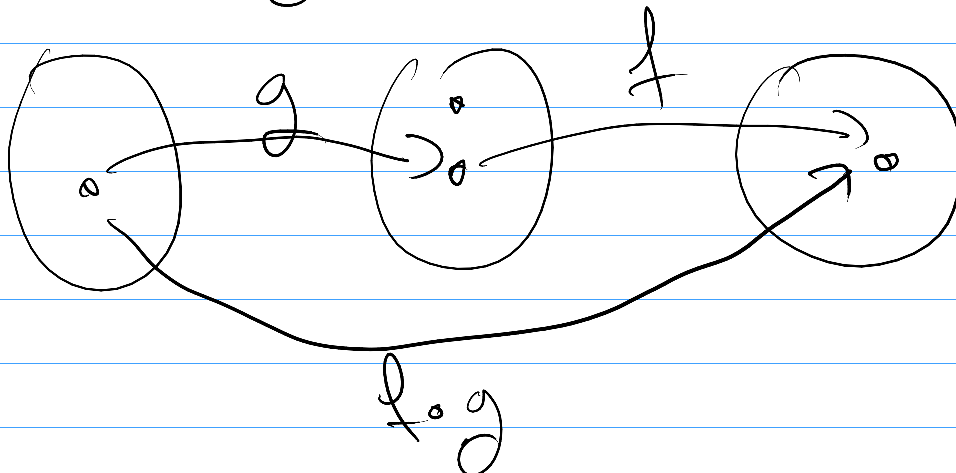
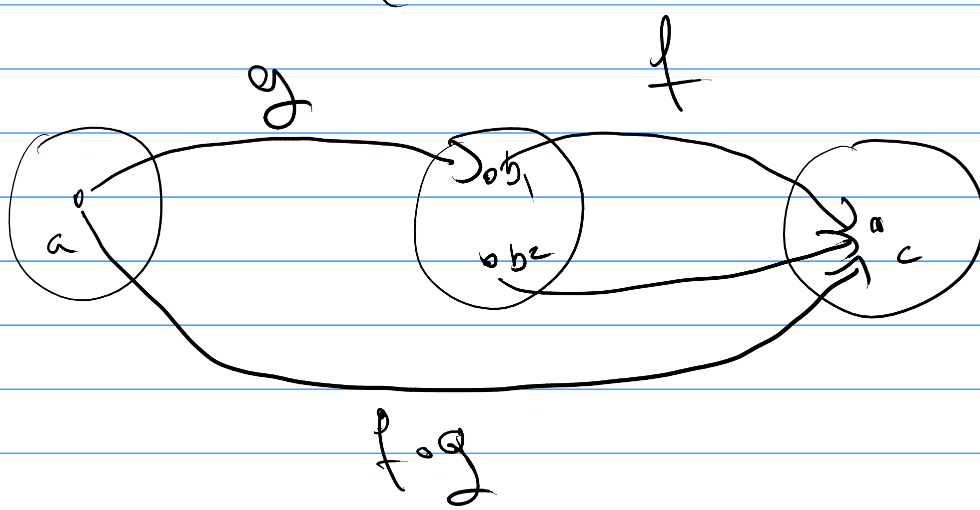
Math 321

Qs ~~f~~ 2.3(31) f and $f \circ g$ are onto

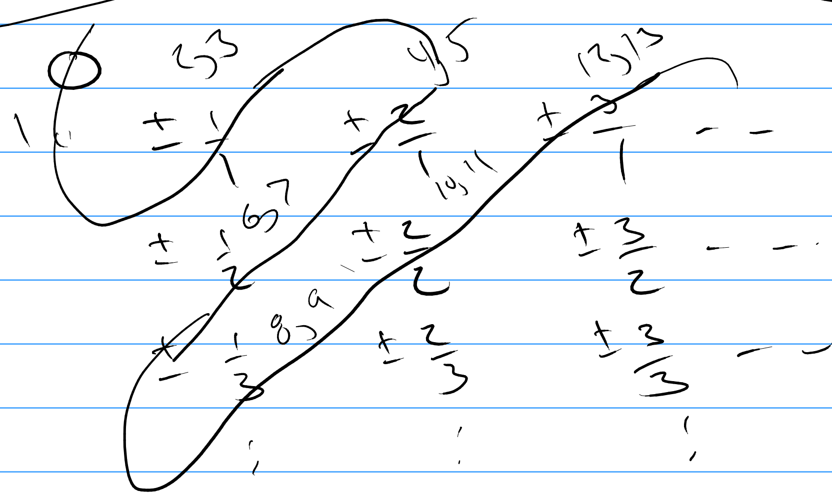
What about g? (onto)



ex



rationals



So $\mathbb{Q}$ is countable.

$\mathbb{R} = \mathbb{Q} \cup$ the set of Irrational numbers

Note: Decimal number

$$321.123$$

$$= 3 \cdot 10^2 + 2 \cdot 10^1 + 1 \cdot 10^0 + 1 \cdot 10^{-1} + 2 \cdot 10^{-2} + 3 \cdot 10^{-3}$$

So $\mathbb{Q}$ is either a decimal that terminates or repeats.

0.12

$3.4\overline{13}$

$\left.\begin{array}{l} 1 \\ 0.\overline{9} \end{array}\right\}$ **Note:** these are equal.

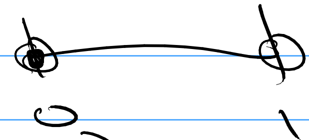
So Irrational doesn't terminate <u>and</u> doesn't repeat

ex $0.101001000100001\ldots$

Conjecture: $\mathbb{R}$ is not countable

pf: (contradiction)

assume $\mathbb{R}$ is countable

consider the interval 

all decimals between $[0,1)$ are

0. something...

and b/c $\mathbb{R}$ is assumed countable then $[0,1)$ is countable.

any real is going to be..

$$r_i = 0.d_{i1}\, d_{i2}\, d_{i3}\, d_{i4} \ldots$$

$$d_{ij} = \{0,1,2,3,\ldots,9\}$$

ex: $r_1 = 0.10209973\ldots$

Countable says a bijection from $\{1,2,3,\ldots\}$ to r_i exists..

let f be

$$\begin{aligned}
1 &\to r_1 = 0.d_{11}\, d_{12}\, d_{13} \ldots \\
2 &\to r_2 = 0.d_{21}\, d_{22}\, d_{23} \ldots \\
3 &\to r_3 = 0.d_{31}\, d_{32}\, d_{33} \ldots \\
4 &\to r_4 = 0.d_{41}\, d_{42}\, d_{43} \ldots \\
&\vdots \qquad \vdots
\end{aligned}$$

if we toss out $\overline{9}$ decimals
each real (r_i) is unique.

consider

$$r_* = 0.d_{*1}d_{*2}d_{*3}d_{*4}\ldots$$

$$d_{*1} = \begin{cases} 3 & d_{11} = 2 \\ 2 & d_{11} \neq 2 \end{cases} \Rightarrow r_1 \neq r_*$$

$$d_{*2} = \begin{cases} 3 & d_{22} = 2 \\ 2 & d_{22} \neq 2 \end{cases} \rightarrow r_2 \neq r_*$$

$$\forall i \quad d_{*i} = \begin{cases} 3 & d_{ii} = 2 \\ 2 & d_{ii} \neq 2 \end{cases} \rightarrow \forall i \; r_i \neq r_*$$

So r_n is one-to-one and onto

all reals are counted

and r_* was not counted. r_n is not onto

$\Rightarrow$ $\not$ (contradiction)

$\boxed{\mathbb{R} \text{ is uncountable}}$

3.8 Matrices: rectangular array of numbers or bits.

$$M = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

i = row
j = col.

Size: $m \times n$

Operations:

① $A = B$ both $m \times n$

$$\forall i \forall j (a_{ij} = b_{ij})$$

② $A + B = [a_{ij} + b_{ij}]$

③ $\underset{m \times k}{A} \, \underset{k \times n}{B} = \underset{m \times n}{[c_{ij}]}$

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{ik} b_{kj}$$

$$[c_{ij}] = \begin{array}{c} \\ i^{th} \text{ row} \\ \\ \end{array} \begin{bmatrix} & & \\ a_{i1} & a_{i2} & \cdots & a_{ik} \\ & & \end{bmatrix} \begin{array}{c} j^{th} \text{ col.} \\ \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{kj} \end{bmatrix} \end{array}$$

Zero-one matrix $0 \equiv$ False

$1 \equiv$ True

$$A \wedge B = [a_{ij} \wedge b_{ij}] \quad \text{(meet)}$$

$$A \vee B = [a_{ij} \vee b_{ij}] \quad \text{(join)}$$

$$A \odot B = [c_{ij}]$$

$$c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \cdots \vee (a_{ik} \wedge b_{kj})$$

Powers:

$$\underset{n \times n}{I_n} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

numbers

$$\underset{n \times n}{A^0} = I_n$$

$$A^r = \underbrace{A \cdot A \cdot A \cdots A}_{r\text{-times}}$$

zero-one matrix

$$A^{[0]} = I_n$$

$$A^{[r]} = \underbrace{A \odot A \odot \cdots \odot A}_{r\text{-times}}$$