Solutions for Homework 4
CIS 770: Formal Language Theory

Problem 1. [Category: Comprehension+Design] Let $L = \mathbf{L}(1^*0(00 \cup 01 \cup 1)(0 \cup 1)^*)$.

1. List all the suffix languages of L, explain why your answer covers all the suffix languages. **[5 points]**
2. Draw the minimum state DFA M^L accepting L. **[5 points]**

Solution:

1. Let's consider suffix languages for some strings first to get an idea.

$$\begin{aligned}
&\text{suffix}(L, \epsilon) = L \\
&\text{suffix}(L, 1) = L \\
&\text{suffix}(L, 0) = \mathbf{L}((00 \cup 01 \cup 1)(0 \cup 1)^*) \\
&\text{suffix}(L, 00) = \mathbf{L}((0 \cup 1)(0 \cup 1)^*) \\
&\text{suffix}(L, 000) = \mathbf{L}((0 \cup 1)^*) \\
&\text{suffix}(L, 001) = \mathbf{L}((0 \cup 1)^*) \\
&\text{suffix}(L, 01) = \mathbf{L}((0 \cup 1)^*)
\end{aligned}$$

   Combine some strings together, we can get a more generalized version like below.

   - For $x \in 1^*$, $\text{suffix}(L, x) = L$
   - For $x \in 1^*0$, $\text{suffix}(L, x) = \mathbf{L}((00 \cup 01 \cup 1)(0 \cup 1)^*)$
   - For $x \in 1^*00$, $\text{suffix}(L, x) = \mathbf{L}((0 \cup 1)(0 \cup 1)^*)$
   - For $x \in L$, $\text{suffix}(L, x) = \mathbf{L}((0 \cup 1)^*)$

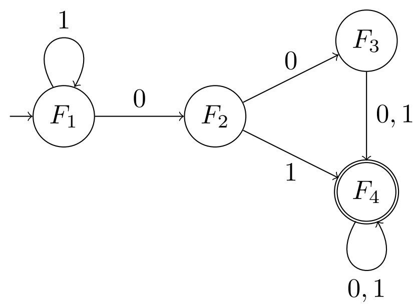
   Observe that $L = \mathbf{L}(1^*0(00 \cup 01 \cup 1)(0 \cup 1)^*)$ is the language that has either one 0 followed by a 1 or two 0s followed by either another 0 or a 1. Therefore $L' = \{x | x \in \Sigma^* \setminus (1^* \cup 1^*0 \cup 1^*00)\}$ is nothing but L , therefore the four suffix languages above covers all the suffix languages of L.

2. The four suffix languages above correspond to the suffix languages of four states of the DFA M^L. So the automaton M^L has $Q^L = \{F_1, F_2, F_3, F_4\}$, where $q_0^L = F_1 = \text{suffix}(L, \epsilon) = L$, $F^L = F_4 = \{\text{suffix}(L, x)\} | \epsilon \in \text{suffix}(L, x)\} = \mathbf{L}((0 \cup 1)^*)$ for $x \in \Sigma^* \setminus (1^* \cup 1^*0 \cup 1^*00)$, $\delta^L(\text{suffix}(L, x), a) = \text{suffix}(L, xa)$ for $x \in \Sigma^*$. The automaton can be drawn as follows.

■

Problem 2. [Category: Comprehension] Given a homomorphism $h : \Sigma^* \to \Delta^*$ and a language $L \subseteq \Sigma^*$, define $h(L) = \{h(w) | w \in L\} \subseteq \Delta^*$.

1. Prove that for all strings $x, y \in \Sigma^*, h(xy) = h(x)h(y)$. **[5 points]**
2. Prove that $h(L_1 \cup L_2) = h(L_1) \cup h(L_2)$. **[5 points]**



3. Prove that $h(L_1 \circ L_2) = h(L_1) \circ h(L_2)$. **[5 points]**

Solution:

1. Let $x = a_1 \dots a_n$ and $y = b_1 \dots b_m$. $h(xy) = h(a_1 \dots a_n b_1 \dots b_m) = h(a_1) \dots h(a_n) h(b_1) \dots h(b_m)$ (using the definition of homomorphism) $= h(a_1) \dots h(a_n) \circ h(b_1) \dots h(b_m) = h(a_1 \dots a_n) \circ h(b_1 \dots b_m) = h(x)h(y)$.

2. $h(L_1 \cup L_2) = h(L_1) \cup h(L_2)$

   $w \in h(L_1 \cup L_2)$ iff $h(x) = w$ for some $x \in L_1 \cup L_2$ iff $h(x) = w$ for some $x \in L_1$ or $x \in L_2$ iff $h(x) = w$ for some $x \in L_1$ or $h(x) = w$ for some $x \in L_2$ iff $w \in h(L_1)$ or $w \in h(L_2)$ iff $w \in h(L_1) \cup h(L_2)$

3. $h(L_1 \circ L_2) = h(L_1) \circ h(L_2)$

   $w \in h(L_1 \circ L_2)$ iff $h(x) = w$ for some $x \in L_1 \circ L_2$ iff $h(x) = w$ for some $y \in L_1, z \in L_2, x = yz$ iff $h(yz) = w$ for some $y \in L_1, z \in L_2$ iff $h(y)h(z) = w$ for some $y \in L_1, z \in L_2$ (from Part 1) iff $w = w_1 w_2$, where $w_1 = h(y)$, $w_2 = h(z)$ for some $y \in L_1, z \in L_2$ iff $w = w_1 w_2$, where $w_1 \in h(L_1)$, $w_2 = h(L_2)$ iff $w \in h(L_1)h(L_2)$

■