Math 321

Getting Out the Extra Credit

→ 8 proofs over the course of the semester.

Q's

(13) $\exists! x\, P(x)$ (p. 37)

$\exists! x\, P(x) \equiv \exists x \forall y\, (P(x) \land P(y) \to y = x)$

a) $\exists x \forall y\, (P(y) \leftrightarrow x = y)$

b) $\exists x\, P(x) \land \forall x \forall y\, (P(x) \land P(y) \to x = y)$

c) $\exists x\, (P(x) \land \forall y\, (P(y) \to x = y))$

#7 consec. pos. integers.

$\boxed{1}, 2, 3, \boxed{4}, 5, 6, 7, 8, \boxed{9}, 10, \ldots$

$\underbrace{5, 6, 7, 8}$

4 consec. pos. integers that are not perfect squares.

$4^2 - 3^2 = 16 - 9 = \boxed{7} \to \boxed{6}$

$5^2 - 4^2 = 25 - 16 = \boxed{9} \to \boxed{8}$

$6^2 - 5^2 = 36 - 25 = \boxed{11} \to \boxed{10}$

$(n+1)^2 - n^2 \longrightarrow (2n) = 100$

$51^2 - 50^2$ $\qquad n = 50$

$2601 - 2500 = 101 \rightsquigarrow 100$ consc. int's w/ no square

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#17 n is odd $\to \exists! k \, (n = (k-2) + (k+3))$

pf: $n = 2k + 1$ k is an integer

$n = k + k + 1$

$n = k + k + 3 - 2$

$n = (k-2) + (k+3)$

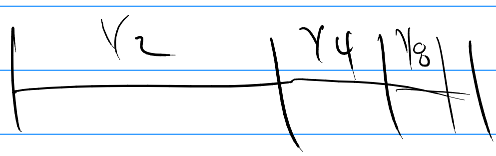
$\boxed{\text{ch 2}}$ Sets.

Paradox vs. $(P \wedge \neg P) \equiv F$

$P \equiv T$

$\neg P \equiv T$

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F



Naive Set Theory

Def: Set $\equiv$ unordered collection of objects

$\leftarrow$ elements or members

Note: a set contains its elements..

Notation:

① list of stuff..

$$S = \{ \square, 1/3, \sqrt{2}, ☺, ☹ \}$$

$$A = \{1,1,1,1,2,2,5,10,20\}$$

② set builder notation.

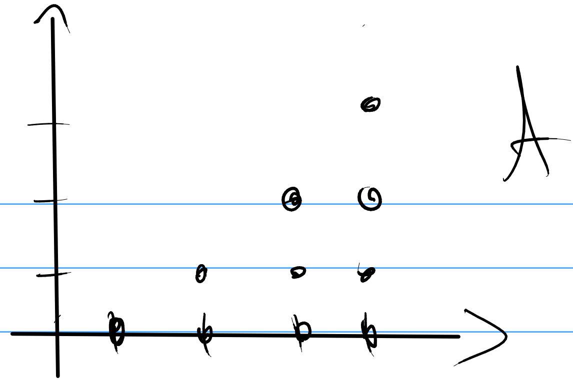
$$S = \{ \text{basic element} \mid \boxed{\text{Propositional func.}} \}$$

"such that"

ex

$$A = \{(a,b) \mid (1 \le a \le 4) \wedge (0 \le b \le 3) \wedge (b < a) \wedge \text{are int.} \}$$

$$A = \{ (1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3) \}$$



Special Sets

① Empty Set

$$\emptyset = \{\ \}$$

② Universal Set

$$\mathcal{U} = \{\underline{\underline{\text{all}}} \text{ elements in univ. of discourse}\}$$

③ Numbers

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

$$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$$

$$\mathbb{Q} = \{\tfrac{a}{b} \mid a \in \mathbb{Z},\ b \in \mathbb{Z},\ b \neq 0,$$

a and b have no common factors $\}$

<u>element of</u>

$\mathbb{R}$ all reals

Comparing Sets.

(1) "Same"

$$A = B \quad \text{iff} \quad \forall x \, (x \in A \Leftrightarrow x \in B)$$

ex $A = \{1, 2, 3\}$ $\quad B = \{2, 3, 1\}$

$$A = B$$

$A = \{1, 1, 2, 10, 10, 10\}$ $\quad B = \{1, 2, 10\}$

$$A = B$$

→ Distinct elements is unique elements.