COMP361: Numerical Methods
Concordia
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Assignment 1

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1. Figures 1 and 2 show the behaviour of the harmonic sum for various values of N. Note that the N values are plotted on a log scale. In both plots we see that the sum continues to grow as N increases.

   The sum of a geometric series with constant ratio r less than 1 sum can be calculated using the formula[1]

   $$\sum_{k=1}^{N} r^k = \frac{r(1-r^N)}{1-r}$$

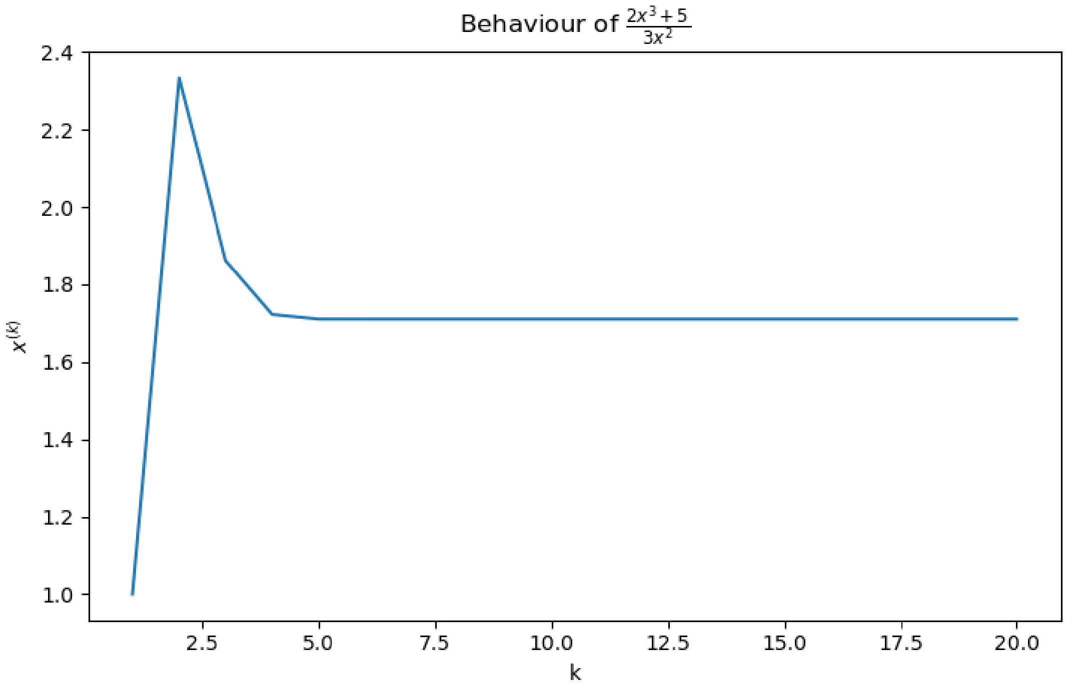
   Our series has $r = \frac{1}{3}$, so the sum is

   $$\frac{\frac{1}{3}(1-(\frac{1}{3})^N)}{1-\frac{1}{3}} = \frac{1}{2}(1-\frac{1}{3^N})$$

   In the limit of large N, the sum of a geometric series with constant ratio less than 1 can be calculated using the formula[1]

   $$\sum_{k=1}^{\infty} = \frac{r}{1-r}$$

   Which for $r = \frac{1}{3}$ is $\frac{1}{2}$.



2.

We see from the above figure that sequence initially rises before settling to a fixed point x^* after $k = 5$. We find this fixed point by solving

$$x = f(x)$$

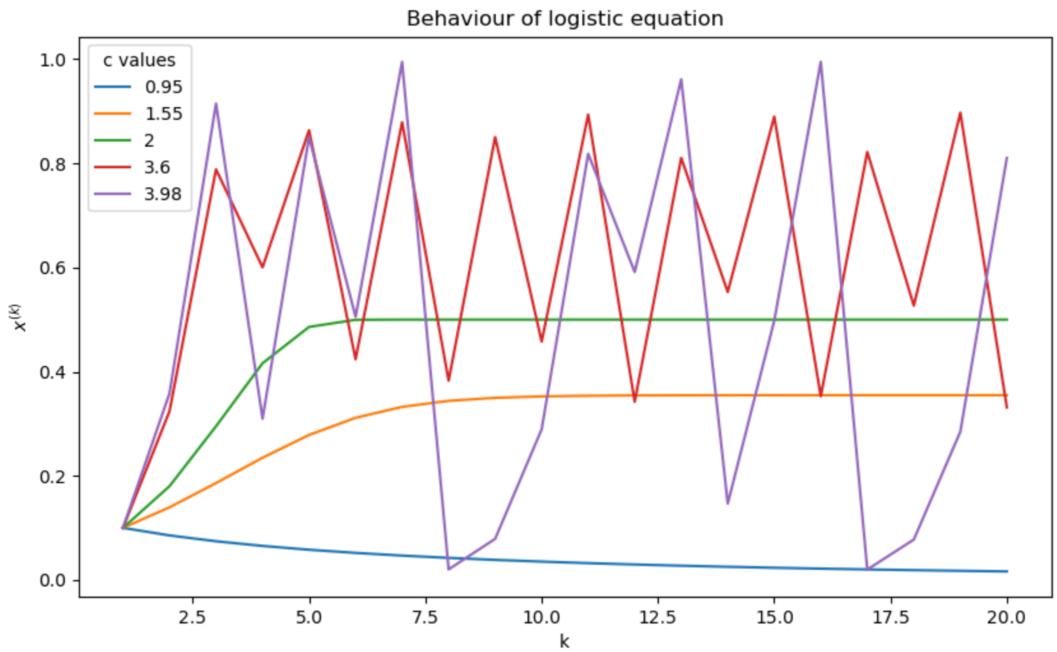
where

$$f(x) = \frac{2x^3 + 5}{3x^2}$$

therefore

$$x^* = \sqrt[3]{5}$$

This fixed point does not depend on $x^{(0)}$.



When $c = 0.95$, the sequence is attacted to the fixed point at 0.
When $c = 1.55$, the sequence is attracted to the fixed point at $1 - \frac{1}{1.55} = 0.355$.
When $c = 2$, the sequence is attracted to the fixed point at $1 - \frac{1}{2} = \frac{1}{2}$.
When $c = 3.6$, $|f'(x^*)| = 1.6 > 1$, so the sequence is repelled from the fixed point at $1 - \frac{1}{3.6} = 0.722$.
When $c = 3.98$, $|f'(x^*)| = 1.98 > 1$, so the sequence is repelled from the fixed point at $1 - \frac{1}{3.98} = 0.749$.

References

[1] Eric W. Weisstein. *Geometric Series. From MathWorld–A Wolfram Web Resource.* Visited on 6/2/21. URL: http://mathworld.wolfram.com/GeometricSeries.html.