4.

(1) $R = \begin{bmatrix} 0.1729 & -0.1468 & 0.9739 \\ 0.9739 & 0.1729 & -0.1468 \\ -0.1468 & 0.9739 & 0.1729 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

by formula 2.15

$$\|w\| = \cos^{-1}\left(\frac{trace(R)-1}{2}\right), \quad \frac{w}{\|w\|} = \frac{1}{2\sin(\|w\|)}\begin{bmatrix} r_{32}-r_{23} \\ r_{13}-r_{31} \\ r_{21}-r_{12} \end{bmatrix}$$

let $w' = \theta w$, w: rotation axis, $\|w\|=1$

plug in 2.15 $\Rightarrow$ $\|w'\| = \cos^{-1}\left(\frac{trace(R)-1}{2}\right) = \cos^{-1}\left(\frac{0.1729\times 3-1}{2}\right) = 1.8138 = \|\theta w\|$
$= \theta\|w\| = \theta$

$$\frac{w'}{\|w'\|} = \frac{\theta w}{\theta\|w\|} = \frac{w}{\|w\|} = \frac{1}{2\sin(\|w'\|)}\begin{bmatrix} r_{32}-r_{23} \\ r_{13}-r_{31} \\ r_{21}-r_{12} \end{bmatrix}$$

$$= \frac{1}{2\sin(\theta\|w\|)}\begin{bmatrix} 0.9739-(-0.1468) \\ 0.9739-(-0.1468) \\ 0.9739-(-0.1468) \end{bmatrix} = \frac{1}{2\sin(\theta)}\begin{bmatrix} 1.1207 \\ 1.1207 \\ 1.1207 \end{bmatrix}$$

$$= \frac{1}{2\cdot\sin(1.8138)}\begin{bmatrix} 1.1207 \\ 1.1207 \\ 1.1207 \end{bmatrix} = \begin{bmatrix} 0.5773 \\ 0.5773 \\ 0.5773 \end{bmatrix} = \frac{w}{\|w\|} = w$$

$\Rightarrow \theta = 1.8138, \quad w = \begin{bmatrix} 0.5773 \\ 0.5773 \\ 0.5773 \end{bmatrix}$

(2) the eigenvector associated with the unit value is $\begin{bmatrix} 0.5773 \\ 0.5773 \\ 0.5773 \end{bmatrix}$, which is the axis

By the definition of eigenvector of a matrix, the eigenvector doesn't change the direction after the matrix transformation. For a rotation, the axis doesn't change its direction after rotation, this meets the definition of eigenvector. $\underbrace{\text{rotation}}_{Rw = w}$

for the unit eigenvalue, since rotation doesn't scale vectors, including the eigenvector, so the eigenvalue is one.

5.

(1) $w \in R^3$, $\|w\| = 1$, $R = e^{\hat{w}\theta}$

let $w = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $\hat{w} = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}$,

eigenvalue of $\hat{w}$: $\det(\hat{w} - \lambda I) = 0 \Rightarrow \det\left(\begin{bmatrix} -\lambda & -c & b \\ c & -\lambda & -a \\ -b & a & -\lambda \end{bmatrix}\right) = 0 \Rightarrow$

$$\det\left(\begin{bmatrix} -\lambda & -c & b \\ c & -\lambda & -a \\ -b & a & -\lambda \end{bmatrix}\right) = -\lambda \begin{vmatrix} -\lambda & -a \\ a & -\lambda \end{vmatrix} - c \begin{vmatrix} -c & b \\ a & -\lambda \end{vmatrix} - b \begin{vmatrix} -c & b \\ -\lambda & -a \end{vmatrix}$$

$$= -\lambda(\lambda^2 + a^2) - c(\lambda c - ab) - b(ac + \lambda b)$$

$$= -\lambda^3 - a^2\lambda - \lambda c^2 + abc - abc - \lambda b^2$$

$$= -\lambda^3 - a^2\lambda - \lambda c^2 - \lambda b^2 = -\lambda(\lambda^2 + a^2 + b^2 + c^2)$$

$\because \|w\| = 1 \Rightarrow a^2 + b^2 + c^2 = 1 \Rightarrow \det(\hat{w} - \lambda I) = -\lambda(\lambda^2 + 1)$

let $\det(\hat{w} - \lambda I) = 0 \Rightarrow \lambda = 0, \pm i$

~~eigenvector for $\lambda = 0$: null vector of $\hat{w}$ = w~~

~~eigenvector for $\lambda = i$, let eigenvector $\underset{\sim}{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \hat{w}\underset{\sim}{x} = i\underset{\sim}{x} \Rightarrow$~~

$-cy + bz = ix$ $\cancel{-acy + abz = iax}$

$cx - az = iy \Rightarrow bcx - abz = iby$

$-bx + ay = iz \Rightarrow \cancel{bcx - a^2 z = iay}$

$-ibx + iay = -z$

$x(bc - ia)(ac - ib) = y(a^2c^2 + b^2)$

$\Uparrow$

$x\cancel{(bc-ia)} = y(ac+ib)$

$\Uparrow$

$$\frac{x}{z} = \frac{(ac+ib)}{(bc-ia)} \cdot \frac{(ab+ic)}{(ac-ib)}$$

$$\Rightarrow x : y : z = 1 : \frac{bc - ia}{ac + ib} : \frac{(bc-ia)(ac-ib)}{(ac+ib)(ab+ic)}$$

$$= ac + ib : bc - ia : \frac{(bc-ia)(ac-ib)}{(ab+ic)}$$

$$= (ac+ib)(ab+ic) : (ab+ic)(bc-ia) : (bc-ia)(ac-ib)$$

$$\Rightarrow x = \begin{bmatrix} (ac+ib)(ab+ic) \\ (bc-ia)(ab+ic) \\ (bc-ia)(ac-ib) \end{bmatrix}$$

5 (2) $R = e^{\hat{\omega}\theta}$

$\because$ $\hat{\omega}$ has eigen value $0, \pm i$ $\Rightarrow$ it has 3 orthogonal eigenvectors (say V_1, V_2, V_3 for $0, +i, -i$)

$\Rightarrow$ $\hat{\omega}$ can be diagonized $\Rightarrow$ $\hat{\omega} = P \begin{bmatrix} 0 & 0 & 0 \\ 0 & +i & 0 \\ 0 & 0 & -i \end{bmatrix} P^{-1} = P D P^{-1}$

$P = [V_1, V_2, V_3]$

for matrix function if $A = PDP^{-1} = P \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} P^{-1}$

then $f(A)$ is defined by $P \begin{bmatrix} f(\lambda_1) & 0 & \cdots & 0 \\ 0 & f(\lambda_2) & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & 0 & f(\lambda_n) \end{bmatrix} P^{-1}$, $f(A)$ has eigenvalues $f(\lambda_1), f(\lambda_2), \ldots f(\lambda_n)$ and the same eigenvectors

$\Rightarrow R = e^{\hat{\omega}\theta}$, $R = P \begin{bmatrix} e^{0\theta} & 0 & 0 \\ 0 & e^{i\theta} & 0 \\ 0 & 0 & e^{-i\theta} \end{bmatrix} P^{-1}$, R has eigenvalues $e^{0\cdot\theta}, e^{i\cdot\theta}, e^{-i\theta}$
$= 1, e^{i\theta}, e^{-i\theta}$

as discussed in problem 4, eigenvector that is corresponding to 1 is the rotation axis

6.

(1) let 2 parallel lines be $L_1 = \underset{\sim}{X_0^1} + \mu \underset{\sim}{V}$, $L_2 = \underset{\sim}{X_0^2} + \mu \underset{\sim}{V}$, $\underset{\sim}{X_0^1} = \begin{bmatrix} X_0^1 \\ Y_0^1 \\ Z_0^1 \\ 1 \end{bmatrix}$, $\underset{\sim}{X_0^2} = \begin{bmatrix} X_0^2 \\ Y_0^2 \\ Z_0^2 \\ 1 \end{bmatrix}$, $\underset{\sim}{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ 0 \end{bmatrix}$, μ: scalar

let the projection matrix be Π, when $\mu \to \infty$ (far away)

$$\left.\begin{array}{l} \Pi L_1 \simeq \Pi \mu V \\ \Pi L_2 \simeq \Pi \mu V \end{array}\right. \Rightarrow \text{they intersect at point } \Pi \mu V$$

(2)

from (1) we can see the vanish point $X' \simeq \Pi \mu V$ when $\mu \to \infty$

(3)

if $V = \begin{bmatrix} V_1 \\ V_2 \\ 0 \\ 0 \end{bmatrix}$, then $\Pi \mu V = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$, this is not a physical point on the imag plane

in this case, the lines are parallel to the image plane