Math 243

Q's / 10.4 #5 $\quad \vec{i}+\vec{j}+\vec{k}, \quad 6\vec{i}+\vec{k}$

$$\langle 1,1,1\rangle \times \langle 6,0,1\rangle$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 6 & 0 & 1 \end{vmatrix} = \vec{i}\cdot 1 - \vec{j}\cdot(-5) + \vec{k}\cdot(-6)$$

$$= \langle 1, 5, -6\rangle$$

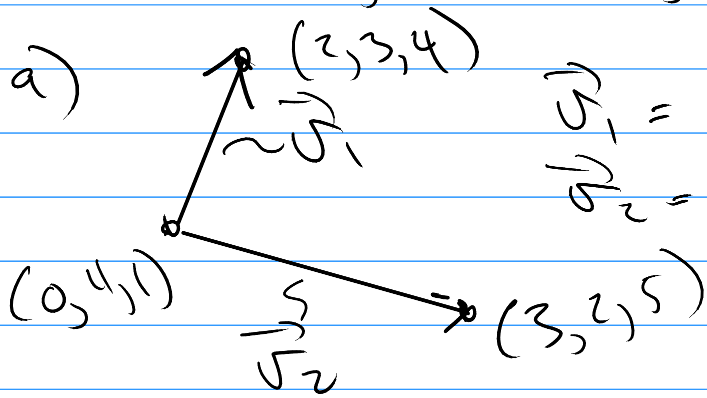
$$= \vec{i} + 5\vec{j} - 6\vec{k}$$

1st unit vector $= \frac{1}{\sqrt{62}}\langle 1, 5, -6\rangle$

2nd unit vector $= \frac{-1}{\sqrt{62}}\langle 1, 5, -6\rangle$

10.4 #7 $\quad (2,3,4) \quad (0,4,1) \quad (3,2,5)$

a)

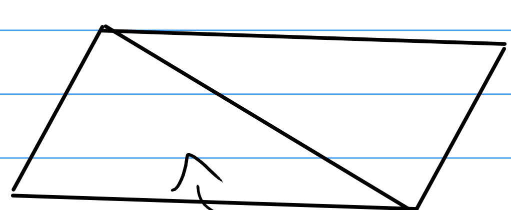


$\vec{v}_1 = \langle 2,-1,3\rangle$

$\vec{v}_2 = \langle 3,-2,4\rangle$

$$\vec{v} = \vec{v}_1 \times \vec{v}_2$$

b)



$\frac{1}{2}|\vec{v}_1 \times \vec{v}_2| =$

Exam (ch10) 12 probs

Open 11pm Tues. } in 105 mins
close 11am Thurs

1. } spheres → find equations
2.
3. Vector ops
4. Vector word problem
5. dot product (do it)
6. use dot product
7. cross product (do it)
8. cross product word problem
9. eqn of line } find
10. eqn of plane
11. } surface?
12.

Integration

1. $(n \neq -1)$ $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

2. $(n = -1)$ $\int \frac{1}{x} dx = \ln|x| + C$

3. $\int e^x dx = e^x + C$

4. $\int a^x dx = \frac{1}{\ln a} a^x + C$

trig

5 $\int \cos x\,dx = \sin x + C$

etc.

$\int \tan x\,dx = ?$

Substitution

chain rule $D_x[f(g(x))] = f'(g(x)) \cdot g'(x)$

$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$

let $u = g(x)$

$du = \boxed{g'(x)\,dx}$

$\to \int f'(u)\,du$

$= f(u) + C$

$= f(g(x)) + C$

$\int \tan x\,dx = \int \frac{\sin x}{\underbrace{\cos x}_{u}}\,dx = \int -\frac{1}{u}\,du$

$\sin x\,dx = -du$

let $u = \cos x$

$du = -\sin x\,dx$

$= -\ln|u| + C$

$= \boxed{-\ln|\cos x| + C}$

$= \boxed{\ln|\sec x| + C}$

$$\int \sec x \, dx = \int \sec x \frac{(\sec x + \tan x)}{(\sec x + \tan x)} dx$$

$$= \int \frac{(\sec^2 x + \sec x \tan x)}{\sec x + \tan x} dx = \int \frac{1}{u} du$$

$u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x) dx$

$= \ln |u| + C$

$= \ln | \sec x + \tan x | + C$

$\int \sinh x \, dx = \cosh x + C$

$\int \cosh x \, dx = \sinh x + C$

$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

$\boxed{6.1}$ Product Rule

$$D_x[f(x)\cdot g(x)] = f'(x)\,g(x) + f(x)\cdot g'(x)$$

$$\int (f'\cdot g + f\cdot g')\,dx = f\cdot g + C$$

$$\int f' g\,dx + \int f\cdot g'\,dx = f\cdot g$$

$$\int f\cdot g'\,dx = \boxed{f\cdot g - \int f'\cdot g\,dx}$$

$$\boxed{\int f(x)\cdot g'(x)\,dx = f(x)\,g(x) - \int g(x)\cdot f'(x)\,dx}$$

Integrate by parts

let $u = f(x) \xrightarrow{\text{deriv.}} du = f'(x)\,dx$

$dv = g'(x)\,dx \xrightarrow[\text{Integral}]{} v = g(x)$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

ex $\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx$

let $u = \ln x \longrightarrow du = \frac{1}{x} dx$

$dv = 1 \cdot dx \longrightarrow v = x$

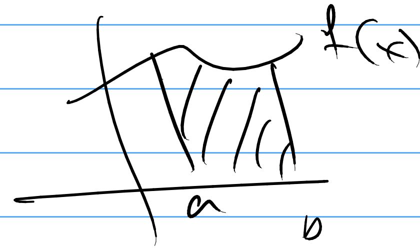
$\int \ln x \, dx = x \ln x - \int dx$

$\int \ln x \, dx = x \ln x - x + C$

Definite Integrals

$$\int f(x) \, dx = A_x[f(x)]$$

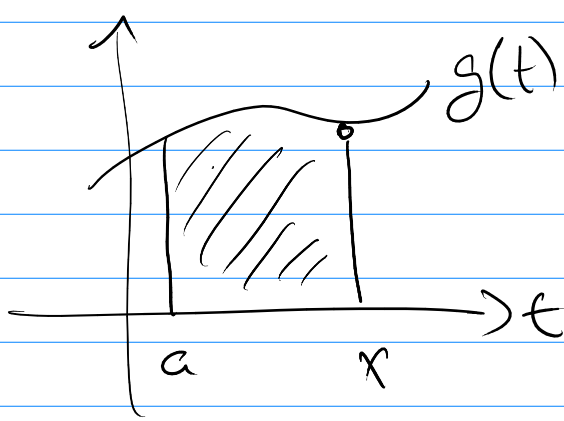
$\int_a^b f(x) \, dx$ = area



2nd Fund. Th^m

$$\int_a^b f(x) \, dx = \left[\int f(x) \, dx \right] \Big|_a^b = F(b) - F(a)$$

1st Fund. Th^m



$$\boxed{f(x) = \int_a^x g(t) \, dt}$$

$$f'(x) = g(x)$$

$\underline{\text{Substitution:}}$ $\displaystyle\int_{x=a}^{x=b} f(g(x))\,g'(x)\,dx$

let $u = g(x)$ $\quad x=a \rightarrow u=g(a)$

$du = g'(x)dx$ $\quad x=b \rightarrow u=g(b)$

$$= \boxed{\int_{u=g(a)}^{u=g(b)} f(u)\,du}$$

by Parts

$$\int_{x=a}^{x=b} u\,dv = u \cdot v \Big|_{x=a}^{x=b} - \int_{x=a}^{x=b} v\,du$$

6.1 #14 $\displaystyle\int e^{-\theta} \cdot \cos 2\theta \, d\theta$

let $u = \cos 2\theta \rightarrow du = -2\sin 2\theta$

$dv = e^{-\theta} d\theta \rightarrow v = -e^{-\theta}$

$$\int = -e^{-\theta}\cos 2\theta - 2\underbrace{\int e^{-\theta} \sin 2\theta \, d\theta}$$

$u = \sin 2\theta \rightarrow du = 2\cos 2\theta$

$dv = e^{-\theta} d\theta \rightarrow v = -e^{-\theta}$

$$= -e^{-\theta}\cos 2\theta \; -2\left[-e^{-\theta}\sin 2\theta + 2\int e^{-\theta}\cos 2\theta \, d\theta\right]$$

$$\overbrace{\int e^{-\theta}\cos 2\theta\, d\theta}^{I} = -e^{-\theta}\cos 2\theta + 2e^{-\theta}\sin 2\theta - 4\overbrace{\int e^{-\theta}\cos 2\theta\, d\theta}^{I}$$

$$5\int e^{-\theta}\cos 2\theta\, d\theta = -e^{-\theta}\cos 2\theta + 2e^{-\theta}\sin 2\theta + C$$

$$\boxed{\int e^{-\theta}\cos 2\theta\, d\theta = \frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C}$$

6.1 #27

$$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\, d\theta = \int_{\pi/2}^{\pi} \theta^{\cancel{3}\,2}\cos u \,\frac{du}{2\cancel{\theta}}$$

$\theta^2 = u$

let $u = \theta^2$ $\qquad \theta = \sqrt{\pi/2} \to u = \pi/2$

$\boxed{du = 2\theta\, d\theta}$ $\qquad \theta = \sqrt{\pi} \to u = \pi$

$d\theta = \dfrac{du}{2\theta}$

$$= \frac{1}{2}\int_{u=\pi/2}^{u=\pi} u \cos u\, du = \frac{1}{2}\left[u\sin u\Big|_{\pi/2}^{\pi} - \int_{\pi/2}^{\pi} \sin u\, du\right]$$

$$M = u \longrightarrow dM = du$$

$$dv = \cos u\, du \longrightarrow v = \sin u$$

$$= \frac{1}{2}\left[u\sin u\Big|_{\pi/2}^{\pi} + \cos u\Big|_{\pi/2}^{\pi}\right]$$

$= \underline{\text{etc}}$ . .

6.1 #35

$$\boxed{\int (x^2+a^2)^n\,dx}$$

$$= \boxed{\frac{x(x^2+a^2)^n}{2n+1}} + \frac{2na^2}{2n+1}\int (x^2+a^2)^{n-1}\,dx$$

$$\int (x^2+a^2)^n\,dx$$

$$\int u\,dv = uv - \int v\,du$$

let $u = (x^2+a^2)^n \to du = n(x^2+a^2)^{n-1} \cdot 2x\,dx$

$dv = dx \to \boxed{v = x}$

$$\int (x^2+a^2)^n\,dx = x(x^2+a^2)^n - 2n\boxed{\int x^2 (x^2+a^2)^{n-1}\,dx}$$

Scratch

$$\int x^2 (x^2+a^2)^{n-1}\,dx = \int \cancel{x}^{\cancel{2}} u^{n-1} \frac{1}{2\cancel{x}}\,du$$

let $\boxed{u = x^2+a^2}$
$du = 2x\,dx$

$$= \frac{1}{2}\int \sqrt{u-a^2} \cdot u^{n-1}\,du$$

$\boxed{?}$

$$\int (x^2+a^2)^n \, dx = \int (x^2+a^2)(x^2+a^2)^{n-1} \, dx$$

$$\boxed{\int (x^2+a^2)^n \, dx} = \int x^2 (x^2+a^2)^{n-1} \, dx + a^2 \int (x^2+a^2)^{n-1} \, dx$$

from above $\quad \| \quad \leftarrow$ they are equal

$$\boxed{\int (x^2+a^2)^n \, dx} = x(x^2+a^2)^n - 2n \int x^2 (x^2+a^2)^{n-1} \, dx$$

$$a^2 \int (x^2+a^2)^{n-1} \, dx = x(x^2+a^2)^n - (2n+1) \int x^2 (x^2+a^2)^{n-1} \, dx$$

$$\frac{a^2}{2n+1} \int (x^2+a^2)^{n-1} \, dx = \frac{x(x^2+a^2)^n}{2n+1} - \int x^2 (x^2+a^2)^{n-1} \, dx$$

Finish!

6.2 Trig Integrals (+) Substitution.

1st know trig identities.

$\int \sin^3 x \cos x \, dx = \int u^3 \, du = \underline{\text{Finish}}$

let $u = \sin x$ $\quad du = \cos x \, dx$

sin/cos probs ex $\sin^n x \cos^m x$

(1) $\cos^m x$ is odd
→ pull out one $\cos x$
→ use $\cos^2 x = 1 - \sin^2 x$
→ let $u = \sin x$

(ex) $\int \sin^4 x \cos^3 x \, dx = \int \sin^4 x (\cos^2 x) \cos x \, dx$

$= \int \sin^4 x (1 - \sin^2 x) \cos x \, dx$

let $u = \sin x$
$du = \cos x \, dx$

$\int u^4 (1 - u^2) \, du = \int u^4 - u^6 \, du$

$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + C = \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$

(2) $\sin^n x$ is odd
→ pull one $\sin x$
→ $\sin^2 x = 1 - \cos^2 x$
→ let $u = \cos x$

(3) both even?

let $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin x \cos x = \frac{1}{2}\sin 2x$

$\tan x$ / $\sec x$?

(1) $\sec^n x$ n is even

→ pull $\sec^2 x$

→ use (for the rest of $\sec^n x$)

$\sec^2 x = 1 + \tan^2 x$

→ let $u = \tan x$

(2) $\tan^n x$ n is odd

→ pull $\sec x \tan x$

→ use $\tan^2 x = \sec^2 x - 1$

→ let $u = \sec x$

Others? be Creative

If you have $\sqrt{a^2-x^2}$ or $\sqrt{a^2+x^2}$

or $\sqrt{x^2-a^2}$

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \boxed{\sin^2\theta = 1 - \cos^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

ex $$\boxed{\sqrt{a^2 - x^2}} = \sqrt{a^2 - a^2\cos^2\theta} = \sqrt{a^2(1-\cos^2\theta)}$$

$$x = a\cos\theta \qquad = \boxed{a\sin\theta}$$

Trig Substitution

① see $\sqrt{a^2 - x^2}$ let $x = a\sin\theta$ $\theta \in [-\pi/2, \pi/2]$

② see $\sqrt{a^2 + x^2}$ let $x = a\tan\theta$ $\theta \in (-\pi/2, \pi/2)$

③ see $\sqrt{x^2 - a^2}$ let $x = a\sec\theta$

$$\theta = [0, \pi/2) \cup [\pi, 3\pi/2)$$

ex $$\int \frac{du}{u\sqrt{4-u^2}}$$ let $\boxed{u = 2\sin\theta}$

$$du = 2\cos\theta\, d\theta$$

$$\rightarrow \int \frac{2\cos\theta\, d\theta}{2\sin\theta\sqrt{4 - 4\sin^2\theta}} = \int \frac{\cancel{2\cos\theta}}{2\sin\theta\ \cancel{2\cos\theta}}\, d\theta$$

$$= \frac{1}{2}\int \csc\theta \, d\theta$$

$$= \frac{1}{2}\int \frac{\csc\theta\,(\csc\theta + \cot\theta)}{(\csc\theta + \cot\theta)}\, d\theta$$

$$= \frac{1}{2}\int \frac{\csc^2\theta + \csc\theta\cot\theta}{\csc\theta + \cot\theta}\, d\theta$$

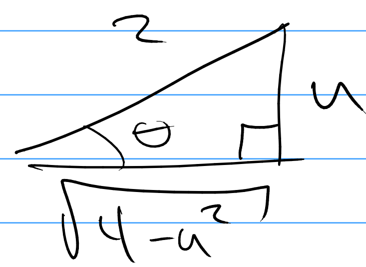
let $u = \csc\theta + \cot\theta$

$du = (-\csc\theta\cot\theta - \csc^2\theta)\,d\theta$

$$= -\frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|\csc\theta + \cot\theta| + C$$

$u = 2\sin\theta$

$\frac{u}{2} = \sin\theta$



$$= -\frac{1}{2}\ln\left|\frac{2}{u} + \frac{\sqrt{4-u^2}}{u}\right| + C$$

$$\int_0^2 x^3\sqrt{x^2+4}\,dx$$

let $x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$$\int_{x=0}^{x=2} 8\tan^3\theta\sqrt{4\tan^2\theta+4}\;2\sec^2\theta\,d\theta$$

$\theta \in (-\pi/2, \pi/2)$

$x=0 \quad 0 = 2\tan\theta \rightarrow \theta = 0$

$x=2 \quad 2 = 2\tan\theta \rightarrow \theta = \pi/4$

$$16\int_0^{\pi/4} \tan^3\theta \left(2\sec\theta\right)\sec^2\theta \, d\theta$$

$$32\int_0^{\pi/4} \tan^3\theta \sec^3\theta \, d\theta$$

$$32\int_0^{\pi/4} \tan^2\theta \boxed{\sec^2\theta} \left(\sec\theta\tan\theta \, d\theta\right)$$

let $u = \sec\theta$ $\qquad \frac{1}{\cos\theta}$

$du = \sec\theta\tan\theta \, d\theta$

$$32\int_0^{\pi/4} \left(\sec^2\theta - 1\right)\sec^2\theta \left(\sec\theta\tan\theta \, d\theta\right)$$

$$32\int_1^{\sqrt{2}} \left(u^2-1\right)u^2 \, du = \text{Finish!}$$