Math 321

Q's / Prove $2+2=4$

$0, 1, 2, 3, 4, 5, \ldots$

$0, S(0)=1, S(1)=S(S(0))=2, \ldots, S(n-1)=n$

$S^2(0) \qquad S^n(0)=n$

Induction

$a+0=a$ Basis.

$a+S(b)=S(a+b)$ inductive.

$a+1=a+S(0)=S(a+0)=S(a)$

$$\begin{aligned}2+2 &= 2+S(1)\\ &= S(2+1)\\ &= S(S(2))\\ &= S(3)\\ &= 4\end{aligned}$$

Peano Postulates.

5.1.3 b) 4 ans. or no ans

$4 + 1 = 5$

5.2.10 $\quad p_1 = (x_1, y_1)$; $p_2 = (x_2, y_2)$ etc.

$$\text{Midpt}(P_i, P_j) = \left(\frac{x_i + x_j}{2}, \frac{y_i + y_j}{2}\right)$$

5 distinct pts $\rightarrow$ at least one midpts has integer coord.

Mid pt $\rightarrow$ $\left(\underbrace{\frac{x_i + x_j}{2}}_{\text{Int.}}, \underbrace{\frac{y_i + y_j}{2}}_{\text{Int.}}\right)$ $\leftarrow$ for this to happen numerator $\equiv$ even.

how can $a+b$ be even?

(1) both odd or (2) both even] same parity

parity: (even, even) or (even, odd) or (odd, even) or (odd, odd)

these are the boxes.

|objects| = 5 points $\qquad$ |boxes| = 4 parities

by pigeonhole principle at least one box has at least two object.

$\rightarrow$ two points have same parity

→ two points have int. midpt.

ex $(4,6)$, $(2,3)$, $(3,2)$, $(3,3)$
$(5,2)$

$$\text{midpt}\big((5,2),(3,2)\big) = (4,2)$$

4D space. (x,y,z,t)

2^4+1 points needed.

Applications of Combinations.

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

Note: $\binom{n}{r}$ vs $\binom{n}{n-r}$

ex $\binom{5}{2}$ vs $\binom{5}{3}$

$$\frac{5!}{2!\,3!} = \frac{5!}{3!\,2!}$$

$\binom{n}{r}$ vs $\binom{n}{n-r}$

$$\frac{n!}{r!\,(n-r)!} \qquad \frac{n!}{(n-r)!\,(n-(n-r))!} = \frac{n!}{(n-r)!\,r!}$$

So $\binom{n}{r} = \binom{n}{n-r}$

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given n-boxes & two things. (x, y)

ex: $\underbrace{\binom{x}{y}}_{b_1}$ $\underbrace{\binom{x}{y}}_{b_2}$ $\underbrace{\binom{x}{y}}_{b_3}$

task: take an object from each box.

take from box 1 $\wedge$ box 2 $\wedge$ box 3

$$2 \cdot 2 \cdot 2 = 2^3 = \underline{\underline{8}} \text{ ways.}$$

So 8 ways to take something from each.

→ each make a triple

x, x, x — 3 x's

x, y, x — 2 x's

1 x

0 x

$$\binom{3}{3} = \frac{3!}{3!\,0!} = 1 \text{ way } 3\ x\text{'s}$$

$$\binom{3}{2} = \frac{3!}{2!\,1!} = 3 \text{ ways } 2\ x\text{'s}$$

$$\binom{3}{1} = \frac{3!}{1!\,2!} = 3 \text{ ways } 1\ x.$$

$$\binom{3}{0} = \frac{3!}{0!\,3!} = 1 \text{ way } 0\ x\text{'s}.$$

8 ways!

$$(x+y)^3 = \underset{\uparrow\ \text{Sum Rule}}{(x+y)} \cdot \underset{\uparrow\ \text{Product Rule}}{(x+y)} \cdot (x+y)$$

$$= \underbrace{x^3} + \underbrace{3x^2y} + \underbrace{3xy^2} + \underbrace{y^3}$$

$$(x+y)^n = (x+y)\cdot(x+y)\cdot \ --- \ \cdot(x+y)$$

$$= \binom{n}{n} x^n + \binom{n}{n-1} x^{n-1} y + \cdots + \binom{n}{n-j} x^{n-j} y^j + \cdots + \binom{n}{0} y^n$$

$$\boxed{(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j}$$

Binomial theorem.

ex

$$(x+y)^7 = \frac{7!}{7!\,0!} x^7 y^0 + \frac{7!}{6!\,1!} x^6 y^1 + \frac{7!}{5!\,2!} x^5 y^2$$

$$+ \frac{7!}{4!\,3!} x^4 y^3 + \frac{7!}{3!\,4!} x^3 y^4 + \frac{7!}{2!\,5!} x^2 y^5 + \frac{7!}{1!\,6!} x^1 y^6$$

$$+ \frac{7!}{0!\,7!} x^0 y^7$$

$(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3$

$+ 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

Pascal's Triangle

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1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

$n=7 \rightarrow$ 1 7 21 35 35 21 7 1