Math 321

Proofs: $\boxed{th^m}$ hyp $\to$ conclusion

premises

is true

or can be shown to be true.

r	s	$r \to s$
T	T	T
T	F	F
F	T	T
F	F	T

if your hypothesis is false th^m is vacuously true.

if conclusion is just true th^m is trivially true.

what is interesting is 1^{st} two lines

h	c	$h \to c$
T	T	T
T	F	F

hypothesis is true

what matters is to now show conclusion is true.

Proof: is the use of rules of inference and tautologies to show the conclusion being true.

techniq.

① **Direct Proof:** (assume hyp, show conclu)

thm: p is even $\to$ p^2 is even

pf: assume p is even.

So $p = 2k$ for some integer k.

$p \cdot p = 2k \cdot p \qquad p \neq 0$

$p^2 = 2k \cdot 2k = 2\underbrace{(2k^2)}_{\text{integer}} = 2(\text{integer})$

So p^2 is even. ∎

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ex **thm:** if p^2 even $\to$ p is even

pf: (direct)

assume $p^2 = 2k$ for some integer k.

$(p^2)^{1/2} = (2k)^{1/2}$

$p = \sqrt{2}\sqrt{k}$?

$p = 2(\text{integer})$

$\boxed{\text{Can't do this.}}$

② indirect proof.

(a) prove the contrapositive directly

Thm: $(p^2 \text{ is even} \to p \text{ is even})$

$\equiv (\neg(p \text{ is even}) \to \neg(p^2 \text{ is even}))$

$\equiv (p \text{ is odd} \to p^2 \text{ is odd})$

Pf: assume p is odd

$p = 2K + 1$ for some integer K

$p \cdot p = (2K+1) \cdot p$

$p^2 = (2K+1)(2K+1)$

$p^2 = 4K^2 + 4K + 1$

$p^2 = 2\underbrace{(2K^2 + 2K)}_{\text{integer}} + 1$

so p^2 is odd.

Note:

$$(p \text{ is even} \rightarrow p^2 \text{ is even}) \equiv (p^2 \text{ is odd} \rightarrow p \text{ is odd})$$

$$(p^2 \text{ is even} \rightarrow p \text{ is even}) \equiv (p \text{ is odd} \rightarrow p^2 \text{ is odd})$$

Thm: $\sqrt{2}$ is irrational

Background:

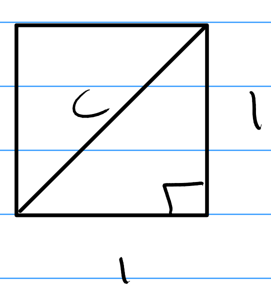
Counting Numbers $\{1, 2, 3, 4, \ldots\}$

rationals $\frac{p}{q}$ and p, q are integers

$q \neq 0$ and p, q have no common factors.

$$\frac{6}{9} = \frac{2}{3}$$

$$= \frac{1}{3} \cdot 2$$



$$\frac{1}{q} \cdot p = \frac{p}{q}$$

$$1^2 + 1^2 = c^2 \rightarrow c^2 = 2$$

$$c = \sqrt{2}$$

Thm: $\sqrt{2}$ is irrational

Pf: proof by contradiction

Normal goal $(h \to c) \equiv T$
$$\left.\begin{array}{l} (h \to c) \equiv T \\ \neg(h \to c) \equiv F \\ \neg(\neg h \vee c) \equiv F \\ \boxed{(h \wedge \neg c) \equiv F} \end{array}\right\} \text{same}$$

$\boxed{(h \wedge \neg c) \equiv F}$ ← proof by contradiction

assume: $\neg(\sqrt{2}$ is irrational$)$

→ $\sqrt{2}$ is rational

means $\sqrt{2} = \frac{p}{q}$ p, q are integers, $q \neq 0$, $\boxed{p, q \text{ have } \underline{\text{no}} \text{ common factors}}$

$\to 2 = \frac{p^2}{q^2} \to \underbrace{2q^2}_{\text{integer}} = p^2$

$\to p^2$ is even $\to p$ is even (by lemma)

So $p = 2 \cdot k$ for some integer k

with $2q^2 = p^2 \to 2q^2 = (2k)^2$

$\rightarrow \quad 2q^2 = 4k^2$

$\rightarrow \quad q^2 = 2\underbrace{k^2}_{\text{integer}} \rightarrow q^2$ is even

$\rightarrow$ (by lemma) q is even

$q = 2 \cdot l$ for some integer l.

p has factors 2 and k
q has factors 2 and l
common factor of 2

and

p, q don't have common factors

$\rightarrow$ **Contradiction**

So $\boxed{\sqrt{2} \text{ is irrational}}$ $\blacksquare$