SINGAPORE POLYTECHNIC

2021/2022 SEMESTER ONE

MID SEMESTER TEST

Diploma in Applied AI and Analytics (DAAA)

2nd Year Full-Time

MATHEMATICS FOR AI Time allowed: 1.5 hrs

Instructions

1. The Singapore Polytechnic Examination rules are to be complied with.
2. This paper consists of 3 printed pages and comprises 6 questions.
3. Answer all the questions.
4. All solutions are to be written in the space provided in the booklet.
5. Unless otherwise stated, all answers given should be correct to at least three significant figures.
6. Except for sketches, graphs and diagrams, no solutions are to be written in pencil.
7. One A4-sized help sheet, handwritten, double-sided, and Python exercises Jupyter notebooks are allowed.
8. No internet access will be allowed.
9. Python working will not be collected at the end of the test.
10. Unless otherwise stated, all working steps should be shown clearly.

1. Given $\mathbf{A}=\begin{pmatrix}2 & 4\\ 1 & 5\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}0 & 1\\ -1 & 6\end{pmatrix}$ and $\mathbf{C}=\begin{pmatrix}-2 & 3\\ 1 & 5\end{pmatrix}$.

(a) Find $\mathbf{Y}$ if $\mathbf{Y}+\mathbf{B}^T=\mathbf{A}$.

(b) Evaluate $\mathbf{A}+\mathbf{B}^T\mathbf{C}$.

(c) Evaluate $2\mathbf{B}^{-1}$. [13 marks]

2. (a) Determine the value of k, if $\begin{pmatrix}k & 2k+1\\ -1 & 3\end{pmatrix}$ is

(i) singular.

(ii) symmetric.

(b) Given $\mathbf{A}=\begin{pmatrix}1 & 0\\ 2 & k\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}1 & k-2\\ -1 & \frac{1}{2}\end{pmatrix}$.

(i) Find $\mathbf{AB}$ in terms of k.

(ii) If $\mathbf{A}$ and $\mathbf{B}$ are inverses of each other, find k. [10 marks]

3. The amount of fat and calcium in milk from three different brands A, B and C are stated in the following table.

Brand	Fat (gram per litre)	Calcium (gram per litre)
A	20	2.4
B	30	1.9
C	16	1.8

The milk from the three brands are to be mixed to form 10 litres of low-fat, high-calcium milk, with 20 gram per litre of fat and 2 gram per litre of calcium. Let a, b and c be the number of litres of milk from brand A, B and C respectively.

(a) Form a system of linear equations.

(b) Use Gaussian elimination method to find the number of litres of milk from each brand to use for the mixture. Show your steps of Gaussian elimination clearly.

[15 marks]

4. (a) Determine the entire solution set of the following linear system:

$$w-6x+2y+2z=0$$
$$3w+2x-y+5z=4$$

(b) Find the values of w and x where $y=z=2$. [12 marks]

5. A fast food restaurant sells 2 combo meals, Combo A and B. The number of meals from each combo sold on three consecutive days were recorded in the following table. The restaurant is interested to know the relationship between combo A & B.

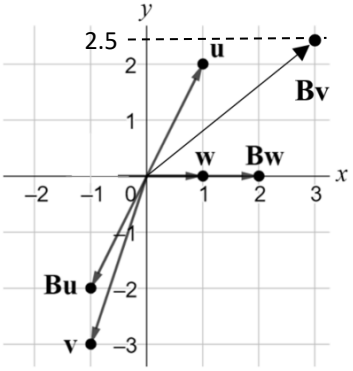
X_1: Number of Combo A	42	48	15
X_2: Number of Combo B	25	40	58

(a) Write the above data set as a data matrix **X**.
(b) Compute the sample mean vector and sample covariance matrix using the matrix method.
(c) Describe the data set using the sample covariance matrix by comparing the two combos and describing their relationship. Justify your answers.
(d) Compute the correlation between the two combos. Is the relationship considered weak, moderate or strong?
(e) Assume bivariate normal data, use Python to find the directions of the radii of the ellipse representing the distribution of the data, clearly indicating the longer and shorter radii. [25 marks]

6. (a) The matrix $\mathbf{A} = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 7 & -2 \\ -5 & 14 & -4 \end{pmatrix}$ has eigenvalues $\lambda_1 = -1, \lambda_2 = 1$ and $\lambda_3 = 3$.

(i) Verify that $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ are eigenvectors of **A**. State their corresponding eigenvalues.
(ii) Hence, find the associated eigenvectors of **A** corresponding to the other eigenvalue.

(b) The diagram below shows the vectors **u, v, w, Bu, Bv,** and **Bw**, where **B** is a square matrix of order 2. Using the Diagonalization Theorem, find **B**.



[25 marks]

END OF PAPER