Quiz 3

1. For a (deterministic or nondeterministic) finite automaton $M = (Q, \Sigma, \delta, q_0, F)$, recall that $\hat{\delta}_M : Q \times \Sigma^* \to 2^Q$ is a function that given a state q and string w returns the set of all states the M could be in after reading w from state q. Formally, $\hat{\delta}_M(q, w) = \{q' \mid q \xrightarrow{w}_M q'\}$. Which of the following statements is true?

   (A) If M is a deterministic finite automaton, then for any $q \in Q$ and $a \in \Sigma$, $\hat{\delta}_M(q, a) = \delta(q, a)$.

   (B) If M is a deterministic finite automaton, then for any $q \in Q$, $\hat{\delta}_M(q, \epsilon) = \{q\}$.

   (C) If M is a nondeterministic finite automaton, then for any $q \in Q$ and $a \in \Sigma$, $\hat{\delta}_M(q, a) = \delta(q, a)$.

   (D) If M is a nondeterministic finite automaton, then for any $q \in Q$, $\hat{\delta}_M(q, \epsilon) = \{q\}$.

   The correct answer is (B). Upon reading ϵ in a DFA, we remain in the same state that we started. Hence $\hat{\delta}_M(q, \epsilon) = \{q\}$.

   (A) is incorrect because the left hand side is a set and the right hand side isn't. (C) and (D) are incorrect because $\delta(q, a)$ and $\{q\}$ do not necessarily include all states reachable from these sets by ϵ transitions, while for any string s $\hat{\delta}_M(q, s)$ must necessarily contain all states reachable from itself by ϵ transitions.

2. Let L be recognized by a DFA M and an NFA N. Which of the following statements is necessarily true?

   (A) M and N are the exact same machines.

   (B) M and N have the same number of states.

   (C) N has transitions on ϵ.

   (D) There is an NFA N' that recognizes L which has the same number of states as M.

   The correct answer is (D). Given any DFA M, it is possible to re-interpret the DFA as an NFA accepting an identical language (just by modifying the definition of δ appropriately). When converting M to an NFA, we preserve the same number of states.

   It is easy to come up with counterexamples for (A) and (B) because we can add states to any NFA without changing the language it accepts, resulting in a family of distinct NFAs that accept the same language. (C) is incorrect because NFAs are not required to have transitions on ϵ.

3. Which of the following statements is true?

   (A) There are languages that can be recognized by an NFA which cannot be recognized by a DFA.

   (B) Languages recognized by NFAs cannot be recognized by DFAs because they can have infinitely many active threads at any given time.

   (C) If L is a language recognized by an NFA then there is a DFA that can recognize L.

   (D) Every language is recognized by an NFA because they are subsets of Σ^*.

   The correct answer is (C). NFAs and DFAs recognize the same class of languages. This means that for every NFA there exists some DFA recognizing the same language, and vice versa.

4. Let M be a DFA with m states, and N be an NFA with n states such that $\mathbf{L}(M) = \mathbf{L}(N)$. Which of the following statements is necessarily true?

(A) $2^n \leq m$
(B) $m < 2^n$
(C) $n \leq m$
(D) None of the above

The correct answer is (D). Recall that our definition of DFA or NFA does not forbid "disconnected" states, that is, states that are unreachable from the start state. Therefore, given any NFA or DFA, we can increase the number of states in it by an arbitrary amount and still have an NFA/DFA recognizing the same language. Thus, starting with a DFA M and and NFA N that recognize the same language, we can construct a DFA M' and an NFA N' such that $\mathbf{L}(M') = \mathbf{L}(N')$ that provides a counterexample to any of (A), (B), or (C), by adding an appropriate amount of disconnected states to M or N (and making appropriate modifications to the transition function δ).

5. Let $L = \{0\}$. Which of the following statements is true?

   (A) $L^* = (LL)^*$
   (B) $L^* = L(L^*)$
   (C) $L^* = (L^*)L$
   (D) $L^* = L^*L^*$

   Correct answer is (D).

   (A) Counterexample: $L = \{0\}$.
   (B) Counterexample: $L \neq \emptyset$.
   (C) Counterexample: $L \neq \emptyset$.
   (D) Let $a = x_1 \dots x_n$, $b = y_1 \dots y_n$, where $x_1, \dots, x_n, y_1, \dots, y_n \in L$. It's easy to see $a, b, ab \in L^*$.

6. Consider $r = a(ab^*a \cup b^*)^*$. Which of the following is true about $\mathbf{L}(r)$?

   (A) $a \in \mathbf{L}(r)$
   (B) $aa \in \mathbf{L}(r)$
   (C) Every string in $\mathbf{L}(r)$ has at least one b.
   (D) None of the above.

   Correct answer is (A).

   (A) a matches the first a and ϵ matches $(ab^*a \cup b^*)^*$.
   (B) a matches the first a, but a doesn't match $(ab^*a \cup b^*)^*$, there need to be at least one more a.
   (C) (A) is a counterexample.
   (D) (A) is true.

7. Let R_1 and R_2 be two regular expressions with $\mathbf{L}(R_1) = \mathbf{L}(R_2)$. Let N_1 and N_2 be the NFA constructed by the inductive algorithm described in lecture 7, for R_1 and R_2, respectively. Which of the following statements is necessarily true about R_1, R_2, N_1, and N_2?

   (A) R_1 and R_2 must be syntactically the same regular expression.
   (B) N_1 and N_2 have the same number of states.
   (C) N_1 and N_2 have the same number of transitions.
   (D) If R_1 and R_2 are syntactically the same then N_1 and N_2 will have the same number of states and transitions.

Correct answer is (D).

(A) Counterexample $R_1 = a^*$, $R_2 = (a^*)^*$.

(B) By repeatedly adding $*$ to R_2 in (A), we can have arbitrary number of states.

(C) By repeatedly adding $*$ to R_2 in (A), we can have arbitrary number of transitions.

(D) The algorithm is determinstic. For the same input, it produces the same output.

8. Which of the following facts is *not* true about GNFAs?

(A) A GNFA has exactly one final state.

(B) The initial state of a GNFA could also be a final state.

(C) The initial state of a GNFA has no incoming transitions.

(D) The final state of a GNFA has no outgoing transitions.

Correct answer is (B).

(A) By definition.

(B) By definition.

(C) By definition.

(D) By definition.