Recursion (Chapter 2)

- Iterative vs. Recursive Algorithms
  - Advantages – Conceptually simpler, Elegant, Easy to read
  - Tradeoffs – execution time and space
- Design of a recursive algorithm
  - Principles for designing recursive algorithms
  - Limitations
- When is recursion appropriate
  - Task is recursively defined
- Example of recursive functions
  - Factorial
  - Fibonacci numbers
  - Prefix to Postfix conversion
  - Towers of Hanoi

2-1 Factorial - A Case Study

We begin the discussion of recursion with a case study and use it to define the concept.
This section also presents an iterative and a recursive solution to the factorial algorithm.

- Recursive Defined
- Recursive Solution

$$\text{Factorial}(n) = \begin{bmatrix} 1 & \text{if } n = 0 \\ n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1 & \text{if } n > 0 \end{bmatrix}$$

FIGURE 2-1 Iterative Factorial Algorithm Definition

$$\text{Factorial}(n) = \begin{bmatrix} 1 & \text{if } n = 0 \\ n \times (\text{Factorial}(n-1)) & \text{if } n > 0 \end{bmatrix}$$

FIGURE 2-2 Recursive Factorial Algorithm Definition

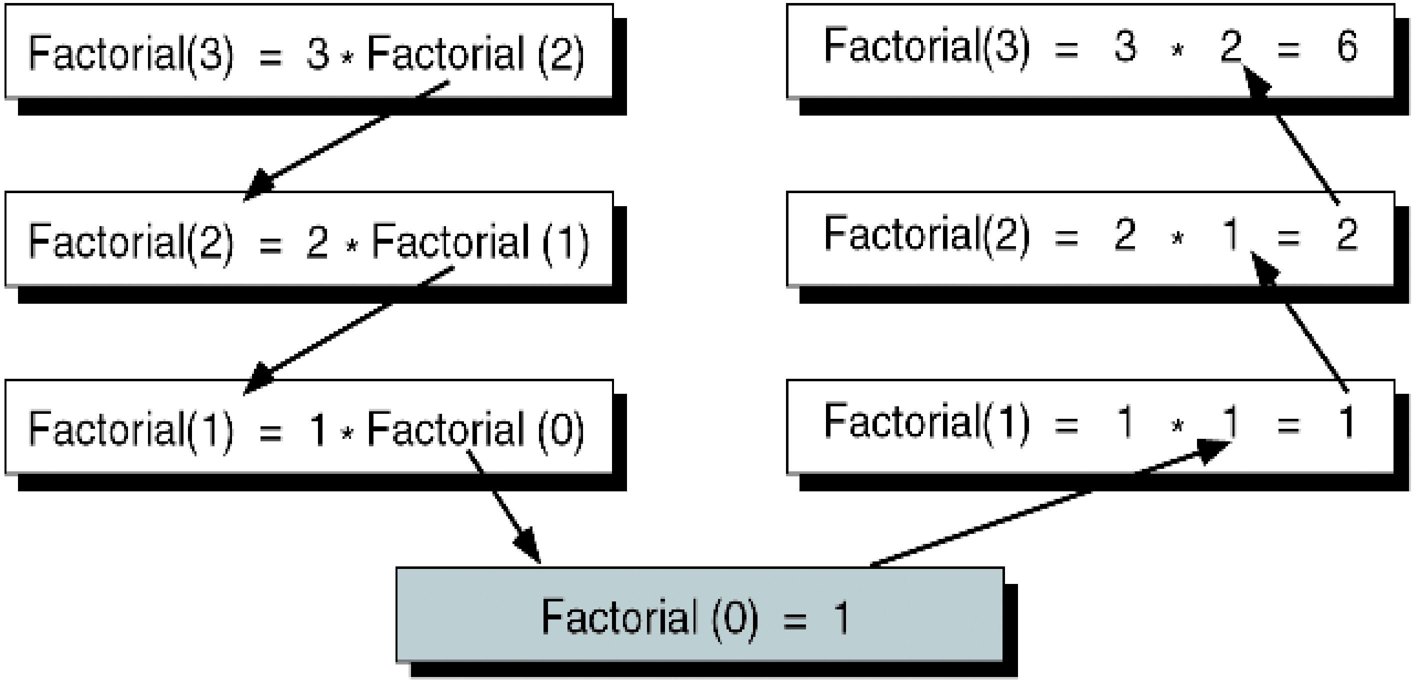


FIGURE 2-3 Factorial (3) Recursively

ALGORITHM 2-1 Iterative Factorial Algorithm

```
Algorithm iterativeFactorial (n)
Calculates the factorial of a number using a loop.
   Pre  n  is the number to be raised factorially
   Post n! is returned
1 set i to 1
2 set factN to 1
3 loop (i <= n)
   1  set factN to factN * i
   2  increment i
4 end loop
5 return factN
end iterativeFactorial
```

ALGORITHM 2-2 Recursive Factorial

```
Algorithm recursiveFactorial (n)
Calculates factorial of a number using recursion.
   Pre    n  is the number being raised factorially
   Post   n! is returned
1 if (n equals 0)
   1  return 1
2 else
   1  return (n * recursiveFactorial (n - 1))
3 end if
end recursiveFactorial
```

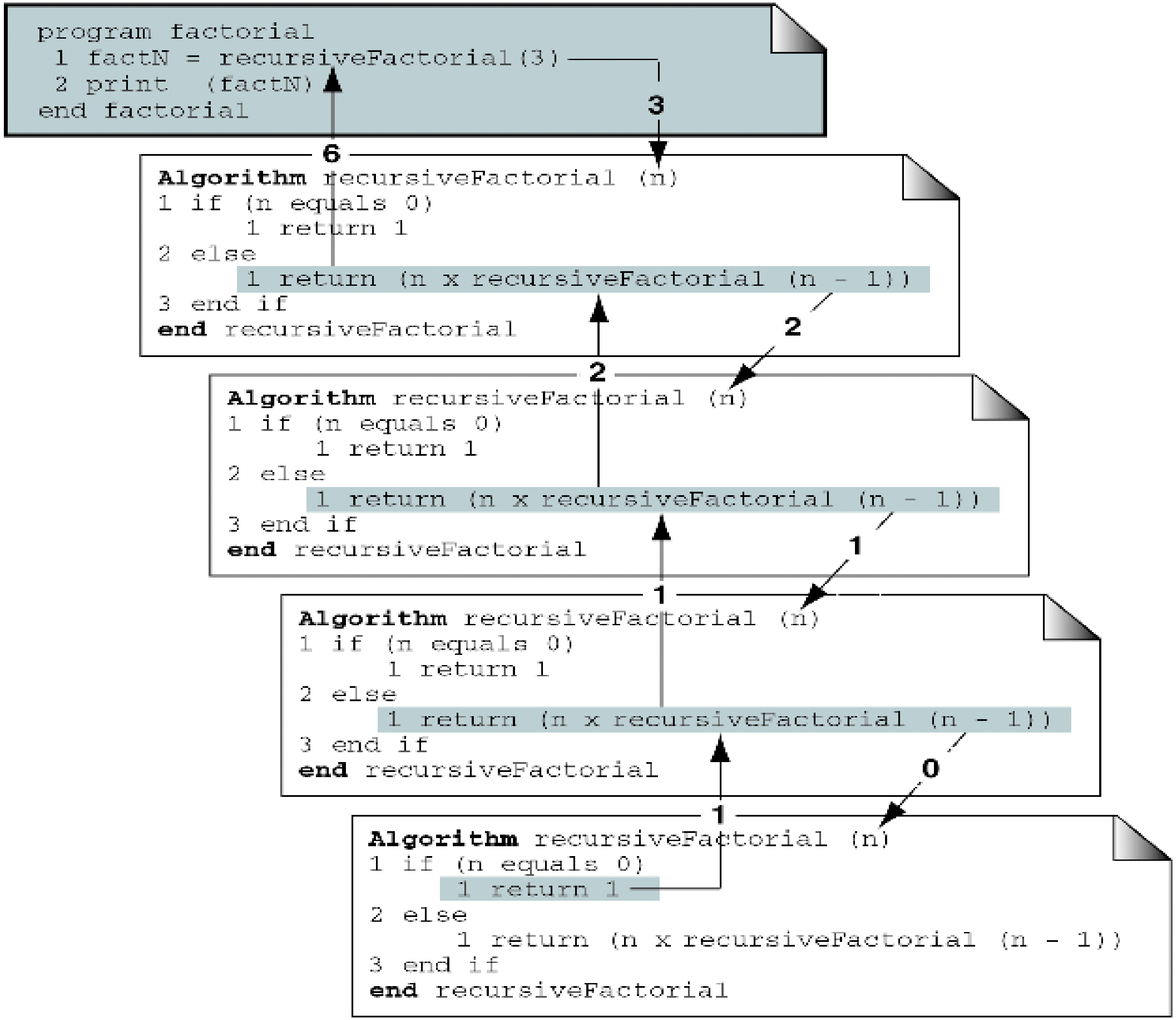


FIGURE 2-4 Calling a Recursive Algorithm

2-2 Designing Recursive Algorithms

In this section we present an analytical approach to designing recursive algorithms.
We also discuss algorithm designs that are not well suited to recursion.

- The Design Methodology
- Limitation of Recursion
- Design Implementation

ALGORITHM 2-3 Print Reverse

```
Algorithm printReverse (data)
Print keyboard data in reverse.
   Pre  nothing
   Post data printed in reverse
1  if (end of input)
   1  return
2  end if
3  read data
4  printReverse (data)
Have reached end of input: print nodes
5  print data
6  return
end printReverse
```

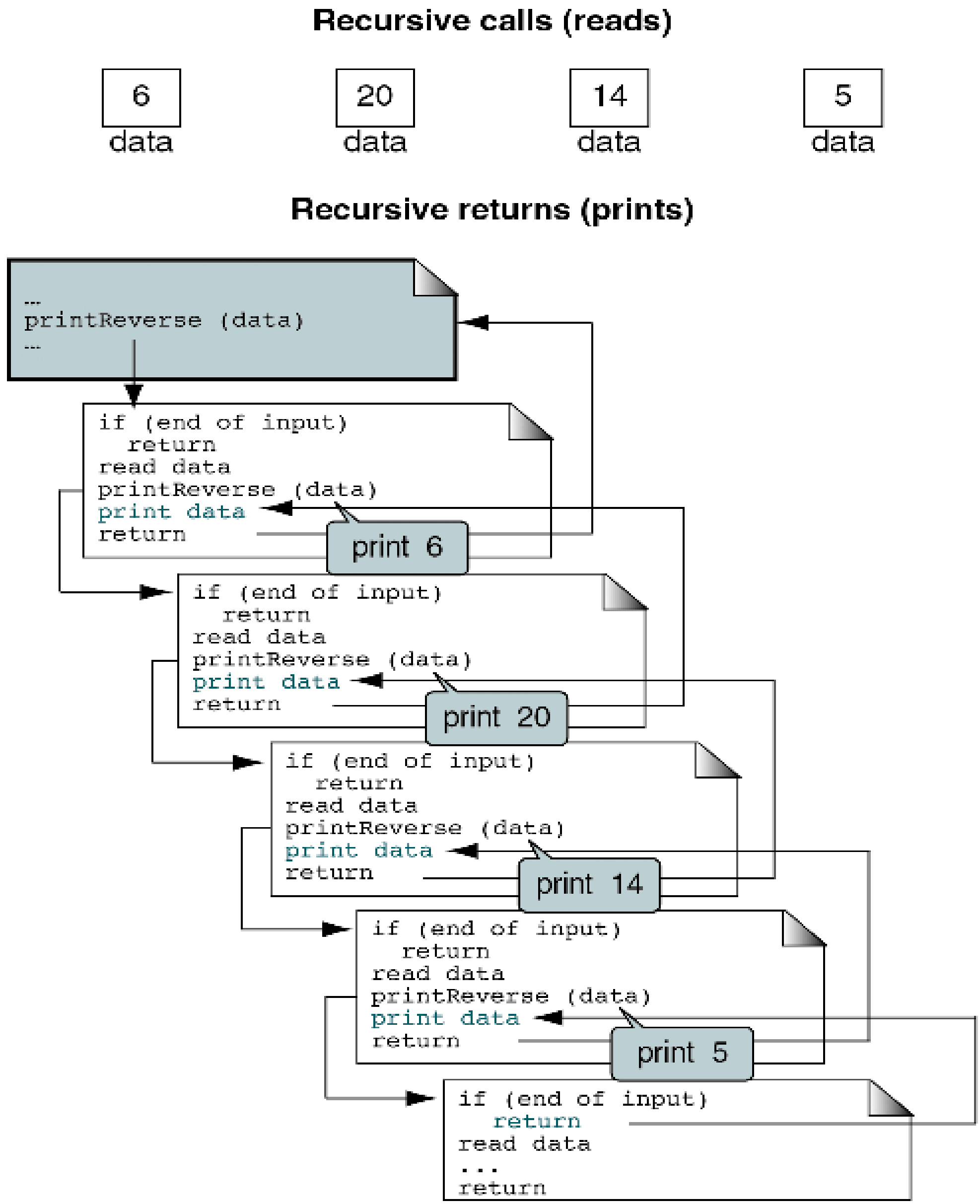


FIGURE 2-5 Print Keyboard Input in Reverse

2-3 Recursive Examples

Four recursive programs are developed and analyzed. Only one, the Towers of Hanoi, turns out to be a good application for recursion.

- Greatest Common Divisor
- Fiboncci Numbers
- Prefix to Postfix Conversion
- The Towers of Honoi

ALGORITHM 2-4 Euclidean Algorithm for Greatest Common Divisor

```
Algorithm gcd (a, b)
Calculates greatest common divisor using the Euclidean algo-
rithm.
   Pre  a and b are positive integers greater than 0
   Post greatest common divisor returned
1 if (b equals 0)
   1  return a
2 end if
3 if (a equals 0)
   2  return b
4 end if
5 return gcd (b, a mod b)
end gcd
```

PROGRAM 2-1 GCD Driver

```
/* This program determines the greatest common divisor
   of two numbers.
      Written by:
      Date:
*/
#include <stdio.h>
```

continued

PROGRAM 2-1 GCD Driver (continued)

```
#include <ctype.h>

// Prototype Statements
int gcd (int a, int b);

int main (void)
{
// Local Declarations
   int  gcdResult;

// Statements
   printf("Test GCD Algorithm\n");

   gcdResult = gcd (10, 25);
   printf("GCD of 10 & 25 is %d", gcdResult);
   printf("\nEnd of Test\n");
   return 0;
}  // main
```

PROGRAM 2-1 GCD Driver (continued)

```
/* ================== gcd ===================
   Calculates greatest common divisor using the
   Euclidean algorithm.
      Pre  a and b are positive integers greater than 0
      Post greatest common divisor returned
*/
int gcd (int a, int b)
{
   // Statements
   if (b == 0)
      return a;
   if (a == 0)
      return b;
   return gcd (b, a % b);
}  // gcd
```

```
Results:
Test GCD Algorithm
GCD of 10 & 25 is 5
End of Test
```

$$\text{Fibonacci}(n) = \begin{bmatrix} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ & \\ \text{Fibonacci}(n-1) + \text{Fibonacci}(n-2) & \text{otherwise} \end{bmatrix}$$

FIGURE 2-7 Fibonacci Numbers Recursive Definition

PROGRAM 2-2 Recursive Fibonacci Series

```
/* This program prints out a Fibonacci series.
      Written by:
      Date:
*/
#include <stdio.h>

// Prototype Statements
   long fib (long num);

int main (void)
{
// Local Declarations
   int seriesSize = 10;

// Statements
   printf("Print a Fibonacci series.\n");

```

PROGRAM 2-2 Recursive Fibonacci Series (Continued)

```
    for (int looper = 0; looper < seriesSize; looper++)
        {
         if (looper % 5)
             printf(", %8ld", fib(looper));
         else
             printf("\n%8ld", fib(looper));
        } // for
    printf("\n");
    return 0;
}   // main

/* ================= fib =================
    Calculates the nth Fibonacci number
        Pre  num identifies Fibonacci number
        Post returns nth Fibonacci number
*/
long fib (long num)
{
// Statements
    if (num == 0 || num == 1)
```

continued

PROGRAM 2-2 Recursive Fibonacci Series *(continued)*

```
        // Base Case
        return num;
    return (fib (num - 1) + fib (num - 2));
} // fib
```

```
Results:
Print a Fibonacci series.

      0,        1,        1,        2,        3
      5,        8,       13,       21,       34
```

fib(*n*)	Calls	fib(n)	Calls
1	1	11	287
2	3	12	465
3	5	13	753
4	9	14	1219
5	15	15	1973
6	25	20	21,891
7	41	25	242,785
8	67	30	2,692,573
9	109	35	29,860,703
10	177	40	331,160,281

TABLE 2-1 Fibonacci Calls

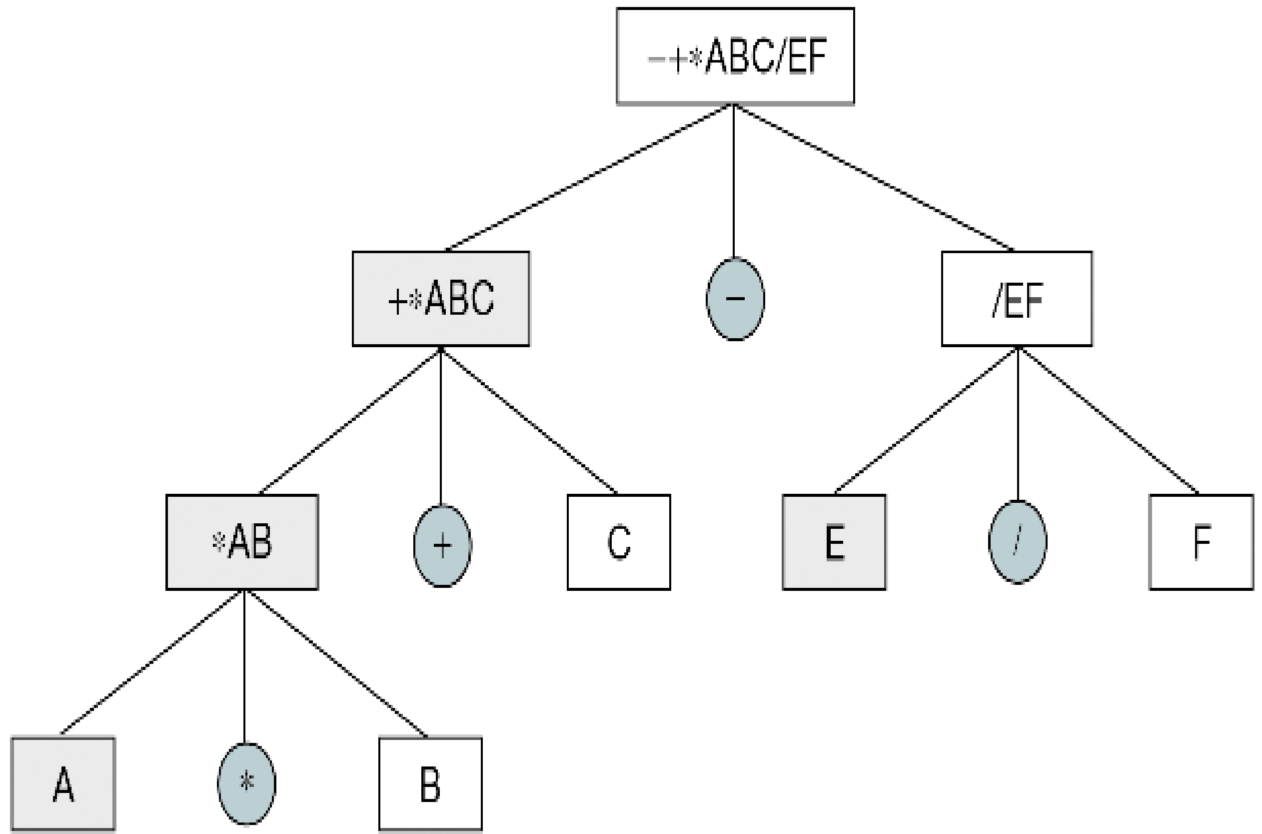


FIGURE 2-9 Decomposition of −+*ABC/EF

ALGORITHM 2-5 Convert Prefix Expression to Postfix

```
Algorithm preToPostFix (preFixIn, postFix)
Convert a preFix string to a postFix string.
   Pre  preFix is a valid preFixIn expression
        postFix is reference for converted expression
   Post postFix contains converted expression
1 if (length of preFixIn is 1)
     Base case: one character string is an operand
   1  set postFix to preFixIn
   2  return
2 end if
  If not an operand, must be an operator
3 set operator to first character of preFixIn
  Find first expression
4 set lengthOfExpr to findExprLen (preFixIn less first char)
5 set temp to substring(preFixIn[2, lengthOfExpr])
6 preToPostFix (temp, postFix1)
  Find second postFix expression
7 set temp to prefixIn[lengthOfExpr + 1, end of string]
8 preToPostFix (temp, postFix2)
  Concatenate postfix expressions and operator
9 set postFix to postFix1 + postFix2 + operator
10 return
end preToPostFix
```

ALGORITHM 2-6 Find Length of Prefix Expression

```
Algorithm findExprLen (exprIn)
Recursively determine the length of a prefix expression.
   Pre  exprIn is a valid prefix expression
   Post length of expression returned
1 if (first character is operator)
  General Case: First character is operator
  Find length of first prefix expression
  1  set len1 to findExprLen (exprIn + 1)
  2  set len2 to findExprLen (exprIn + 1 + len2)
2 else
  Base case--first char is operand
  1  set len1 and len2 to 0
3 end if
4 return len1 + len2 + 1
end findExprLen
```

PROGRAM 2-3 Prefix to Postfix

```
/* Convert prefix to postfix expression.
      Written by:
      Date:
*/
#include <stdio.h>
#include <string.h>

#define OPERATORS "+-*/"

// Prototype Declarations
void preToPostFix  (char* preFixIn, char* exprOut);
int  findExprLen   (char* exprIn);

int main (void)
{
```

continued

PROGRAM 2-3 Prefix to Postfix *(continued)*

```
// Local Definitions
   char  preFixExpr[256]  = "-+*ABC/EF";
   char  postFixExpr[256] = "";

// Statements
   printf("Begin prefix to postfix conversion\n\n");

   preToPostFix (preFixExpr, postFixExpr);
   printf("Prefix expr:  %-s\n", preFixExpr);
   printf("Postfix expr: %-s\n", postFixExpr);

   printf("\nEnd prefix to postfix conversion\n");
   return 0;
}  // main

```

PROGRAM 2-3 Prefix to Postfix *(continued)*

```
/* =================== preToPostFix ====================
   Convert prefix expression to postfix format.
      Pre  preFixIn is string prefix expression
           expression can contain no errors/spaces
           postFix is string variable for postfix
      Post expression has been converted
*/
void preToPostFix (char* preFixIn, char* postFix)
{
// Local Definitions
   char  operator [2];
   char  postFix1[256];
   char  postFix2[256];
   char  temp    [256];
   int   lenPreFix;

```

PROGRAM 2-3 Prefix to Postfix *(continued)*

```
// Statements
   if (strlen(preFixIn) == 1)
      {
       *postFix          = *preFixIn;
       *(postFix + 1) = '\0';
       return;
      } // if only operand

   *operator         = *preFixIn;
   *(operator + 1) = '\0';

   // Find first expression
   lenPreFix = findExprLen (preFixIn + 1);
   strncpy (temp, preFixIn + 1, lenPreFix);
   *(temp + lenPreFix) = '\0';
   preToPostFix (temp, postFix1);
```

continued

PROGRAM 2-3 Prefix to Postfix *(continued)*

```

    // Find second expression
    strcpy (temp, preFixIn + 1 + lenPreFix);
    preToPostFix (temp, postFix2);

    // Concatenate to postFix
    strcpy (postFix, postFix1);
    strcat (postFix, postFix2);
    strcat (postFix, operator);

    return;
} // preToPostFix

```

PROGRAM 2-3 Prefix to Postfix *(continued)*

```
/* ==================== findExprLen ====================
   Determine size of first substring in an expression.
      Pre  exprIn contains prefix expression
      Post size of expression is returned
*/
int findExprLen (char* exprIn)
{
// Local Definitions
   int  len1;
   int  len2;

```

PROGRAM 2-3 Prefix to Postfix (continued)

```
// Statements
   if (strcspn (exprIn, OPERATORS) == 0)
         // General Case: First character is operator
         // Find length of first expression
       {
         len1 = findExprLen(exprIn + 1);

         // Find length of second expression
         len2 = findExprLen(exprIn + 1 + len1);
       } // if
   else
         // Base case--first char is operand
         len1 = len2 = 0;
   return len1 + len2 + 1;
}  // findExprLen
```

```
Results:
Begin prefix to postfix conversion

Prefix expr:  -+*ABC/EF
Postfix expr: AB*C+EF/-

End prefix to postfix conversion
```

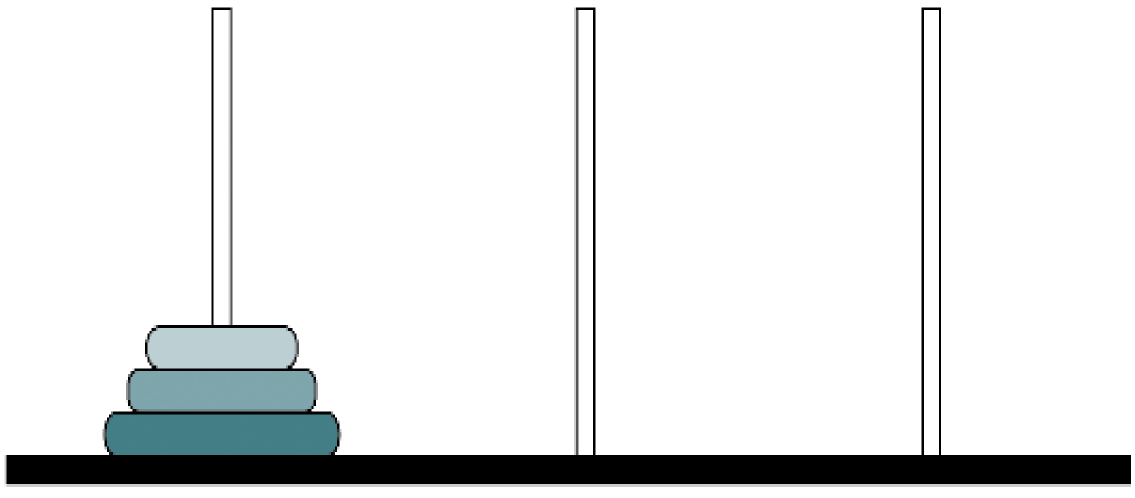


FIGURE 2-11 Towers of Hanoi—Start Position

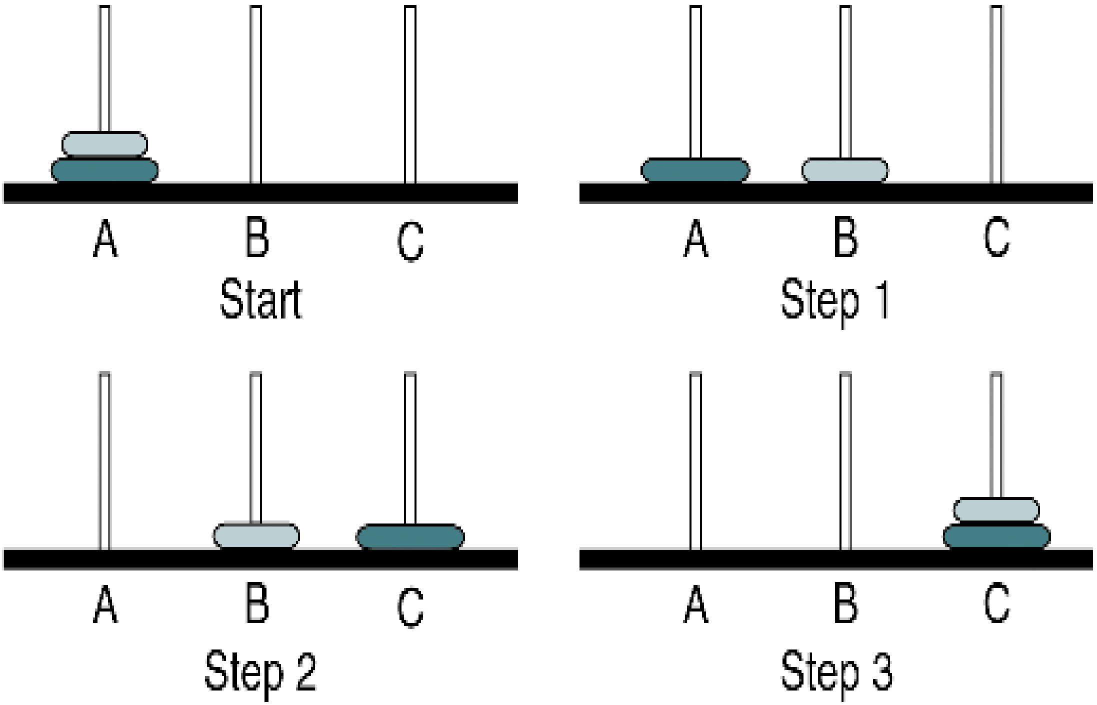


FIGURE 2-12 Towers Solution for Two Disks

FIGURE 2-13 Towers Solution for Three Disks

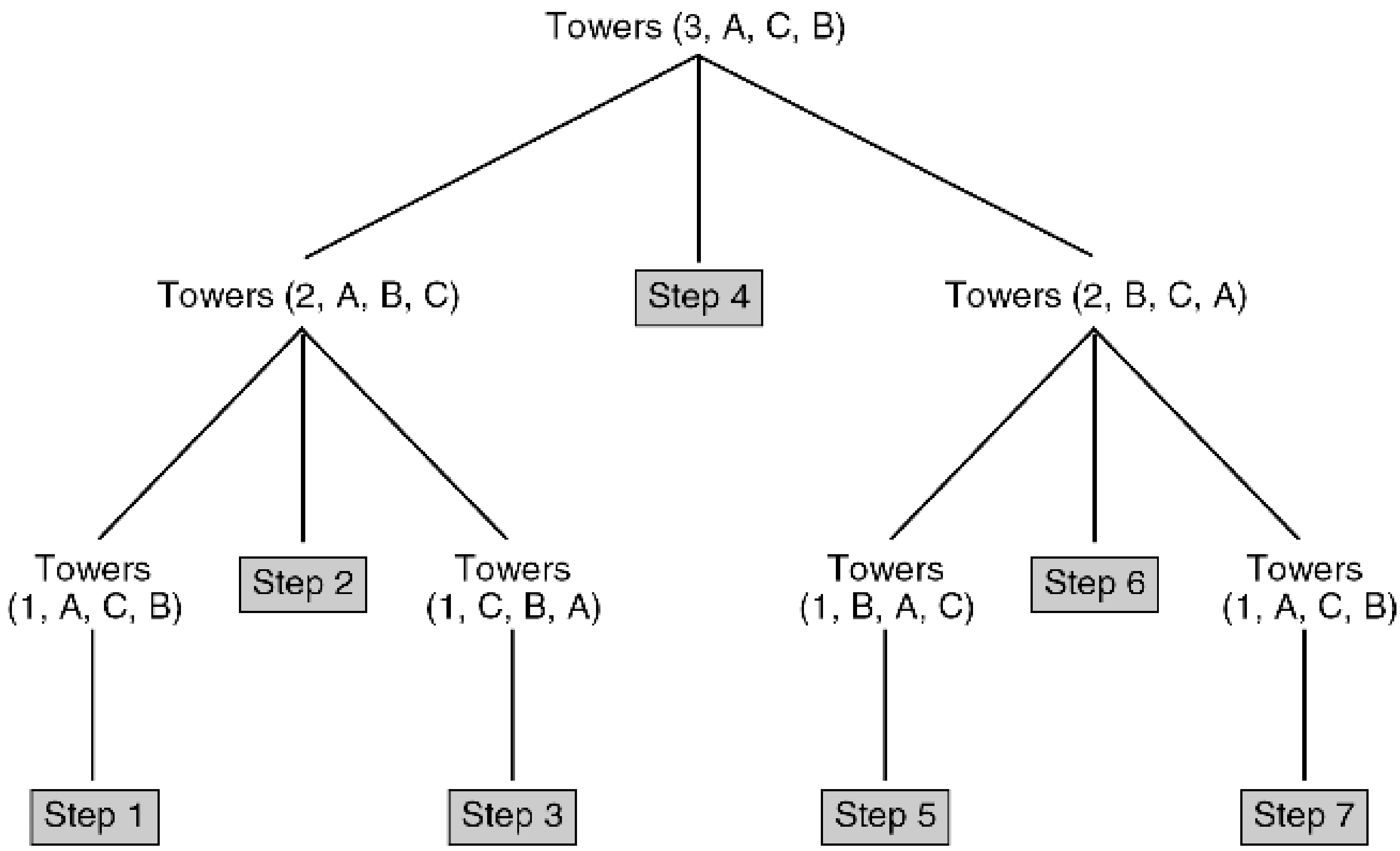
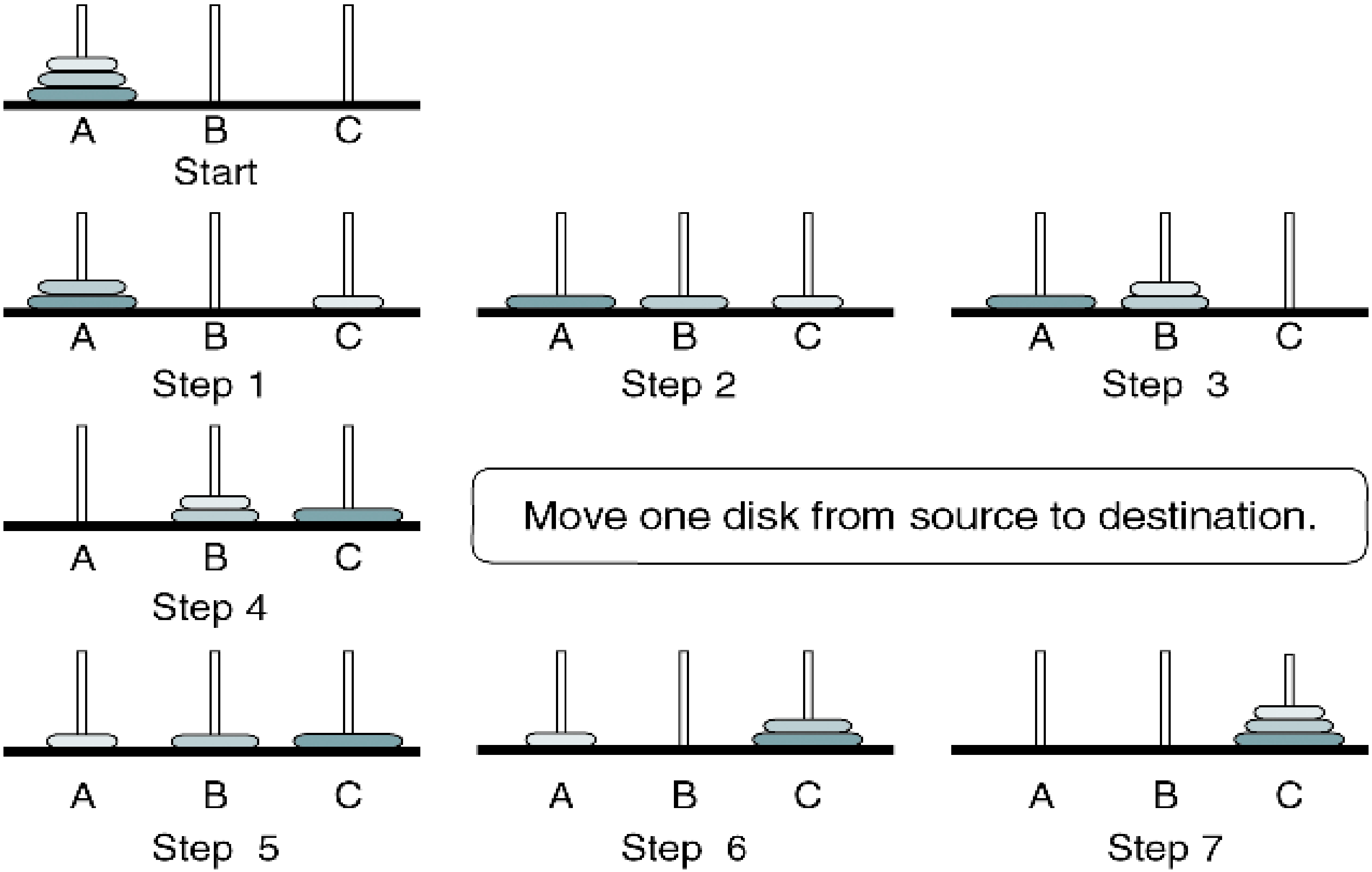


FIGURE 2-13 Towers Solution for Three Disks (Continued)



ALGORITHM 2-7 Towers of Hanoi

```
Algorithm towers (numDisks, source, dest, auxiliary)
Recursively move disks from source to destination.
   Pre  numDisks is number of disks to be moved
        source, destination, and auxiliary towers given
   Post steps for moves printed
1 print("Towers: ", numDisks, source, dest, auxiliary)
2 if (numDisks is 1)
   1 print ("Move from ", source, " to ", dest)
3 else
   1 towers (numDisks - 1, source, auxiliary, dest, step)
   2 print ("Move from " source " to " dest)
   3 towers (numDisks - 1, auxiliary, dest, source, step)
4 end if
end towers
```

Calls:	Output:
Towers (3, A, C, B)	
Towers (2, A, B, C)	
Towers (1, A, C, B)	
	Move from A to C
	Move from A to B
Towers (1, C, B, A)	
	Move from C to B
	Move from A to C
Towers (2, B, C, A)	
Towers (1, B, A, C)	
	Move from B to A
	Move from B to C
Towers (1, A, C, B)	
	Move from A to C

FIGURE 2-14 Tracing Algorithm 2-7, Towers of Hanoi

PROGRAM 2-4 Towers of Hanoi

```
/* Test Towers of Hanoi
        Written by:
        Date:
*/
#include <stdio.h>

// Prototype Statements
   void towers (int  n,     char source,
                char dest,  char auxiliary);

int main (void)
{
// Local Declarations
   int numDisks;

```

continued

PROGRAM 2-4 Towers of Hanoi (Continued)

```
// Statements
   printf("Please enter number of disks: ");
   scanf ("%d", &numDisks);

   printf("Start Towers of Hanoi.\n\n");

   towers (numDisks, 'A', 'C', 'B');

   printf("\nI Hope you didn't select 64 "
            "and end the world!\n");
   return 0;
}  // main

```

PROGRAM 2-4 Towers of Hanoi (Continued)

```
/* ===================== towers =====================
   Move one disk from source to destination through
   the use of recursion.
      Pre  The tower consists of n disks
           Source, destination, & auxiliary towers
      Post Steps for moves printed
*/
void towers (int   n,    char  source,
                char  dest, char  auxiliary)
{
// Local Declarations
   static int step = 0;

```

PROGRAM 2-4 Towers of Hanoi (Continued)

```
// Statements
   printf("Towers (%d, %c, %c, %c)\n",
                    n, source, dest, auxiliary);
   if (n == 1)
      printf("\t\t\tStep %3d: Move from %c to %c\n",
             ++step, source, dest);
   else
      {
       towers (n - 1, source, auxiliary, dest);
       printf("\t\t\tStep %3d: Move from %c to %c\n",
              ++step, source, dest);
       towers (n - 1, auxiliary, dest, source);
      } // if … else
   return;
}  // towers
```

$$\text{squareRoot}(num, ans, tol) = \left[\begin{array}{ll} ans & \text{if } |\, ans^2 - num \,| \le tol \\ \text{squareRoot}(num, (ans^2 + num)/(2 \times ans), tol) & \text{otherwise} \end{array}\right]$$

FIGURE 2-15 Newton's Method for Exercise 4

$$C(n, k) = \begin{bmatrix} 1 & \text{if } k = 0 \text{ or } n = k \\ C(n, k) = C(n-1, k) + C(n-1, k-1) & \text{if } n > k > 0 \end{bmatrix}$$

FIGURE 2-16 Selection Algorithm for Exercise 5

$$\text{Ackerman}(m,\ n) = \begin{bmatrix} n+1 & \text{if } m = 0 \\ \text{Ackerman}(m-1, 1) & \text{if } n = 0 \text{ and } m > 0 \\ \text{Ackerman}(m-1, \text{Ackerman}(m, n-1)) & \text{otherwise} \end{bmatrix}$$

FIGURE 2-17 Ackerman Formula for Problem 6