KANSAS STATE UNIVERSITY | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 10

Created by Prof. Diego Maldonado and Prof. Virginia Naibo

Name: _____

Due date: Thursday, April 11th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

and their corresponding eigenvectors.

2. Find the eigenvalues of the matrix

$$B = \begin{bmatrix} -2 & 1 & 3 \\ 0 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

and their corresponding eigenvectors.

3. Find an orthonormal basis of $\mathbf{R}^2$ consisting of eigenvectors of the matrix

$$C = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}.$$

4. Suppose that a square matrix A has the polynomial

$$p(\lambda) = (3 + \lambda)(2 - \lambda)(1 + \lambda)$$

as its characteristic polynomial. Answer the following questions:

(i) What is the size of A?

(ii) What are the eigenvalues of A?

(iii) Is A invertible?

(iv) Is A diagonalizable?

(v) What is the value of $\det(A)$?

(vi) What is the value of $\det(A - 4I)$? Here I is the identity matrix.

(vii) What are the eigenvalues of A^3?

(viii) Can A be an orthogonal matrix?

(ix) Can A be a lower-triangular matrix?

5. Write a Matlab function that takes 3 vectors in $\mathbf{R}^3$ and outputs the volume of the tetrahedron that has those three vectors as edges.

6. Consider the matrices

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & -\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find a 3×3 matrix P such that

$$D = P^{-1}AP.$$

7. Find the eigenvalues of the matrix

$$N = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and their corresponding eigenvectors.

8. True or False - **Circle the right one** (1 point each)

   T or **F**. If A is diagonalizable, then A is invertible.

   T or **F**. If A is invertible, then A is diagonalizable.

   T or **F**. $\det(A)$ equals the product of the eigenvalues of A.

   T or **F**. The characteristic polynomial of a matrix A always has real roots.

   T or **F**. If A is a symmetric $n \times n$ matrix, then there exists a basis of $\mathbf{R}^n$ consisting of eigenvectors of A.

Points obtained in this assignment (out of 16): _____