GPINNs : Exploring Gravitational Physics Informed Neural Networks

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Context

Solve physics problems like:
➔ **Extrapolation** of set of data points
➔ **Interpolation** without overfitting
➔ **Encoding** solutions of a problem

Methodology: Add physics prior knowledge to the training as a loss function to optimize.

Regular setting : The loss is only a term of fit to the data
PINN setting: We add a regularization term (often the PDE that models our problem:

PINN total loss ⟶ $\mathcal{L}_{total} = \mathcal{L}_{fit} + \mathcal{L}_{reg}$

Regular loss (MSE) ⟶ $\mathcal{L}_{fit} = \|\phi(\mathbf{x}) - \mathbf{y}\|_2^2$

Dampen Spring

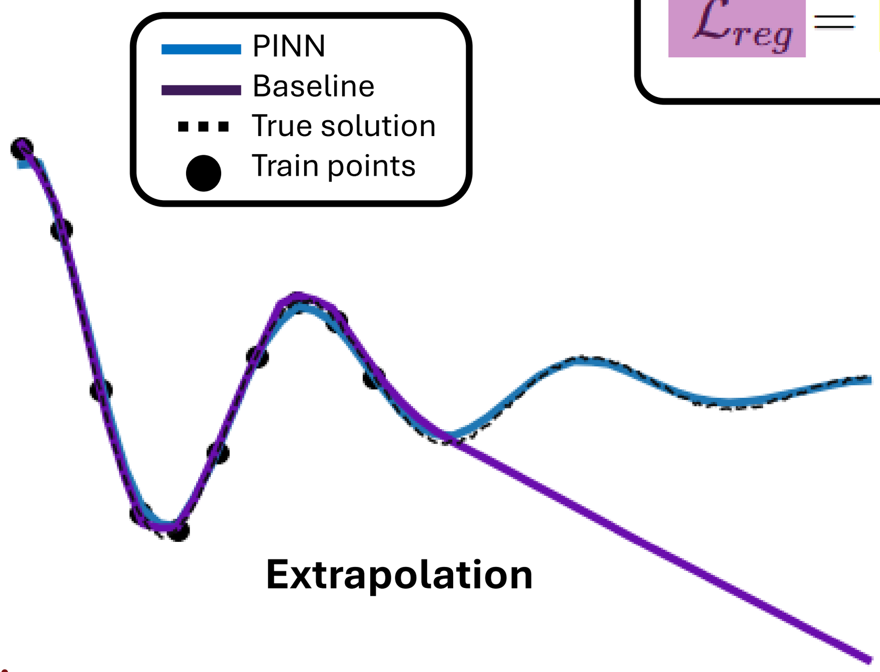
Model :
- A mass m
- A spring k
- Friction μ
- No gravity

$$\omega = \sqrt{\frac{k}{m}} \qquad \xi = \frac{\mu}{2\sqrt{m \cdot k}}$$

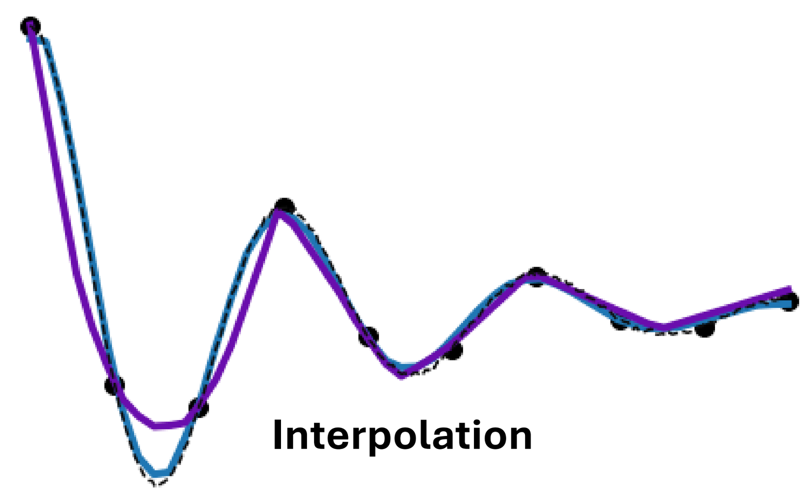
The position of the spring is solution of:

$$\ddot{x}(\mathbf{t}) + 2\xi\omega\dot{x}(\mathbf{t}) + \omega^2 = 0$$

$$\mathcal{L}_{reg} = \|\ddot{\phi}(\mathbf{x}) + 2\xi\omega\dot{\phi}(\mathbf{x}) + \omega^2\|_2^2$$



Extrapolation



Interpolation

SInusoïdal REpresentation Networks (SIRENs)

Dense Neural Network with :

$$\Phi(\mathbf{x}) = \mathbf{W}_n(\phi_{n-1} \circ \phi_{n-2} \circ \ldots \circ \phi_0)(\mathbf{x}) + \mathbf{b}_n$$

➔ **Periodic** activation function $\mathbf{x}_i \mapsto \phi_i(\mathbf{x}_i) = \sin(\mathbf{W}_i\mathbf{x}_i + \mathbf{b}_i)$

➔ Specific **Initialization** for stability $w_i \sim \mathcal{U}(-\sqrt{6/n}, \sqrt{6/n})$

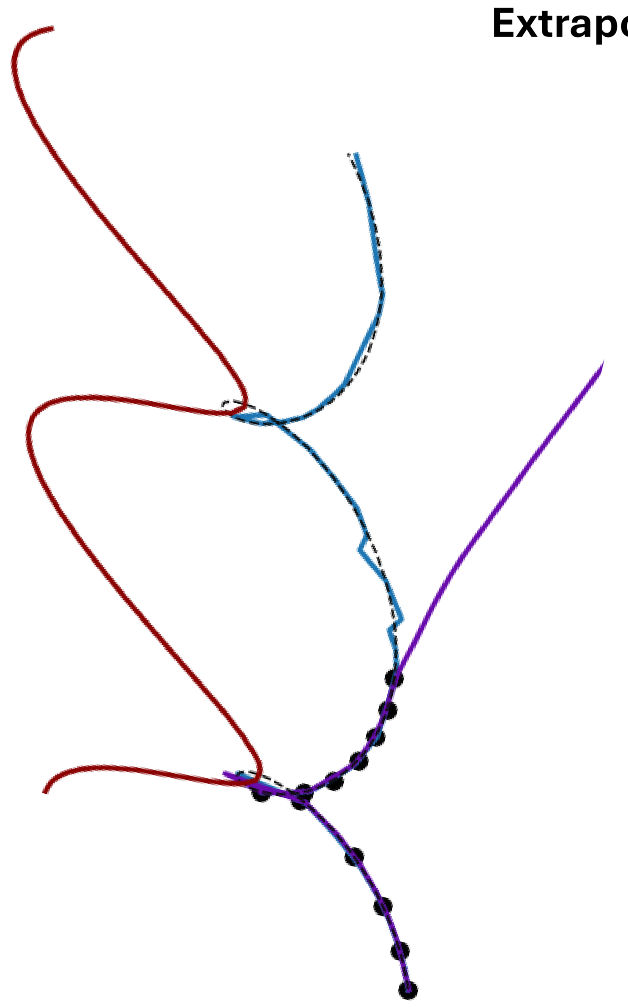
Advantages :
➔ Derivatives of the activation have the same characteristics
➔ Well suited for fitting the **derivatives** of the network

Multiple body problem

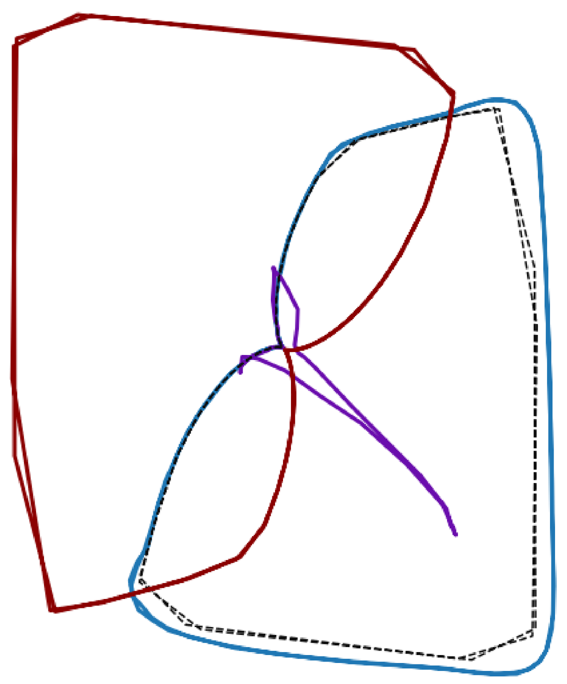
Model :
- Multiple bodies m
- Pairwise attraction force
➔ Newton 2nd law

$$\mathcal{L}_{reg} = \sum_{i=1}^{N} \|\ddot{\phi}(\mathbf{x}_i) - \sum_{j \neq i} \frac{Gm_j}{\|\mathbf{x}_j - \mathbf{x}_i\|^3}(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

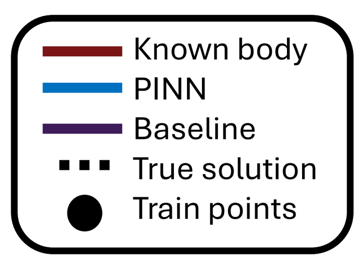
Extrapolation



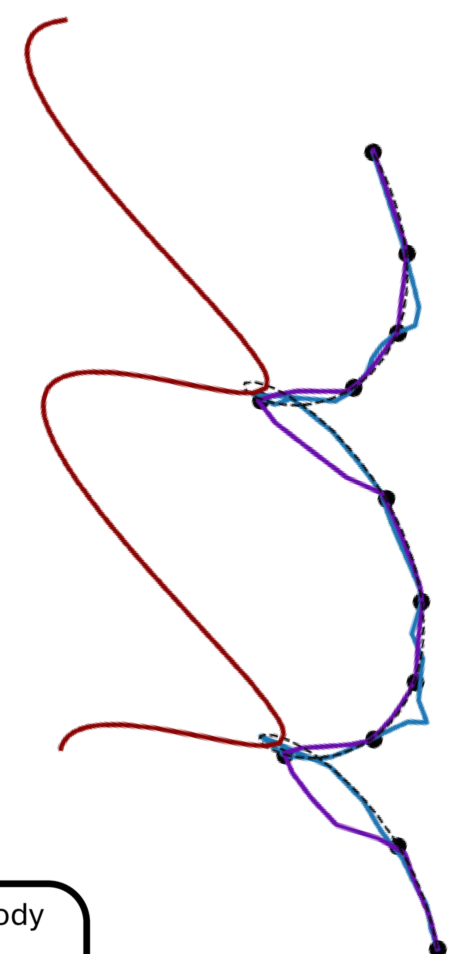
Position



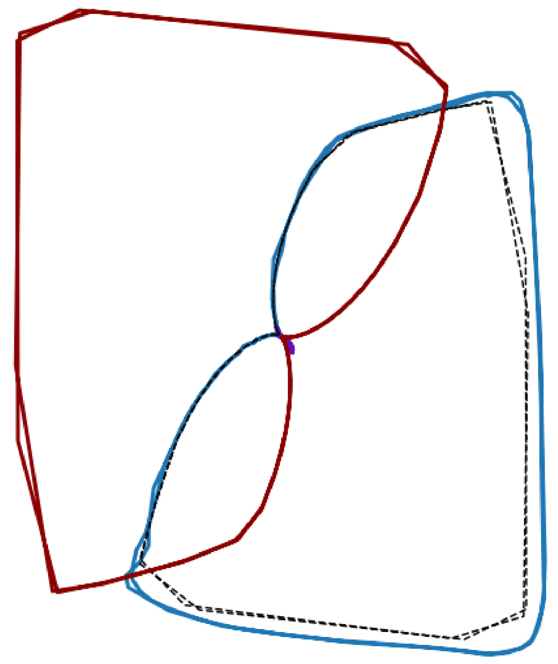
Acceleration



Interpolation



Position

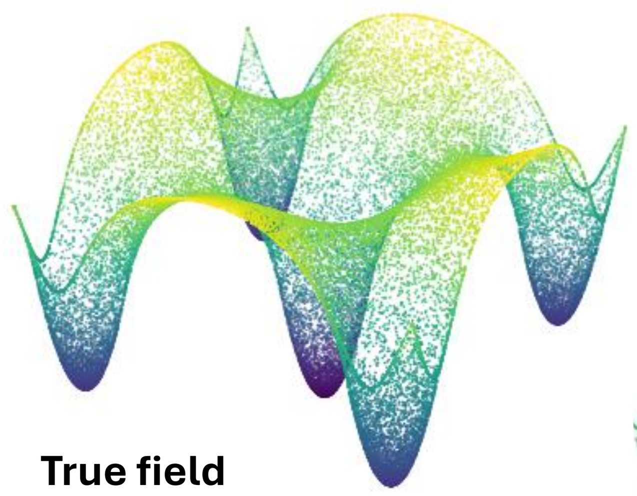


Acceleration

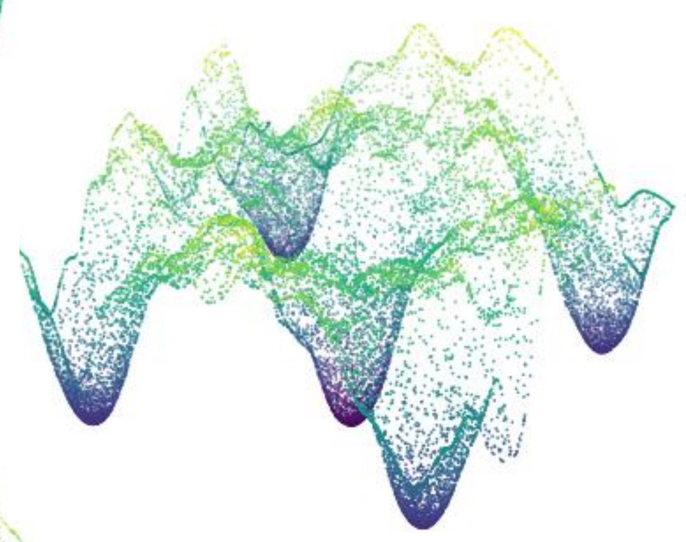
Gravitational Field

Model :
- Multiple bodies **m** of radius **R**
- Additive gravitational field
➔ Poisson's equation

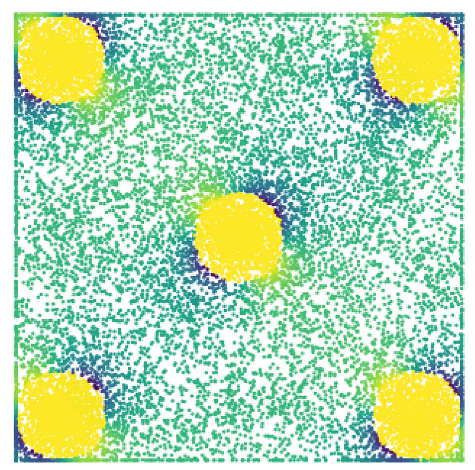
$$\mathcal{L}_{reg} = \|\Delta\phi(\mathbf{x}) - \frac{4}{3R^2}\|_2^2$$



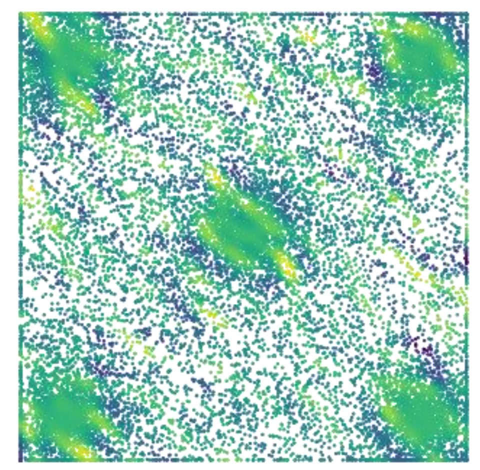
True field



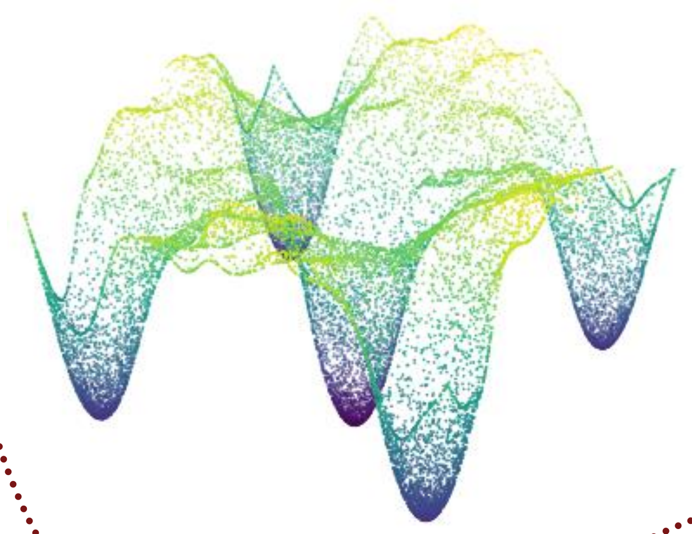
Baseline field



True Laplacian

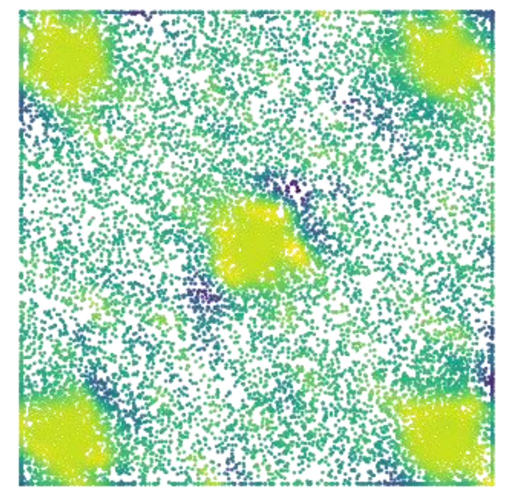


Baseline Laplacian



PINN field

	PINN	Baseline
Val. MSE	5e-3	4e-2



PINN Laplacian

Conclusion

➔ Significant improvements with physics prior
➔ Can handle a variety of tasks, types of equations

Next step : 3 bodies problem (chaotic)

GitHub code

References

1. Breen, P., Foley, C., Boekholt, T., & Zwart, S. (2020). Newton versus the machine: solving the chaotic three-body problem using deep neural networks. *Monthly Notices of the Royal Astronomical Society, 494(2), 2465–2470.*
2. Vincent Sitzmann, Julien N. P. Martel, Alexander W. Bergman, David B. Lindell, & Gordon Wetzstein (2020). Implicit Neural Representations with Periodic Activation Functions. *CoRR, abs/2006.09661*