# Math 321

Q's / $f_0 f_1 + f_1 f_2 + \dots + f_{2n-1} f_{2n} = f_{2n}^2$ #15 p. 308

$f_0 = 0$
$f_1 = 1$
$f_2 = 1$
$f_3 = 2$
$\vdots$

**Basis:** $n=1$ Right $= f_{2(1)}^2 = f_2^2 = 1$

left $= f_0 f_1 + f_1 f_2 = 0 \cdot 1 + 1 \cdot 1 = 1$

True!

**Inductive:** (show $P(k) \to P(k+1)$)

assume $P(k)$: $f_0 f_1 + f_1 f_2 + \dots + f_{2k-1} f_{2k} = f_{2k}^2$

show $P(k+1)$: $f_0 f_1 + \dots + f_{2k-1} f_{2k} + f_{2k} f_{2k+1} + f_{2k+1} f_{2k+2} = f_{2k+2}^2$

given $f_0 f_1 + f_1 f_2 + \dots + f_{2k-1} f_{2k} = f_{2k}^2$

$$f_0 f_1 + \dots + f_{2k-1} f_{2k} + f_{2k} f_{2k+1} + f_{2k+1} f_{2k+2} = f_{2k}^2 + f_{2k} f_{2k+1} + f_{2k+1} f_{2k+2}$$

$$f_0 f_1 + \dots + f_{2k+1} f_{2k+2} = f_{2k}\underbrace{(f_{2k} + f_{2k+1})}_{f_{2k+2}} + f_{2k+1} f_{2k+2}$$

$$" \quad = f_{2k+2}[f_{2k} + f_{2k+1}]$$

$$" \quad = f_{2k+2}^2$$

True

7b $a_n = 2n+1 \quad n = 1, 2, 3, \ldots$

$a_1 = 3$
$a_2 = 5$ $+2$
$a_3 = 7$
$\vdots$

$$\begin{cases} a_1 = 3 \\ a_n = a_{n-1} + 2 \end{cases}$$

5a $f(0) = 0$

$f(n) = 2f(n-2) \quad n = 1, 2, \ldots$

$f(1) = 2f(-1) = ?$

$\boxed{\text{Not Valid}}$

## Counting Basics.

Counting is a thought experiment.
Find a rule that will "count" your set.

Counting a task that has two parts.
You do one $\boxed{\text{and}}$ you then do part two.

$A_1$ is the set of ways for task 1
$A_2$ is the set of ways for task 2

task one **and** task 2

pick one & $A_1 \times A_2 = \{(a,b) \mid a \in A_1 \wedge b \in A_2\}$

**Product Rule** $|A_1 \times A_2| = |A_1| \cdot |A_2|$

ex: 5 ways to mow, 2 ways to trim

to finish lawn.. mow **and** trim.

$$5 \cdot 2 = \boxed{10}$$

$(\equiv, \circlearrowright), (\equiv, \circlearrowleft)$, etc.

**Product Rule** for $|A_1 \times A_2 \times \dots \times A_n| = |A_1||A_2| \cdots |A_n|$

ex: pick a password that can have lowercase letters or numbers that contains 10 symbols.

(a) $b_1 \wedge b_2 \wedge \dots \wedge b_{10}$

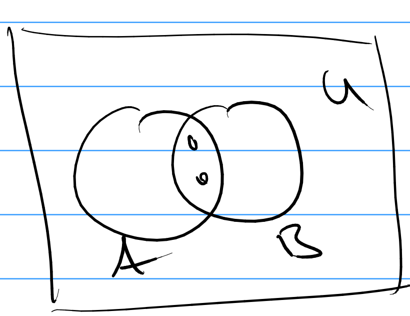
$36 \cdot 36 \cdot \dots \cdot 36 = 36^{10}$

(b) lower, upper, numbers, and no repeats.

$b_1 \wedge b_2 \wedge \dots \wedge b^{10} = 62 \cdot 61 \cdot 60 \cdot 59 \cdot 58 \cdot 57 \cdot 56 \cdot 55 \cdot 54 \cdot 53$

## Sum Rule: (or)

task one or task two




① $A$ and $B$ are disjoint

$$|A \cup B| = |A| + |B|$$

② Not Disjoint! (Inclusion/Exclusion Principle)

$$|A \cup B| = |A| + |B| - |A \cap B|$$

## Lots of Sets.

① Disjoint

$$|A_1 \cup A_2 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n|$$

② Non Disjoint?

| $A_1$ | $A_2$ |
|---|---|
| 1 | 1 |
| 1 | 0 |
| 0 | 1 |
| 0 | 0 |

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

| $A_1$ | $A_2$ | $A_3$ | |
|---|---|---|---|
| 1 | 1 | 1 | |
| 1 | 1 | 0 | d |
| 1 | 0 | 1 | d |
| 1 | 0 | 0 | s |
| 0 | 1 | 1 | d |
| 0 | 1 | 0 | s |
| 0 | 0 | 1 | s |
| 0 | 0 | 0 | |

$$|A_1 \cup A_2 \cup A_3|$$
$$= |A_1| + |A_2| + |A_3|$$
$$- |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3|$$
$$+ |A_1 \cap A_2 \cap A_3|$$

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Simple: How many bubble answer sheets are possible from 10 question w/ 4 answers and you can leave blank.

$q_1$ ○ ○ ○ ○ $\qquad |q_1| = 5$

$q_2$ $\qquad |q_2| = 5$

$\vdots$

$q_1 \wedge q_2 \wedge \ldots \wedge q_{10}$

$5 \cdot 5 \cdot \ldots \cdot 5 = 5^{10}$

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$$|A_1 \cup A_2| = |A_1| + |A_2|$$

$$|A_1 \cup A_2| - |A_1| = |A_2|$$

all = some a + no a's

$$\boxed{36^{10} - 35^{10}} = |\text{some } a\text{'s}|$$

$$|\overset{\text{exactly}}{1\ a\text{'s}}| + |\overset{\text{exactly}}{2\ a\text{'s}}| + \cdots + |\text{exactly } 10\ a\text{'s}|$$