## Plastic Bistable Recurrent Cells

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## Abstract

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### Introduction

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## Background

#### Recurrent Neural Networks

A recurrent neural network (RNN) is a type of neural network that excels at temporal prediction tasks. Inputs are fed into the network one at a time, and the model maintains a memory in the form of a hidden state vector  $\mathbf{h}_t$  that is updated at each timestep. The model can then make predictions based on the contents of the input as well as the memory. At each timestep, the model updates the hidden state using the rule

$$\mathbf{h}_t = \sigma(W_h \mathbf{h}_{t-1} + W_x \mathbf{x}_t + b_h)$$

The output of the model can then be computed as

$$\mathbf{y}_t = \sigma(W_u \mathbf{h}_t + b_u)$$

where  $\sigma$  is a nonlinear activation function,  $W_h$ ,  $W_x$  and  $W_y$  are weight matrices learned by the model,  $b_h$  and  $b_y$  are learned bias vectors and  $\mathbf{x}_t$  is the input at time t. These weight matrices and bias vectors are typically learned using gradient descent.

While RNNs have proven effective for many tasks, in this standard form their ability to capture long-term dependencies is limited. This is because of the so-called vanishing/exploding gradient problem, which is a result of the repeated application of the weight matrix  $W_h$  at each timestep. Various approaches have been proposed to address this problem. One approach is to use a modified

update rule that avoids changing the hidden state unless necessary. This is the approach used by the Gated Recurrent Unit.

#### Gated Recurrent Units

Gated Recurrent Units (GRUs) [1] are a type of RNN that preserve the gradient using a modified update rule. At each timestep the model calculates

$$\mathbf{z}_t = \sigma(W_{zh}\mathbf{h}_{t-1} + W_{zx}\mathbf{x}_t + \mathbf{b}_z)$$
  
$$\mathbf{r}_t = \sigma(W_{rh}\mathbf{h}_{t-1} + W_{rx}\mathbf{x}_t + \mathbf{b}_r)$$

known as the update and reset gate, respectively. These are then used to calculate

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{r}_t \odot W_{hh} \mathbf{h}_{t-1} + W_{hx} \mathbf{x}_t + \mathbf{b}_h)$$
$$\mathbf{h}_t = \mathbf{z}_t \odot \tilde{\mathbf{h}}_t + (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1}$$

where  $W_{zh}$ ,  $W_{zx}$ ,  $W_{rh}$ ,  $W_{rx}$ ,  $W_{hh}$ ,  $W_{hx}$  are weight matrices,  $b_z$ ,  $b_r$ ,  $b_h$  are bias vectors, and  $\sigma$  is a nonlinear activation function. Since at each timestep  $\mathbf{h}_t$  is updated using linear interactions only, the vanishing gradient problem is much less pernicious, and the model is able to learn much longer term dependencies than the standard RNN model.

#### Bistable Recurrent Cells

Bistable Recurrent Cells (BRCs) [2] are another recurrent model similar to the GRU that allows for each individual unit of the memory to hold onto memories for an arbitrarily long time. The BRC modifies the update and reset gate equations to

$$\mathbf{z}_t = \sigma(\mathbf{w}_z \odot \mathbf{h}_{t-1} + W_z \mathbf{x}_t + \mathbf{b}_z)$$
  
$$\mathbf{r}_t = 1 + \tanh(\mathbf{w}_r \odot \mathbf{h}_{t-1} + W_r \mathbf{x}_t + \mathbf{b}_r$$

where  $\mathbf{w}_z$  and  $\mathbf{w}_r$  are now weight vectors multiplied elementwise with the previous hidden state. The equation for  $\tilde{\mathbf{h}}_t$  is also modified, to

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{r}_t \odot \mathbf{h}_{t-1} + W_h \mathbf{x}_t + \mathbf{b}_h)$$

Since these equations change all matrix multiplications with the hidden state vector to elementwise multiplications, all interactions between elements of the hidden state are removed. Instead, each unit of the hidden state interacts only with itself. Since it features only local computations, the BRC in this form is a much more plausible model of how biological neural networks might function than either RNNs or GRUs.

#### Neuromodulated Bistable Recurrent Cells

[2] also introduced another form of the BRC that adds back in the interaction between elements of the hidden state in the equations for the update and reset gates. By relaxing the biological plausibility requirement, this modified BRC showed improved performance on a number of tasks. The update and reset gate equations are now

$$\mathbf{z}_t = \sigma(W_{zh}\mathbf{h}_{t-1} + W_{zx}\mathbf{x}_t + \mathbf{b}_z)$$
  
$$\mathbf{r}_t = \sigma(W_{rh}\mathbf{h}_{t-1} + W_{rx}\mathbf{x}_t + \mathbf{b}_r)$$

whereas the equations for  $\tilde{\mathbf{h}}_t$  and the update to  $\mathbf{h}_t$  are the same as the standard BRC.

#### Differentiable Plasticity

#### Plastic Recurrent Cells

#### Method

#### Plastic Bistable Recurrent Cells

In this paper we propose the Plastic Bistable Recurrent Cell (PBRC) model, which is a combination of the BRC and diffentiable plasticity methods.

#### The Copy First Task

The copy first task is a temporal prediction task that is an effective test of a model's ability to maintain long term memories. In this task, the model is presented with a sequence of T inputs drawn from a standard multivariate normal distribution. At time T the model must output the first input of the series. All other outputs are discarded. This the model is presented with a new random input at each timestep, it must learn not only to remember the first input, but also to ignore all subsequent inputs. When T is large (i.e.  $\xi$  5), this task is very difficult for most recurrent models, including the GRU.

### Results

These are the results.

### Conclusion

This is the conclusion.

# References

- [1] Kyunghyun Cho, Bart Van Merriënboer, Caglar Gulcehre, Dzmitry Bahdanau, Fethi Bougares, Holger Schwenk, and Yoshua Bengio. Learning phrase representations using rnn encoder-decoder for statistical machine translation. arXiv preprint arXiv:1406.1078, 2014.
- [2] Nicolas Vecoven, Damien Ernst, and Guillaume Drion. A bio-inspired bistable recurrent cell allows for long-lasting memory. *Plos one*, 16(6):e0252676, 2021.