KANSAS STATE UNIVERSITY | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 3

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Name: ____

Due date: Thursday February 14th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises

1. Find at least one 2×2 matrix C such that

$$C \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}. \tag{1}$$

Hint: Express (1) as a system having the entries of C as unknowns. There might be infinitely many choices for C, provide one.

2. In No-Mid-Class City there are two neighborhoods: Fancy Woods and Shanty Hills. Each year, 40% of the residents of Fancy Woods move to Shanty Hills and 10% of the residents of Shanty Hills move to Fancy Woods. If this year there are exactly 1,000 residents in each neighborhood. What will be the distribution of residents in 5 years? What will be the long term distribution of residents?

3. (a) For what values of k is the matrix

$$B = \begin{bmatrix} k^2 & 2k \\ 8 & k \end{bmatrix}$$

(lower or upper) triangular?

(b) For what values of k is the matrix B symmetric?

(c) Solve the system $Bx = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ when $k = 1$.

(d) If $k = 3$, what is B^k?

4. My aunt Lila's house is infested with ants. My friend Rainman and I visit my aunt daily because she makes a really good stew. The stew usually has ants in it, but it's no big deal for us. Anyway, about a month ago, right after lunch time, good old Rainman took a glance at the house and, almost immediately, he said to me: "There are exactly 6,000 ants in this house". "And how many anthills?", I asked. A couple of hours later, he replied: "Two". "Where are they?" I inquired. "There is one in the basement and one in the master bedroom. The one in the basement has 2,000 ants", he said. Right away, Rainman and I took up a meticulous study of the circulation of the ants in the house. After a month of research here is what we got: each day, 40% of the ants in the anthill located in the basement moved to the anthill in the master bedroom. In turn, 30% of the ants in the master bedroom would move to the anthill in the basement. Coincidentally enough, 10% of the ants in the master bedroom would patiently find their way into the stew pot. A few moments ago I asked Rainman: "How many ants do you think we ate today?". He couldn't answer because he suddenly felt very sick. Was he overreacting?

5. A town's economy is based on the following three sectors: Administration *(A)*, Housing *(H)*, and Transportation *(T)*. These sectors are related as follows: The production of one dollar of A requires 72 cents of A, 18 cents of H, and 10 cents of T. Each dollar produced by H requires 35 cents of A, 45 cents of H, and 20 cents of T. Each dollar of T requires 20 cents of A, 25 cents of H, and 55 cents of T. Is this an open or a close economy model? Find the production schedule and identify the consumption matrix. If this economy is worth 1 million dollars, how much A, H, and T must be produced? Which sector consumes the least H?

   Hint: Begin with the key questions: how much A is consumed? How much H is consumed? How much T is consumed?.

6. Let (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4) be points in the plane. Let $y = ax^3 + bx^2 + cx + d$ be a cubic curve passing through all points (assuming there is one). What system of linear equations must be solved in order to find the coefficients a, b, c, and d? Identify the corresponding augmented matrix.

7. An $n \times n$ matrix is called *a column stochastic matrix* if its entries are nonnegative real numbers and each column adds up to 1. Check that the following matrix U is column stochastic

$$U = \begin{bmatrix} .2 & .5 & .1 \\ .1 & 0 & .6 \\ .7 & .5 & .3 \end{bmatrix}$$

and find a vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

such that $Ux = x$ and $x_1 + x_2 + x_3 = 1$.

Hint: Use the definition of multiplication to spell out the identity $Ux = x$ and obtain three equations for x_1, x_2, and x_3. To those, add the equation $x_1 + x_2 + x_3 = 1$ and solve the resulting system.

8. Graphs and websites. Suppose that a 5-page web is represented by the graph in Figure 1 in such a way that an edge going from vertex v_i to vertex v_j means that there is a link from page v_i to page v_j. By departing from page 2 (at vertex v_2) and clicking exactly 8 times, in how many ways can we end up on page 3? Explain using **powers of the adjacency matrix**. Any other type of explanation (such as just "counting") will be worth 0 points.

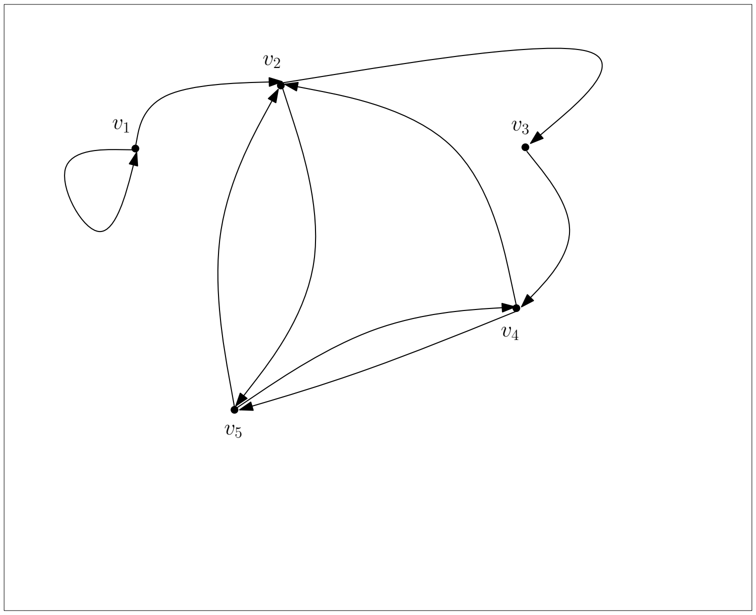

Figure 1: Directed graph representing a 5-page web

9. Consider an economy which has a steel plant, a coal mine and a transportation industry. To produce \$1 value of steel requires inputs of 50 cents from the steel plant, 30 cents from the coal mine, and 10 cents from transportation. To produce \$1 value of coal requires inputs of 10 cents from the steel plant, 20 cents from the coal mine, and 30 cents from transportation. Each dollar's worth of transportation output requires inputs of 10 cents from the steel plant, 40 cents from the coal mine, and 5 cents from transportation. Assume that the outside demand for the current production period is 2 million dollars for steel, 1.5 million dollars for coal, and \$500,000 for transportation.

   For this situation, identify whether this is an open or closed economy, determine the production schedule, the consumption matrix, and how much each industry must produce in order to satisfy the demands. Which sector consumes the most steel?

   Hint: Begin with the key questions: how much steel is consumed? How much coal is consumed? How much transportation is consumed?.

10. True or False - **Circle the right one** (One point each)

T or **F**. In a discrete dynamical system with step matrix A, the identity

$$u_{k+1} = A^k u_1, \quad k \in \mathbf{N}$$

is equivalent to the identity

$$u_{k+1} = Au_k, \quad k \in \mathbf{N}.$$

T or **F**. Given a system $Ax = b$, if $rank(A) < rank([A, b])$, then the system has at least one solution.

T or **F**. If A has an inverse, then A^{-1} has an inverse.

T or **F**. There exist real numbers x_1, x_2, and x_3 such that

$$x_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -2 \end{bmatrix}.$$

T or **F**. If U, V, and W are $n \times n$ matrices such that

$$UV = W$$

and U has an inverse, then it must be

$$V = WU^{-1}.$$

Points obtained in this assignment (out of 16): ______________