Quiz 4

1. Recall that regular languages are closed under union. Based on this observation which of the following is necessarily true?

   (A) If L_1 and L_2 are regular then $L_1 \cup L_2$ is regular.

   (B) If $L_1 \cup L_2$ is regular then L_1 and L_2 is regular.

   (C) $L_1 \cup L_2$ is regular.

   (D) All of the above.

   Correct answer is (A). (B) is not true because if L_1 be a non-regular language, and L_2 be its complement, then their union is the set of all strings over the alphabet, which is regular. (C) is not true because we do not know whether L_1 and L_2 are regular in the first place.

2. Recall that regular languages are closed under complementation. Based on this observation, we have three statements: (1) If L is regular then $\overline{L}$ is regular; (2) If $\overline{L}$ is regular then L is regular; (3) $L \cup \overline{L}$ is regular. Which of the following is necessarily true about these three statements?

   (A) (1)(2) is true, but (3) is false.

   (B) (2)is true, but (1)(3) are false.

   (C) (3) is true, but (1)(2) are false.

   (D) (1)(2)(3) are all true.

   Correct answer is (D). The first statement essentially states that regular languages are closed under complementation. The second statement is correct, because $L = \overline{\overline{L}}$, and since $\overline{L}$ is known to be regular, so must be $\overline{\overline{L}}$. The third statement is right since for any L (regular or non-regular), $L \cup \overline{L}$ is always the set of all strings over the alphabet, which is regular.

3. Consider the following alternate proof that regular languages are closed under Kleene closure that uses other closure properties. "Since regular languages are closed under concatenation, if L is regular then so is L^i for any i. Next, since regular languages are closed under union, it follows that $L^* = \cup_{i \geq 0} L^i$ is regular, if L is regular."

   (A) The proof is correct.

   (B) The proof is incorrect because closure under concatenation does not imply that if L is regular then so is L^i for any i.

   (C) The proof is incorrect because closure under union does not imply that if each L^i is regular then $\cup_{i \geq 0} L^i$ is regular.

   (D) The proof is incorrect because $L^* \neq \cup_{i \geq 0} L^i$.

   Correct answer is (C). The proof fails because closure under union doesn't mean that regular languages are closed under infinite union.

4. Let $h : \{0,1\}^* \to \{a\}^*$ be a homomorphism defined as follows: $h(0) = a$ and $h(1) = \epsilon$. Let $L_{0n1n} = \{0^n 1^n \mid n \geq 0\}$. Taking $A \subset B$ to mean A is a proper subset of B, which of the following is true?

   (A) $h^{-1}(h(L_{0n1n})) = L_{0n1n}$

(B) $h^{-1}(h(L_{0n1n})) \subset L_{0n1n}$

(C) $L_{0n1n} \subset h^{-1}(h(L_{0n1n}))$

(D) $h^{-1}(h(L_{0n1n})) \cap L_{0n1n} = \emptyset$

The correct answer is (C). By definition, $h^{-1}(h(L_{0n1n}))$ is the set of all strings w such that $h(w) \in h(L_{0n1n})$. It is easy to see that L_{0n1n} is contained in $h^{-1}(h(L_{0n1n}))$, because by definition, for any string $w \in L_{0n1n}$, $h(w)$ is in $h(L_{0n1n})$. This shows (D) is false. Because $h(1) = \epsilon$, we cannot say anything in general about the occurrence of the symbol 1 in any string w such that $h(w) \in h(L_{0n1n})$. In particular this means that w is not necessarily in L_{0n1n}. For instance, $h(00) = h(0011) = aa$, so that $00 \in h^{-1}(h(L_{0n1n}))$ even while $00 \notin L_{0n1n}$. This shows that (A) and (B) are false, and that L_{0n1n} is a proper subset of $h^{-1}(h(L_{0n1n}))$.

5. For $n \geq 0$, let $K_n = \{a^i b^k \mid i \geq n,\ 0 < k < n\}$. Which of the following is true?

   (A) K_n is regular for all values of n

   (B) K_n is not regular for any value of n

   (C) There is an N_0 such that K_n is regular for all $n \leq N_0$ but not regular for $n > N_0$

   (D) The regularity of K_n depends on the value of n and cannot be described in a simple manner.

   Correct answer is (A). K_n can be defined as a finite union of regular languages as follows:

$$K_n = \bigcup_{0<k<n} L(a^n a^* b^k)$$

   ,

   where x^m denotes m consecutive occurences of symbol x.

6. Consider the following proof showing that $L = \mathbf{L}(0^*1^*)$ does not satisfy the pumping lemma. Let p be the pumping length. Consider the string $w = 001^p \in L$. Consider a $x = 0$, $y = 01$ and $z = 1^{p-1}$. Now observe that $xy^2z = 001011^{p-1} \notin L$. Hence, L does not satisfy the pumping lemma.

   (A) This proof demonstrates that L does not satisfy the pumping lemma.

   (B) This proof only shows that one particular w cannot be pumped. That is not enough to show that L does not satisfy the pumping lemma.

   (C) This proof only shows that a specific division of w into x, y, and z cannot be pumped. That is not enough to prove that L does not satisfy the pumping lemma.

   (D) This proof only shows that a specific value of the pumping length p is not correct. That is not enough to show that L does not satisfy the pumping lemma.

   Correct answer is (C). The pumping lemma only states that there must exist some division for each string of length p or more, such that the pumping criterion is satisfied. 001^p can be divided into $x = 00$, $y = 1$, $z = 1^{p-1}$, which will satisfy criterion. So, the string doesn't viloate pumping lemma.

7. Let $L \subseteq \Sigma^*$ be a language such that L satisfies the pumping lemma. What can we say about L?

   (A) L is regular.

   (B) L is not regular.

   (C) L may or may not be regular.

   (D) $\Sigma^* \setminus L$ is regular.

   Correct answer is (C). Pumping lemma is a necessary but not sufficient condition for regularity. For eg., the language L_1 over $\Sigma = \{0, a, b\}$, defined as $L_1 = \{0a^n b^n | n \geq 0\} \cup L((\epsilon \cup 000^*)(a \cup b)^*)$ satisfies pumping lemma, but is not regular. (The part on the RHS of the union is regular. The part on the LHS is not, but any string from the LHS can be pumped into a string in the RHS).