# Math 321

Q's 9.1 #29 $H_n = \frac{1}{n}$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$H_{2^n} = \frac{1}{2^n}$

Prove:

$$\frac{1}{2^n} \leq 1 + n$$

$n = 0, 1, 2, \ldots$

Pf: Basis: $P(0)$

Left: $\frac{1}{2^0} = 1$

Right: $1 + 0 = 1$

$1 \leq 1$ True.

Inductive assume: $\frac{1}{2^k} \leq 1 + k$

(show $\frac{1}{2^{k+1}} \leq 2 + k$)

Given: $\frac{1}{2^k} \leq 1 + k$

$$\frac{1}{2}\left(\frac{1}{2^k}\right) \leq \frac{1}{2}(1+k)$$

$$\frac{1}{2^{k+1}} \leq \frac{1}{2} + \frac{1}{2}k \leq 2 + \frac{1}{2}k \leq 2 + k$$

b/c $\frac{1}{2} \leq 2$ b/c $\frac{1}{2}k \leq k$

True!

4.3

Ex 4.2 #3 3¢ & 5¢ stamps

→ any amount $\geq 8$¢

**Basis:** 8¢ = (1) 3¢ + (1) 5¢

9¢ = (3) 3¢ + (0) 5¢ ✓

10¢ = (0) 3¢ + (2) 5¢

**Strong Induction:**

assume you can use 3¢ & 5¢ stamps to make 8¢, 9¢, 10¢, 11¢, .., K¢

Show (K+1)¢

(K-2)¢ + (1) 3¢ = (K+1)¢

I can form this!

**True.**

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Sequences and Induction.

Recursive Definitions

**Step 1** **Basis:** specify initial value(s)

**Recursive Step:** A rule that gives new values based on older values ex $a_0 = 1$

$$\boxed{a_n = 2 \cdot a_{n-1}} \quad n = 1, 2, 3, \ldots$$

$a_1 = 2 \cdot a_0 = 2$

$a_2 = 2 \cdot a_1 = 4$

$a_3 = 2 \cdot a_2 = 8$

$\vdots$

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$a_1 = 1$

$$\boxed{a_{n+1} = 2 \cdot a_n} \quad n = 1, 2, 3, \ldots$$

$a_2 = 2 \cdot a_1 = 2 \cdot 1 = 2$

$a_3 = 4$

$a_4 = 8$

$\vdots$

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Seq: $1, 2, 4, 8, 16, \ldots$

| Recursive Def: | Function Def: |
| --- | --- |
| $a_0 = 1$ | $a_n = 2^n$ |
| $a_n = 2 \cdot a_{n-1} \quad n = 1, 2, \ldots$ | $n = 0, 1, 2, \ldots$ |
| Open form | Closed form |

Fibbonacci Numbers

$$f_0 = 0 \qquad f_1 = 1$$

$$f_n = f_{n-1} + f_{n-2} \qquad n = 2, 3, 4, \ldots$$

Seq: $0, 1, 1, 2, 3, 5, 8, 13, \ldots$

Set $= \{3, 6, 9, 12, 15, \ldots\}$

ex

Basis: $a = 3$ is in my set

Inductive: $a$ and $b$ are in the set then $a+b$ is in the set.

ex Prove: $\forall n \quad f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$

$f_0 = 0$
$f_1 = 1$
$f_2 = 1$
$f_3 = 2$
3
5
8
13
21

pf: $\boxed{\text{Basis:}}$ $P(1)$

left $f_1^2 = 1^2 = 1$

right $f_1 f_2 = 1 \cdot 1 = 1$

$\boxed{\text{Induction}}$ assume $f_1^2 + \cdots + f_k^2 = f_k f_{k+1}$

(show $f_1^2 + \cdots + f_{k+1}^2 = \boxed{f_{k+1} f_{k+2}}$)

5(c $f_1^2 + \dots + f_k^2 = f_k f_{k+1}$

$$f_1^2 + \dots + f_k^2 + f_{k+1}^2 = f_k f_{k+1} + f_{k+1}^2$$

$$= f_{k+1}(\underbrace{f_k + f_{k+1}}_{f_{k+2}})$$

$$= f_{k+1} \cdot f_{k+2}$$

True!