Math 321

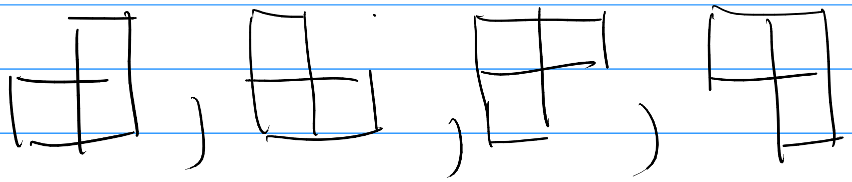
Thm: all $2^n \times 2^n$ $(n=1,2,\ldots)$ boards with one block missing can be tiled with ▛

Pf: (weak induction)

Basis Step (show $P(1)$ is true)

$P(1)$: "a $2^1 \times 2^1$ board with one piece missing can be tiled by ▛"

we would have 4 possible boards →



obviously each can be tiled by ▛. **True!**

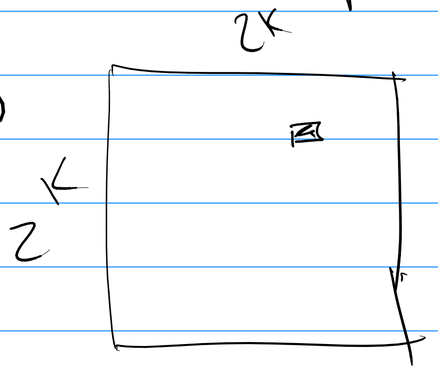
"weak"

Inductive Step (show $P(k) \to P(k+1)$ is true)

$P(k)$: "a $2^k \times 2^k$ board with one piece missing can be tiled by ▛"

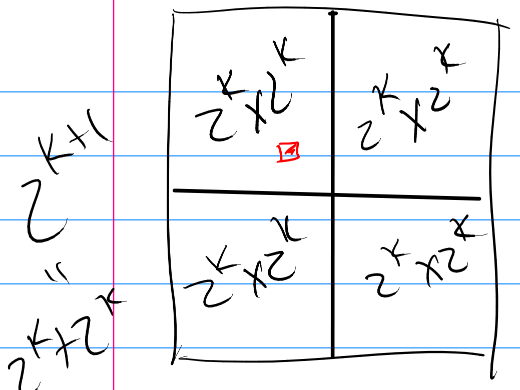
$P(k+1)$: "a $2^{k+1} \times 2^{k+1}$ board with one piece missing can be tiled by ▛"

assume $P(k)$



can be tiled

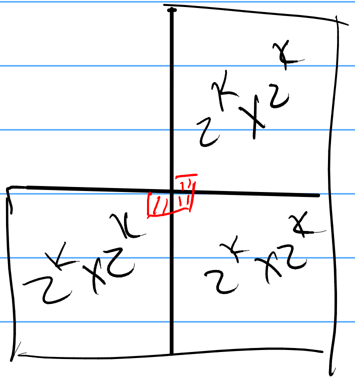
$$2^{k+1} = 2\cdot 2^k = 2^k + 2^k$$

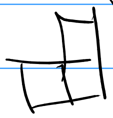


$2^{k+1} \times 2^{k+1}$ has one piece missing.

This missing piece is in one of the $2^k \times 2^k$. We can tile that one.

This leaves three full $2^k \times 2^k$ boards. take out the shared corner and they can be tiles.



And the last three pieces are "  " our tile.

so $P(k) \to P(k+1)$ is true.

∴ thm is true.

p291 #4

thm: any amount $n \geq 18$ can be formed by 4¢ and 7¢ stamps.

7¢

		0	1	2	3	4	
	0	0	7	14	21	28	..
4¢	1	4	11	18	25	32	..
	2	8	15	22	29	36	..
	3	12	19	26	33	40	..
	4	16					
	5	20					

$$n = a\cdot 7 + b\cdot 4$$

Diophantine Equations

$\boxed{18, 19, 20, 21}$ ✓

$+4 \quad +4 \quad +4 \quad +4$

22 23 24 25

$\hookrightarrow$ 26

Pf: $\boxed{\text{Basis Step}}$ (show P(1) is true)

P(1): "18, 19, 20, and 21 can be formed w/ 4 & 7"

$18 = 2 \cdot 7 + 1 \cdot 4$

$19 = 1 \cdot 7 + 3 \cdot 4$

$20 = 0 \cdot 7 + 5 \cdot 4$

$21 = 3 \cdot 7 + 0 \cdot 4$

True.

"strong" $\boxed{\text{Inductive Step}}$ show $((P(1) \land P(2) \land \ldots \land P(k)) \to P(k+1))$ is true.

$P(1) \xrightarrow{+4} P(2) \xrightarrow{+4} P(3)$, $P(1) \xrightarrow{+2 \cdot 4} P(3)$

P(1)	P(2)	P(3)	P(k)	P(k+1)
18	22	26	18 + (k-1)4	18 + k·4
19	23	27	19 + (k-1)4	19 + k·4
20	24	28	20 + (k-1)4	20 + k·4
21	25	29	21 + (k-1)4	21 + k·4

assume:

18 + (k-1)·4

19 + (k-1)·4 can be formed by

20 + (k-1)·4 7's & 4's

21 + (k-1)·4

add 1 4& to each give P(k+1) is true.

Stamps.

$n = a \cdot c + b \cdot d$

4¢ & 7¢ stamps. $n \geq 18$

Basis: P(1) "18¢"

$18 = 2 \cdot 7 + 1 \cdot 4$

Inductive: $k = a \cdot 7 + b \cdot 4$, if you have 7's

Case 1 remove a 7 and add 2·4's

$k - 7 + 2 \cdot 4 = k + 1$ is true.

Case 2 you have no 7's you have at least 5·4's.

remove 5·4's add 3·7's

$k - 5 \cdot 4 + 3 \cdot 7 = k + 1$ is true.

$18 = 2 \cdot 7 + 1 \cdot 4$

$19 = 1 \cdot 7 + 3 \cdot 4$

$20 = 0 \cdot 7 + 5 \cdot 4$

$21 = 3 \cdot 7 + 0 \cdot 4$

$22 = 2 \cdot 7 + 2 \cdot 4$

$23 = 1 \cdot 7 + 4 \cdot 4$

$24 = 0 \cdot 7 + 6 \cdot 4$

$25 = 3 \cdot 7 + 1 \cdot 4$

$\vdots$