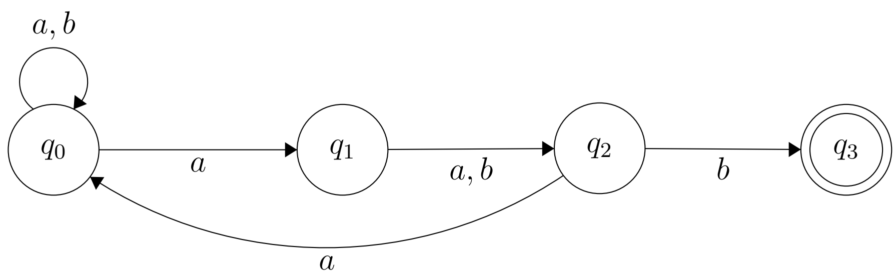
CIS770 Homework 2

Andre Gregoire

February 16, 2016

Problem1

1.1.



1.2.

Proof by cases:

I first choose 5 strings that all go to different states:

w1 = abba where $q_0 \xrightarrow{w1}_M$ A
w2 = a where $q_0 \xrightarrow{w2}_M$ B
w3 = b where $q_0 \xrightarrow{w3}_M$ C
w4 = ab where $q_0 \xrightarrow{w4}_M$ D
w5 = abb where $q_0 \xrightarrow{w5}_M$ E

w1 $\notin$ L, however w2 $\in$ L, w3 $\in$ L, w4 $\in$ L, w5 $\in$ L

B $\neq$ C consider bbb *w2*bb $\in$ L, *w3*bbb $\notin$ L
B $\neq$ D consider bb *w2*bb $\in$ L, *w4*bb $\notin$ L
B $\neq$ E consider bb *w2*bb $\in$ L, *w5*bb $\notin$ L
C $\neq$ D consider bb *w3*bb $\in$ L, *w4*bb $\notin$ L
C $\neq$ E consider bb *w3*bb $\in$ L, *w5*bb $\notin$ L
D $\neq$ E consider a *w4*bb $\in$ L, *w5*a $\notin$ L

Since *w1, w2, w3, w4, w5* must all go to unique states there must be at least 5 states for a DFA to recognize the language L

Problem 2

2.1.

$M = (Q, \Sigma, \delta, q_0, F)$
$M^{\mathrm{R}} = (Q^{\mathrm{R}}, \Sigma^{\mathrm{R}}, \delta^{\mathrm{R}}, {q_0}^{\mathrm{R}}, F^{\mathrm{R}})$
    $Q^{\mathrm{R}} = 2^{\mathrm{Q}}$
    ${q_0}^{\mathrm{R}} = F$
    $F^{\mathrm{R}} = \{\mathrm{S} \subseteq \mathrm{Q} \mid q_0 \in \mathrm{S}\}$
    $\delta^{\mathrm{R}}(\mathrm{S}, \mathrm{s}) = \{\mathrm{q} \in \mathrm{Q} \mid \delta(\mathrm{q,s}) \in \mathrm{S}\}$

2.2.

Note: Come back to this
Forward transition:
    $\delta_R(q_1, \mathrm{xa}) = \delta_R(\delta_R(q_1, \mathrm{x}), \mathrm{a})$
Reverse transition:
    $\delta_R(q_2, \mathrm{ax}) = \delta_R(\delta_R(q_2, \mathrm{x}), \mathrm{a})$

$\delta(q_1, \mathrm{x}) = q_2 \Leftrightarrow \delta_R(q_2, \mathrm{x}) = q_1$ given the string x

Problem 3

3.1.

An all-NFA M accepts w iff there is q $\in$ F such that $q_0 \xrightarrow{w}_M$ q and for every q' if $q_0 \xrightarrow{w}_M q'$ then $q' \in$ F.

The language recognized by M:

$$\mathrm{L(M)} = \{w \in \Sigma^* \mid \mathrm{M\ accepts\ } w\}$$

3.2.

DFA dfa(M) $= (2^{\mathrm{Q}}, \Sigma, \delta', {q_0}', F')$
    ${q_0}' = \hat{\delta}_M\ (q_0, \epsilon)$
    $F' = 2^{\mathrm{F}} \setminus \{\emptyset\}$ Note: all subsets of F minus the empty sets, wasnt sure if it was denoted right
    $\delta'(\mathrm{S}, \mathrm{s}) = \cup_{q \in S}\ \hat{\delta}_M(q, s)$

Problem 4

4.1.a.
The set of all binary strings.

4.1.b.
The set of all binary strings with a leading 0 and ending 1

4.1.c.
The set of all binary strings that no 0 can follow a 01 sequence. i.e. 010 is impossible

4.2.a.
$1^*(0 \cup \epsilon)1^*(0 \cup \epsilon)1^*(0 \cup \epsilon)1^*$

4.2.b.
$(01 \cup 10)^*(0 \cup 1 \cup \epsilon)$

4.2.c.
$(1 \cup 01 \cup 001)^*000(1 \cup 10 \cup 100)^*$