Math 243

$\boxed{\text{Exam 2}}$ Ch. 6 (12 probs)

Tues 10pm – Thurs 7pm @ $\boxed{\text{105 mins to finish}}$

Integration by Parts (2)

Integrate Trig (1)

Trig. Substitution (1)

Partial Fraction (2)

Table of Integrals (2)

Mid pt. Approx (1)

Simpson's Approx (1)

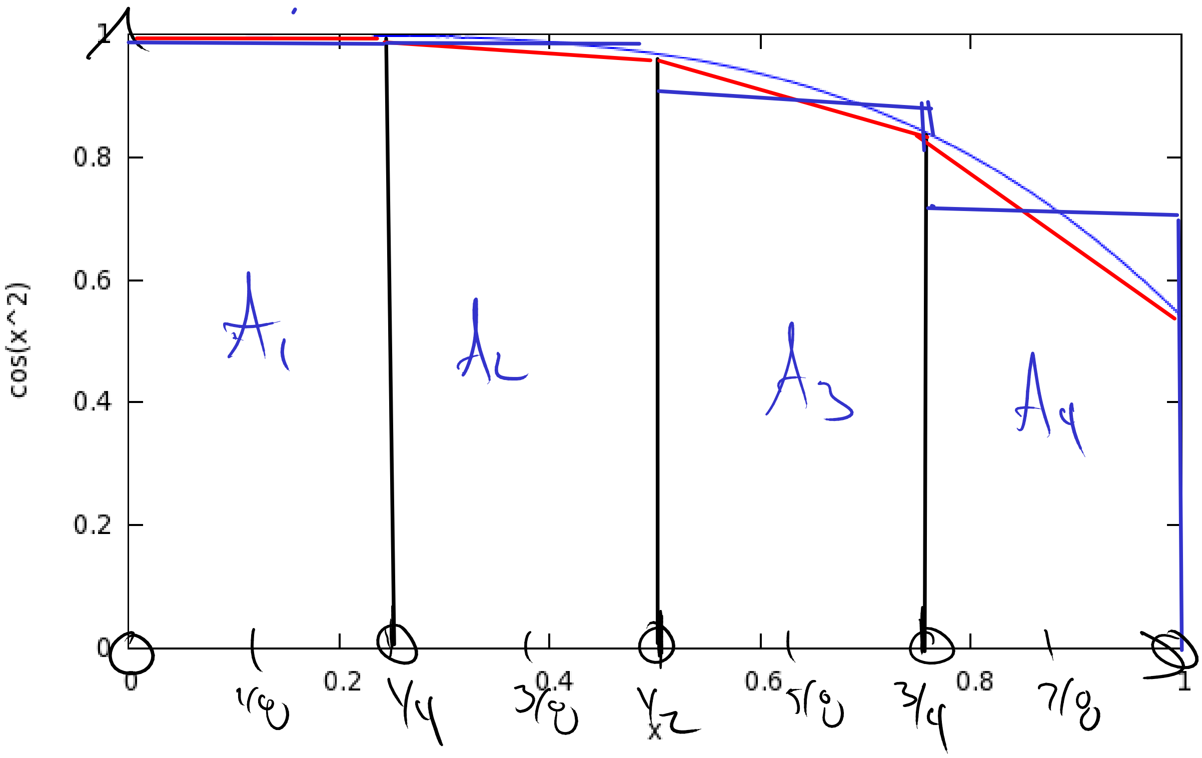
Type 1 Improper Integral (1)

Type 2 Improper Integral (1)

$\boxed{\#13 \quad \#14}$ ← extra credit (same type of problem)

$\boxed{\text{Q's}}$ 6.5 (#2 on webassign
#3 in text)

$\int_0^1 \cos(x^2)\,dx \qquad n=4$



$$A_M = (.25)\left(\cos\left(\tfrac{1}{8}^2\right) + \cos\left(\tfrac{3}{8}^2\right) + \cos\left(\tfrac{5}{8}^2\right) + \cos\left(\tfrac{7}{8}^2\right)\right)$$

$$A_T = \tfrac{1}{2}(.25)\Big(\cos(0^2) + 2\cos\left(\tfrac{1}{4}^2\right) + 2\cos\left(\tfrac{1}{2}^2\right)$$
$$+ 2\cos\left(\tfrac{3}{4}^2\right) + \cos(1^2)\Big)$$

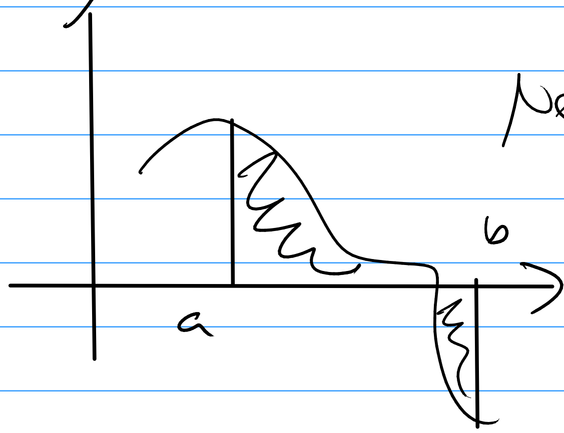
A_M is probably overestimating

A_T is underestimating.

6.6 Type 1 Improper Integral

(definate V.I.)

$\int_a^b f(x)\,dx \leftarrow$ definate integral



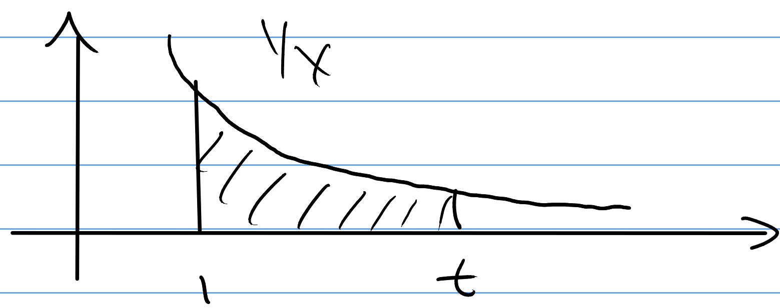
Net signed area

$= \int_a^b f(x)\,dx$

$= \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$

Type 1 what if a is ∞ and/or b is $-\infty$?

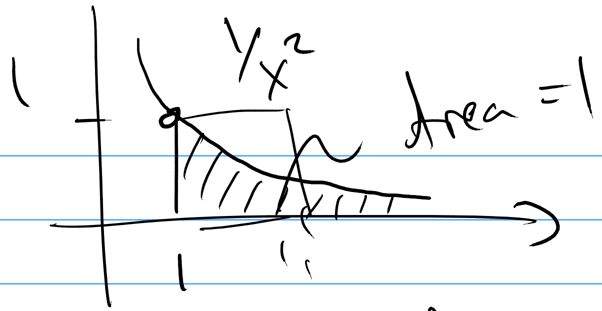
ex

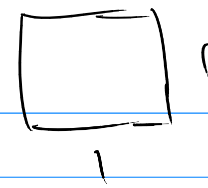


$\int_1^t \frac{1}{x}\,dx = \ln x \Big|_1^t = \ln t - \ln 1$

$= \ln t$

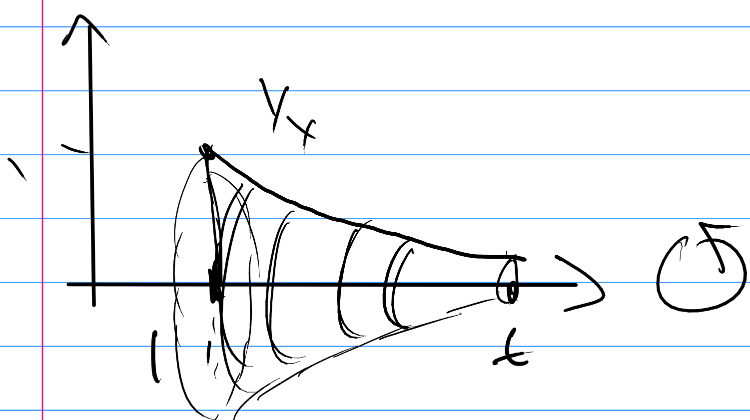
$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} \ln t = \infty$

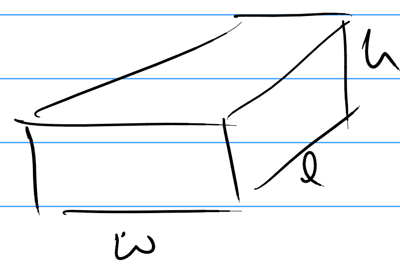




$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty}\int_1^t x^{-2}\,dx = \lim_{t\to\infty} -\frac{1}{x}\Big|_1^t$$

$$= \lim_{t\to\infty} \overset{0}{\cancel{-\frac{1}{t}}} - -\frac{1}{1} = \lim_{t\to\infty}\left(1-\frac{1}{t}\right)$$





$$V = \int_1^t \pi\left(\frac{1}{x}\right)^2 dx$$

$$= \pi\left(1-\frac{1}{t}\right)$$

$$V = \int_0^h w\cdot l\,dx$$

$$= wlx\Big|_0^h$$

$$= wlh$$

$$V = \int_1^\infty \pi\left(\frac{1}{x}\right)^2 dx = \lim_{t\to\infty}\int_1^t \pi\left(\frac{1}{x}\right)^2 dx$$

$$= \lim_{t\to\infty}\pi\left(1-\frac{1}{t}\right) = \pi$$

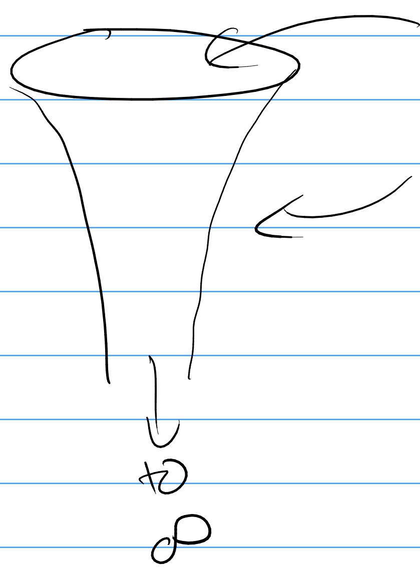
$$SA = \int_1^\infty 2\pi\left(\frac{1}{x}\right)\sqrt{1 + \left(\left(\frac{1}{x}\right)'\right)^2}\,dx$$

$$= \lim_{t\to\infty} \int_1^t 2\pi \frac{\sqrt{1 + 1/x^4}}{x}\,dx$$

$$= \lim_{t\to\infty} 2\pi \int_1^t \boxed{\frac{\sqrt{1+1/x^4}}{x}}\,dx \quad \sqrt{1+1/x^4} \geq \sqrt{1} = 1$$

$$\boxed{SA} > \lim_{t\to\infty} 2\pi \int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} 2\pi \ln t^{\infty}$$

So Surface Area $\to \infty$



Fill it with π units^3

can't paint it without ∞ units^2 of paint.

Type 1 improper integral

a) if $\int_a^t f(x)\,dx$ exists for all

t from a to ∞

then $\int_a^\infty f(x)\,dx = \lim_{t \to \infty} \int_a^t f(x)\,dx$

b) if $\int_t^b f(x)\,dx$ exists for all

t from $-\infty$ to b

then $\int_{-\infty}^b f(x)\,dx = \lim_{t \to -\infty} \int_t^b f(x)\,dx$

<u>Note:</u> if the limit exists we say

$\int_{-\infty}^b f(x)\,dx$ or $\int_a^\infty f(x)\,dx$ $\boxed{\text{converge}}$

They are called $\boxed{\text{divergent}}$ otherwise.

c) if $\int_a^\infty f(x)\,dx$ <u>and</u> $\int_{-\infty}^a f(x)\,dx$

converge

$\rightarrow \int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$

Ex $\int_{-\infty}^{\infty} x^2 e^{-x^3}\,dx = \int_{-\infty}^{1} x^2 e^{-x^3}\,dx + \int_{1}^{\infty} x^2 e^{-x^3}\,dx$ (1)

(1) $\int_{1}^{\infty} x^2 e^{-x^3}\,dx = \lim_{t\to\infty} \int_{1}^{t} x^2 e^{-x^3}\,dx$

let $u = -x^3$

$du = -3x^2\,dx$

$= \lim_{t\to\infty} \int_{-1}^{-t^3} -\frac{1}{3} e^u\,du$

$= \frac{1}{3} \lim_{t\to\infty} \int_{-t^3}^{-1} e^u\,du = \frac{1}{3} \lim_{t\to\infty} \left(\frac{1}{e} - \cancel{\frac{1}{e^{t^3}}}^{\,0}\right)$

$= \frac{1}{3e}$

---

other side: $\int_{-\infty}^{1} x^2 e^{-x^3}\,dx = \lim_{t\to-\infty} \int_{t}^{1} x^2 e^{-x^3}\,dx$

let $u = -x^3$

$du = -3x^2\,dx$

$= \lim_{t\to-\infty} -\frac{1}{3} \int_{-t^3}^{-1} e^u\,du = \frac{1}{3} \lim_{t\to-\infty} \int_{-1}^{-t^3} e^u\,du$

$= \frac{1}{3} \lim_{t\to-\infty} \cancel{e^{-t^3}}^{\,\infty} - \frac{1}{e} = \infty$ $\boxed{\text{divergent}}$

So $\int_{-\infty}^{\infty} x^2 e^{-x^3}\,dx$ is divergent

$$\int_1^{\infty} \frac{1}{x^p}\,dx = \lim_{t\to\infty} \int_1^t x^{-p}\,dx$$

$$= \lim_{t\to\infty} \frac{1}{1-p} x^{1-p} \Big|_1^t = \frac{1}{1-p} \lim_{t\to\infty} \left(t^{1-p} - 1\right)$$

$$= \frac{1}{1-p}\left[\underbrace{\left(\lim_{t\to\infty} t^{1-p}\right)} - 1\right]$$

$(p \neq 1) \leftarrow \ln(\;)$ special case (diverges)

$(p > 1)$ $\lim_{t\to\infty} t^{1-p}$ negative $= \lim_{t\to\infty} \frac{1}{t^{p-1}} \to 0 = 0$ (converges)

$(p < 1)$ $\lim_{t\to\infty} t^{1-p}$ positive $= \infty$ (diverges)

So $\int_1^{\infty} \frac{1}{x^p}\,dx$ $p \le 1$ diverges

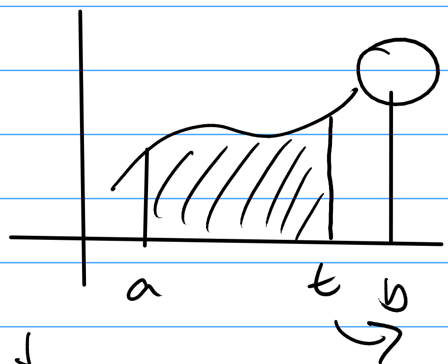
$p > 1$ converges

Type 2 does $f(x)$ have a discontinuity in $[a,b]$

$$\int_a^b f(x)\,dx$$

a) is $f(x)$ discontinuous @ $x=b$?

idea



if $\int_a^t f(x)\,dx$ exists as t goes to b

$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$$

b) $f(x)$ is discontinuous @ $x=a$?

if $\int_t^b f(x)\,dx$ exists from a to t

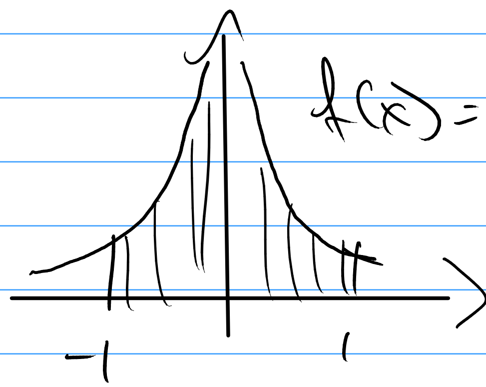
$$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$$

c) $f(x)$ is discontinuous @ $x=c \in (a,b)$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

if convergent

ex



$f(x) = 1/x^{2/3}$ $\quad \int_{-1}^{1} x^{-2/3}\,dx$

$$\underbrace{\int_{-1}^{0} x^{-2/3}\,dx}_{1} + \underbrace{\int_0^1 x^{-2/3}\,dx}_{2}$$

1 $\int_{-1}^{0} x^{-2/3}\,dx = \lim_{t\to 0^-} \int_{-1}^{t} x^{-2/3}\,dx$

$= \lim_{t\to 0^-} 3x^{1/3}\Big|_{-1}^{t} = \lim_{t\to 0^-} 3t^{1/3} + 3$

$= 3$

2 $\int_0^1 x^{-2/3}\,dx = \lim_{t\to 0^+} \int_t^1 x^{-2/3}\,dx$

$$= \lim_{t\to 0^+} 3x^{1/3} \Big|_t^1 = \lim_{t\to 0^+} 3 - 3t^{1/3}$$

$$= \underline{\underline{3}}$$

$\boxed{50}$ $$\int_{-1}^{1} x^{-2/3}\,dx = \boxed{6}$$

$\boxed{353 \quad \#54}$

$$\bar{v} = \underbrace{\frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2}} \underbrace{\int_0^\infty v^3 e^{-\frac{M}{2RT}v^2}\,dv}$$

Integrate: $$\int_0^\infty x^3 e^{-cx^2}\,dx$$

$$= \lim_{t\to\infty} \int_0^t x^3 e^{-cx^2}\,dx$$

let $u = -cx^2$

$du = -2cx\,dx$

$$= \lim_{t\to\infty} \frac{1}{2c^2} \int_0^{-ct^2} u\,e^u\,du$$ Integration by Parts (table)

$$= \lim_{t\to\infty} \frac{1}{2c^2} (u-1)e^u \Big|_0^{-ct^2}$$

$$= \frac{1}{2c^2} \lim_{t\to\infty} \left[(-ct^2 - 1)e^{-ct^2} - (-1) \right]$$

$$= \frac{1}{2c^2} \lim_{t\to\infty} \left[\cancel{\frac{-ct^2}{e^{ct^2}}}^{\,0} - \cancel{\frac{1}{e^{ct^2}}}^{\,0} + 1 \right]$$

$$= \frac{1}{2c^2} \qquad c = M/2RT$$

$$\bar{v} = \frac{4}{\sqrt{\pi}} \left(\frac{M}{2RT} \right)^{3/2} \boxed{\int_0^\infty v^3 e^{-\frac{M}{2RT} v^2} \, dv}$$

$$\bar{v} = \frac{\cancel{4}^{\,2}}{\sqrt{\pi}} \left(\frac{M}{2RT} \right)^{\cancel{3/2}\,-1/2} \cdot \frac{1}{\cancel{2} \cancel{\left(\frac{M}{2RT} \right)^2}}$$

$$\bar{v} = \frac{\sqrt{4}}{\sqrt{\pi}} \sqrt{\frac{2RT}{M}} = \boxed{\sqrt{\frac{8RT}{\pi M}}}$$

Q's/

$$\int_0^\infty 3 \frac{(2 + e^{-x})}{x} \, dx$$

$$3 \int_0^1 \frac{2+e^{-x}}{x} \, dx + 3 \int_1^\infty \frac{2+e^{-x}}{x} \, dx$$

type 2   type 1

ex

$$3\int_1^{\infty} \frac{2+e^{-x}}{x}\,dx = 3\lim_{t\to\infty}\int_1^t \frac{2+e^{-x}}{x}\,dx$$

$$= 3\lim_{t\to\infty}\left[\int_1^t \frac{2}{x}\,dx + \int_1^t \frac{e^{-x}}{x}\,dx\right]$$

$$= 3\lim_{t\to\infty}\left[2\ln|x|\Big|_1^t + \int_1^t x^{-1}e^{-x}\,dx\right]$$

$$= 3\lim_{t\to\infty}\left[\ln t^2 + \int_1^t x^{-1}e^{-x}\,dx\right]$$

let $u = e^{-x}$. $du = -e^{-x}dx$

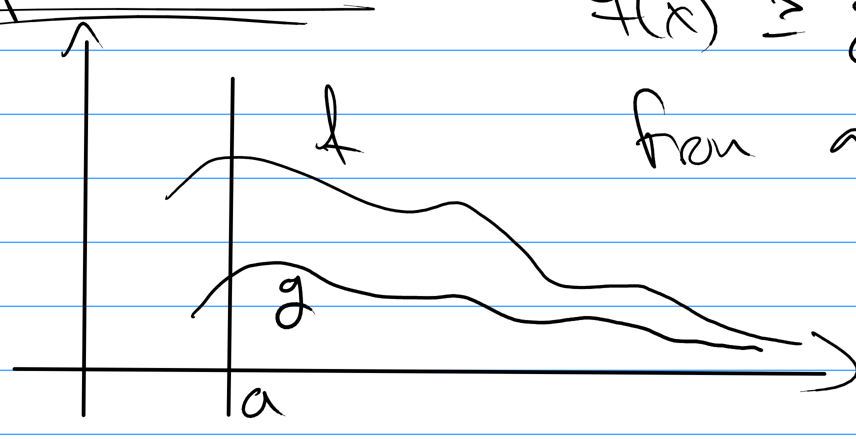
$dv = \frac{1}{x}dx \quad v = \ln|x|$

$$\int \frac{1}{xe^x}\,dx$$ "can't" be integrated.

Comparison thm:

$f(x) \geq g(x)$

from a to ∞



$$\int_a^\infty f(x)\,dx \geq \int_a^\infty g(x)\,dx$$

if $\int_a^\infty f(x)\,dx$ converges

$\rightarrow \int_a^\infty g(x)\,dx$ converges

if $\int_a^\infty g(x)\,dx$ diverges

$\rightarrow \int_a^\infty f(x)\,dx$ diverges.

$$\int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx$$ b/c $\frac{1}{x^2}$ converges

$p > 1$

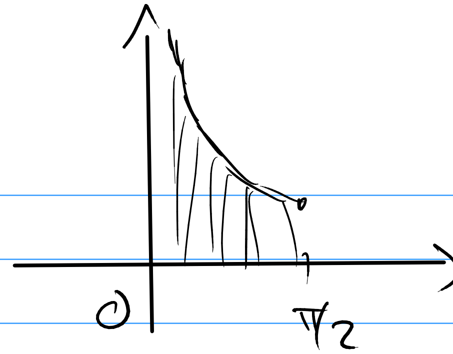
$$\frac{\cos^2 x}{1+x^2} \leq \frac{1}{1+x^2} \leq \frac{1}{x^2}$$

greater

(b/c we are looking for a convergent & larger function)

So b/c $\int_1^\infty \frac{1}{x^2}\,dx$ converges so does our problem!

$$\int_0^{\pi/2} \frac{dx}{x \sin x}$$



$$\int_0^1 \frac{1}{x^p}\,dx = \lim_{t\to 0^+} \int_t^1 x^{-p}\,dx \quad (p \neq 1)$$

$$= \lim_{t\to 0^+} \frac{1}{1-p} x^{1-p} \Big|_t^1 = \frac{1}{1-p} \lim_{t\to 0^+} \left(1 - t^{1-p}\right)$$

$p > 1$ diverges

$p < 1$ converges

$$p = 1 \qquad \int_0^1 \frac{1}{x}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{1}{x}\,dx$$

$$= \lim_{t\to 0^+} \ln x \Big|_t^1 = \lim_{t\to 0^+} -\ln t = +\infty$$

div.

$$\int_1^\infty \frac{1}{x^p}\,dx \qquad \begin{array}{ll} p > 1 & \text{conv.} \\ p \le 1 & \text{div.} \end{array}$$

$$\int_0^1 \frac{1}{x^p}\,dx \qquad \begin{array}{ll} p \ge 1 & \text{div.} \\ p < 1 & \text{conv.} \end{array}$$

so.. our problem $\int_0^1 \frac{1}{x \cdot \sin x}\, dx$

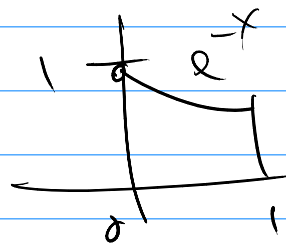
guess x^p $p = 1$

goal!: find small divergent function → divergent?

$$\frac{1}{x \boxed{\sin x}} \geq \frac{1}{x \cdot \boxed{1}} = \frac{1}{x} \text{ diverges}$$

so $\int_0^1 \frac{1}{x \sin x}\, dx$ diverges as well

$\int_0^1 \frac{e^{-x}}{\sqrt{x}}\, dx$ guess converges



$$\frac{e^{-x}}{\sqrt{x}} \leq \frac{1}{\sqrt{x}}$$

b/c $\int_0^1 \frac{1}{x^{1/2}}\, dx$ converges so does the smaller area

$\rightarrow \quad \int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx \quad \boxed{\text{converges.}}$