Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 8

Created by Prof. Diego Maldonado and Prof. Virginia Naibo

Name: ______________________________

Due date: Thursday, March 28th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. Find the values of a, b, and c that make the matrix Q below an orthogonal matrix

$$Q = \begin{bmatrix} a & 0 & 2c \\ -a & b & c \\ a & b & -c \end{bmatrix}.$$

2. Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be a linear transformation and $\beta = \{u_1, u_2, u_3\}$ be a basis of $\mathbf{R}^3$ such that

$$u_1 = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

and

$$T(u_1) = \begin{bmatrix} 4 \\ 6 \\ -1 \\ 5 \end{bmatrix}, \quad T(u_2) = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad T(u_3) = \begin{bmatrix} 7 \\ 3 \\ 6 \\ -4 \end{bmatrix}.$$

Find $T(u)$ where

$$u = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}.$$

3. Find the orthogonal projection of the vector $v = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ onto the subspace $\mathcal{U} = span(u_1, u_2)$ where $u_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$.

4. Find the (shortest) distance between the vector $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and the subspace $\mathcal{U} = span(u_1, u_2)$ where $u_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$.

5. Use *branching* (that is, the “if-else-end” syntax) to write a Matlab function that takes an arbitrary $n \times n$ matrix Q and returns a 1 if Q is orthogonal and a 0 otherwise.

6. Find a decomposition of the vector $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ of the form

$$v = \bar{v} + w$$

such that $\bar{v}$ belongs to the subspace $\mathcal{U} = span(u_1, u_2)$, where $u_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$, and $w \in \mathcal{U}^{\perp}$.

7. Write a Matlab function that takes an $m \times n$ matrix A and a vector $v \in \mathbf{R}^m$ and returns an $m \times n$ matrix B so that, for $1 \leq j \leq n$, the j-th column of B is the projection of v onto the subspace generated by the j-th column of A. The code should account for the case in which some of columns of A are the zero vector.

8. Consider the vectors

$$w_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \\ -2 \\ 1 \\ 5 \end{bmatrix} \quad w_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad w_3 = \begin{bmatrix} 4 \\ -1 \\ 1 \\ 1 \\ 5 \\ -8 \end{bmatrix}.$$

Find a basis for the subspace of all the vectors in $\mathbf{R}^6$ which are orthogonal to w_1, w_2, and w_3.

9. True or False - **Circle the right one** (1 point each)

T or **F**. The linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ represented by the matrix

$$C = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 5 & 0 \\ 4 & 2 & -1 \end{bmatrix}$$

is one-to-one.

T or **F**. An $n \times n$ matrix Q is orthogonal if and only if $QQ^T = I$, here I stands for the $n \times n$ identity matrix.

T or **F**. If A is an $m \times n$ matrix with $rank(A) = n$, then the linear transformation represented by A is onto.

T or **F**. An $n \times n$ matrix P is orthogonal if and only if its rows are mutually orthonormal vectors.

T or **F**. The linear transformation represented by the matrix

$$N = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

is onto.

Points obtained in this assignment (out of 16): ______________