Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 5

Created by Prof. Diego Maldonado and Prof. Virginia Naibo

Name: ____

Due date: Thursday, February 28 at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises

1. Find a basis for the subspace $\mathcal{V} = \{x = (x_1, x_2, x_3) \in \mathbf{R}^3 : 2x_1 + x_2 = 0, x_1 - x_2 - x_3 = 0\}$.

   Hint: Express $\mathcal{V}$ as the null space of a 2×3 matrix A and then find a basis for $null(A)$.

2. Find bases for the column space, the row space, and the null space of the matrix

$$N = \begin{bmatrix} 5 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & -1 \\ -1 & -1 & -1 & -5 & 0 \\ 0 & 2 & -1 & 1 & 1 \end{bmatrix}.$$

Indicate the corresponding dimensions of those subspaces.

3. Prove that if $\mathcal{U}$ and $\mathcal{V}$ are subspaces of $\mathbf{R}^m$ so is the intersection $\mathcal{U} \cap \mathcal{V}$. Provide a counter-example to the analogous statement involving $\mathcal{U} \cup \mathcal{V}$ instead of $\mathcal{U} \cap \mathcal{V}$. That is, the intersection of any two subspaces is always a subspace, but the union need not be.

4. Find a basis for the subspace

$$\mathcal{U} = \{x = (x_1, x_2, x_3, x_4) \in \mathbf{R}^4 : x_1 + 2x_3 = 0, x_1 - x_2 + x_4 = 0\}.$$

Hint: Express $\mathcal{U}$ as the null space of a 2×4 matrix A and then find a basis for $null(A)$.

5. Find bases for the column space, the row space, and the null space of the matrix

$$P = \begin{bmatrix} 5 & 3 & 1 & 0 \\ 4 & 4 & 6 & 6 \\ -1 & -9 & -1 & -7 \\ 0 & 2 & -4 & 1 \end{bmatrix}.$$

Indicate the corresponding dimensions of those subspaces.

6. Find bases for the column space, the row space, and the null space of the matrix

$$Q = \begin{bmatrix} 4 & 3 & 1 & 0 & -1 \\ 4 & 4 & 0 & 2 & 1 \\ -1 & -3 & -1 & -1 & 1 \\ 2 & 1 & 0 & 3 & 0 \end{bmatrix}.$$

Indicate the corresponding dimensions of those subspaces.

7. True or False - **Circle the right one** (One point each)

T or **F**. The vectors

$$u_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \text{and } u_3 = \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$$

span the vector

$$u = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

T or **F**. The vector

$$u = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

belongs to the column space of the matrix

$$D = \begin{bmatrix} 5 & 3 & 1 \\ 4 & 4 & 4 \\ -1 & -9 & -17 \end{bmatrix}.$$

T or **F**. The column space of the matrix

$$H = \begin{bmatrix} 5 & 3 & 1 & 0 \\ 4 & 4 & 6 & 6 \\ -1 & -9 & -1 & -7 \\ 0 & 2 & -4 & 1 \end{bmatrix}.$$

has dimension 3.

T or **F**. The vector

$$v = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

belongs to the null space of the matrix

$$B = \begin{bmatrix} 3 & -6 & 0 \\ -1 & 2 & 6 \\ 2 & -4 & -7 \\ 2 & -4 & 1 \end{bmatrix}.$$

T or **F**. The vectors

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \text{ and } \quad v_3 = \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$$

form a basis for the column space of the matrix

$$C = \begin{bmatrix} 4 & 3 & 2 & 0 & 1 \\ 1 & 4 & 1 & 2 & 0 \\ 8 & -3 & 4 & -1 & 2 \end{bmatrix}.$$

Points obtained in this assignment (out of 16): ______________