Assignment 9 of Math6321

Due: April 22, 2015, in class

1. From the textbook, page 418, Exercises 14.1.

2. Consider the linear programming problem

$$\begin{aligned} \min \quad & x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 + x_3 = 1, \\ & (x_1, x_2, x_3) \geq 0. \end{aligned}$$

(a) Find the optimal solution x^*.
(b) Find the central path, i.e., the each fixed $\tau > 0$, find the central path point $(x_\tau, \lambda_\tau, s_\tau)$.
(c) Show that as τ decreases to 0, x_τ converges to the unique optimal solution x^*.
(d) Draw the projections of the feasible region and the central path in the x_1-x_2 plane.

3. Sparse signal recovery (Compressed Sensing application). Let $x_{true} \in \mathbb{R}^n$ be a sparse signal, that is, only a few coordinates of x are nonzero. We would like to recover x_{true} from the measurements $y \in \mathbb{R}^m$. Assume $y = Ax_{true}$, where A is a given $m \times n$ matrix. We can recover x_{true} by solving the following l_1 minimization problem:

$$\begin{aligned} \min \quad & \|x\|_1 \\ \text{subject to} \quad & Ax = y, \\ & x \in \mathbb{R}^n, \end{aligned}$$

where $\|x\|_1 = \sum_{i=1}^{n} |x_i|$.

This problem can be reformulated as a Linear Programming problem as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} t_i \\ \text{subject to} \quad & Ax = y, \\ & x + t \geq 0, \\ & -x + t \geq 0. \end{aligned}$$

In this assignment, you will try to recover x_{true} from the given y and matrix A using the Matlab linear programming routine "linprog". The file "dataHW9.mat" contains A, y and x_{true}, where $A \in \mathbb{R}^{25\times 100}$, $y \in \mathbb{R}^{25}$ and $x_{true} \in \mathbb{R}^{100}$.

Reformulate the problem so that you can use the Matlab routine. State that formulation in your report.

Report your recovered signal x. It should be pretty close to x_{true}.