CIS 560 – Database System Concepts

Lecture 10

Functional Dependencies and Normalization

September 18, 2013

Credits for slides: Suciu, Chang, Ullman.

Announcements

- HW3 due Friday
- HW4 will be posted Friday, due September 27th
- Exam 1 – October 7th (sample exam posted on KSOL)
- Project information posted on KSOL
- Project proposals
  - Information about team members and English description due September 24th
  - E/R diagram and relational schema due October 4th
- Proposal presentations October 9th and 11th

Outline

Last time:

- DB Design: Functional Dependencies (3.1 – 3.2)

Today:

- DB Design: Functional Dependencies (3.1 – 3.2)
- DB Design: Normalization (3.3-3.4)

Next:

- Transactions in SQL

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Review

- Data anomalies?
- Functional dependencies?
- Armstrong's rules?

Goal: Find ALL Functional Dependencies

Anomalies occur when certain “bad” FDs hold

- We know some of the FDs
- Need to find *all* FDs, then look for the bad ones
- With *closure* we can find all FD’s easily

Closure of a set of Attributes

Given a set of attributes $A_1, \ldots, A_n$

The **closure**, $\{A_1, \ldots, A_n\}^+$ = the set of attributes B s.t. $A_1, \ldots, A_n \rightarrow B$

Example:

name → color
category → department
color, category → price

Closures:

name$^+$ = {name, color}
{name, category}$^+$ = {name, category, color, department, price}
color$^+$ = {color}

Closure Algorithm

$X=\{A_1, \ldots, A_n\}$.

Repeat until X doesn't change **do**:

if $B_1, \ldots, B_n \rightarrow C$ is a FD **and** $B_1, \ldots, B_n$ are all in X
then add C to X.

Example:

name → color
category → department
color, category → price

$\{name, category\}^+ =$
{ }

Closure Algorithm

$X=\{A_1, \ldots, A_n\}$.

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Example:

name → color
category → department
color, category → price

$\{name, category\}^+ =$
{ name, category, color, department, price }

Closure Algorithm

$X = \{A_1, \ldots, A_n\}$.

Repeat until X doesn't change **do**:

if $B_1, \ldots, B_n \rightarrow C$ is a FD **and**
$B_1, \ldots, B_n$ are all in X
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Example:

name → color
category → department
color, category → price

$\{\text{name, category}\}^+ =$
{ name, category, color, department, price }

Hence: name, category → color, department, price

Example

In class:

R(A,B,C,D,E,F)

A, B → C
A, D → E
B → D
A, F → B

Compute $\{A,B\}^+$ X = {A, B, }

Compute $\{A, F\}^+$ X = {A, F, }

Example

In class:

R(A,B,C,D,E,F)

$$
\begin{array}{l}
A, B \rightarrow C \\
A, D \rightarrow E \\
B \rightarrow D \\
A, F \rightarrow B
\end{array}
$$

Compute $\{A,B\}^+$ $X = \{A, B, C, D, E\}$

Compute $\{A, F\}^+$ $X = \{A, F, B, C, D, E\}$

Why Do We Need Closure

- With closure we can find all FDs easily
- To check if $X \rightarrow A$
  - Compute X^+
  - Check if $A \in X^+$

Using Closure to Infer ALL FDs

Example:

$A, B \rightarrow C$
$A, D \rightarrow B$
$B \rightarrow D$

Step 1: Compute X^+, for every X:

Step 2: Enumerate all FD's $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \varnothing$:

Using Closure to Infer ALL FDs

Example:

$A, B \rightarrow C$
$A, D \rightarrow B$
$B \rightarrow D$

Step 1: Compute X^+, for every X:

$A^+ = A$, $B^+ = BD$, $C^+ = C$, $D^+ = D$
$AB^+ = ABCD$, $AC^+ = AC$, $AD^+ = ABCD$,
$BC^+ = BCD$, $BD^+ = BD$, $CD^+ = CD$
$ABC^+ = ABD^+ = ACD^+ = ABCD$ (no need to compute– why ?)
$BCD^+ = BCD$, $ABCD^+ = ABCD$

Step 2: Enumerate all FD's $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \varnothing$:

Using Closure to Infer ALL FDs

Example:

A, B	$\rightarrow$	C
A, D	$\rightarrow$	B
B	$\rightarrow$	D

Step 1: Compute X^+, for every X:

$A^+ = A$, $B^+ = BD$, $C^+ = C$, $D^+ = D$
$AB^+ = ABCD$, $AC^+ = AC$, $AD^+ = ABCD$,
$BC^+ = BCD$, $BD^+ = BD$, $CD^+ = CD$
$ABC^+ = ABD^+ = ACD^+ = ABCD$ (no need to compute– why ?)
$BCD^+ = BCD$, $ABCD^+ = ABCD$

Step 2: Enumerate all FD's $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \emptyset$:

$AB \rightarrow CD$, $AD \rightarrow BC$, $BC \rightarrow D$, $ABC \rightarrow D$, $ABD \rightarrow C$, $ACD \rightarrow B$

Another Example

- Enrollment(student, major, course, room, time)

  student $\rightarrow$ major
  major, course $\rightarrow$ room
  course $\rightarrow$ time

What else can we infer ? [at home]

Keys

- A **superkey** is a set of attributes $A_1, \ldots, A_n$ s.t. for any other attribute B, we have $A_1, \ldots, A_n \rightarrow B$
- A **key** is a minimal superkey
  - I.e. set of attributes which is a superkey and for which no subset is a superkey

Computing (Super)Keys

- Compute X^+ for all sets X
- If X^+ = all attributes, then X is a (super)key
- List only the minimal X's

Example

Product(name, price, category, color)

name, category $\rightarrow$ price
category $\rightarrow$ color

What is the key?

Example

Product(name, price, category, color)

name, category $\rightarrow$ price
category $\rightarrow$ color

What is the key ?

$(\text{name, category})^{+} = \{\text{name, category, price, color}\}$

Hence (name, category) is a key

Examples of Keys

Enrollment(student, address, course, room, time)

student → address
room, time → course
student, course → room, time

Examples of Keys

Enrollment(student, address, course, room, time)

student → address
room, time → course
student, course → room, time

Keys: {student, room, time}, {student, course} and all supersets

Key or Keys?

Can we have more than one key?

Given R(A,B,C) define FD's s.t. there are two or more keys

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Can we have more than one key?

Given R(A,B,C) define FD's s.t. there are two or more keys

$AB \rightarrow C$ $BC \rightarrow A$	or	$A \rightarrow BC$ $B \rightarrow AC$

What are the keys here?

Can you design FDs such that there are *three* keys?

Eliminating Anomalies

Main idea:

- $X \to A$ is OK if X is a (super)key
- $X \to A$ is not OK otherwise

Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Topeka
Fred	123-45-6789	206-555-6543	Topeka
Joe	987-65-4321	908-555-2121	Manhattan
Joe	987-65-4321	908-555-1234	Manhattan

SSN → Name, City

What is the key?

{SSN, PhoneNumber}

Hence SSN → Name, City is a "bad" dependency

Boyce-Codd Normal Form

A simple condition for removing anomalies from relations:

A relation R is in BCNF if:

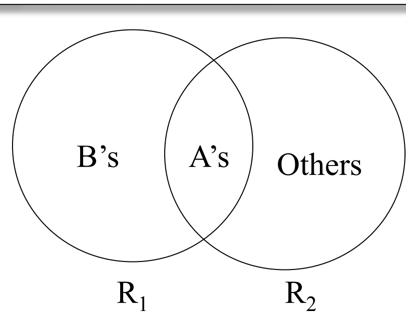
If $A_1, ..., A_n \rightarrow B$ is a non-trivial dependency in R, then $\{A_1, ..., A_n\}$ is a superkey for R

In other words: there are no "bad" FDs

Equivalently:
$\forall$ X, either $(X^+ = X)$ or $(X^+ = \text{all attributes})$

BCNF Decomposition Algorithm

repeat
choose $A_1, \ldots, A_m \rightarrow B_1, \ldots, B_n$ that violates BNCF
split R into $R_1(A_1, \ldots, A_m, B_1, \ldots, B_n)$ and $R_2(A_1, \ldots, A_m, [\text{others}])$
continue with both R_1 and R_2
until no more violations



Is there a 2-attribute relation that is not in BCNF?

In practice, we have a better algorithm (coming up)

Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Topeka
Fred	123-45-6789	206-555-6543	Topeka
Joe	987-65-4321	908-555-2121	Manhattan
Joe	987-65-4321	908-555-1234	Manhattan

SSN → Name, City

What is the key?

{SSN, PhoneNumber}

use SSN → Name, City to split

Example

Name	SSN	City
Fred	123-45-6789	Topeka
Joe	987-65-4321	Manhattan

SSN → Name, City

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Let's check anomalies:

- Redundancy?
- Update?
- Delete?

BCNF Decomposition Algorithm

BCNF_Decompose(R)

find X s.t.: $X \neq X^+ \neq$ [all attributes]

<u>if</u> (not found) **<u>then</u>** "R is in BCNF"

<u>let</u> $Y = X^+ - X$
<u>let</u> $Z =$ [all attributes] $- X^+$
decompose R into $R1(X \cup Y)$ and $R2(X \cup Z)$
continue to decompose recursively R1 and R2

Find X s.t.: $X \neq X^+ \neq$ [all attributes]

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN → name, age
age → hairColor

Find X s.t.: $X \neq X^+ \neq$ [all attributes]

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN → name, age

age → hairColor

Iteration 1: Person

SSN^+ = {SSN, name, age, hairColor}

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)

Find X s.t.: $X \neq X^+ \neq$ [all attributes]

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN → name, age

age → hairColor

Iteration 1: Person

SSN^+ = {SSN, name, age, hairColor}

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)

Iteration 2: P

age^+ = {age, hairColor}

Decompose: People(SSN, name, age)

Hair(age, hairColor)

Phone(SSN, phoneNumber)