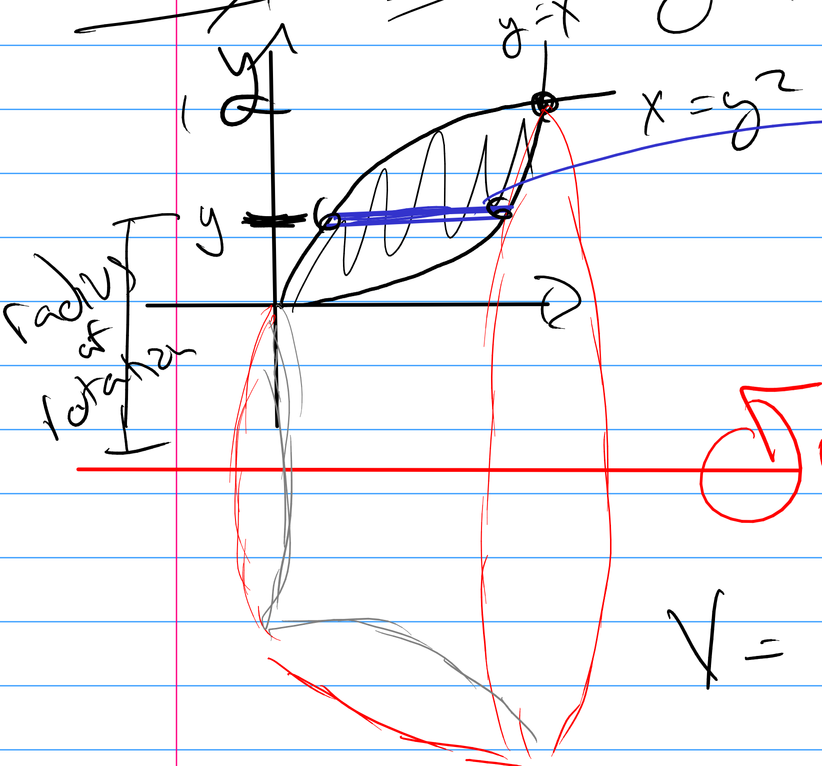
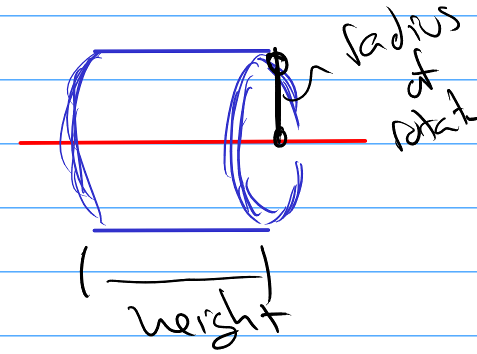
Math 243

Q's of 7.3 $\quad y=x^2 \quad x=y^2$ about $y=c$





$$V = \int_0^1 2\pi \left(\text{radius of rotation}\right)\left(\text{height}\right) dy$$

$$V = \int_0^1 2\pi (y-c)(\sqrt{y} - y^2)\, dy$$

$$= 2\pi \int_0^1 \left(-y^3 + cy^2 + y^{3/2} - cy^{1/2}\right) dy$$

$$= 2\pi \left[-\frac{1}{4} y^4 + \frac{c}{3} y^3 + \frac{2}{5} y^{5/2} - \frac{2c}{3} y^{3/2} \right]\Bigg|_0^1$$

$$V = 2\pi \left[-\frac{1}{4} + \frac{c}{3} + \frac{2}{5} - \frac{2c}{3} \right]$$

Watch movie for explaination of

Webassign 7.4 #1 (book 7.3(42))

$\boxed{\text{Midterm}}$ 16 probs + 1 extra credit

Chapter 10 (5 probs)

1. Vector ops problem (word problem?)
2. Dot Product (work?)
3. Cross Product (torque?)
4. Eqn of line
5. Eqn of plane.

$\boxed{\text{Ch 6}}$ 6 probs.

1. Integration by Parts

$$\int u\,dv = uv - \boxed{\int v\,du}$$

ex $\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \boxed{\int \frac{1}{2} x \, dx}$

let $u = \ln x$ $\qquad du = \frac{1}{x} dx$

$dv = x\,dx \qquad v = \frac{1}{2}x^2$

$=$ finish...

② Trig Substitution $(u = a\sin\theta)$

$$\int 3x^2 \sqrt{4-(x^3)^2}\, dx = \int \boxed{\sqrt{4-u^2}\, du}$$

let $u = x^3$ $\qquad$ let $u = 2\sin\theta$

$du = 3x^2 dx \qquad du = 2\cos\theta \, d\theta$

$$= \int 4\cos^2\theta \, d\theta = \text{finish.}$$

③ Partial Fraction Decomposition

$$\int \frac{x^3}{x^2-1}\, dx$$

$$\begin{array}{r} x \\ x^2 + 0x - 1 \overline{\smash{)}\, x^3 + 0x^2 + 0x + 0} \\ \underline{x^3 + 0x^2 - x} \\ x \end{array}$$

$$\int x + \frac{x}{x^2 - 1}\, dx$$

$$\int x\,dx + \int \frac{x}{(x+1)(x-1)}\, dx$$

$$\frac{1}{2}x^2 + \int \frac{x}{(x+1)(x-1)}\, dx$$

$$\frac{x}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$x = A(x-1) + B(x+1)$$

$$B = 1/2 \qquad A = 1/2$$

$$\frac{1}{2}x^2 + \int \left(\frac{1/2}{x+1} + \frac{1/2}{x-1} \right) dx$$

$$\frac{1}{2}x^2 + \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$$

$$\frac{1}{2}x^2 + \ln\sqrt{x^2-1} + C$$

(4) given several table integrals use a u-substitution followed by table integral.

(5) Simpson's Approx.

ex: ans $.1\left(\sqrt{1} + 4\sqrt{1.3} + 2\sqrt{1.6} + 4\sqrt{1.9} + 2\sqrt{2.2} + 4\sqrt{2.5} + \sqrt{2.8}\right)$

$$\int_1^{2.8} \sqrt{x}\, dx \quad \text{using } n = 6$$

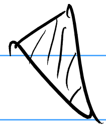
(6) Improper Integral. (type 1 + 2)

$$\int_0^{\infty} \frac{1}{x-1}\, dx$$

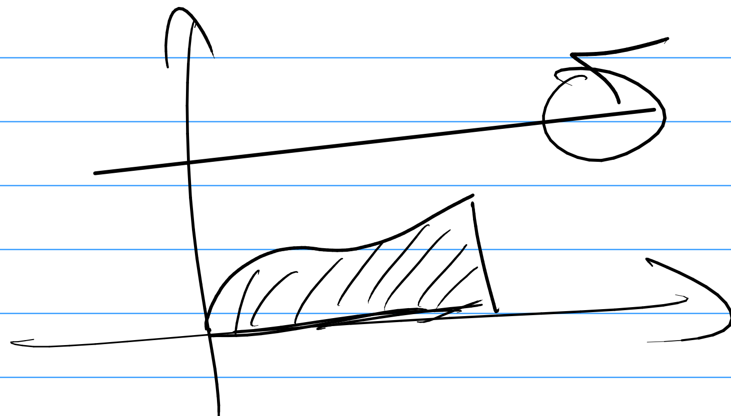
$$\int_0^1 \frac{1}{x-1}\, dx + \boxed{\int_1^{\infty} \frac{1}{x-1}\, dx}$$

$$\int_0^{t} \frac{1}{x-1}\, dx + \int_{t}^{2} \frac{1}{x-1}\, dx + \int_2^{t} \frac{1}{x-1}\, dx$$

Ch 7   5 probs

1. Volume by slicing
2.  disk? washer? shell? (you pick 'best' method)

ex



3. Arclength

ex $f(x) = \sin x$ how long is f from $x=0$ to $\pi/2$

$$\text{Arclength} = \int_0^{\pi/2} \sqrt{1+\cos^2 x}\, dx$$

$=$

4. Work problem

Spring or chain

5 Seperable Diff Eq.
with initial conditions.

---

extra credit

trig integral of $\sec^n x$ and/or $\tan^m x$
and/or product.

$$\int \sec^m x \tan^n x \, dx$$

7.6 assignment $x = 2$

$f(x) = x^3 - 1$

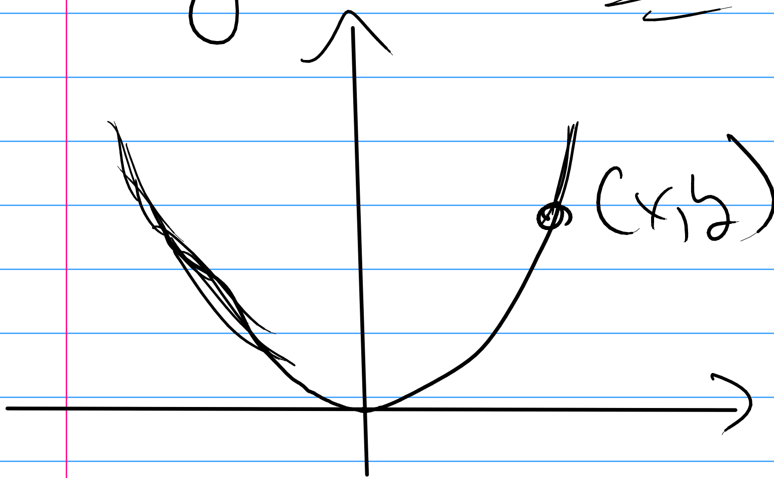
Propositional (yes / no question)

equation $x^2 - 3 = 2x + 1$

Is it equal?

$y = x^2$

<u>ex's</u>



$3 = 1^2$

False

$1 = 1^2$

<u>True.</u>

$f'(x)$ <u>rates</u>

$f(x)$ <u>amounts.</u>

$$f'(x) \alpha f(x)$$

$$f'(x) \alpha \left(M - f(x)\right)$$

$$f'(x) \alpha f(x)\left(M - f(x)\right)$$

ex

$$\boxed{P' = kP(M-P)}$$

Differential Equation

(any equation of functions and their derivatives)

ex $$y'' + y = 0$$

$$\frac{d^2 y}{dx^2} + y = 0$$

$$f''(x) + \boxed{f(x)} = 0$$

guess

ex $$\boxed{f(x) = \sin x}$$

b/c $$\frac{d^2}{dx^2}\{\sin x\} + \{\sin x\} \stackrel{?}{=} 0$$

$$\{-\sin x\} + \{\sin x\} \stackrel{?}{=} 0$$

$$0 \stackrel{?}{=} 0$$

$\boxed{\text{Yes}}$

Seperable Equ's

$$\frac{dy}{dx} = g(x)\,h(y)$$

$$\frac{1}{h(y)}\,dy = g(x)\,dx$$

$$\int \frac{1}{h(y)}\,dy = \int g(x)\,dx$$

ex

$$\boxed{\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}} \qquad \boxed{u(0) = -5}$$

initial value

$$\int 2u\,du = \int 2t + \sec^2 t\,dt$$

$$\rightarrow \boxed{u^2 = t^2 + \tan t + C}$$

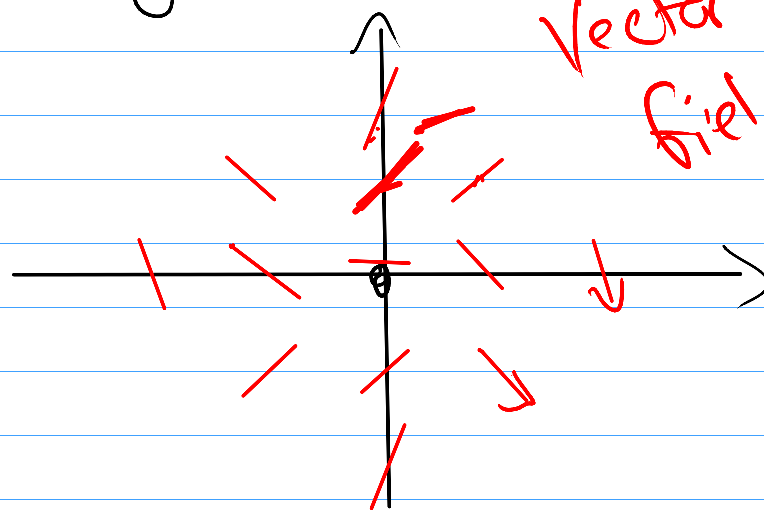
@ $t=0$ $u=-5$

$$25 = C$$

$$\boxed{\underline{\underline{\text{Curve}}} \quad u^2 = t^2 + \tan t + 25}$$

$$\boxed{\frac{dy}{dx}} = yx + y^2 - x^2$$

slope



Vector field.

Apps

$$\frac{dP}{dt} = kP(M-P)$$

$$\int \frac{1}{P(M-P)}\,dP = \int k\,dt$$

$$\underbrace{\int \frac{1}{P(M-P)}\,dP}_{} = kt + C$$

* Use partial fraction decomposition

$$\frac{1}{P(M-P)} = \frac{A}{P} + \frac{B}{M-P}$$

etc (Marina)

$$\longrightarrow \frac{\log(P)}{M} - \frac{\log(P-M)}{M}$$

$$\frac{1}{M} \ln|P| - \ln|P-M| = Kt + C$$

$$\frac{1}{M} \ln\left|\frac{P}{P-M}\right| = Kt + C$$

$$\ln\left|\frac{P}{P-M}\right| = M(Kt + C)$$

makes positive

$$\frac{P}{M-P} = e^{MKt + C}$$

$$\frac{P}{M-P} = C e^{MKt}$$

$$P = Ce^{MKt}(M-P)$$

$$P + PCe^{MKt} = Ce^{MKt}$$

$$\boxed{P = \frac{Ce^{Mkt}}{1+Ce^{Mkt}}}$$

$$t=0 \quad P = P_0$$

$$P_0 = \frac{C}{1+C}$$

$$P_0 + P_0 C = C$$

$$P_0 = C(1-P_0)$$

$$C = \frac{P_0}{1-P_0}$$

---

ex

#34

p405

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

let $S = \frac{dT}{dr}$ $\quad \frac{dS}{dr} = \frac{d^2T}{dr^2}$

$$\frac{dS}{dr} + \frac{2}{r}S = 0$$

$$\frac{dS}{dr} = -\frac{2}{r}S$$

$$\int \frac{dS}{S} = \int -\frac{2}{r}\,dr$$

$$\ln|S| = \ln(r)^{-2} + C$$

$$S = e^{\ln\frac{1}{r^2} + C}$$

$$S = \frac{C_1}{r^2}$$

$$\frac{dT}{dr} = \frac{C}{r^2}$$

$$\int dT = \int \frac{C}{r^2}\,dr$$

$$T = \frac{C_1}{r} + C_2$$

$r = 1 \qquad T = 15$

$r = 2 \qquad T = 25$

$$15 = C_1 + C_2$$

$$50 = C_1 + 2C_2$$

$$35 = C_2$$

$$\boxed{T = \frac{-20}{r} + 35}$$

$\boxed{\text{For Exam}}$ (type difficulty)

$$\frac{dP}{dt} = \sqrt{Pt} \qquad \text{when } t=1 \; P=2$$

$$\frac{dP}{\sqrt{P}} = \sqrt{t}\, dt$$

$$\int P^{-1/2}\, dP = \int t^{1/2}\, dt$$

$$2P^{1/2} = \frac{2}{3}t^{3/2} + C$$

$$2p^{1/2} = \frac{2}{3}t^{3/2} + C$$

and $t = 1 \quad P = 2$

$$2\sqrt{2} = \frac{2}{3}(1)^{3/2} + C$$

$$C = 2\sqrt{2} - \frac{2}{3}$$

$$\boxed{2p^{1/2} = \frac{2}{3}t^{3/2} + 2\sqrt{2} - 2/3}$$

---

diff eq. $\frac{dy}{dx} = \frac{-x}{y}$ @ $x = 1 \rightarrow y = 1$

$$x\,dx = -y\,dy$$

$$\int (2x)\,dx = \int(-2y)\,dy$$

$$x^2 = -y^2 + C$$

$$x^2 + y^2 = C \rightarrow (1,1) \Rightarrow C = 2$$

$$\boxed{x^2 + y^2 = 2}$$