Final Exam

Exam 1 $\longrightarrow$ 4 probs

Exam 2 $\longrightarrow$ 4 probs

Exam 3 $\longrightarrow$ 4 probs

Exam 4 $\longrightarrow$ 4 probs

16

Exam 1

1a) Let p : "You have the flu virus", q : "You miss the first Exam", and r : "You fail Math 321". Express the $(p \wedge q) \to (\neg q \vee r)$ as an English sentence.

1b) Express "It is necessary for you to eat cheese for you to not have bad breath" using symbols and logical operators

5-i) Let $S(u, v)$ mean that "u slapped v" and let $M(r, t)$ mean that "r and y mash potatoes together". Where the domain for all variables is students in this class. Translate into English the following compound propositions.

a) $\forall x \exists y S(x, y)$

b) $\exists y \forall x S(x, y)$

1 symbols $\rightleftarrows$ english

ex p: Mark is dumb.

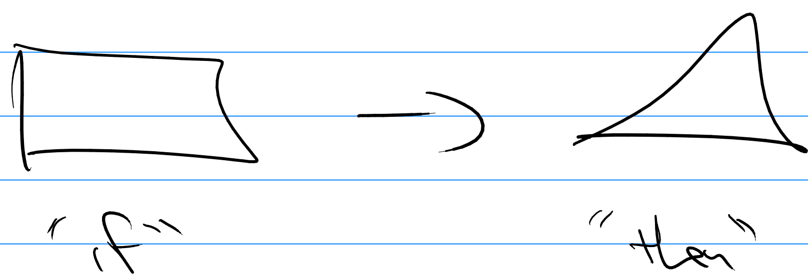
$F(x)$: x gets fired.
x from set of all teachers.

$$p \to F(\text{Mark}) \to \text{`Sentence'?}$$

ex If all dumb teachers get fired then Mark is fired.

$D(x)$: "x is dumb"
$F(x)$: "x gets fired"

$$[\forall x\,(D(x) \to F(x))] \to F(\text{Mark})$$



"if" — "suf."

"then" — "only if" — "nec."

2) Construct the truth table everyone should know.

operators: $\neg, \vee, \wedge, \oplus, \rightarrow, \leftrightarrow$

truth tables

1

3) Verify: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$ using a truth table.

$\leftrightarrow$

tautology.

like this?

two variable.

4) Show that the statement $[\neg p \wedge (p \vee q)] \rightarrow q$ is logically a tautology using logical equivalences. Bonus for 1 point ... What is the name of this Rule of Inference?

2.

Maybe.

$$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$$

$$\equiv \neg(\neg p) \wedge \neg q$$

$$\equiv p \wedge \neg q$$

Study tables 6, 7, 8 p. 24, 25

p. 41 table 2

6) Come up with three valid conclusions for the set of premises: "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."

Use rules of inference.

~~7) What rules of inference are used in this famous argument? "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."~~

probs 8-10 Proofs:

1. $\sqrt{2}$ is irrational ? Maybe ?

Types:

1. Contradiction
2. Direct Proof

Exam 2

Exam 2

1) Use a Venn diagram, set builder notation, and a list to illustrate the set of odd integers from 3 to 9, the set of all integers divisible by 3 from -3 to 12, among the universe of discourse of all integers.

2) For $A = \{-2, 1, 3, 5, 6, 7\}$, $B = \{-1, 0, 3, 4, 5\}$, and $U = \{i \mid i \epsilon \mathbb{Z} \wedge -5 \leq i < 10\}$ find ...

a) $A \cup B$

b) $A \cap B$

c) $A - B$

d) $\bar{A}$

3) Show $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by using a membership table and a Venn Diagram.

1-3 (from exam) → Set probs.

$\boxed{\text{1 prob from Sets}}$ → elements and/or Venn Diagram

4) a) Is $f(n) = \sqrt{n^2 - 2}$ a function from $\mathbb{Z}$ to $\mathbb{R}$?

b) If g and $f \circ g$ are onto, does it follow that f is onto? Justify your answer.

Functions:

1 prob.

5) Give an example of a function from $\mathbb{Z}$ to $\mathbb{N}$ that is ...

a) neither one-to-one nor onto.

b) one-to-one and onto.

Countability → prove $\mathbb{Q}$ is countable

→ prove $\mathbb{R}$ is not countable

Series: (1 prob)

Know:

① telescoping

② $\Sigma 1$, Σk, Σk^2

③ geometric

Matrices?. ~~None.~~

Exam 3

Number Theoretic?.

1) Show that if $a, b,$ and c are integers with $c \neq 0$, such that $ac \mid bc$, then $a \mid b$.

2) For $a = (2,0,2)_3$ and $b = (1,2)_3$.

Calculate $a + b$ and $a \cdot b$ using only base 4 notation.

3) State the three equivalent statements for $a \equiv b \pmod{m}$ and prove they are equivalent.

→ Know: divisible??.

congruent?.

~~base b~~?.

4) Prove there are infinitely many primes. ★ ← on test!.

5

7 gcd using Euclid's Alg.?.

~~6~~

8-10 Induction!

Strong? Weak? (Numeric Only)

ex: $1+2+\dots+n = \frac{n(n+1)}{2}$

Exam 4

~~1)~~ A brand of shirt comes in 7 colors for men in four sizes, 13 colors for women in five sizes, or 9 colors for children in six sizes. How many different types of shirt are made?

$7\cdot4 + 13\cdot5 + 9\cdot6$

2) How many strings of 5 digits {0,1,2,...,9} contain at least one 2?

$\boxed{S_1 S_2 S_3 S_4 S_5}$

2? 5?

all = no 2's + 1-2 + 2-2's + ... + 5-2's

$10^5 = 9^5 + \boxed{\text{at least } 1\text{-}2}$

$\boxed{10^5 - 9^5} =$

$\binom{5}{1}9^4 + \binom{5}{2}9^3 + \dots +$

3) How many positive integers between 5 and 51 are divisible by 2 or 3?

6 to 50 $\longrightarrow$ $50 - 6 + 1 = 45$

div by 2 $\quad \frac{45}{2} = 22.5 \rightarrow \underline{23}$

div by 3 $\quad \frac{45}{3} = 15$

div by 6 $\quad \frac{45}{6} = 7.5 \rightarrow \underline{8}$

$23 + 15 - 8 = \underline{30}$

? 4) State the generalized pigeonhole principle. Use it to find the minimum number of students who
0 ~~have to come to class to be sure that~~ at least five have the same grade in an A, B, C, D, and F grade system.

5

5

bleh (look it up)

$\left\lceil \frac{N}{5} \right\rceil = 5 \rightarrow \boxed{N = 21}$

2 5) Sixteen people on a baseball team show up for a game.

0 a) How many ways are there to choose 9 players to take the field? $\binom{16}{9} = \frac{16!}{9!\,7!}$

b) How many ways to assign the 9 positions? $P(16,9) = \frac{16!}{7!}$

c) Five of the sixteen players are Amish. How many ways to choose the 9 players if at least one of these players must be Amish?

all = $\boxed{0\text{-Amish}}$ + 1-Amish + ... + 5 Amish

$\boxed{\binom{16}{9} - \binom{11}{9}}$

6) Prove Pascal's Identity by using a combinatorical argument. Let n and k be positive integers with $n \geq k$. Then

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

n people choose K $= \binom{n}{K}$

given the n-people one is 'Mark'

$\Rightarrow$ this is two disjoint sets.

$\binom{n}{K} = \left(\text{with Mark} \right) + \left(\text{without Mark} \right)$

$$\binom{n}{K} = \binom{n-1}{K-1} + \binom{n-1}{K}$$

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5) $H_n = 2\boxed{H_{n-1}} + 1$ given $n \geq 2$, $H_1 = 1$

$2H_{n-2}+1$

$2H_{n-3}+1$

$$H_n = 2(2(2H_{n-3}+1)+1)+1$$

$$H_n = 2^3 H_{n-3} + 2^2 + 2 + 1$$

$\vdots$

$$H_n = 2^{n-1} \overset{1}{H_1} + 2^{n-2} + \dots + 2^2 + 2 + 1$$

$$H_n = \boxed{2^n - 1}$$

$a_n = 3 \cdot a_{n-1} + 1 \qquad a_1 = 1$

$= 3(3a_{n-2} + 1) + 1$

$= 3^2 a_{n-2} + 3 + 1$

$= 3^{\boxed{3}} a_{\boxed{n-3}} + 3^2 + 3 + 1$

$\vdots$

$= 3^{n-1} a_1 + 3^{n-2} + \cdots + 3^2 + 3 + 1$

$= \sum_{j=0}^{n-1} 3^j \qquad \sum_{j=0}^{M} a \cdot r^j = a\left(\frac{r^{M+1} - 1}{r - 1}\right)$

$= 1 \cdot \left(\frac{3^n - 1}{3 - 1}\right) = \frac{1}{2}(3^n - 1)$

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8) Find a recurrence relation for the number of ways to give someone n dollars if you have 1 dollar coins, 1 dollar bills, and 5 dollar bills. What are the initial conditions?

8 $a_n = 2a_{n-1} + a_{n-5}$

$a_0 = 1$

$a_1 = 2$ or

$a_2 = 2^2 = 4 \to 3$

$a_3 = 2^3 = 8 \to 4$

$a_4 = 2^4 = 16 \to 5$ 2_0

$d_0 \mid \underbrace{d_1 \mid d_2 \mid d_3}_{4} \mid d_4$

d d d – 1
d d c – 3
d c d – 3
d c c – 4
c d d – 3
c d c → 4
c c d → 4
c c c – 2

9 - 10 Look up.   7, 2

last prob

Solve rec. relation (given? like 9, 10
or
found? like 8.)

ex. $1 bill / coin $2 bill

$a_n = 2a_{n-1} + a_{n-2}$ $a_0 = 1$
$a_1 = 2$

Solve!

$r^2 - 2r - 1 = 0$

etc