Quiz 1

1. Let $L = \{010, 101, 001, 011\}$, and $K = \{w \mid 0w \in L\}$. Which of the following strings is a member of K?
   (A) 0101
   (B) 01
   (C) 011
   (D) 0110

   The correct answer is (B). K is the set of strings formed by removing the leading 0 from a string in L. Thus, $K = \{10, 01, 11\}$.

2. Let Σ_1 and Σ_2 be two alphabets, with $\Sigma_1 \neq \Sigma_2$. Which of the following is necessarily true?
   (A) $\Sigma_1^* = \Sigma_2^*$
   (B) $\Sigma_1^n = \Sigma_2^n$ for all n
   (C) $|\Sigma_1^n| = |\Sigma_2^n|$, for all n. Here $|A|$ denotes the number of elements in the set A.
   (D) $\Sigma_1^0 = \Sigma_2^0$

   The correct answer is (D). $\Sigma^0 = \{\epsilon\}$ for any Σ. The first two options are wrong because $\Sigma_1 \neq \Sigma_2$ and hence, there is at least one symbol (say x) in one of them which is not present in the other. Strings containing x will be present in sets described in options (A) and (B) for one of the alphabets, but not the other. (C) is incorrect because the number of elements in the 2 alphabets may differ.

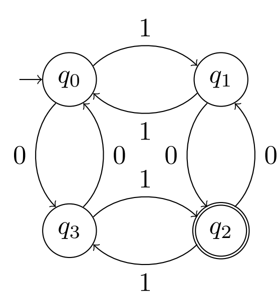

Figure 1: DFA M for questions 4 and 5

3. Consider the DFA M over the alphabet $\{0, 1\}$ shown in Figure 1. Which of the following strings is accepted by M?
   (A) ϵ
   (B) 0011
   (C) 1111000
   (D) 1011

   The correct answer is (D). $\hat{\delta}_M(q_0, \epsilon) = \{q_0\}$, $\hat{\delta}_M(q_0, 0011) = \{q_0\}$, $\hat{\delta}_M(q_0, 1111000) = \{q_3\}$, $\hat{\delta}_M(q_0, 1011) = \{q_2\}$. The only accepting state is q_2.

4. The language recognized by DFA M in Figure 1 is

   (A) $\{w \mid w$ has even length$\}$
   (B) $\{w \mid w$ has an odd number of 1s and an odd number of 0s$\}$
   (C) $\{w \mid w$ has an equal number of 0s and 1s$\}$
   (D) $\{w \mid w$ has an odd number of 1s$\}$.

   The correct answer is (B). It can be established by proving the following system of equivalences by induction on length of w.

   - $\hat{\delta}_M(q_0, w) = \{q_0\} \Leftrightarrow w$ has even number of 1s and even number of 0s
   - $\hat{\delta}_M(q_0, w) = \{q_3\} \Leftrightarrow w$ has even number of 1s and odd number of 0s
   - $\hat{\delta}_M(q_0, w) = \{q_1\} \Leftrightarrow w$ has odd number of 1s and even number of 0s
   - $\hat{\delta}_M(q_0, w) = \{q_2\} \Leftrightarrow w$ has odd number of 1s and odd number of 0s

   Observe that options (A) and (D) describe sets that are *strict* supersets of the set of strings accepted by the DFA.

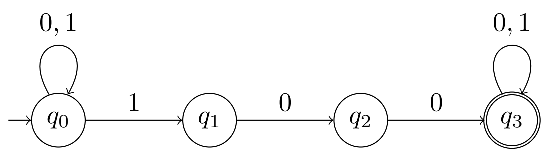

Figure 2: NFA N for problem 5

5. Consider the NFA N shown in Figure 2. Which of the following strings is *not* accepted by N?

   (A) 001
   (B) 001100
   (C) 10011001
   (D) 1001

   The correct answer is (A). Observe that any string accepted by N must contain a 1 followed by two 0s.

6. Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Recall that $\hat{\Delta} : Q \times \Sigma^* \to 2^Q$ is a function that given a state q and string w returns the set of all states that N could be in after reading w from state q. Formally, $\hat{\Delta}(q, w) = \{q' \mid q \xrightarrow{w}_N q'\}$. We can say that N accepts a string w iff

   (A) $\hat{\Delta}(q_0, w) \in F$
   (B) $\hat{\Delta}(q_0, w) = F$
   (C) $\hat{\Delta}(q_0, w) \cap F \neq \emptyset$
   (D) $\hat{\Delta}(q_0, w) \subseteq F$

   The correct answer is (C). An NFA accepts a string if and only if there is any way to reach a final state after reading the string. Therefore, all that we require is that the intersection of $\hat{\Delta}(q_0, w)$ and F is nonempty.