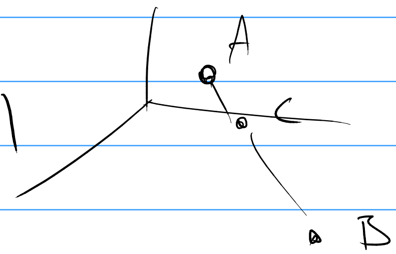
Math 243

Q's/ $A(2,4,2)$ $B(3,7,-2)$ $C(1,3,3)$

$|AC| + |BC| \geq |AB|$



Properties of vectors

① $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

② $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

③ $\vec{a} + \vec{0} = \vec{a}$

④ $\vec{a} + (-\vec{a}) = \vec{0}$

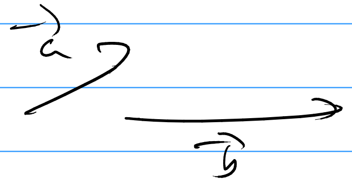
⑤ $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$

⑥ $(a+b)\vec{c} = a\vec{c} + b\vec{c}$

⑦ $(cd)\vec{a} = c(d\vec{a})$

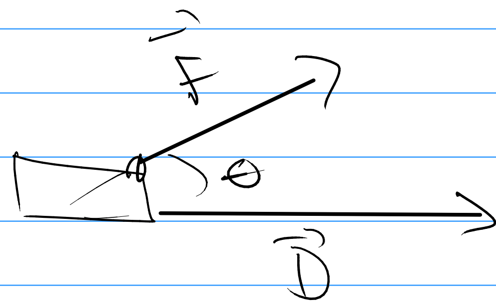
⑧ $1 \cdot \vec{a} = \vec{a}$

10.3 / 10.4 Like Multiplication?



$\vec{a}$ (times?) $\vec{b}$

ex



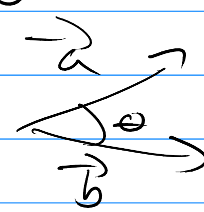
$$\text{Work} = \vec{F} \cdot \vec{D}$$

$$\vec{F} \cdot \vec{D} = |\vec{F}||\vec{D}| \cos\theta$$

Def.

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

dot product



by Components $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

Note: Dot Product

vector $\cdot$ vector = Scalar

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properties

① $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

② $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

③ $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

④ $(c\vec{a})\cdot\vec{b} = c(\vec{a}\cdot\vec{b}) = \vec{a}\cdot c\vec{b}$

⑤ $\vec{a}\cdot\vec{0} = 0$

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b/c $\quad \underset{\overset{\shortparallel}{a_1b_1+a_2b_2+a_3b_3}}{\vec{a}\cdot\vec{b}} = |\vec{a}||\vec{b}|\cos\theta$

$$\rightarrow \boxed{\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \cos\theta}$$

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

use this to find θ!

Note: $\vec{a}\cdot\vec{b} = 0$ then $\vec{a}, \vec{b}$ are orthogonal

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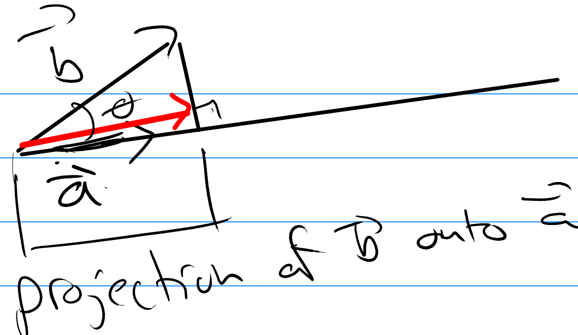
Applications:

① Work

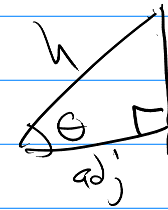
② Finding θ

③ Projections of $\vec{v}$ onto $\vec{u}$

Projections



$$|\text{proj}_{\vec{a}}\vec{b}| = \text{comp}_{\vec{a}}\vec{b}$$



$$= |\vec{b}|\cos\theta$$

$$\cos\theta = \frac{adj}{hyp}$$

$$= \frac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{a}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

Vector projection: (right length) $\cdot$ (unit vector in right direction)

$$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\left(\frac{\vec{a}}{|\vec{a}|}\right) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$$

Q's/ projection of $\vec{b}$ onto $\vec{a}$

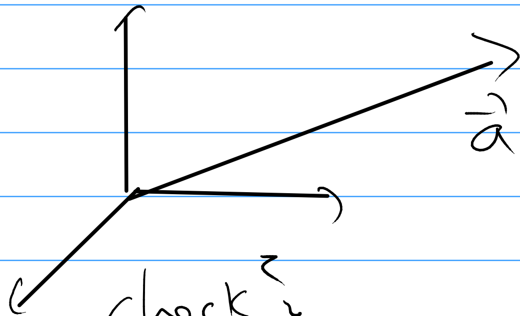
Scalar Comp: is $|\vec{b}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

10.3

27 given $\vec{a} = \langle 3, 0, -1\rangle$, scalar comp $= 2$ $\Big\}$ $\vec{b} = ?$

$$\rightarrow \frac{3b_1 + 0 \cdot b_2 + (-1)b_3}{(3^2+0^2+(-1)^2)^{1/2}} = 2$$

$$3b_1 - b_3 = 2\sqrt{10}$$



$$\vec{b} = \left\langle \frac{t_2 + 2\sqrt{10}}{3}, t_1, t_2 \right\rangle$$

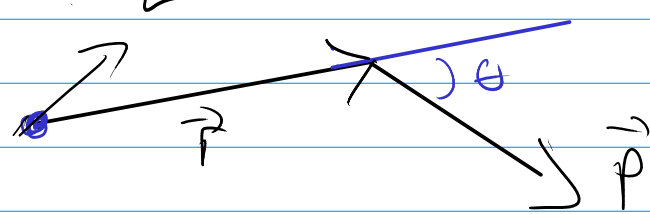
<u>check:</u>

$$\vec{a} = \langle 3, 0, -1 \rangle$$

$$\vec{b} = \left\langle \frac{t_2 + 2\sqrt{10}}{3}, t_1, t_2 \right\rangle$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{\cancel{t_2} + 2\sqrt{10} + 0 + (-\cancel{t_2})}{\sqrt{10}} = 2 \quad ✓$$

$\boxed{\text{Cross Product}}$



$$|\vec{\tau}| = |\vec{r}||\vec{p}| \sin\theta$$

$$\vec{r} \times \vec{p} = \langle r_2 p_3 - r_3 p_2, r_3 p_1 - r_1 p_3, r_1 p_2 - r_2 p_1 \rangle$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_1 & r_2 & r_3 \\ p_1 & p_2 & p_3 \end{vmatrix} = \vec{i}\begin{vmatrix} r_2 & r_3 \\ p_2 & p_3 \end{vmatrix} - \vec{j}\begin{vmatrix} r_1 & r_3 \\ p_1 & p_3 \end{vmatrix} + \vec{k}\begin{vmatrix} r_1 & r_2 \\ p_1 & p_2 \end{vmatrix}$$

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th^m: $\vec{a} \times \vec{b}$ is $\perp$ to $\vec{a}$ and $\vec{b}$

proof: $\vec{a} = \vec{r}$ $\quad \vec{b} = \vec{p}$

$$(\vec{r} \times \vec{p}) \cdot \vec{p} = \langle r_2 p_3 - r_3 p_2,\ r_3 p_1 - r_1 p_3,\ r_1 p_2 - r_2 p_1 \rangle \cdot \langle p_1, p_2, p_2 \rangle$$

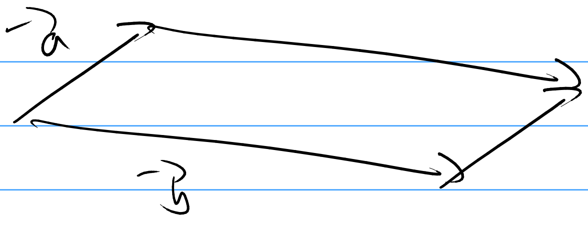
$$= \ldots = 0$$

$$(\vec{r} \times \vec{p}) \cdot \vec{r} = 0$$

$\boxed{th^m}$ $\quad |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$\boxed{th^m}$ $\quad \vec{a} \times \vec{b} = \vec{0}$ if and only if parallel

$\boxed{th^m}$



$|\vec{a} \times \vec{b}| = \text{area}$

Properties

(1) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(2) $c \cdot (\vec{a} \times \vec{b}) = (c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$

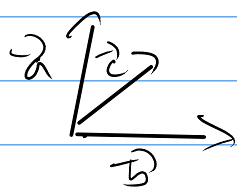
(3) $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$

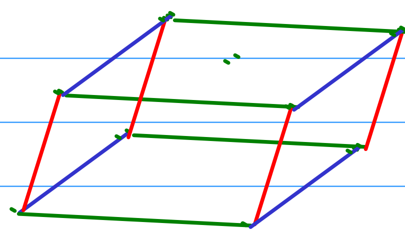
(4) $(\vec{a} + \vec{b}) \times \vec{c} = (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{c})$

☆ (5) $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$

(6) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Triple Product

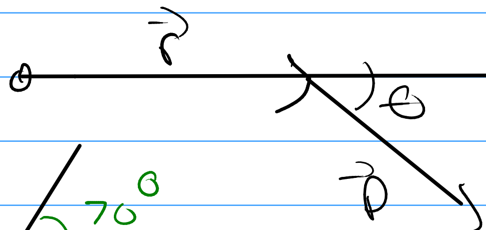




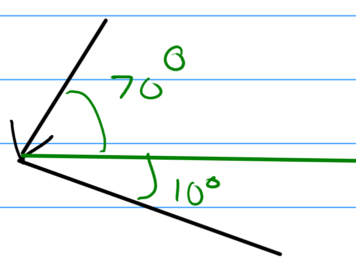
$$\left|\vec{a} \cdot (\vec{b} \times \vec{c})\right| = \text{Volume}$$

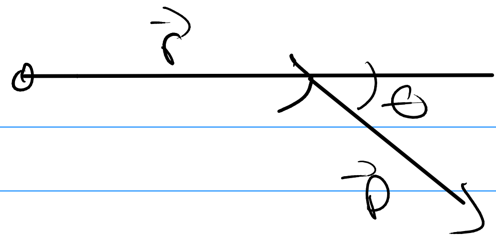
Apps

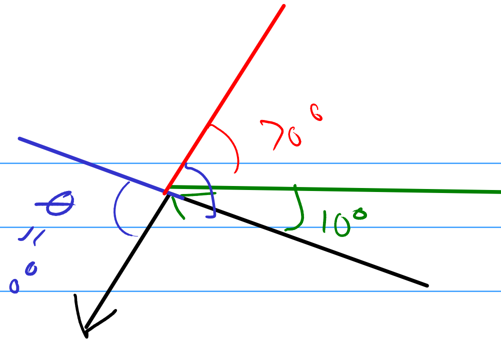
$$|\vec{\tau}| = |\vec{r}||\vec{p}| \sin\theta$$



10.4 (35)







$$|\vec{\tau}| = (60)\cdot(.18)\sin 80^\circ$$

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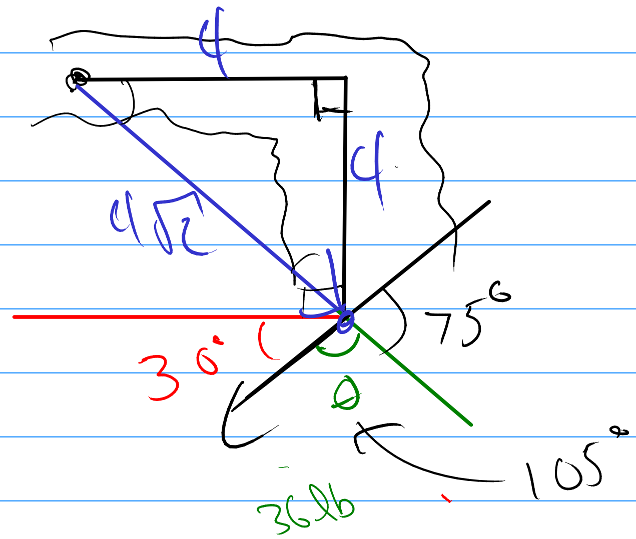

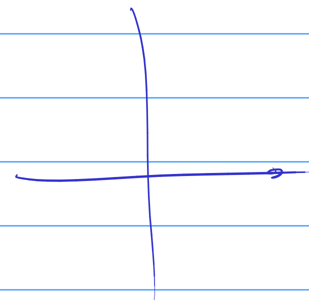

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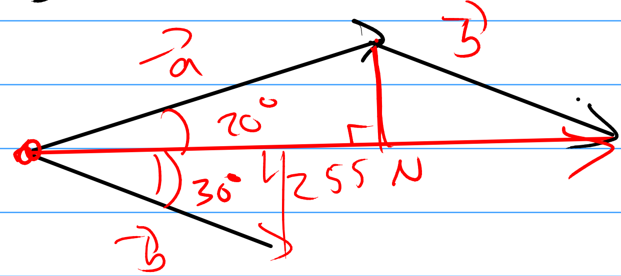
p. 589 #13

$\vec{R} = \langle 255, 0 \rangle$



$\vec{a} = \langle |\vec{a}|\cos 20^\circ, |\vec{a}|\sin 20^\circ \rangle$

$+$

$\vec{b} = \langle |\vec{b}|\cos 30^\circ, -|\vec{b}|\sin 30^\circ \rangle$

$\vec{R} = \langle 255, 0 \rangle$

Solve system of equations

$a \cos 20° + b \cos 30° = 255$

$a \sin 20° - b \sin 30° = 0$ ✓

$a = b \dfrac{\sin 30}{\sin 20}$

$b \sin 30 \cdot \cot 20 + b \cos 30 = 255$

let $b = \dfrac{255}{\sin 30 \cot 20 + \cos 30}$

let $a = \left(\dfrac{255}{\sin 30 \cot 20 + \cos 30} \right) \left(\dfrac{\sin 30}{\sin 20} \right)$

$$\boxed{\begin{aligned} \vec{a} &= a \langle \cos 20, \sin 20 \rangle \\ \vec{b} &= b \langle \cos 30, -\sin 30 \rangle \end{aligned}}$$