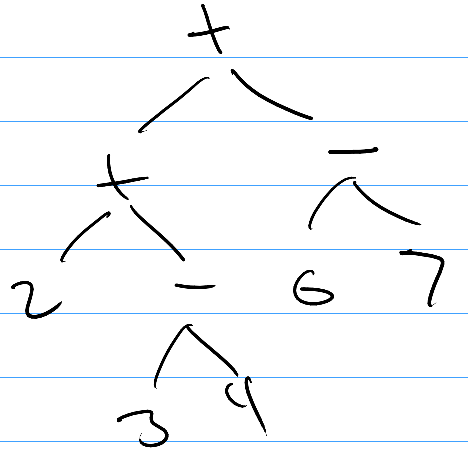
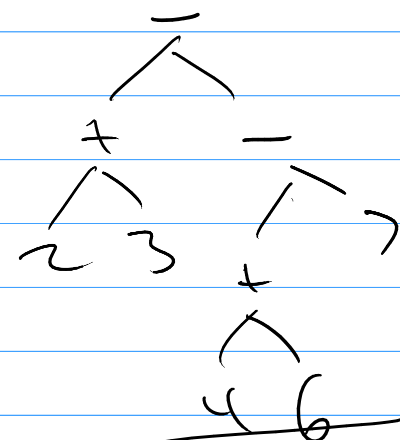
Math 322

Q's | 10.3 (21) $A \wedge B - A \wedge B - A$

$(2 + (3 - 4)) + (6 - 7)$ $(2 + 3) - ((4 + 6) - 7)$





So # of ways to use paren. = $\boxed{\#\text{ of trees}}$

#3 #4

Counting:

$A \wedge B - A \wedge B - A$

root #2 (case 2)

root #1 (case 1)

but |ways on 1| = |ways on 4|

|ways on 2| = |ways on 3|

case 1 $(A) \wedge (B - A \wedge B - A)$

T_1 T_2

$|T_1| = 1$ $|T_2| = ?$

$T = B - A \wedge B - A$

$-$: B , $A \wedge B - A$ (or) $\wedge$: $B - A$, $B - A$ (or) $-$: $B - A \wedge B$, A

$|S_1| = 2$ $\quad$ $|S_2| = 1$ $\quad$ $|S_3| = 2$

$|T_2| = 5$

$\boxed{\text{Case 1}}$ $1 \cdot 5 = 5$ ways on case 1

$\boxed{\text{Case 2}}$

$-$: $A \wedge B$, $A \wedge B - A$

$|T_1| = 1$ $\quad$ $|T_2| = 2$

$1 \cdot 2 = 2$ ways on case 2

total: $\boxed{14}$

Boolean Algebra

<u>Algebra</u>: Study of the rules, operation, and relations on stuff and what you get from them.

<u>Elementary Algebra</u>:

stuff = $\mathbb{R}$

→ properties of operations on $\mathbb{R}$

→ rules of expressions

→ rules of equality.

Abstract Algebra

→ Not just numbers → sets of elements.

→ Not just number ops → binary and unary operations on the elements.

→ Not just mult. or add identities or inverses → Identity, Inverse elements

tied to ops.

what about laws? rules?

Assoc? commutative? etc

Note: Boolean Algebra is based on the study of logic.

But it is a generalization of it.

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① Our set $\{0,1\} = B$

② Our operations

a) Unary: complement $\bar{0}=1$, $\bar{1}=0$

b) binary:

OR $0+1=1$, $1+0=1$, $0+0=0$, $1+1=1$

AND $0\cdot 1=0$, $1\cdot 0=0$, $0\cdot 0=0$, $1\cdot 1=1$

Rules: expressions

$$\bar{1}+\left(\overline{(0\cdot 1)}\cdot(1+1)\right) = 0+\left(\overline{(0)}\cdot(1)\right)$$

$$= 0+(1\cdot 1) = 0+1 = 1$$

Algebraic Variables

$x \in \{0,1\}$ is a Boolean Variable.

Boolean Function:

$$(x_1, x_2, \ldots, x_n) \in B^n$$

$$f(x_1, x_2, \ldots, x_n) : B^n \to B$$

Boolean Expression:

$$(x \cdot y) + z$$

$$x \cdot (y+z)$$

$$(\overline{x+y}) \cdot (s+\bar{t})$$

Function:

$$f(x,y,z,s,t) = (\bar{x}+\bar{y}) \cdot (z+s+t)$$

$$f(0,0,0,1,1) = (\bar{0}+\bar{0}) \cdot (0+(1+1))$$
$$= 1 \cdot 1 = 1$$

functions: $f = g$

ex: $f(x,y) = ...$ $g(x,y) = ...$

x	y	f
1	1	1
1	0	0
0	1	1
0	0	0

x	y	g
1	1	1
1	0	0
0	1	1
0	0	0

$f(x_1, x_2, \ldots, x_n)$

2^n rows

x_1	x_2	x_3	$\cdots$	x_n	f	
1	1	1		1	0 or 1	= 2 ways and
1	1	1		0		= 2 ways and
1						⋮
⋮						
0	0	0	$\cdots$	0	0 or 1	= 2 ways and

total functions $B^n \to B$

$$\underbrace{2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2}_{2^n} = \boxed{2^{2^n}}$$

unique tables for $f: B^n \to B$

Study operations → laws p. 753

$\overline{\overline{x}} = x$

$x + x = x \quad x \cdot x = x$

$x + 0 = x$

$x \cdot 1 = x$

$x + 1 = 1$

$x \cdot 0 = 0$

etc. compare to logical laws.

Now: given $\overline{0} = 1$, $\overline{1} = 0$, $0 + 0 = 0$, etc.

$B = \{0, 1\}$, all the laws $\boxed{\text{p. 753}}$

and ask: what is really needed?

ex. $x \cdot (x + y) = x$ absorption.

show: $x \cdot (x + y) = (x + 0) \cdot (x + y)$

$= x + (0 \cdot y)$

$= x + 0$

$= x$

So absorption is a byproduct of other laws.

Boolean Algebra Abstract Def.

① $B = \{0, 1\}$

② given two binary and one unary operation. symbols $\{\underbrace{\vee, \wedge}_{\text{binary}}, \underbrace{\overline{\ \ }}_{\text{unary}}\}$

③ You have to have..

a) Identity laws

$$b \vee 0 = b$$
$$b \wedge 1 = b$$

b) Complement

$$b \vee \bar{b} = 1$$
$$b \wedge \bar{b} = 0$$

c) Assoc.

d) Distrib

e) Commutative.