Discussion Section

Week 4

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August 29, 2019

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Admin

Test

- I haven't started grading them. Hopefully I will be done with it and the homeworks by the middle of next week
- Study the asnwer key, understand the questions that you got wrong, none of these concepts are going away

Office Hours

- **I WILL NOT** have office hours on Monday as it is Labor Day. I will have make-up office hours in the same room (SSH 2143) from 10-11 on Tuesday

Hypothesis Testing

What is hypothesis testing in a nutshell?

Hypothesis Testing

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- Remember that we talked about b_0 and b_k being random variables
  - That is, they will change with each sample we take from the population
  - The population parameter β will not change and is thus not a random variable
  - This is the same for all random variables and their population parameters (like $\bar{X}$ and μ_X)
- Since these estimates can very, we want to understand construct some range for b that we think is "believable"

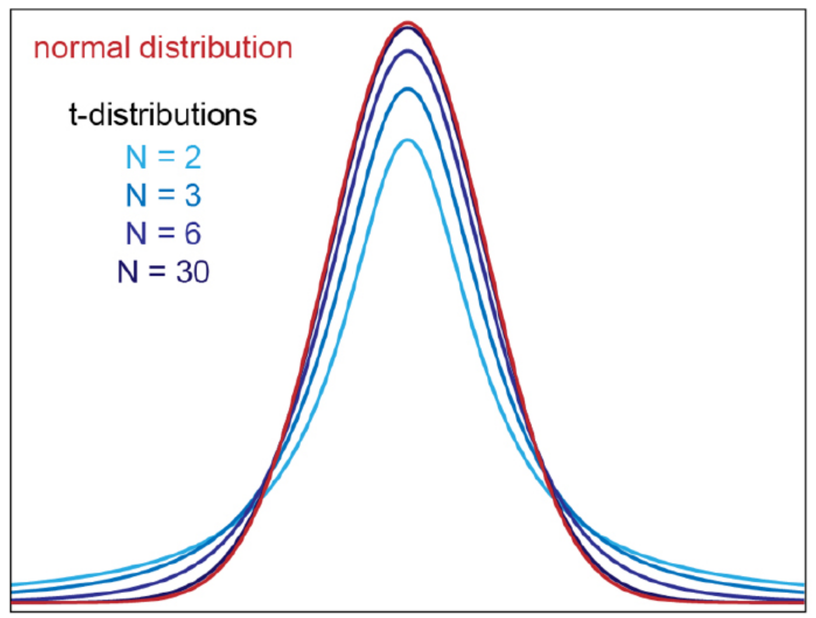
How do we find this range of believable values?

Hypothesis Testing

How do we find this range of believable values?

- There are a couple of different methods, that will tell us the same result
  - Its good to know all of them since they will tell you the same answer, but explain it in a different way
- All three of these methods revolve around the t-distribution
  - The t-distribution is very similar to a normal distribution, but its a bit “fatter”
  - This distribution is supposed to be like a normal distribution that accounts for sample size
  - The larger the sample, the more the t-distribution looks like the normal distribution

Hypothesis Testing



Hypothesis Testing

Why would we base this on the normal distribution?

Hypothesis Testing

Why would we base this on the normal distribution?

- There is a statistical theorem which essentially states that the average of a random variable is normally distributed
- The OLS b coefficients can be understood as a type of average
  - This means that the theorem can apply to OLS b coefficients
- This means that if we collected many samples from a population and ran the same regression on all of them, then plotted the values of the bs we found it would look like a bell curve
- We can then treat our b values as a point on a bell curve

Hypothesis Testing

What can we do with this knowledge?

Hypothesis Testing

What can we do with this knowledge?

- Let's say we run a regression with 100 observations and get the following results:

$$b_0 = 2 \qquad b_1 = 2$$
$$s.e.(b_0) = 2 \qquad s.e.(b_1) = 1$$

- We know that b_0 and b_1 are random variables, what if we want to know the likelihood that $b_1 = 5$?
- We can use the fact that b is a random variable and is essentially normally distributed to understand how likely this is
- A better way to phrase this question is: "If 5 is the true population β, how likely would it be that I find $b_1 = 3$ in a random sample?"
  - This type of question is what we call a "hypothesis test"
  - $b_1 = 5$ is called our "null hypothesis (H_0)" where $b_1 \neq 5$ is our alternative hypothesis (H_A)

Hypothesis Testing

How do we conduct a hypothesis test?

Hypothesis Testing

How do we conduct a hypothesis test?

- To find how likely it is to see our sample estimate, given that our hypothesis is true, we want to "normalize" our data
  - This takes that distribution of *b*s we talked about before and makes its mean = 0 and its variance = 1
  - This will allow us to translate our b_1 estimate into "standard deviations from the mean" as opposed to just a number
  - This will let us find the probability of getting $b_1 = 3$ given the "fact" that $\beta = 5$
- Since we are asking "If 5 is the true population parameter..." our unnormalized hypothetical distribution would be centered on 5
  - To make its mean = 0, we would have to subtract 5
- We then want to make the variance of the distribution = 1
  - To do this, we would want to just divide by the population standard deviation
  - We don't have this, so we will divide by the sample standard deviation and use the t-distribution instead of the normal distribution

Hypothesis Testing

What does this process look like in math?

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- From our example:

$$\frac{3-5}{1} = -2$$

- From this process we get a "t-score". Our t-score is -2. This means that **IF** the true β were 5, our estimate would be -2 standard deviations from the mean
- Since we normalized our distribution, we can actually calculate the probability of getting an estimate that is -2 standard deviations from the mean

Hypothesis Testing

- If we were to look at a t-table and find this probability we would find that there is a roughly 5% chance of this happening
  - This is called a "p-value"
  - We can define a p-value as: "The probability that we would find our estimate (or bigger) from a random sample, conditional on our null hypothesis being true"
  - We would denote this as $p = .05$
- We then have to ask ourselves if this seems reasonable?
- If our hypothesis is true, we would only expect to see a value this far from the mean one in every 20 samples
- In economics, this is the standard cut-off, although this is up for debate as it is totally subjective
- Typically, if a t-test yields a p-value that is less than or equal to .05, we say that we "reject the null hypothesis"

Hypothesis Testing

The logic of hypothesis testing is a bit like this:

- Pretend our hypothesis is true
- See how likely/unlikely our sample would have to be to get the estimate that we actually found
- If it seems very unlikely that we would get a sample that would give us the result we found, we say that our hypothesis must be wrong
  - We then “reject” the null hypothesis
- If it seems likely that we would get a sample that would give us the result we found, we say that we can’t necessarily reject the null hypothesis
  - We then “fail to reject” the null hypothesis
- We never say that we “accept” the null hypothesis, this is because we will fail to reject a lot of null hypothesis

Hypothesis Testing

To understand this, lets look at b_0 from our example regression:

$$b_0 = 2 \qquad\qquad b_1 = 2$$
$$s.e.(b_0) = 2 \qquad\qquad s.e.(b_1) = 1$$

- Lets test if $b_0 = 1$
- we can find our t-score first:
$$\frac{2-1}{2} = .5$$
- If we were to look up the probability of being .5 standard deviations from the mean on a t-distribution with 99 degrees of freedom we would find $p = .618$
- This means that in about 60% of samples we would expect to see an estimate this far from the mean
- This falls short of the standard $p = .05$ cut-off and thus, we would fail to reject the null hypothesis.

Hypothesis Testing

- What if we instead tested of $b_0 = 1.1$?

$$\frac{2 - 1.1}{2} = .45 \Rightarrow p = .653$$

- We would again fail to reject the null hypothesis
- In fact, we will fail to reject a range of null hypotheses, this range is called a "confidence interval", it gives us all of the values that we would fail to reject
- To calculate a two sided confidence interval we find:

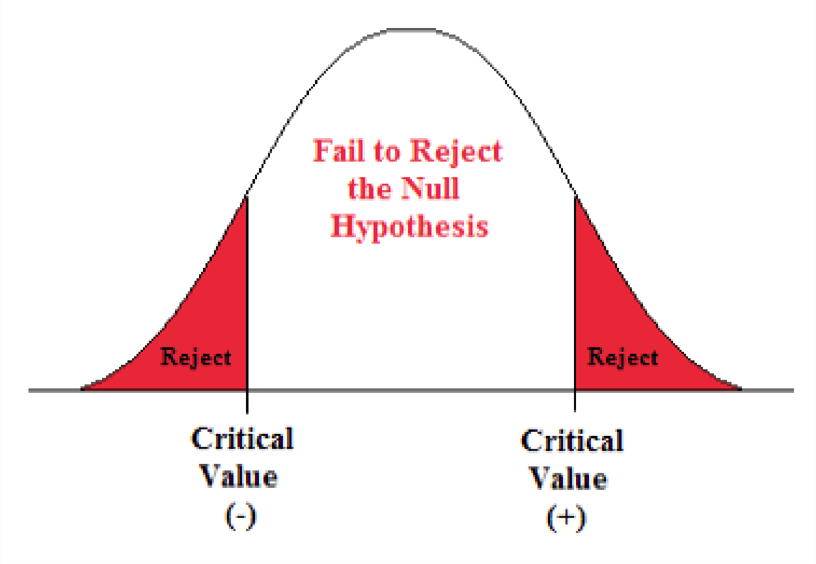
$$b \pm t_{crit} \times s.e.(b)$$

  In our example the confidence interval for b_0 is:

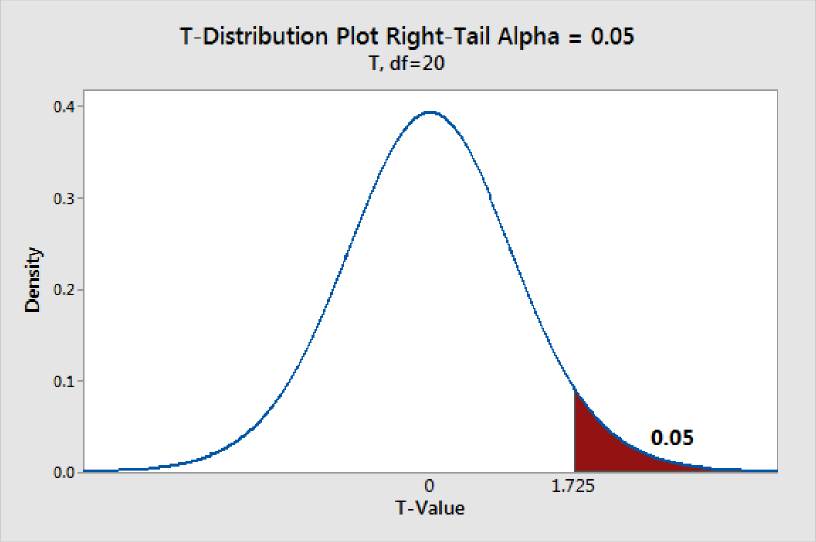
$$2 \pm 2 * 1.96 = (1.92, 5.92)$$

- That means, if you test if b_0 is any number between 1.92 and 5.92, you will fail to reject the null hypothesis
- On the other hand, if you test if b_0 is any number outside of this interval, you will reject the null hypothesis

Hypothesis Testing



Hypothesis Testing



Hypothesis Testing

All of this is so interesting and cool, but why exactly would we do this?

Hypothesis Testing

All of this is so interesting and cool, but why exactly would we do this?

- We typically test to see if a coefficient that we are interested in is "statistically different from 0"
- If b is statistically different from 0, then if we make 0 our null hypothesis, we would reject the null
- Remember that b represents the effect of x on y, thus if we fail to reject that $b = 0$ we are failing to reject that **there is no effect**
- When we reject the null we say that the result is *statistically significant*
    - This means that we don't think that the true β could be 0, thus there is some effect of x on y

Hypothesis Testing

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	3.9650	0.711	5.577	0.000	2.552	5.378
LSAT	0.0329	0.006	5.165	0.000	0.020	0.046
GPA	0.4152	0.141	2.955	0.004	0.136	0.695

Hypothesis Testing

Another helpful test is the Wald Statistic

- This test allows us to compare two different regression models
- To do this, we essentially compare the R^2 of the regression:

$$\frac{R^2_{alt} - R^2_{null}}{(1 - R^2_{alt})/(N - K_{alt} - 1)}$$

- Instead of following a t-distribution, this test statistic will follow a Chi-squared distribution where the degrees of freedom is the number of variables that are different between the regressions
- What this test is actually doing is simultaneously doing a t-test on all of the variables that are in the alt regression but not the null

BIG TAKEAWAYS

- t-test gives us how many standard deviations our estimate would be away from the mean **if** our null hypothesis were true
  - Critical value at 5% confidence level is 1.96 for a two-sided test and 1.645 for a one-sided test (assuming a large sample)
- p-value gives us the probability of getting a sample which would give us our estimate or a more extreme one
  - For either test, the critical p-value is always just $p = .05$ at the 5% confidence level
- Confidence interval tells us all of the null hypotheses we would not reject (and all of the ones we would)
- Wald statistic lets us compare regressions