Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 9

Created by Prof. Diego Maldonado and Prof. Virginia Naibo

Name: ______________________________

Due date: Thursday, April 4th at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. Let
$$A = \begin{bmatrix} -2 & 2 \\ 4 & 6 \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$
Solve for λ in the equation
$$\det(A - \lambda I) = 0.$$
Hint: Spell out all the operations, compute the 2×2 determinant, and solve the quadratic equation in λ.

2. Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be a linear transformation and $\beta = \{u_1, u_2, u_3\}$ be a basis of $\mathbf{R}^3$ such that

$$u_1 = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

and

$$T(u_1) = \begin{bmatrix} 4 \\ 6 \\ -1 \\ 5 \end{bmatrix}, \quad T(u_2) = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad T(u_3) = \begin{bmatrix} 3 \\ 7 \\ -2 \\ 5 \end{bmatrix}.$$

Find $T(u)$ where

$$u = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}.$$

Find a basis for $kernel(T)$. Is T one-to-one? Find a basis for $range(T)$. Is T onto?

3. Find the orthogonal projection of the vector $v = \begin{bmatrix} 2 \\ 4 \\ 6 \\ -2 \\ 4 \\ 8 \end{bmatrix}$ onto the subspace $\mathcal{U} = span(u_1, u_2, u_3)$, where $u_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, and $u_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. Compute the distance from v to $\mathcal{U}$.

4. Let N be the 2×5 matrix

$$N = \begin{bmatrix} -2 & 2 & 1 & 5 & 2 \\ 4 & 6 & -1 & 3 & 1 \end{bmatrix}$$

Find an **orthonormal** basis for $null(N)$ by first finding any basis and then applying the Gram-Schmidt process to it.

5. Write a Matlab function that takes an arbitrary 3×3 non-singular matrix U and performs and outputs the Gram-Schmidt process to its columns.

6. Find a decomposition of the vector $v = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 4 \\ 1 \end{bmatrix}$ of the form $v = \bar{v} + w$ such that $\bar{v}$ belongs to the subspace $\mathcal{U} = span(u_1, u_2)$, where $u_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ 3 \\ 4 \\ -1 \\ 1 \end{bmatrix}$, and $w \in \mathcal{U}^{\perp}$.

   Hint: Notice that u_1 and u_2 are **not** orthogonal (check out their dot product).

7. Consider the vectors

$$w_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \\ -2 \\ 1 \end{bmatrix} \quad w_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad w_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 5 \\ -8 \end{bmatrix}.$$

Find an **orthonormal** basis for the subspace of all the vectors in $\mathbf{R}^5$ which are orthogonal to w_1, w_2, and w_3.

Hint: Find a regular basis and then apply the Gram-Schmidt process to it.

8. True or False - **Circle the right one** (1 point each)

   T or **F**. The angle between the vectors w and v is the same as the angle between the vectors $2w$ and $4v$.

   T or **F**. If P and Q are $n \times n$ orthogonal matrices then PQ is also orthogonal.

   T or **F**. If Q is an orthogonal matrix, so is Q^T.

   T or **F**. For any $n \times n$ matrix A and any pair of vectors u and v in $\mathbf{R}^n$ we have

$$Au \cdot v = u \cdot A^T v$$

   T or **F**. Orthogonal matrices can only have determinant equal to 1 or -1.

Points obtained in this assignment (out of 16): ______________