Math 321

Q31 $\boxed{5.3\ (15)}$ $\underbrace{H, H, \dots, H}$, 10 H's or T's

$|all| = |0\ H's| + |1\ H| + \dots + |10\ H's|$

$\quad\quad\quad\ \ \| \quad\quad\quad\quad \| \quad\quad\quad\quad\quad\quad \|$

$\quad\quad\quad |10\ T's| + |9\ T's| + \dots + |0\ T's|$

a) 2^{10}

b) $|\text{exactly } 2\ H's| = \dfrac{10!}{2!\,8!} = 45$

arrange 10 things

but 2 are H's

8 are T's

c) at most 3 tails

$|\text{exactly } 10\ H's| + |\text{exactly } 9\ H's| + \dots + |\text{exactly } 7\ H's|$

$\binom{10}{10} + \binom{10}{9} + \binom{10}{8} + \binom{10}{7} = ?$

d) $\binom{10}{5} = ?$

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21 (c) $L = \{A, B, C, D, E, F, G\}$

all ways $= 7!$

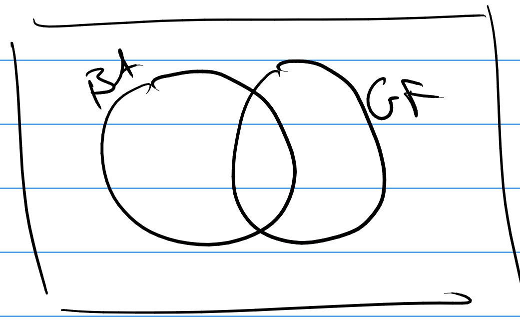
how many have BA and GF

$C = \{ BA, GF, C, D, E \}$

$\boxed{5!}$

ex BA or GF

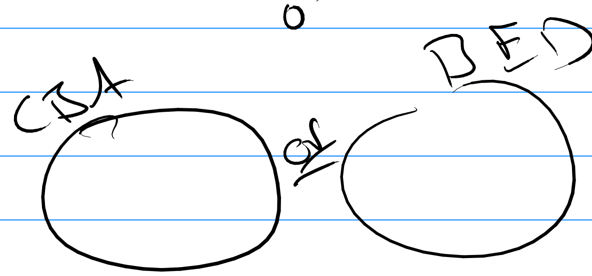
$\boxed{6! + 6! - 5!}$



21e $L = \{ A, B, C, D, E, F, G \}$

$C = \{ ABCDE, F, G \}$ $3!$

21f $\underbrace{CBA \wedge BED}_{?} \rightarrow \boxed{0}$



$\rightarrow \boxed{5! + 5!}$

7.1

Sequences

Type 1) Closed / functionally defined

ex $a_n = 2^n \quad n = 0, 1, 2, \ldots$

$2^0, 2^1, 2^2, 2^3, \ldots$

$1, 2, 4, 8, \ldots$

Type 2) open / inductively defined
recursively defined

ex Basis: $a_0 = 1$

Inductive or Recursive Step : $\boxed{a_n = 2 \cdot a_{n-1}}$ $n = 1, 2, \ldots$

recurrence relation

$a_0 = 1$
$a_1 = 2 \cdot a_0 = 2$
$a_2 = 2 \cdot a_1 = 4$
$a_3 = 2 \cdot a_2 = 8$
$\vdots$

Def: A recurrence relation is an equation that expresses a_n in terms of previous values of the sequence.

ex

$a_n = a_{n-1} + a_{n-2}$

$a_n = a_{n-4} + n \cdot a_{n-6} + \pi$

$f(n) = f(n-1) + f(n-2)$

<u>Def:</u> A sequence is a solution of a recurrence relation if its terms satisfy the equation.

ex

$\boxed{a_n = 2 \cdot a_{n-1}}$ $\boxed{a_n = 2^n}$

recurrence relation function

$a_\square = 2^\square$

is it a solution?

$2^n = 2^1 \cdot 2^{n-1}$

$2^n = 2^n$ $\boxed{\text{true}}$

so $a_n = 2^n$ <u>is</u> a solution.

Solving Recurrence Relations.

given the $\boxed{\text{recurrence relation}}$

an equation...

how do you find a function that make the equation true?

ex

$$\sum_{i=0}^{n} a \cdot r^i = a + ar + ar^2 + \cdots + ar^n$$

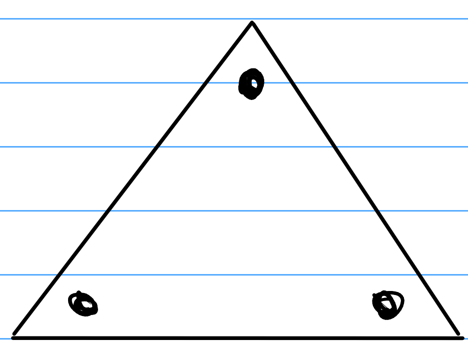
$$S = a + ar + ar^2 + \cdots + ar^n$$

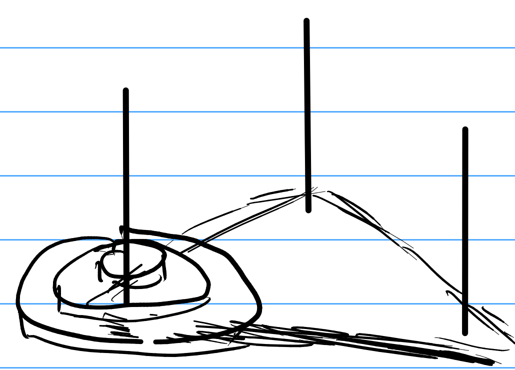
$$rS = ar + ar^2 + ar^3 + \cdots + ar^{n+1}$$

$$rS - S = ar^{n+1} - a$$

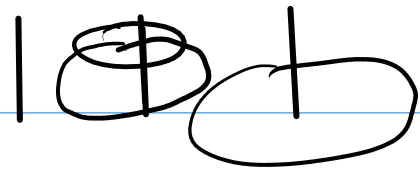
$$S(r-1) = a(r^{n+1} - 1)$$

★ $$\sum_{i=0}^{n} ar^i = a\left(\frac{r^{n+1}-1}{r-1}\right) \quad \boxed{r \neq 1}$$





Tower of Hanoi



$H_1 = 1$

$H_2 = 2(H_1) + 1$

$H_2 = 2(1) + 1 = 3$

$H_3 = 2(H_2) + 1$

$= 2(3) + 1 = 7$

$$H_n = 2(H_{n-1}) + 1$$

recurrence relation!

Seq: 1, 3, 7, 15, 31, 63, ...

Technique 1

recursive nesting.

$$H_n = 2(H_{n-1}) + 1$$

$$H_n = 2(2H_{n-2} + 1) + 1$$

$$= 2^2 H_{n-2} + 2^1 + 2^0$$

$$= 2^2(2H_{n-3} + 1) + 2^1 + 2^0$$

$$= 2^3 H_{\boxed{n-3}} + 2^2 + 2^1 + 2^0$$

$$\vdots$$

$$H_n = 2^{n-1} H_{\underbrace{n-(n-1)}} + 2^{n-2} + \cdots + 2^2 + 2^1 + 2^0$$

$H_1 = 1$

$H_n = 2^{n-1} + 2^{n-2} + \cdots + 2^2 + 2^1 + 2^0$

$H_n = \sum_{i=0}^{n-1} 1 \cdot 2^i = 1 \cdot \left(\frac{2^n - 1}{2 - 1} \right) = \boxed{2^n - 1}$

$\boxed{H_n = 2^n - 1}$ check!