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# PROBLEM SET 1

## CIS 770: FORMAL LANGUAGE THEORY

Assigned: January 26, 2016    Due on: February 2, 2016

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**Instructions:** This homework has 4 problems that must be solved **individually**. In addition, 2 more optional problems that will not be graded are given to you for practicing, you don't need to submit solutions for these two problems. Please follow the homework guidelines given on the class website. Solutions not following these guidelines will not be graded.

**Recommended Reading:** Lectures 1 and 2.

**Problem 1.** [Category: Design] Design a DFA for the language  $L_{A1} = \{w \in \{a, b\}^* \mid \text{number of } a\text{'s in } w \text{ is not divisible by 3}\}$ . [5 points]

**Problem 2.** [Category: Design] Design a DFA for the language  $L_{A3} = \{w \in \{a, b\}^* \mid \text{if } w \text{ starts with an } a \text{ then it does not end with a } b\}$ . [5 points]

**Problem 3.** [Category: Design] Design a DFA for the language  $L_{A4} = \{w \in \{a, b\}^* \mid ba \text{ appears exactly twice as a substring}\}$ . [5 points]

**Problem 4.** [Category: Design+Proof] Let  $A_k \subseteq \{a, b\}^*$  be the collection of strings  $w$  where there is a position  $i$  in  $w$  such that the symbol at position  $i$  (in  $w$ ) is  $a$ , and the symbol at position  $i + k$  is  $b$ . For example, consider  $A_2$  (when  $k = 2$ ).  $baab \in A_2$  because the second position ( $i = 2$ ) has an  $a$  and the fourth position has a  $b$ . On the other hand,  $bb \notin A_2$  (because there are no  $as$ ) and  $aba \notin A_2$  (because none of the  $as$  are followed by a  $b$  2 positions away).

1. Design a DFA for language  $A_k$ . Your formal description (by listing states, transitions, etc. and not “drawing the DFA”) will depend on the parameter  $k$  but should work no matter what  $k$  is; see lecture 2, last page for such an example. [5 points]
2. Prove that your DFA is correct when  $k = 2$ . [5 points]

### Optional

**Problem 5.** [Category: Design] Design a DFA for the language  $L_{A2} = \{w \in \{a, b\}^* \mid \text{number of } a\text{'s in } w \text{ is at least 2 or the number of } b\text{'s is at most 1}\}$ .

**Problem 6.** [Category: Design] Design a DFA for the language  $L_B = \{w \in \{a, b\}^* \mid w \text{ has at least 2 } as \text{ and ends with } ab\}$