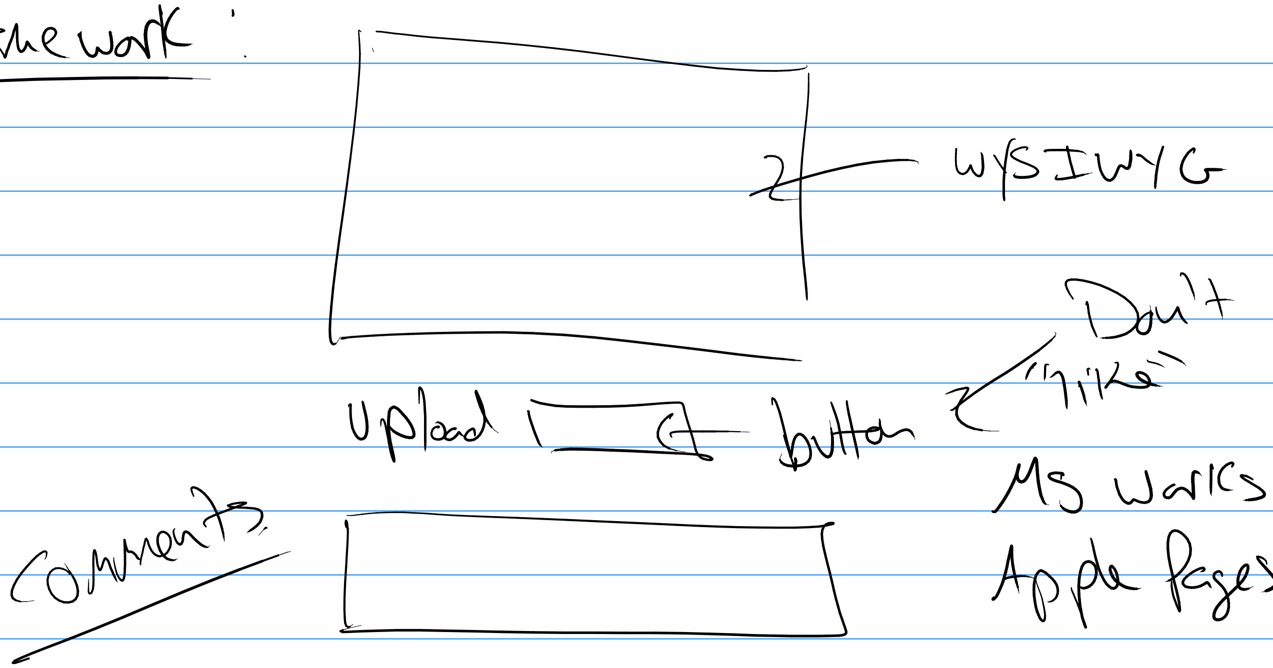
Math 322

Homework:



Q's / Relations on $\mathbb{R}^2$ $(\mathbb{R} \times \mathbb{R})$

8.1 (32-35)

aR_1b if $a>b$

aR_2b if $a \geq b$

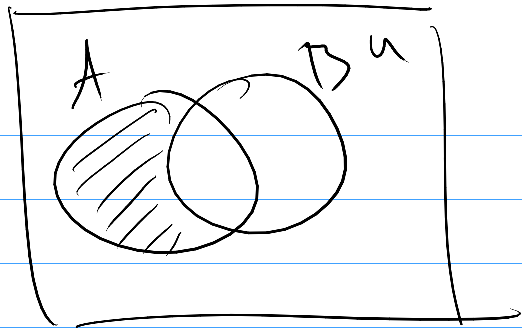
aR_3b if $a<b$

aR_4b if $a \leq b$

aR_5b if $a=b$

aR_6b if $a \neq b$

33 f) $R_6 - R_3$



$$A - B = \{x \mid x \in A \land x \notin B\}$$

$$R_6 - R_3 = \{x \mid x \in R_6 \land x \notin R_3\}$$

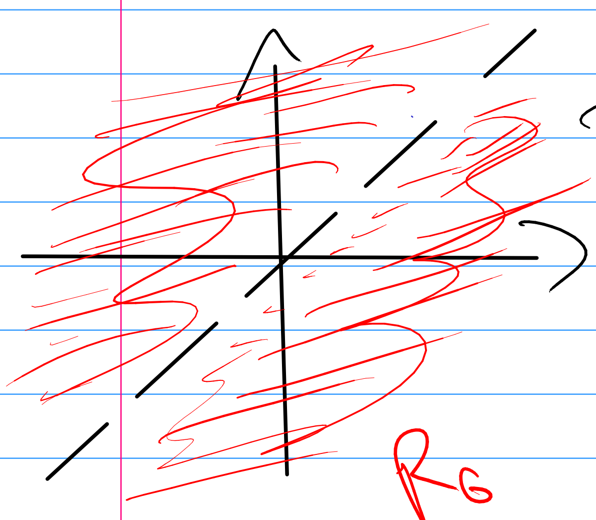
$$= \{(a,b) \mid a R_6 b \land a \not{R}_3 b\}$$

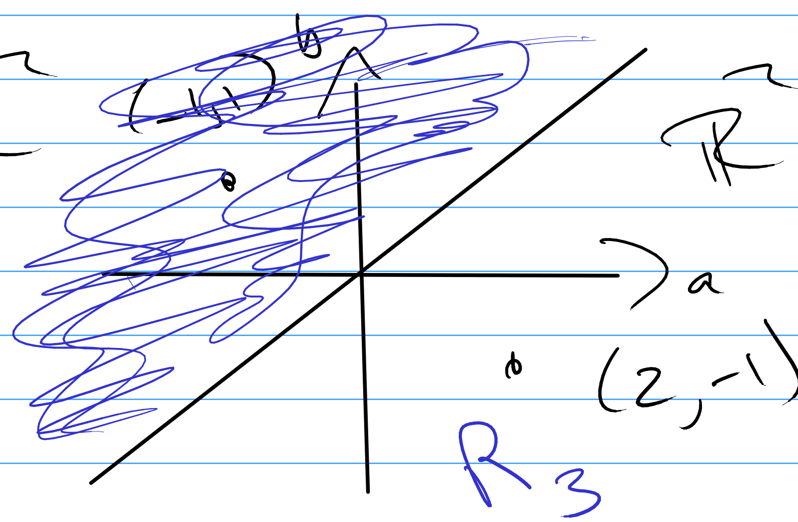
$$= \{(a,b) \mid \underbrace{a R_6 b}_{a \neq b} \land \neg(\underbrace{a R_3 b}_{a < b})\}$$

$$= \{(a,b) \mid a \neq b \land a \geq b\}$$

$$= \{(a,b) \mid a > b\} = R_1$$

R_6 $a \neq b$ | R_3 $a < b$ $\mathbb{R}^2$





$\boxed{8.1}$ $\boxed{\text{Reflexive}}$, Sym., Antisym, $\boxed{\text{Trans.}}$

R on set $A = \{a, b, c, \ldots, z\}$

$R = \{$ list of ordered pairs $\}$

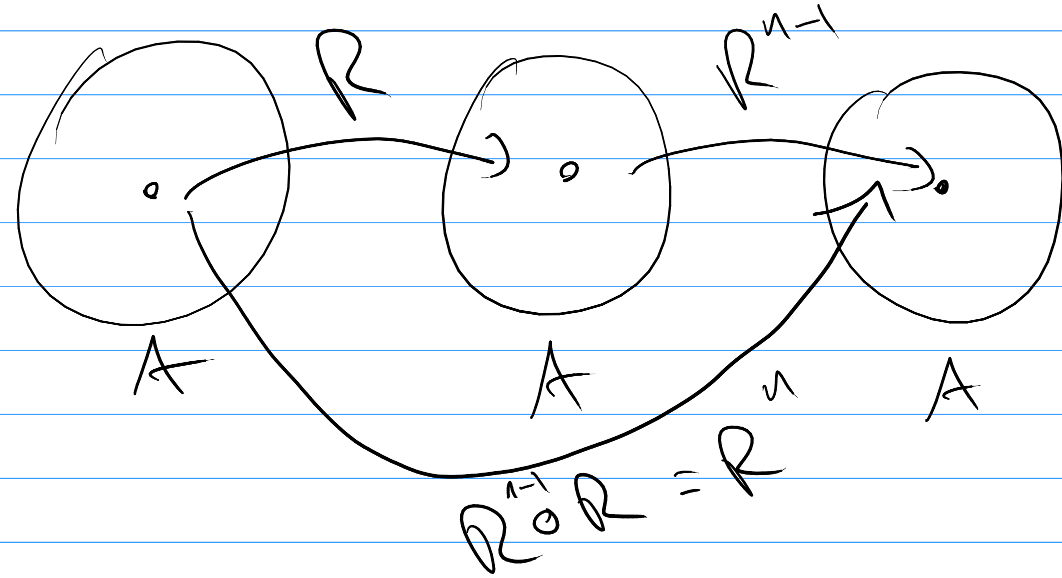
$\boxed{\text{Th}^{\underline{m}}}$ R on set A is transitive

iff $\forall n \; R^n \subseteq R$

<u>Note:</u> (Inductive Def)

$R^1 = R$

$R^n = R^{n-1} \circ R \qquad n = 1, 2, \ldots$

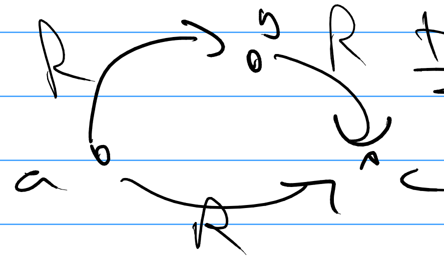


Remember: $(p \leftrightarrow q)$ is T

$$p \equiv q$$

Thm: R is trans $\leftrightarrow \forall n\ R^n \subseteq R$

Pf: Case 1 $\forall n\ R^n \subseteq R \rightarrow R$ is trans



transitive

if $\forall n\ R^n \subseteq R$

then $R^2 \subseteq R$

$\rightarrow R \circ R \subseteq R$

$aRb \wedge bRc \rightarrow aRc$

if $aRb \wedge bRc \rightarrow aRc$ def. of trans.

(this is if $(a,c) \in R^2$ then $(a,c) \in R$

b/c $R^2 \subseteq R$) ∎

Case 2 R is trans $\rightarrow \forall n\ R^n \subseteq R$

Basis Step $(n=1)$

prove: R is trans $\rightarrow \boxed{R^1 \subseteq R}$ true

So Basis is true trivially.

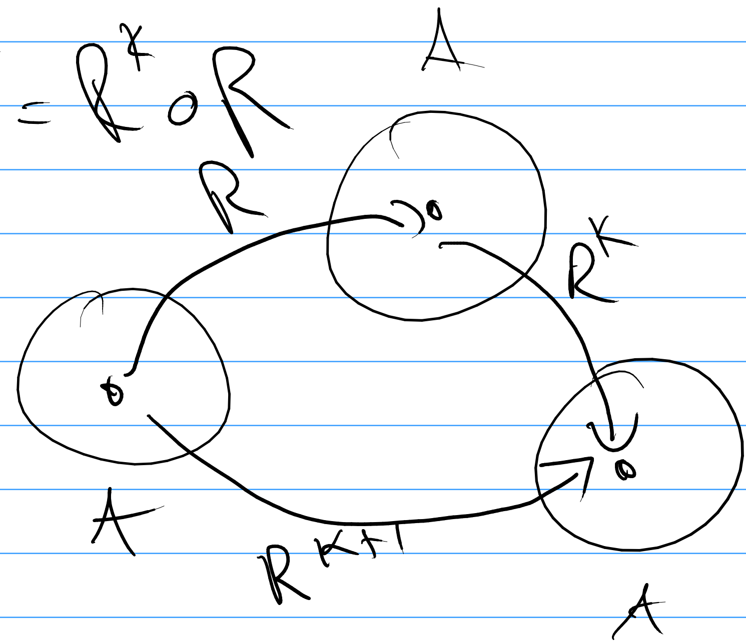
□	△	□ → △
T	T	T
T	F	F
F	T	T
F	F	T

Inductive:

Show $\underbrace{P(K)}_{\forall_K R^K \subseteq R} \rightarrow \underbrace{P(K+1)}_{\forall K\, R^{K+1} \subseteq R}$

assume $\forall_K R^K \subseteq R$ show $\forall K\, R^{K+1} \subseteq R$

$R^{K+1} = R^K \circ R$



Idea: $R^{K+1} \subseteq R$ show!

means: $aR^{K+1}b \rightarrow aRb$

$aR^{K+1}b \rightarrow aRc \land cR^K b$

b/c $R^K \subseteq R \rightarrow aRc \land cRb$

b/c R is trans: → aRb

$\boxed{\text{Q.E.D.}}$

8.2 n-ary Relations

R, an n-ary relation on sets $A_1, A_2, \dots, A_n$, is a subset of $A_1 \times A_2 \times \dots \times A_n$.

Def: A_i are domains of R

n is R's degree

Application: Relational Data Model

n-tuple ≡ $\boxed{\text{record}}$

A_i ≡ field

relation R ≡ table

set of R ≡ database

If one domain of R is defined such that its values uniquely determine a record, then it is a $\boxed{\text{primary key}}$.

If a cross product of A_i's uniquely determines n-tuples it is a $\boxed{\text{composite key.}}$

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table 1	table 2
(n-tuple)	(m-tuple)
⋮	⋮
(n-tuples)	(m-tuple)

Operations:

① Add / Remove records.

② Selection (S_c)

R is n-ary

C is a condition elements in R may have.

$$S_c = \{(a_1, .., a_n) \mid (a_1, .., a_n) \text{ satisfy } C\}$$

(3) given R, take some domains.

Projection:

$P_{i_1, i_2, \ldots, i_m}$ maps n-tuple $(a_1, \ldots, a_n)$

to m-tuple $(a_{i_1}, a_{i_2}, \ldots, a_{i_m})$

(4) Join n-tuple to m-tuple

a) shared domains?

$A_1, A_2, \ldots, A_p$ are shared.

b) $(m+n-p)$-tuples

c) $(\underbrace{a_1, a_2, \ldots, a_{n-p}, \overbrace{c_1, c_2, \ldots, c_p}^{\text{shared} \in \text{field}}}_{\text{in } R_1}, b_1, b_2, \ldots, b_{m-p})$

$c_1, \ldots, c_p, b_1, \ldots, b_{m-p}$ in R_2

$\boxed{J_p(R_1, R_2)}$

p. 374

Teaching

prof $\underbrace{\text{Dept.}}$ $\underbrace{\text{Course}}$

Class

$\underbrace{\text{Dept}}$ $\underbrace{\text{Course}}$ Room Time

$J_2(T_5, T_6)$