

Math 321

Induction

Base: Show $P(1)$ is true for whatever the first part is

Weak

Strong

Inductive step

Show $(P(k) \rightarrow P(k+1))$

Show $(\sum_{i=1}^k P(i) \rightarrow P(k+1))$

by: assume $P(k)$ is true
Show $P(k+1)$ is true

by: assume $P(1) \wedge P(2) \wedge \dots \wedge P(k)$ true.
Show $P(k+1)$ true.

Ex:

$$\sum_{i=1}^n i^5 < \frac{n^5 - n^4 - 3}{n^6} \leq \frac{n^5}{n^6} = \frac{1}{n}$$

Show $\forall n \quad \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n} \quad n \geq 2$

a) $P(2): 1 + \frac{1}{4} < 2 - \frac{1}{2}$

b) is $P(2)$ true? $\frac{5}{4} < \frac{3}{2}$

$5(\frac{1}{4}) < 6(\frac{1}{4})$ yes.

if you would prove the statement

$$\forall n \geq 2 \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n} \right) \quad \text{is true}$$

Pf. Base: $P(2)$ is true (See above)

Inductive: (show $P(k) \rightarrow P(k+1)$)

assume $P(k): 1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{k^2} < 2 - \frac{1}{k}$

have to
show

$$P(k+1): 1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k+1}$$

given $1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{k^2} < 2 - \frac{1}{k}$ is true.

$$1 + \dots + \frac{1}{(k+1)^2} < 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$$

$$1 + \dots + \frac{1}{(k+1)^2} < 2 - \left(\frac{1}{k} - \frac{1}{(k+1)} \right) = 2 - \left[\frac{(k+1)^2 - k}{k(k+1)^2} \right]$$

we are here!

$$1 < 2 - \left[\frac{k^2 + k + 1}{k(k+1)^2} \right]$$

$$< 2 - \left[\frac{k^2 + k}{k(k+1)^2} \right] = 2 - \frac{1}{k+1}$$

True.

want to get 1 here!

$$\frac{1}{k+1} = \frac{k(k+1)}{k(k+1)^2}$$
$$\frac{k^2 + k}{k(k+1)^2}$$

Win: you take stuff. last player to take something loses.

q/c #11 you take at most 3 sticks.

one pile of n sticks.. $n = \{1, 2, 3, 4, \dots\}$

Base:

$n=1$	player one loses	$n \bmod 4 = 1$ player 1 loses
$n=2$	player two loses	
$n=3$	player two loses	
$n=4$	player two loses	$n \bmod 4 = 0, 2, 3$
$n=5$	player one loses	player 2 loses
$n=6$	player two loses.	

$n = 4j + 0, n = 4j + 2, n = 4j + 3$ player 2 loses
 $n = 4j + 1$ player one loses

Inductive: assume the pattern is true for $n = 1, 2, 3, \dots, K$
 show pattern works for $K+1$

$K+1 \bmod 4 = 0, 1, 2 \text{ or } 3$

Case 1

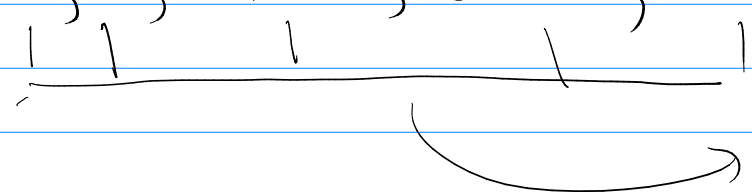
$K+1 \bmod 4 = 1$
$K \bmod 4 = 0$
$K-1 \bmod 4 = 3$
$K-2 \bmod 4 = 2$

Case 2

$K+1 \bmod 4 = 0, 2, 3$
$K+1 \bmod 4 = 0$ take 3
$K \bmod 4 = 1$
$K-1 \bmod 4 = 0$ take 1
$K-2 \bmod 4 = 3$
$K-3 \bmod 4 = 2$ take 2
$K-4 \bmod 4 = 1$

4.2 #7

\$2 and \$5

$$2, 2+2=4, 5, 3+2=6, 5+2=7, 4+2=8, 2+2+5=9, 2+5=10,$$


Basis:

$$2+2=\boxed{4}, \boxed{5}, 3+2=\boxed{6}, 5+2=\boxed{7}, 4+2=\boxed{8}$$

+ \$5

true.

Inductive:

assume \$4, \$5, \$6, \$7, \$8, ..., \$K

$$\text{take: } \underbrace{\$(K-4)}_{\uparrow} + 1.\$5 = \$(K+1)$$

this is
formed by \$2 and \$5

true.