Probability

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Introduction

What is Probability?

Probability deals with uncertainty and quantifies how likely an event is to occur.

- Many real-life situations involve uncertainty rather than certainty
- Probability helps us make informed decisions under uncertainty
- It provides a numerical measure of chance, between 0 and 1
- A probability close to 0 indicates a rare event
- A probability close to 1 indicates a highly likely event

Examples

Probability concepts appear naturally in daily activities.

- Weather forecasting: chance of rain tomorrow
- Medical testing: likelihood that a test result is correct
- Games and sports: chances of winning or losing
- Traffic planning: probability of congestion at a given time
- Finance: risk assessment and expected returns

Example: Tossing a Coin

- The experiment consists of tossing a fair coin once
- Possible outcomes are Head (H) and Tail (T)
- Each outcome has an equal chance of occurring
- Probability of Head = 0.5
- Probability of Tail = 0.5

Key Concept and Terms

Basic Principal of Counting

- If an event can occur in m possible ways and for each of the m possible ways that the first event can occur, there are n possible ways that a second event can occur, then there are in total $m \times n$ possible ways that the two events can occur together
- For example, if a person can go from place A to place B in three possible ways, and B to C in two ways, then there are a total of six ways to go from A to C

Generalized Basic Principle of Counting

- If an event can occur in m_1 possible ways and for each of the possible ways that the first event can occur, there are m_2 possible ways that a second event can occur, and again for each of the $m_1 \times m_2$ possible ways that the first two events can occur, there are m_3 possible ways that a third event can occur, and so on, then there are in total $m_1 \times m_2 \times m_3 \ldots$ possible ways that all these events can occur together

Permutation

A permutation is an arrangement of objects where the order matters.

- Number of permutations of r objects chosen from n distinct objects:

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

- Used when positions or order are important
- Example:
  - Number of ways to arrange 3 students out of 5 in a row:

$$ {}^{5}P_{3} = \frac{5!}{2!} = 60 $$

Combination

A combination is a selection of objects where the order does not matter.

- Number of combinations of r objects chosen from n distinct objects:

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

- Used when only selection matters, not arrangement
- Example:
  - Number of ways to choose 3 students from 5:

$$^{5}C_{3} = \frac{5!}{3!2!} = 10$$

Experiment

- An experiment is any process that can be repeated under certain conditions and that produces an observable result
- The result of an experiment is called an outcome
- Example:
  - Tossing a coin or a dice
  - Measuring daily rainfall
  - Conducting chemical reactions

Outcome

- An outcome is a single possible result of an experiment
- There can be one or more *potential* outcomes
- Each experiment produces exactly one outcome
- Outcomes may be numerical or categorical
- Example:
  - Getting a head when tossing a coin
  - Getting a 4 when throwing a dice

Types of Experiment

Experiments can be categorized in to two types based on the nature of their outcome(s):

- Deterministic: outcome is known or can be predicted with certainty
- Random: outcome is unknown and cannot be predicted with certainty

Random Experiment

A random experiment is an experiment whose outcome cannot be predicted with certainty.

- The same experiment may produce different outcomes on repetition
- Potential outcomes are known, but which one will occur is uncertain
- Examples:
  - Tossing a coin
  - Rolling a dice
  - Drawing a card from a shuffled deck

Deterministic Experiment

A deterministic experiment is an experiment whose outcome can be predicted with certainty.

- Repeating the experiment under identical conditions gives the same result
- No randomness is involved
- Examples:
  - Calculating the sum of two fixed numbers
  - Measuring the boiling point of pure water at standard pressure

Iteration (Trial or Repetition)

An iteration refers to repeating an experiment under identical conditions.

- Each repetition is called a trial
- Iterations help study long-run behavior of outcomes
- Examples:
  - Tossing a coin 100 times
  - Rolling a die repeatedly and recording outcomes

Sample Space

The sample space is the set of all possible outcomes of a random experiment.

- Denoted by S.
- Each outcome is called a sample point
- Example:
  - Tossing a coin once: $S = \{H, T\}$
  - Rolling a die: $S = \{1, 2, 3, 4, 5, 6\}$
  - Tossing a coin twice: $S = \{HH, HT, TH, TT\}$

Event

An event is any *subset* of the sample space.

- An event may contain one or more outcomes
- A *simple (elementary) event* contains exactly one outcome
- Example (dice roll):
  - Event:
    - Getting an even number: $\{2, 4, 6\}$
    - Getting four or higher: $\{4, 5, 6\}$
  - Simple event: getting a 4: $\{4\}$

Mutually Exclusive Events

Two or more events are mutually exclusive if they cannot occur simultaneously.

- They have no common outcomes
- For events A and B:

$$A \cap B = \emptyset$$

- Example (dice roll):
  - A: getting an even number
  - B: getting an odd number

Collectively Exhaustive Events

Events are collectively exhaustive if their union covers the entire sample space.

- At least one of the events must occur
- For events $A_1, A_2, \dots, A_n$:

$$A_1 \cup A_2 \cup \cdots \cup A_n = S$$

- Example:
  - Tossing a coin: $A_1 = \{H\}$ and $A_2 = \{T\}$

Impossible and Sure Events

- An impossible event is an event that cannot occur
  - Probability is 0
  - Example: getting a 7 on a fair die
- A sure (certain) event is an event that always occurs
  - Probability is 1
  - Example: getting a number less than 7 on a fair die

Equally Likely Events

Events are equally likely if each has the same chance of occurring.

- Common in experiments with symmetry
- Example:
  - Tossing a fair coin: $P(H) = P(T) = 0.5$
  - Rolling a fair die: each outcome has probability $1/6$

Definition of Probability

Classical Definition of Probability

The classical definition applies when outcomes are equally likely.

- If A is an event:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

- Example:
  - Probability of getting an even number on a dice:

$$P(\text{getting an even number}) = \frac{3}{6} = \frac{1}{2}$$

Frequency (Empirical) Definition of Probability

Probability is defined as the long-run relative frequency of an event.

- Based on repeated experiments
- If an event A occurs f times in n trials:

$$P(A) \approx \lim_{n \to \infty} \frac{f}{n}$$

- Becomes more accurate as n increases
- For example, if a coin is tossed $1{,}000$ times and 520 heads are seen, then probability of getting a head is $520/1000 = 0.52$

Axiomatic Definition of Probability

Probability is defined using a set of axioms.

- Proposed by Kolmogorov
- For any event A:
  - $0 \leq P(A) \leq 1$ *(probability is a number between* 0 *and* 1*)*
  - $P(S) = 1$ *(probability of sample space is* 1*)*
  - For a sequence of disjoint (mutually exclusive) events $A_1, A_2, ..., A_k$:

$$P(A_1, A_2, ..., A_k) = P(A_1) + P(A_2) + ... + P(A_k)$$

- Forms the foundation of modern probability theory

Thank you.