Math 322

Modeling Computations

① Number Theoretic Functions

$$f : \mathbb{Z}^{+n} \to \mathbb{Z}^{+}$$

ex: $f(x) = x^3 + 3 = y$

$(0,3) \in f$

$(1,3) \notin f$

② Decision Problem (Yes-No problem)

Is it English?

Is it PEARL?

Natural Language vs Formal Language

Hard.

Make the rules first

→ Language?

syntax : form

semantics : meaning ← ignore

12.1 Languages and Grammars

↖ rules!

Def:

1. V : Vocabulary / Alphabet
   non-empty set of elements (symbols)

2. a sentence / word over V is a string of finite length of symbols from V.

3. λ is the string of no symbols

4. V^* : set of <u>all</u> sentences / words over V.

5. a Language over V is a subset of V^*

↖ set builder notation?

Is there a propositional function?
↳ yes-no problem.

Notation:

1. Vocabulary will be broken to two sets.

$$V = T \cup N$$

T: terminal elements (symbols that are not replaced)

N: non-terminal elements (symbols that can be replaced by other symbols)

2. $S \in N$, start symbol.

3. P set of productions

$$p: z_0 \rightarrow z_1$$

string z_0 can be replaced by string z_1

(replacement rules)

$\boxed{\text{p. 7.93}}$ $S \equiv$ Sentence

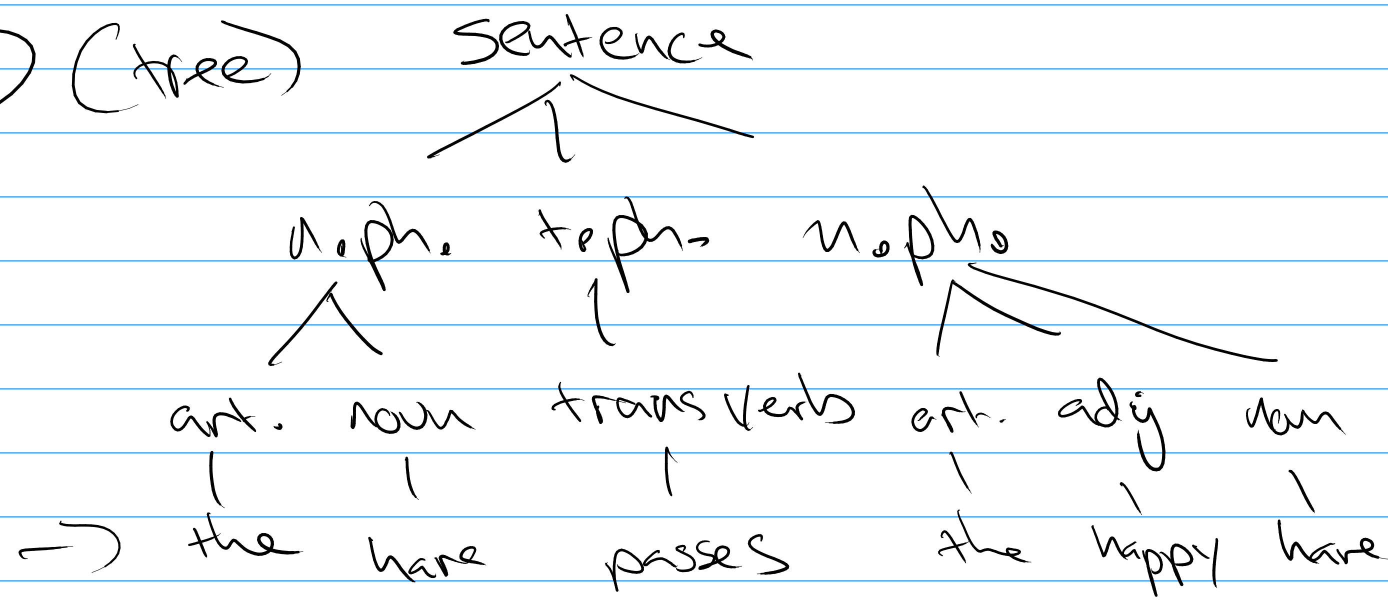
$T = \{$ the, sleepy, happy, ..., slowly $\}$

$N = \{$ Sentence, noun phrase, ... $\}$

$P = \{$

sentence → noun phrase trans verb ph. n. ph.

ex (tree)



Def: Phrase Structure Grammar

$G = (V, T, S, P)$

rules make the grammar.

Note: ① $T \subseteq V$

② $N = V - T$

③ $S \in N$ (start symbol)

④ all $p_i \in P$ $\quad p_i : z_0 \to z_1$

a) z_0, z_1 are strings

b) z_0 has at least <u>one</u> non-terminal.

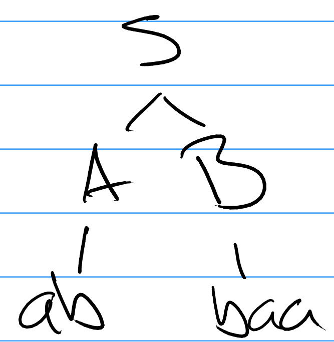
<u>ex's</u>

$$p_1: aAb \to aaab$$

<u>ex.</u> $V = \{\underbrace{a, b}_{T}, \underbrace{S, A, B}_{N}\}$

$$P = \{S \to AB,\ S \to aA,\ A \to ab,\ B \to baa,\ AB \to ba\}$$

<u>tree:</u>



<u>derivation:</u>

string w_0 derives to w_1

$(w_0 \Rightarrow w_1)$

if a production replaces string(s) in w_0 to make w_1

$$\underbrace{w_0 \Rightarrow w_1}_{\text{directly derivable}} \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n$$

entire chain $\quad w_0 \stackrel{*}{\Rightarrow} w_n$
is a derivation.

ex $P = \{ S \stackrel{1}{\to} AB,\ S \stackrel{2}{\to} aA,\ A \stackrel{3}{\to} ab,$
$B \stackrel{4}{\to} baa,\ AB \stackrel{5}{\to} ba \}$

$$S \underset{p_2}{\Rightarrow} aA \underset{p_3}{\Rightarrow} aab$$

$\boxed{S \stackrel{*}{\Rightarrow} aab}$ derivation.

Languages:

$$L(G) = \{ w \mid w \in T^* \wedge S \stackrel{*}{\Rightarrow} w \}$$

string

of only terminals

Because $\underline{P}$ makes the Grammar

what are the types?

$w_0 \to w_1$

type	Name	Restrictions
0	Phrase Structure Grammar	None
1	Context Sensitive	$w_1 = \boxed{l A r}$, $w_2 = \boxed{l w r}$, $w \in (N \cup T)^+$, $\boxed{w \neq \lambda}$ or non-dec. replacement $S \to \lambda$ (if S is never on right)
2	Context free	$w_1 = A$ (left is single non-term) e $A \to ab$
3	regular	$w_2 = aB$ or $w_2 = a$