Math 321

Extra Credit Proofs

← Just stop by my office (Jabara 219) to take them.

1. Due Dec 8
2. Due Nov. 5
3. Due Dec 8
4. Due Nov. 5
5. Due Dec. 8
6. Due Dec. 8
7. Due Oct. 6
8. Due Nov. 5

2.1 Sets:

Comparison $A = B$ iff $\forall x\,(x \in A \leftrightarrow x \in B)$

Subset $A \subseteq B$ iff $\forall x\,(x \in A \rightarrow x \in B)$

Note:

1. $\phi \subseteq S$
2. $S \subseteq S$

ex $A = \{a, b, c\}$

What are all the subsets of A?

Sets with 0 elements $\emptyset$

1 element $\{a\}, \{b\}, \{c\}$

2 elements $\{a,b\}, \{a,c\}, \{b,c\}$

3 elements $\{a,b,c\}$

Operations $P(S)$ is the power set

a new set that has as elements

all the subsets of S.

ex $P(\{a,b,c\}) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

In general: how many subsets?

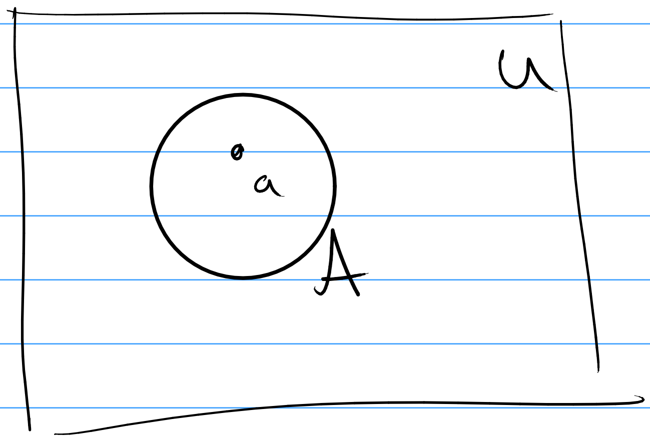
later

Representing Sets

① list

② Set builder notation

③ Venn Diagrams



④ Bit Strings.

ex: $U = \{a, e, i, o, u\}$ $A = \{e, o\}$

$U = 11111$

$\phi = 00000$

$A = 01010$

$B = 01101$ $B = \{e, i, u\}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

How many subsets? 2^n n is number of elements in S.

$P(\{a, b, \{a,b\}\}) \rightarrow 2^3 = 8$

$P(\underbrace{P(\phi)}_{2^0 = 1 \text{ element}}) \rightarrow 2^1 = \boxed{2}$

$P(\phi) = \{\phi\}$

Cardinality of S ($|S|$)

$|S| = n$ is the number of distinct elements in S.

① if n is a non-neg integer S is said to be finite.

② if not finite then it is infinite.

Operations:

① $P(S)$ Power set

② Cross Product.

Cross Product

ex $A = \{\square, \triangle, \bigcirc\}$

$B = \{\nearrow, \sim, \wedge\}$

$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

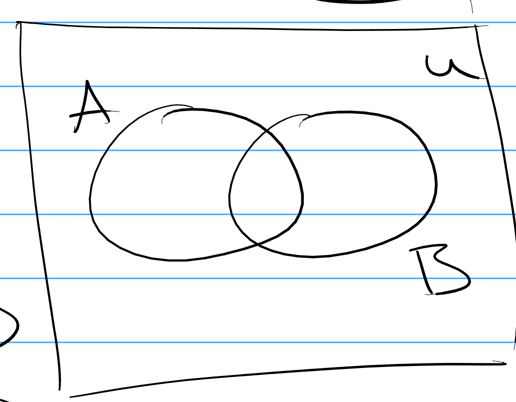
$|A \times B| = |A| \cdot |B|$

n-tuples

$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid \forall i\, a_i \in A_i\}$

$|A_1 \times A_2 \times \cdots \times A_n| = |A_1|\,|A_2| \cdots |A_n|$

More Operations



1. **Union**

$A \cup B = \{x \mid x \in A \vee x \in B\}$

2. $A \cap B = \{x \mid x \in A \wedge x \in B\}$

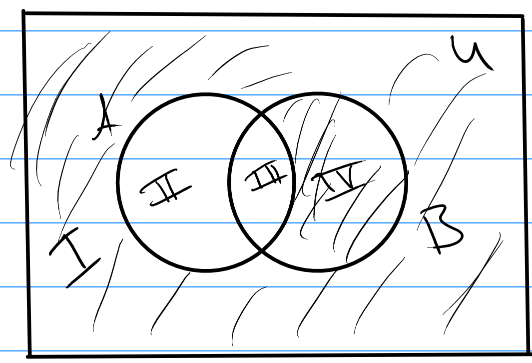
if $A \cap B = \emptyset$ they are disjoint.

3. $A - B = \{x \mid x \in A \wedge x \notin B\}$

(4) $\overline{A} = U - A = \{x \mid x \notin A\}$

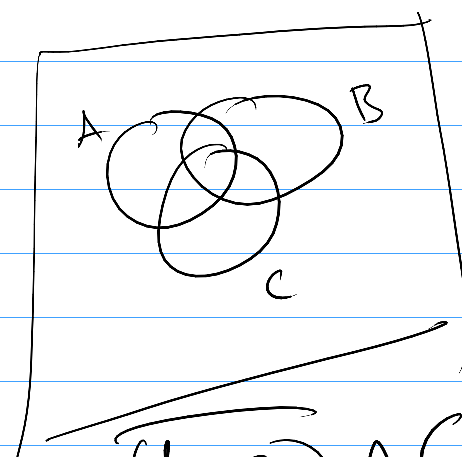
Membership Tables

	A	B	$A \cup B$	$A \wedge B$	$\overline{A}$
III	1	1	1	1	0
II	1	0	1	0	0
IV	0	1	1	0	1
I	0	0	0	0	1



$\overline{A}$ is shaded.

A	B	C
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0



$(\overline{A \cup B}) \wedge C$

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$$\begin{aligned}\overline{A \cap B} &= \{x \mid x \notin (A \cap B)\} \\ &= \{x \mid \neg (x \in A \cap B)\} \\ &= \{x \mid \neg (x \in A \wedge x \in B)\} \\ &= \{x \mid x \notin A \vee x \notin B\} \\ &= \{x \mid x \in \overline{A} \vee x \in \overline{B}\}\end{aligned}$$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$ De Morgan's law for sets

<u>p. 124</u> Set laws

based on logic laws.

So to show $A = B$

① Membership Table

② $\forall x (x \in A \leftrightarrow x \in B)$

<u>show:</u> $A \subseteq B \wedge B \subseteq A$.

③ use the laws.