

Math 321

Q5 / Knight \rightarrow always tells truth
Knaave \rightarrow always lies

55 A says "At least one of us is a Knaave"

B says nothing.

a) if A was a knight what he says is true.

A: Knight B: Knaave

b) if A was a Knaave what he says is false

"At least one of us is a Knaave" = ~~F~~ \rightarrow T

So for a Knaave \rightarrow is true!

56 Focus on B
if B is a Knaave \rightarrow A is a Knight
if B is a Knight \rightarrow A is a Knaave \leftarrow

A says "we are both knights"

but by \leftrightarrow we know they are not the same.

So A is a Knaave \wedge B is a Knight.

23c Given $\boxed{p \Rightarrow q}$ (implication)

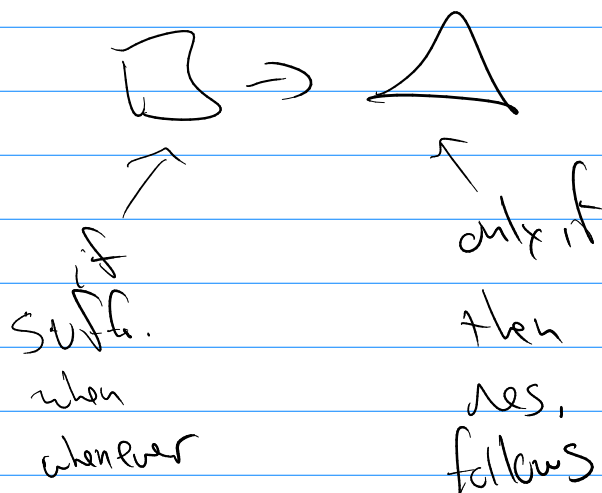
converse $q \Rightarrow p$

inverse $\neg p \Rightarrow \neg q$

contrapos $\neg q \Rightarrow \neg p$

pos. int. is prime only if only 1 and itself divides it.

Ex:



prime \Rightarrow 1 & itself are only divisors

converse: (prime) \Leftarrow (1 & itself are only divisors)

it is suff. for a number to have only 1 and itself as divisors for it to be prime.

etc.

1.2

Sameness & compound propositions

Idem: two compound propositions are the "Same" if they have the same truth values when the individual proposition truth values match.

Def: ① a compound proposition that is always true (no matter the truth values of the propositions in it) is a tautology.

② Always false? contradiction

③ Sometimes True. Sometimes false.

Contingency

ex	$\neg p$	p	q	$p \supset q$	$p \wedge \neg p$	$p \vee \neg p$
F	T	T	T	T	F	T
F	T	T	F	F	F	T
T	F	F	T	T	F	T
T	F	F	F	T	F	T
				<u>Contingency</u>	<u>Contradiction</u>	<u>tautology</u>

(ex)

$$p \rightarrow q$$

vs

$$\neg p \vee q$$

p	q	$p \rightarrow q$
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T	T
T	F
F	T
F	F

T
F
T
T

p	q	$\neg p$
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T	T	F
T	F	F
F	T	T
F	F	T

$\neg p \vee q$

T
F
T
T

Def: if $p \leftrightarrow q$ is a tautology
then $p \equiv q$ are logically equivalent.

(ex) show $(p \rightarrow q) \equiv (\neg p \vee q)$

means show $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$ is a tautology

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
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T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

tautology

so $(p \rightarrow q) \equiv (\neg p \vee q)$

Logical Equivalence to know

[p. 21] table 6, table 7 (1st two), table 8

ex $\neg(p \wedge q) \equiv (\neg p \vee \neg q)$

De Morgan's law

$$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$$

Using Logical Equivalences.

ex $(p \wedge q) \rightarrow p \quad \square \rightarrow \Delta$

$$\begin{aligned} &\equiv \neg(p \wedge q) \vee p && \equiv \neg \square \vee \Delta \\ &\equiv \neg p \vee \neg q \vee p \\ &\equiv \neg p \vee p \vee \neg q \\ &\equiv (\neg p \vee p) \vee \neg q \\ &\equiv \top \vee \neg q \\ &\equiv \top \end{aligned}$$