Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 14

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Name: ____

Due date: Thursday, May 9th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. The 2D stress state of a material point is given by the following data: $\sigma_x = 8$, $\tau_{xy} = \tau_{yx} = 2$ and $\sigma_y = 4$. Find the principal directions and principal stresses.

2. The 3D stress state of a material point is given by the following data: $\sigma_x = 16$, $\sigma_y = 10$, $\sigma_z = 4$, $\tau_{xy} = \tau_{yx} = 2$, $\tau_{xz} = \tau_{zx} = 1$, and $\tau_{yz} = \tau_{zy} = 3$. Find the principal directions and principal stresses.

3. Find a decomposition of the vector $v = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$ of the form $v = \bar{v} + w$ such that $\bar{v}$ belongs to the subspace $\mathcal{U} = span\{u_1, u_2\}$, where $u_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$, and $w \in \mathcal{U}^{\perp}$.

4. Find a 2×2 matrix A that represents a linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ such that

$$T\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ -5 \end{bmatrix}.$$

Is T one-to-one? Is T onto? Justify.

5. Find a decomposition of the vector $v = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 4 \\ 1 \end{bmatrix}$ of the form $v = \bar{v} + w$ such that $\bar{v}$ belongs to the subspace $\mathcal{U} = span\{u_1, u_2\}$, where $u_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \\ 1 \end{bmatrix}$, and $w \in \mathcal{U}^{\perp}$.

   Hint: Notice that u_1 and u_2 are **not** orthogonal and the Gram-Schmidt orthonormalization process comes in handy.

6. Write a Matlab function that takes an arbitrary $n \times n$ matrix A and two indices j and k, with $1 \leq j < k \leq n$, and returns the sum of the angles between the last column of A and the l-th column of A for all l with $j \leq l \leq k$. The code should display an error message if the input matrix is not a square matrix.

7. A town's economy is based on the following three sectors: Administration *(A)*, Housing *(H)*, and Transportation *(T)*. These sectors are related as follows: The production of one dollar of A requires 72 cents of A, 18 cents of H, and 10 cents of T. Each dollar produced by H requires 35 cents of A, 45 cents of H, and 20 cents of T. Each dollar of T requires 20 cents of A, 25 cents of H, and 55 cents of T. Is this an open or a close economy model? Find the production schedule and identify the consumption matrix. If this economy is worth 1 million dollars, how much A, H, and T must be produced? Which sector consumes the least H?

   Hint: Begin with the key questions: how much A is consumed? How much H is consumed? How much T is consumed?.

8. True or False - **Circle the right one** (1 point each)

   T or **F**. The system $Ax = b$ has a solution if and only if $b \in col(A)$.

   T or **F**. If P is an $n \times n$ column-stochastic matrix, then its columns form a basis of $\mathbf{R}^n$.

   T or **F**. The system $Mx = 0$ has at least one solution only if the columns of M are linearly independent.

   T or **F**. If v_1 is orthogonal to v_2 and v_2 is a multiple of v_3, then v_1 is orthogonal to v_3.

   T or **F**. If $\bar{v}$ is the projection of v onto the subspace $\mathcal{U}$, then $\|\bar{v} - v\|$ equals the (shortest) distance from v to $\mathcal{U}$.

Points obtained in this assignment (out of 16): ______________