KANSAS STATE UNIVERSITY | Department of Mathematics

Applied Matrix Theory - Math 551

Homework assignment 13

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Name: _____

Due date: Thursday, May 2nd at 5:00pm. Use the drop box adjacent to CW120. No late homework will be accepted.

Instructions: Unless indicated otherwise, you are strongly encouraged to use your calculator or Matlab to complete this assignment. Write legibly, use extra sheets of paper if needed, and **staple your work**. Also, try to do a two-sided printing of this assignment.

Honor pledge: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work."

Exercises. All answers must be justified by using matrix theory

1. A web of 6 sites is described by the adjacency matrix

```
B =

     0     0     0     1     0     1
     1     1     0     1     1     1
     1     1     0     1     1     0
     1     0     1     1     0     1
     1     0     0     0     0     1
     0     0     1     1     0     0
```

   Use Google's PageRank algorithm to rate the importance of each site.

2. A web of 8 sites is described by the adjacency matrix

```
B =

     1     0     0     1     1     1     0     1
     0     0     0     1     0     1     1     1
     1     1     1     1     0     1     0     0
     1     1     1     0     1     0     0     1
     1     0     1     1     1     1     0     1
     0     0     0     0     1     1     0     1
     0     0     0     0     1     0     0     0
     0     0     1     0     0     1     1     0
```

Use Google's PageRank algorithm to rate the importance of each site.

3. Find the singular value decomposition of the matrix

```
A =
     2     2     1     2     3     2
     1     2     2     3     1     1
     0     2     1     2     0     3
     2     1     2     2     3     1
```

4. Consider the matrix

```
C =

     3     1     1     3     1
     2     0     3     3     4
     2     2     1     2     1
```

and compare the singular values of C and the square roots of the eigenvalues of CC^T. What do you observe?

5. Consider the vectors

$$w_1 = \begin{bmatrix} 2 \\ -1 \\ -2 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad w_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \\ 4 \end{bmatrix}.$$

Find an **orthonormal** basis for the subspace of all the vectors in $\mathbf{R}^5$ which are orthogonal to w_1 and w_2.

6. Write a Matlab function that takes an $m \times n$ matrix A and determines whether the input matrix A is column stochastic or row stochastic. If it is column stochastic, the function outputs a message indicating so. Similarly for a row-stochastic input. If it is neither, the function must also display a message indicating so.

   Note. Recall that a column-stochastic matrix is a *square matrix* with *all non-negative entries* and whose *columns add up to 1*. Similarly for row-stochastic matrices.

7. True or False - **Circle the right one** (1 point each)

T or **F**. The singular values of any given $m \times n$ matrix A, with $n \leq m$, are the square roots of the eigenvalues of $A^T A$.

T or **F**. If A and B are column-stochastic matrices, so is $A + B$.

T or **F**. If Q is a column-stochastic matrix, then $\lambda = 1$ is always an eigenvalue of Q.

T or **F**. Given any $n \times n$ matrix M, the matrices M and M^T share the same eigenvalues.

T or **F**. If P is a column-stochastic matrix, then P is orthogonal.

Points obtained in this assignment (out of 16): _____