COMP361: Numerical Methods

Concordia
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Assignment 1

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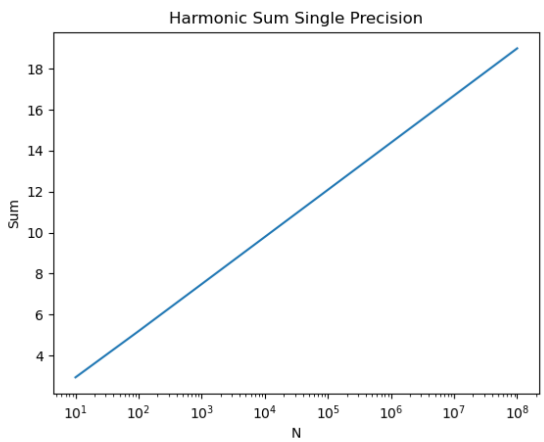
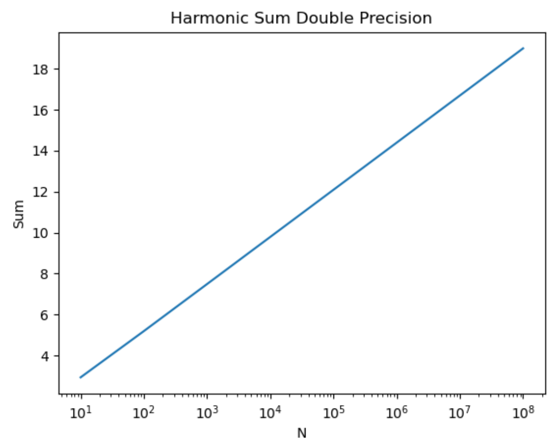
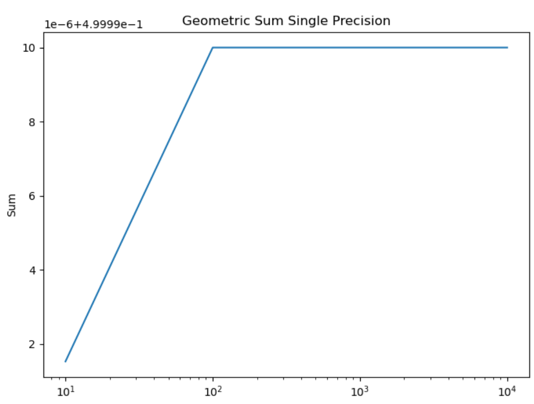
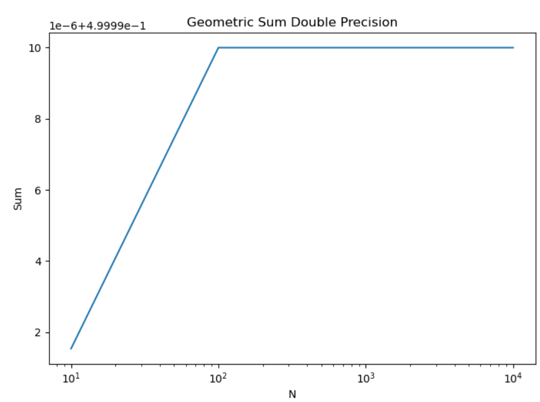




1.

Figures 1 and 2 show the behaviour of the harmonic sum for various values of N. Note that the N values are plotted on a log scale. In both plots we see that the sum continues to grow as N increases.





In both cases the sum has converged when $N = 100$.
The sum of a geometric series with constant ratio r less than 1 sum can be calculated using the formula[2]

$$\sum_{k=1}^{N} r^k = \frac{r(1-r^N)}{1-r}$$

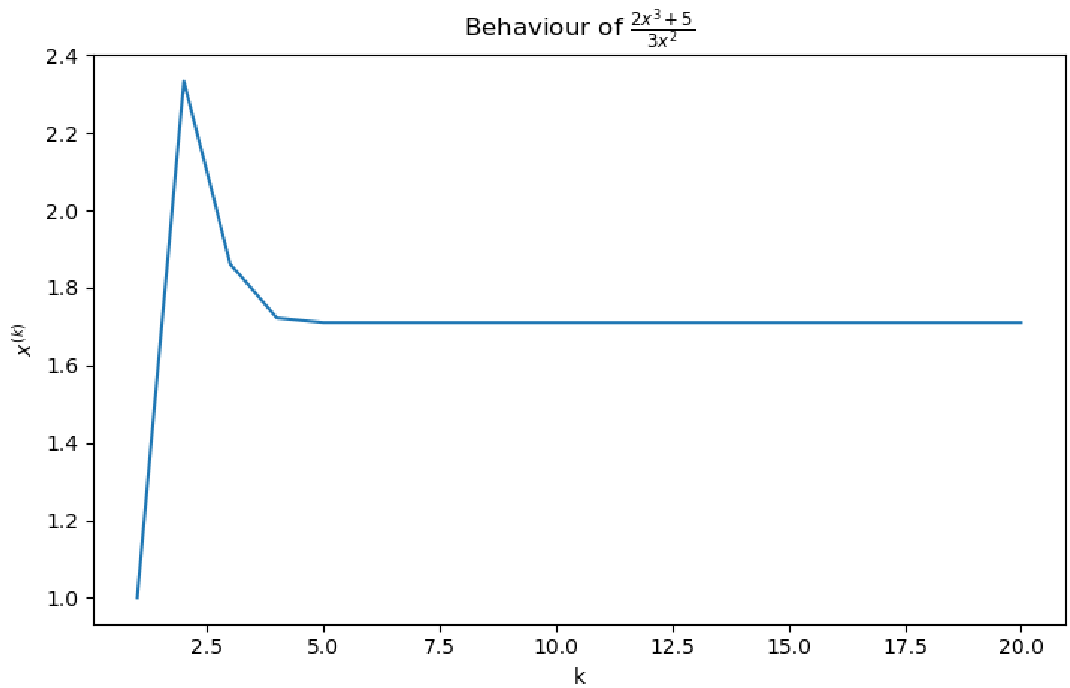
Our series has $r = \frac{1}{3}$, so the sum is

$$\frac{\frac{1}{3}(1 - (\frac{1}{3})^N)}{1 - \frac{1}{3}} = \frac{1}{2}(1 - \frac{1}{3^N})$$

In the limit of large N, the sum of a geometric series with constant ratio less than 1 can be calculated using the formula[2]

$$\sum_{k=1}^{\infty} = \frac{r}{1-r}$$

Which for $r = \frac{1}{3}$ is $\frac{1}{2}$.



2.

We see from the above figure that sequence initially rises before settling to a fixed point x^* after $k = 5$. We find this fixed point by solving

$$x = f(x)$$

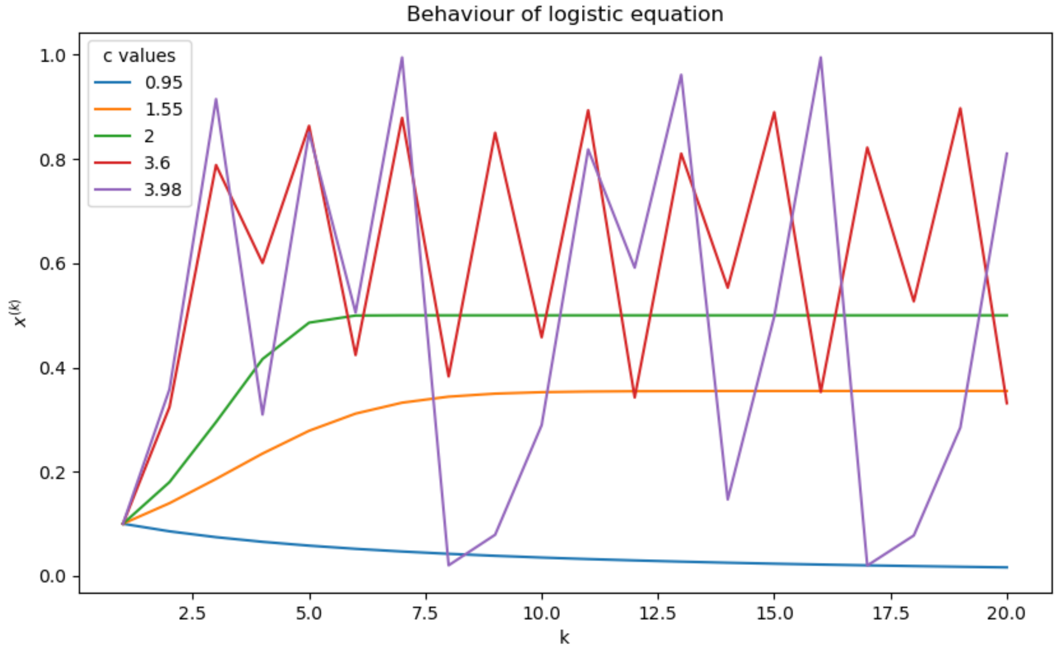
where

$$f(x) = \frac{2x^3 + 5}{3x^2}$$

therefore

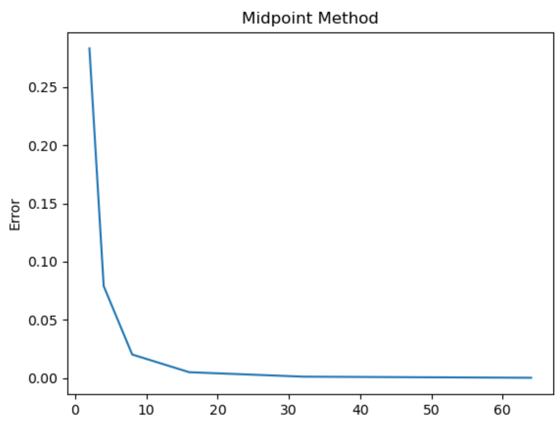
$$x^* = \sqrt[3]{5}$$

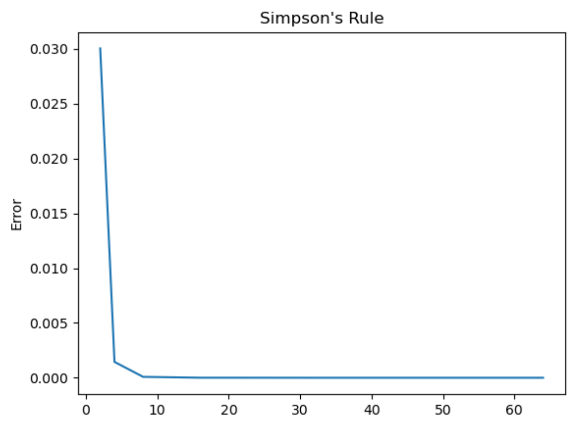
This fixed point does not depend on $x^{(0)}$.



When $c = 0.95$, the sequence is attacted to the fixed point at 0.
When $c = 1.55$, the sequence is attracted to the fixed point at $1 - \frac{1}{1.55} = 0.355$.
When $c = 2$, the sequence is attracted to the fixed point at $1 - \frac{1}{2} = \frac{1}{2}$.
When $c = 3.6$, $|f'(x^*)| = 1.6 > 1$, so the sequence is repelled from the fixed point at $1 - \frac{1}{3.6} = 0.722$.
When $c = 3.98$, $|f'(x^*)| = 1.98 > 1$, so the sequence is repelled from the fixed point at $1 - \frac{1}{3.98} = 0.749$.

3.





For the Midpoint Method, the smallest M for which the error is less than 10^{-7} is 2^{11}. This corresponds to 2048 function evaluations.
For Simpson's Method, the smallest M for which the error is less than 10^{-7} is 2^6. This corresponds to 65 function evaluations.

For both methods, the error decreases sharply with h. As M gets very large the error tends towards 0.

a) The error bound for the Midpoint Rule is given by[1]

$$|E_M| \leq \|f''(x)\|_\infty \frac{(b-a)^3}{24M^2}$$

In our case, $|f''(x)| = |-\pi^2 \sin(\pi x)|$, which is greatest when the argument to the sin term is $\frac{\pi}{2}$, i.e. at $x = \frac{1}{2}$. So

$$\|f''(x)\|_\infty = \pi^2$$

Since we seek an error less than 10^{-7},

$$\begin{aligned} 10^{-7} &\geq \pi^2 \frac{1}{24M^2} \\ M &\geq \pi\sqrt{\frac{1}{24(10^{-7})}} \\ M &\geq 2027.889 \end{aligned}$$

So the smallest M such that the error is less than 10^{-7} is 2028.

b) The error bound for Simpson's Rule is given by[1]

$$|E_S| \leq \|f^{(4)}(x)\|_\infty \frac{(b-a)^5}{180M^4}$$

Similar to above, $\|f^{(4)}(x)\|_\infty = \|-\pi^4 \sin(\pi x)\|_\infty = \pi^4$, so

$$\begin{aligned} 10^{-7} &\geq \pi^4 \frac{1}{180M^4} \\ M &\geq \pi\sqrt[4]{\frac{1}{180(10^{-7})}} \\ M &\geq 48.232 \end{aligned}$$

So the smallest M such that the error is less than 10^{-7} is 50.

References

[1] Aaron Leclair. *Error Bound Theorems*. Visited on 6/2/21. URL: `https://www.math.cmu.edu/~mittal/Recitation_notes.pdf`.

[2] Eric W. Weisstein. *Geometric Series. From MathWorld–A Wolfram Web Resource*. Visited on 6/2/21. URL: `http://mathworld.wolfram.com/GeometricSeries.html`.