Kansas State University | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 1

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Name: ____

Due date: Thursday, January 31st at 5:00pm. Use the drop box adjacent to CW120. Pay attention as there several drop boxes, one for each section of the course. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises

1. Find a 3×3 matrix P such that for a general 3×3 matrix A, i.e.,

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

P satisfies

$$AP = \begin{bmatrix} A_{13} & A_{12} & A_{11} \\ A_{23} & A_{22} & A_{21} \\ A_{33} & A_{32} & A_{31} \end{bmatrix}.$$

(That is, P permutes the first and third columns of any A.)

$$P = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Once you got P, compute, by hand, PA,

$$PA = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

What do you observe?

2. Draw the graph G whose adjacency matrix is given by

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A =

     1     0     0     0     0     0
     0     1     1     0     1     0
     1     0     1     0     0     1
     1     1     1     1     0     1
     0     0     1     0     0     1
     0     1     1     1     1     1
```

Is G a directed or undirected graph? Why?

3. Compute **by hand** the following product

$$RQ = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 3 & 2 & -8 & 3 \\ 10 & -21 & 13 & 21 \\ 3 & -3 & -9 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & -8 \\ 0 & 5 \\ 1 & 3 \end{bmatrix}$$

4. Find, by hand and without computing the whole product, the values of x and y

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ 3 & -3 & -9 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 4 \\ -2 & -8 & -1 \\ 0 & 5 & 2 \\ 1 & 3 & 3 \end{bmatrix} = \begin{bmatrix} * & * & x \\ * & y & * \end{bmatrix}$$

$x =$

$y =$

5. (Matrix form of a system) The following system of linear equations

$$(S_1) \left\{ \begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & 4x_3 & = & 1 \\ 3x_1 & + & 5x_2 & + & x_3 & = & 8 \\ x_1 & - & x_2 & - & 3x_3 & = & 7 \end{array} \right.$$

can be written in *matrix form* as

$$Ax = b,$$

where A is the *matrix of coefficients* of (S_1) given by

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 5 & 1 \\ 1 & -1 & -3 \end{bmatrix},$$

b is the *right-hand side vector of* (S_1) given by

$$b = \begin{bmatrix} 1 \\ 8 \\ 7 \end{bmatrix},$$

and x is the *vector of unknowns* given by

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(a) Use the definition of matrix product Ax to verify that the equality $Ax = b$ indeed yields the system (S_1) (Hint: Spell out the equality $Ax = b$).

(b) Write the following system in matrix form (specify the matrix of coefficients, the right-hand side vector, and the vector of unknowns).

$$(S_2)\left\{\begin{array}{rcrcrcrcr} x_1 & + & & + & 2x_3 & + & x_4 & = & 10 \\ x_1 & + & 7x_2 & - & x_3 & - & x_4 & = & -1 \\ x_1 & - & x_2 & - & 8x_3 & + & x_4 & = & 4 \end{array}\right.$$

6. (Vector form of a system) The following system of linear equations

$$(S_1)\left\{\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 4x_3 & = & 1 \\ 3x_1 & + & 5x_2 & + & x_3 & = & 8 \\ x_1 & - & x_2 & - & 3x_3 & = & 7 \end{array}\right.$$

can be written in *vector form* as

$$x_1u_1 + x_2u_2 + x_3u_3 = b, \tag{1}$$

where the vectors u_1, u_2, and u_3 are the columns of the matrix of coefficients of (S_1), that is,

$$u_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix},$$

and b is the right-hand side vector of (S_1).

(a) Verify that the expression in (1) indeed yields the system (S_1).

(b) Write the following system in vector form.

$$(S_3) \begin{cases} \begin{array}{rcrcrcrcr} x_1 & + & & + & 2x_3 & + & 5x_4 & = & 10 \\ x_1 & + & 7x_2 & - & x_3 & - & 10x_4 & = & -1 \\ 3x_1 & - & x_2 & - & 8x_3 & + & 6x_4 & = & 14 \\ 2x_1 & - & x_2 & + & 8x_3 & - & x_4 & = & 40 \\ 8x_1 & + & x_2 & & & & & = & -4 \end{array} \end{cases}$$

7. True or False - **Circle the right one** (One point each)

   T or **F**. There exists a one to one correspondence between graphs and binary square matrices. That is, given a graph G there exists exactly one square binary matrix associated to G and given a square binary matrix A there is exactly one graph whose adjacency matrix is A.

   T or **F**. If S is a 15×15 symmetric matrix, then the matrix M created with Matlab by doing

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>> M=S(1:10,1:10)
```

   is also symmetric.

   T or **F**. Given an $m \times n$ matrix A and an $n \times r$ matrix B, we always have $AB = BA$.

   T or **F**. If $v = [2,\ 4]$ and $w = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, then

$$v * w = 2$$

   and

$$w * v = \begin{bmatrix} 6 & 12 \\ -2 & -4 \end{bmatrix}.$$

   T or **F**. If U is a 2×2 matrix such that

$$U \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

   then it must be

$$U = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Points obtained in this assignment (out of 16): ______________