LECTURE 25 OF 42

Reasoning under Uncertainty: Knowledge Representation in Uncertain Domains Discussion: Diagnosis and Causal Reasoning

William H. Hsu
Department of Computing and Information Sciences, KSU

KSOL course page: **http://snipurl.com/v9v3**
Course web site: **http://www.kddresearch.org/Courses/CIS730**
Instructor home page: **http://www.cis.ksu.edu/~bhsu**

Reading for Next Class:
Chapter 14, p, 492 – 499, Russell & Norvig 2nd edition
Fuzzy Logic (Wikipedia, Scholarpedia): **http://bit.ly/2rEMHe**, **http://bit.ly/4wYnAQ**
Dempster-Shafer Theory (Wikipedia): **http://bit.ly/2Y9FoS**
Ch. 13 notes (C. Dyer, U. Wisconsin – Madison): **http://bit.ly/53sLn**

LECTURE OUTLINE

- **Reading for Next Class: Chapter 14 (p. 492 – 499), R&N 2e**
- **Last Class: Robust Planning, 12.5 – 12.8 (p. 441 – 454), R&N 2e**
  - **Monitoring and replanning (12.5)**
  - **Continuous planning (12.6)**
  - **Need for representation language for uncertainty**
- **Today: Reasoning under Uncertainty, Probability, 13 (p. 462-486), R&N 2e**
  - **Where uncertainty is encountered**
    - **Reasoning**
    - **Planning**
    - **Learning (later)**
  - **Sources of uncertainty**
    - **Sensor error**
    - **Incomplete or faulty domain theory**
    - **"Nondeterministic" environment**
- **Coming Week: More Applied Probability, Graphical Models**

Acknowledgements

Stuart J. Russell
Professor of Computer Science
Chair, Department of Electrical Engineering and Computer Sciences
Smith-Zadeh Prof. in Engineering

Peter Norvig
Director of Research

© 2004-2005

Russell, S. J.
University of California, Berkeley
http://www.eecs.berkeley.edu/~russell/

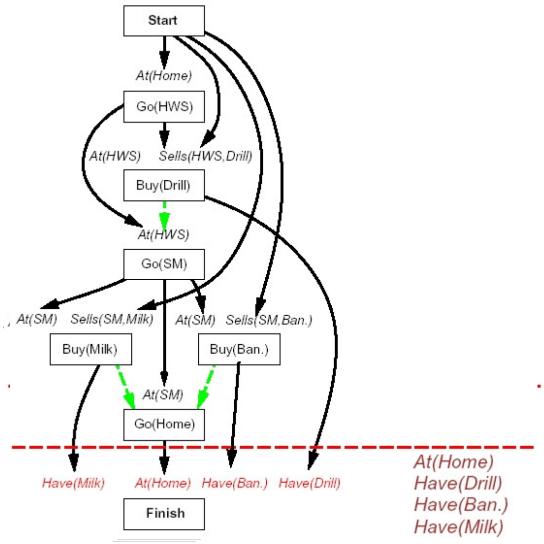
Norvig, P.
http://norvig.com/

Slides from:
http://aima.eecs.berkeley.edu

Taking Stock – Robust Planning: Review

- Bounded Indeterminacy: "Uncertainty Abounds" (12.3)
- Four Techniques for Dealing with Uncertain Domains
- 1. Sensorless *aka* Conformant: "Be Prepared" (12.3)
  - Idea: be able to respond to any situation (universal planning)
  - Coercion
- 2. Conditional *aka* Contingency: "Review the Situation" (12.4)
  - Idea: be able to respond to many typical alternative situations
  - Actions for sensing
- 3. Monitoring, Replanning: "The Show Must Go On" (12.5)
  - Idea: be able to resume momentarily failed plans
  - Plan revision
  - Monitoring: execution (present postcondition) *vs.* action (next precondition)
- 4. Continuous Planning: "Always in Motion, The Future Is" (12.6)
  - Lifetime planning (and learning!)
  - Formulate new goals

Replanning & Continuous Planning: Review



© 2004 S. Russell & P. Norvig. Reused with permission.

How Things Go Wrong in Planning: Review

Incomplete information

Unknown preconditions, e.g., $Intact(Spare)$?

Disjunctive effects, e.g., $Inflate(x)$ causes

$Inflated(x) \vee SlowHiss(x) \vee Burst(x) \vee BrokenPump \vee \ldots$

Incorrect information

Current state incorrect, e.g., spare NOT intact

Missing/incorrect postconditions in operators

Qualification problem:

can never finish listing all the required preconditions and possible conditional outcomes of actions

Based on slide © 2004 S. Russell & P. Norvig. Reused with permission.

Uncertainty

Let action A_t = leave for airport t minutes before flight
Will A_t get me there on time?

Problems:
1) partial observability (road state, other drivers' plans, etc.)
2) noisy sensors (KCBS traffic reports)
3) uncertainty in action outcomes (flat tire, etc.)
4) immense complexity of modelling and predicting traffic

Hence a purely logical approach either
1) risks falsehood: "A_{25} will get me there on time"
or 2) leads to conclusions that are too weak for decision making:
"A_{25} will get me there on time if there's no accident on the bridge and it doesn't rain and my tires remain intact etc etc."

(A_{1440} might reasonably be said to get me there on time but I'd have to stay overnight in the airport ...)

© 2004 S. Russell & P. Norvig. Reused with permission.

Methods for Handling Uncertainty

Default or nonmonotonic logic:
Assume my car does not have a flat tire
Assume A_{25} works unless contradicted by evidence
Issues: What assumptions are reasonable? How to handle contradiction?

Rules with fudge factors:
$A_{25} \mapsto_{0.3} AtAirportOnTime$
$Sprinkler \mapsto_{0.99} WetGrass$
$WetGrass \mapsto_{0.7} Rain$
Issues: Problems with combination, e.g., $Sprinkler$ causes $Rain$??

Probability
Given the available evidence,
A_{25} will get me there on time with probability 0.04
Mahaviracarya (9th C.), Cardamo (1565) theory of gambling

(Fuzzy logic handles **degree of truth** NOT uncertainty e.g.,
$WetGrass$ is true to degree 0.2)

© 2004 S. Russell & P. Norvig. Reused with permission.

Probability

Probabilistic assertions **summarize** effects of
- laziness: failure to enumerate exceptions, qualifications, etc.
- ignorance: lack of relevant facts, initial conditions, etc.

Subjective or Bayesian probability:
Probabilities relate propositions to one's own state of knowledge
e.g., $P(A_{25} | \text{no reported accidents}) = 0.06$

These are **not** claims of a "probabilistic tendency" in the current situation (but might be learned from past experience of similar situations)

Probabilities of propositions change with new evidence:
e.g., $P(A_{25} | \text{no reported accidents, 5 a.m.}) = 0.15$

(Analogous to logical entailment status $KB \models \alpha$, not truth.)

© 2004 S. Russell & P. Norvig. Reused with permission.

Making Decisions Under Uncertainty

Suppose I believe the following:

$$
\begin{aligned}
P(A_{25} \text{ gets me there on time} | \ldots) &= 0.04 \\
P(A_{90} \text{ gets me there on time} | \ldots) &= 0.70 \\
P(A_{120} \text{ gets me there on time} | \ldots) &= 0.95 \\
P(A_{1440} \text{ gets me there on time} | \ldots) &= 0.9999
\end{aligned}
$$

Which action to choose?

Depends on my preferences for missing flight vs. airport cuisine, etc.

Utility theory is used to represent and infer preferences

Decision theory = utility theory + probability theory

© 2004 S. Russell & P. Norvig. Reused with permission.

Probability Basics

Begin with a set Ω—the sample space

e.g., 6 possible rolls of a die.

$\omega \in \Omega$ is a sample point/possible world/atomic event

A probability space or probability model is a sample space with an assignment $P(\omega)$ for every $\omega \in \Omega$ s.t.

$0 \le P(\omega) \le 1$

$\Sigma_\omega P(\omega) = 1$

e.g., $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$.

An event A is any subset of Ω

$$P(A) = \Sigma_{\{\omega \in A\}} P(\omega)$$

E.g., $P(\text{die roll} < 4) = P(1) + P(2) + P(3) = 1/6 + 1/6 + 1/6 = 1/2$

© 2004 S. Russell & P. Norvig. Reused with permission.

Random Variables

A random variable is a function from sample points to some range, e.g., the reals or Booleans

e.g., $Odd(1) = true$.

P induces a probability distribution for any r.v. X:

$$P(X = x_i) = \Sigma_{\{\omega : X(\omega) = x_i\}} P(\omega)$$

e.g., $P(Odd = true) = P(1) + P(3) + P(5) = 1/6 + 1/6 + 1/6 = 1/2$

© 2004 S. Russell & P. Norvig. Reused with permission.

Propositions

Think of a proposition as the event (set of sample points) where the proposition is true

Given Boolean random variables A and B:
- event a = set of sample points where $A(\omega) = true$
- event $\neg a$ = set of sample points where $A(\omega) = false$
- event $a \wedge b$ = points where $A(\omega) = true$ and $B(\omega) = true$

Often in AI applications, the sample points are **defined** by the values of a set of random variables, i.e., the sample space is the Cartesian product of the ranges of the variables

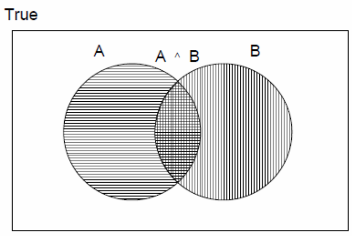
With Boolean variables, sample point = propositional logic model
e.g., $A = true$, $B = false$, or $a \wedge \neg b$.
Proposition = disjunction of atomic events in which it is true
e.g., $(a \vee b) \equiv (\neg a \wedge b) \vee (a \wedge \neg b) \vee (a \wedge b)$
$\Rightarrow P(a \vee b) = P(\neg a \wedge b) + P(a \wedge \neg b) + P(a \wedge b)$

© 2004 S. Russell & P. Norvig. Reused with permission.

Why Use Probability?

The definitions imply that certain logically related events must have related probabilities

E.g., $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$



de Finetti (1931): an agent who bets according to probabilities that violate these axioms can be forced to bet so as to lose money regardless of outcome.

© 2004 S. Russell & P. Norvig. Reused with permission.

Syntax for Propositions

Propositional or Boolean random variables
  e.g., $Cavity$ (do I have a cavity?)
  $Cavity = true$ is a proposition, also written $cavity$

Discrete random variables (finite or infinite)
  e.g., $Weather$ is one of $\langle sunny, rain, cloudy, snow\rangle$
  $Weather = rain$ is a proposition
  Values must be exhaustive and mutually exclusive

Continuous random variables (bounded or unbounded)
  e.g., $Temp = 21.6$; also allow, e.g., $Temp < 22.0$.

Arbitrary Boolean combinations of basic propositions

© 2004 S. Russell & P. Norvig. Reused with permission.

Prior Probability

Prior or unconditional probabilities of propositions
  e.g., $P(Cavity = true) = 0.1$ and $P(Weather = sunny) = 0.72$
correspond to belief prior to arrival of any (new) evidence

Probability distribution gives values for all possible assignments:
  $\mathbf{P}(Weather) = \langle 0.72, 0.1, 0.08, 0.1\rangle$ (normalized, i.e., sums to 1)

Joint probability distribution for a set of r.v.s gives the probability of every atomic event on those r.v.s (i.e., every sample point)
  $\mathbf{P}(Weather, Cavity)$ = a 4×2 matrix of values:

$Weather =$	$sunny$	$rain$	$cloudy$	$snow$
$Cavity = true$	0.144	0.02	0.016	0.02
$Cavity = false$	0.576	0.08	0.064	0.08

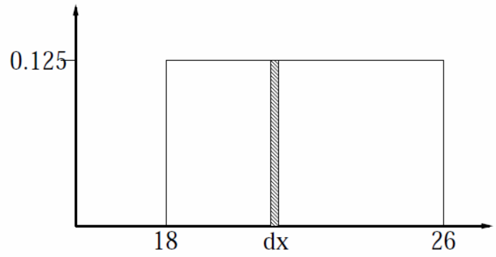
Every question about a domain can be answered by the joint distribution because every event is a sum of sample points

© 2004 S. Russell & P. Norvig. Reused with permission.

Probability for Continuous Random Variables

Express distribution as a parameterized function of value:

$P(X=x) = U[18,26](x) =$ uniform density between 18 and 26



Here P is a density; integrates to 1.

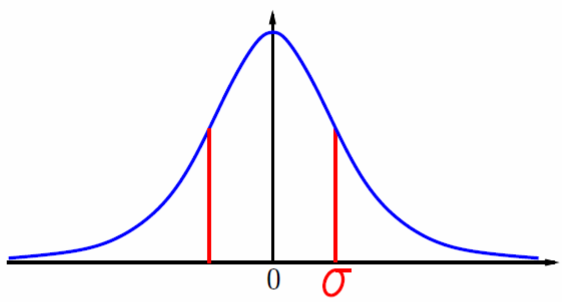
$P(X=20.5) = 0.125$ really means

$$\lim_{dx \to 0} P(20.5 \leq X \leq 20.5 + dx)/dx = 0.125$$

© 2004 S. Russell & P. Norvig. Reused with permission.

Gaussian Density *aka* Normal Density

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$



Based on slide © 2004 S. Russell & P. Norvig. Reused with permission.

Conditional Probability [1]: Intuition & General Concepts

Conditional or posterior probabilities

e.g., $P(cavity|toothache) = 0.8$

i.e., **given that** $toothache$ **is all I know**

NOT "if $toothache$ then 80% chance of $cavity$"

(Notation for conditional distributions:

$\mathbf{P}(Cavity|Toothache)$ = 2-element vector of 2-element vectors)

If we know more, e.g., $cavity$ is also given, then we have

$P(cavity|toothache, cavity) = 1$

Note: the less specific belief **remains valid** after more evidence arrives, but is not always **useful**

New evidence may be irrelevant, allowing simplification, e.g.,

$P(cavity|toothache, 49ersWin) = P(cavity|toothache) = 0.8$

This kind of inference, sanctioned by domain knowledge, is crucial

Based on slide © 2004 S. Russell & P. Norvig. Reused with permission.

Conditional Probability [2]: Definition

Definition of conditional probability:

$$P(a|b) = \frac{P(a \wedge b)}{P(b)} \text{ if } P(b) \neq 0$$

Product rule gives an alternative formulation:

$P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a)$

A general version holds for whole distributions, e.g.,

$\mathbf{P}(Weather, Cavity) = \mathbf{P}(Weather|Cavity)\mathbf{P}(Cavity)$

(View as a 4×2 set of equations, **not** matrix mult.)

Chain rule is derived by successive application of product rule:

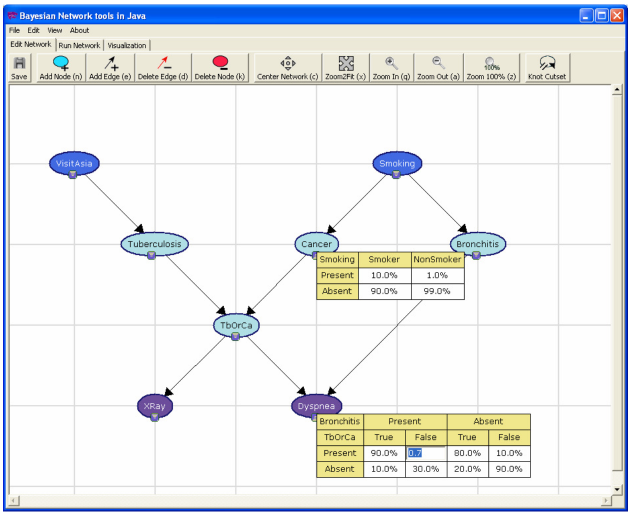
$$\begin{aligned}\mathbf{P}(X_1, \ldots, X_n) &= \mathbf{P}(X_1, \ldots, X_{n-1})\ \mathbf{P}(X_n|X_1, \ldots, X_{n-1}) \\ &= \mathbf{P}(X_1, \ldots, X_{n-2})\ \mathbf{P}(X_{n_1}|X_1, \ldots, X_{n-2})\ \mathbf{P}(X_n|X_1, \ldots, X_{n-1}) \\ &= \ldots \\ &= \Pi_{i=1}^{n} \mathbf{P}(X_i|X_1, \ldots, X_{i-1})\end{aligned}$$

Based on slide © 2004 S. Russell & P. Norvig. Reused with permission.

Looking Ahead: Uncertain Reasoning Roadmap

- **Framework: Interpretations of Probability [Cheeseman, 1985]**
  - Bayesian subjectivist view
    - Measure of agent's belief in proposition
    - Proposition denoted by random variable (range: sample space Ω)
    - e.g., *Pr*(*Outlook* = *Sunny*) = 0.8
  - Frequentist view: probability is *frequency of observations* of event
  - Logicist view: probability is inferential evidence in favor of proposition
- **Some Applications**
  - HCI: learning natural language; intelligent displays; decision support
  - Approaches: prediction; sensor and data fusion (e.g., bioinformatics)
- **Prediction: Examples**
  - Measure *relevant parameters*: temperature, barometric pressure, wind speed
  - Make statement of the form *Pr*(*Tomorrow's-Weather* = *Rain*) = 0.5
  - College admissions: *Pr*(*Acceptance*) ≡ *p*
    - Plain beliefs: unconditional acceptance (*p*=1), categorical rejection (*p*=0)
    - Conditional beliefs: depends on reviewer (use probabilistic model)

Looking Ahead: Graphical Models of Probability



Asia (Chest Clinic) Network
© 2004 KSU BNJ Development Team

Terminology

- **Uncertain Reasoning**
  - Ability to perform inference in presence of uncertainty about
    - premises
    - rules
  - Nondeterminism
- **Representations for Uncertain Reasoning**
  - Probability: measure of belief in sentences
    - Founded on Kolmogorov axioms
    - prior, joint *vs.* conditional
    - Bayes's theorem: P(A | B) = (P(B | A) * P(A)) / P(B)
  - Graphical models: graph theory + probability
  - Dempster-Shafer theory: upper and lower probabilities, reserved belief
  - Fuzzy representation (sets), fuzzy logic: degree of membership
  - Others
    - Truth maintenance system: logic-based network representation
    - Endorsements: evidential reasoning mechanism

Summary Points

- **Last Class: Robust Planning**
  - Monitoring and replanning (12.5)
  - Continuous planning (12.6)
  - Need for representation language for uncertainty
- **Today: Reasoning under Uncertainty and Probability**
  - Uncertainty is pervasive
    - Planning
    - Reasoning
    - Learning (later)
  - What are we uncertain about?
    - Sensor error
    - Incomplete or faulty domain theory
    - "Nondeterministic" environment
- **Coming Week: More Applied Probability**
  - Graphical models as KR for uncertainty: Bayesian networks, *etc.*
  - Some inference algorithms for Bayes nets