Quiz 4

Name: **Time:** Feb 18, 2016

Instructions: Please write down the correct answer for each question in the following box.

1	2	3	4	5	6	7	Total Score

1. Recall that regular languages are closed under union. Based on this observation which of the following is necessarily true?

   (A) If L_1 and L_2 are regular then $L_1 \cup L_2$ is regular.

   (B) If $L_1 \cup L_2$ is regular then L_1 and L_2 is regular.

   (C) $L_1 \cup L_2$ is regular.

   (D) All of the above.

2. Recall that regular languages are closed under complementation. Based on this observation, we have three statements: (1) If L is regular then $\overline{L}$ is regular; (2) If $\overline{L}$ is regular then L is regular; (3) $L \cup \overline{L}$ is regular. Which of the following is necessarily true about these three statements?

   (A) (1)(2) is true, but (3) is false.

   (B) (2)is true, but (1)(3) are false.

   (C) (3) is true, but (1)(2) are false.

   (D) (1)(2)(3) are all true.

3. Consider the following alternate proof that regular languages are closed under Kleene closure that uses other closure properties. "Since regular languages are closed under concatenation, if L is regular then so is L^i for any i. Next, since regular languages are closed under union, it follows that $L^* = \cup_{i \geq 0} L^i$ is regular, if L is regular."

   (A) The proof is correct.

   (B) The proof is incorrect because closure under concatenation does not imply that if L is regular then so is L^i for any i.

   (C) The proof is incorrect because closure under union does not imply that if each L^i is regular then $\cup_{i \geq 0} L^i$ is regular.

   (D) The proof is incorrect because $L^* \neq \cup_{i \geq 0} L^i$.

4. Let $h : \{0,1\}^* \rightarrow \{a\}^*$ be a homomorphism defined as follows: $h(0) = a$ and $h(1) = \epsilon$. Let $L_{0n1n} = \{0^n 1^n \mid n \geq 0\}$. Taking $A \subset B$ to mean A is a proper subset of B, which of the following is true?

   (A) $h^{-1}(h(L_{0n1n})) = L_{0n1n}$

   (B) $h^{-1}(h(L_{0n1n})) \subset L_{0n1n}$

   (C) $L_{0n1n} \subset h^{-1}(h(L_{0n1n}))$

   (D) $h^{-1}(h(L_{0n1n})) \cap L_{0n1n} = \emptyset$

5. For $n \geq 0$, let $K_n = \{a^i b^k \mid i \geq n,\ 0 < k < n\}$. Which of the following is true?

(A) K_n is regular for all values of n

(B) K_n is not regular for any value of n

(C) There is an N_0 such that K_n is regular for all $n \leq N_0$ but not regular for $n > N_0$

(D) The regularity of K_n depends on the value of n and cannot be described in a simple manner.

6. Consider the following proof showing that $L = \mathbf{L}(0^*1^*)$ does not satisfy the pumping lemma. Let p be the pumping length. Consider the string $w = 001^p \in L$. Consider a $x = 0$, $y = 01$ and $z = 1^{p-1}$. Now observe that $xy^2z = 001011^{p-1} \notin L$. Hence, L does not satisfy the pumping lemma.

   (A) This proof demonstrates that L does not satisfy the pumping lemma.

   (B) This proof only shows that one particular w cannot be pumped. That is not enough to show that L does not satisfy the pumping lemma.

   (C) This proof only shows that a specific division of w into x, y, and z cannot be pumped. That is not enough to prove that L does not satisfy the pumping lemma.

   (D) This proof only shows that a specific value of the pumping length p is not correct. That is not enough to show that L does not satisfy the pumping lemma.

7. Let $L \subseteq \Sigma^*$ be a language such that L satisfies the pumping lemma. What can we say about L?

   (A) L is regular.

   (B) L is not regular.

   (C) L may or may not be regular.

   (D) $\Sigma^* \setminus L$ is regular.