CIS 721 - Real-Time Systems
Homework #2 Solution
Fall 2015

Due: Friday, September 25, 2015, by 1:20 pm (turn in on paper)

Twenty-five points.

1. A system consists of four preemptive, periodic tasks, A, B, C, and D, assigned static priorities using the Rate Monotonic (RM) algorithm, with the following properties:

Task	Run Time (C_i)	Period (T_i)
A	1	8 **(high)**
B	2	16
C	8	24
D	8	48 **(low)**

(a) What is the total utilization (U) of these four tasks?

U = 1/8 + 2/16 + 8/24 + 8/48 = 1/8 + 1/8 + 1/3 + 1/6 = 3/4 = 0.75

(b) Does Liu and Layland's Utilization-Based Test ensure that the task set is schedulable? What is the schedulable utilization for these n=4 tasks?

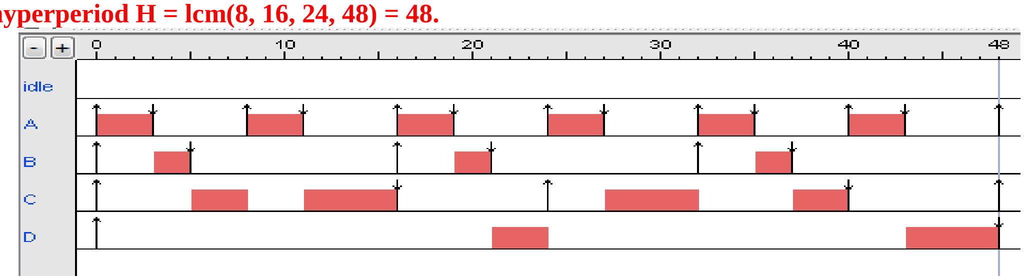
The number of tasks n = 4, so U_{RM} = 4 (2^1/4 – 1) = 0.7568, since U <= U_{RM}, the task set is schedulable.

(c) What if the run time of task A is increased to 3, is the task set still schedulable? Which test did you use? Show work.

U = 1.0, since U > U_{RM}, Liu and Layland's Test is inconclusive.

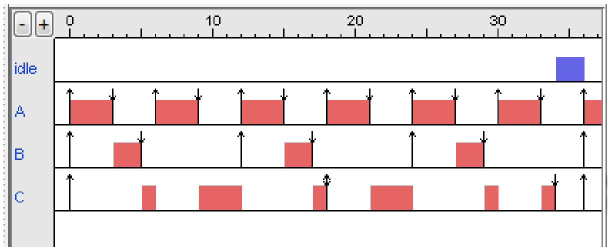
WCRT of task A: R_A = 3 <= 8 = D_A
WCRT of task B: $W_B(0)$ = C_B = 2, $W_B(1)$ = 2 + ceil($W_B(0)$/8) * 3 = 5,
$W_B(2)$ = 2 + ceil(5/8) * 3 = 5, so R_B = 5 <= 16 = D_B
WCRT of task C: $W_c(0)$ = C_c = 8, $W_c(1)$ = 8 + ceil($W_c(0)$/8) * 3 + ceil($W_c(0)$/16) * 2 = 13,
$W_C(2)$ = 8 + ceil(13/8) * 3 + ceil(13/16) * 2 = 16,
$W_C(3)$ = 8 + ceil(16/8) * 3 + ceil(16/16) * 2 = 16, so R_C = 16 <= 24 = D_C
WCRT of task D: $W_D(0)$ = C_D = 8, $W_D(1)$ = 8+ceil(8/8)*3+ceil(8/16)*2+ceil(8/24)*8 = 21,
$W_D(2)$ = 8+ceil(21/8)*3+ceil(21/16)*2+ceil(21/24)*8 = 29,
$W_D(3)$ = 8+ceil(29/8)*3+ceil(29/16)*2+ceil(29/24)*8 = 40,
$W_D(4)$ = 8+ceil(40/8)*3+ceil(40/16)*2+ceil(40/24)*8 = 45,
$W_D(5)$ = 8+ceil(45/8)*3+ceil(45/16)*2+ceil(45/24)*8 = 48,
$W_D(6)$ = 8+ceil(48/8)*3+ceil(48/16)*2+ceil(48/24)*8 = 48, so R_D = 48 <= 48 = D_D

(d) As in (c), if the run time of task A is 3, draw a Gannt Chart to show how these tasks would be scheduled using the rate monotonic scheduling algorithm in the interval [0, 48]. Does checking the tasks over this interval ensure that the task set is feasible? **Yes** If not, what is the minimum interval length that must be checked to ensure that the task set is schedulable? Explain briefly. **Since the tasks all have a common release time of 0, we only need to check over one hyperperiod H = lcm(8, 16, 24, 48) = 48.**



2. Which of the following task sets are schedulable if priorities are assigned using the rate-monotonic algorithm? Which of the following task sets are **simply periodic**? Justify your answer – specify which test(s) you used and show work.

(a) **Schedulable, Not Simply Periodic, apply Response Time Analysis, $R_A = 3$, $R_B = 5$, $R_C = 18$, or Leung's Test over one hyperperiod [0,36]:**

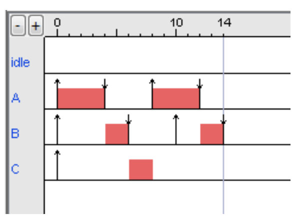


	Run Time (C_i)	Period (T_i)
A	3	6
B	2	12
C	5	18

(b) **Schedulable, Simply Periodic, U = 2/5 + 3/10 + 6/20 = 1.0 <= 1.0 via Utilization-Based Test.**

	Run Time (C_i)	Period (T_i)
A	2	5
B	3	10
C	6	20

(c) **Not Schedulable, Not Simply Periodic, Utilization-Based Test, U = 4/8 + 2/10 + 3/14 < 1.0, but $U > U_{RM}$ => inconclusive. Apply Response Time Analysis or Leung's Test:**



$WCRT_C$: $W_C(0) = 3$, $W_C(1) = 3 + ceil(3/8) * 4 + ceil(3/10) * 2 = 9$,
$W_C(2) = 3 + ceil(9/8) * 4 + ceil(9/10) * 2 = 13$,
$W_C(3) = 3 + ceil(13/8) * 4 + ceil(13/10) * 2 = 15 > D_C$ => not feasible.

	Run Time (C_i)	Period (T_i)
A	4	8
B	2	10
C	3	14

3. A set of n independent, preemptable periodic tasks with relative deadlines equal to their respective periods are schedulable rate-monotonically if their utilizations $u_1, u_2, \ldots, u_n$ satisfy the inequality $(1 + u_1)(1 + u_2) \ldots (1 + u_n) <= 2$.

(a) Show that this test is strictly stronger than Liu and Layland's Utilization-Based Test.

First, we need to show that it is as strong as Liu and Layland's Test, and then we just need a single example that satisfies the new test, but does not satisfy Liu and Layland's Test to show "strictly" stronger than. Such an example is shown below in 3(b), U = 0.84 > U_{RM} so it doesn't satisfy Liu and Layland's Test, but it does satisfy the new test and is consequently feasible.

So, assume that a given task set satisfies Liu and Layland's Test; e.g., $U <= U_{RM}$. Then, $U = u_1 + u_2 + .. + u_n <= n\ (2^{1/n} - 1)$.

We want to show that $(1 + u_1)(1 + u_2) \ldots (1 + u_n) <= 2$.

Since the sum, $u_1 + u_2 + .. + u_n$ equals some constant U, we know that the product $(1 + u_1)(1 + u_2) \ldots (1 + u_n) <= (1+U/n)(1+U/n) .. (1+U/n) = (1+U/n)^n$; that is, replacing each u_i with the average over all i will result in a product that is at least as large as the original.

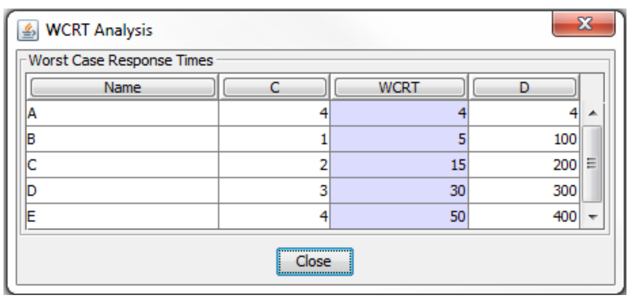
Since $U <= n\ (2^{1/n} - 1)$, $(1+U/n) <= (1 + n\ (2^{1/n} - 1)/n) = 1 + 2^{1/n} - 1 = 2^{1/n}$

So, $(1+U/n)^n <= (2^{1/n})^n = 2$. Finally, by the transitive property, we have the desired result $(1 + u_1)(1 + u_2) \ldots (1 + u_n) <= 2$.

(b) Apply both tests to a task set consisting of five periodic tasks with processor utilizations given by $u_1 = 0.8$, $u_2 = u_3 = u_4 = u_5 = 0.01$. What conclusions can be drawn?

$U = 0.84$, $U_{RM} = 5(2^{1/5} - 1) = 0.74349..$, so Liu and Layland's Test is INCONCLUSIVE. BUT,
$(1 + u_1)(1 + u_2) \ldots (1 + u_n) = 1.8*1.01*1.01*1.01*1.01 = 1.873087.. <= 2$, so by the new test, the task set is feasible!

NOT REQUIRED: Example feasible task set: (5, 4), (100, 1), (200, 2), (300, 3), (400, 4)


WCRT Analysis

Worst Case Response Times

Name	C	WCRT	D
A	4	4	4
B	1	5	100
C	2	15	200
D	3	30	300
E	4	50	400

Close