Covariance as a flux: a word problem

When you put a pot of water on a heated stove, the water convects. The temperature at the top of the pan increases because a vertical mass flux ρw carries specific enthalpy $C_p T$ upward, comprising an energy flux $\rho C_p \overline{wT}$ which we can summarize by its *kinematic temperature flux* wT.

To study this flux, you set up a vertical anemometer to measure w, and a thermometer to measure T, in the mid-depth of the pan. The devices collect the following statistics, where the overbar is a time average that you may assume is the same as a spatial average over the area of the pan:

$\overline{w'T'}$ = 1 K m/s

σ_T = 3 K

σ_w = 2 m/s

a. Show that since $\overline{w}$ = 0 (explain why), $\overline{\overline{wT}} = \overline{\overline{w'T'}}$.

b. Somebody gets your data and uses it to test their proposition that T "explains" w because warm water rises:

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T' = a w' + e1
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Somebody else gets your data and uses it to test the proposition that w "explains" T because rising water carries heat:

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w' = b T' + e2
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What are the values of a and b, assuming the usual *principle of least squares* (traditional linear regression) is used to define their values?

Is a = 1/b? Why or why not? Carry units on all quantities.

c. What fraction of $\overline{T'^2}$ does the a term explain? Show your work.

d. What fraction of $\overline{w'^2}$ does the b term explain?

e. Suppose your thermometer breaks, and you buy a cheaper one, with large random measurement errors. That increases σ_T to 5 K. **Answer the table below:**

Quantity: increase, decrease, same?	Quantity: increase, decrease, same?
var(T)	var(w)
cov(w,T)	cor(w,T)
a	b
e1	e2
explained variance of T	explained variance of w