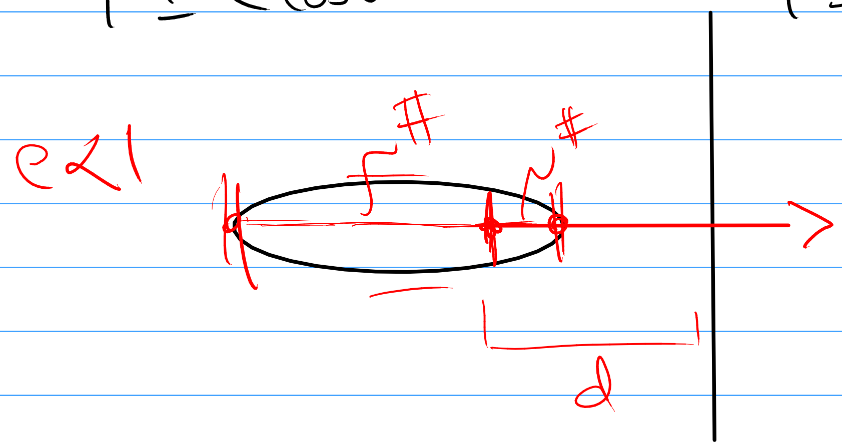
Math 243

Q's / 9.5 (23a, 24a, 25)

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad r = \frac{ed}{1 \pm e\sin\theta}$$



Final Exam

5 probs based on Midterm

11 probs from ch 8, 9

1 extra credit

① Vector based

→ operations on vectors

$+, -, |\ |, \cdot, \times$

→ Word Problem based on ops.

easy $\vec{v} = \langle 2, x, -1 \rangle \quad \vec{u} = \langle x^2, \frac{1}{x}, 6 \rangle$

$\vec{v} - \vec{u} = \langle 2 - x^2, x - \frac{1}{x}, -7 \rangle$

$|2\vec{v} + \vec{u}| = |\langle 4 + x^2, 2x + \frac{1}{x}, 4 \rangle|$

$= \sqrt{(4+x^2)^2 + (2x + \frac{1}{x})^2 + 16}$

$\vec{u} \cdot \vec{v} = 2x^2 - 5$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & x & -1 \\ x^2 & \frac{1}{x} & 6 \end{vmatrix}$$

$= \langle 6x + \frac{1}{x}, -(12 + x^2), \frac{2}{x} - x^3 \rangle$

ex <u>Word problem:</u> $\vec{F} = \vec{u} \quad \vec{r} = \vec{v}$

$|\hat{\tau}| = |\vec{F} \times \vec{r}| = |\vec{F}||\vec{r}| \sin\theta$

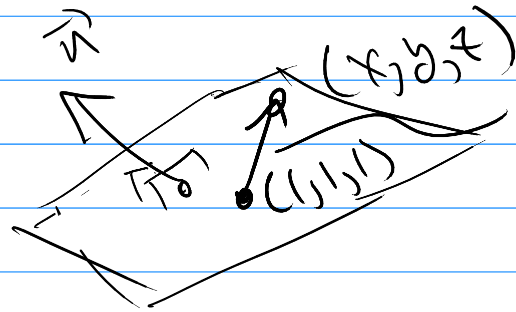
② Eqn's of lines & planes

to get a line need → ① pt
→ ② parallel vector

to get a plane need → ① pt
→ ② perpendicular vector

ex $\vec{n} = \langle 6, -3, 2 \rangle$

pt $= (1, 1, 1)$

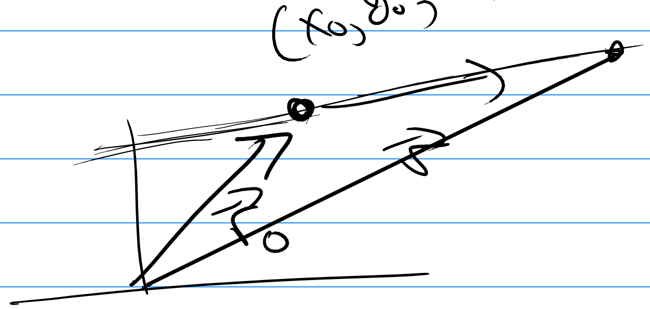


$\vec{v} = \langle x-1, y-1, z-1 \rangle$

$$6(x-1) - 3(y-1) + 2(z-1) = 0$$

$$6x - 3y + 2z = 5$$

$(0, 0, 5/2)$



$(1, 1, 1)$

ex $\langle x, y, z\rangle = \langle 1+t, 1+2t, 1+3t\rangle$

$= \langle 1,1,1\rangle + t\langle 1,2,3\rangle$

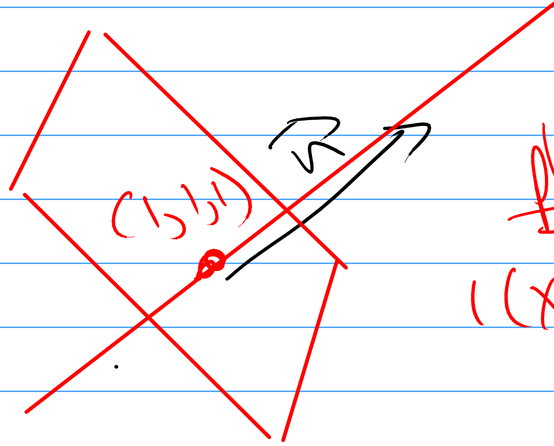
pt. $(1,1,1)$

$\vec{v} = \langle 1,2,3\rangle$ (parallel to)

$x = 1+t$, $y = 1+2t$, $z = 1+3t$

Find a plane through $(1,1,1)$ and perpendicular to the given line.

$\vec{n} = \langle 1,2,3\rangle$



Plane!

$1(x-1) + 2(y-1) + 3(z-1) = 0$

3 Integration by Parts.

ex $\int e^x \sin x \, dx$

$u = e^x \qquad du = e^x dx$

$dv = \sin x \, dx \qquad v = -\cos x$

$= -e^x \cos x + \int e^x \cos x \, dx$

$u = e^x \qquad du = e^x dx$

$dv = \cos x \, dx \qquad v = \sin x$

$= -e^x \cos x + \left[e^x \sin x - \int e^x \sin x \, dx \right]$

so:

$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$

$\rightarrow \quad 2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

$\int e^x \sin x \, dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + C$

(4) type 1 or type 2 Improper Integral

$\int_a^b f(x)\,dx$

$a=-\infty$ and/or $b=\infty$

$f(x)$ has a discontinuity on interval.

$$\int_{-\infty}^{\infty} \frac{1}{(x+1)^3}\,dx$$

$$\int_{-\infty}^{-2} \qquad \int_{-2}^{-1} \qquad \int_{-1}^{0} \qquad \int_{0}^{\infty}$$

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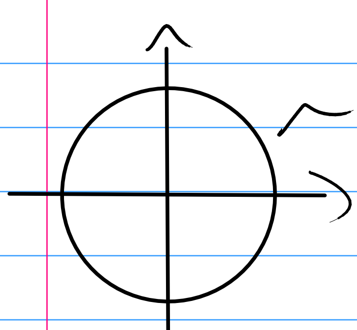
(5) Area or Volume by rotation.

⇝ Use trig sub.

area inside a circle of radius r.



$x^2+y^2=r^2$

$$2\int_{-r}^{r}\sqrt{r^2-x^2}\,dx$$

let $x=r\sin\theta$

$dx=r\cos\theta\, d\theta$

$$\rightarrow 2\int_{x=-r}^{x=r} \sqrt{r^2 - r^2\sin^2\theta}\; r\cos\theta\, d\theta$$

$$= 2r^2 \int_{x=-r}^{x=r} \sqrt{\underbrace{1-\sin^2\theta}_{\cos^2\theta}}\, \cos\theta\, d\theta$$

$$= 4r^2 \int_{x=0}^{x=r} \cos^2\theta\, d\theta$$

$$= \ldots \text{ finish.}$$

$x = r\sin\theta$

$x = r\tan\theta$ — p. 315

$x = r\sec\theta$

6 conv. seq → find the limit.

ex $\lim_{n\to\infty} \frac{n^3}{4n^3+n^2} = \lim_{n\to\infty} \frac{1}{4+\cancel{1/n}^{\,0}} = \frac{1}{4}$

$\lim_{n\to\infty} \frac{\ln(n)}{n}$ 0 $\triangleleft$

$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

(7) find the value of a series.

(ex) $\sum_{i=1}^{\infty} a_i = S$ $S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$\vdots$

$S = \lim_{n\to\infty} S_n$ $S_n = a_1 + a_2 + \cdots + a_n$

(8) (9) (10) Series conv.? divergent?

Know: divergence test, comparison test, p-series, Integral test, alt. series, etc.

(11) Maclaurin (+) radius of conv.

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots$$

$$= \boxed{\sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} x^n}$$

ex $f(x) = \sin x$ $\qquad f(0) = 0$

$f'(x) = \cos x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x$ $\qquad f''(0) = 0$

$f'''(x) = -\cos x$ $\qquad f'''(0) = -1$

$f^{(4)}(x) = \sin x$ $\qquad f^{4}(0) = 0$

$$\sin x = 0 + 1 \cdot x^1 + \frac{0 \cdot x^2}{2!} - \frac{1 \cdot x^3}{3!} + \frac{0 \cdot x^4}{4!} + \frac{1 \cdot x^5}{5!} + \ldots$$

$$\sin x = \sum_{n=0}^{\infty} \boxed{\frac{(-1)^n x^{2n+1}}{(2n+1)!}}$$

Consider $\lim_{n\to 0} \left| \frac{a_{n+1}}{a_n} \right| < 1$ Convergent

ex of Radius of conv

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \{ \} = \lim_{n\to\infty} |x| \cdot \frac{n}{2n+1}$$

miracle happens

$$= \boxed{\frac{|x|}{2} < 1}$$ to be conv.

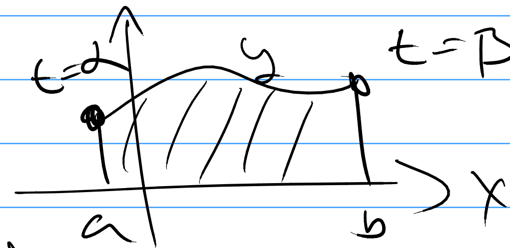
$|x| < 2 = R$

(12) Taylor (+) radius of conv.

(13) Parametric Curve

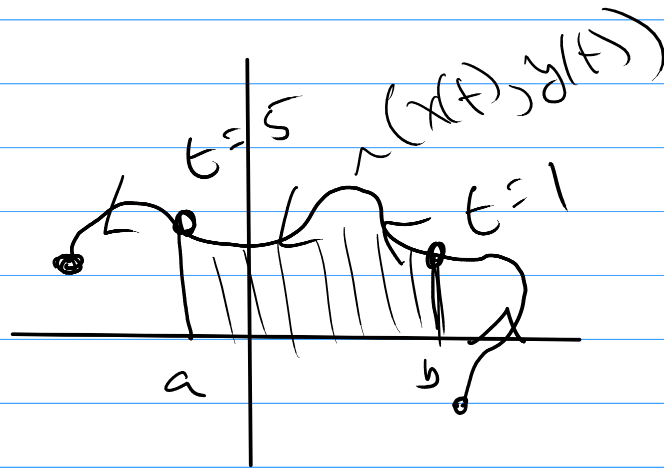
Area or Arc Length

$$A = \int_a^b y \, dx$$



$$A = \int_\alpha^\beta \overbrace{y(t)}^{y} \, \overbrace{x'(t)\, dt}^{dx}$$

ex



$$A = \int_5^1 y(t)\, x'(t)\, dt$$

arc length / $$L = \int_\alpha^\beta \sqrt{(x'(t))^2 + (y'(t))^2} \, dt$$

(14) Derivatives of parametric curves

$\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

$y = t^{1/2} \cdot \tan(t)$

$x = \sec(t)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{2}t^{-1/2}\tan t + t^{1/2}\sec^2 t}{\sec t \tan t}$$

$$= \frac{1}{2}t^{-1/2}\cos t + t^{1/2}\csc t$$

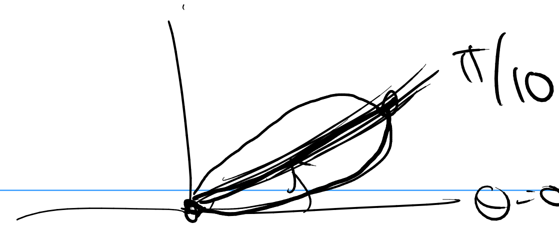
$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{1}{2}t^{-1/2}\cos t + t^{1/2}\csc t\right]}{\frac{d}{dt}[\sec t]}$$

$= \underline{\text{finish}}$

(15) Area of Polar Curve

$$A = \int_{\theta=\alpha}^{\theta=\beta} \frac{1}{2}r^2\,d\theta \quad \text{where } r = f(\theta)$$

$r = \sin(5\theta)$



$1 = \sin\left(\frac{\pi}{2}\right)$

$\frac{\pi}{2} = 5\theta$

$\underbrace{\theta = 0 \quad \text{to} \quad \theta = \pi/10}_{1/2 \text{ of one petal}}$

$$\text{Area petal} = 2\int_0^{\pi/10} \frac{1}{2}\sin^2(5\theta)\, d\theta$$

(16) Arc length of Polar Curve

$$L = \int_{\theta=\alpha}^{\theta=\beta} \sqrt{r^2 + r'^2}\, d\theta$$

ex $r = \sin 5\theta$

$$\text{arc length of one petal} = 2\int_0^{\pi/10} \sqrt{\sin^2 5\theta + 25\cos^2 5\theta}\, d\theta$$

$\boxed{\text{extra credit}}$ p. 514 (23a + 24a + 25)