Math 243

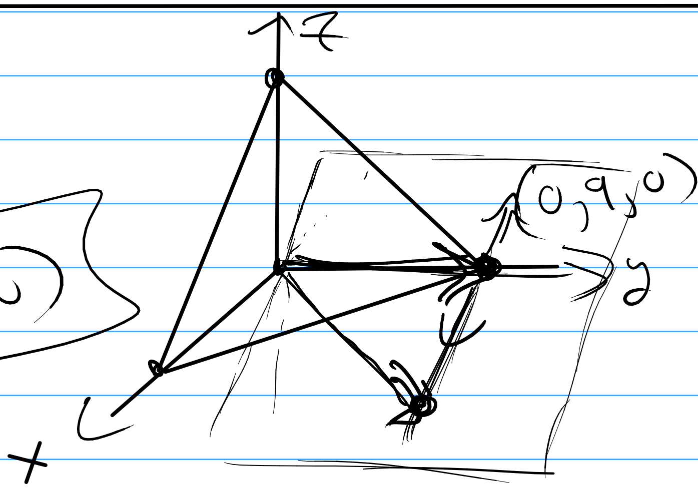
Q's/ P_1 P_2

Pt: let $z = 0$

$x + y = 9$

$x = 0$

$(0,9,0)$



$\langle 1,1,1\rangle \times \langle 1,0,1\rangle = \vec{v}$ that line is parallel to

$= \langle a,b,c\rangle = \langle 1,0,-1\rangle$

$\vec{r} = \vec{r}_0 + t\vec{v}$

$\langle x,y,z\rangle = \langle 0,9,0\rangle + t\langle 1,0,-1\rangle$

Parametric.

$x = t$

$y = 9$ ← no t!

$z = -t$

Solve for t.

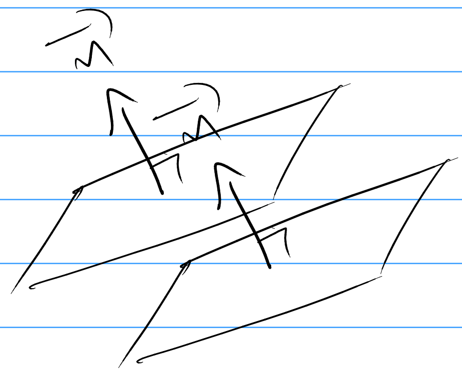
Sym $\frac{x}{1} = \frac{z}{-1}$

eqn of a plane: need pt. & normal

($\perp$ vector to plane)

$(0,0,0)$ [illegible] $= 4$

parallel to this



so... same normal

$\vec{n} = \langle 4,-1,3 \rangle$

vector $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$

$\langle 4,-1,3 \rangle \cdot \langle x-0, y-0, z-0 \rangle = 0$

$\boxed{4x - y + 3z = 0}$

or

$ax + by + cz = d$

$4x - y + 3z = d$

if $x=0, y=0, z=0 \Rightarrow d=0$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & e^{t} & e^{-t} \\ c & e^{t} & -e^{-t} \end{vmatrix} = \langle -1-1,\ -(-e^{-t} - ce^{-t}),\ e^{t} - ce^{t} \rangle$$

$$= \langle -2,\ (c+1)e^{-t},\ (1-c)e^{t} \rangle$$

$$4x^2 + 4y^2 + 4z^2 - 16x + 8y = 1$$

$$x^2 + y^2 + z^2 - 4x + 2y = 1/4$$

$$(x^2 - 4x + 4) + (y^2 + 2y + 1) + z^2 = 1/4 + 4 + 1$$

$$(x-2)^2 + (y+1)^2 + z^2 = \left(\frac{\sqrt{21}}{2}\right)^2$$

Induction:

Basis Step: basic (primitive) elements

Inductive Step: rules to make new elements.

Elementary Functions

Basis: one variable functions
1. polynomials
2. rationals
3. x^a
4. a^x
5. log.
6. trig
7. inv. trig

Inductive: apply a finite number of

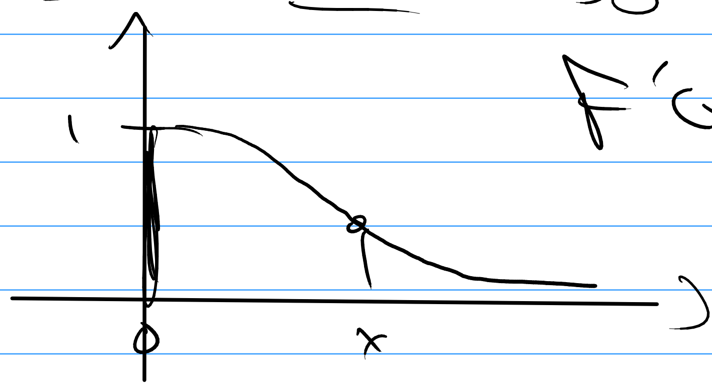
$+, -, \times, \div,$ composition

$$\sin\sqrt{x + \sqrt[3]{x^4}} - e^{x + 1/x} + \ln(\tan x)$$

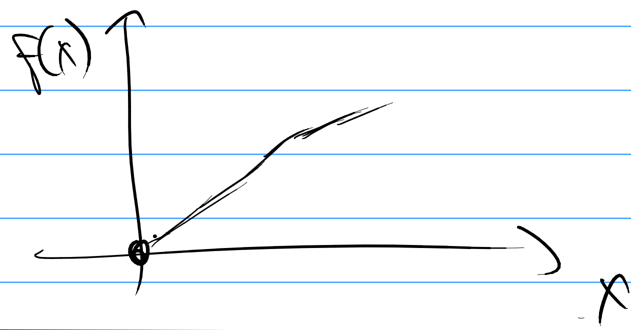
$D_x[f(x)]$ is an elementary function if f is elementary.

what about $A_x[f(x)]$?

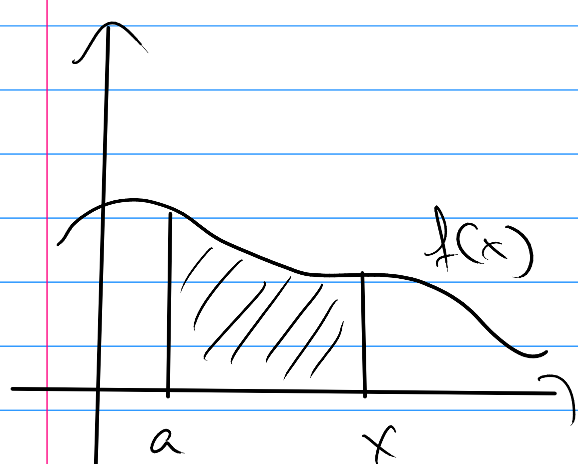
<u>ex:</u> $\underline{F}(x) = \int_0^x e^{-t^2}\,dt$



$F'(x) = \boxed{e^{-x^2}}$



but $F(x)$ is not elementary

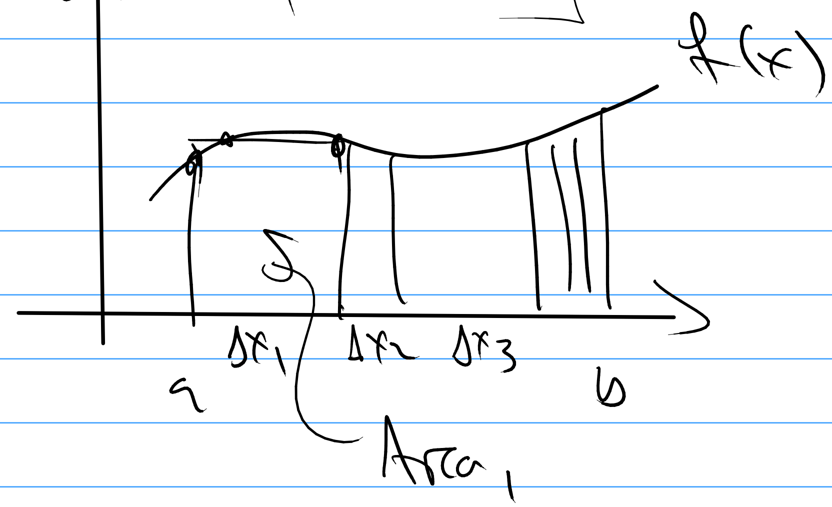


$F(x) = \int_a^x f(x)\,dx$

$\to$ we can <u>plot</u> $F(x)$ if we can <u>approx.</u> area.

$$\boxed{\int_a^b f(x)\,dx} = \boxed{\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n}} \underbrace{f(x_i^*)}_{\text{height}} \underbrace{\Delta x_i}_{\text{width}}$$

True Area

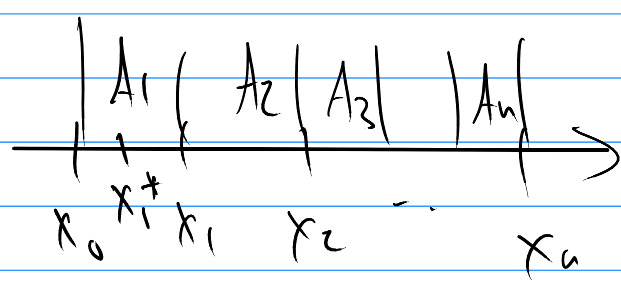


if you don't take the limit

① Area $\approx \sum_{i=1}^{n} f(x_i^*)\,\underline{\Delta x_i}$

② let $\Delta x_i = \Delta x = \dfrac{b-a}{n}$

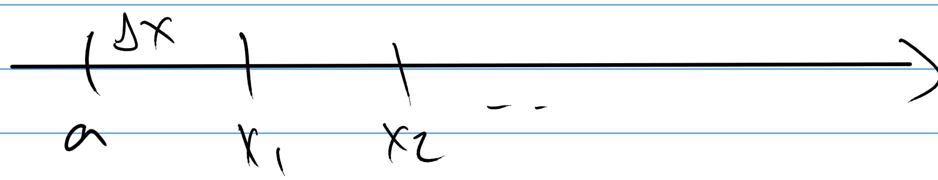
③ $\boxed{\begin{matrix} x_i^* = ? \\ i = 1 \ldots n \end{matrix}}$



Tech. #1 left end pont.

$x_i^* = x_0, x_1, x_2, \ldots, x_{n-1}$

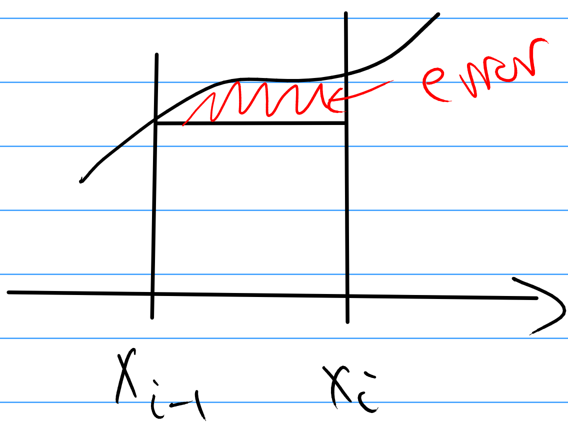
$x_i^* = a + (i-1)\Delta x \qquad i = 1, 2, 3, \ldots n$



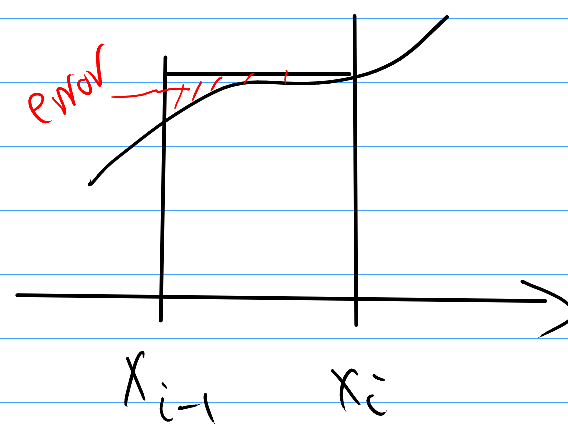
Tech #2 right end pt.

$x_i^* = x_1, x_2, \ldots, x_n$

$x_i^* = a + i\Delta x \qquad i = 1, 2, \ldots, n$

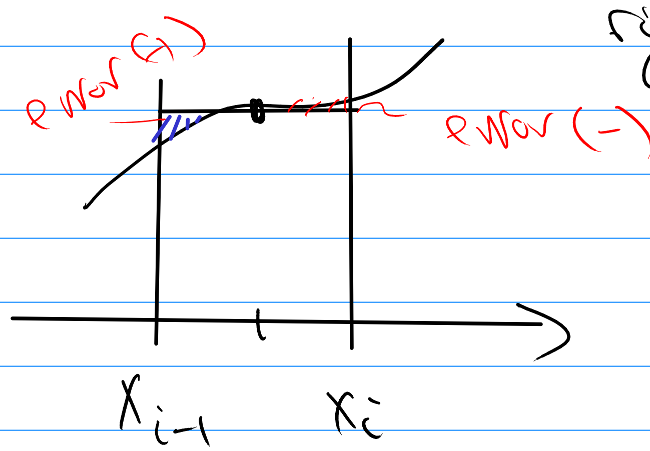


left



right

Tech #3 Midpt.

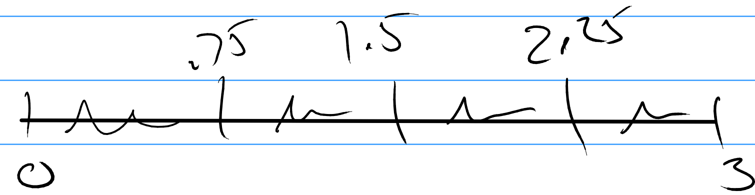


$$x_i^* = \frac{x_0 + x_1}{2}, \quad \frac{x_1 + x_2}{2}, \ldots, \quad \frac{x_{n-1} + x_n}{2}$$

$$x_i^* = a + \left(i - \frac{1}{2}\right)\Delta x$$

In Application:

$$\int_0^3 x^3\,dx = \frac{1}{4}x^4\Big|_0^3 = \frac{81}{4} = \underline{\underline{20.25}}$$



$n = 4 \qquad \Delta x = \frac{3-0}{4} = 0.75$

$\underline{\underline{\text{Left}}}\ \sum_{i=1}^{4} f(x_i^*)\Delta x = .75\left(f(0) + f(.75) + f(1.5) + f(2.25)\right)$

$= .75\left(0 + .75^3 + 1.5^3 + 2.25^3\right)$

See Movie

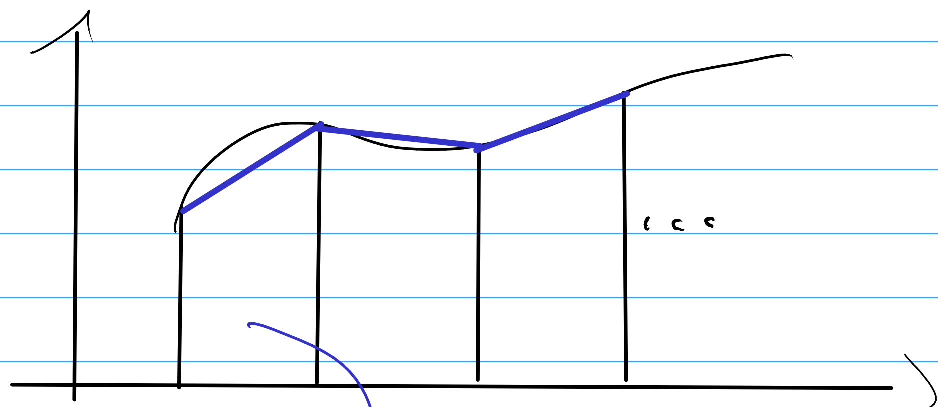
$\frac{1}{10} \to \underbrace{\frac{1}{5} \to \frac{2}{5} \to \frac{4}{5} \to \frac{8}{5} \to \frac{3}{5} \to \frac{6}{5}}_{\text{repeat}} \to \frac{1}{5} \to$

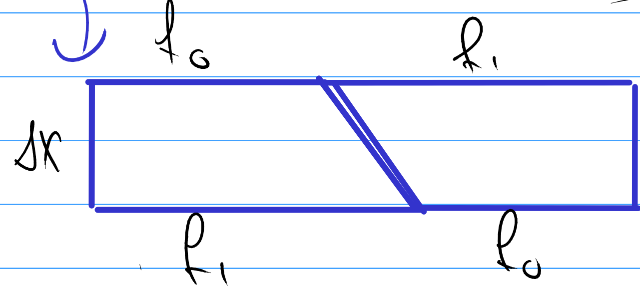
$f_n = f(x_n)$

$A_L = \Delta x(f_0 + f_1 + f_2 + \cdots + f_{n-1})$

$A_R = \Delta x(f_1 + f_2 + \cdots + f_n)$

$A_m = \Delta x(f_{m_1} + f_{m_2} + \cdots + f_{m_n})$





trapezoidal Rule

$T_1 = \frac{1}{2}\Delta x(f_0 + f_1)$

$T_2 = \frac{1}{2}\Delta x(f_1 + f_2)$

$\vdots$

$T_n = \frac{1}{2}\Delta x(f_{n-1} + f_n)$

$\text{Area}_T \approx \frac{1}{2}\Delta x(f_0 + 2f_1 + 2f_2 + \cdots + 2f_{n-1} + f_n)$

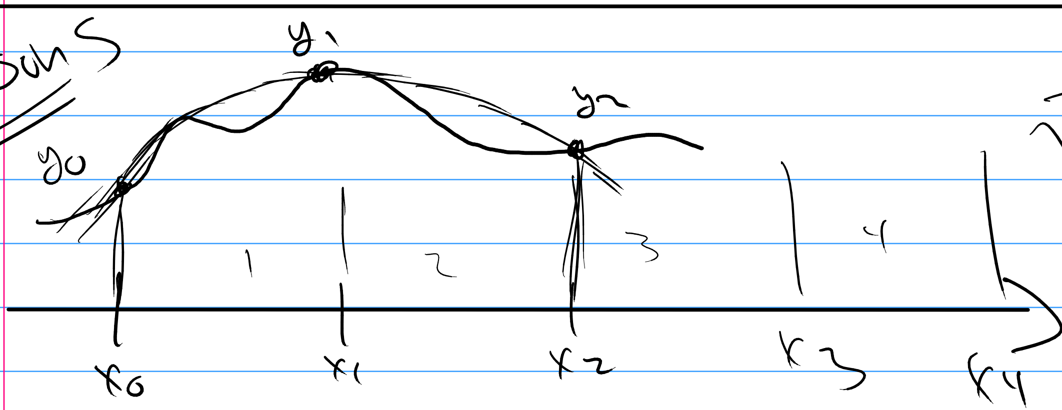
$A_T \approx \Delta x(f_0 + f_1 + \cdots + f_n) - \frac{1}{2}\Delta x(f_0 + f_n)$

$\boxed{\text{Error}}$ $|f''(x)| \le K \quad x \in [a,b]$

$$|E_M| \le K \frac{(b-a)^3}{24n^2}$$

$$|E_T| \le K \frac{(b-a)^3}{12n^2}$$

Simpsons



n must be even

$$y = ax^2 + bx + c$$

$$\int_{x_0}^{x_2} (ax^2 + bx + c)\,dx = A_{P_1} = \frac{\Delta x}{3}(f_0 + 4f_1 + f_2)$$

$$P_1 = \frac{\Delta x}{3}(f_0 + 4f_1 + f_2)$$

$$P_2 = \frac{\Delta x}{3}(f_2 + 4f_3 + f_4)$$

$$\vdots$$

$$P = \frac{\Delta x}{3}(f_{n-2} + 4f_{n-1} + f_n)$$

$$\text{Area}_{\text{Simpsons}} \approx \frac{\Delta x}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + \cdots + 2f_{n-2} + 4f_{n-1} + f_n)$$

$$|E_S| \le K\frac{(b-a)^5}{180n^4}$$

$$|f^{(4)}(x)| \le K \qquad x \in [a,b]$$

$$\int_0^4 \sqrt{1+\sqrt{x}}\,dx \qquad \text{error} \le 10^{-8}$$

$\boxed{\text{Mid pt.}}$ $$|E_M| \le K\frac{(b-a)^3}{24n^2} \le 10^{-8}$$

$$|f''(x)| \le K \quad \text{on } (0,4)$$

$$f''(x) = \frac{\sqrt{\sqrt{x}+1}\,(3\sqrt{x}+2)}{\sqrt{x}\left(16x^2+16x\right)+32x^2}$$

$$= \frac{\sqrt{\sqrt{x}+1}\,(3\sqrt{x}+2)}{x\left(\sqrt{x}(16x+16)+32x\right)}$$

let $n = 10$ $\qquad \Delta x = \frac{4}{10} = \frac{2}{5} = .4$

$$\int_0^4 \sqrt{1+\sqrt{x}}\,dx$$

0 $\quad$ .4 $\quad$ .8 $\quad$ 1.2 $\quad \ldots \quad$ 3.6 $\quad$ 4

$$A_n \approx .4\left(f(.2)+f(.6)+\cdots+f(3.8)\right)$$

$$\text{Ans} \approx 4\left(\sqrt{1+\sqrt{2}} + \dots\right)$$

$$\approx \underline{6.08}$$