CIS 770: Formal Language Theory

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Kansas State University

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General Computing Machines

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  - No limitation on what they can compute?
  - No! There are far too many languages over $\{0, 1\}$ than there are "machines" or programs (as long as machines can be represented digitally)
  - Come up with a model that describes all "conceivable" computation

General Computing Machines

Alonzo Church, Emil Post, and Alan Turing (1936)

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- In this course: Turing Machines

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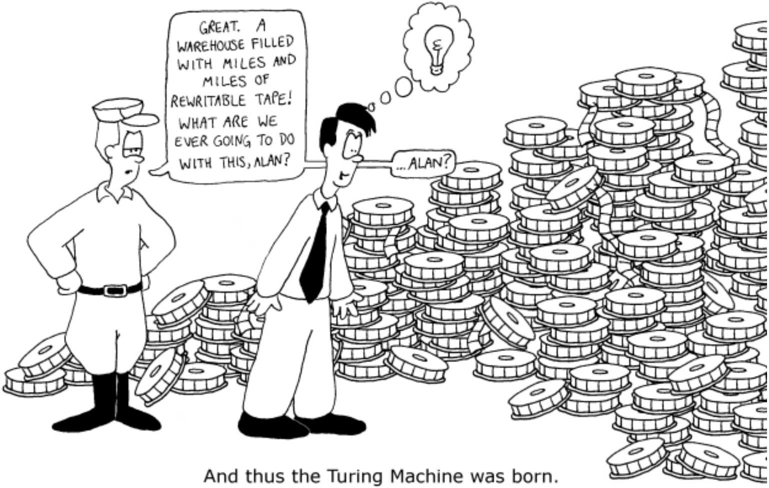
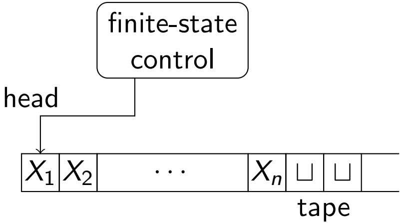


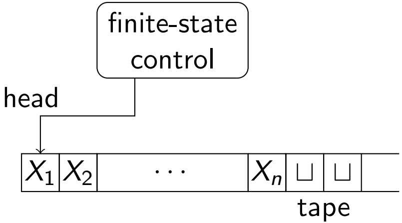
And thus the Turing Machine was born.

Turing Machines



- Unrestricted memory: an infinite tape

Turing Machines



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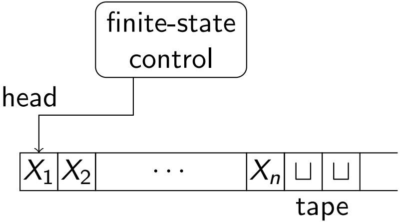

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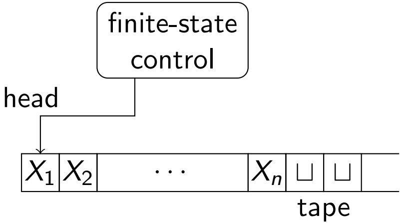

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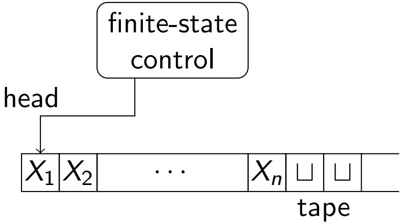

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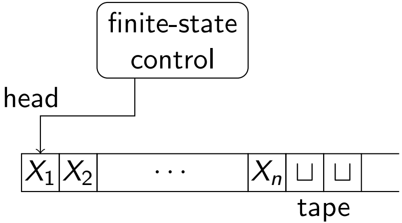

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- Initially, tape has input and the machine is reading (i.e., tape head is on) the leftmost input symbol.
- Transition (based on current state and symbol under head):
  - Change control state
  - Overwrite a new symbol on the tape cell under the head
  - Move the head left, or right.

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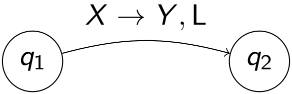
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}\}$ is the transition function. Given the current state and symbol being read, the transition function describes the next state, symbol to be written and direction (left or right) in which to move the tape head.

Transition Function



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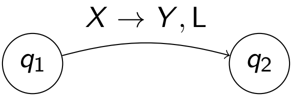

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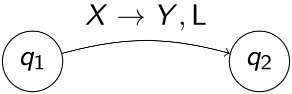

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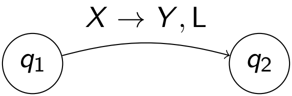

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$$q_1 \xrightarrow{X \to Y, \mathrm{L}} q_2$$

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- Transitions are deterministic
- Convention: if $\delta(q, X)$ is not explicitly specified, it is taken as leading to q_{rej}, i.e., say $\delta(q, X) = (q_{\mathrm{rej}}, \sqcup, \mathrm{R})$

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Single Step

Definition

We say one configuration (C_1) yields another (C_2), denoted as $\mathrm{C}_1 \vdash \mathrm{C}_2$, if one of the following holds.

- If $\delta(q, X_i) = (p, Y, \mathrm{L})$ then

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Acceptance and Recognition

Definition

A Turing machine M accepts w iff $q_0 w \vdash^* \alpha_1 q_{\mathsf{acc}} \alpha_2$, where α_1, α_2 are some strings. In other words, the machine M when started in its intial state and with w as input, reaches the accept state.

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High level description

```
On input string w
    while there are unmarked 0s, do
        Mark the left most 0
        Scan right till the leftmost unmarked 1;
            if there is no such 1 then crash
        Mark the leftmost 1
    done
    Check to see that there are no unmarked 1s;
        if there are then crash
    accept
```

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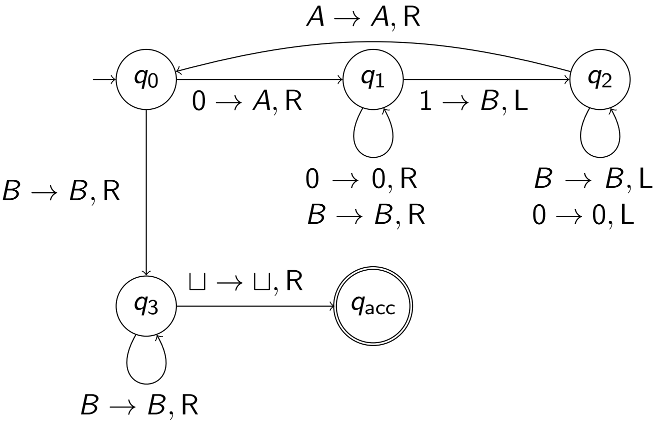

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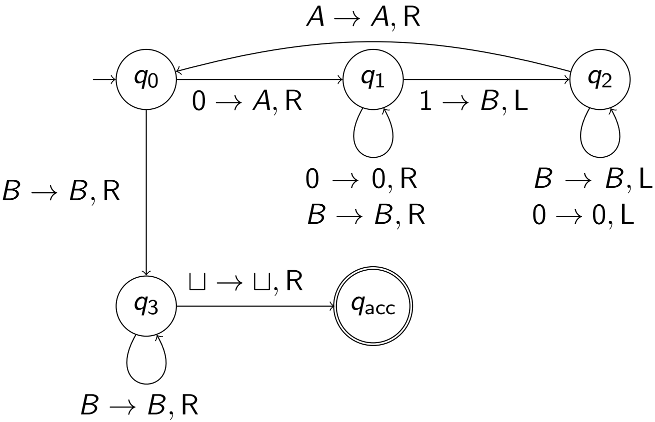

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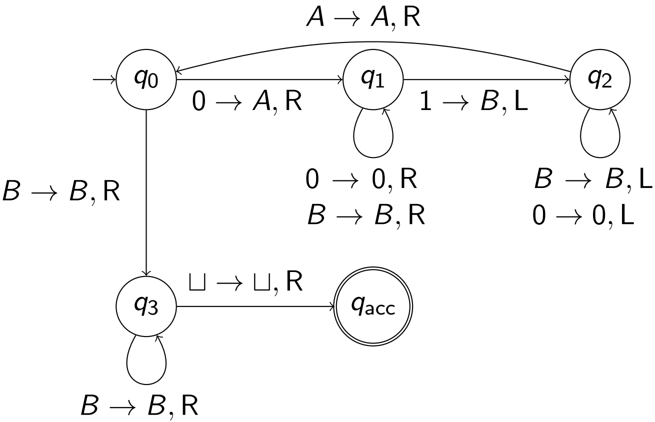

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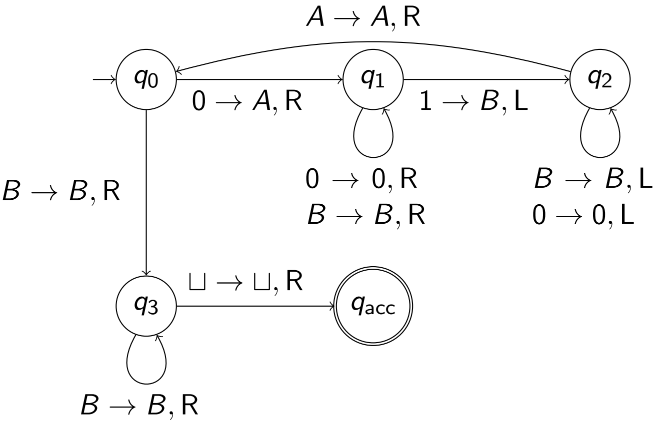

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash$

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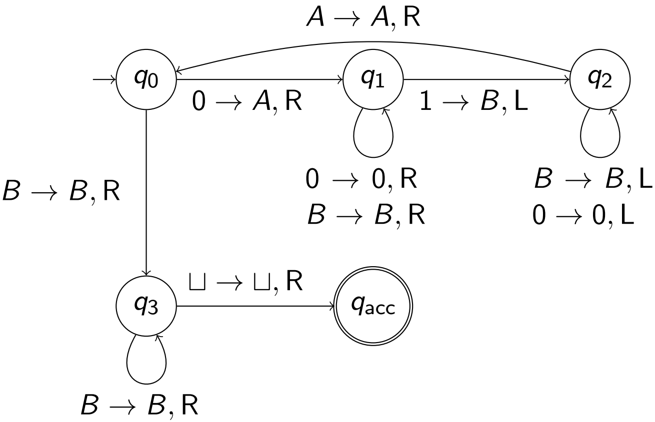

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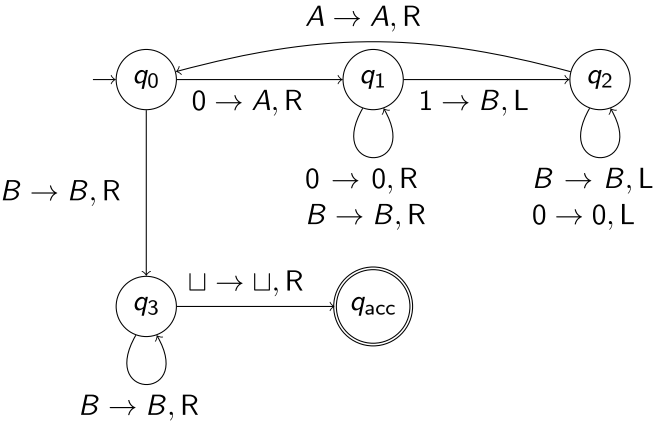

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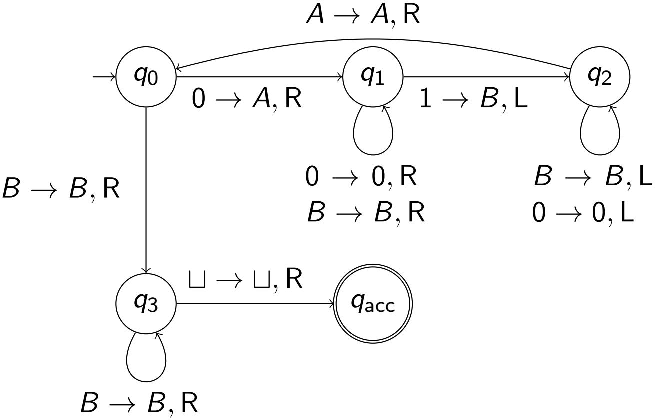

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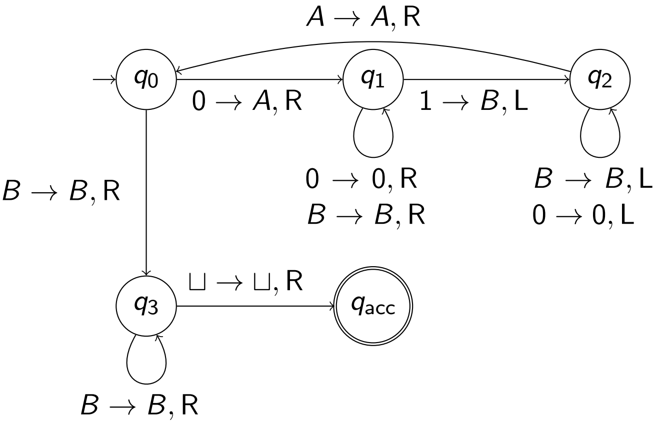

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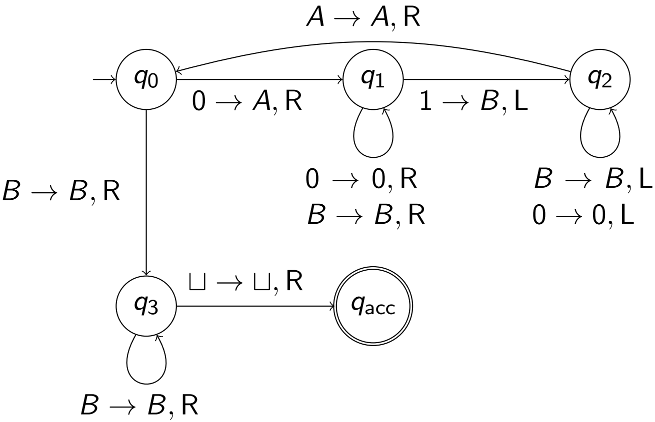

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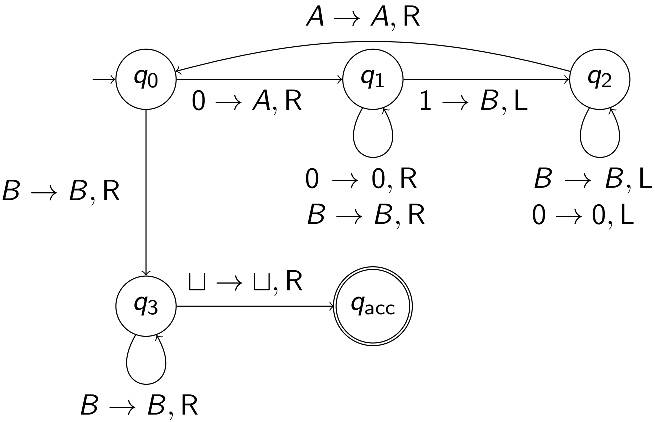

- Accepts input 0011: $q_00011 \vdash Aq_1011 \vdash A0q_111 \vdash Aq_20B1 \vdash q_2A0B1 \vdash Aq_00B1 \vdash AAq_1B1 \vdash AABq_11 \vdash AAq_2BB \vdash$

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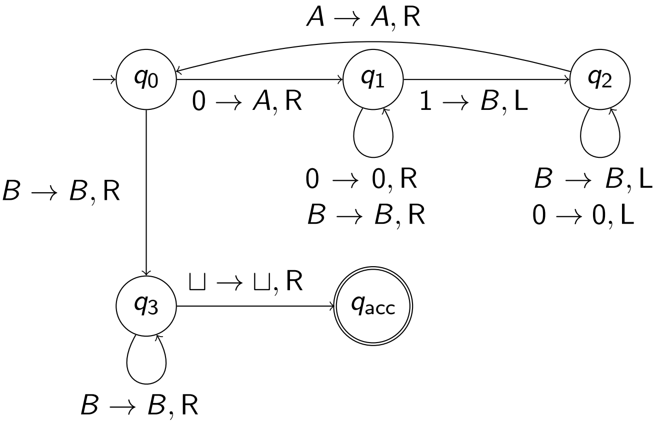

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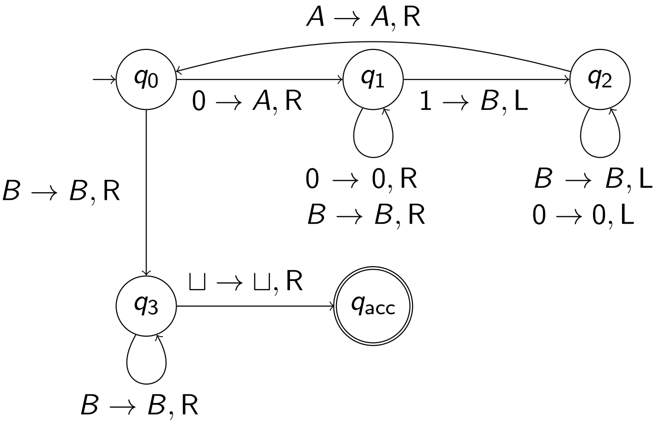

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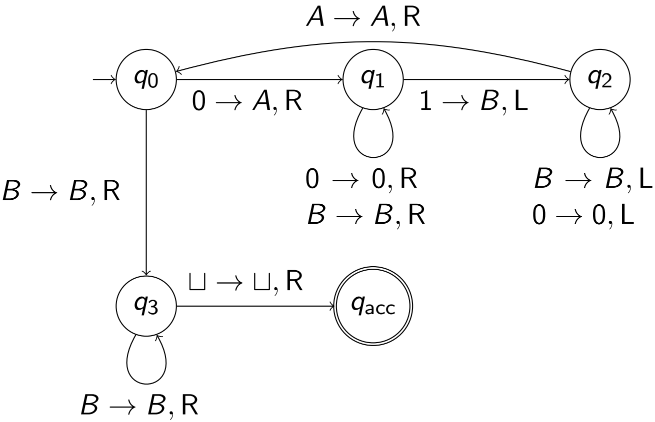

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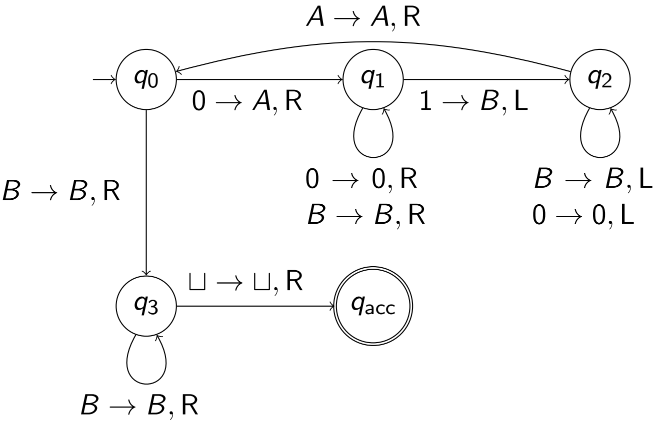

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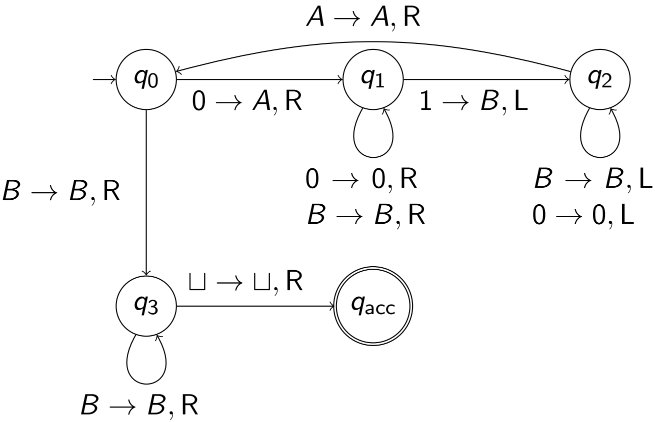

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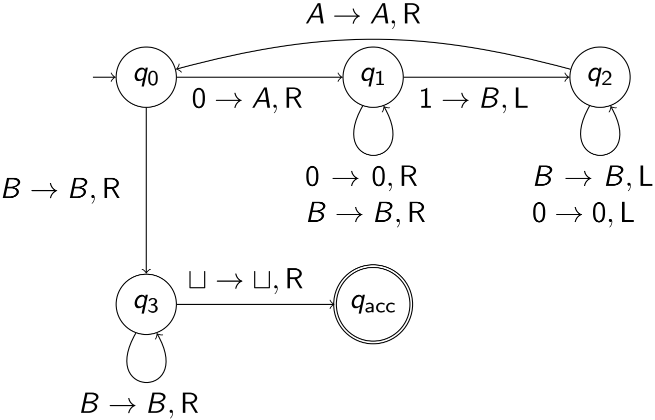

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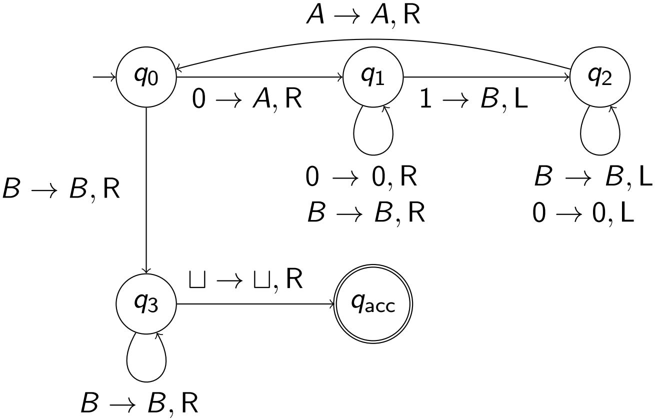

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- Rejects input 00: $q_0 00 \vdash Aq_1 0 \vdash A0q_1 \sqcup \vdash A0 \sqcup q_{rej} \sqcup$

The machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$ where

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Formal Definition

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$$
\begin{array}{ll}
\delta(q_0, 0) = (q_1, A, R) & \delta(q_0, B) = (q_3, B, R) \\
\delta(q_1, 0) = (q_1, 0, R) & \delta(q_1, B) = (q_1, B, R) \\
\delta(q_1, 1) = (q_2, B, L) & \delta(q_2, B) = (q_2, B, L) \\
\delta(q_2, 0) = (q_2, 0, L) & \delta(q_2, A) = (q_0, A, R) \\
\delta(q_3, B) = (q_3, B, R) & \delta(q_3, \sqcup) = (q_{acc}, \sqcup, R)
\end{array}
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In all other cases, $\delta(q, X) = (q_{rej}, \sqcup, R)$.

Example: $\{0^n 1^n \mid n > 0\}$

Formal Definition

The machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ where

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\end{array}
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In all other cases, $\delta(q, X) = (q_{\text{rej}}, \sqcup, R)$. So for example, $\delta(q_0, 1) = (q_{\text{rej}}, \sqcup, R)$.

Example 2: TM for $\{0^n1^n2^n \mid n > 0\}$

Design a TM to accept the language $L = \{0^n1^n2^n \mid n > 0\}$

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High level description

```
On input string w
    while there are unmarked 0s, do
        Mark the left most 0
        Scan right to reach the leftmost unmarked 1;
            if there is no such 1 then crash
        Mark the leftmost 1
        Scan right to reach the leftmost unmarked 2;
            if there is no such 2 then crash
        Mark the leftmost 2
    done
    Check to see that there are no unmarked 1s or 2s;
        if there are then crash
    accept
```

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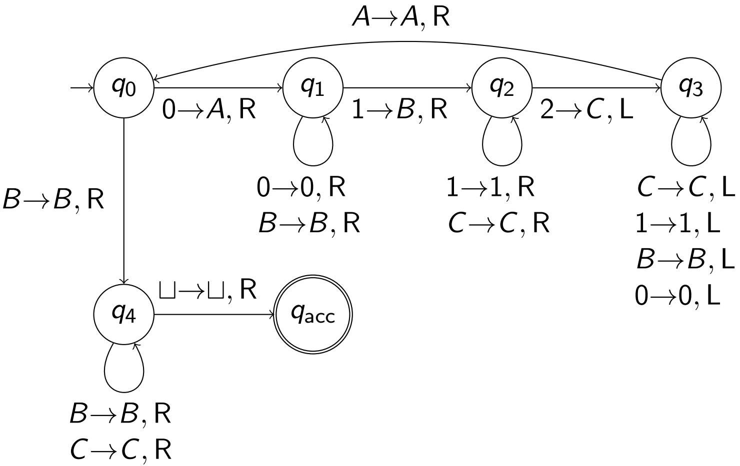

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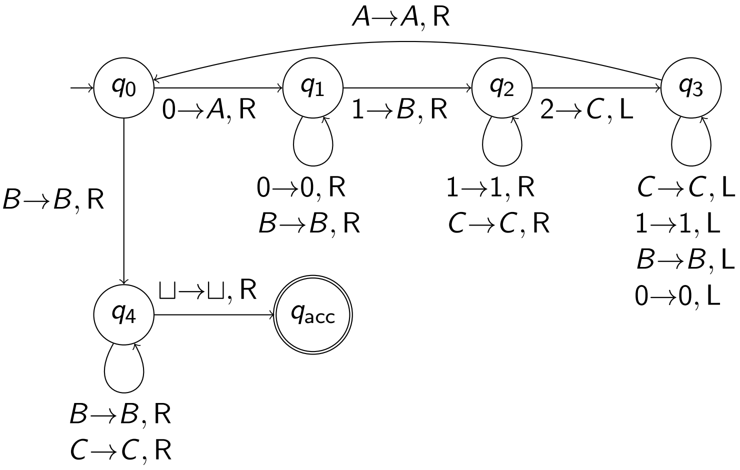

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2$

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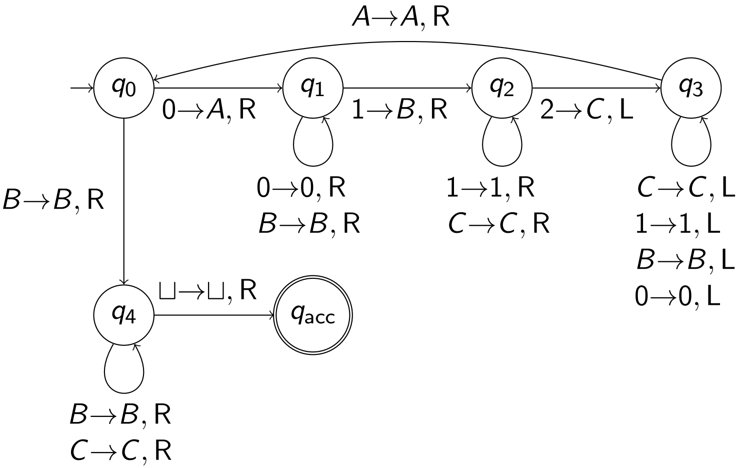

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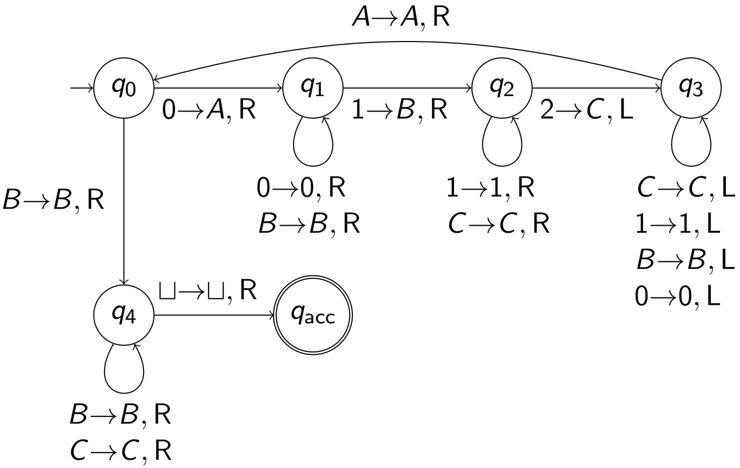

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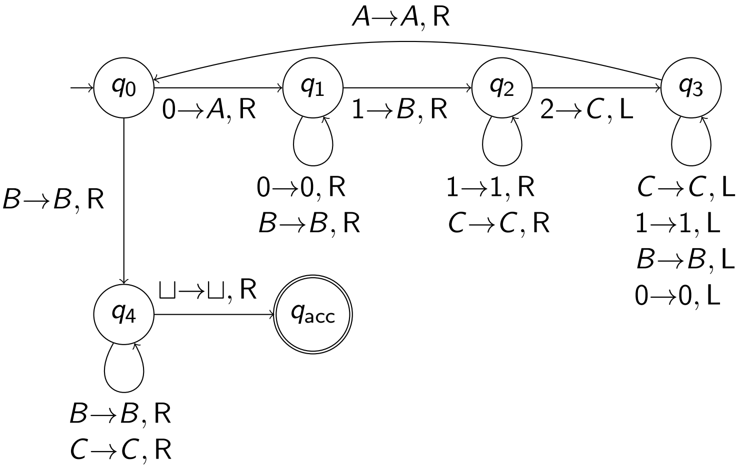

e.g.: $q_0001122 \vdash^* A0Bq_31C2 \vdash^* q_3A0B1C2 \vdash Aq_00B1C2$
$\vdash^* AAq_0BBCC$

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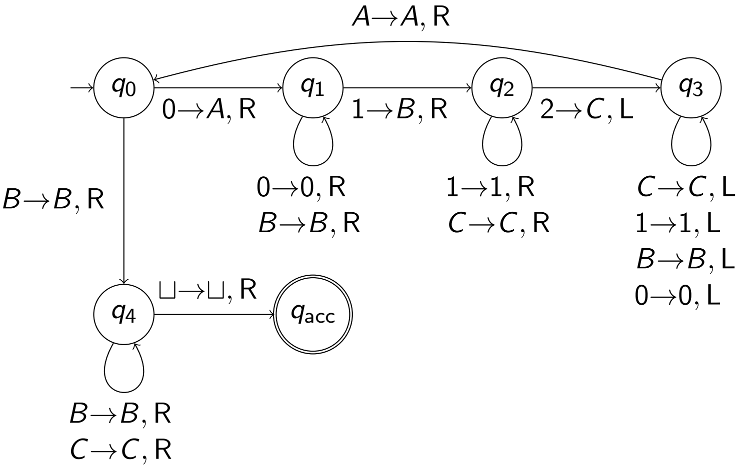

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$
$\vdash^* AAq_0 BBCC \vdash^* AABBCCq_4 \sqcup$

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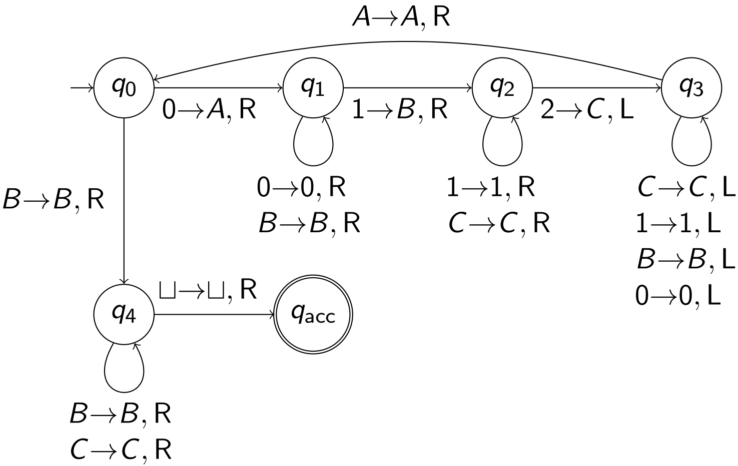

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$
$\vdash^* AAq_0 BBCC \vdash^* AABBCCq_4 \sqcup \vdash AABBCC\sqcup q_{\mathrm{acc}} \sqcup$

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Definition

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Deciding a language is more than recognizing it. There are languages which are recognizable, but not decidable.