Math 293

Q's/ Arclength $= \int_a^b \sqrt{1+(f')^2}\,dx$

Simpsons: Def. Integral $= \int_a^b F(x)\,dx$

Def. Int. $\approx \frac{\Delta x}{3}(F_0 + 4F_1 + 2F_2 + \cdots + 4F_{n-1} + F_n)$

$y = e^x \qquad x \in [0, 3]$

$y' = e^x$

Arclength $= \int_0^3 \sqrt{1+(e^x)^2}\,dx$

Try #1

$u = e^x$

$du = e^x\,dx$

$$\int_{x=0}^{x=3} \frac{\sqrt{1+u^2}}{u}\,du$$

table #23

Finish

or

let $u = 1 + e^{2x}$

$du = 2\underbrace{e^{2x}}_{u-1}\,dx$

$$\frac{1}{2}\int_{x=0}^{x=3} \frac{\sqrt{u}}{u-1}\,du$$

$$\frac{1}{2}\int_{x=0}^{x=3} \frac{\sqrt{u}}{(\sqrt{u})^2 - 1}\,du$$

let $w = \sqrt{u} \qquad dw = \frac{1}{2\sqrt{u}}du$

Try #2 $\frac{1}{2}\int_{x=0}^{x=3} \frac{w}{w^2-1}\, 2w\, dw$

$$\int_{x=0}^{x=3} \boxed{\frac{w^2}{w^2-1}}\, dw$$

use partial fraction decomposition

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\, \Delta x_i$$

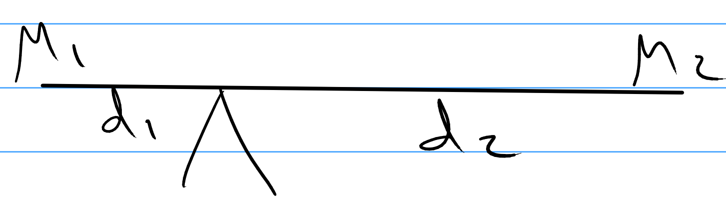
Physics Applications

① Work $\quad W = (\text{Force})(\text{displacement})$

Force ← ?

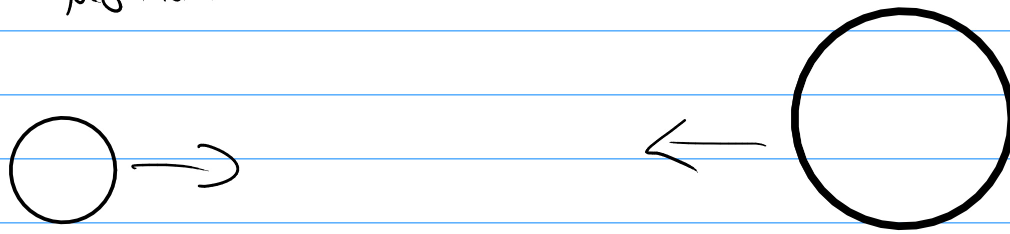
a) Momentum $\quad M \cdot v$

vs Moment



$$M_1 d_1 = M_2 d_2$$

In motion



Momentum (Mass)(velocity)

$$\text{Force} = \frac{d}{dt}\left[(\text{mass})(\text{velocity})\right]$$

$$= (\text{Mass})\frac{d}{dt}\left[\text{velocity}\right]$$

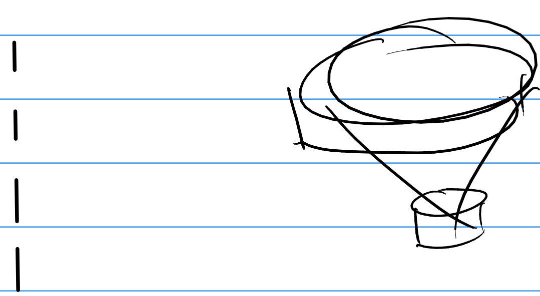
$$= (\text{Mass})\cdot(\text{accel})$$

$$F = ma$$

typically: ① S.I. we keep m and a seperate,

② U.S. measure we put them together (ma) = lbs.

$$\text{Work} = (\text{Force})(\text{displacement})$$

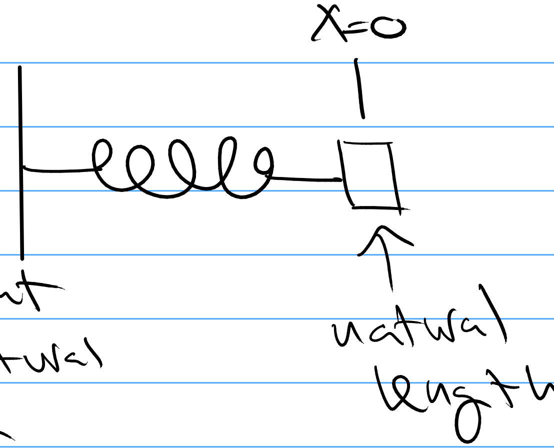


Typically: $W \approx \sum_{i=1}^{n} W_i$

$$W_i = \underbrace{F(x_i^*)}_{\text{changing force}} \underbrace{\Delta x_i}_{\text{small displacement}}$$

Force Examples

① Spring



$F_{spring} \propto$ displacement from natural length

$$F(x) = K \cdot x$$

Ex: work to stretch a spring from 2m to 3m if it takes 25 N to keep it stretched @ 2m?

$N = kg \cdot m/s^2$

$$W = \int_2^3 (Kx)(dx)$$

$25\text{ N} = k \cdot 2\text{ m}$

$k = 12.5\text{ N/m}$

$$W = \int_2^3 12.5x\,dx = 6.25x^2\Big|_2^3$$

$$= 6.25(9-4) = \boxed{31.25\text{ N}\cdot\text{m}}$$

---

ex 10 lb to hold @ 4 in

$10\text{ lb} = k \cdot 4\text{ in} \rightarrow k = 2.5\text{ lb/in}$

Work from natural to 6 in

$$W = \int_0^6 2.5x\,dx$$

$$= 1.25x^2\Big|_0^6 = 1.25(36) = 45\text{ lb}\cdot\text{in}$$

---

(2) Work due to gravity (pumping fluid, lifting chain, etc)

<u>density</u>:

(a) mass density

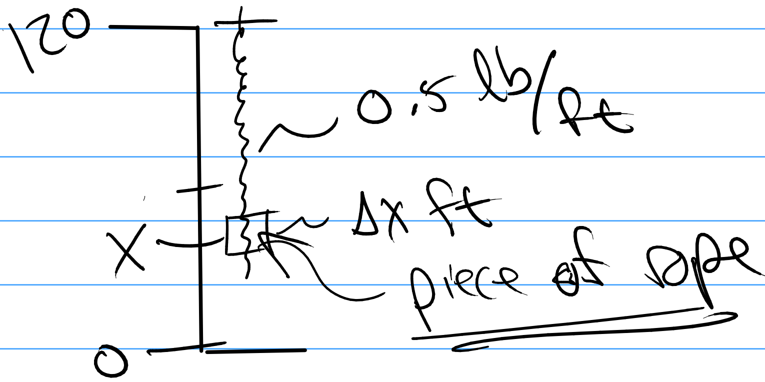
$$= \frac{(\text{units mass})}{(\text{units measure})}$$

(b) weight density $= \dfrac{\text{units weight}}{\text{units measure}}$

Force

ex) heavy rope, 50 ft long, 0.5 lb/ft hangs over building 120 ft high.

Work to pull up the rope?



$$W = \underbrace{F}_{\left(.5 \frac{lb}{ft}\right)(\Delta x \text{ ft})} \cdot \overbrace{d}^{120-x}$$

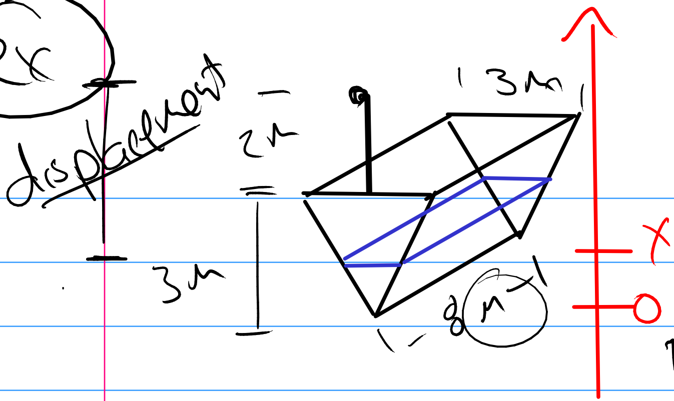
$$W_i = (.5)\Delta x\,(120-x)$$

$$W = \lim \sum .5(120-x)\,\Delta x$$

$$W = \int_{70}^{120} .5(120-x)\,dx$$

$$W = \int_{70}^{120} (60 - .5x)\,dx = \boxed{\text{finish}}$$

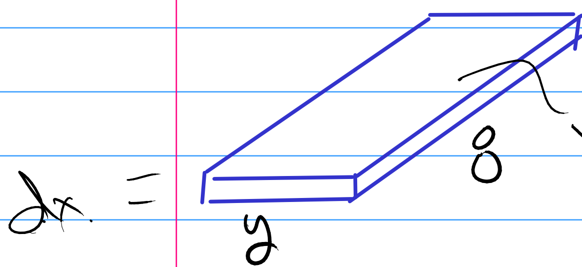
ex



Work to Empty?

$W = (\text{Force})(\underbrace{\text{displacement}}_{5-x})$

$\text{Force} = \underbrace{M}_{\text{Mass}} \cdot \underbrace{\text{accel}}_{9.8\ m/sec^2}$

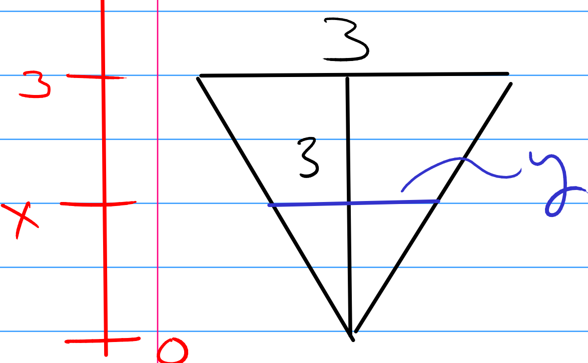


$V = 8y\,dx$

For H_2O US. 62.5 lb/ft³

SI. 1000 kg/m³

to get mass $1000\ \frac{kg}{m^3} \cdot \text{Volume}\ m^3$



$$W = \int_0^3 (\text{Force})(5-x)$$

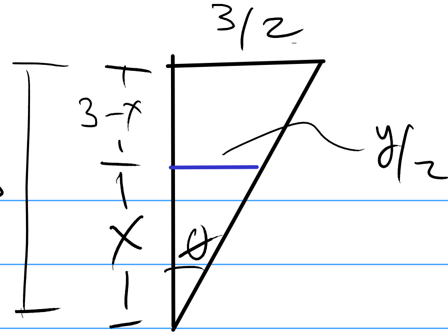
$(\text{Mass})(\text{accel}) = (\text{Mass})(9.8)$

$\text{Force} = \left(1000\ \frac{kg}{m^3} \cdot \text{Volume}\right)(9.8)$

of the blue slice

$$W = \int_0^3 \overbrace{9800\,(8y\,dx)}^{\text{Force}}\,(5-x)$$

but... what is y?



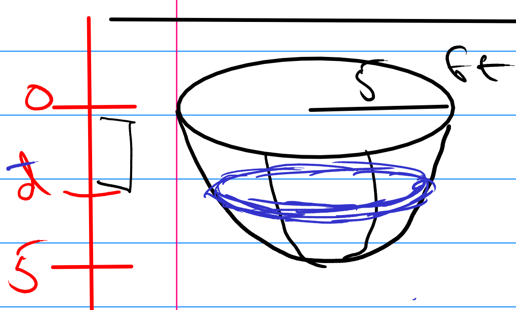
$$\frac{y/2}{x} = \frac{3/2}{3}$$

$$y = x$$

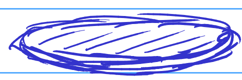
$$W = \int_0^3 9800\,(8\,x\,dx)(5-x)$$

$$W = 9800 \cdot 8 \int_0^3 x(5-x)\,dx$$

$= \underline{\text{finish!}}$



$$\delta = 62.5\ \frac{lb}{ft^3}$$



$$W = \int_0^5 \underbrace{(\text{Force})}_{?}\underbrace{(\text{displacement})}_{d}$$

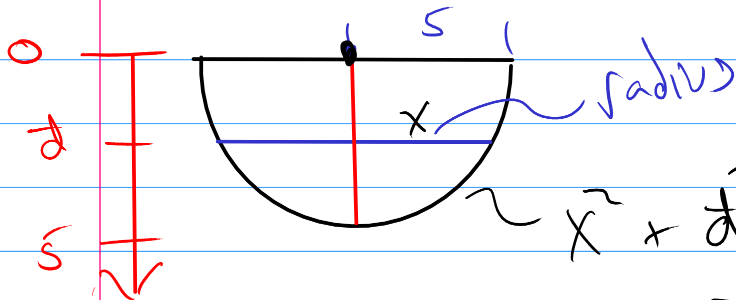
$$\left(62.5\ \frac{lb}{ft^3}\right)(\text{Volume of slice})$$

$$W = \int_0^5 (62.5)(\text{Volume of slice})\ d$$

$d\,d$  ← Volume?

$= (\text{Area of slice})\, d\,d$

Volume of slice $\pi (\text{radius})^2\, d\,d$



$x^2 + d^2 = 5^2$

$\text{radius} = x = \sqrt{25 - d^2}$

$W = \int_0^5 (62.5)(\text{Volume of slice})\, d$

$= \left(\pi(\text{radius}^2)\, d\,d\right)$

$W = \int_0^5 62.5 \cdot \pi (25 - d^2)\, d \;\; d\,d$

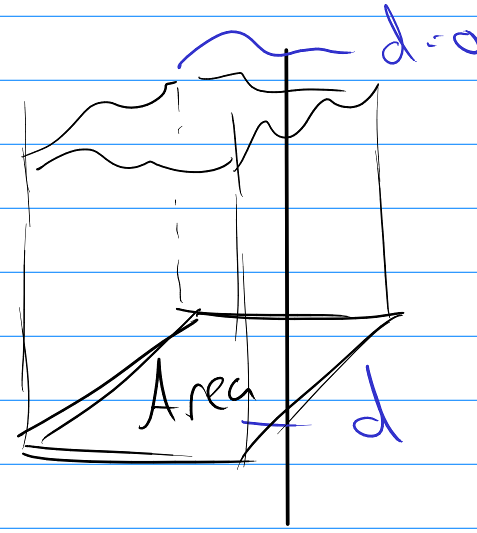
$W = 62.5\pi \int_0^5 (25d - d^3)\, d\,d$

$W =$ finish!

Pressure

$$\frac{\text{Force}}{\text{Area}} = \text{Pressure}$$

ρ = mass density



$$F = (M)\cdot(\text{gravity})$$

$$F = (A \cdot d \cdot \rho) \cdot g$$

$$P = \frac{F}{A} = \rho g d$$

Note: at any point of depth d

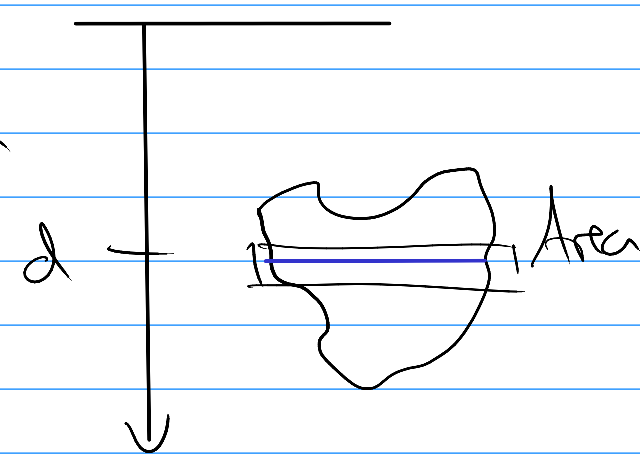
Pressure is same in all directions

$$\underbrace{\boxed{\rho} g}_{\text{mass density}} = \underbrace{\delta}_{\text{force density}}$$

Vertical Plate?

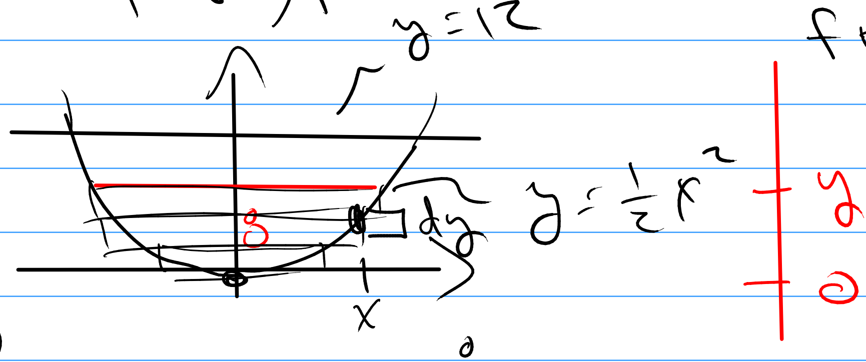
Pressure varies with d. $P(d)$

$$P = F/A$$



$F = P \cdot A$ $\qquad \delta = 42 \frac{lb}{ft^3}$

ex 28



$y = \frac{1}{2}x^2$

$$\int_0^8 (\text{Force}) = \int_0^8 (P)(A)$$

$A = (2x \cdot dy) = (2(\sqrt{2y})\, dy)$

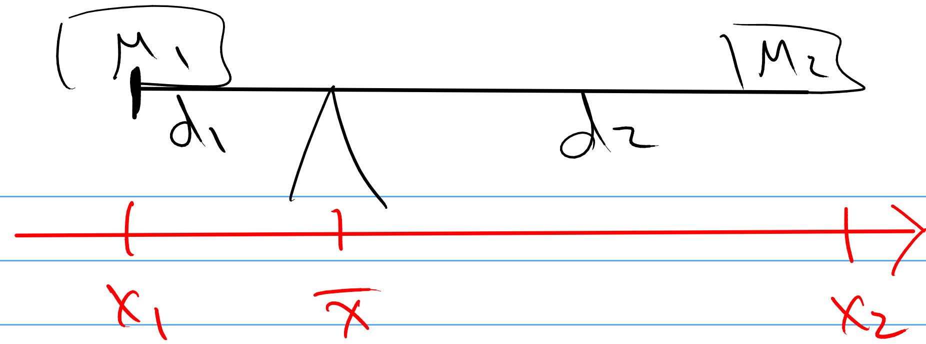
$$= 2\sqrt{2} \int_0^8 P \cdot \sqrt{y}\, dy$$

know: $P = \rho g d = \boxed{\delta d} = 42(8-y)$

$$F = 2\sqrt{2} \int_0^8 42(8-y)\sqrt{y}\, dy$$

$= \text{finish}$

Moments



$$M_1 d_1 = M_2 d_2$$

$$M_1(\bar{x} - x_1) = M_2(x_2 - \bar{x})$$

$$M_1\bar{x} - M_1 x_1 = M_2 x_2 - M_2\bar{x}$$

$$\bar{x} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2}$$

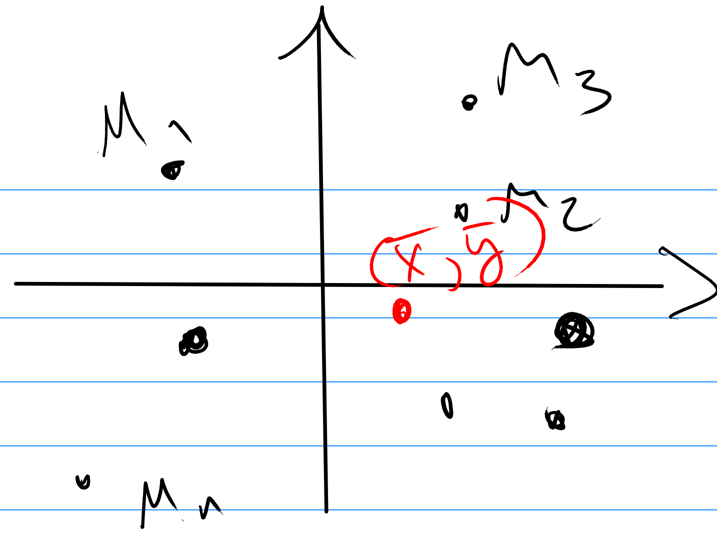
given $\underline{\underline{\text{many}}}$ $M_1, M_2, \ldots, M_n$

$$\bar{x} = \frac{M_1 x_1 + M_2 x_2 + \cdots + M_n x_n}{M_1 + M_2 + \cdots + M_n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} (M_i x_i)}{\sum_{i=1}^{n} M_i}$$

Moment of M_i

2D's



so x's balance is around $x=0$ (y-axis)
y's balance is around $y=0$ (x-axis)

$$M_{(x=0)} = M_{y\text{-axis}} = M_y = \sum_{i=1}^{n} m_i x_i$$

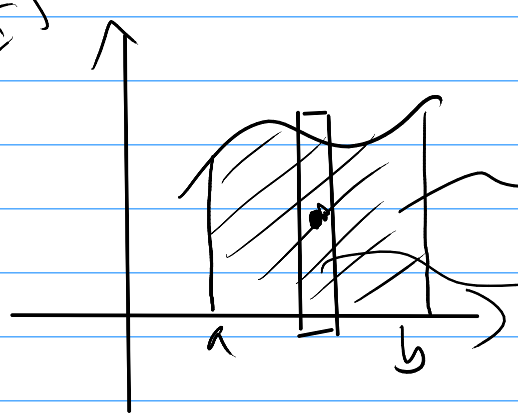
$$M_{(y=0)} = M_{x\text{-axis}} = M_x = \sum_{i=1}^{n} m_i y_i$$

balance

$$\bar{x} = \frac{M_{y\text{-axis}}}{M} \qquad \bar{y} = \frac{M_{x\text{-axis}}}{M}$$

$$M = \sum_{i=1}^{n} m_i$$

sheet



ρ = constant density

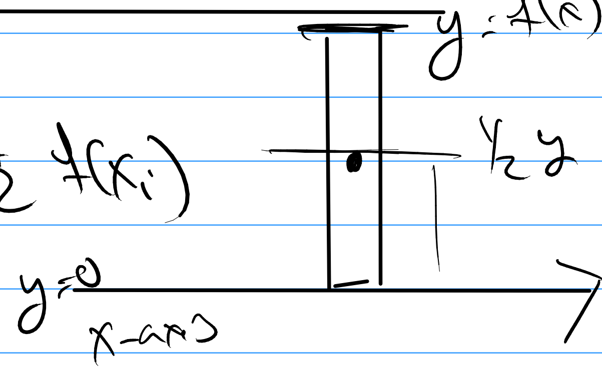
crush slice to the point.

$$\text{Mass} = \overbrace{\left(\frac{\text{Mass}}{\text{Area}}\right)}^{\rho} \text{Area}$$

$$M_{y\text{-axis}} \text{ of } R_i = \underbrace{(\rho \cdot f(x_i)\,\Delta x)}_{\text{Mass}}\ \underbrace{x_i}_{\text{position}}$$

$$M_{y\text{-axis}} = \lim \sum_{i=1}^{n} \rho\, f(x_i)\,\Delta x\, x_i$$

$$M_{y\text{-axis}} = \int_a^b \rho\, x\, f(x)\, dx$$



$$M_{x\text{-axis}} \text{ of } R_i = \left(\rho\, f(x_i)\,\Delta x\right) \frac{1}{2} f(x_i)$$

$$M_{x\text{-axis}} = \int_a^b \frac{1}{2}\rho \left[f(x)\right]^2 dx$$

$$\text{Mass} = M = \rho \int_a^b f(x)\,dx$$

$$\bar{x} = \frac{\int_a^b x\, f(x)\, dx}{\int_a^b f(x)\, dx}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2}\left[f(x)\right]^2 dx}{\int_a^b f(x)\, dx}$$