i) Recap

Lyptohiz condition |f(tixi)-f(tixe) | 2 |xi-xe)

Jolea, I = i focal uniquenes

Globally S. = 7 Global uniquenos

12 F/ KC

. Stability of on equilibrium point

YE>0 3 S(E): 120(0)/2 SE => (204)/2E

· Asymptolic stability: S. Din XE, 20

. Randobnon 1000 10

. Implications of those proportion

Boundodness autoriar V= = |x|2=> V < 0 (x1) S=> |x(1)|(S

is tability initerial for equilibria of autonomous systems

Consider the outermores system

 $(i) \quad sic = f(x)$

where f: D -> IR" is a locally Liptchie map from a domain
DCIR" into IR" Suppose that EED is an equilibrium point of (1),

i. s. P(x)=0 and, without how of generally, assume in.

About Rollines presents themans to ensure the stability of

the equilibrian point is: o.

Theorem Let x=0 be an equilibrium point for (1) and DCIR" be a domain containing x = 0. Let U: D -> IR be a continuously differentiable function such that i) V(0) = 0 and $V(\infty) > 0$ in $D - \{0\}$ ii) \$\int (x) \(\rho \) in D. Then, x=0 is (Socally) stable. Monopour, if iii) V(x) <0 in D-{0}, then x=0 is asymptotically stable. Proof Given Ezo, choose ne (0, E] such that $B_n = \left\{ x \in \mathbb{R}^m : |x| \le n \right\} \subset D$ that is a sphere of nadious r contouried in D. Let d = min V(x) >0, Ixl = n and consider $\Omega_{\beta} = \{ x \in B_n : V(x) \neq \beta \}, \beta \in (0, \infty),$

Then $\Omega_{\beta} \subset B_n$. The set Ω_{β} has the property that any trajectory stantars in Ω_{β} at t=0 stays in Ω_{β} $\forall t \geq 0$. In fact, $\mathcal{V}(x(t)) = 0 \Rightarrow \mathcal{V}(x(t)) = \mathcal{V}(x(t)) = \mathcal{V}(x(t)) = \mathcal{V}(x(t)) = 0$.

Because Ip is a compact set (Ip is closed and bounded),

By repetions the previous arguments, we can choose by a such that $\Omega b \in Ba$. Therefore, it is sufficient to show that $V(X(t)) \rightarrow 0$ as $t \rightarrow +\infty$.

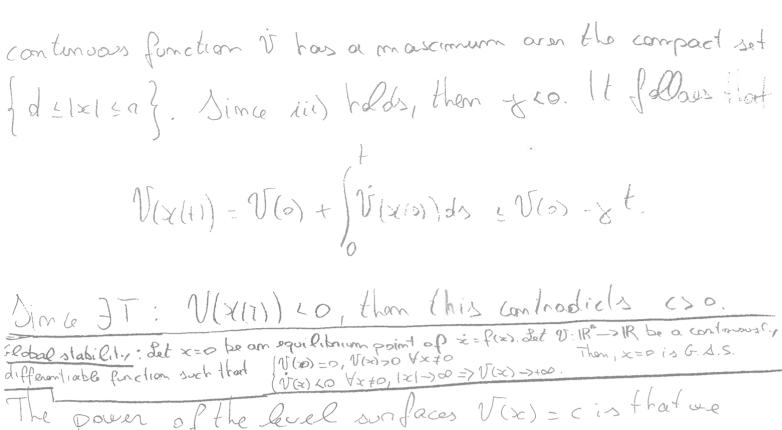
Since V(X(H)) is monotonically decreasing and bounded from below by zero, V(X(H)) meansarily converges to a constant C20, i.e.

lim V(x(+)) = < 20.

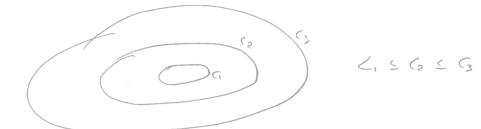
It is left to show that (= 0. So, proceed by contrastiction. Assume cro. By contensity of V(x), \exists dro such that $Bd \in \Omega$. Then

Jim V(x) = c => X(t) & Bd.

Let -8 = max vix), which necessainly exists because the



The power of the level surfaces V(x) = c is that we can deduce



The condition V 50 implies that islam a trajectory noses a Lyapunov surface V(x) = c, it moves inside the set

12 = { scell: V(x) & c}

and can never come out. When it is, the Englectory moves from one level surface to an immer one with a smoth r. ii) Quadratic forms & Define meaning of positive do facts The A class of scalar functions for which the sign definiteness can be easily checked, and consequently easily used to venil, V Lo, is the class of functions of quadratic forms: