Math 321

Q's ~~1~~ $S = \{100, 101, \ldots, 999\}$

$|S| = 999 - 100 + 1 = 900$

$7 \mid x \qquad x \bmod 7 = 0$

98 $\quad \frac{1}{7} \cdot 900 \approx 128.5\ldots$

$$\begin{array}{r} 128.5\ldots \\ 7\overline{)900.0} \\ \underline{7} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 4.0 \end{array}$$

$\boxed{128}$ or 129 are div. by 7

$7 \mid 105$

$7 \mid 7 \cdot 15 \longrightarrow 7 \mid 7 \cdot (142)$

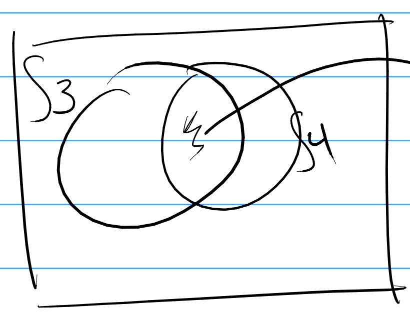
$142 - 15 + 1 = 128$

div. by 7 $= 128$

div by 3 $= \frac{1}{3}(900) = 300$

div by 4 $= \frac{1}{4}(900) = 225$

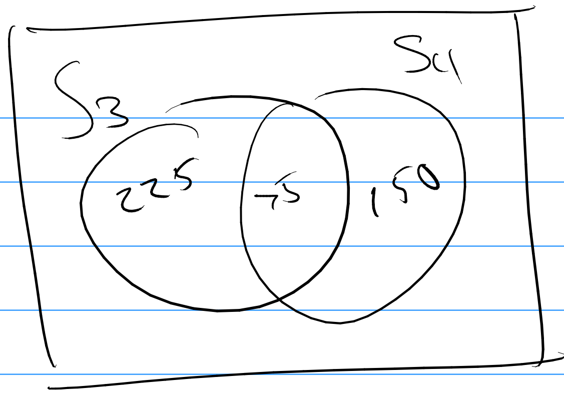
(a)



$3 \wedge 4$ div by 12

div by 12 $= \frac{1}{12} \cdot 900 = 75$

div by 3 $\wedge$ $\neg$ div. by 4 $= 300 - 75 = \boxed{225}$



3.3 Ways to take stuff. (without replacement)

Product Rule: 8 charc. none repeated (ascii)

128 charc.

$b_1 \wedge b_2 \wedge b_3 \wedge \cdots \wedge b_8$

$128 \wedge 127 \wedge 126 \wedge 125 \wedge 124 \wedge 123 \wedge 122 \wedge 121$

$\text{total} = 128 \cdot 127 \cdot \ldots \cdot 121$

1 **Factorial.**

$128 \cdot 127 \cdot 126 \cdot \ldots \cdot 1 = 128!$

ex: $\boxed{\dfrac{12!}{9!}} = \dfrac{12 \cdot 11 \cdot 10 \overbrace{(9 \cdot 8 \cdot 7 \cdot \ldots \cdot 1)}}{9 \cdot 8 \cdot 7 \cdot \ldots \cdot 1}$

$= \dfrac{12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{9!}} = 12 \cdot 11 \cdot 10$

$1! = 1$ $\qquad 2 \cdot 1 = 2!$ $\qquad t_1 \wedge t_2$

$2 \cdot 1 = 2!$

$1! = 1$

$$\boxed{0! = 1}$$

(2) taking stuff when order matters.

(Picking / Permutations)

ex: $n = 21$ pick $1^{st}, 2^{nd}, 3^{rd}$ place.

$$\text{total} = 21 \cdot 20 \cdot 19 = \frac{21!}{18!}$$

$$\boxed{P(n,r) = \frac{n!}{(n-r)!} \qquad 0 \leq r \leq n}$$

use when order matters.

(3) taking stuff when order does not matter.

ex $n = 21$ 3 on comm. and 18 not on comm.

$$\frac{21!}{3!\,18!}$$

(Choosing / Combinations) $C(n,r) = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$0 \leq r \leq n$

ex $\binom{21}{0} = \dfrac{21!}{0!(21-0)!} = 1$

$\binom{21}{21} = \dfrac{21!}{21!(21-21)!} = \dfrac{21!}{21!\,0!} = 1$

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(4) Counting Proofs / Combinatorical Proofs

show an identity by counting the same problem in two ways.

ex 3 ascii characters.
How many have at least one '@'?

$$|\text{all}| = |\text{no '@'s}| + \underbrace{|\text{exactly } 1| + |\text{exactly } 2| + |\text{exactly } 3|}_{\text{at least one '@'}}$$

$$128^3 = 127^3 + (\text{ all this })$$

$$128^3 - 127^3 = |\text{exactly } 1| + |\text{exactly } 2| + |\text{exactly } 3|$$

$$48769 = 1\cdot\binom{3}{1}\cdot 127^2 + 1\cdot\binom{3}{2}\cdot 127 + 1\cdot\binom{3}{3}127^0$$

$$= \frac{3!}{1!\,2!}127^2 + \frac{3!}{2!\,1!}127 + 1 = 48769$$

Scale up to $\underline{\underline{n}}$ ascii characters.

$$128^n - 127^n = \binom{n}{1} 127^{n-1} + \binom{n}{2} 127^{n-2} + \cdots + \binom{n}{n} 127^{n-n}$$

$$128^n = \binom{n}{0} 127^n + \binom{n}{1} 127^{n-1} + \cdots + \binom{n}{n} 127^{n-n}$$

$$\boxed{\sum_{i=0}^{n} \binom{n}{i} 127^{n-i} = 128^n}$$

$$\sum_{i=0}^{n} \binom{n}{i} (1)^{n-i} = 2^n$$

$$\boxed{\sum_{i=0}^{n} \binom{n}{i} = 2^n}$$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$$

$p_1 \quad p_2 \ldots \quad p_n$