Homework 3
CS 770: Formal Language Theory

Assigned: Febuary 17, 2016 Due on: March 1, 2016

Instructions: This homework has 4 problems that can be solved individually. Please follow the homework guidelines given on the class website. Solutions not following these guidelines will not be graded.

Recommended Reading: Lectures 5,6,7, 8 and 9.

Problem 1. [Category: Comprehension+Design] Consider the language $L = \overline{\mathbf{L}((abb^*)^*)}$.

1. Construct a DFA recognizing L. You need not prove that your construction is correct. If you construct it using the algorithms described in class then you should show all your steps. If you construct the automaton directly then you should explain the intuition behind your construction clearly. **[5 points]**
2. Construct a regular expression for the language L. Again you don't need to prove your regular expression to be correct, but you should show all the steps in the construction. **[5 points]**

Problem 2. [Category: Comprehension+Design+Proof] For a language $A \subseteq \Sigma^*$ define

$$\text{left}(A) = \{w \in \Sigma^* \mid ww^R \in A\}$$

where w^R denotes the reverse of w.

1. Taking $A = \{\epsilon, 01, 10, 1001\}$, what is left$(A)$? **[1 points]**
2. Taking $A = \mathbf{L}(0^*110^*)$, what is left(A)? **[1 points]**
3. Prove that if A is regular then left(A) is regular. You can either construct a DFA/NFA/regular expression for left(A) (and then you don't have to prove that your construction is correct) or use previously established closure properties to prove this result. *Hint:* Look at the construction of halving a language in lecture 3 (starting from page 26). **[8 points]**

Problem 3. [Category: Proof] Let $C = \{1^k x \mid x \in \{0,1\}^*,\ k \geq 1,\ \text{and } x \text{ contains at most } k \text{ 1s}\}$. Using the pumping lemma, prove that C is not regular. **[10 points]**

Problem 4. [Category: Comprehension+Proof] Consider the language $F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$

1. Prove that F is not regular. **[5 points]**
2. Prove that F satisfies the pumping lemma. *Hint:* Take the pumping length to be $p = 3$ and show that F satisfies the pumping lemma for this length. **[5 points]**