Math 243

$\boxed{8.4}$ is $\sum a_n$ conv? divergent?

Alt. Series Test $(b_n > 0)$

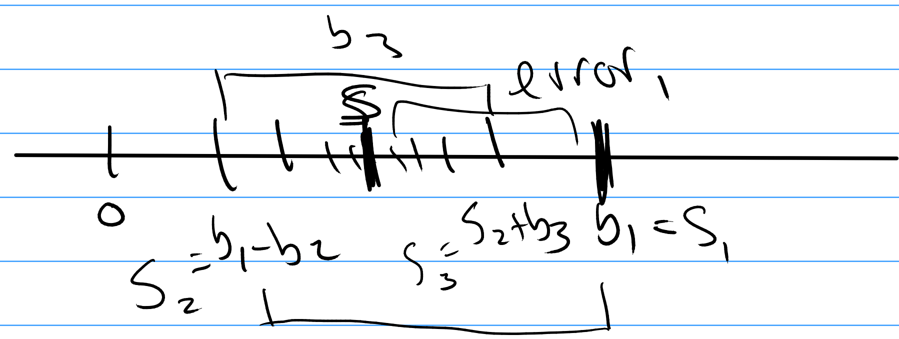
$$\sum_{n=1}^{\infty} (-1)^{n-1} \boxed{b_n} = b_1 - b_2 + b_3 - b_4 + \cdots$$

if ① $0 \le b_{n+1} \le b_n$ (non-inc.)

② $\lim_{n\to\infty} b_n = 0$

then $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ converges.

Estimation:



$$|R_n| = |S - S_n| \le b_{n+1}$$

b_2

$\boxed{ex}$ $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ alternating Harmonic series

$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \cdots$$

a) it is an alt. series

so ① is it non-inc? ② $\lim_{n\to\infty} \frac{1}{n} = 0$?

① is $\frac{1}{n}$ non-inc.? yes

② $\lim_{n\to\infty} \frac{1}{n} = 0$? yes

so $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ is conv.

(Find an approx. ans)

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \boxed{\frac{1}{4}} + \frac{1}{5} - \frac{1}{6} + \cdots$$

$S \approx 1 - \frac{1}{2} + \frac{1}{3}$ error ≤ 0.25

or $S \approx 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9}$

error ≤ 0.1

What about $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$?

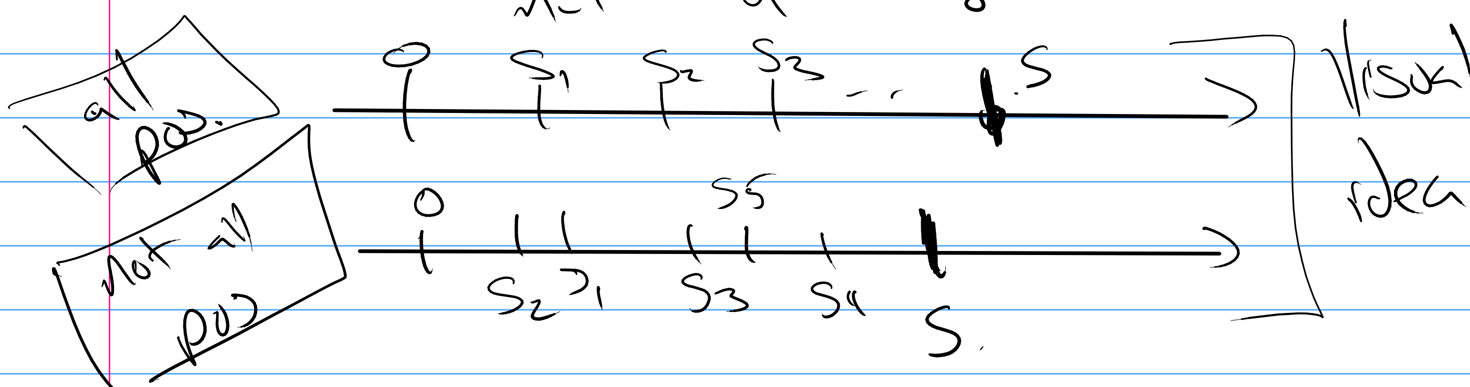
well $\sum_{n=1}^{\infty} \left| \frac{\sin(n)}{n^3} \right|$ is one we can do.

$\sum_{n=1}^{\infty} \frac{|\sin(n)|}{n^3}$ guess conv. & use comparison test

$$\frac{|\sin(n)|}{n^3} \le \frac{1}{n^3} \leftarrow \text{conv. by p-series}$$

$\Rightarrow$ $\sum_{n=1}^{\infty} \left| \frac{\sin(n)}{n^3} \right|$ is conv. by comparison.

What does this say about the $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$?



Thm: if $\sum |a_n|$ converges $\Rightarrow$ $\sum a_n$ converges

Def: if $\sum |a_n|$ converges we say $\sum a_n$ is <u>abs. convergent</u>

[ex] know $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ converges

what about abs. convergence?

$$\sum_{n=1}^{\infty} \left| (-1)^{n-1} \frac{1}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$$

divergent.

So far $\sum_{n=1}^{\infty} a_n$

you have 3 possible answers.

① $\sum_{n=1}^{\infty} a_n$ diverges

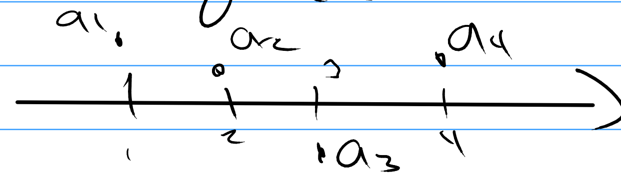
② $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges — conditionally convergent

③ $\sum_{n=1}^{\infty} |a_n|$ converges $\rightarrow$ $\sum a_n$ conv.

(abs. convergence)

Tests for abs. convergence of

$$\sum_{n=1}^{\infty} a_n$$



① <u>Ratio Test</u>

a) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$

$\Rightarrow \sum a_n$ is abs. conv.

b) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$

$\Rightarrow \sum a_n$ is divergent

c) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$

<u>test fails</u>! (try something else)

ex's $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2}{2^{n+1}}}{\frac{n^2}{2^n}} \right| = \lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{\cancel{2^n}}{n^2}$$

$$= \frac{1}{2} \lim_{n\to\infty} \frac{n^2+2n+1}{n^2} = \frac{1}{2}\cdot 1 = \frac{1}{2} < 1$$

$$\Rightarrow \boxed{\sum \frac{n^2}{2^n} \text{ is abs. conv.}}$$

ex $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$

abs conv?

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} \frac{n+1}{(n+1)^2+1}}{(-1)^n \frac{n}{n^2+1}} \right|$$

$$= \lim_{n\to\infty} \frac{(n+1)}{((n+1)^2+1)} \cdot \frac{(n^2+1)}{(n)} = 1$$ ratio test failed

try $\sum_{n=1}^{\infty} \left| (-1)^n \frac{n}{n^2+1} \right| = \sum_{n=1}^{\infty} \frac{n}{n^2+1}$ guess divergent

use limit comparison

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{n}{n^2+1}} = \lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$$

pos. const.

$\rightarrow$ $\frac{1}{n}$ is like $\frac{n}{n^2+1}$

so $\sum \left| (-1)^n \frac{n}{n^2+1} \right|$ diverges.

what about $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$?

it is an alt. series.

alt. series test.

① $\frac{n}{n^2+1}$ is non-inc. true

② $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$ true

$\therefore \sum (-1)^n \frac{n}{n^2+1}$ is conv.

$\therefore$ $\boxed{\text{conditionally conv.}}$

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② $\boxed{\text{Root Test}}$

a) $\lim_{n\to\infty} |a_n|^{1/n} = L < 1 \rightarrow \sum a_n$ is abs. conv.

b) $\lim_{n\to\infty} |a_n|^{1/n} = L > 1 \rightarrow \sum a_n$ is divergent

c) $\lim_{n\to\infty} |a_n|^{1/n} = 1 \rightarrow$ test fails.

ex $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

try $\lim_{n\to\infty} \left| \frac{(-1)^n}{(\ln n)^n} \right|^{1/n} = \lim_{n\to\infty} \frac{1}{\ln n} = 0$

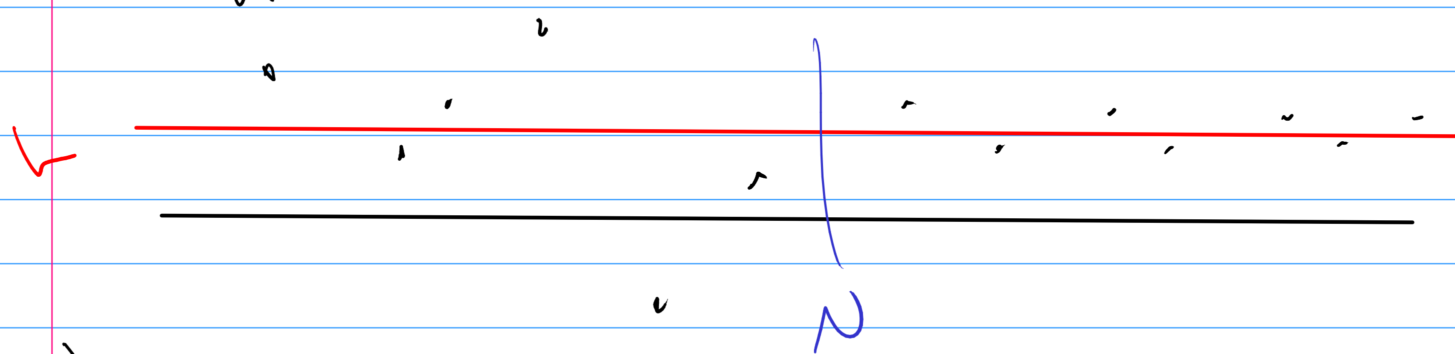
$\rightarrow \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$ is abs. conv.

(ex) $\sum_{n=1}^{\infty} (-1)^{n+1} = 1 - 1 + 1 - 1 + 1 - 1 + \cdots$

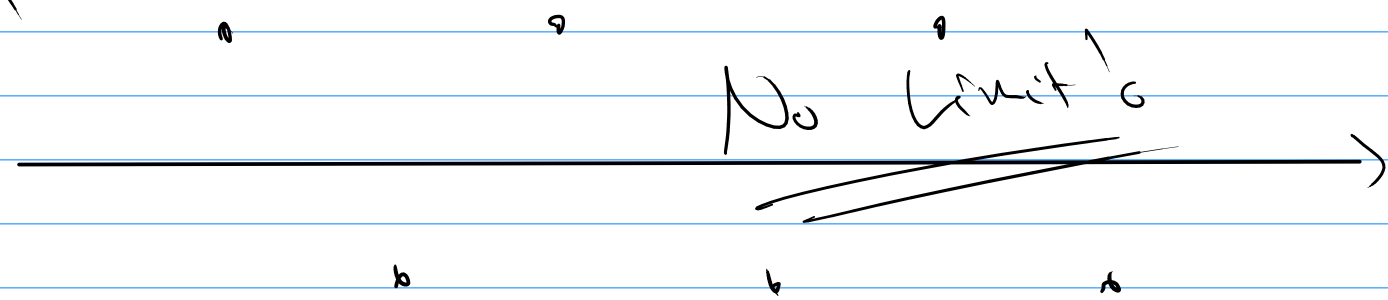
div. test.

$$\boxed{\sum a_n \text{ if } \lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum a_n \text{ is divergent}}$$

$\lim_{n\to\infty} (-1)^{n+1}$ <u>dne.</u>



$(-1)^{n+1}$



10.5 Power Series

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + \cdots$$

all $c_n \in \mathbb{R}$ (Real Constants) or coefficients let $f(x) = c_0 + c_1 x + c_2 x^2 + \dots$

$$f(x) = \boxed{\sum_{n=0}^{\infty} c_n x^n}$$

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ex $c_n = 1$

$$\sum_{n=0}^{\infty} (x)^n$$

test for abs. convergence

ratio test $\lim_{n\to\infty} \left| \frac{x^{n+1}}{x^n} \right| = \lim_{n\to\infty} |x| = \boxed{|x| = L}$

a) if $|x| < 1$ abs. conv.

b) if $|x| > 1$ divergent

c) if $|x| = 1$

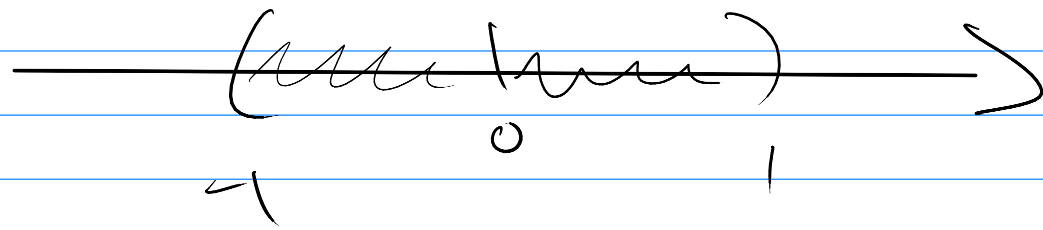
@ $x = 1$ or $x = -1$ test fails

$\sum_{n=0}^{\infty} 1^n$ $\sum_{n=0}^{\infty} (-1)^n$

divergent. divergent.

let $f(x) = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 +$

abs. conv. if $|x| < 1$



Note: building lost power @ this point several times so no movie.