Math 322

Q's

12.1 #7

p_1: $S \to 0S1$

p_2: $S \to \lambda$

type 0 not type 1 b/c the $S \to \lambda$ along with $0S1$ on a right makes a decreasing produc.

$S \overset{*}{\Rightarrow} 000111$

$S \underset{p_1}{\Rightarrow} 0S1 \underset{p_1}{\Rightarrow} 00S11$

$\underset{p_1}{\Rightarrow} 000S111 \underset{p_2}{\Rightarrow} 000111$

$S \overset{*}{\Rightarrow} 000111$ because of this

$000111 \in L(G)$

Using same productions $S \to 0S1$, $S \to \lambda$

$L(G) = \{\lambda, 01, 0011, 000111, \ldots\}$

$\int f(x)\,dx = A_x(f(x)) = F(x) + C$

$\int_a^b f(x)\,dx$

12.2 Finite-State Machine with output.

$$M = (S, I, O, f, g, s_0)$$

$S \equiv$ Finite set of states
$I \equiv$ Input Alphabet (finite)
$O \equiv$ Output Alphabet (finite)

$f \equiv$ transition function (ordered triples)

$$f : S \times I \to S$$

$g \equiv$ output function (ordered triple)

$$g : S \times I \to O$$

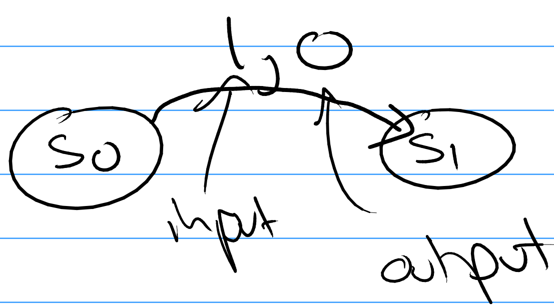
$s_0 \equiv$ start state

Visualize:

State diagram: ① State $\equiv$ (label)

② f/g $\quad (s_0, 1, s_1) \in f$

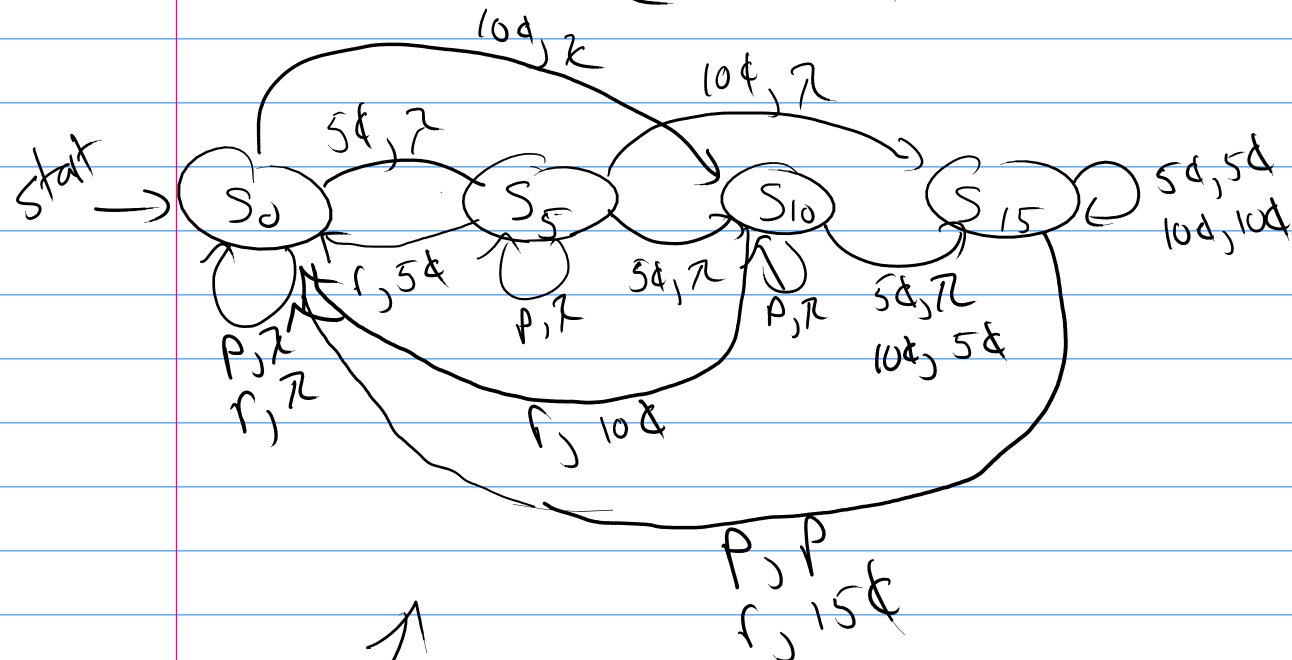
$(s_0, 1, 0) \in g$



15¢ Pop Machine

$I = \{5¢,\ 10¢,\ pop,\ return\}$

$O = \{pop,\ \lambda,\ 5¢,\ 10¢,\ 15¢\}$



M

feed it 10, 10, 5, 5, p, p, p, r, 10, 5, r

Input

Output: λ, 5, 5, 5, p, λ, λ, λ, λ, λ, 15¢

State Table

	f				g			
state	5	10	p	r	5	10	p	r
S_0	S_5	S_{10}	S_0	S_0	λ	λ	λ	λ
S_5	S_{10}	S_{15}	S_5	S_0	λ	λ	λ	5
S_{10}	S_{15}	S_{15}	S_{10}	S_0	λ	5	λ	10
S_{15}	S_{15}	S_{15}	S_0	S_0	5	10	p	15

$I = \{0,1\} \quad O = \{0,1\}$

Def: $L \subseteq I^*$

M recognizes a string x of input iff the last output symbol of M given x is 1.

L(M) is the set of all such x.

(12.3) Finite-State Machine with no output.

$M = (S, I, f, s_0, F)$

S = states

I = inputs

$f : S \times I \to S$

s_0 = start state

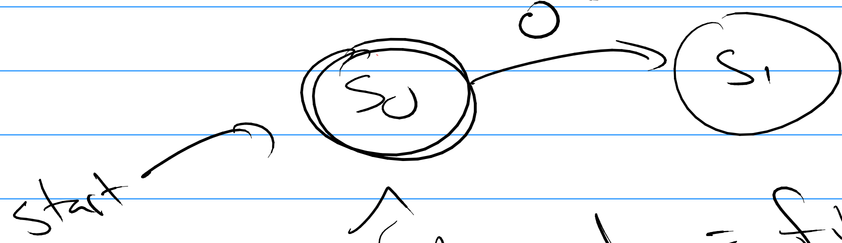
$F \subseteq S$ (final states)

states that an input string should take M to.

Visualize

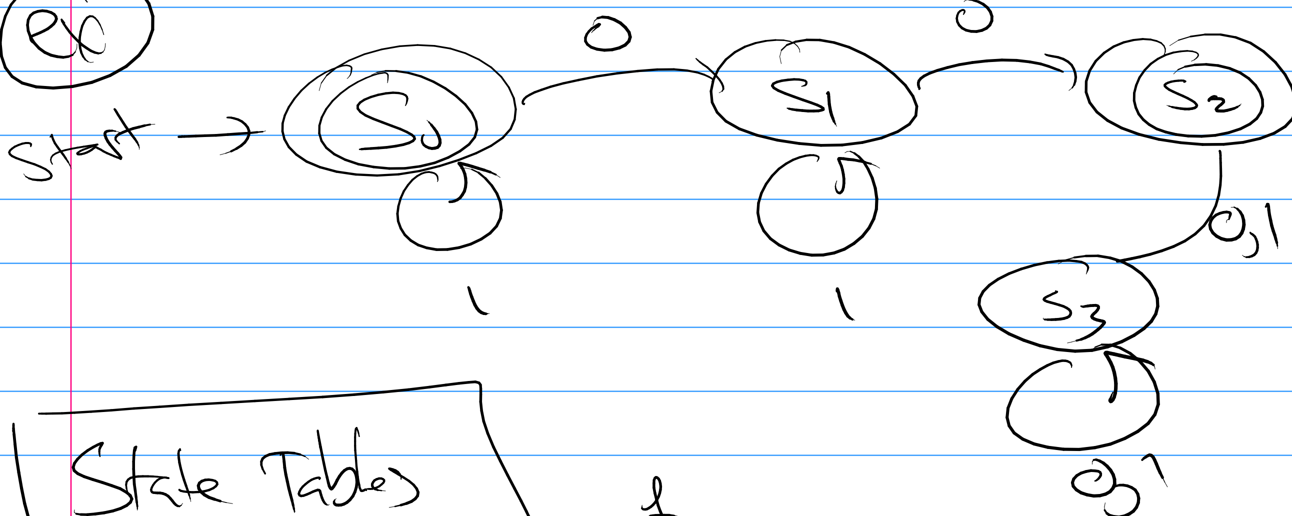
① State Diagrams

all edge labels are inputs



double circle = final state

ex



State Tables

	f	
	Inputs	
States	0	1
S_0	S_1	S_0
S_1	S_2	S_1
S_2	S_3	S_3
S_3	S_3	S_3

Input Strings of symbols.

Symbol ops.

① concatnation

$S_1 = 00 \quad S_2 = 10$

$S_1 S_2 = 0010$

② Union

$S_1 \cup S_2 = \{00, 10\}$

Symbol Sets Ops

① Concatnation

$A = \{0, 01\} \quad B = \{1, 10\}$

$AB = \{01, 010, 011, 0110\}$

$S_1 S_2 = \{ s_1 s_2 \mid s_1 \in S_1 \wedge s_2 \in S_2 \}$

② Union

$A \cup B = \{0, 01, 1, 10\}$

③ Power

$A^n = \overbrace{AAA \cdots A}^{n \text{ concatnations}}$

(4) Kleene Closure

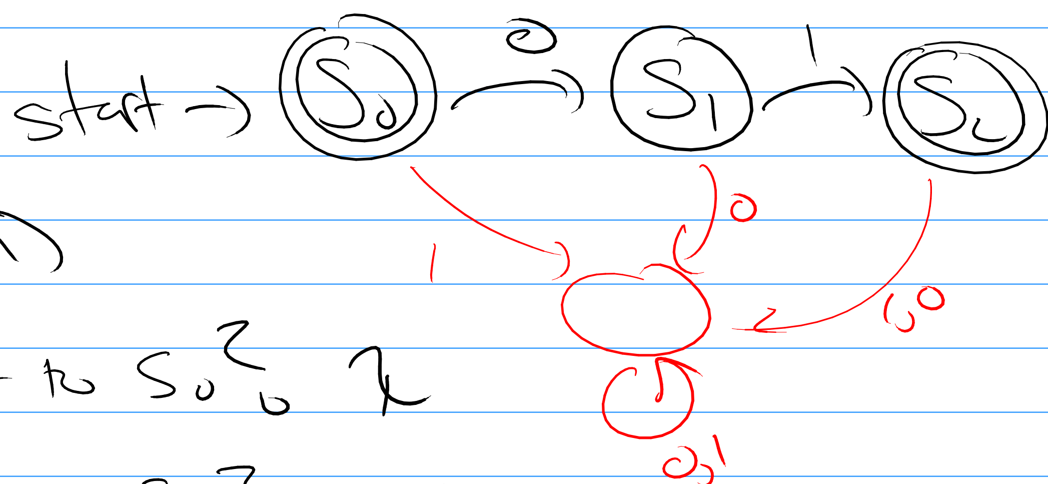
$$A^* = A^0 \cup A^1 \cup A^2 \cup \cdots$$

ex $\{0,1\}^* = \{\lambda, 0, 1, 00, 01, 10, 11, \ldots\}$

<u>all</u> bit strings

$L(M)$ is set of all $x \in I^*$ iff it takes S_0 to a final state.

ex



$L(M)$

how to get to S_0? λ

how to get to S_2? 01

$$L(M) = \{\lambda, 01\}$$

$$L(M) = \{0\{0,1\}^*, \boxed{10\{0,1\}^*, 11\{0,1\}^*}\}$$

$$= \{0, 10, 11\}\{0,1\}^*$$

$$= \{0\{0,1\}^*, 1\{0,1\}^n \mid n \geq 1\}$$

Deterministic vs Non-Deterministic