Probability

Md. Aminul Islam Shazid

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Introduction

What is Probability?

Probability deals with uncertainty and quantifies how likely an event is to occur.

- Many real-life situations involve uncertainty rather than certainty
- Probability helps us make informed decisions under uncertainty
- It provides a numerical measure of chance, between 0 and 1
- A probability close to 0 indicates a rare event
- A probability close to 1 indicates a highly likely event

Examples

Probability concepts appear naturally in daily activities.

- Weather forecasting: chance of rain tomorrow
- Medical testing: likelihood that a test result is correct
- Games and sports: chances of winning or losing
- Traffic planning: probability of congestion at a given time
- Finance: risk assessment and expected returns

Example: Tossing a Coin

- The experiment consists of tossing a fair coin once
- Possible outcomes are Head (H) and Tail (T)
- Each outcome has an equal chance of occurring
- Probability of Head = 0.5
- Probability of Tail = 0.5

Key Concept and Terms

Basic Principal of Counting

- If an event can occur in m possible ways and for each of the m possible ways that the first event can occur, there are n possible ways that a second event can occur, then there are in total $m \times n$ possible ways that the two events can occur together
- For example, if a person can go from place A to place B in three possible ways, and B to C in two ways, then there are a total of six ways to go from A to C

Generalized Basic Principle of Counting

- If an event can occur in m_1 possible ways and for each of the possible ways that the first event can occur, there are m_2 possible ways that a second event can occur, and again for each of the $m_1 \times m_2$ possible ways that the first two events can occur, there are m_3 possible ways that a third event can occur, and so on, then there are in total $m_1 \times m_2 \times m_3 \ldots$ possible ways that all these events can occur together

Permutation

A permutation is an arrangement of objects where the order matters.

- Number of permutations of r objects chosen from n distinct objects:

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

- Used when positions or order are important
- Example:
  - Number of ways to arrange 3 students out of 5 in a row:

$$ {}^{5}P_{3} = \frac{5!}{2!} = 60 $$

Combination

A combination is a selection of objects where the order does not matter.

- Number of combinations of r objects chosen from n distinct objects:

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

- Used when only selection matters, not arrangement
- Example:
  - Number of ways to choose 3 students from 5:

$$^{5}C_{3} = \frac{5!}{3!2!} = 10$$

Experiment

- An experiment is any process that can be repeated under certain conditions and that produces an observable result
- The result of an experiment is called an outcome
- Example:
  - Tossing a coin or a dice
  - Measuring daily rainfall
  - Conducting chemical reactions

Outcome

- An outcome is a single possible result of an experiment
- There can be one or more *potential* outcomes
- Each experiment produces exactly one outcome
- Outcomes may be numerical or categorical
- Example:
  - Getting a head when tossing a coin
  - Getting a 4 when throwing a dice

Types of Experiment

Experiments can be categorized in to two types based on the nature of their outcome(s):

- Deterministic: outcome is known or can be predicted with certainty
- Random: outcome is unknown and cannot be predicted with certainty

Random Experiment

A random experiment is an experiment whose outcome cannot be predicted with certainty.

- The same experiment may produce different outcomes on repetition
- Potential outcomes are known, but which one will occur is uncertain
- Examples:
  - Tossing a coin
  - Rolling a dice
  - Drawing a card from a shuffled deck

Deterministic Experiment

A deterministic experiment is an experiment whose outcome can be predicted with certainty.

- Repeating the experiment under identical conditions gives the same result
- No randomness is involved
- Examples:
  - Calculating the sum of two fixed numbers
  - Measuring the boiling point of pure water at standard pressure

Iteration (Trial or Repetition)

An iteration refers to repeating an experiment under identical conditions.

- Each repetition is called a trial
- Iterations help study long-run behavior of outcomes
- Examples:
  - Tossing a coin 100 times
  - Rolling a die repeatedly and recording outcomes

Thank you.