Math 321

Extra Credit Proofs (http://chaos.math.wichita.edu/)

"Due" by Dec. 15 (Final)

Block 3 (Due by Final Exam)

1) Find a recurrence relation for the number of ways to parenthesize the n products of $n+1$ numbers.

2) Prove the Fundamental Theorem of Arithmetic.

3) Prove there are infinitely many primes.

4) Prove Pascal's Identity.

5) How many possible values to m and n are there so that 18^{10} is the least common multiple of m, n, and 6^{10} where m and n are distinct.

Q's?

S_1 costs 1 micro sec

S_2 costs 2 micro sec.

7.2 #5

7.1 #35

1st

$t_n = t_{\text{before}} + \text{signal}$

$t_n = t_{n-1}$ and 1 (got S_1 right before)

or

$t_n = t_{n-2}$ and 2 (got S_2 right befor)

$$t_n = (t_{n-1})(1) + (t_{n-2})(1)$$

$$t_n = t_{n-1} + t_{n-2} \quad n \geq 3$$

$$t_0 = 1 \quad t_1 = 1 \quad t_2 = 2$$

You could just look this up.

Solve: $t_n = t_{n-1} + t_{n-2}$

$$r^2 - r - 1 = 0$$

$$r_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot -1}}{2 \cdot 1}$$

$$r_1 = \frac{1+\sqrt{5}}{2} \qquad r_2 = \frac{1-\sqrt{5}}{2}$$

$$t_n = a\left(\frac{1+\sqrt{5}}{2}\right)^n + b\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Use: $t_0 = 1$ & $t_1 = 1$

$$1 = a + b \rightarrow a = 1 - b$$

$$1 = a\left(\frac{1+\sqrt{5}}{2}\right) + b\left(\frac{1-\sqrt{5}}{2}\right)$$

$$2 = a(1+\sqrt{5}) + b(1-\sqrt{5})$$

$$2 = (1-b)(1+\sqrt{5}) + b(1-\sqrt{5})$$

$$2 = (1+\sqrt{5}) - b(\cancel{1}+\sqrt{5}) + b(\cancel{1}-\sqrt{5})$$

$$1 - \sqrt{5} = -2\sqrt{5}\, b$$

$$b = -\frac{1-\sqrt{5}}{2\sqrt{5}}$$

$$a = 1 - b = \frac{2\sqrt{5} + 1 - \sqrt{5}}{2\sqrt{5}}$$

$$a = \frac{1+\sqrt{5}}{2\sqrt{5}}$$

$$t_n = \frac{1+\sqrt{5}}{2\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1-\sqrt{5}}{2\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Note:

$$\frac{1+\sqrt{5}}{2} \qquad\qquad \frac{1-\sqrt{5}}{2}$$

$$\sim \left(\frac{3}{2}\right)^n \xrightarrow{n\to\infty} \infty \qquad\qquad \sim \left(-\frac{1}{2}\right)^n \xrightarrow{n\to\infty} 0$$

7.2 #9

	Jan 1		Dec 31
Year 1	100,000	$\longrightarrow$	(+) $A_1 = .20(100,000)$
			(+) $A_2 = .45(0)$
Year 2	120,000	$\dashrightarrow$	(+) $A_1 = .20(120,000)$
			(+) $A_2 = .45(100,000)$

$$P_n = P_{n-1} + .20(P_{n-1}) + .45(P_{n-2})$$

$$P_n = 1.2(P_{n-1}) + .45(P_{n-2})$$

$$P_0 = 100{,}000 \qquad P_1 = 120{,}000$$

$$r^2 - 1.2r - .45 = 0$$

$$r = \frac{1.2 \pm \sqrt{1.44 + 4(.45)}}{2} \leftarrow r_1 \,\&\, r_2$$

$$P_n = a r_1^n + b r_2^n$$

$P_0 = 100{,}000$ $\qquad 100{,}000 = a + b$

$P_1 = 120{,}000$ $\qquad 120{,}000 = r_1 a + r_2 b$

Solve!

Exam 4 $\quad$ 10 probs + 1 ec.

5.1 (3 probs)

3 applications of
1. sum rule
2. prod. rule
3. over counting (sum-Rule)

mixed.

5.2 (1 prob)

Word Problem application of
Gen. Pigeonhole Principle
(State it!)

5.3 (1 prob)

Word problem using $C(n,r)$ and/or $P(n,r)$

5.4 (1 prob)

① Combinatorical Proof of Pascal's Identity

or

① Application of binomial thm.

ex $(x^2 - \sqrt{x})^{17}$ what is the 13th term?

7.1 (2 probs)

① Solve for a_n using iteration.

② Find rec. relation.

ex $1 bill $1 coin $2 bill $5 bill

a_n is a_{n-1} and $1 amount

$(a_{n-1})(2)$

or a_n is a_{n-2} and \$2 amount

$(a_{n-2})(1)$

or a_n is a_{n-5} and \$5 amount

$(a_{n-5})(1)$

$\to a_n = 2a_{n-1} + a_{n-2} + a_{n-5}$

$a_0 = 1 \quad a_1 = 2 \quad a_2 = 5 \quad a_3 = 10 \quad a_4 = ?$

7.2 (2 probs)

(1) no mult. and you solve it.

ex $\boxed{a_n = 2a_{n-1} + 3a_{n-2} \quad a_0 = 0 \quad a_1 = 5}$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

$r = 3 \quad r = -1$

$a_n = a\,3^n + b\,(-1)^n$

(2) Find a_n if you have mult. roots.
(not an initial value prob.)

$r^5 \sim \quad = 0$

r_1, r_2, r_3

$\nwarrow$ mult 3

$$a_n = a r_1^n + b r_2^n + (c + dn + en^2) r_3^n$$

extra credit?

prove $\sqrt{2}$ is irrational.