Math 321

Qs/ f: All bit Strings $\to \{0, 1, 3, \ldots\}$

(3e) $f(s) = i$ $\quad i$ 1st time the bit is 1

Def: $f(\ ) = 0$

(Ex) $\cancel{f(001111)} = 3$

$\boxed{f(000) = 0} \leftarrow$ is a function

(or) $f(010) = 2$

not a function: $f(000)$ has no 1st bit with a 1 so it doesn't get mapped.

Sums

$$\underbrace{1 + 1 + 1 + 1 + \cdots + 1}_{n \text{ of those}} = n$$

$$\sum_{k=1}^{n} 1 = n \quad \text{or} \quad \sum_{k=1}^{n} 3 = 3n$$

Note: $k^2 - (k-1)^2 = k^2 - (k^2 - 2k + 1)$

$= 2k - 1$

$$\sum_{k=1}^{n} \underline{k^2} - \underline{(k-1)^2} = \sum_{k=1}^{n} (2k-1)$$

$$\to \sum_{k=1}^{n} k^2-(k-1)^2 = (\cancel{1^2}-0^2)+(\cancel{2^2}-\cancel{1^2}) + (\cancel{3^2}-\cancel{2^2})+(\cancel{4^2}-\cancel{3^2}) + \cdots + (n^2-\cancel{(n-1)^2})$$

$$= n^2 - 0^2 = n^2$$

So
$$n^2 = \sum_{k=1}^{n} (2k-1)$$

$$n^2 = 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 \quad = n$$

$$n^2 = 2\boxed{\sum_{k=1}^{n} k} - n$$

$$\frac{n^2+n}{2} = \sum_{k=1}^{n} k$$

$$\boxed{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}}$$

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to find $\sum_{k=1}^{n} k^2$ $\qquad (k-1)^3 = k^3-3k^2+3k-1$

use $k^3-(k-1)^3 = 3k^2-3k+1$

$$\boxed{\sum_{k=1}^{n} k^3-(k-1)^3} = 3\boxed{\sum_{k=1}^{n} k^2} - 3\boxed{\sum_{k=1}^{n} k} + \boxed{\sum_{k=1}^{n} 1}$$

$$\underbrace{}_{\text{know} \to n^3} \qquad\qquad\qquad \underbrace{}_{\frac{n(n+1)}{2}} \qquad \underbrace{}_{n}$$

$$n^3 = 3\boxed{\sum_{k=1}^{n} k^2} - \frac{3(n)(n+1)}{2} + n$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}\,\frac{n(n+1)}{2}\,(2n+1)$$

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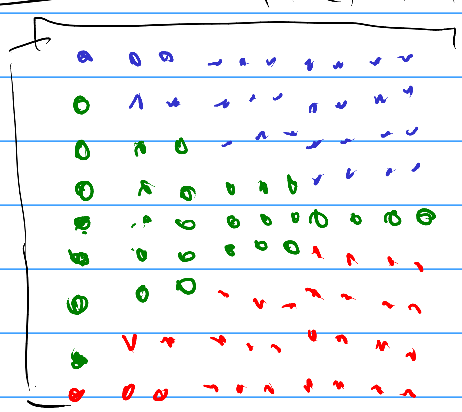
same idea to find. use $\boxed{k^4-(k-1)^4}$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Graphical. $\sum k^2$

$1+2+\cdots+n = \frac{n(n+1)}{2}$

$2n+1$



$$3\cdot S = \left(\frac{n(n+1)}{2}\right)(2n+1)$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Ex $a = 0.999\ldots$

$10a = 9.999\ldots$

$10a - a = 9a = 9.\cancel{99}\ldots - 0.\cancel{999}\ldots$

$= 9$

$9a = 9$

$\boxed{a = 1}$

Show: $0.123 = 0.122999\ldots$

$$\sum_{k=0}^{n} ar^k = a + ar + ar^2 + ar^3 + \ldots + ar^n$$

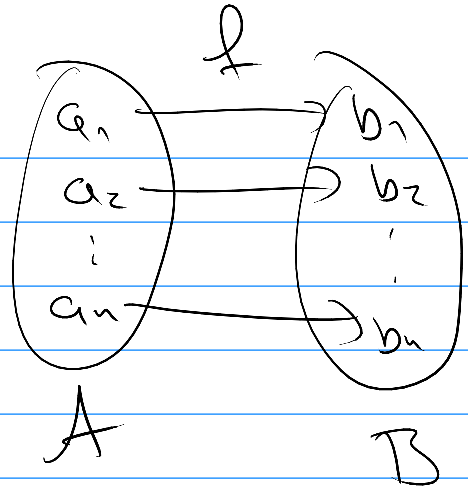
$$= \begin{cases} a(n+1) & r = 1 \\ a\left(\frac{r^{n+1} - 1}{r - 1}\right) & r \neq 1 \end{cases}$$

Cardinality if $|S| = n$ where n is non-neg

the S is called finite

and n is S's cardinality

Bijection:



$f(a_1) = b_1$

Def: if there is a bijection from A to B then they have the same cardinality.

ex

$$\begin{array}{ccccccc} 1, & 2, & 3, & 4, & 5, & \ldots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ 2, & 4, & 6, & 8, & 10, & \ldots \end{array}$$

Def: $|\mathbb{Z}^+| = \aleph_0$ "aleph null"

is the cardinality of the counting numbers.

Def: given $|S|$

if (1) $|S| \in \mathbb{Z}^+$ S is countable

(2) $|S| = \aleph_0$ S is countable.

ex

$1, 2, 3, 4, 5, \ldots$

$$\begin{array}{ccccc} P_{1,1} & P_{1,2} & P_{1,3} & P_{1,4} & \cdots \\ P_{2,1} & P_{2,2} & \cdots & & \end{array}$$

$\mathbb{Q}$

$$\begin{array}{ccccc} \pm\frac{1}{1} & \pm\frac{2}{1} & \frac{3}{1} & \frac{4}{1} & \cdots \\ \pm\frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} & \cdots \\ \frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} & \cdots \\ \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$