Assignment_6

October 25, 2023

1 Computational Methods in Stochastics - Assignment 6

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from tqdm import tqdm
```

1.0.1 a)

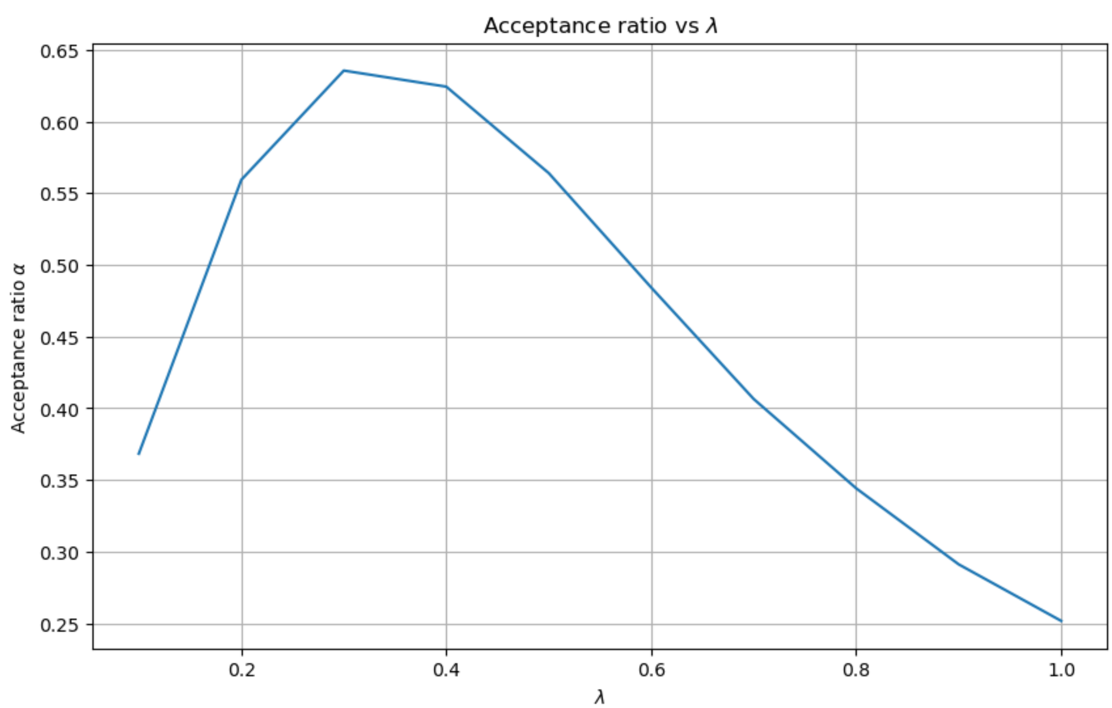
[2]:
```
a = 3
b = 1
l = b/a

def f(x, l):
    return l * np.exp(-l * x)

def f_inv(r, l):
    return -np.log(1 - r)/l

def x_exp_random_variates(x, l):
    samples = np.empty(x)
    for i in range (x):
        random_value = np.random.random()
        samples[i] = f_inv(random_value, l)
    return samples

def exponential(x, l):
    return l * np.exp(-l * x)

def gamma(x, a, b):
    ba = b**a
    x_pow = x**(a - 1)
    exp_term = np.exp(-b * x)
    factorial_a_minus_1 = np.math.factorial(a - 1)
    result = ba * x_pow * exp_term / factorial_a_minus_1
    return result

def plot_acceptance_ratio(start, end, step, N):
```

```
    interval  = np.arange(start, end, step)
    accs = np.empty(len(interval))
    for i, l in tqdm(enumerate(interval)):
        theta = x_exp_random_variates(1, l)
        acc = [theta]
        for j in range(N):
            phi = x_exp_random_variates(1, l)
            numerator = gamma(phi, a, b) / gamma(theta, a, b)
            denominator = exponential(phi, l) / exponential(theta, l)
            acc_prob = min(1, numerator / denominator)
            if(np.random.random() < acc_prob):
                theta = phi
                acc.append(theta)
        accs[i] = len(acc)/N
    plt.figure(figsize=(10, 6))
    plt.plot(interval, accs)
    plt.xlabel('$\lambda$')
    plt.ylabel('Acceptance ratio $\\alpha$')
    plt.title('Acceptance ratio vs $\lambda$')
    plt.grid()
    plt.show()
    return accs
```

[3]:

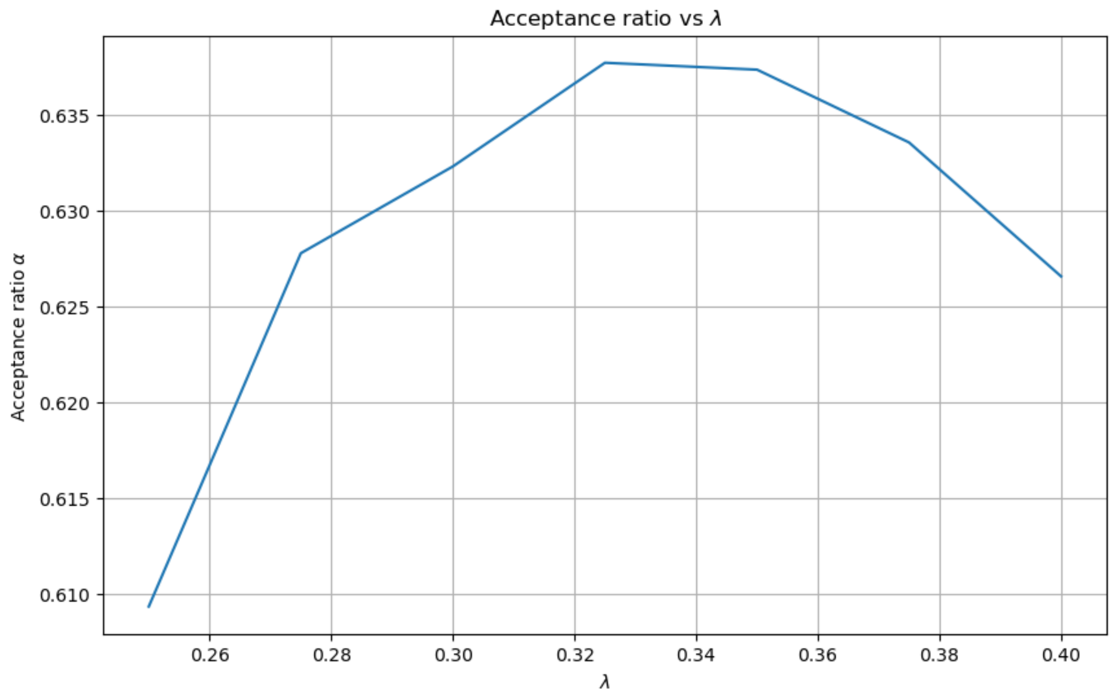
```
N = 100000
ans1 = plot_acceptance_ratio(0.1, 1.1, 0.1, N)
```

```
10it [00:30,  3.03s/it]
```



[4]:
```python
ans2 = plot_acceptance_ratio(0.25, 0.425, 0.025, N)
```

```
7it [00:21,  3.01s/it]
```



[5]:
```python
idx = np.argmax(ans2)
lambdas = np.arange(0.25, 0.425, 0.025)
print(f"The value of the optimal lambda is: {lambdas[idx]} ± 0.05")
```

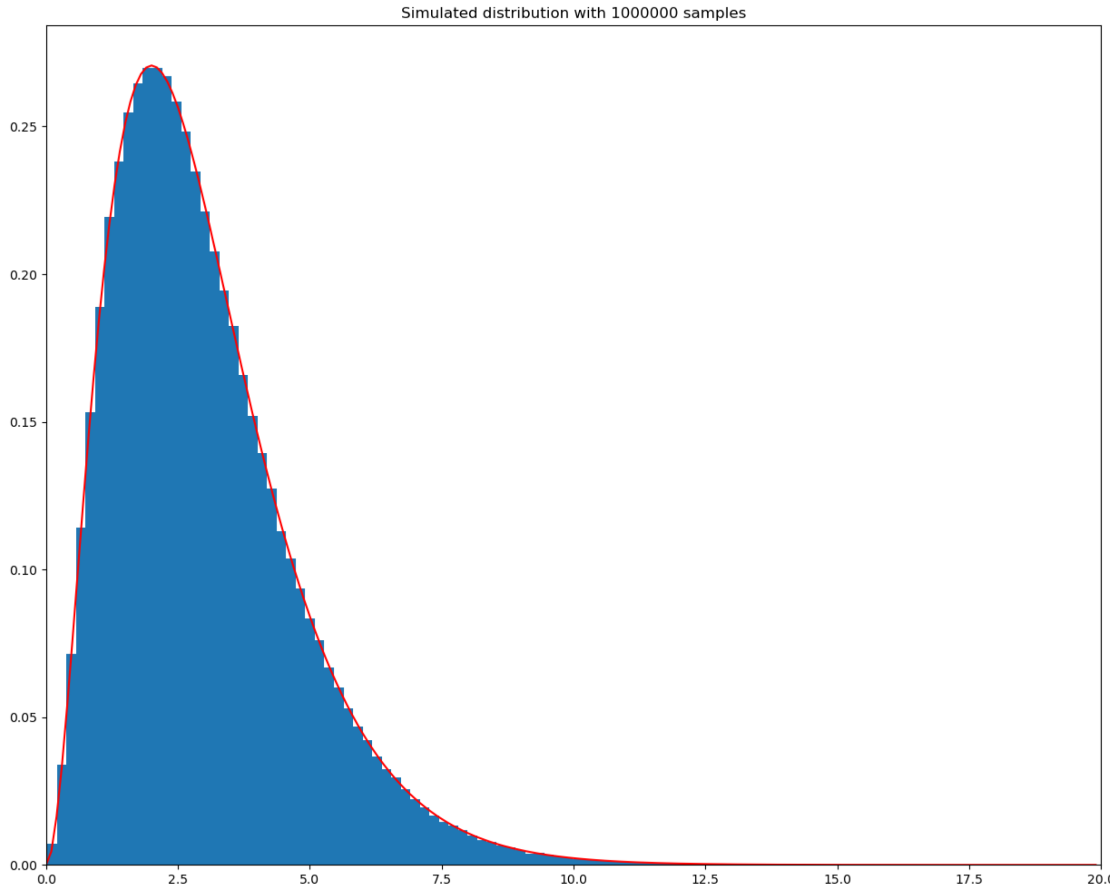
```
The value of the optimal lambda is: 0.32500000000000007 ± 0.05
```

1.0.2 b)

[6]:
```python
N = 10**6
theta = x_exp_random_variates(1, l)
acc = [theta]
for i in tqdm(range(N)):
    phi = x_exp_random_variates(1, l)
    numerator = gamma(phi, a, b) / gamma(theta, a, b)
    denominator = exponential(phi, l) / exponential(theta, l)
    acc_prob = min(1, numerator / denominator)
    if(np.random.random() < acc_prob):
        theta = phi
```

```
    acc.append(theta)

interval = np.arange(0,20,0.1)
plt.figure(figsize=(15, 12))
plt.hist(np.asarray(acc), bins = 100, density = True)
plt.plot(interval, gamma(interval, a, b), color = "red")
plt.title("Simulated distribution with " + str(N) + " samples")
plt.xlim(0,20)
plt.show()
```

```
100%|      | 1000000/1000000 [00:31<00:00, 32198.47it/s]
```



1.0.3 c)

[7]:
```
mean_acc = np.mean(acc)
variance_acc = np.var(acc)

beta = mean_acc / variance_acc
alpha = beta * mean_acc
```

```python
print(f"Mean = {mean_acc:.4f}")
print(f"Variance = {variance_acc:.4f}")
print(f"Simulated alpha = {alpha:.4f}")
print(f"Simulated beta = {beta:.4f}")
```

```
Mean = 2.9957
Variance = 2.9926
Simulated alpha = 2.9987
Simulated beta = 1.0010
```

1.0.4 d)

The Gibbs Sampler algorithm requires the ability to simulate random variables from conditional distributions, along with one of the marginals. Conversely, the Metropolis-Hastings algorithm does not have this specific requirement, as it involves simulating random variables from a proposal distribution. While the choice of the proposal distribution is critical for the efficiency of the Metropolis-Hastings algorithm, it theoretically allows for the use of various distributions. This is why the Metropolis-Hastings algorithm is generally considered more versatile compared to the Gibbs Sampler.