

QUIZ 2

Name:

Time: Feb 4, 2016

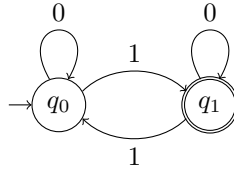
Instructions: Please write down the correct answer for each question in the following box.

1	2	3	4	Total Score

1. Consider the sequence defined inductively as follows: $a_0 = 0$, and $a_n = a_{\lceil n/2 \rceil} + a_{\lfloor n/2 \rfloor}$. We claim that $a_n = n$ for all n . We prove this by induction. For the base case observe that $a_0 = 0$ by definition. Assume that for all $n < k$, we have $a_n = n$. Now $a_k = a_{\lceil k/2 \rceil} + a_{\lfloor k/2 \rfloor}$ by definition. From the induction hypothesis, we have $a_{\lceil k/2 \rceil} = \lceil k/2 \rceil$ and $a_{\lfloor k/2 \rfloor} = \lfloor k/2 \rfloor$. Thus, $a_k = a_{\lceil k/2 \rceil} + a_{\lfloor k/2 \rfloor} = \lceil k/2 \rceil + \lfloor k/2 \rfloor = k$. Thus, the claim is established by induction.

Notes: $\lceil k/2 \rceil$ is the largest integer not greater than $k/2$ and $\lfloor k/2 \rfloor$ is the smallest integer not less than $k/2$. For example, if $k = 5$, then $\lfloor 5/2 \rfloor = 2$, $\lceil 5/2 \rceil = 3$.

- (A) The proof is correct.
- (B) For some values of k , the induction hypothesis does not allow us to conclude $a_{\lceil k/2 \rceil} = \lceil k/2 \rceil$
- (C) For some values of k , the induction hypothesis does not allow us to conclude $a_{\lfloor k/2 \rfloor} = \lfloor k/2 \rfloor$.
- (D) For some values of k , $\lceil k/2 \rceil + \lfloor k/2 \rfloor \neq k$.



2. Consider the automaton M_1 shown above. We will show that $w \in \mathbf{L}(M_1)$ iff w has an odd number of 1s. We will prove this by induction on the length of w . For the base case, observe that when $w = \epsilon$, w has an even number of 1s. Further, $q_0 \xrightarrow{\epsilon} M_1 q_0$, and so M_1 does not accept w . For the induction step, consider two cases. When $w = 0u$, w has an odd number of 1s iff u has an odd number of 1s, iff (by ind. hyp.) $q_0 \xrightarrow{u} M_1 q_1$ iff $q_0 \xrightarrow{w=0u} M_1 q_1$ (since $\delta(q_0, 0) = q_0$). When $w = 1u$, w has an odd number of 1s iff u has an even number of 1s iff (by ind. hyp.) $q_1 \xrightarrow{u} M_1 q_1$ iff $q_0 \xrightarrow{w=1u} M_1 q_1$ (since $\delta(q_0, 1) = q_1$). Thus, the claim is established by induction.

- (A) The proof is correct.
- (B) The proof is incorrect because not all possible strings w have been considered in the induction step.
- (C) The proof is incorrect because there is no basis for concluding that $q_1 \xrightarrow{u} M_1 q_1$ by induction hypothesis in the case when $w = 1u$.
- (D) The proof is incorrect because the base case has not been proved.

3. Recall the language $L_2 \subseteq \{0, 1\}^*$ defined as

$$L_2 = \{u10 \mid u \in \{0, 1\}^*\} \cup \{u11 \mid u \in \{0, 1\}^*\}$$

That is, L_2 contains all strings that have a 1 as the second last symbol. Consider the following proof that any DFA recognizing L_2 must have at least 4 states: Let M be a DFA with initial state q_0 recognizing L_2 having less than 4 states. Let $q_0 \xrightarrow{00}_M A$, $q_0 \xrightarrow{01}_M B$, $q_0 \xrightarrow{10}_M C$, $q_0 \xrightarrow{11}_M D$, where A, B, C, D are some states of M . Now, if M has less than 4 states, then two out of A, B, C, D must be the same state. If $A = B$ then M either accepts both 000 and 010 or neither; but only 010 should be accepted. If $A = C$ then M either accepts both 00 and 10 or neither; again only 10 must be accepted. If $A = D$ then M either accepts both 00 and 11 or neither; however, only 11 must be accepted. If $B = C$ then M either accepts both 01 and 10 or neither; again only 10 should be accepted. If $B = D$ then M either accepts both 01 and 11 or neither; only 11 should be accepted. Finally, if $C = D$ then M either accepts both 100 and 110 or neither; however, only 110 should be accepted. Thus, A, B, C, D must all be different, and M cannot have less than 4 states.

- (A) The proof is correct.
 - (B) The proof is incorrect because there is a DFA with 3 states that recognizes L_2 .
 - (C) The proof does not show that there is no DFA with less than 4 states accepting L_2 . It only shows that a specific DFA M that has 5 states q_0, A, B, C , and D cannot accept L_2 .
 - (D) The proof is incorrect because there is no basis for assuming that two out of A, B, C, D must be the same.
4. Recall the language $L_2 \subseteq \{0, 1\}^*$ defined as

$$L_2 = \{u10 \mid u \in \{0, 1\}^*\} \cup \{u11 \mid u \in \{0, 1\}^*\}$$

That is, L_2 contains all strings that have a 1 as the second last symbol. Consider the following proof that any DFA recognizing L_2 must have at least 4 states: Any DFA recognizing L_2 has to remember the last two symbols of the input string. Since there are 4 strings of length 2, it must have 4 states.

- (A) The proof is correct.
- (B) The proof is incorrect because there is a DFA with 3 states that recognizes L_2 .
- (C) The proof is incorrect because there is no basis for assuming that a DFA recognizing L_2 has to remember that last two symbols of the input string.
- (D) The proof is incorrect because this statement has to be proved by induction.