Math 321

Q's $\quad a_n = a_{n-1} + n \qquad a_0 = 1$

$\{a_n\} = 1, 2, 4, 7, 11, \ldots$

$a_1 = a_0 + 1 = 1 + 1 = 2$
$a_2 = a_1 + 2 = 2 + 2 = 4$
$a_3 = a_2 + 3 = 4 + 3 = 7$
$a_4 = a_3 + 4 = 7 + 4 = 11$

$a_n = a_{n-1} + n$

$a_n = a_{n-2} + (n-1) + n$

$a_{\square} = a_{\square - 1} + \square$

$a_{n-1} = a_{n-2} + (n-1)$

$a_n = a_{\boxed{n-3}} + \boxed{(n-2)} + (n-1) + n$

$\vdots$

$a_n = a_{\boxed{0}} + (1 + 2 + \cdots + n) = a_0 + \frac{n(n+1)}{2}$

$\boxed{a_n = 1 + \frac{n(n+1)}{2}} = \frac{n^2 + n + 2}{2}$

Check: $\quad a_n = a_{n-1} + n$

$\left(1 + \frac{n(n+1)}{2}\right) = \left(1 + \frac{(n-1)(n)}{2}\right) + n$

$1 + \frac{n(n+1)}{2} = 1 + \frac{(n-1)n + 2n}{2}$

$1 + \frac{n(n+1)}{2} = 1 + \frac{n(n-1+2)}{2}$

$1 + \frac{n(n+1)}{2} = 1 + \frac{n(n+1)}{2}$

equal

7.2 Solve

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

$c_i \equiv$ constants.

ex's are:

$$a_n = 3a_{n-7} + a_{n-11}$$
$$a_n = \pi a_{n-5} - a_{n-7}$$

are not:

$a_n = (a_{n-1})^2$ ← not linear

$a_n = 3a_{n-7} + a_{n-11} + \boxed{n}$ ← not homogeneous

$a_n = n\, a_{n-1}$ ← not const. coeff.

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

Solve? (ans. has $\boxed{r^n}$)

Solve: $r^k - c_1 r^{k-1} - c_2 r^{k-2} - \cdots - c_k = 0$

solve a polynomial? Sure... easy!

we will have k roots.

ex:

$$(r-1)(r-1)(r-2)(r+4)=0$$

$\underbrace{r_1 = 1 \quad r_2 = 1}$ $\quad r_3 = 2 \quad r_4 = -4 \quad$ (4 roots)

$r_1 = 1$ is a distinct root of mult. 2

$r_2 = 2$

$r_3 = -4$

Case 1 $\quad r_k$ are all distinct.

soln: $a_n = \alpha_1(r_1^n) + \alpha_2(r_2^n) + \cdots + \alpha_k(r_k^n)$

ex: $a_n = a_{n-1} + a_{n-2}$ $\quad \boxed{a_0 = 0 \quad a_1 = 1}$

$$r^2 - 1 \cdot r^1 - 1 \cdot r^0 = 0$$

$$r^2 - r - 1 = 0$$

$$r = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \boxed{\frac{1 \pm \sqrt{5}}{2}} \; r_k$$

Soln: $a_n = a\left(\frac{1+\sqrt{5}}{2}\right)^n + b\left(\frac{1-\sqrt{5}}{2}\right)^n$

$a_0 = 0 \qquad 0 = a + b \Rightarrow a = -b$

$a_1 = 1 \qquad 1 = a\left(\frac{1+\sqrt{5}}{2}\right) + b\left(\frac{1-\sqrt{5}}{2}\right)$

$$1 = -b\left(\frac{1+\sqrt{5}}{2}\right) + b\left(\frac{1-\sqrt{5}}{2}\right)$$

$$2 = -b(1+\sqrt{5}) + b(1-\sqrt{5})$$

$$2 = \cancel{-b} - \sqrt{5}b + \cancel{b} - \sqrt{5}b$$

$$2 = -2\sqrt{5}b \rightarrow b = -\frac{1}{\sqrt{5}}$$

$$a = -b \rightarrow a = \frac{1}{\sqrt{5}}$$

recursive: $a_0 = 0 \quad a_1 = 1 \quad a_n = a_{n-1} + a_{n-2}$

function: $a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

$$a_{10^{10}} = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{10^{10}} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{10^{10}}$$

Multiplicity?: r_1 has m_1 mult.
r_2 has m_2 mult.
(r_t distinct)

$$a_n = (\alpha_{1,0} + \alpha_{1,1} n + \cdots + \alpha_{1,m_1-1} n^{m_1-1})(r_1)^n$$
$$+ (\alpha_{2,0} + \alpha_{2,1} n + \cdots + \alpha_{2,m_2-1} n^{m_2-1})(r_2)^n$$
$$\vdots$$
$$+ (\alpha_{t,0} + \alpha_{t,1} n + \cdots + \alpha_{t,m_t-1} n^{m_t-1})(r_t)^n$$