Math 321

Q's

Logical Equiv.

$(\neg rain \vee \neg fog) \to (race \wedge demo)$

$(race) \to (trophy)$

$\neg trophy$

$\neg race$

deny the conclusion
Modus Tollens

$\neg(rain \wedge fog) \to (race \wedge demo)$

Simplification

$\frac{p \wedge q}{\therefore p}$

$\neg(rain \wedge fog) \to race$

$\neg race$

$rain \wedge fog$

$\therefore$ rain

Proofs:

Vacuous

if $1+1=3$, then Mark ~~is~~ smart.

tautology

Trivial

if Mark ~~is the~~ pres, then $1+1=2$

tautology

Direct Proof:

hyp $\to$ conclusion

assume hyp and show conclusion.

Indirect.

① Contrapositive. (show $\neg$conclusion $\to$ $\neg$hyp)

② Contradiction

(show: $\neg(\text{hyp} \to \text{con}) \equiv F$)

$(\text{hyp} \wedge \neg \text{con}) \equiv F$

1.7 ① Exhaustive Proofs

show: $\forall x P(x)$ is true.

if domain is "small"

$\boxed{\forall x P(x)} \equiv P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)$

ex 3x+1 $\boxed{\text{Conjecture}}$ ← "maybe theorem"

start with any pos. integer ≥ 2.

① if it is even divide by 2.

② if it is odd → $3x+1$

loop.

all numbers will go to 1.

$2 \to 1$

$3 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$

$4 \to 2 \to 1$

$5 \to 16 \to 8 \to 4 \to 2 \to 1$

this is a $\boxed{\text{th}^{\underline{m}}}$ if you say...

3x+1 holds for all integers between 2 and 56,000,000,000,000

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② Proof by cases..

$$(p_1 \vee p_2 \vee p_3 \vee \cdots \vee p_n) \to q$$

$$\equiv \underbrace{(p_1 \to q)}_{\text{case } 1} \wedge \underbrace{(p_2 \to q)}_{\text{case } 2} \wedge \cdots \wedge \underbrace{(p_n \to q)}_{\text{case } n}$$

③ Existance Proofs: $\exists x P(x)$

two techniques...

① Find the special c such that $P(c)$ is true.

→ A constructive proof.

② Show it exists don't find it.

ex th$^{\underline{m}}$: there is an $\text{irrational}^{\text{irrational}} = \text{rational}$

pf: given $\sqrt{2}$ is irrational.

so $\sqrt{2}^{\sqrt{2}}$

case 1 → rational

case 2 → irrational

so.. $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{2} = 2$ rational!

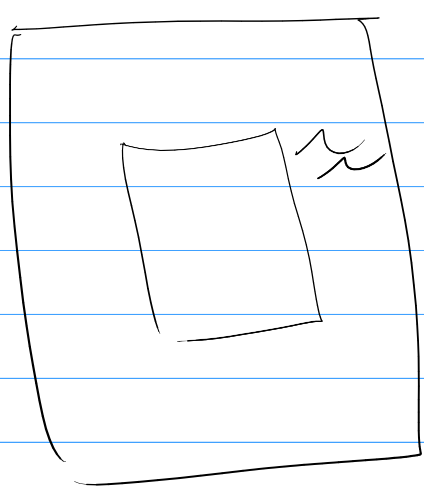
Hence:

$\sqrt{2}^{\sqrt{2}}$ is rational = irrational$^{\text{irrational}}$

xor

$\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ is rational = irrational$^{\text{irrational}}$

Conjectures.



$x^2 + y^2 = z^2$

$3^2 + 4^2 = 5^2$

$x^n + y^n = z^n$ $\quad n > 2$

$x^3 + y^3 = z^3$ $\quad n = 2, 3, 4,$