Random Variable and Probabiilty Distribution

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Outline

1. Random Variable
2. Probability Distribution

Random Variable

Introduction

A variable which takes numerical values resulting from random experiments, is called a random variable (RV).

- There is a probability associated with each possible values
- Random variables are denoted by capital letters such as X, Y, Z etc.
- Possible values are denoted by small letters such as x, y, z etc.
- Example:
  - Height of students
  - Number of heads when tossing a coin three times

Types of Random Variable

- **Discrete random variable**: A random variable defined over a discrete sample space
  - Number of students in a class, sample space, $S = 0, 1, 2, ..., \infty$
  - Number of correct answers in among 50 questions, $S = 0, 1, 2, ..., 50$
- **Continuous random variable**: A random variable defined over a continuous sample space
  - Monthly income, $S = X : 0 \leq x < \infty$
  - Monthly profit, $S = X : -\infty < x < \infty$

Probability Distribution

Probability Distribution

- Probability distribution means the distribution of the probabilities among the different values of a random variable
- For example, when tossing a coin twice, the sample space, $S = \{HH, HT, TH, TT\}$
- Let an RV, $X =$ number of heads in two coin tosses, then X can take the values: $0, 1, 2$
- Then the probability of each value of X:

x	$P(X = x)$
0	$1/4$
1	$2/4$
2	$1/4$

Types of Probability Distributions

Depending on the type of variable, distributions are of two types:

- **Discrete probability distribution**: probability distribution of a discrete random variable
- **Continuous probability distribution**: probability distribution of a continuous random variable

Discrete probability distribution

- The probability distribution of a discrete random variable is a table, graph, formula, or other device used to specify all possible values of a discrete random variable along with their respective probabilities
- If we let the discrete probability distribution be represented by the function $p(x)$, then $p(x) = P(X = x)$ is the probability of the discrete random variable X to assume a value x
- $p(x)$ is called a probability mass function (PMF)

Probability Mass Function (PMF)

A function, $p(x)$, of a discrete random variable X will be called a PMF, if and only if all the following conditions are satisfied:

1. $p(x) \geq 0; \ \forall x$
2. $\sum_x p(x) = 1$
3. $P(X = a) = p(a)$

Example: PMF

In page 3, the probability mass function of the variable X representing the number of heads when a coin is tossed twice is given. It is an example of a PMF.

x	$P(X = x)$
0	$1/4$
1	$2/4$
2	$1/4$

Thank you.