Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 4

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Name: ____

Due date: Thursday February 21 at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises

1. **The inverse matrix and cryptography**. Suppose that the coded message

$$P = \begin{bmatrix} 122 & 123 & 87 & 127 \\ 84 & 95 & 106 & 89 \\ 124 & 79 & 53 & 99 \end{bmatrix}$$

was intercepted and our code breakers assure us that it means **No way Jose**. Later, the message

$$Q = \begin{bmatrix} 131 & 21 & 47 & 37 & 122 & 109 & 70 & 113 \\ 101 & 26 & 98 & 34 & 76 & 50 & 65 & 69 \\ 95 & 55 & 17 & 79 & 108 & 147 & 60 & 157 \end{bmatrix}$$

is intercepted. What does it mean?

2. **Lanchester's warfare model**. Set up the equations (as a system and also in matrix form) of a warfare model for a three-way battle involving units from armies X, Y, and Z.

3. **Richardson's arms race model and the cost of peace by having the bigger stick**. Consider the following situations for countries X and Y and their corresponding fear and fatigue factors:

   (a) $a = .4$, $m = .4$ and $b = .2$, $n = .2$. Notice that the corresponding Richardson matrix is row stochastic; thus, peace is possible. Find a non-zero peace vector $p = [p_1, p_2]'$. Look at the ratio p_2/p_1

   (b) $a = .3$, $m = .6$ and $b = .6$, $n = .3$. Notice that the corresponding Richardson matrix is column stochastic; thus, peace is possible here too. Find a non-zero peace vector $p = [p_1, p_2]'$. Look at the ratio p_2/p_1.

   Between those two peaceful scenarios, which one is more convenient for country Y? Justify.

4. A certain economy consists of three sectors: Agriculture, Manufacturing, and Labor. \$1 of Agriculture requires 40 cents in agriculture, 20 cents in Manufacturing, and 60 cents in labor. \$1 of Manufacturing uses 70 cents in Manufacturing and 40 cents Labor. \$1 worth of Labor takes 25 cents of in Agriculture and 10 cents of Manufacturing. Suppose that the external demand is for \$200 Agriculture, \$500 Manufacturing, and \$600 Labor. Set up an input-output model for this economy, determine the consumption matrix, and find the production schedule. Which sector consumes the most Labor?

5. Determine all the columns of P that are spanned by the remaining ones. Justify.

$$P = \begin{bmatrix} 5 & 3 & 1 & 0 \\ 4 & 4 & 6 & 6 \\ -1 & -9 & -1 & -7 \\ 0 & 2 & -4 & 1 \end{bmatrix}.$$

6. The following matrix

```
A =

     0     1     0     0     1     0     0     0
     1     0     1     1     0     1     1     1
     1     1     0     0     0     0     0     0
     0     0     1     0     0     1     1     1
     0     0     1     1     0     1     1     1
     0     0     0     1     1     0     1     1
     1     1     1     1     0     0     0     0
     0     0     1     0     1     0     0     0
```

is the adjacency matrix of a graph describing the outcomes of a basketball tournament. The vertices represent the basketball teams and the edges represent that team i "beat" team j. What team won the most games? What team was defeated the most? Justify by using properties of the matrix A.

7. Consider the matrix

```
W =

     1    -4     5     6     5     2
     2    -4    -5     0     1    -4
     3     9     1     1    -7    10
     1    -4     5     6     5     2
     2    -4    -5     0     1    -4
     3     9     1     1    -7    10
```

and let w_1, w_2, w_3, w_4, w_5, and w_6 denote the columns of W (in the specified order). Show that set $\mathcal{M} = \{w_1, w_4, w_6\}$ is a *maximal set* of linearly independent columns of W, in the sense that if we add to $\mathcal{M}$ any other column of W, the resulting set is a set of linearly dependent vectors.

Hint. First check that the vectors in $\mathcal{M}$ are linearly independent. Then, add to $\mathcal{M}$ the columns of W, one at a time, and check that in each case you obtain a linearly dependent set (that is, a set of linearly dependent vectors).

8. True or False - **Circle the right one** (One point each)

   T or **F**. There exist real numbers x_1, x_2, and x_3 such that

$$x_1 \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

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   T or **F**. If A and B are $n \times n$ invertible matrices, then $A + B$ is also invertible.

   T or **F**. An $n \times n$ matrix A is invertible if and only if $rank(A) = n$.

   T or **F**. The vector

$$v = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

   belongs to $span(u_1, u_2, u_3)$, where

$$u_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \text{ and } \quad u_3 = \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}.$$

Points obtained in this assignment (out of 16): ______________