Math 322 / 16 probs 4 per exam

FINAL REVIEW

EXAM 1

(4 Problems)

1) Is the relation reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and/or transitive. Also, state the definitions of the property as you consider it.

a) $R = \{\}$ on the set of all integers.

b) $R = \{(a,b) \mid 1/b \le a\}$ where $a, b \in \mathbb{Z}$

Reflexive: $\forall a (aRa)$

irreflexive: $\forall a (a \not R a)$

2) Prove the Theorem:

The relation R on a set A is transitive iff $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$

trans $\leftarrow$ $\forall n\ R^n \subseteq R$

$\rightarrow$

3) Represent the relation on set $A = \{a, b, c, d, e, f\}$ with a matrix and digraph.

$R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a), (e,f)\}$

4) For the relations $R_1 = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a)\}$ and $R_2 = \{(a,a), (a,d), (b,b), (b,c), (c,c), (c,a), (d,d), (d,c)\}$ on the set $A = \{a, b, c, d, e\}$ use matrix operations find the zero-one matrices for $R_1 \cup R_2$ and $R_1 \circ R_2$. $\rightarrow$ $M_{R_2} \odot M_{R_1}$

$\vee$ $\wedge$

5) For $R = \{(a,a), (a,c), (a,d), (b,a), (b,d), (c,a), (c,d), (d,a), (d,c)\}$ on the set $A = \{a, b, c, d\}$ find the:

a) Reflexive Closure as a digraph.

b) Symmetric Closure as a digraph.

6) one problem: given R find ref., sym, trans closures

"however" $\rightarrow$

6) Find the Transitive Closure of $R = \{(1,1),(1,3),(2,1),(3,1),(3,3)\}$ on the set $A = \{1,2,3\}$ using the method of joining the boolean powers of M_R.

7) Find the Transitive Closure of $R = \{(1,1),(1,3),(2,1),(3,1),(3,3)\}$ on the set $A = \{1,2,3\}$ using Warshall's Algorithm.

8) Is the relation on the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$ where function f is related to function g if the graph of f is one unit above the graph g an equivalence relation? State all properties needed for an equivalence relation and why, or why not, the given relation has each property.

$$R = \{(f,g) \mid f(x) - g(x) = 1 \text{ for all } x \in \mathbb{Z}\}$$

Show:

$\forall a (aRa)$

$\forall a \forall b (aRb \to bRa)$

$\forall a \forall b \forall c (aRb \wedge bRc \to aRc)$

9) What is the equivalence class of an integer z for the given relation on the set of integers.

$$R = \{(x,y) \mid x + y = 3 \vee x - y = 3\}$$

10) Are the given relations partial orderings? State all properties needed for an partial order and why, or why not, the given relation has each property.

a) $(\mathbb{Z}, =)$

b) $(\mathbb{Z}^+, |)$

Show:

$\forall a (aRa)$

$\forall a \forall b (a \neq b \to \neg(aRb \wedge bRa))$

$\forall a \forall b \forall c (aRb \wedge bRc \to aRc)$

11) Draw the Hasse diagram for the poset $(\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, |)$.

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join?

12) State the maximal, minimal, greatest, and least elements for the Hasse diagram.

0 (+) bounds.

13) Prove: Let A be a set with n elements, and let R be a relation on A. If there is a path of length at least one in R from a to b, then there is such a path with length not exceeding n. Moreover, when $a \neq b$, if there is a path of length at least one in R from a to b, then there is such a path with length not exceeding $n - 1$.

Exam 2

(4 Problems)

1) Determine the type of graph and state why it isn't the more restrictive type(s).

a)

b)

c) 0

d)

2) In a round-robin tournament the Tigers beat the Blue Jays, the Tigers beat the Cardinals, the Tigers beat the Orioles, the Blue Jays beat the Cardinals, the Blue Jays beat the Orioles, and the Cardinals beat the Orioles. Model the outcome with a directed graph.

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or.. any application type in this section.

3) For Q_3 state the number of vertices, the number of edges, the degree of each vertex, show that the handshaking theorem applies, and is the graph bipartite?

4) Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph. Show that the sum of the in-degrees of the vertices and the sum of the out-degrees of the vertices are both equal to the number of edges.

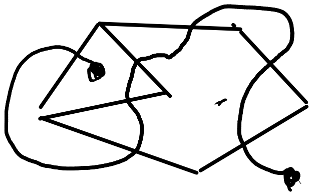
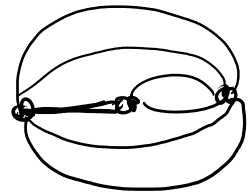
5) Represent W_4 as sets of vertices and edges, an adjacency matrix, and a simple graph.

6) Provide a rigorous argument why the graphs are not isomorphic.

7) Use paths either show that the graphs are not isomorphic or to find an isomorphism. If you think they are isomorphic DO NOT form the adjacency matrices. Just show the isomorphism.

8) Determine if the graph is strongly connected and/or weakly connected. For the underlying undirected graph what are the cut vertices?

9) For the given puzzle you need to draw a continuous connected curve that cuts each line segment exactly once. Draw the representative graph for the puzzle. Construct the path using the proof of the theorem for Euler Circuits. Explain your reasoning.

10) State Dirac's Theorem. Does Dirac's Theorem apply to $K_{3,4}$? Does $K_{3,4}$ have a Hamilton Circuit? Explain your reasoning.

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or Ore's thm:

11) Find the length of a shortest path between a and every vertex in the graph.

12) Solve the given traveling salesman problem.

13) Show that every connected graph with n vertices has at least $n-1$ edges.

Exam 3

(4 Problems)

1) Prove: There are at most m^h leaves in an $m-$ary tree of height h.

2) You receive a chain text and notice that it has six "FW"-forward notices at the front. Also, the text message tells you to send it to ten other people. Give an estimate of the number of people who do not send out copies (explain your reasoning). Using your estimate, how many people have sent and/or received the text? How many sent it out? If it costs 20 cents to send or receive a text how much money will be collected by the cell-phone companies?

$h \geq \lceil \log_m l \rceil$

3) In a best case situation how many weighings of a balance scale are needed to determine if one of four coins ~~may be a light~~ or heavy counterfeit? Construct a decision tree that uses this number of weighings. If your tree does not use that number of weighings explain why.

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$h \geq \lceil \log_4 16 \rceil = \lceil 2 \rceil = 2$

4) Use Huffman coding to encode the symbols with the given frequencies: a:0.30, b:0.25, c:0.15, d:0.10, and e:0.20.

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or tick-tack-toe

or ? ?

5) Draw a game tree with the values for every vertex of a Game of Nim variant. The starting position consists of one pile of 5 stones, player one can take 1 or 2 stones at a time (always), and player two can take 1 to 3 stones at a time (always).

6) Construct the universal address system for the tree of the infix expression $(2 + 3) - (4 \times (3 - 1))$. Then use that to order the vertices using the lexicographic order of universal address labels.

7) For the prefix expression $+, -, \times, 2, 3, 5, \div, \uparrow, 2, 3, 4$ construct its rooted tree, write the expression in postfix notation, and write it in infix notation with parentheses.

$C_0 = 1$

$C_1 = 1$

1, 1, 2, $\boxed{5}$, 14, 42

14, 5, 2, 1, 1

8) In how many ways can $p \wedge q \vee r \wedge s$ be fully parenthesized to yield an infix expression?

$((X_0) \circ (X_1) \circ (X_2 \circ \cdots \circ X_n))$

n operators

$C_n = C_0 C_{n-1} + C_1 C_{n-2} + C_2 C_{n-3} + \cdots + C_{n-1} C_0$

9) Use a table to express the values of the Boolean function $F(x,y,z) = \overline{xy} + yz + xz$

10) Use a table to verify De Morgan's law $\overline{(x \cdot y)} = \bar{x} + \bar{y}$.

11) Find the sum-of-products expansion of $F(x,y,z) = x$ with AND without using a table.

12) Find the product-of-sums expansion of $F(x,y,z) = \bar{x}$ with AND without using a table.

13) Show that in a Boolean algebra the idempotent and double complement laws hold.

Exam 4

(4 Problems)

1) For the given grammar with start symbol ... *etc.* Construct derivations for two valid sentences.

$S \Rightarrow$ $\Rightarrow$

p_1 p_2 p_3 p_4 p_5

2) For the productions $S \to AB,\ A \to aAb,\ B \to bBa,\ A \to \lambda,$ and $B \to \lambda$ construct a derivation for $aabbba$ using the productions.

$S \underset{1}{\Rightarrow} AB \underset{2}{\Rightarrow} aAbB \underset{2}{\Rightarrow} aaAbbB \underset{4}{\Rightarrow} aabbB$

$\underset{3}{\Rightarrow} aabbbBa \underset{5}{\Rightarrow} aabbba$

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3) Name the grammar type and explain your reasoning.

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a) $S \to AB$, $A \to Ab$, $B \to bB$, $A \to a$, and $B \to b$

b) $S \to AB$, $A \to aAb$, $B \to bBa$, $A \to \lambda$, and $B \to \lambda$

c) $S \to AB$, $B \to aAb$, $A \to a$, and $aAb \to b$

d) $S \to AB$, $\boxed{AB \to aAb}$, $B \to b$, and $A \to a$

type 1 not type 2

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4) Find the output and sequence of states generated from the input string 01110 for the finite-state machine with the given state table.

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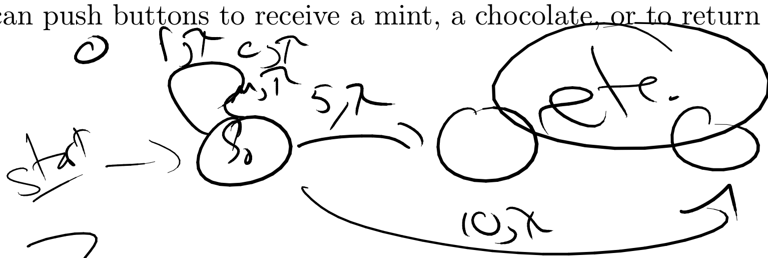
		f			g	
		Input			Output	
State	0		1	0		1
s_0	s_1		s_0	0		1
s_1	s_2		s_0	1		0
s_2	s_0		s_3	0		1
s_3	s_1		s_2	1		0

5) Construct a finite-state machine that models a candy machine that accepts nickels and dimes. A mint candy costs 15 cents and a chocolate candy costs 10 cents. The machine accepts change until 15 cents has been put in. It then gives change back for any amount greater than 15 cents. The customer can push buttons to receive a mint, a chocolate, or to return change at any time.



6) Determine the language recognized by the deterministic finite-state automaton.

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7) Determine the language recognized by the non-deterministic finite-state automaton.

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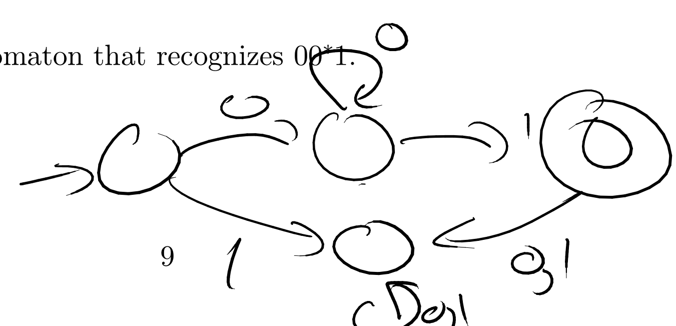
8) Find a deterministic finite-state automaton that recognizes the same language as the non-deterministic finite-state automaton using the techniques from the proof of Theorem 1 of Section 12.3.

9) Using the constructions ~~described~~ in the proof of Kleene's Theorem, find a non-deterministic finite-state automaton that recognizes 00*1.

10) Find a deterministic finite-state automaton that recognizes 00*1.



11) Let T be the Turing machine defined by five-tuples: $(s_0, 0, s_0, 0, R)$, $(s_0, 1, s_1, 1, R)$, (s_0, B, s_3, B, R), $(s_1, 0, s_0, 0, R)$, $(s_1, 1, s_2, 0, L)$, (s_1, B, s_3, B, R), and $(s_2, 1, s_3, 0, R)$. For the initial tape

$$\ldots B, B, 0, 1, 0, 1, 1, 1, 0, B, B, B, \ldots$$

determine the final tape when T halts. Show each of the iterations until the machine halts.

merge

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12) Construct a Turning machine that computes the function $f(n) = n \bmod 2$.

13) Construct a non-deterministic F.S.A. that recognizes the language generated by the regular grammar:

$$G = (V = \{0, 1, A, S\}, T = \{0, 1\}, S, P)$$

with productions $P = \{S \to \lambda, S \to 1, S \to 0A, S \to 1B, A \to 0A, A \to 1B, A \to 1, B \to 0A, B \to 1B, B \to 1\}$