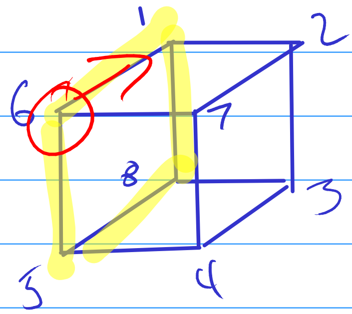
Math 322



U
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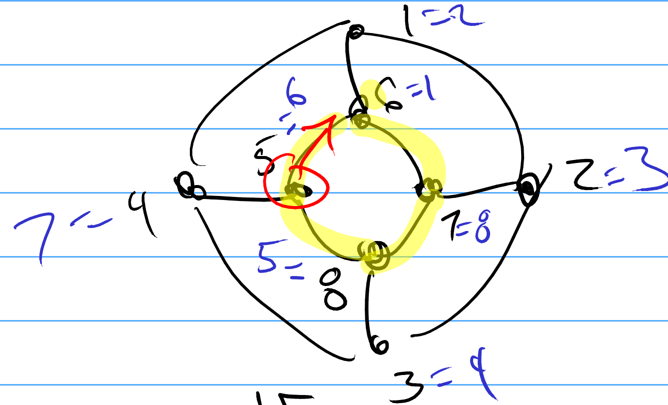

V
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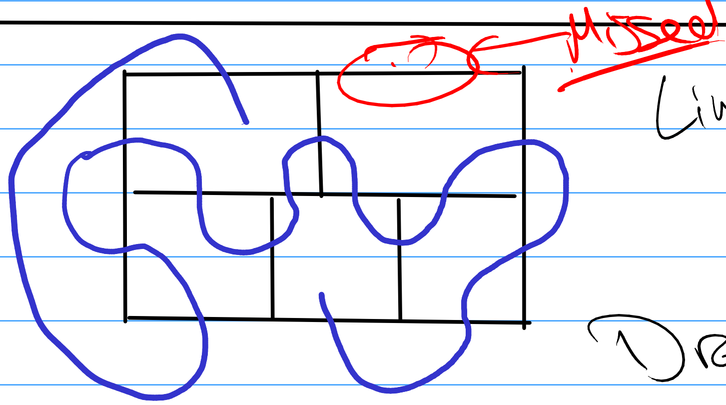
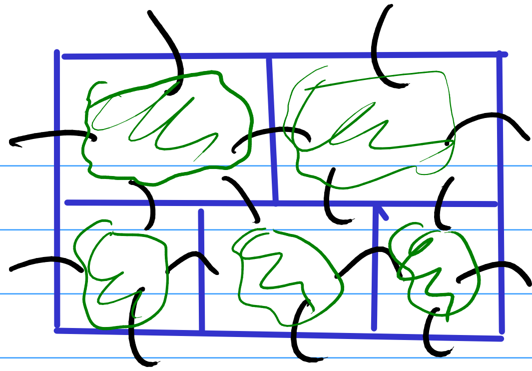


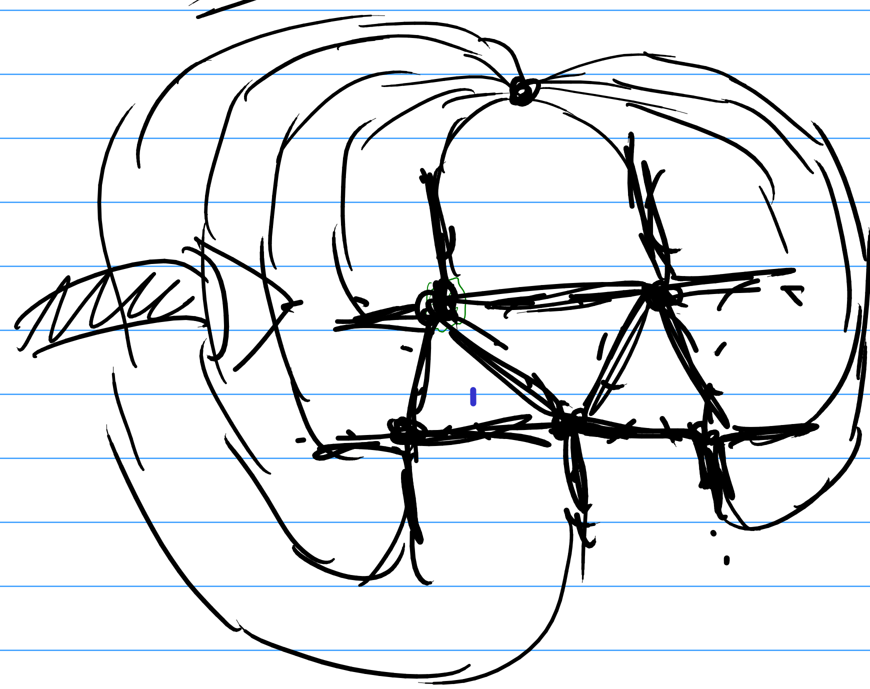
Line Segment Puzzle

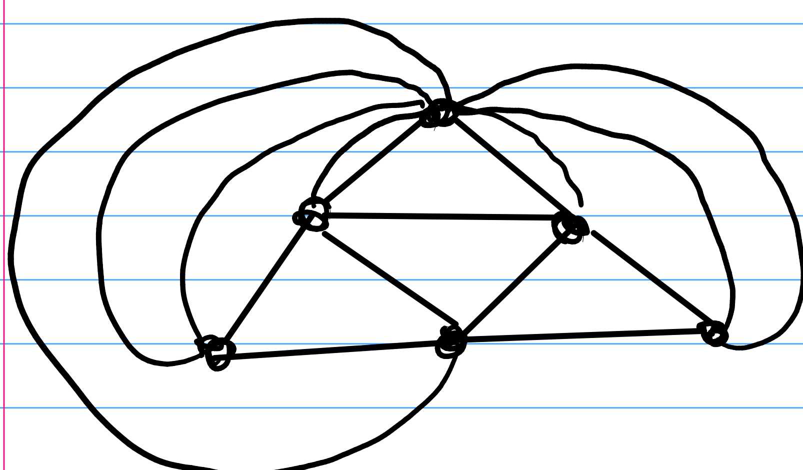
Draw a continuous curve that crosses each segment once.

Bridge Problem: cross each bridge exactly once.



Segment Puzzle = Bridge Puzzle





this graph is the same problem.

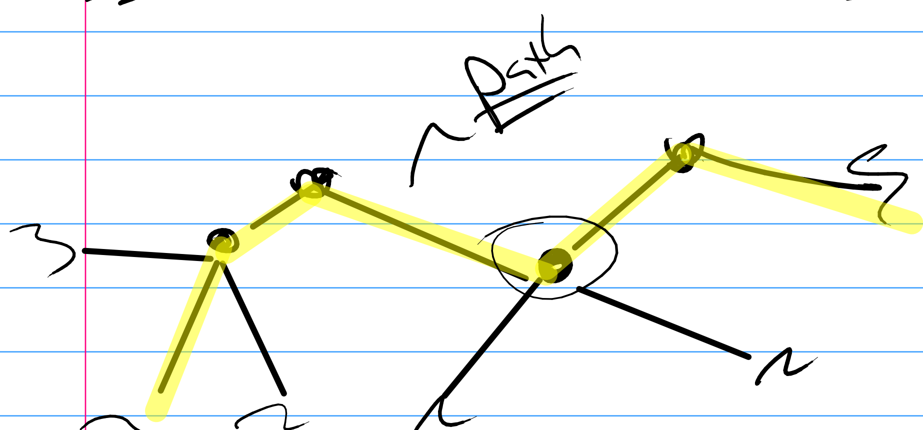
circuit? is there a simple path that has every edge in it?

☆ **Euler Circuit**: simple circuit that contains every edge in E.

Euler Path: simple path that is not a circuit that contains every edge in E.

Euler Circuit:

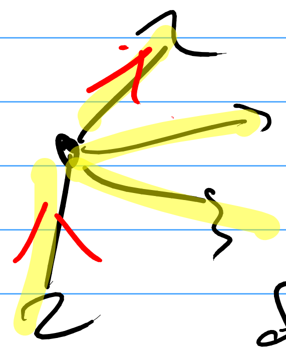
Want: Euler circuit iff (something you can check)



Consider a part of a graph

Notice a path that visits and leaves a vertex it adds 2 to v's degree.

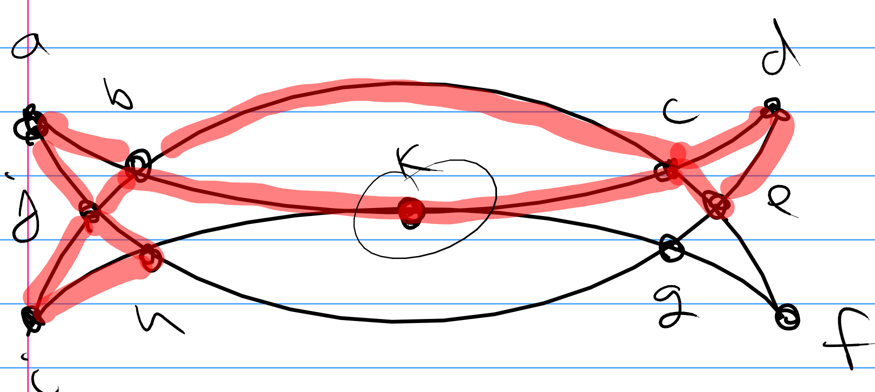
If G had an Euler Circuit



B/c the path always adds 2 to the degree of any v...

Euler Circuit → all even degrees.

← _ _ ? is it biconditional?

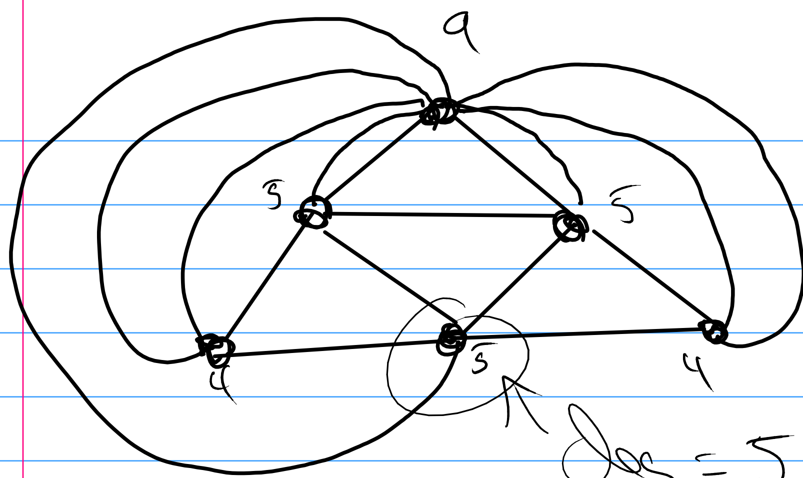


a b j a

a b c d e c k b j a

a b c d e c k b j i h j a

h g f e g k h



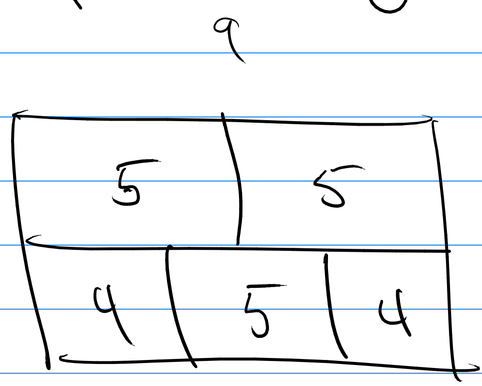
Line Segment Puzzle as a graph

deg = 5

no Euler Circuit

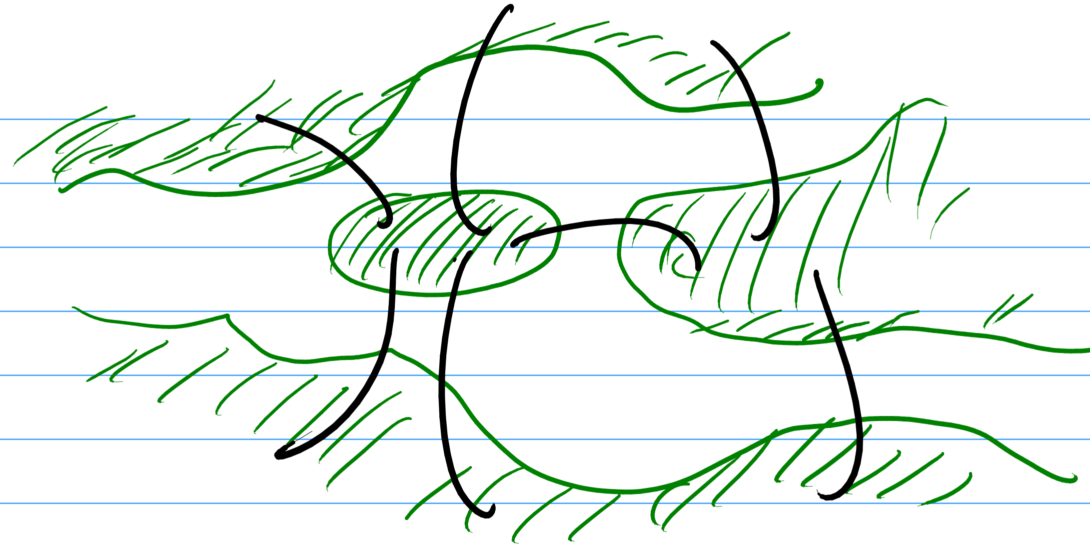
Thm: Euler circuit **iff** all deg(v) are even.

Thm: Euler path (not circuit) iff exactly two odd vertices (path begins and ends @ them)



No Euler Path.

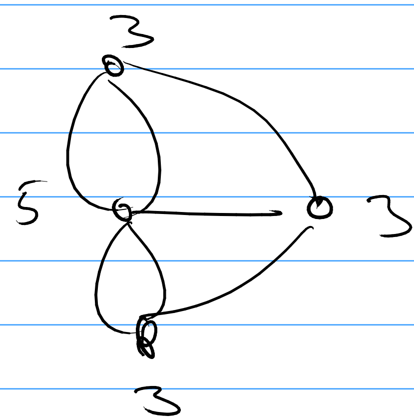
No Solution.



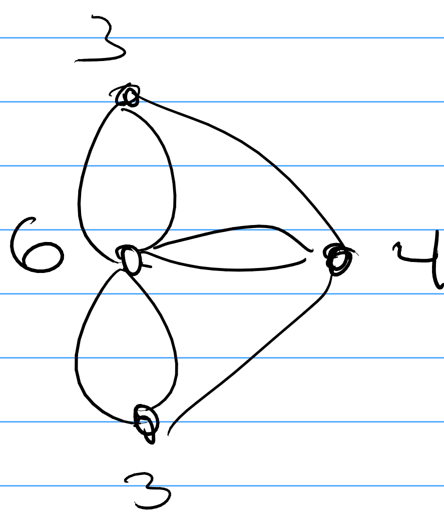
No Euler Circuit

No Euler Path

Graph version

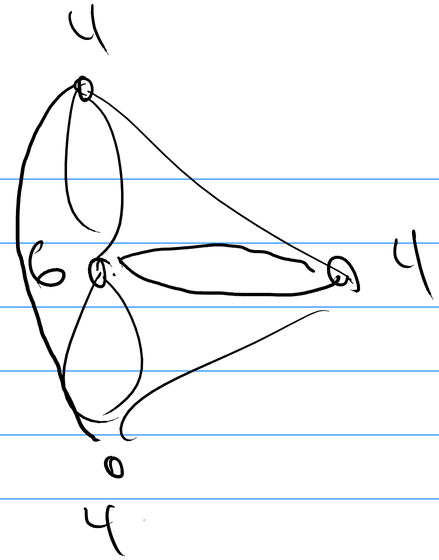


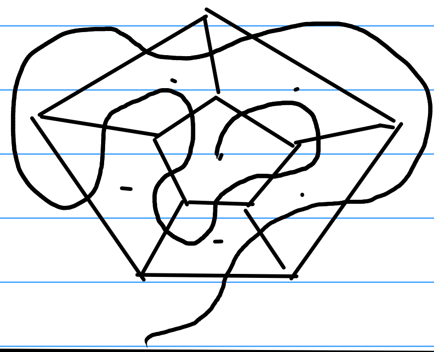
Variation:



Has Euler path

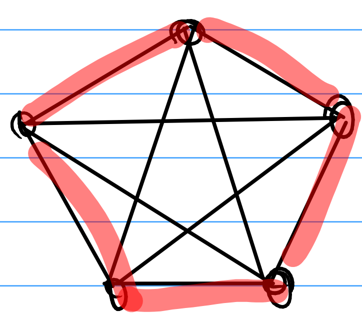
Has Euler circuit

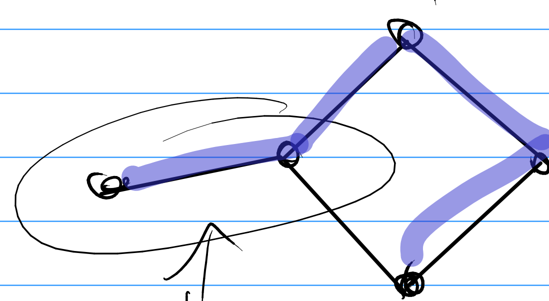




Hamilton Circuit / Path

Simple circuit / path that visits each vertex exactly once.





problem: no Hamilton circuit.

Does have a Hamilton path.

Thm: Dirac's thm ($n \geq 3$)

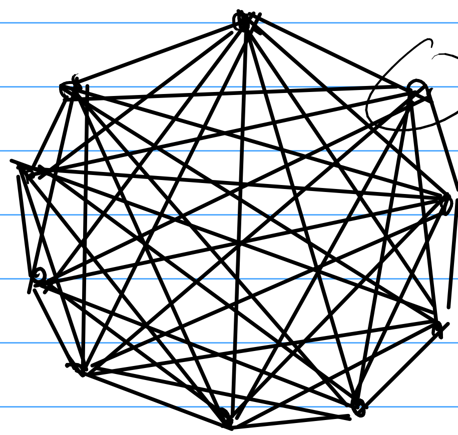
if $\forall v \; \deg(v) \geq n/2 \rightarrow$ Hamilton circuit.

Thm: Ore's thm ($n \geq 3$)

if $\forall v \; \forall u$ non-adj.

$\deg(u) + \deg(v) \geq n \rightarrow$ Hamilton circuit.

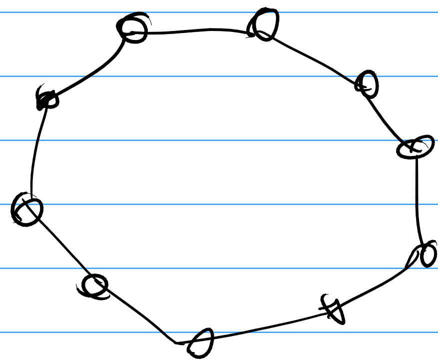
ex K_{10}



$\deg(v) = 9$

Has Ham. Circ. by Dirac's.

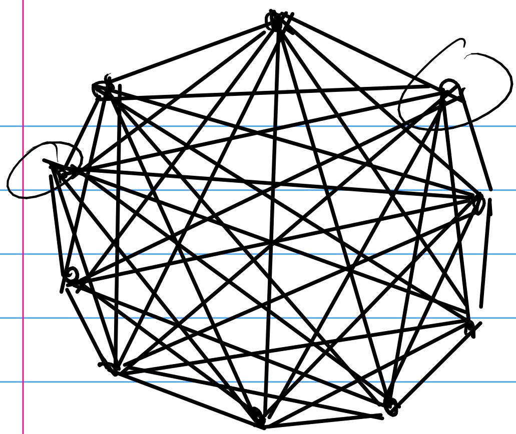
C_{10}



Dirac's Does not apply

but it does have a Ham. Circ.

#1



K_{10} (Complete Graph)

Travel. Salesman.

$$(9)(8)(7)\cdots(1) = 9!$$