KANSAS STATE UNIVERSITY | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 12

Created by Prof. Diego Maldonado and Prof. Virginia Naibo

Name: _____

Due date: Thursday, April 25th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. Find the singular value decomposition of the matrix

$$A = \begin{bmatrix} 2 & 1 & 4 & -1 & 3 \\ 4 & -1 & -1 & 0 & 5 \\ 9 & -2 & 7 & 8 & 4 \end{bmatrix}$$

2. Find the eigenvalues of the matrix $B^T B$ and the singular values of B, where

$$B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}.$$

Compute the square roots of the eigenvalues of $B^T B$ and compare them to the singular values of B.

3. Find an orthonormal basis of $\mathbf{R}^3$ consisting of eigenvectors of the matrix

$$C = \begin{bmatrix} -2 & 9 & 1 \\ 9 & -3 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

Why does such orthonormal basis exist?

4. Suppose that a square matrix A has the polynomial

$$p(\lambda) = (2 + \lambda)(4 - \lambda)(1 + \lambda)$$

as its characteristic polynomial. Answer the following questions:

(i) What is the size of A?

(ii) What are the eigenvalues of A?

(iii) Is A invertible?

(iv) Is A diagonalizable?

(v) What is the value of $\det(A)$?

(vi) What is the value of $\det(A - 4I)$? Here I is the identity matrix.

(vii) What are the eigenvalues of A^3?

(viii) What are the eigenvalues of A^{-1}?

(ix) Can A be an orthogonal matrix?

(x) Can A be a lower-triangular matrix?

5. Write a Matlab function that takes an $n \times n$ matrix A and returns a 1 if A is diagonalizable and a 0 otherwise. If the input matrix is not square, the function should display a message indicating so.

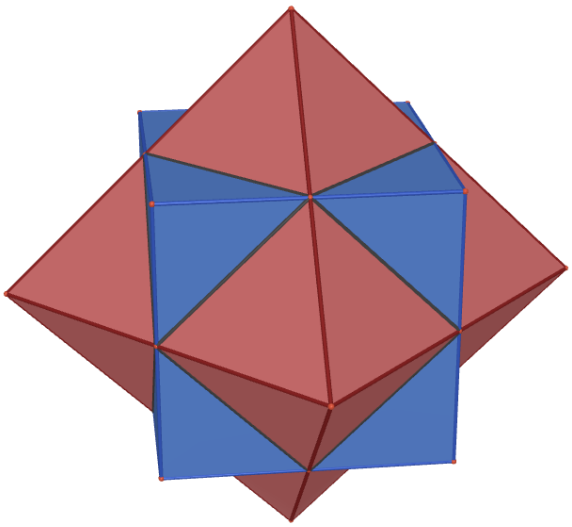
6. Find the singular values of the matrix

$$A = \begin{bmatrix} 1 & 7 & 3 & 5 & -9 & 5 & -3 & 2 \\ -6 & -1 & 0 & -2 & 3 & 2 & 2 & -6 \\ 4 & 0 & 1 & 9 & -5 & 8 & -8 & -4 \\ 4 & 3 & -2 & 0 & 0 & 4 & 3 & -2 \end{bmatrix}$$

7. Find the eigenvalues and the singular values of the matrix

$$N = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

8. Use the cross product to find the volume of the solid shown in the picture. Assume that the cube has side length equal to 1 (unit of length).



9. True or False - **Circle the right one** (1 point each)

   T or **F**. If A is symmetric, then A is invertible.

   T or **F**. If A is symmetric, then A is diagonalizable.

   T or **F**. The singular values of A are always non-negative real numbers.

   T or **F**. The singular value decomposition can be implemented only for square matrices.

   T or **F**. The singular values of A are the eigenvalues of A.

Points obtained in this assignment (out of 16): _____