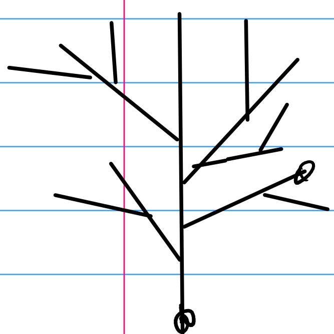
Math 322

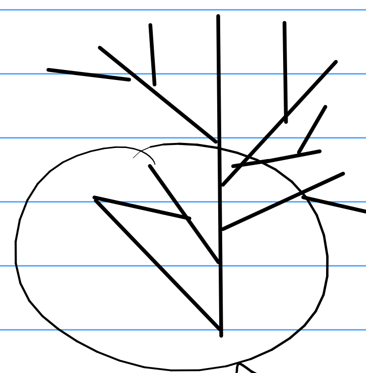
- Homework will be modified tonight. None due till <u>after</u> break.
- Thurs. class will be recorded tomorrow b/c I'll be driving to <u>Denver</u> (Math conference)

<u>Trees</u>



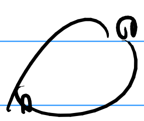
<u>tree</u>

<u>Def:</u> A tree is a connected undirected graph with no simple circuits.

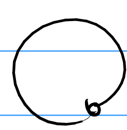


<u>not a tree</u>

simple circuit



multigraph



pseudograph

← simple circuits!

No simple circuit → simple graph

Thm: An undirected graph is a tree iff there is a unique simple path between any two vertices.

Pf: (nec)

assume uniq. simple path → $\boxed{\text{tree}}$?

a) connected?

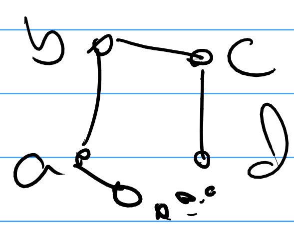
Mean a path between any two vertices. True.

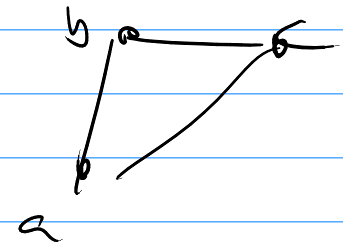
b) uniq. simple path → no simple circ.

try contradiction:

assume uniq. simp. path ∧ simple circ.

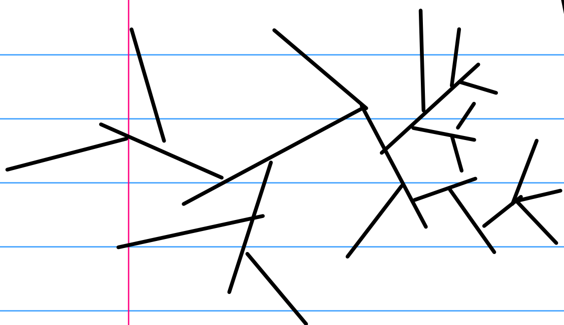
& there is a simple circuit from a to a





blah ⇐ listen to lecture.

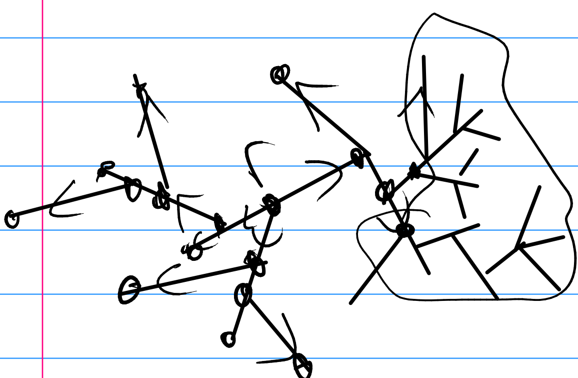
∴ contradiction.



$\boxed{\text{tree}}$

Def:

A rooted tree is a tree in which you designate a "root" and and every edge is directed away from it.

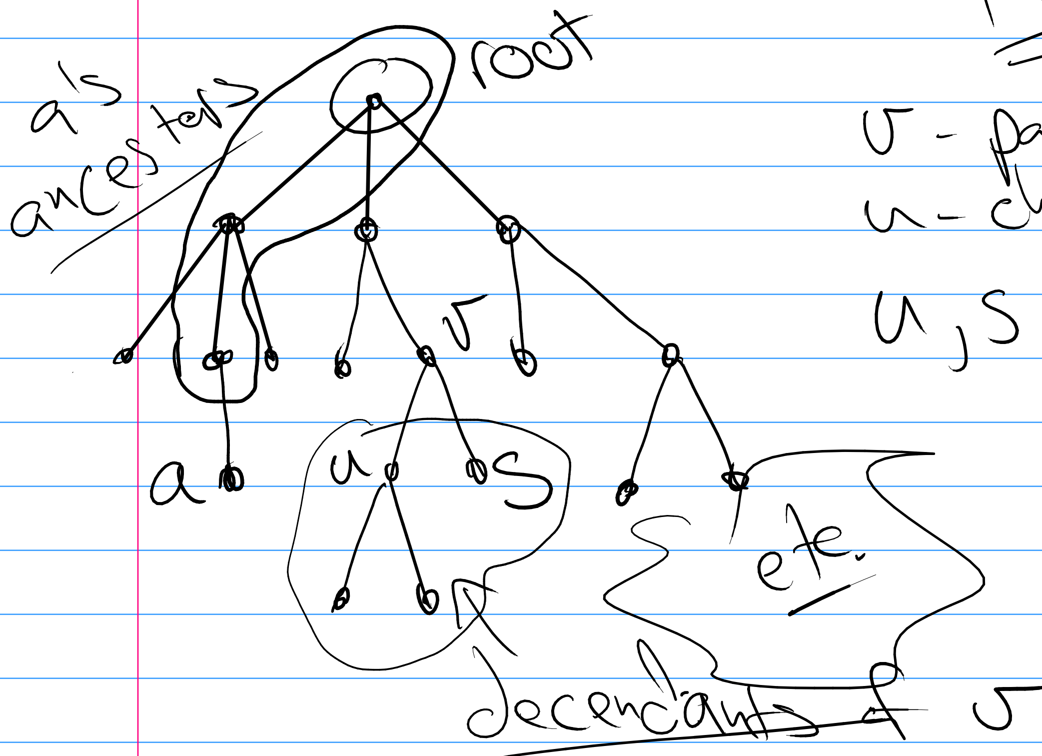


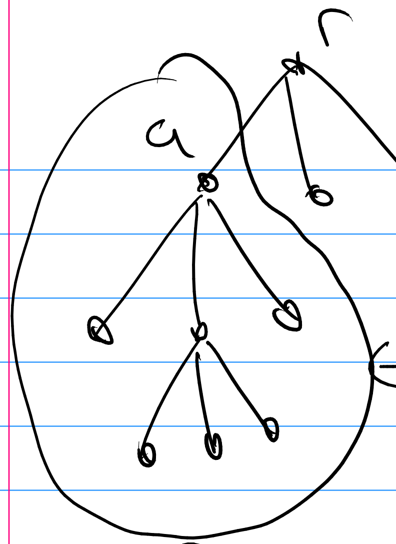
terms.

v - parent of u

u - child of v

u, s are siblings

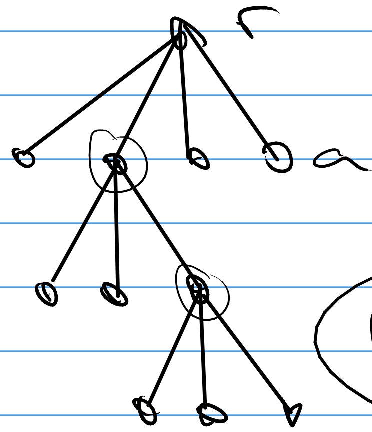




subtree rooted at a.

if you have children = internal vertex

if you do not have children = leaf



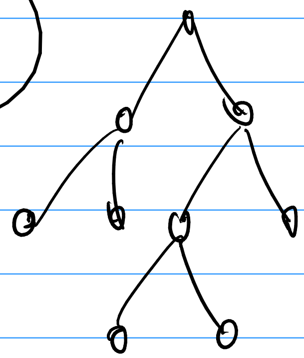
height = max path from root to all vertices

h=3

- balanced tree has all leaves at h or h-1
- m-ary tree has all internal vertices with <u>at most</u> m - children

- full m-ary tree. Internal vertices have <u>exactly</u> m children

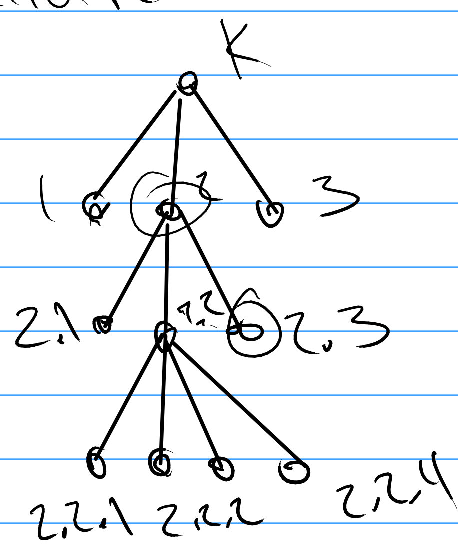
ex



full balanced 2-ary tree.

- ordered rooted tree. You order the children.

ex



Properties of trees

n = all vertices
i = internal vertices
l = leaves

Property $\boxed{n = i + l}$

Th^m: A tree with n vertices has $n-1$ edges. (= children)

Application:

1000 players come to a single elimination tourny.

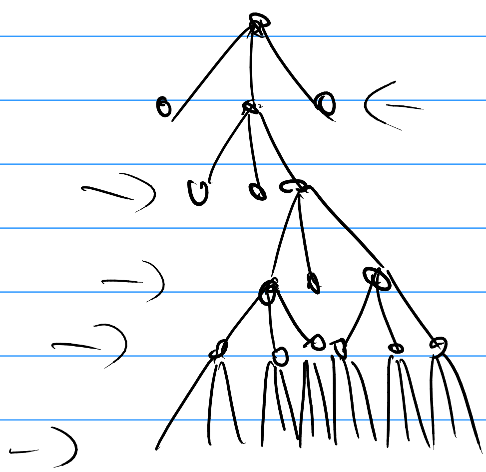
How many games must be played?

$\boxed{999}$ (edges = losers)

Th^m: full m-ary tree

$n - 1 = i \cdot m$

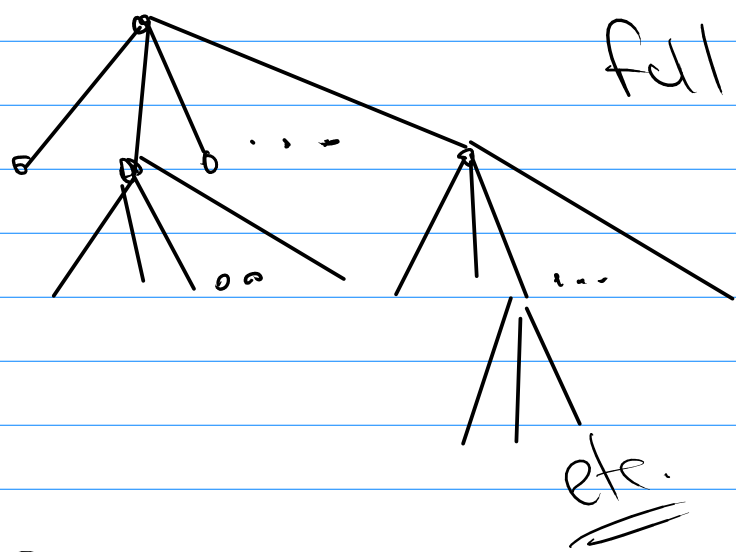
edges = children



$$\boxed{n = mi + 1}$$

$\boxed{\text{thm } 4}$ $\underline{n = i + l}$ $\wedge$ $\underline{n = mi + 1}$

given one of the unknowns $\{n, i, l\}$
you can find the other two.



full 10-ary tree

<u>chain_text.</u>

i = sender

l = non-sender

n = has text

$n-1$ = receivers

$m = 10$
$i = 10{,}000$

$n = i + l$ $\quad$ $n = mi + 1$

$n = 10(10000) + 1 = 100{,}001$

$l = n - i = 100{,}001 - 10{,}000 = 90{,}001$

edges $= n - 1 = 100{,}000$

cost $= \$200{,}000$

height

th^m: m-ary tree of height h

$$l \leq m^h$$

th^m: $h \geq \lceil \log_m l \rceil$

if full and balanced

$$h = \lceil \log_m l \rceil$$

ex $h \geq \lceil \log_{10} 9 \cdot 10^4 \rceil$

$$\lceil \log_{10} 9 + \log_{10} 10^4 \rceil$$

$$h \geq 5$$

$h = 20$

$$l \leq 10^{10} = 10{,}000{,}000$$