

# Towers Of Hanoi

3 needles: src, aux, dest.

64 disks of decreasing sizes stacked on the  
of src needle.  
↳ variable. (n)

Goal: Move all disks from src to dest  
(preserve the order)

Rules: ① Only one disk can be moved at a time.  
② A larger disk can not be stacked above a  
smaller disk.

How many (minimum) number of moves needed?  
 $O(2^n)$

Case I:  $n = 1$

$$f(n) = 1 \quad (1 \text{ step})$$

Case II:  $n = 2$

$$f(n) = 3 \quad (3 \text{ steps})$$

Case III:  $n = 3$

$$f(n) = 7 \quad \boxed{f(n) = 2^n - 1}$$

$n = 3$  Steps.

- ① Move 2 disks from src to aux.
- ② Move 1 disk from src to dest.
- ③ Move 2 disks from aux to dest.

General N:

- ① Move  $N-1$  disks from src to aux.
- ② Move 1 disk from src to dest
- ③ Move  $N-1$  disks " aux to dest.

$$f(N) = 2f(N-1) + 1$$

Assume that  $f(N-1) = 2^{N-1} - 1$

$$\text{Then, } f(N) = \cancel{2(2^{N-1} - 1)} + 1$$

$$= 2(2^{N-1} - 1) + 1$$

$$= 2^N - 2 + 1 = 2^N - 1$$

Recursive Sol<sup>n</sup>: - easier to code  
- also efficient [ $O(2^N) = \# \text{ steps}$ ]

Algo. Design.

Towers.

arguments.

1) # disks.

2) src.

3) dest.

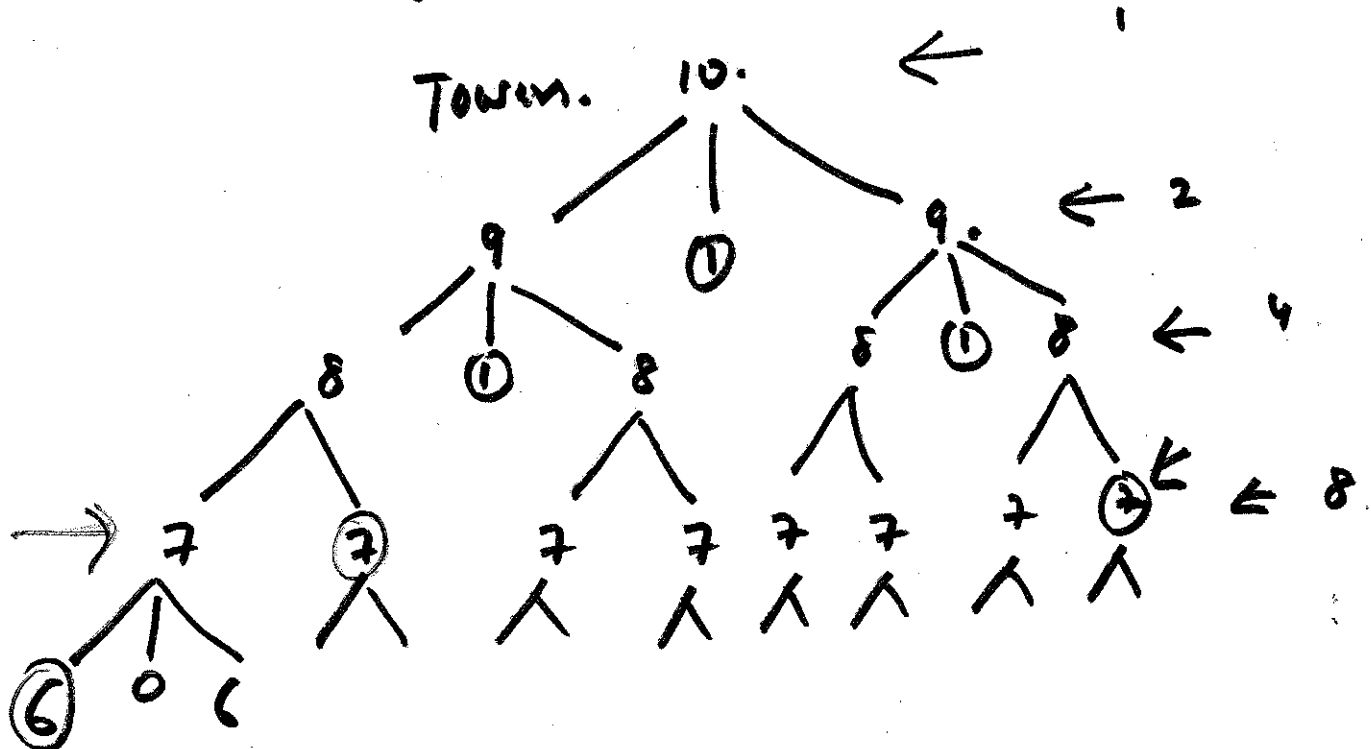
4) aux.

Towers (N, src, dest, aux)

① Towers (N-1, src, aux, dest)

② Move 1 disk from src to dest. [Base case]  
[Towers(1, src, dest, aux)]  $\leftarrow$  X.

③ Towers (N-1, aux, dest, src)



# Stacks. (Ch. 3)

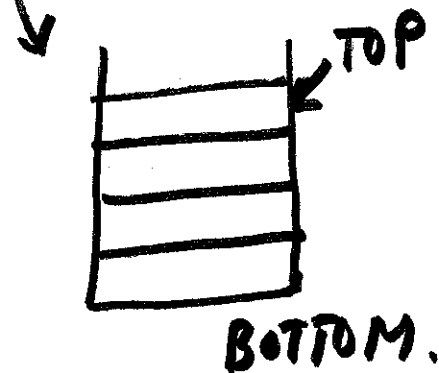
## Linear lists.

In a linear list, each element has a unique successor.

General list - data (element-) can be inserted or deleted anywhere in the list.

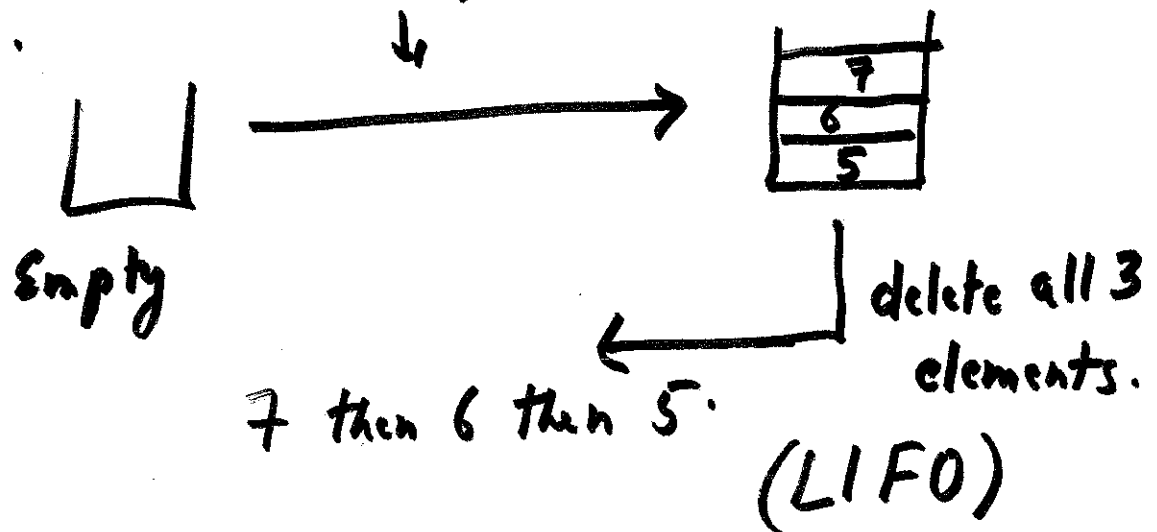
Restricted list - data can only be inserted or deleted at the ends of the list.

Stacks (LIFO)      Queues (FIFO)

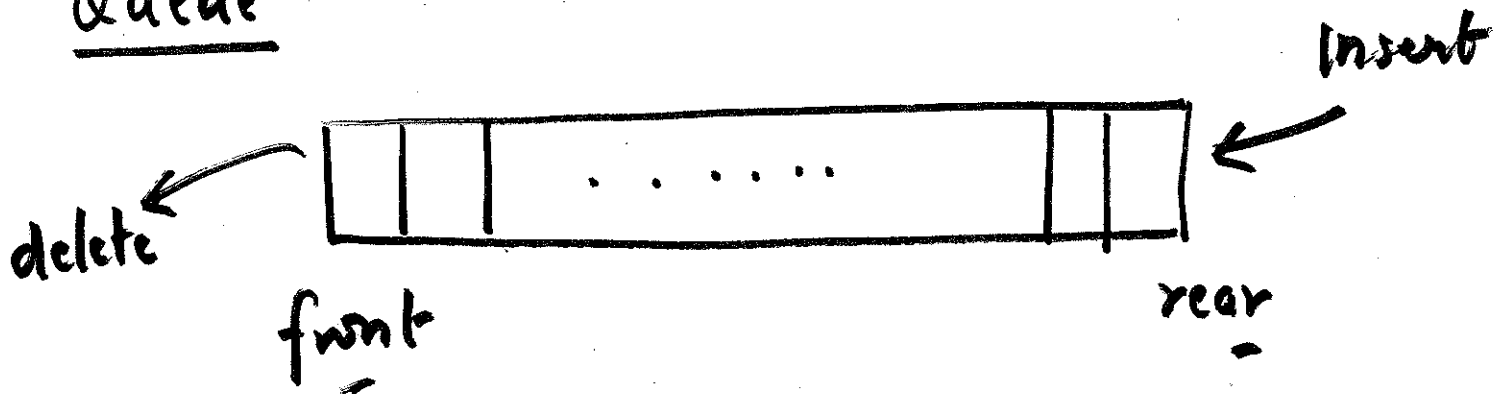


Insert & delete allowed from the TOP only.

Insert 5, then 6, then 7.



## Queue



Insert 5, then 6, then 7



FIFO

delete all 3.

5 then 6 then 7

(Queue Empty)

## DeQueue: (Double Ended Queue)

↳ allows insertion and deletion at both ends.

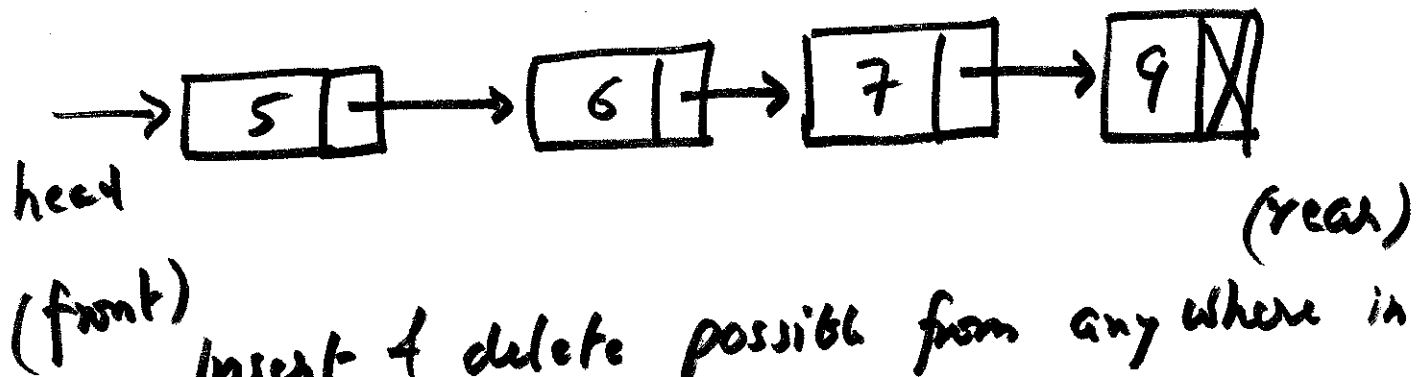
Insert 5, then 6, then 7.



delete all 3

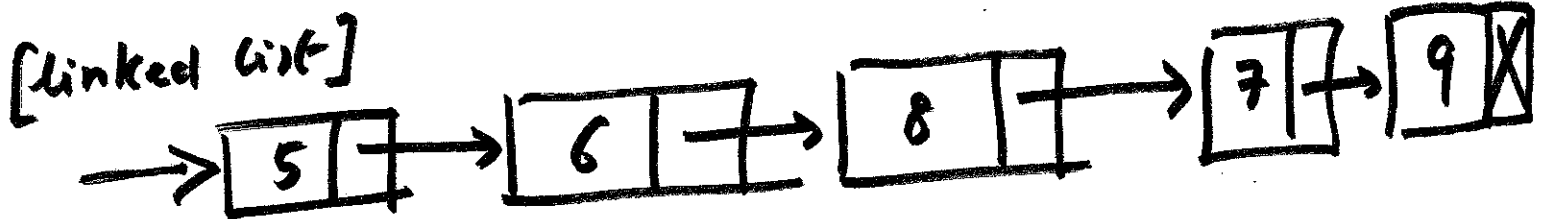
↳ 5 7 6 (feasible)

## General List:-



Insert & delete possible from anywhere in the list.

Insert 8 bet<sup>n</sup> 6 and 7



## Stacks:

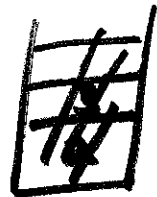
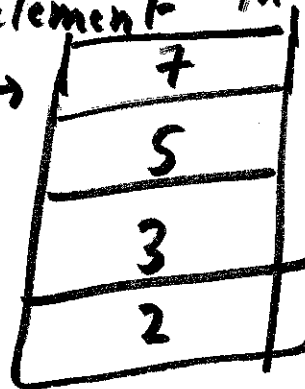
Q. How to remove an element in the middle!

Remove 3.

delete → 7 → temp.

delete → 5 → storage.

delete → 3 → use it.



insert back on stack.

## Basic Operations:

PUSH, POP, Stack TOP, ....