CIS770 Homework 1

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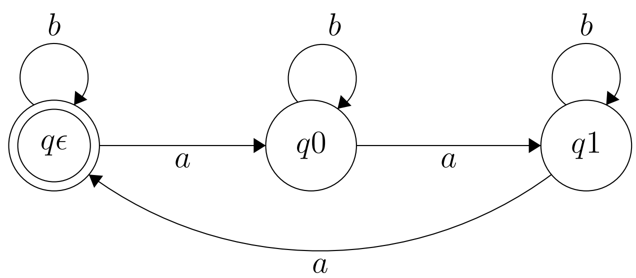
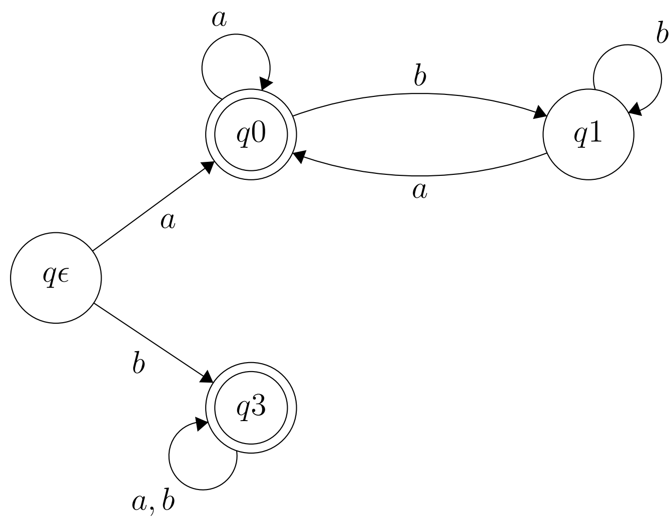
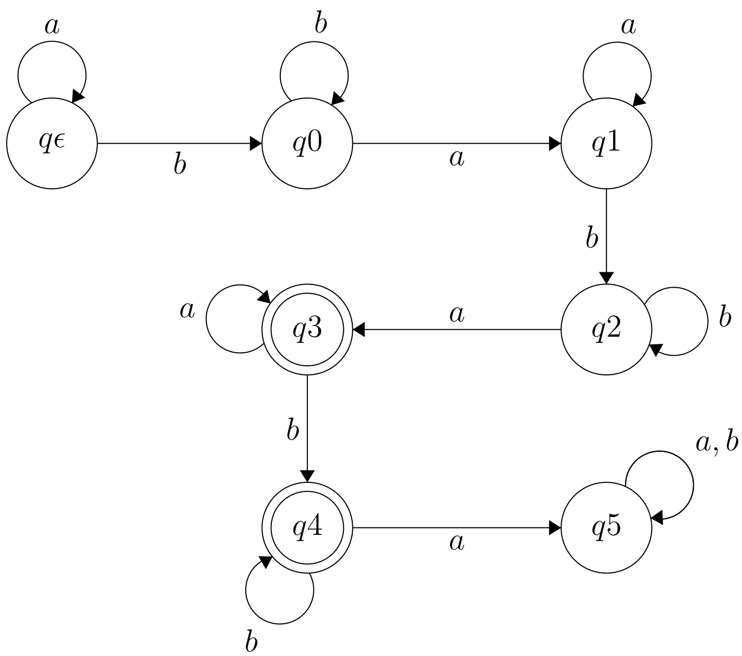
1. Design a DFA for the language L_{A1} = w $\in$ {a,b}|number of as in w is not divisible by 3.



2. Design a DFA for the language L_{A3} = w $\in$ {a,b}|if w starts with an a then it does not end with a b.



3. Design a DFA for the language $L_{A4} =$ w $\in$ {a,b} $|ba$ appears exactly twice as a substring.

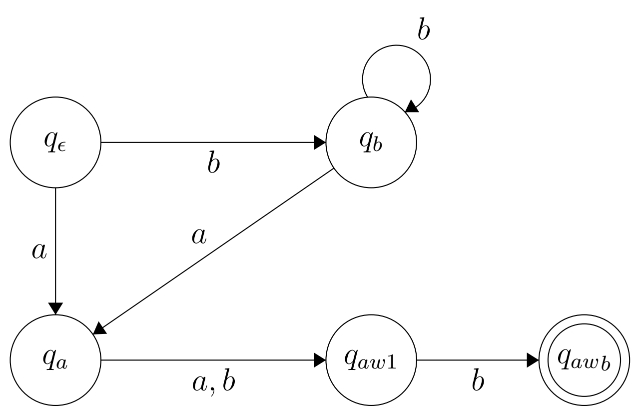


4. Let $A_{\mathrm{k}} \subseteq \{a, b\}^*$ be the collection of strings w where there is a position i in w such that the symbol at position i (in w) is a, and the symbol at position $i + k$ is b. For example, consider A_2 (when $k = 2$). $b$aab $\in A_2$ because the second position ($i = 2$) has an$a$ and the fourth position has ab. On the other hand, $bb \notin A_2$ (because there are no as) and $a$ba $\notin A_2$ (because none of the as are followed by a b 2 positions away).

4-1. Design a DFA for language A_{k}. Your formal description (by listing states, transitions, etc. and not drawing the DFA) will depend on the parameter k but should work no matter what k is; see lecture 2, last page for such an example.

$M_k = (Q, \{a, b\}, \delta, q_0, F)$
$\delta(q_\epsilon, a) = q_a$
$\delta(q_\epsilon, b) = q_b$
$\delta(q_b, a) = q_a$
$\delta(q_b, b) = q_b$
$\delta(q_a, a) = q_{aw}$
$\delta(q_a, b) = q_{aw}$
where $q_{aw} = q_{aw}$1 ... q_{aw}k-1
$\delta(q_{aw}, a) = q_{aw}$ if |aw|< k-1
$\delta(q_{aw}, b) = q_{aw}$ if |aw|< k-1 otherwise q_{awb}
$q_0 = q_\epsilon$
$F = q_{awb}$

4-2. Prove that your DFA is correct when $k = 2$.
Below is a DFA for the K = 2 case



Assume $i = 3$ and $K = 2$
Base: $w = \epsilon$
Because $|w|= 0$ the word is rejected

Hypothesis:
$w = ba\mathbf{a}bbaaab$
$w_i = a$ noted in bold above

If we choose a position i where w[i] = a, it is only neccesary to test the substring w[i] to w[i+k]. Therefore if w[i] is assumed to be the first character read into the DFA the following transition sequence is followed:
$(q_e, a) = a$
$(q_a, b) = q_{aw1}$
$(q_{aw}, b) = q_{awb}$ because |aw|= k-1

Because we reached the finish state following the set transitions, the DFA holds true.