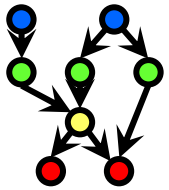

Lecture 37 of 42

Genetic Programming
Discussion: Schema Theory

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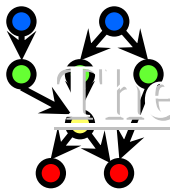
KSOL course page: http://snipurl.com/v9v3

Course web site: http://www.kddresearch.org/Courses/CIS730

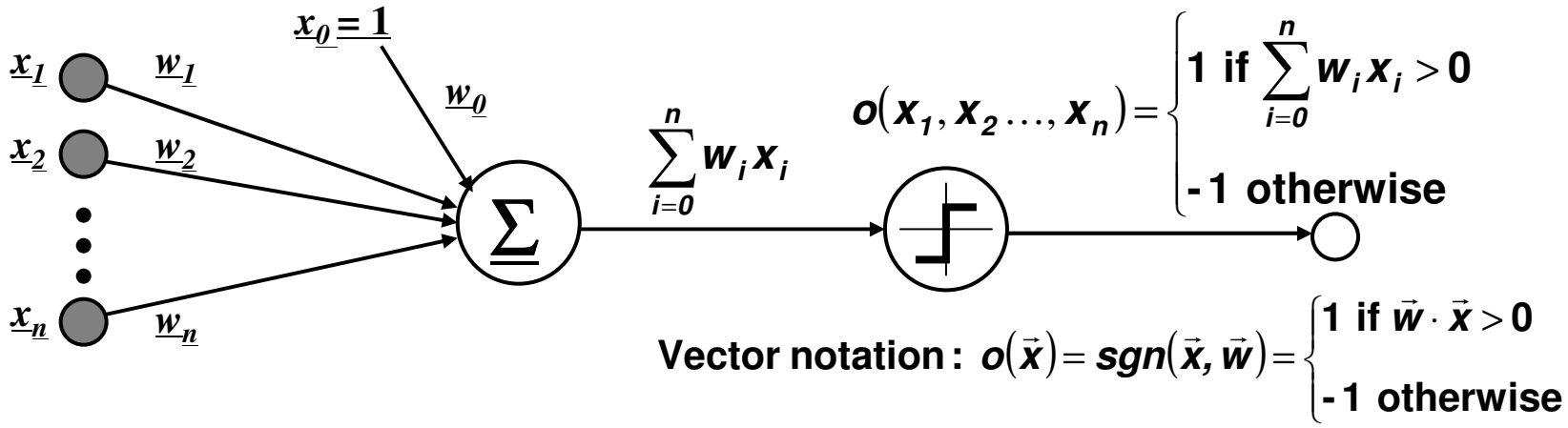
Instructor home page: http://www.cis.ksu.edu/~bhsu

Reading for Next Class:

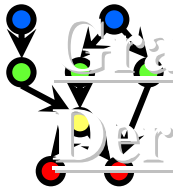
Chapter 20, Russell and Norvig



The Perceptron



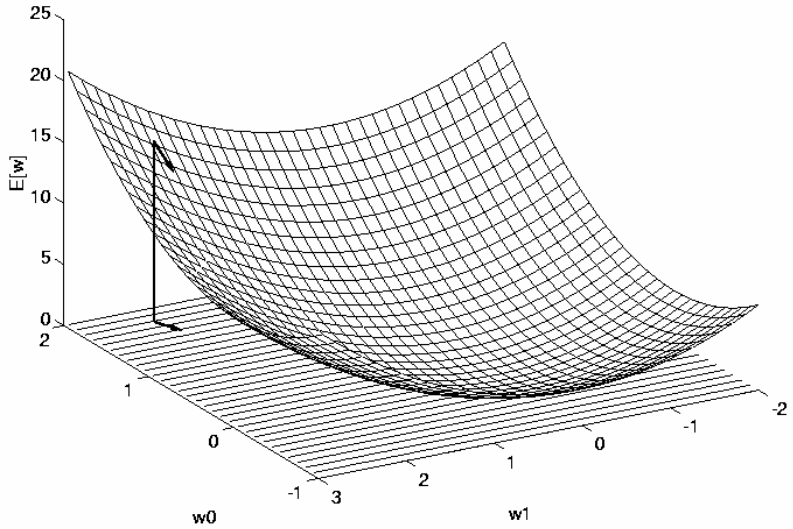
- **Perceptron: Single Neuron Model**
  - *aka* Linear Threshold Unit (LTU) or Linear Threshold Gate (LTG)
  - Net input to unit: defined as linear combination $net = \sum_{i=0}^{n} w_i x_i$
  - Output of unit: threshold (activation) function on net input (threshold $\theta = w_0$)
- **Perceptron Networks**
  - Neuron is modeled using a unit connected by weighted links w_i to other units
  - Multi-Layer Perceptron (MLP): next lecture



Gradient Descent: Derivation of Delta/LMS (Widrow-Hoff) Rule

- **Definition: Gradient**

$$\nabla E[\vec{w}] \equiv \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \ldots, \frac{\partial E}{\partial w_n} \right]$$



- **Modified Gradient Descent Training Rule**

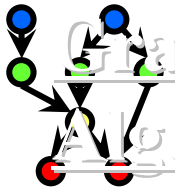
$$\Delta \vec{w} = -r \nabla E[\vec{w}]$$

$$\Delta w_i = -r \frac{\partial E}{\partial w_i}$$

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i} \left[\frac{1}{2} \sum_{x \in D} (t(x) - o(x))^2 \right] = \frac{1}{2} \sum_{x \in D} \left[\frac{\partial}{\partial w_i} (t(x) - o(x))^2 \right]$$

$$= \frac{1}{2} \sum_{x \in D} \left[2(t(x) - o(x)) \frac{\partial}{\partial w_i} (t(x) - o(x)) \right] = \sum_{x \in D} \left[(t(x) - o(x)) \frac{\partial}{\partial w_i} (t(x) - \vec{w} \cdot \vec{x}) \right]$$

$$\frac{\partial E}{\partial w_i} = \sum_{x \in D} [(t(x) - o(x))(-x_i)]$$



Gradient Descent: Algorithm using Delta/LMS Rule

- **Algorithm *Gradient-Descent* (*D*, *r*)**
  - Each training example is a pair of the form $\langle x, t(x) \rangle$, where x is the vector of input values and $t(x)$ is the output value. r is the learning rate (e.g., 0.05)
  - Initialize all weights w_i to (small) random values
  - UNTIL the termination condition is met, DO

    Initialize each Δw_i to zero

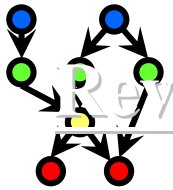
    FOR each $\langle x, t(x) \rangle$ in D, DO

      Input the instance x to the unit and compute the output o

      FOR each linear unit weight w_i, DO

        $\Delta w_i \leftarrow \Delta w_i + r(t - o)x_i$

        $w_i \leftarrow w_i + \Delta w_i$
  - RETURN final w
- **Mechanics of Delta Rule**
  - Gradient is based on a derivative
  - Significance: later, will use nonlinear activation functions (*aka* transfer functions, squashing functions)



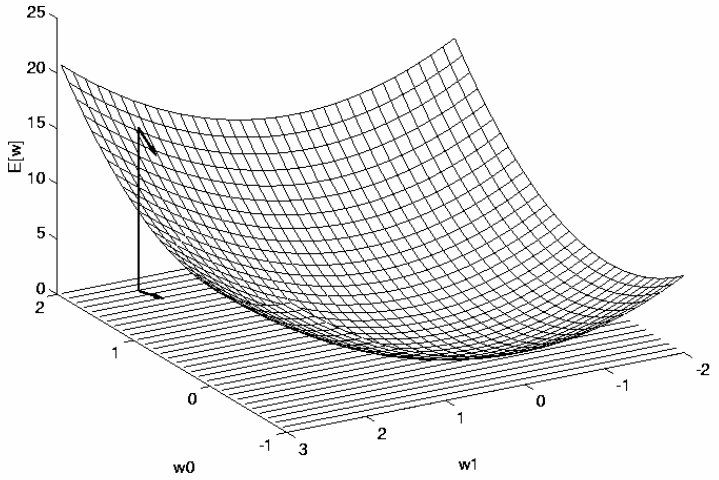
Review: Derivation of Backprop

- **Recall: Gradient of Error Function** $\nabla E[\vec{w}] \equiv \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \ldots, \frac{\partial E}{\partial w_n}\right]$
- **Gradient of Sigmoid Activation Function**

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i}\left[\frac{1}{2}\sum_{\langle \vec{x}, t(\vec{x})\rangle \in D}\left(t(\vec{x}) - o(\vec{x})\right)^2\right] = \frac{1}{2}\sum_{\langle \vec{x}, t(\vec{x})\rangle \in D}\left[\frac{\partial}{\partial w_i}\left(t(\vec{x}) - o(\vec{x})\right)^2\right]$$

$$= \frac{1}{2}\sum_{\langle \vec{x}, t(\vec{x})\rangle \in D}\left[2\left(t(\vec{x}) - o(\vec{x})\right)\frac{\partial}{\partial w_i}\left(t(\vec{x}) - o(\vec{x})\right)\right] = \sum_{\langle \vec{x}, t(\vec{x})\rangle \in D}\left[\left(t(\vec{x}) - o(\vec{x})\right)\left(-\frac{\partial o(\vec{x})}{\partial w_i}\right)\right]$$

$$= -\sum_{\langle \vec{x}, t(\vec{x})\rangle \in D}\left[\left(t(\vec{x}) - o(\vec{x})\right)\frac{\partial o(\vec{x})}{\partial net(\vec{x})}\frac{\partial net(\vec{x})}{\partial w_i}\right]$$

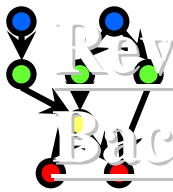


- **But We Know:**

$$\frac{\partial o(\vec{x})}{\partial net(\vec{x})} = \frac{\partial \sigma(net(\vec{x}))}{\partial net(\vec{x})} = o(\vec{x})(1 - o(\vec{x}))$$

$$\frac{\partial net(\vec{x})}{\partial w_i} = \frac{\partial(\vec{w} \cdot \vec{x})}{\partial w_i} = x_i$$

- **So:** $\frac{\partial E}{\partial w_i} = -\sum_{\langle \vec{x}, t(\vec{x})\rangle \in D}\left[\left(t(\vec{x}) - o(\vec{x})\right) \cdot \left(o(\vec{x})(1 - o(\vec{x}))\right) \cdot x_i\right]$



Review: Backprop, Feedforward

- **Intuitive Idea: Distribute *Blame* for Error to Previous Layers**
- **Algorithm *Train-by-Backprop* (D, r)**
  - Each training example is a pair of the form $\langle x, t(x) \rangle$, where x is the vector of input values and $t(x)$ is the output value. r is the learning rate (e.g., 0.05)
  - Initialize all weights w_i to (small) random values
  - UNTIL the termination condition is met, DO
    - FOR each $\langle x, t(x) \rangle$ in D, DO
      - Input the instance x to the unit and compute the output $o(x) = \sigma(net(x))$
      - FOR each output unit k, DO

$$\delta_k = o_k(x)(1 - o_k(x))(t_k(x) - o_k(x))$$

Output Layer

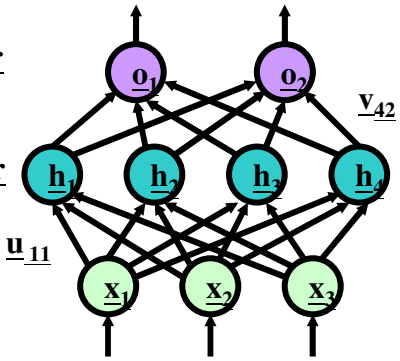
      - FOR each hidden unit j, DO

$$\delta_j = h_j(x)(1 - h_j(x)) \sum_{k \in outputs} v_{j,k} \delta_j$$

Hidden Layer

      - Update each $w = u_{i,j}$ $(a = h_j)$ or $w = v_{j,k}$ $(a = o_k)$

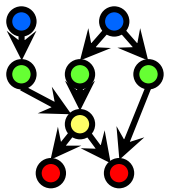
Input Layer



$$w_{start\text{-}layer,\ end\text{-}layer} \leftarrow w_{start\text{-}layer,\ end\text{-}layer} + \Delta w_{start\text{-}layer,\ end\text{-}layer}$$

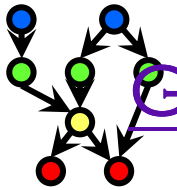
$$\Delta w_{start\text{-}layer,\ end\text{-}layer} \leftarrow r\, \delta_{end\text{-}layer}\, a_{end\text{-}layer}$$

  - RETURN final u, v



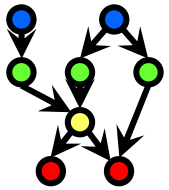
Simple Genetic Algorithm (SGA)

- **Algorithm *Simple-Genetic-Algorithm* (*Fitness*, *Fitness-Threshold*, *p*, *r*, *m*)**
  - // *p*: population size; *r*: replacement rate (*aka* generation gap width), *m*: string size
  - *P* ← *p* random hypotheses // initialize population
  - FOR each *h* in *P* DO *f*[*h*] ← *Fitness*(*h*) // evaluate *Fitness*: *hypothesis* → R
  - WHILE (*Max*(*f*) < *Fitness-Threshold*) DO
    - 1. Select: Probabilistically select (1 - *r*)*p* members of *P* to add to P_S
    - $$P(h_i) = \frac{f[h_i]}{\sum_{j=1}^{p} f[h_j]}$$
    - 2. Crossover:
      - Probabilistically select ($r \cdot p$)/2 pairs of hypotheses from *P*
      - FOR each pair <h_1, h_2> DO
        - P_S += *Crossover* (<h_1, h_2>) // $P_S[t+1] = P_S[t] + \langle offspring_1, offspring_2 \rangle$
    - 3. Mutate: Invert a randomly selected bit in $m \cdot p$ random members of P_S
    - 4. Update: *P* ← P_S
    - 5. Evaluate: FOR each *h* in *P* DO *f*[*h*] ← *Fitness*(*h*)
  - RETURN the hypothesis *h* in *P* that has maximum fitness *f*[*h*]



GA-Based Inductive Learning (*GABIL*)

- GABIL System [Dejong *et al*, 1993]
  - Given: concept learning problem and examples
  - Learn: disjunctive set of propositional rules
  - Goal: results competitive with those for current decision tree learning algorithms (e.g., C4.5)
- Fitness Function: $Fitness(h) = (Correct(h))^2$
- Representation
  - Rules: IF $a_1 = T \wedge a_2 = F$ THEN $c = T$; IF $a_2 = T$ THEN $c = F$
  - Bit string encoding: a_1 [10] . a_2 [01] . c [1] . a_1 [11] . a_2 [10] . c [0] = 10011 11100
- Genetic Operators
  - Want variable-length rule sets
  - Want only well-formed bit string hypotheses



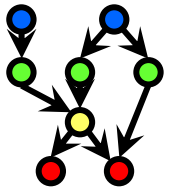
Crossover: Variable-Length Bit Strings

- Basic Representation
  - Start with

	a_1	a_2	c	a_1	a_2	c
h_1	1[0	01	1	11	1]0	0
h_2	0[1	1]1	0	10	01	0

  - Idea: allow crossover to produce variable-length offspring
- Procedure
  - 1. Choose crossover points for h_1, e.g., after bits 1, 8
  - 2. Now restrict crossover points in h_2 to those that produce bitstrings with well-defined semantics, e.g., <1, 3>, <1, 8>, <6, 8>
- Example
  - Suppose we choose <1, 3>
  - Result

h_3	11	10	0					
h_4	00	01	1	11	11	0	10	01	0

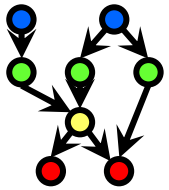


GABIL Extensions

- New Genetic Operators
  - Applied probabilistically
  - 1. *AddAlternative*: generalize constraint on a_i by changing a 0 to a 1
  - 2. *DropCondition*: generalize constraint on a_i by changing every 0 to a 1
- New Field
  - Add fields to bit string to decide whether to allow the above operators

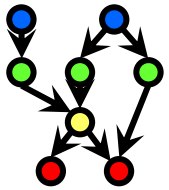
a_1	a_2	c	a_1	a_2	c	*AA*	*DC*
01	11	0	10	01	0	1	0

  - So now the learning strategy also evolves!
  - *aka* genetic wrapper



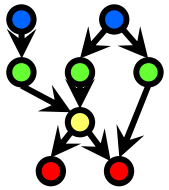
GABIL Results

- Classification Accuracy
  - Compared to symbolic rule/tree learning methods
    - *C4.5* [Quinlan, 1993]
    - *ID5R*
    - *AQ14* [Michalski, 1986]
  - Performance of *GABIL* comparable
    - Average performance on a set of 12 synthetic problems: 92.1% test accuracy
    - Symbolic learning methods ranged from 91.2% to 96.6%
- Effect of Generalization Operators
  - Result above is for *GABIL* without *AA* and *DC*
  - Average test set accuracy on 12 synthetic problems with *AA* and *DC*: 95.2%



Building Blocks (Schemas)

- Problem
  - How to characterize evolution of population in GA?
  - Goal
    - ⇨ Identify basic building block of GAs
    - ⇨ Describe *family of individuals*
- Definition: Schema
  - String containing 0, 1, * (“don’t care”)
  - Typical schema: 10**0*
  - Instances of above schema: 101101, 100000, …
- Solution Approach
  - Characterize population by number of instances representing each possible schema
  - $m(s, t)$ ≡ number of instances of schema s in population at time t



Selection and Building Blocks

- **Restricted Case: Selection Only**
  - $\bar{f}(t)$ ≡ average fitness of population at time *t*
  - $m(s, t)$ ≡ number of instances of schema *s* in population at time *t*
  - $\hat{u}(s,t)$ ≡ average fitness of instances of schema *s* at time *t*
- **Quantities of Interest**
  - Probability of selecting *h* in one selection step

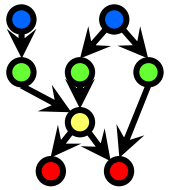
$$P(h) = \frac{f(h)}{\sum_{i=1}^{n} f(h_i)}$$

$$P(h \in s) = \sum_{h \in (s \cap p_t)} \frac{f(h)}{n \cdot \bar{f}(t)} = \frac{\hat{u}(s,t)}{n \cdot \bar{f}(t)} \cdot m(s,t)$$

  - Probability of selecting an instance of *s* in one selection step

$$E[m(s, t+1)] = \frac{\hat{u}(s,t)}{\bar{f}(t)} \cdot m(s,t)$$

  - Expected number of instances of *s* after *n* selections

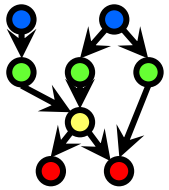


Schema Theorem

- Theorem

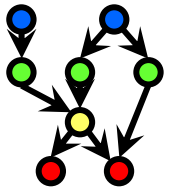
$$E[m(s, t+1)] \geq \frac{\hat{u}(s, t)}{\bar{f}(t)} \cdot m(s, t) \cdot \left(1 - p_c \frac{d_s}{l - 1}\right) \cdot (1 - p_m)^{o(s)}$$

  - $\frac{m(s, t)}{\bar{f}(t)}$ ≡ number of instances of schema s in population at time t
  - $\hat{u}(s, t)$ ≡ average fitness of population at time t
  - ≡ average fitness of instances of schema s at time t
  - p_c ≡ probability of single point crossover operator
  - p_m ≡ probability of mutation operator
  - l ≡ length of individual bit strings
  - $o(s)$ ≡ number of defined (non "*") bits in s
  - $d(s)$ ≡ distance between rightmost, leftmost defined bits in s
- Intuitive Meaning
  - "The expected number of instances of a schema in the population tends toward its relative fitness"
  - A fundamental theorem of GA analysis and design



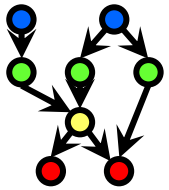
Terminology

- Evolutionary Computation (EC): Models Based on Natural Selection
- Genetic Algorithm (GA) Concepts
  - Individual: single entity of model (corresponds to hypothesis)
  - Population: collection of entities in competition for survival
  - Generation: single application of selection and crossover operations
  - Schema *aka* building block: descriptor of GA population (e.g., 10**0*)
  - Schema theorem: *representation of schema proportional to its relative fitness*
- Simple Genetic Algorithm (SGA) Steps
  - Selection
    - ⇨ Proportionate reproduction (*aka* roulette wheel): $P(individual) \propto f(individual)$
    - ⇨ Tournament: let individuals compete in pairs or tuples; eliminate unfit ones
  - Crossover
    - ⇨ Single-point: 11101001000 × 00001010101 → { 11101010101, 00001001000 }
    - ⇨ Two-point: 11101001000 × 00001010101 → { 11001011000, 00101000101 }
    - ⇨ Uniform: 11101001000 × 00001010101 → { 10001000100, 01101011001 }
  - Mutation: single-point (“bit flip”), multi-point



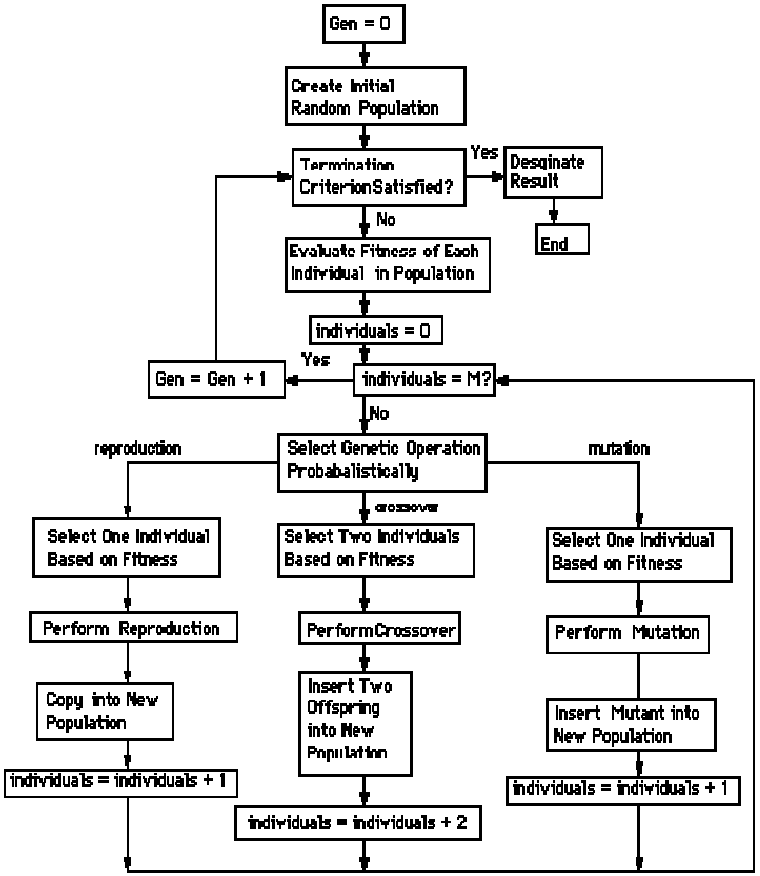
Lecture Outline

- Readings / Viewings
  - View GP videos 1-3
    - GP1 – *Genetic Programming: The Video*
    - GP2 – *Genetic Programming: The Next Generation*
    - GP3 – *Genetic Programming: Human-Competitive*
  - Suggested: Chapters 1-5, Koza
- Previously
  - Genetic and evolutionary computation (GEC)
  - Generational vs. steady-state GAs; relation to simulated annealing, MCMC
  - Schema theory and GA engineering overview
- Today: GP Discussions
  - Code bloat and potential mitigants: types, OOP, parsimony, optimization, reuse
  - Genetic programming vs. human programming: similarities, differences
- Thursday: Course Review

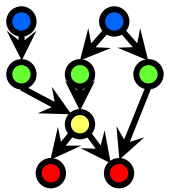


GP Flow Graph

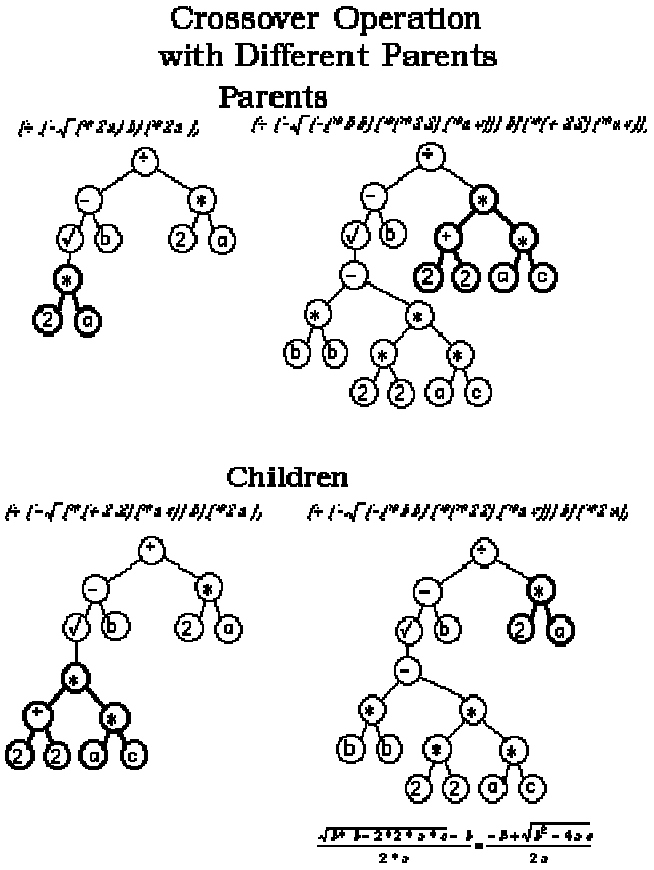
Flowchart for Genetic Programming

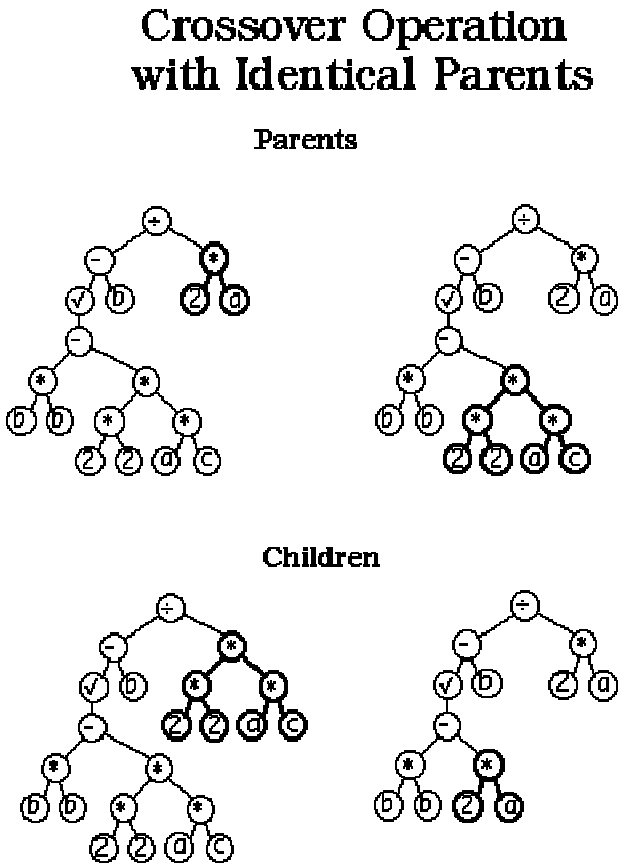


Adapted from The Genetic Programming Notebook © 2002 Jaime J. Fernandez
http://www.geneticprogramming.com



Structural Crossover





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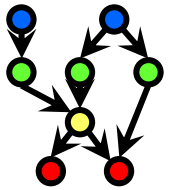
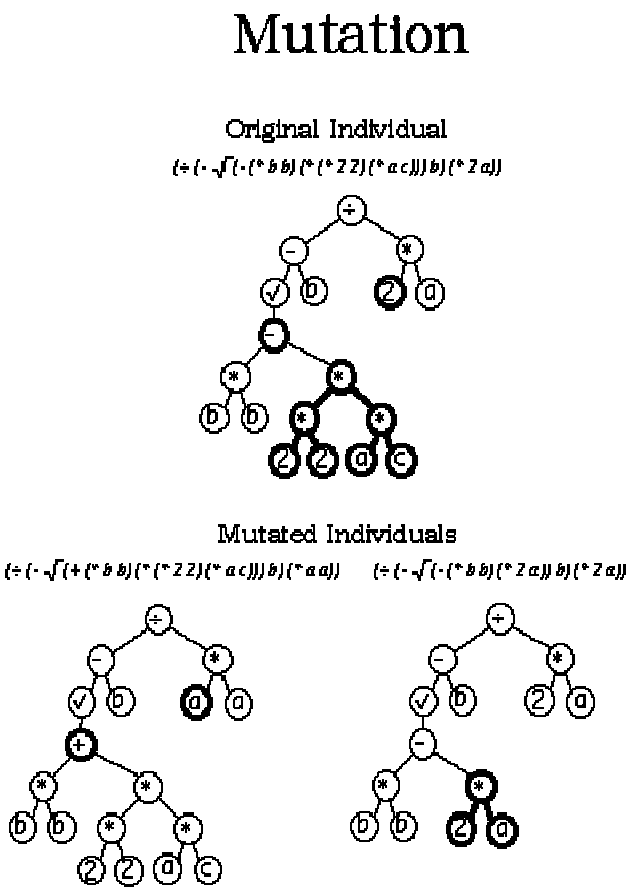


Structural Mutation



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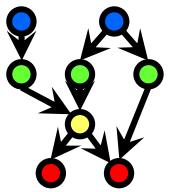
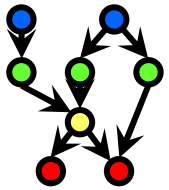


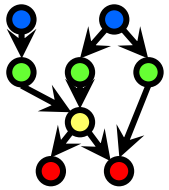
Genetic Programming: The Next Generation (Synopsis and Discussion)

- Automatically-Defined Functions (ADFs)
  - *aka* macros, anonymous inline functions, subroutines
  - Basic method of software reuse
- Questions for Discussion
  - *What are advantages, disadvantages of learning anonymous functions?*
  - *How are GP ADFs similar to and different from human-produced functions?*
- Exploiting Advantages
  - Reuse
  - Innovation
- Mitigating Disadvantages
  - Potential lack of meaning – semantic clarity issue (and topic of debate)
  - Redundancy
  - Accelerated bloat – scalability issue



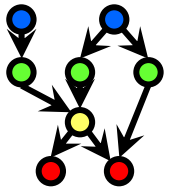
Code Bloat [1]: Problem Definition

- Definition
  - Increase in program size not commensurate with increase in functionality (possibly as function of problem size)
  - Compare: structural criteria for overfitting, overtraining
- Scalability Issue
  - Large GPs will have this problem
  - Discussion: *When do we expect large GPs?*
  - Machine learning: large, complex data sets
  - Optimization, control, decision making / DSS: complex problem
- What Does It Look Like?
- What Can We Do About It?
  - ADFs
  - Advanced reuse techniques from software engineering: e.g., design patterns
  - Functional, object-oriented design; theory of types
  - Controlling size: parsimony (MDL-like), optimization (cf. compiler)



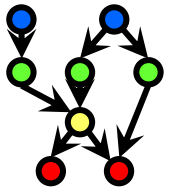
Code Bloat [2]: Mitigants

- Automatically Defined Functions
- Types
  - Ensure
    - ⇨ Compatibility of functions created
    - ⇨ Soundness of functions themselves
  - Define: abstract data types (ADTs) – object-oriented programming
  - Behavioral subtyping – still “future work” in GP
  - Generics (cf. C++ templates)
  - Polymorphism
- Advanced Reuse Techniques
  - Design patterns
  - Workflow models
  - Inheritance, reusable classes



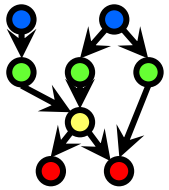
Code Bloat [3]: More Mitigants

- Parsimony (cf. Minimum Description Length)
  - Penalize code bloat
  - Inverse fitness = loss + cost of code (evaluation)
  - May include terminals
- Target Language Optimization
  - Rewriting of constants
  - Memoization
  - Loop unrolling
  - Loop-invariant code motion



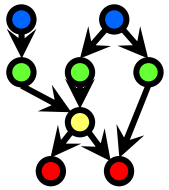
Genetic Programming 3 (Synopsis and Discussion [1])

- **16 Criteria for Automatic Program Synthesis by Computational Intelligence**
  - **1. Specification: starts with *what needs to be done***
  - **2. Procedural representation: tells us how to do it**
  - **3. Algorithm implementation: produces a computer program**
  - **4. Automatic determination of program size**
  - **5. Code reuse**
  - **6. Parametric reuse**
  - **7. Internal storage**
  - **8. Iteration (while / for), recursion**
  - **9. Self-organization of hierarchies**
  - **10. Automatic determination of architecture**
  - **11. Wide range of programming constructs**
  - **12. Well-defined**
  - **13. Problem independent**



Genetic Programming 3 (Synopsis and Discussion [2])

- 16 Criteria for Automatic Program Synthesis …
  - 14. Generalization: wide applicability
  - 15. Scalability
  - 16. Human-competitiveness
- Current Bugbears: Generalization, *Scalability*
- Discussion: Human Competitiveness?



More Food for Thought and Research Resources

- Discussion: Future of GP
- Current Applications
- Conferences
  - *GECCO: ICGA + ICEC + GP*
  - *GEC*
  - *EuroGP*
- Journals
  - *Evolutionary Computation Journal (ECJ)*
  - *Genetic Programming and Evolvable Machines (GPEM)*