

Probability

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Introduction

What is Probability?

Probability deals with uncertainty and quantifies how likely an event is to occur.

- Many real-life situations involve uncertainty rather than certainty
- Probability helps us make informed decisions under uncertainty
- It provides a numerical measure of chance, between 0 and 1
- A probability close to 0 indicates a rare event
- A probability close to 1 indicates a highly likely event

Examples

Probability concepts appear naturally in daily activities.

- Weather forecasting: chance of rain tomorrow
- Medical testing: likelihood that a test result is correct
- Games and sports: chances of winning or losing
- Traffic planning: probability of congestion at a given time
- Finance: risk assessment and expected returns

Example: Tossing a Coin

- The experiment consists of tossing a fair coin once
- Possible outcomes are Head (H) and Tail (T)
- Each outcome has an equal chance of occurring
- Probability of Head = 0.5
- Probability of Tail = 0.5

Key Concept and Terms

Basic Principal of Counting

- If an event can occur in m possible ways and for each of the m possible ways that the first event can occur, there are n possible ways that a second event can occur, then there are in total $m \times n$ possible ways that the two events can occur together
- For example, if a person can go from place A to place B in three possible ways, and B to C in two ways, then there are a total of six ways to go from A to C

Generalized Basic Principle of Counting

- If an event can occur in m_1 possible ways and for each of the possible ways that the first event can occur, there are m_2 possible ways that a second event can occur, and again for each of the $m_1 \times m_2$ possible ways that the first two events can occur, there are m_3 possible ways that a third event can occur, and so on, then there are in total $m_1 \times m_2 \times m_3 \dots$ possible ways that all these events can occur together

Permutation

A permutation is an arrangement of objects where the order matters.

- Number of permutations of r objects chosen from n distinct objects:

$${}^n P_r = \frac{n!}{(n-r)!}$$

- Used when positions or order are important
- Example:
 - Number of ways to arrange 3 students out of 5 in a row:

$${}^5 P_3 = \frac{5!}{2!} = 60$$

Combination

A combination is a selection of objects where the order does not matter.

- Number of combinations of r objects chosen from n distinct objects:

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

- Used when only selection matters, not arrangement
- Example:
 - Number of ways to choose 3 students from 5:

$${}^5C_3 = \frac{5!}{3!2!} = 10$$

Experiment

- An experiment is any process that can be repeated under certain conditions and that produces an observable result
- The result of an experiment is called an outcome
- Example:
 - Tossing a coin or a dice
 - Measuring daily rainfall
 - Conducting chemical reactions

Outcome

- An outcome is a single possible result of an experiment
- There can be one or more *potential* outcomes
- Each experiment produces exactly one outcome
- Outcomes may be numerical or categorical
- Example:
 - Getting a head when tossing a coin
 - Getting a 4 when throwing a dice

Types of Experiment

Experiments can be categorized into two types based on the nature of their outcome(s):

- Deterministic: outcome is known or can be predicted with certainty
- Random: outcome is unknown and cannot be predicted with certainty

Random Experiment

A random experiment is an experiment whose outcome cannot be predicted with certainty.

- The same experiment may produce different outcomes on repetition
- Potential outcomes are known, but which one will occur is uncertain
- Examples:
 - Tossing a coin
 - Rolling a dice
 - Drawing a card from a shuffled deck

Deterministic Experiment

A deterministic experiment is an experiment whose outcome can be predicted with certainty.

- Repeating the experiment under identical conditions gives the same result
- No randomness is involved
- Examples:
 - Calculating the sum of two fixed numbers
 - Measuring the boiling point of pure water at standard pressure

Iteration (Trial or Repetition)

An iteration refers to repeating an experiment under identical conditions.

- Each repetition is called a trial
- Iterations help study long-run behavior of outcomes
- Examples:
 - Tossing a coin 100 times
 - Rolling a die repeatedly and recording outcomes

Sample Space

The sample space is the set of all possible outcomes of a random experiment.

- Denoted by S .
- Each outcome is called a sample point
- Example:
 - Tossing a coin once: $S = \{H, T\}$
 - Rolling a die: $S = \{1, 2, 3, 4, 5, 6\}$
 - Tossing a coin twice: $S = \{HH, HT, TH, TT\}$

Event

An event is any *subset* of the sample space.

- An event may contain one or more outcomes
- A *simple (elementary) event* contains exactly one outcome
- Example (dice roll):
 - Event:
 - ▶ Getting an even number: {2, 4, 6}
 - ▶ Getting four or higher: {4, 5, 6}
 - Simple event: getting a 4: {4}

Mutually Exclusive Events

Two or more events are mutually exclusive if they cannot occur simultaneously.

- They have no common outcomes
- For events A and B :

$$A \cap B = \emptyset$$

- Example (dice roll):
 - A : getting an even number
 - B : getting an odd number

Collectively Exhaustive Events

Events are collectively exhaustive if their union covers the entire sample space.

- At least one of the events must occur
- For events A_1, A_2, \dots, A_n :

$$A_1 \cup A_2 \cup \dots \cup A_n = S$$

- Example:
 - Tossing a coin: $A_1 = \{H\}$ and $A_2 = \{T\}$

Impossible and Sure Events

- An impossible event is an event that cannot occur
 - Probability is 0
 - Example: getting a 7 on a fair die
- A sure (certain) event is an event that always occurs
 - Probability is 1
 - Example: getting a number less than 7 on a fair die

Equally Likely Events

Events are equally likely if each has the same chance of occurring.

- Common in experiments with symmetry
- Example:
 - Tossing a fair coin: $P(H) = P(T) = 0.5$
 - Rolling a fair die: each outcome has probability $1/6$

Thank you.
