a)\}$

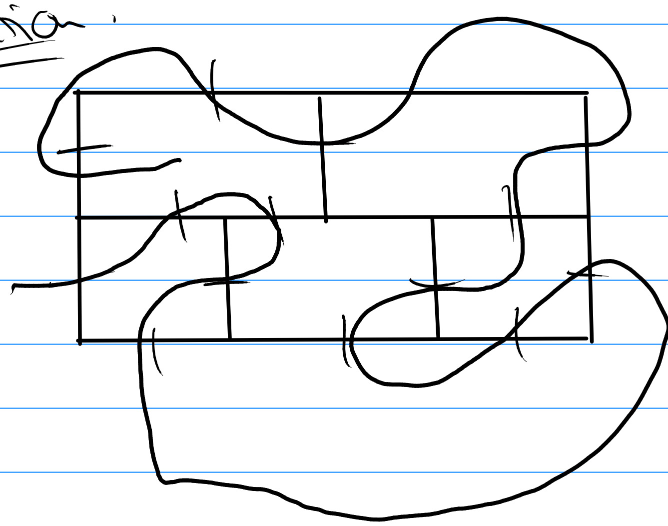
Generalize + Study $\longrightarrow$ Graph Theory

$G = (V, E)$

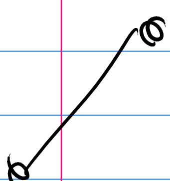
Types: Undirected vs. Directed

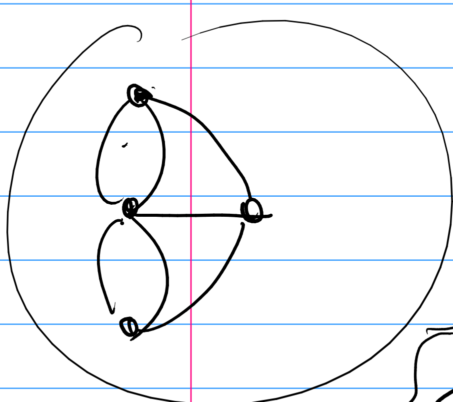
edge $= \{a,b\}$ edge $= (a,b)$

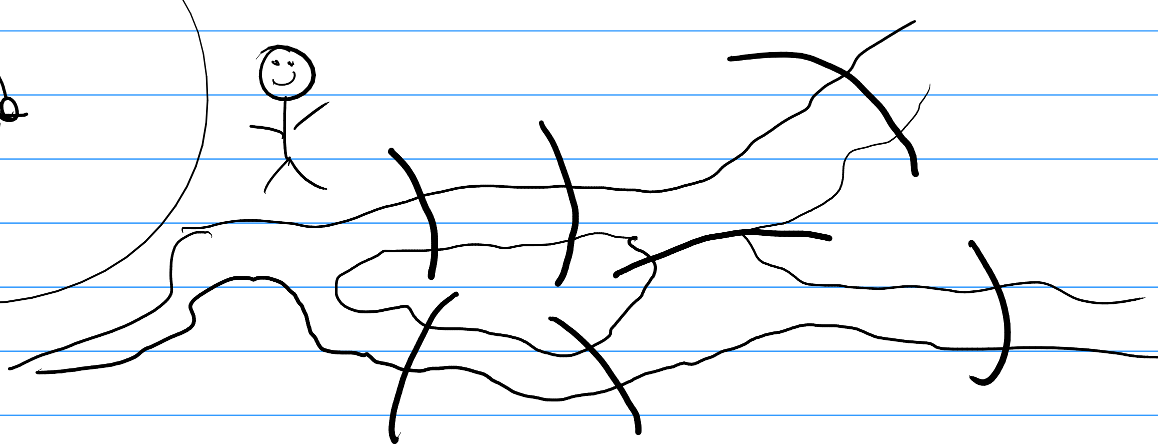
Side Application:



not a solution.



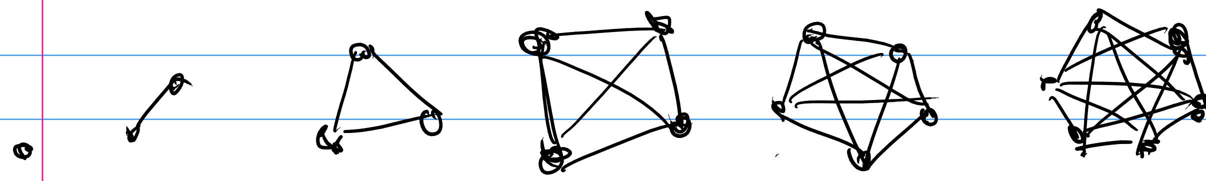




Special Graphs (Simple)

① Complete Graphs K_n $(n \geq 1)$

all n vertices connect to each other



K_1 K_2 K_3 K_4 K_5 K_6

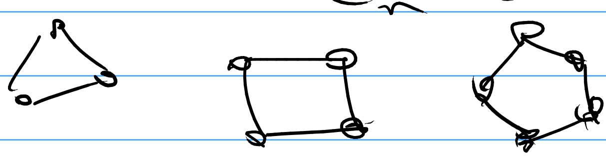
<u>Note</u>: $\deg(v) = (n-1)$ in a complete graph.

② Cycle C_n $(n \geq 3)$

$V = \{v_1, v_2, v_3, \ldots, v_n\}$

$e_1 = \{v_1, v_2\}$, $e_2 = \{v_2, v_3\}$, ...

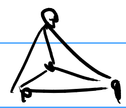
$e_n = \{v_n, v_1\}$



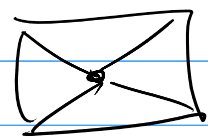
C_3 C_4 C_5

③ Wheel W_n $(n \geq 3)$

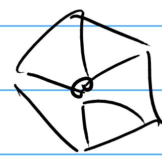
C_n + 1 vertex that connects to each vertex in C_n



W_3



W_4



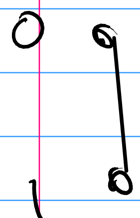
W_5

④ Q_n n-dimensional hypercube.

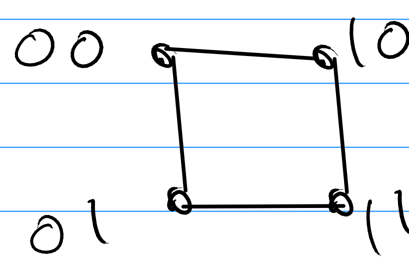
a) Form the 2^n bit strings of length n.

b) Connect strings that differ in 1 bit.

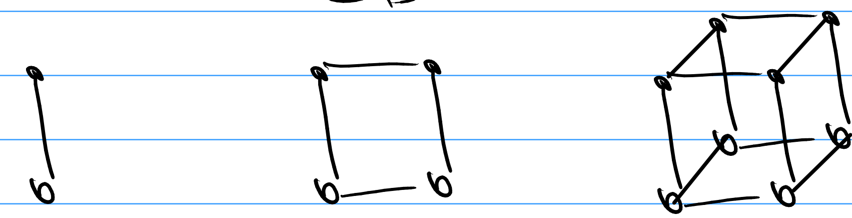
$n=1$ $2^1 = 2$ vertices

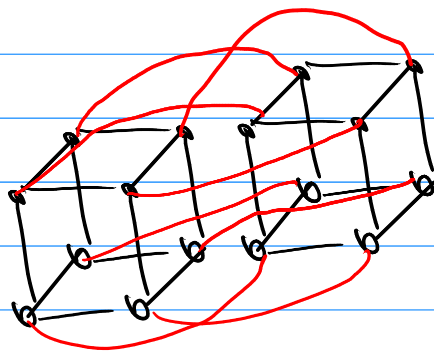


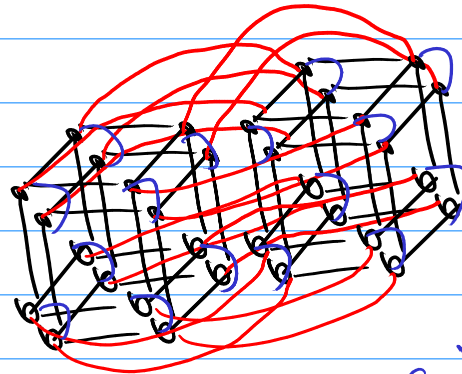
$n=2$ $2^2 = 4$ vertices



1D 2D 3D



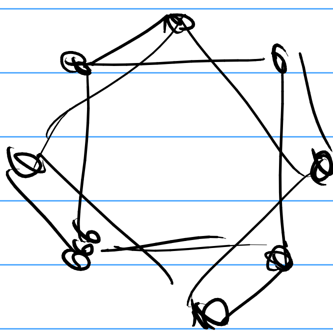




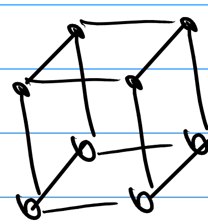
4D - Cube
hypercube

5D - Cube

All of these are $G = (V, E)$



3D - Cube

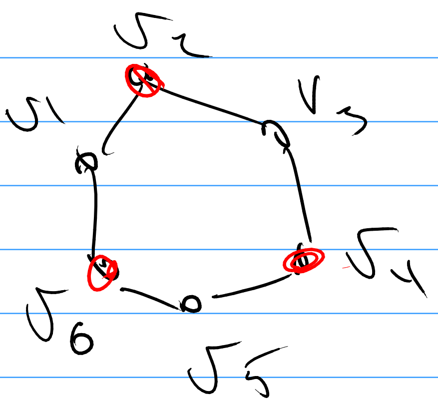


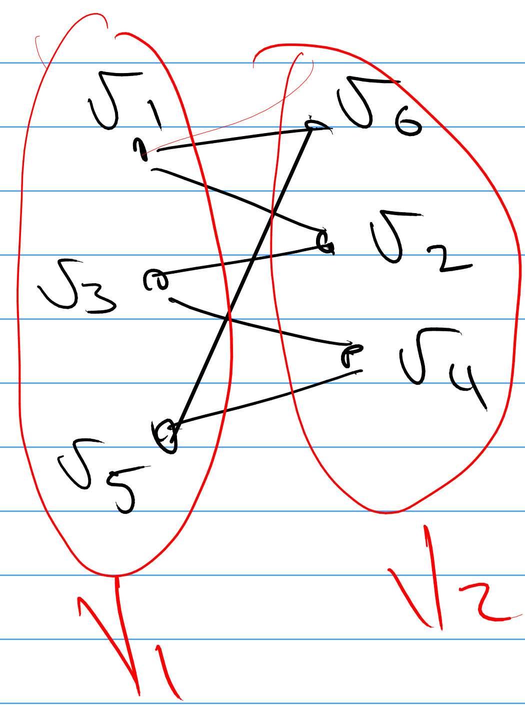
Same?

~~Type:~~ bipartite.

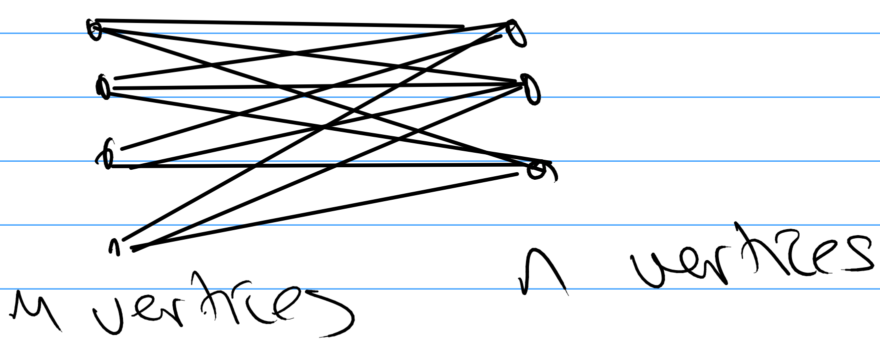
G is bipartite if V can be partitioned into V_1 and V_2 such that all edge go between them.

C_6





$K_{m,n}$ Complete bipartite.

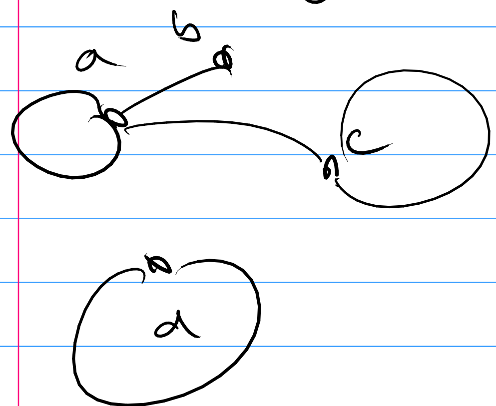


$G = (V, E)$

Representing G outside of points and lines.

① list V and E

② Adj. Lists.



Vertices	Adj
a	a, b, c
b	a
c	a, c
d	d

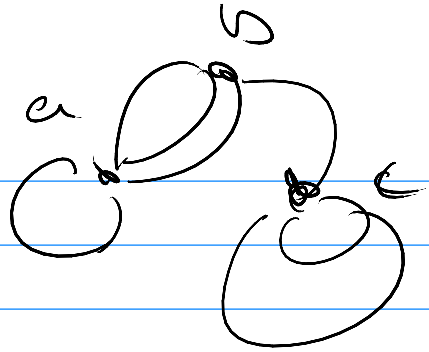
	a	b	c	d
a	✓	✓	✓	
b	✓			
c	✓		✓	
d				✓

③ Adj. Matrix

Same example:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

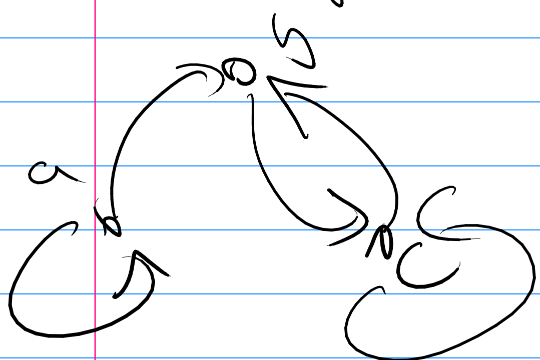
ex



$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 3 & 0 \\ b & 3 & 0 & 1 \\ c & 0 & 1 & 2 \end{array}$$

Directed graphs.

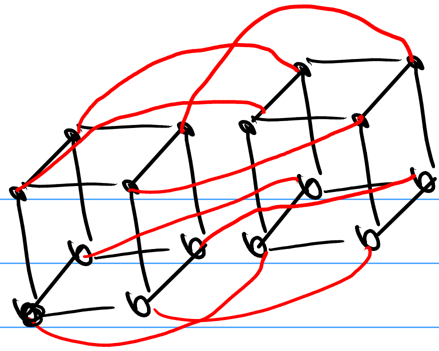
① Adj. to/from lists



initial	terminals
a	a, b
b	c
c	b, c

② adj. matrix

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 1 & 0 \\ b & 0 & 0 & 1 \\ c & 0 & 1 & 1 \end{array}$$



4D - Cube

$|V| = 2^4 = 16$

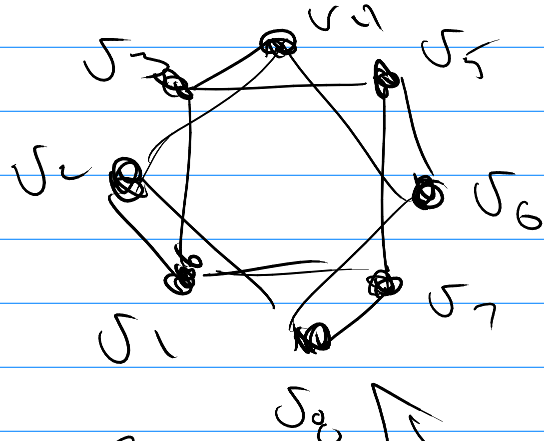
Handshake thm

Matrix?

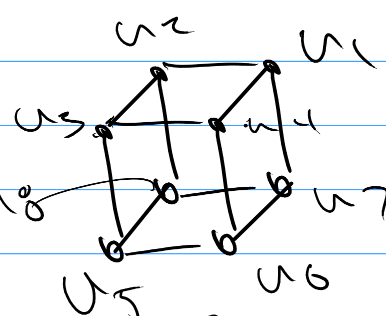
$$\sum \deg(v) = 2|E|$$

$$16 \cdot 4 = 2|E|$$

$$16 \cdot 2 = |E| = \underline{\underline{32}}$$



3D - Cube



G_1 Same? G_2

$G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$

Can you find a function (bijective) from V_1 to V_2 that preserves edges.

$\{a,b\} \in E_1 \longleftrightarrow \{f(a), f(b)\} \in E_2$

to compare:

Adj. Matrix of G_1 in order of

$v_1, v_2, v_3, \ldots$

vs

Adj. Matrix of G_2 in order of

$f(v_1), f(v_2), \ldots$

if matrices are equal

G_1 is isomorphic to G_2

and f is the isomorphism.

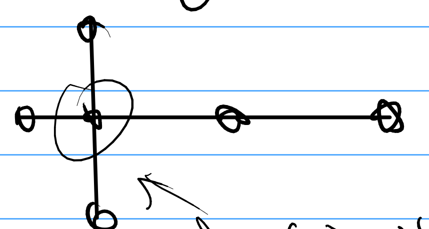
Note: Some things never change under an isomorphism. (invariants)

If an invariant is broken

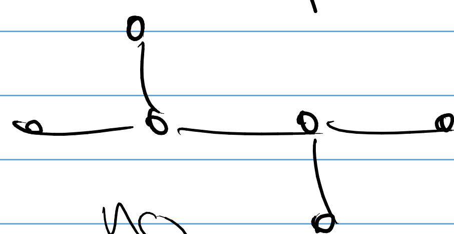
$\rightarrow$ it can not be isomorphic.

1. $|V_1| = |V_2|$
2. $|E_1| = |E_2|$
3. degrees all "match up"

ex

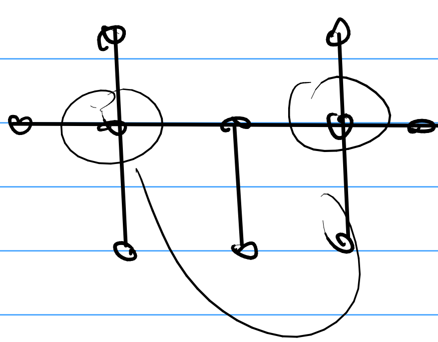


$\deg(v)=4$

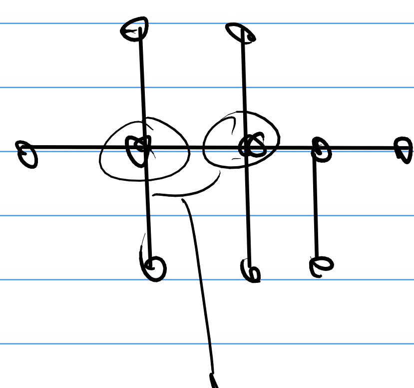


no $\deg(v)=4$!

Not isomorphic.



not adj.



adj.

Not isomorphic.