Model Checking Task Sets with Preemption Thresholds

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Periodic Task Model

- Each task τ_i is characterized by a 5-tuple of natural numbers denoted:

$$(C_i, T_i, D_i, \pi_i, \gamma_i)$$

where

  - C_i is the run-time of task τ_i,
  - T_i is the period of task τ_i,
  - D_i is its relative deadline,
  - π_i is its static priority, and
  - γ_i is its preemption threshold.

Example Periodic Task Set

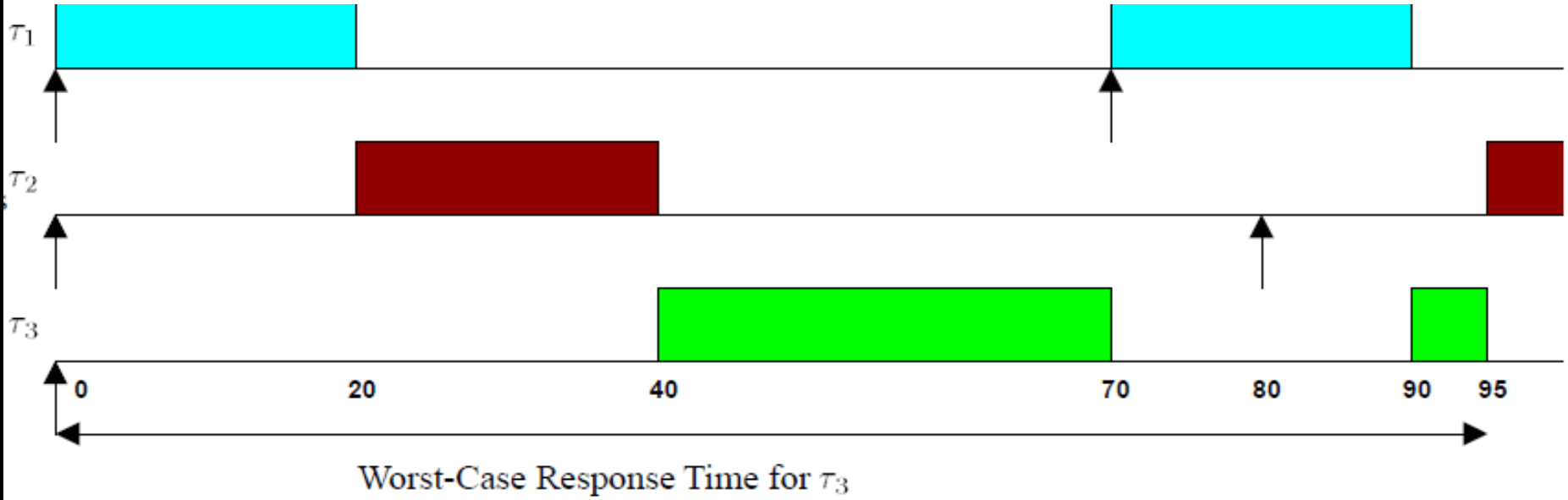
- From Wang and Saksena's original paper on preemption thresholds

Table 1. Periodic task set

i	C_i	T_i	D_i	π_i	γ_i
0	20	70	50	3	3
1	20	80	80	2	3
2	35	200	100	1	2

Task	π_i	WCRT Preemptive $\gamma_i = \pi_i$	WCRT Non-Preemptive $\gamma_i = 3$	γ_i	WCRT Preemption-Threshold
τ_1	3	20	**55**	3	40
τ_2	2	40	75	3	75
τ_3	1	**115**	75	2	95

Table 2. Response Times for Tasks under Different Schedulers.



Symbolic Schedulability Analysis

- Construct a formal model consisting of two automata:
  - PERIODIC_TASKS - to generate jobs
  - SCHEDULER - to model scheduling algorithm
- Perform symbolic analysis on SCHEDULER automaton to see if an ERROR state is reachable
- UPPAAL model checker is used to perform analysis

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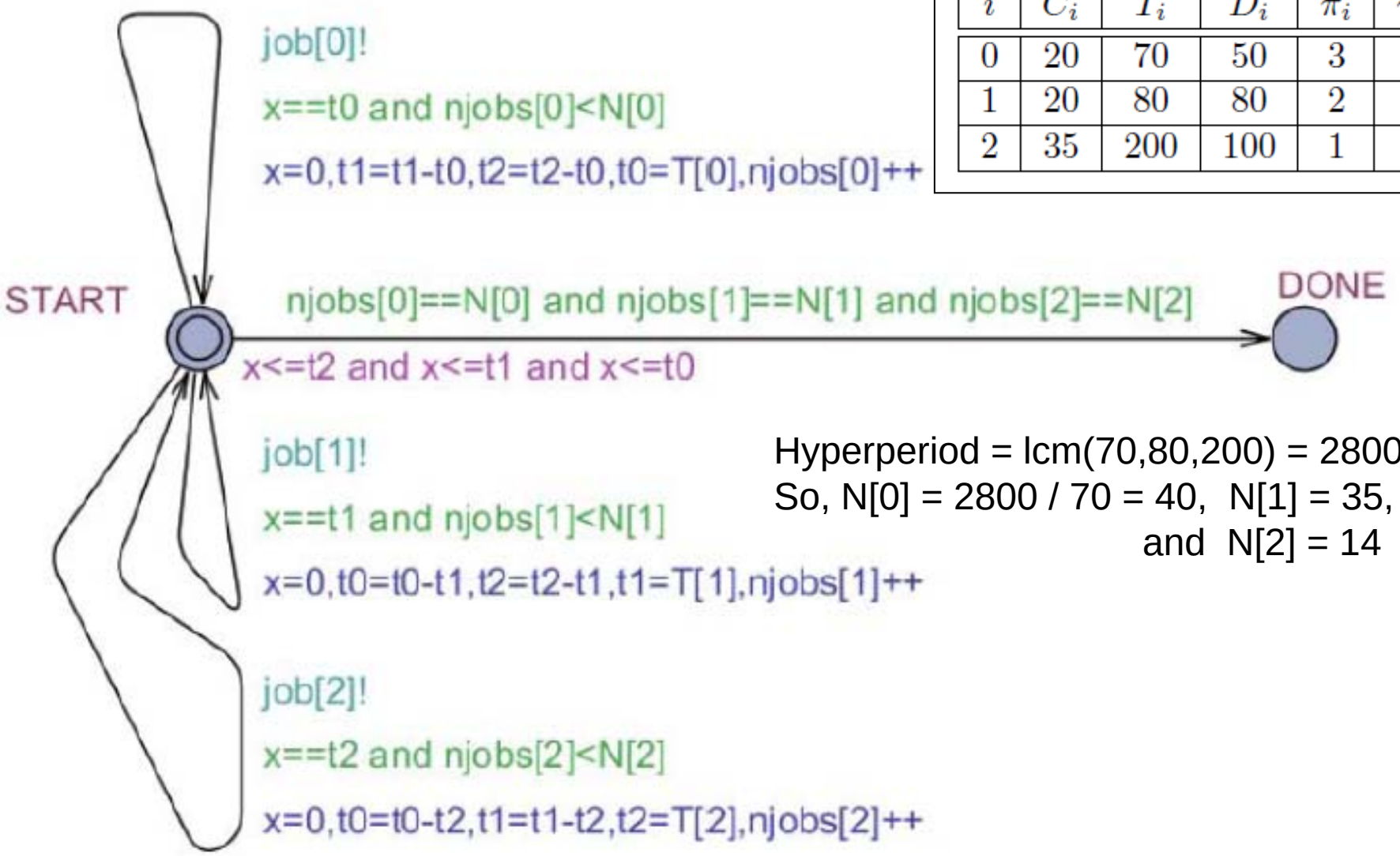
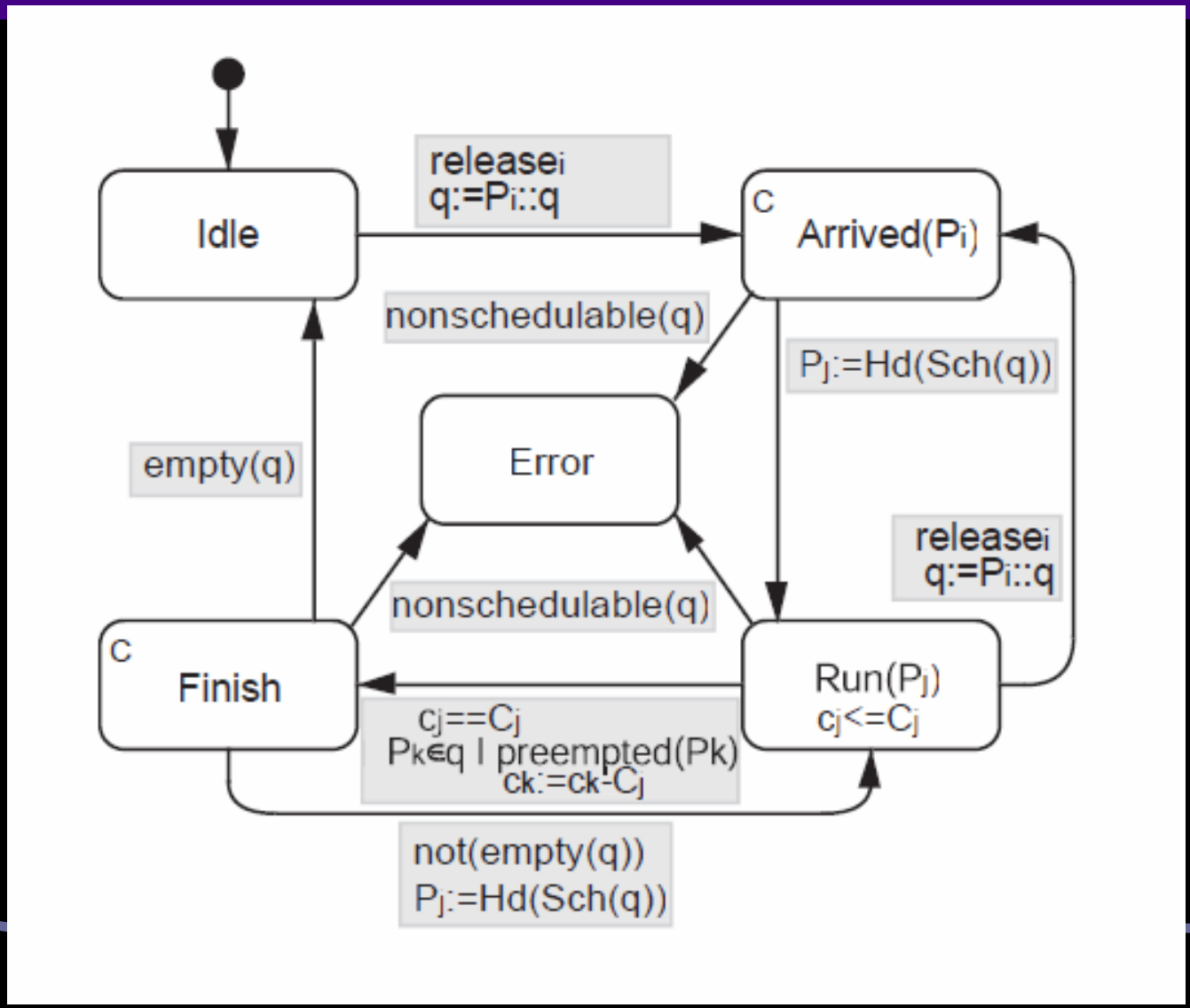


Hyperperiod = lcm(70,80,200) = 2800
So, N[0] = 2800 / 70 = 40, N[1] = 35,
and N[2] = 14

Figure 1. PERIODIC_TASKS automaton

General O(n)-Clocks Scheduler

- E. Fersman, et. al



New 2-Clocks Scheduler

- Correctly detects when an arbitrary task set is feasible.
- Only requires two clocks for ***n*** tasks:
  - One clock to check if the currently scheduled task has missed its deadline, **d**, and
  - One clock to check that the running task has completed it’s execution, **c**.
- For simplicity, we consider Wang and Saksena's example with just ***n=3*** tasks, but it is trivial to generalize to any ***n***.

SCHEDULER Automaton

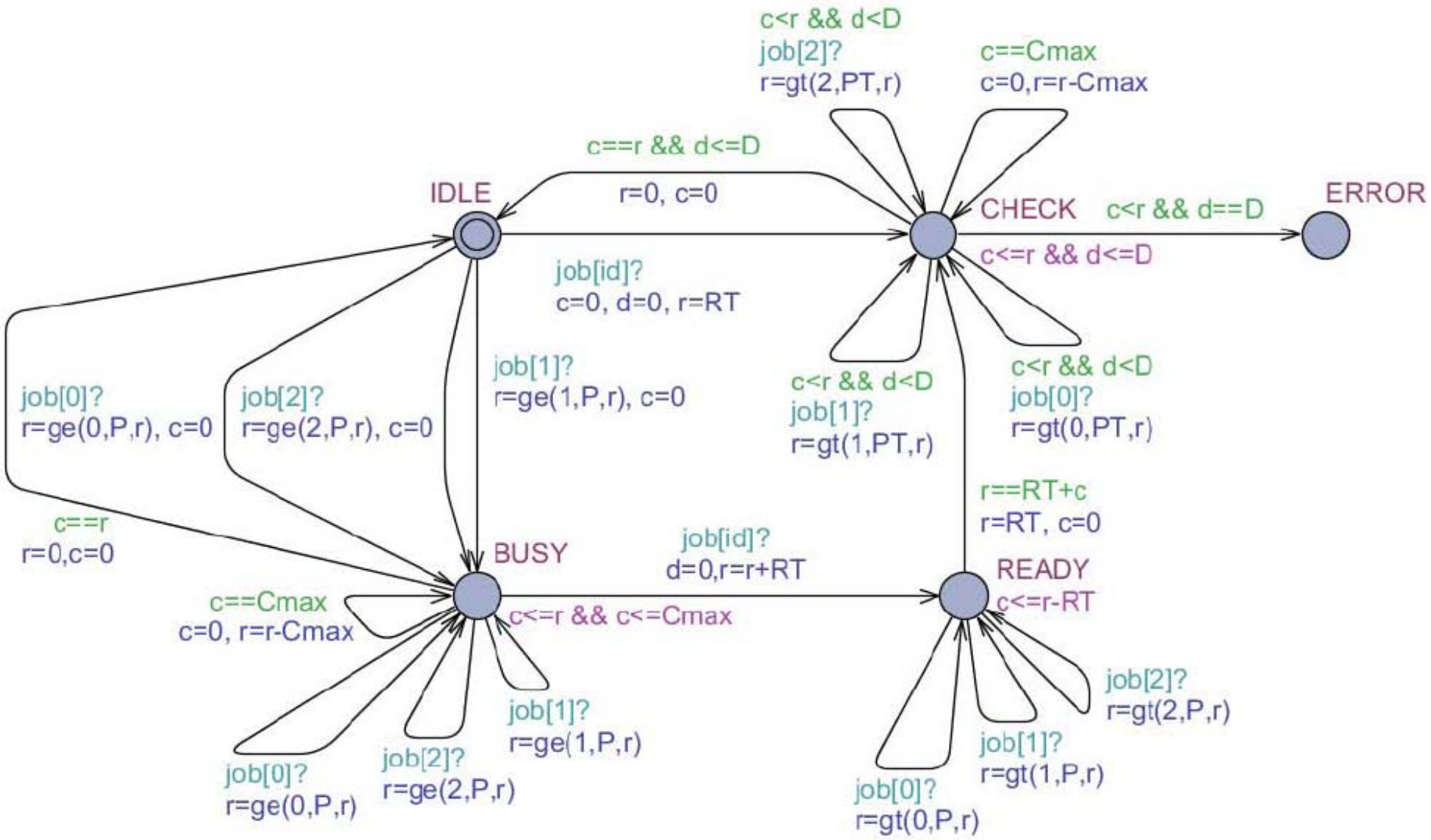


Figure 2. SCHEDULER automaton

System Declarations

```
S0 = SCHEDULER(PR[0],PRT[0],C[0],D[0],0);
S1 = SCHEDULER(PR[1],PRT[1],C[1],D[1],1);
S2 = SCHEDULER(PR[2],PRT[2],C[2],D[2],2);

system PERIODIC_TASKS,S0,S1,S2;
```

… and SCHEDULER prototype:

```
SCHEDULER( const int PR,  const int PRT,  const int RT,
                    const int D,  const int id );
```

SCHEDULER Automaton

```
clock c, d;
int r;
int Cmax = 1;

int gt(int i, int x, int r)
{
  if (PR[i]>x)
    return (r+C[i]);
  else
    return (r);
}
```

```
int ge(int i, int x, int r)
{
  if (PRT[i]>=x)
    return (r+C[i]);
  else
    return (r);
}
```

SCHEDULER States

- $Idle_i$ - the ready job queue is empty,
- $Busy_i$ - jobs with priority greater than or equal to task i have arrived for execution,
- $Ready_i$ - the job in task i to be checked for feasibility has been released for execution, but has not started executing, and
- $Check_i$ - the job in task i to be checked for feasibility has started executing, and
- $Error_i$ - the checked job in task i missed its deadline.

Preemptive Task Set

```
const int PR[3] = { 3, 2, 1 };
const int PRT[3] = { 3, 2, 1 };
```

	Task	π_i	WCRT Preemptive $\gamma_i = \pi_i$	WCRT Non-Preemptive $\gamma_i = 3$	γ_i	WCRT Preemption-Threshold
S0	τ_1	3	20	**55**	3	40
S1	τ_2	2	40	75	3	75
S2	τ_3	1	**115**	75	2	95

Table 2. Response Times for Tasks under Different Schedulers.

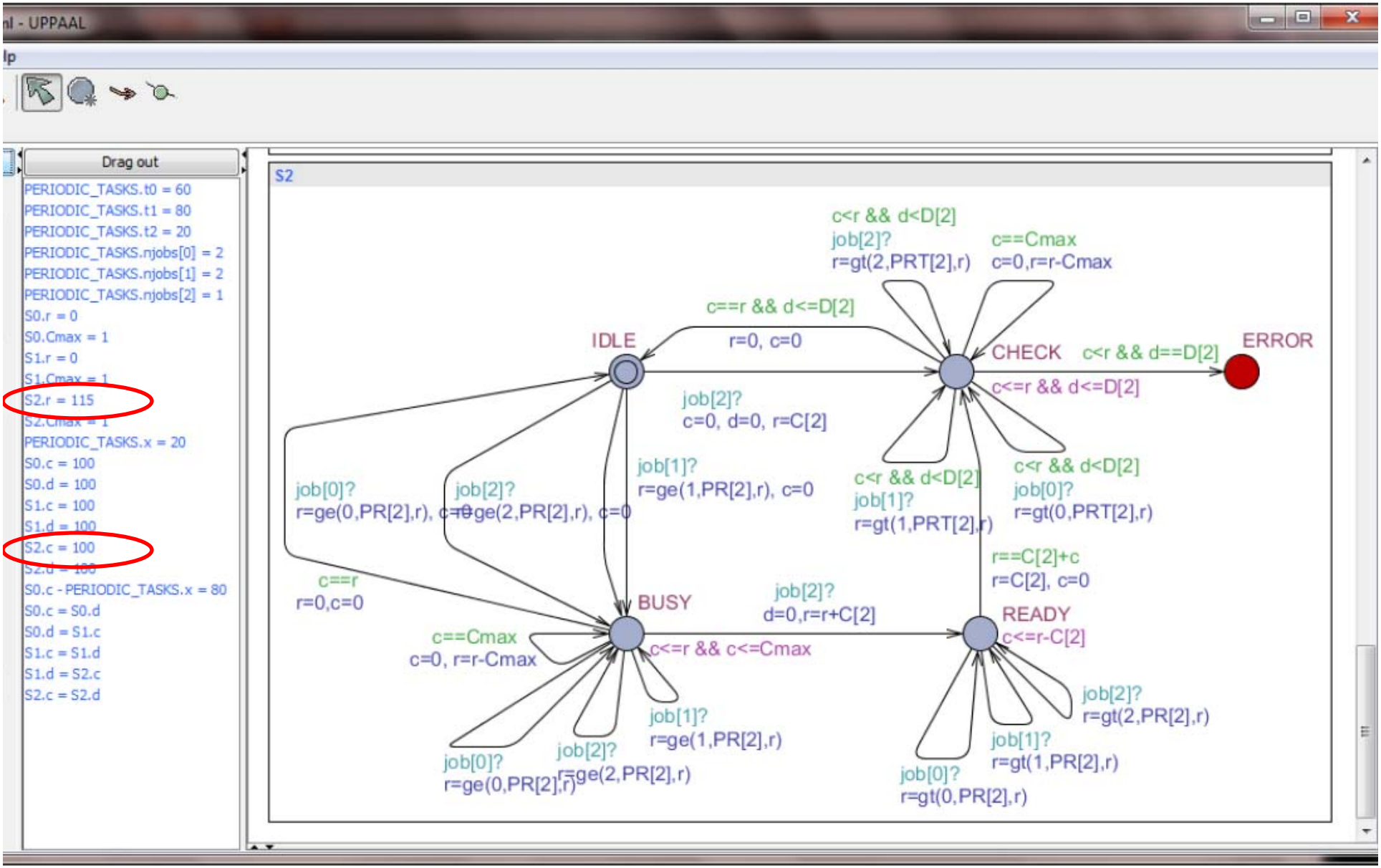


Figure 3. Missed deadline

Task Set is not Feasible

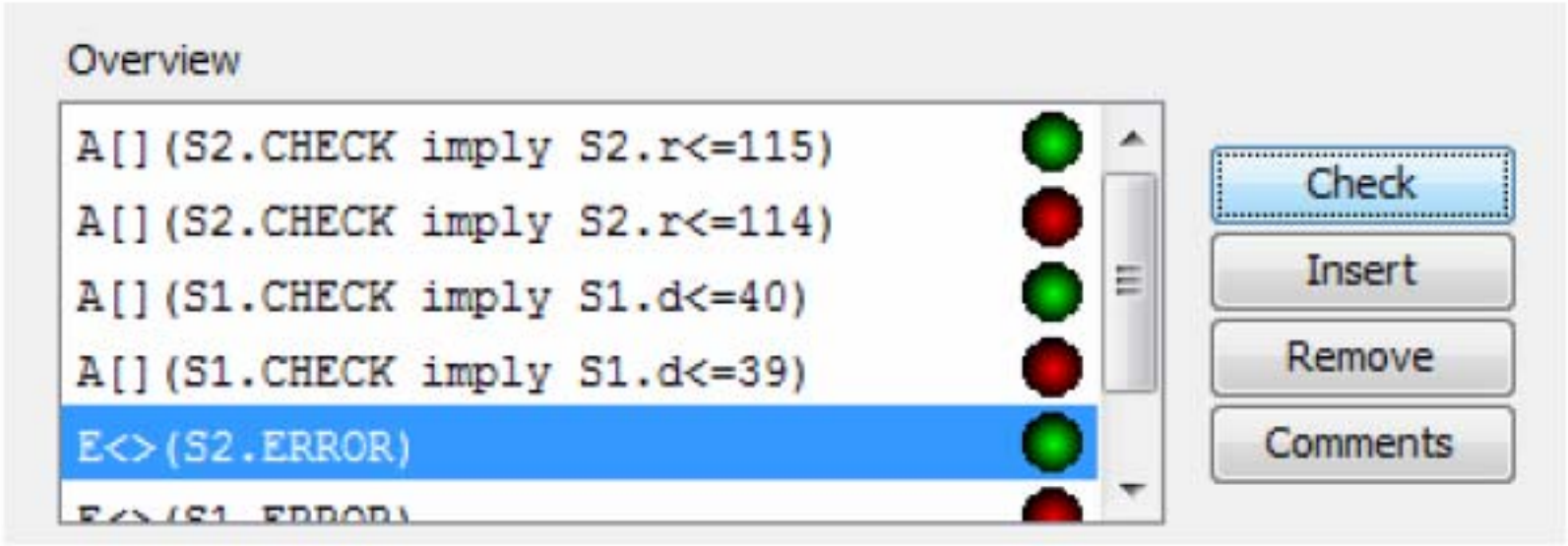


Figure 4. Preemptive tasks output

S0 = scheduler automaton for task τ_1

S1 = scheduler automaton for task τ_2

S2 = scheduler automaton for task τ_3

Non-Preemptive Task Set

const int PR[3] = { 3, 2, 1 };

const int PRT[3] = { 3, 3, 3 };

Task	π_i	WCRT Preemptive $\gamma_i = \pi_i$	WCRT Non-Preemptive $\gamma_i = 3$	γ_i	WCRT Preemption-Threshold
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In this case, E<> S0.Error is satisfied

Preemption-Threshold Task Set

$$\mathrm{PRT}[3] = \{\ 3,\ 3,\ 2\ \}$$

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Verifier Output

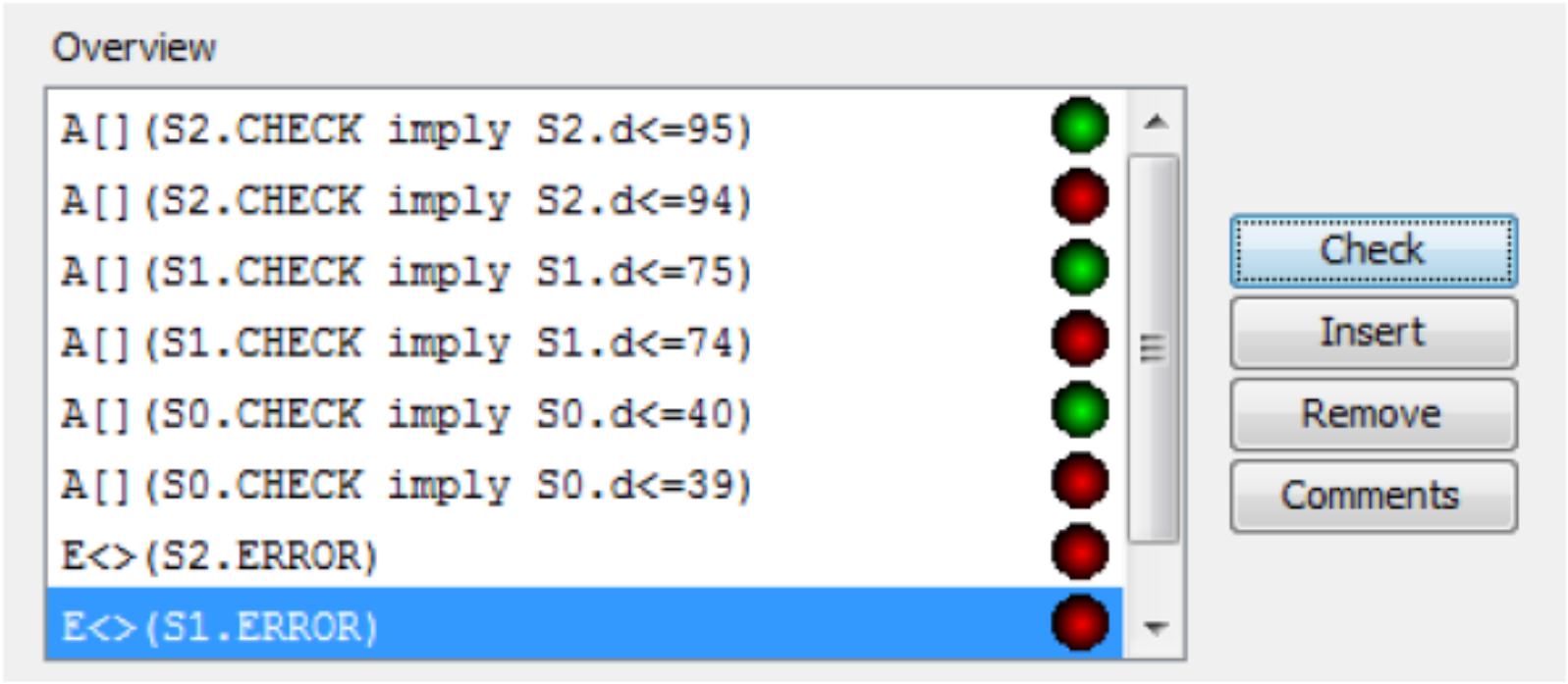


Figure 5. Verification output

SCHEDULER Automaton

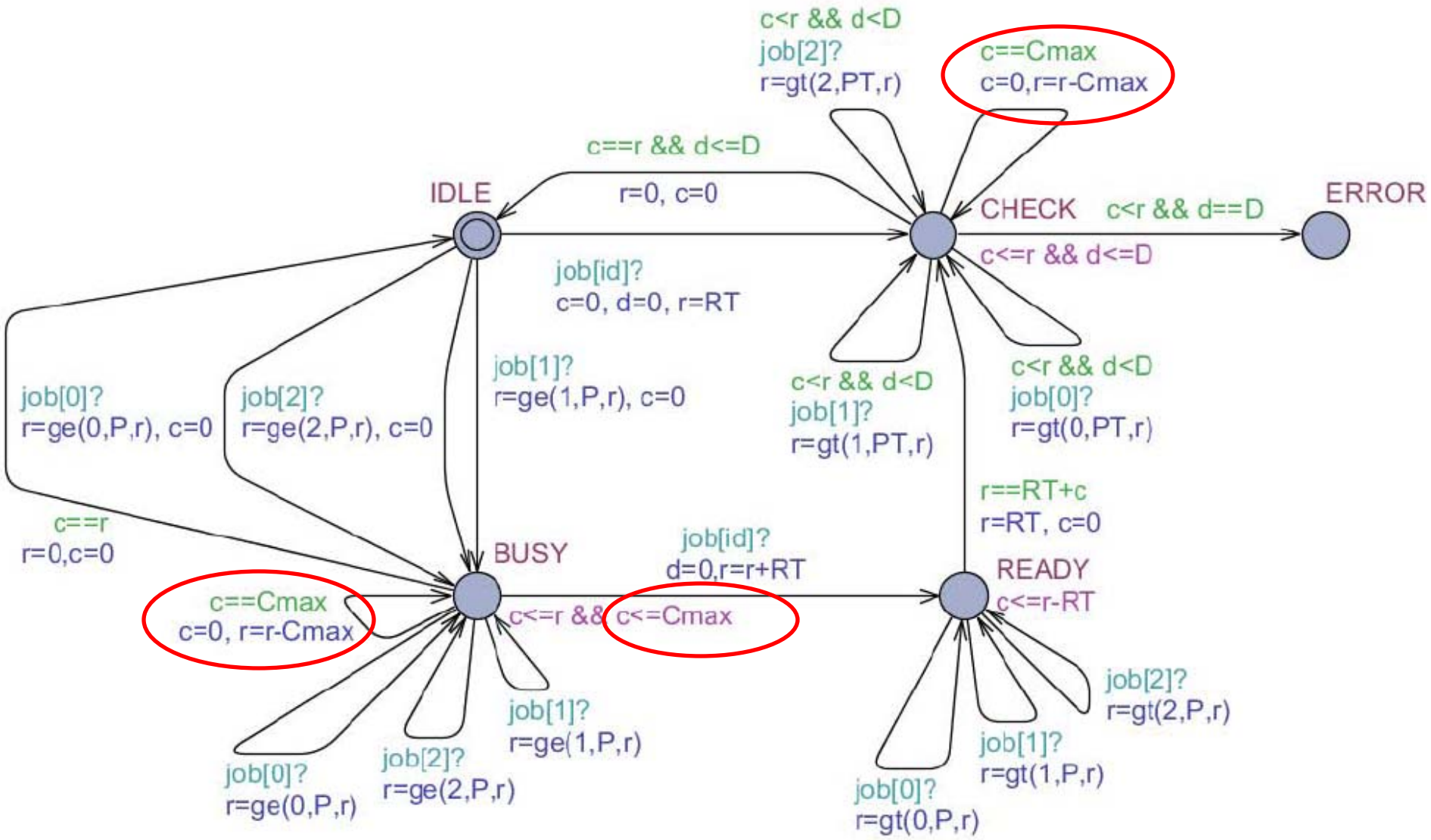


Figure 2. SCHEDULER automaton

Effect of Changing Cmax

Table 2. Bounds on number of states.

Cmax	States	Cmax	States
1	150,180	50	1,793
3	20,820	100	1,635
6	8,155	250	1,617
12	3,805	500	1,617

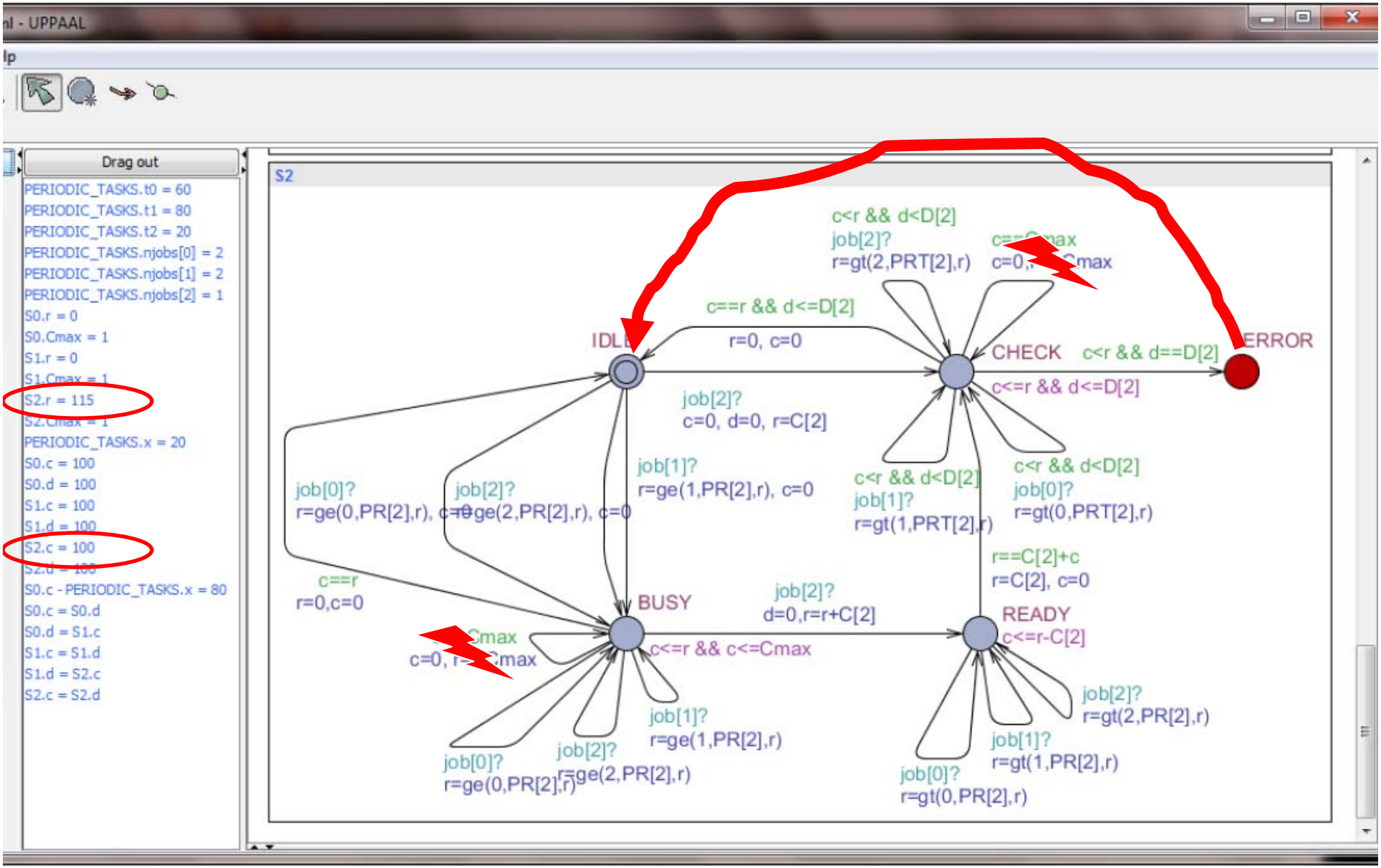
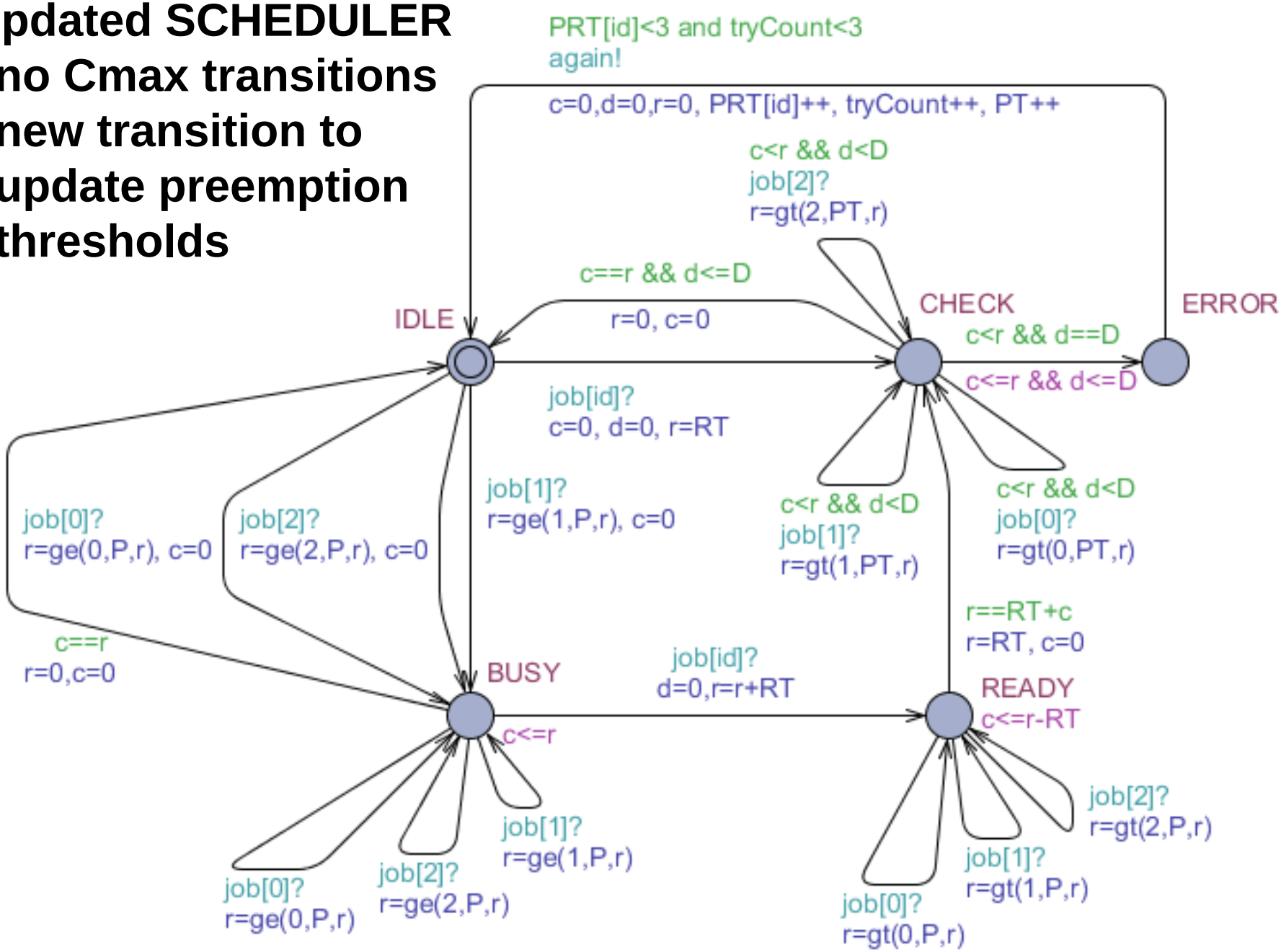


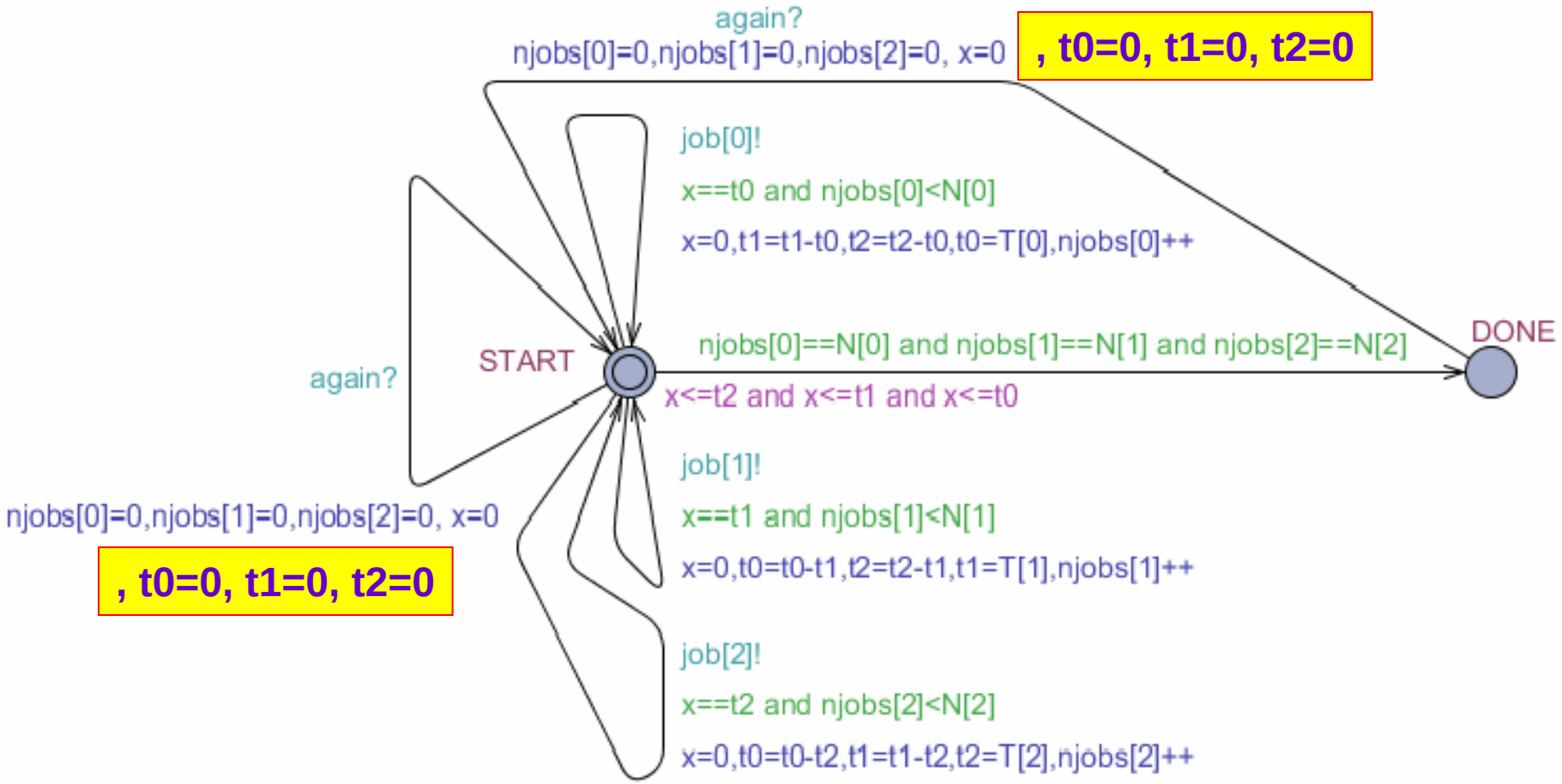
Figure 3. Missed deadline

Updated SCHEDULER

- no Cmax transitions
- new transition to update preemption thresholds

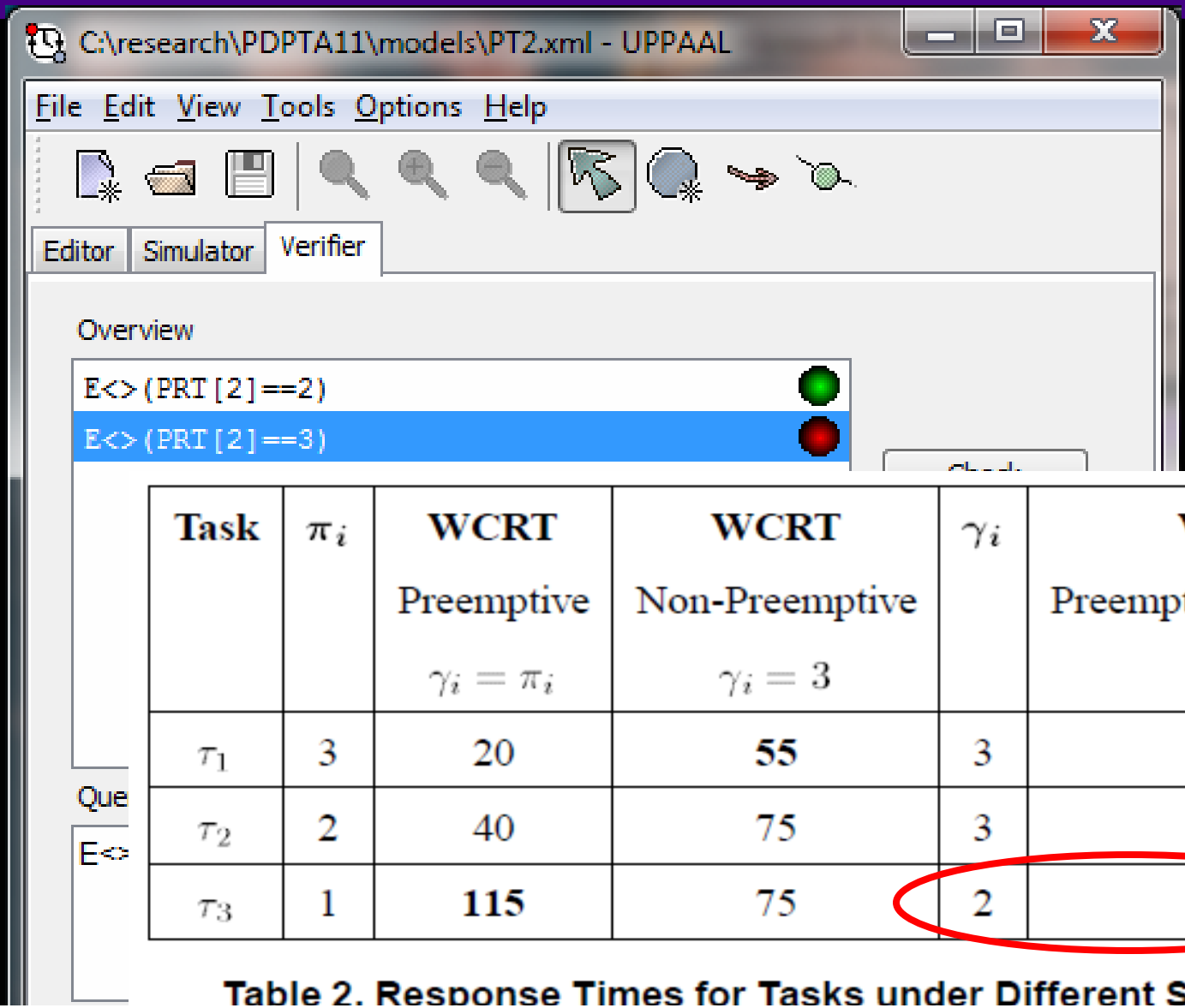


Updated PERIODIC_TASKS



*** oops, typo in paper**

Verification



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Summary

- The 2-Clocks Scheduler presented in this paper can be used to correctly test the feasibility of periodic task sets with preemption thresholds.
- The Updated Scheduler can be used to determine if a feasible set of preemption thresholds exist.
- All models and source code are available on-line at: www.cis.ksu.edu/~neilsen/pdpta11/