CIS770 Homework 8

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Problem1

1. N

for w $\in \{0,1\}^*$
simulate M on w
if M accepts w then:
    write the word w on output tape

note: the words tested by the enumerator should be in lexicographical order i.e. 0,1,00,01,11..

2. M

The turing machine M that decides E(N), on input w will run N until N outputs our input w, where we should accept and stop N, or another word that is greater than our input w, where we should reject and stop N. If neither of these are the case we should keep running N until one of the above cases is true.

Run N on w (input)
Every time N writes a word w_1 compare w_1 with w.
    If w_1 = w then accept and stop N
    else if w_1 > w then reject and stop N
    else continue running N

Problem2

Given some grammar G we will convert G into Chomsky Normal Form and the resulting grammar will be called G_1. All words of length that are less than or equal to the number of variables in $G_1(|G_1|)$, which will be called L_2 is a finite language, thus regular. Since regular languages are closed under complementation the language L_2 is also regular. Also because L(G_1) is context free and L_2 is regular L(G_1) $\cap$ $L_2 \neq \emptyset$ because context free languages are closed under intersection with regular languages.

Note: need to finish

Problem3

Let M_A be a Turing machine recognizing $\bar{A}$ and let M_B be a Turing machine recognizing $\bar{B}$. Since A $\cap$ B = $\emptyset$, A $\cup$ $\bar{B} = \Sigma^*$. Consider some program M that simulates the previous two turing machines on some input w. We can step through each of these machines 1 step at a time and when one completes we check which one completed and accept or reject accordingly. If M_A accepts the input w it should reject and if M_B accepts the input w it should accept.

Because $\bar{A} \cup \bar{B} = \Sigma^*$ the program described above will always terminate making the language decidable.

Finally, if we let w $\in$ A. M_A will reject w however, M_B will accept w thus M will accept w showing A $\subseteq$ L. Now if we let w $\in$ B. M_A will accept w and M_B would reject w thus M will reject w showing B $\subseteq$ $\bar{L}$

Program described above
M:

Input w
  for i = 1,2...
    run M_A on w for i steps
    run M_B on w for i steps
    if M_A or M_B accepts exit loop
  if M_A accepts reject
  if M_B accepts accept