Math 322

Exam on thurs. (12 probs + lec)

8.1 Relations (2 probs)

① does it have ____ ← ref? irref? etc.

give $R = \{ (a,b) \mid a,b \text{ have a property} \}$

ex/ (a is taller than b)

reflexive: $\forall a \; aRa$

$\equiv \forall a$ (a is taller than a)

No? give counterexample.

$\boxed{\text{etc.}}$

② prove thm 1 on page 527

8.2 n-ary Relations ($\emptyset$ probs)

8.3 Representing Relations (2 probs)

① list ⟷ Matrix

list ⟷ digraph

digraph ⟷ Matrix

(2) Matrix Ops. for Relations

$R_1 \cup R_2 \quad R_1 \cap R_2 \quad R_1 \circ R_2$

with matrices.

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_1 \circ R_2 \qquad M_{R_2} \odot M_{R_1}$

$$M_{R_1 \circ R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

8.4 Closures (3 probs)

(1) Find Reflexive closure
Find Symmetric closure

(2) Find Trans. closure using...

$$M_{R^*} = M_R \vee M_R^{[2]} \vee \cdots \vee M_R^{[n]}$$

③ Find trans closure using Warshall's Algorithm.

8.5 Equiv. Relations (2 probs)

① Is it one?

ex of trans. check.

$R = \{(a,b) \mid a \mid b\}$ a,b are integers

$\forall a \forall b \forall c\,(aRb \wedge bRc \rightarrow aRc)$

$\equiv \forall a \forall b \forall c\,(a \mid b \wedge b \mid c \rightarrow a \mid c)$

$\equiv \forall a \forall b \forall c\,(\underbrace{b = c_1 \cdot a \wedge c = c_2 \cdot b} \rightarrow c = c_3 \cdot a)$

$c = c_2 \cdot (c_1 \cdot a)$

$c = \underbrace{(c_2 \cdot c_1)}_{\text{let } c_3 = c_2 \cdot c_1} \cdot a$

$\rightarrow c = c_3 \cdot a$

$a \mid c$

Still need to check reflexive & sym.

(2) Find equivalence classes.

$$[a]_R = \{ s \mid aRs \}$$

$\boxed{8.6}$ Partial Orderings (3 probs)

(1) Is it one?

(2) Make a Hasse diagram

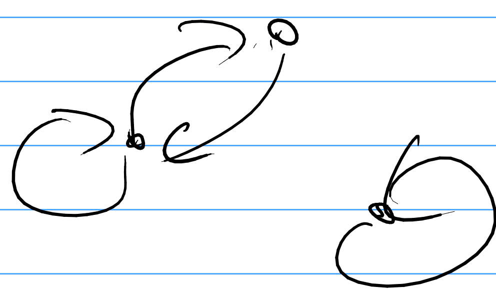
(3) Use a Hasse diagram

$\boxed{\text{extra credit}}$

prove lemma 1 p. 548-549

$\boxed{\text{chapter 9 - Graphs}}$

Nd: digraphs



Consider it has two main features.

(1) non-empty set of vertices

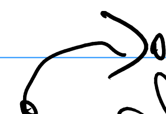
(2) set of associations between one or two vertices.

$G = (V, E)$

G is a graph has a non-empty set V of vertices and a set E of edges.

two types of associations

no order (undirected)

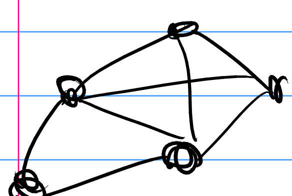


order (directed)

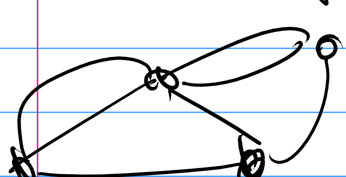
Undirected

① Simple Graph
- no loops
- no mult. edges

ex

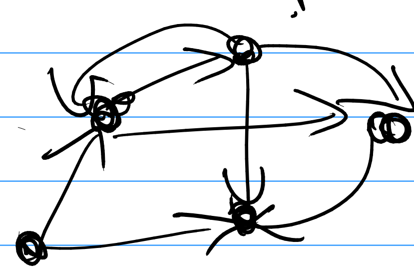


② Multigraph
- no loops

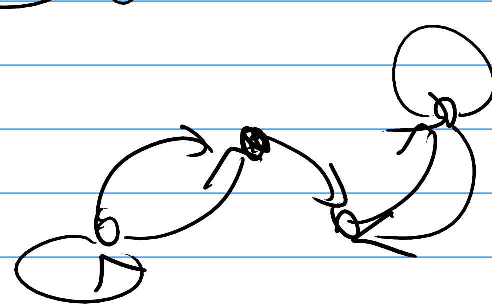


Directed

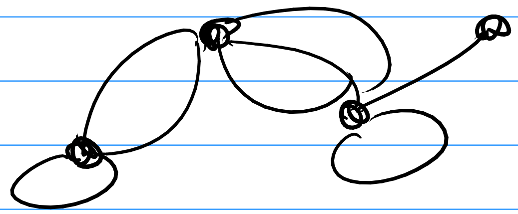
① Simple directed
- no loops
- no mult. edges



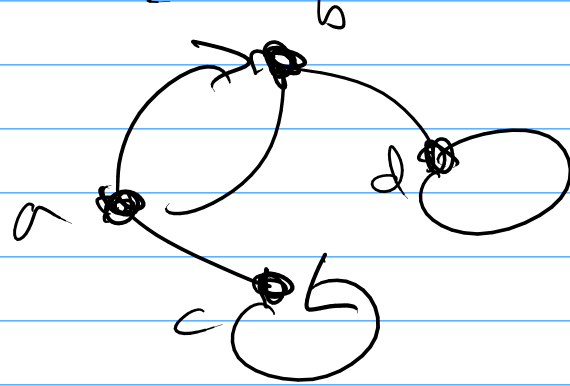
② directed multigraph



③ Pseudograph



Mixed directed <u>and</u> undirected.



$$G = (\{a,b,c,d\}, \{(a,b), \{a,b\}, \{a,c\}, (c,c), \{b,d\}, \{d,d\}\})$$

end of 9.1 has "real life" applications.

9.2 terms / types

For Undirected Graphs

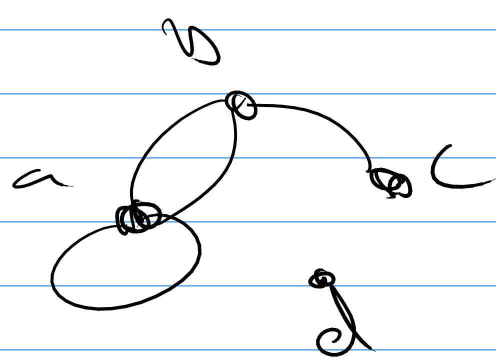
a) $e = \{u, v\}$

edge   end points

u and v are adjacent

e is incident to u and v

$\deg(v)$ = number of edges incident to it, but loops count as 2



$|V| = 4 \quad |E| = 4$

$\deg(a) = 4 \quad \deg(b) = 3$

$\deg(c) = 1 \quad \deg(d) = 0$

pendent   isolated

$$\sum \deg(v) = 4 + 3 + 1 + 0 = 8 = 2 \cdot 4$$

$\underline{\underline{Th^m}}$: $\sum_v \deg(v) = 2|E|$

$\underline{\underline{Th^m}}$: $\sum_v \deg(v) = \sum_{\text{even's}} \deg + \underbrace{\sum_{\text{odd's}} \deg}_{\text{even number of odd deg. vertices}}$

For Directed Graphs

$e = (u, v)$ u is adj. to v

initial   terminal v is adj. fom u

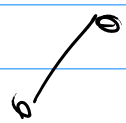
$\deg^-(u)$ the in-degree is number of edges with u as terminal.

$\deg^+(u)$ the out-degree is number of edges with u as initial.

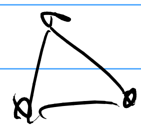
$\underline{\underline{Th^m}}$: $\sum_v \deg^-(u) = \sum_v \deg^+(u) = |E|$

Special Simple Graphs

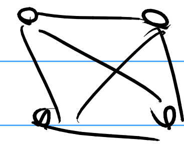
① K_n (complete graph)



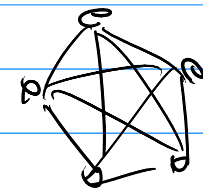
$n=2$



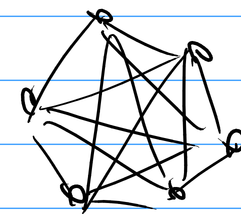
$n=3$



$n=4$

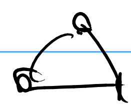


$n=5$

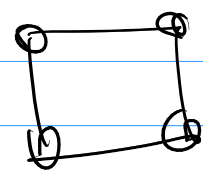


$n=6$

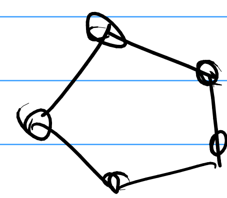
② C_n (cycle)



$n=3$



$n=4$



$n=5$

...