Math 243

Q's

9.2 #1

tangent to line

need ① slope ② point

$x = t^4 + 2$ @ $t = 1$
$y = t^3 + t$

$y - y_0 = m(x - x_0)$

$x(1) = 3$
$y(1) = 2$] $(3, 2)$ pt.

slope $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{3t^2 + 1}{4t^3}$

$\dfrac{dy}{dx}(1) = \dfrac{3(1)^2 + 1}{4(1)^3} = \dfrac{4}{4} = \boxed{1 = m}$

$$\boxed{y - 2 = 1(x - 3)}$$

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tangent

$x = e^{\sqrt{t}}$ $y = t - 5\ln t$ @ $t = 1$

pt: $(e, 1)$

slope: $\left.\dfrac{dy}{dx}\right|_{t=1} = \left.\dfrac{1 - 5/t}{(e^{\sqrt{t}})\left(\frac{1}{2\sqrt{t}}\right)}\right|_{t=1} = \dfrac{-4}{\frac{e}{2}}$

$m = -8/e$

$\rightarrow$ eqn. $\boxed{y - 1 = \frac{-8}{e}(x - e)}$

$\boxed{9.3.0.54}$ horz. / vert. tangent

$\boxed{r^2 = \sin 2\theta}$

$x = r\cos\theta$
$y = r\sin\theta$

$\frac{dy}{dx} = 0$ or d.n.e

$r = (\sin 2\theta)^{1/2}$ or $r = -(\sin 2\theta)^{1/2}$

$y = (\sin 2\theta)^{1/2} \cdot \sin\theta$
$x = (\sin 2\theta)^{1/2} \cdot \cos\theta$

$x = -(\sin 2\theta)^{1/2} \cos\theta$
$y = -(\sin 2\theta)^{1/2} \sin\theta$

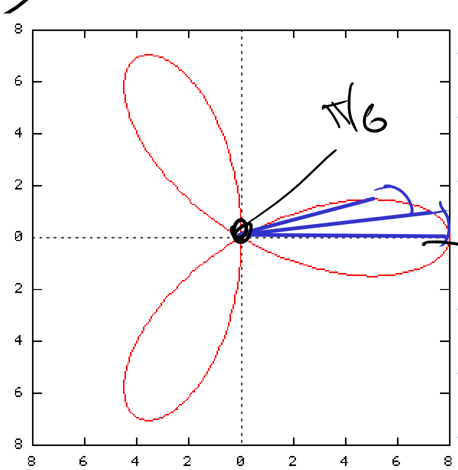
$$\frac{dy}{dx} = \frac{\cos 2\theta \sin\theta + (\sin 2\theta) \cos\theta}{\cos 2\theta \cos\theta - (\sin 2\theta) \sin\theta}$$

set top to zero → horz. tangent

set bottom to zero → vert. tangent

9.4.0.11

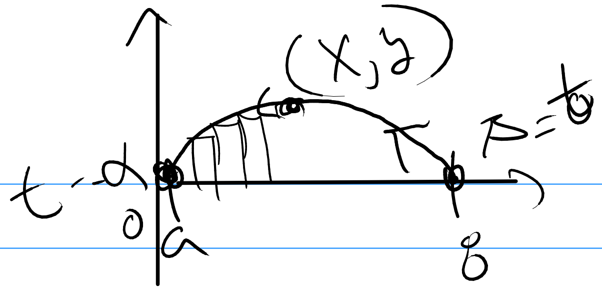
#3 $r = 8\cos 3\theta$ Area in curve.



$$A = \int_\alpha^\beta \frac{1}{2} r^2 \, d\theta$$

$$\text{Total} = 6 \int_0^{\pi/6} \frac{1}{2} (8\cos 3\theta)^2 \, d\theta$$

$$\text{Total} = 198 \int_0^{\pi/6} \cos^2 3\theta \, d\theta = \ldots$$



$A = \int_a^b y\,dx$

$A = \int_\alpha^\beta y(t)\,x'(t)\,dt$

Same prob. as parametric.

$r = 8\cos 3\theta$

$y(\theta) = 8\cos 3\theta \sin\theta$

$x(\theta) = 8\cos 3\theta \cos\theta$

$$A = \int_{\pi/6}^{0} (8\cos 3\theta \sin\theta)\, x'(\theta)\, d\theta$$

$= \ldots$ (finish)

☆ $x = t + \ln t \qquad y = t - \ln t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 - 1/t}{1 + 1/t} = \frac{t-1}{t+1} \quad (t>0)$$

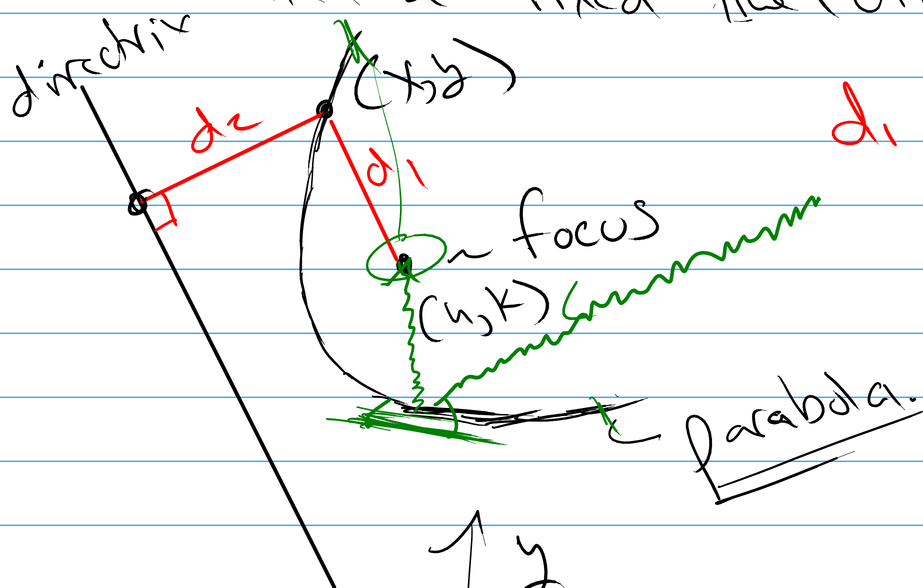
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{t-1}{t+1}\right]$$

$$= \frac{\frac{d}{dt}\left[\frac{t-1}{t+1}\right]}{\frac{dx}{dt}} = \frac{\frac{(t+1)-(t-1)}{(t+1)^2}}{1 + 1/t}$$

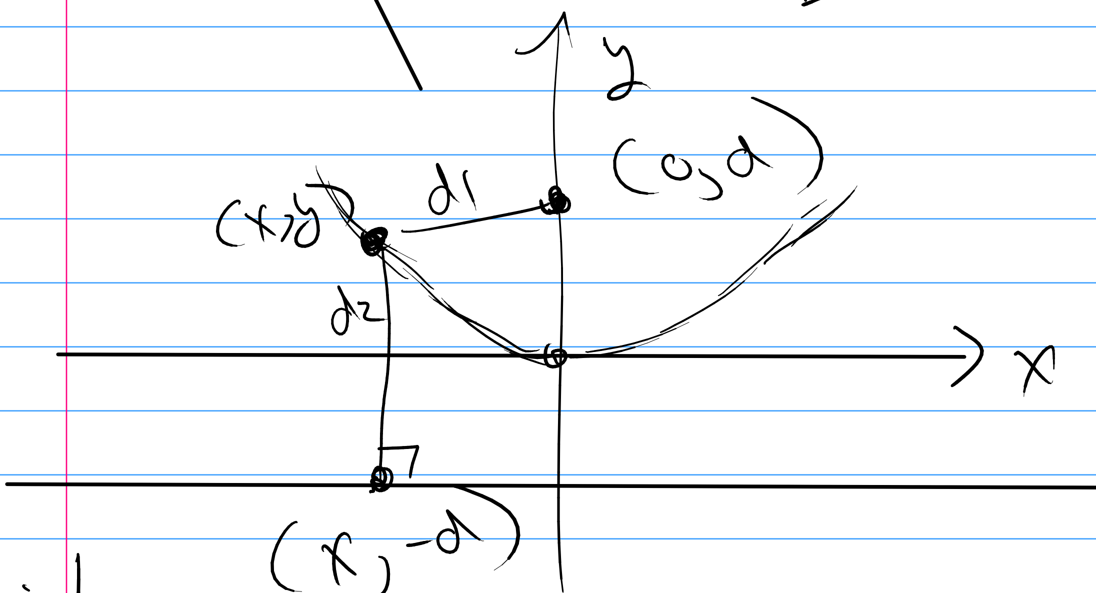
$$= \boxed{\frac{2t}{(t+1)^3}} \quad (t>0)$$

9.5 Conic Sections.

Parabola set of all points equidistant from a fixed point (focus) and a fixed line (directrix)



$d_1 = d_2$



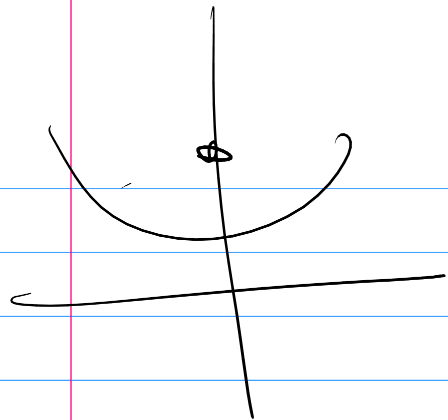
$d>0$

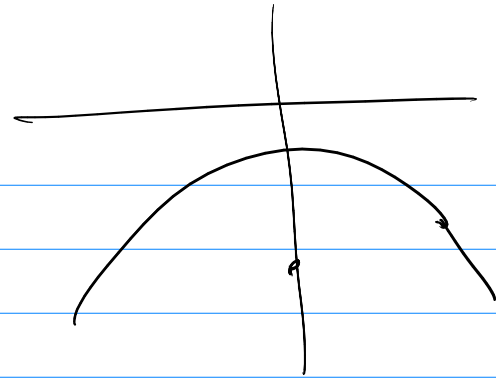
equi-distant $\sqrt{(x-0)^2+(y-d)^2} = \sqrt{(x-x)^2+(y+d)^2}$

$$x^2 + \cancel{y^2} - 2yd \cancel{+d^2} = \cancel{y^2} + 2yd \cancel{+d^2}$$

$$x^2 = 4d\,y \qquad d>0$$

$$y = \frac{1}{4d}x^2$$



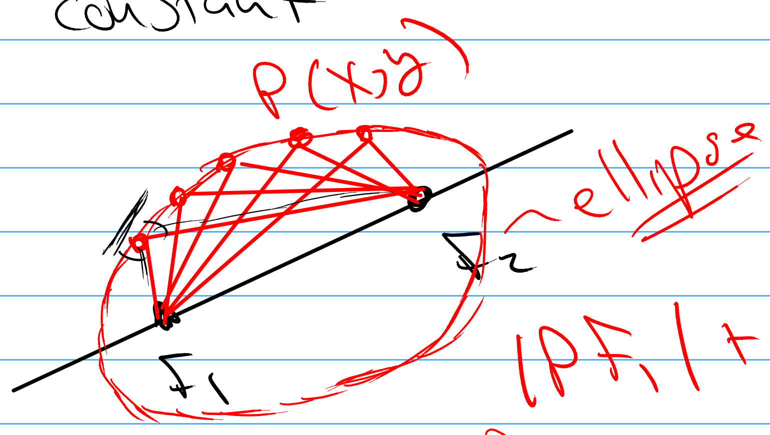


$$y = -\frac{1}{4a}x^2$$

$$y = ax^2 + bx + c$$

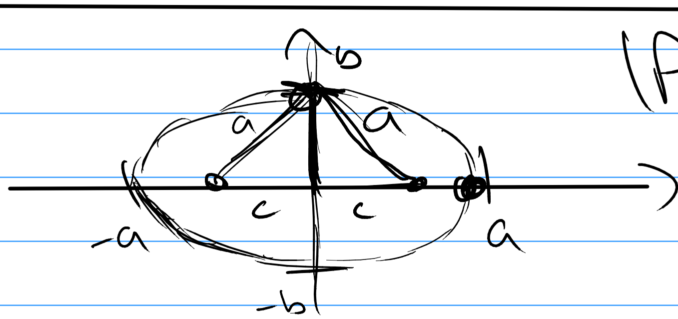
$$x = ay^2 + by + c$$

<u>Ellipse</u> set of all points whose sum of distances from two foci are constant



$|PF_1| + |PF_2| = \text{const.}$

ellipse.



$$|PF_1| + |PF_2| = 2a$$

$$c^2 + b^2 = a^2$$

$\boxed{\text{eqn}}$ $\quad \dfrac{x}{a^2} + \dfrac{y^2}{b^2} = 1 \qquad a \geq b > 0$

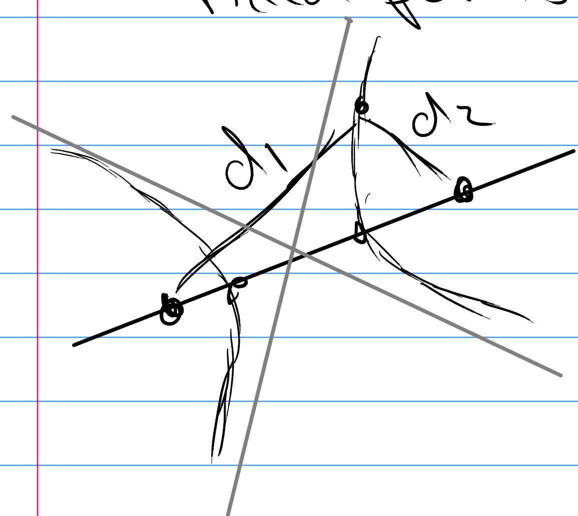
foci $\quad (c,0) \quad (-c,0)$

vertices $\quad \boxed{(a,0) \quad (-a,0)} \quad (0,b)\,(0,-b)$

$$c^2 + b^2 = a^2$$

$$c^2 = a^2 - b^2$$

Hyperbola!! set of all points whose difference in distances from two fixed points is a constant.



$d_1 - d_2 =$ const.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

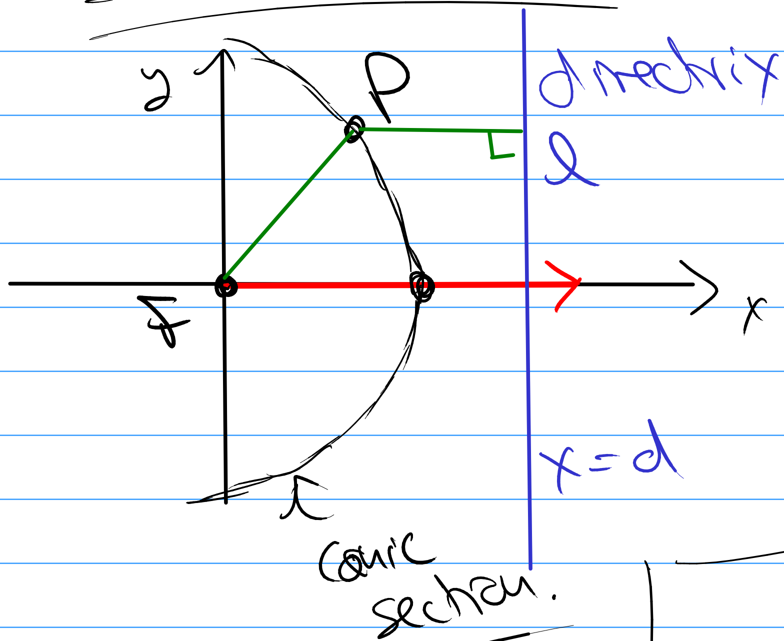
foci $\quad (c,0) \quad (-c,0)$

$$c^2 = a^2 + b^2$$

vertices $\quad (a,0) \quad (-a,0)$

asymptotes $\quad y = \pm \dfrac{b}{a} x$

Into Polar System



F = fixed pt

$P = (x,y)$
conic section

ℓ = directrix

for a conic section

$$\boxed{\begin{gathered} |PF| = e|P\ell| \\ e > 0 \\ e \equiv \text{a real number} \end{gathered}}$$

$$|PF| = e|P\ell|$$

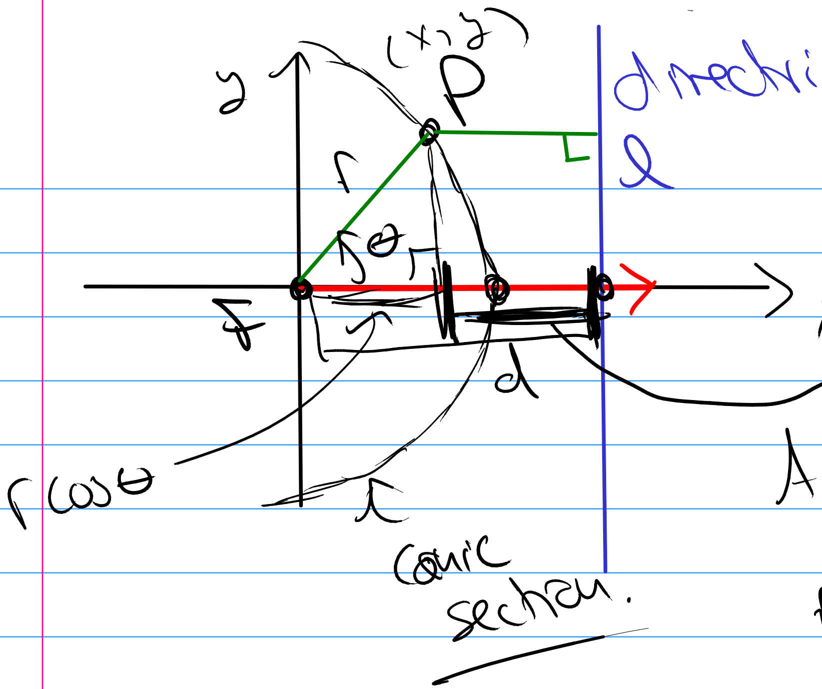
or

$$\frac{|PF|}{|P\ell|} = e \quad (e > 0)$$

① $e = 1$ parabola

② $e < 1$ ellipse

③ $e > 1$ hyperbola



$|PF| = r$

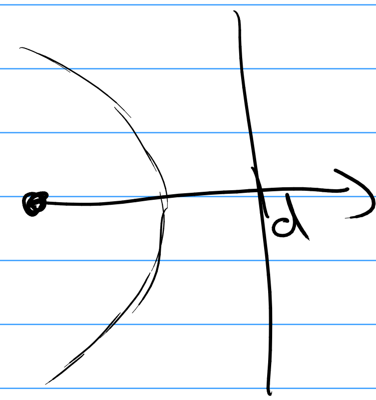
$\to |Pl| = d - r\cos\theta$

Any conic section

$r = e(d - r\cos\theta)$

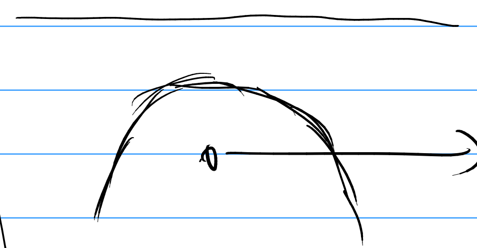
$\to r = ed - re\cos\theta$

$\to \boxed{r = \dfrac{ed}{1 \pm e\cos\theta}}$



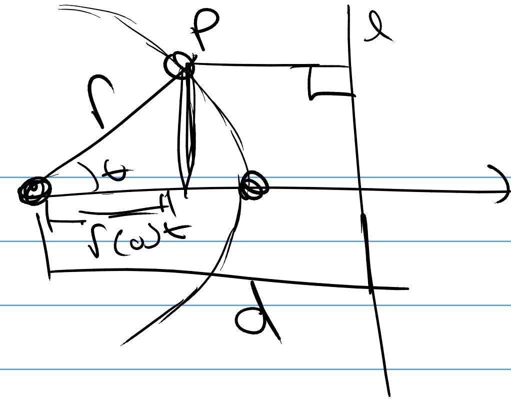
(+) left (−) right

$\to \boxed{r = \dfrac{ed}{1 \pm e\sin\theta}}$



(+) down (−) up

$$\underset{|PF|}{r} = e\underbrace{(d - r\cos\theta)}_{|P\ell|}$$



x's & y's?

$$(r)^2 = (e(d-x))^2$$

$$r^2 = e^2(d-x)^2$$

$$x^2+y^2 = e^2(d-x)^2$$

if $e=1$ $\quad |PF| = |P\ell|$

this <u>is</u> a parabola by definition

$$x^2+y^2 = (d-x)^2$$

$$\cancel{x^2}+y^2 = d^2 - 2dx + \cancel{x^2}$$

$$x = -\frac{1}{2d}y^2 + \frac{1}{2}d$$

left opening parabola

if $e \neq 1$

$$x^2+y^2 = e^2(d-x)^2$$

$$x^2+y^2 = e^2(d^2-2dx+x^2)$$

$$x^2+y^2 = e^2d^2 - 2de^2x + e^2x^2$$

$$(1-e^2)x^2 + 2de^2x \quad + y^2 = e^2d^2$$

$$\underbrace{x^2 + \frac{2de^2}{1-e^2}x + \left(\frac{de^2}{1-e^2}\right)^2} + \frac{y^2}{1-e^2} = \boxed{\frac{e^2d^2}{1-e^2}} + \left(\frac{de^2}{1-e^2}\right)^2$$

$$\left(x + \frac{de^2}{1-e^2}\right)^2 + \frac{y^2}{\boxed{1-e^2}} = \boxed{\text{that}}$$

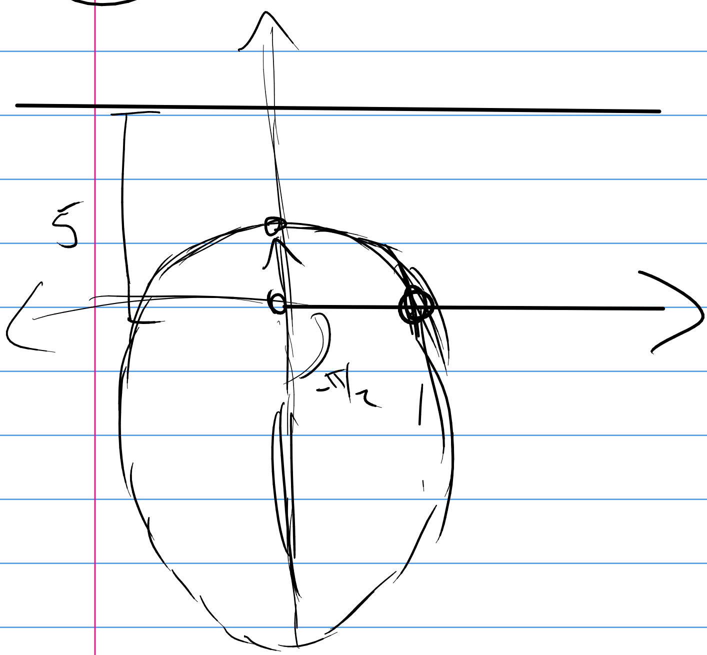
$1-e^2 > 0 \Rightarrow$ ellipse, $e<1$

$1-e^2 < 0 \Rightarrow$ hyperbola, $e>1$

$$r = \frac{ed}{1 \pm e\sin\theta} \qquad r = \frac{ed}{1 \pm e\cos\theta}$$

$\boxed{\text{vs}}$ $y = \frac{1}{4d}x^2$ vs $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$

ex) eccentricity $e = 3/4$ directrix $y = 5$



$$r = \frac{ed}{1 + e\sin\theta}$$

$$r = \frac{(3/4)\cdot 5}{1 + 3/4\sin\theta}$$

$$\boxed{r = \frac{15}{4 + 3\sin\theta}}$$