

Announcement

HW1 online : Due. 02/17

Programming Assignment 2 online : Due 02/22

Designing Recursive Algorithms.

Recursive Algorithms.

2 components.

① solves one part of the problem.
Ex. $\text{fact}(0)$

Ex. $\text{fact}(n) = n * \text{fact}(n-1)$

② Reduces the size of the problem.

① Base Case \rightarrow should not call the algo.

② General Case. \rightarrow does call the algo again (recursive)

Ex. $\text{fact}(n) = \frac{\text{fact}(n+1)}{n+1}$

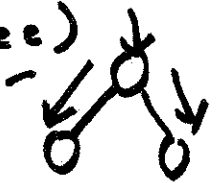
Rules for designing a recursive algo. $\left[\begin{array}{l} \text{① Determine the base case.} \\ \text{② " the general case.} \\ \text{③ combine them into an algorithm.} \end{array} \right.$

Limitations: Extensive overhead. (time & memory)

- slower than iterative (loop) approach
- function calls. (how many?)
- how deep. (l).

Advantages: Elegant, Easy to read,

- some algo are inherently recursive. [$O(\log(N))$]
- some DS " " " (tree)



fact(10) - recursive design is better.

fact(100) - iterative " " " ✓

Ex. Print Reverse.

- Reads ~~at~~ data from keyboard (input)
- once input finishes, then prints the data in reverse order.

Q. Is DS or Algo. naturally suited for recursion?

list \swarrow $O(N)$ \searrow [O(N) function calls]

Q. Is the algo simpler to understand? Y.

Iterative approach is more suitable.

Fibonacci Numbers.

0 1 1 2 3 5 8 ...

$$F_{i6}(0) = 0$$
$$Fib(1) = 1$$
$$Fib(2) = 1 = Fib(0) + Fib(1) \leftarrow$$
$$Fib(n) = Fib(n-2) + Fib(n-1) \quad \forall n \geq 2$$

recursive.

(Iterative)

0 1 1 2 3 5

second last last

$$f'(0) = 0$$
$$f_3(1) = 1$$

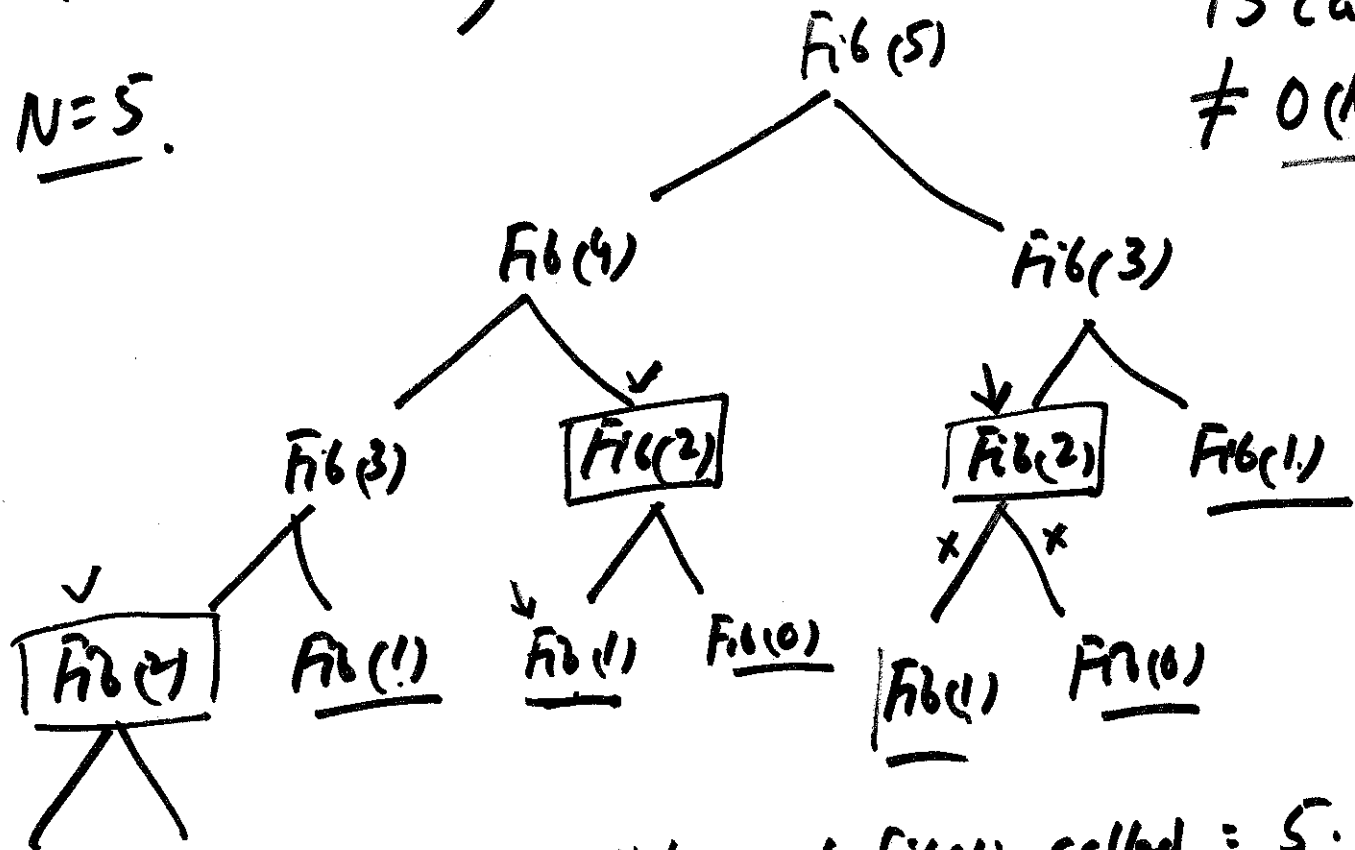
white (i)

```
{
    result = secondLast + last;
    secondLast = last;
    last = result;
}
```

Q. How many function calls. are made to function fib. to compute fib(5). using recursive design.

$N=5$.

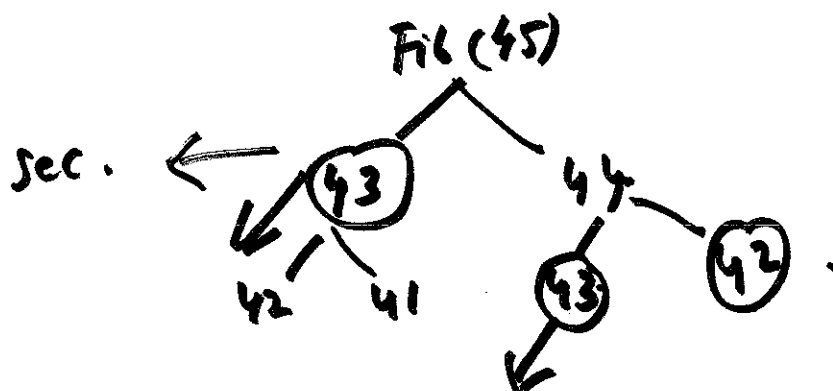
15 calls.
 $\neq O(N)$



times of fib(1) called = 5.
(Redundant calls increase with N)

fib(1) fib(0).

Idea: keep recursive approach.
(good idea) However, store all ~~computed~~ Fib numbers.
computed once.



original recursive approach

- # calls to fib. grows exponentially with N .

new recursive approach.

- # calls to ~~fib~~ fib is linear in N .

iterative approach is still better.

- for large n , recursive f^n is not efficient.
($O(N)$ efficiency, $O(N) f^n$ calls)

1.13. for ($i=1$; $i < n$; $i * = 2$) (Multiply loop)
doIt(\dots) \rightarrow (Given) efficiency $O(n^2)$

Q. What is the efficiency of the above code segment?

for \rightarrow multiply loop
divide loop } logarithmic loops. $O(\log N)$

Ans: $n^2 \cdot \log n$

(2.1). α

2.1. algorithm fun1(x)

1. if (x < 5)

2. return (3 * x)

3 else.

4 return (2 * fun1(x-5) + 7)

5 end fun1

(A) fun1(10) = ?

(21)

return (2 * fun1(5) + 7)

↓ ↑ 7

fun1(5) return (2 * fun1(0) + 7)

fun1(0) return 0

↑

(B) fun1(2) = ? (6).

x
0
7
14
17
41
54.

$$(c) \quad \text{fun } 1(11) = ? \quad (33)$$