CS–E4850 Computer Vision
Week 08

Amirreza Akbari

1 Problem 1

Part c)

One of the main reason is that the tracking algorithm uses RANSCA to track liners and eliminate outliers from the last frame, and the key points would be missed the since the image is rotated or the camera moves at a fast pace. Because rotation can cause drop in high amount of the tracked features, and regarding to high speed of camera, it can cause that the program process cannot process the image and looses track of the points.

Part d)

As a good improvement, we can try to avoid large movements in a short period of time, thus, it can result new features and track them. Furthermore, keeping outliers instead of eliminating them and use them in further tracking, could be helpful, although the points always exist even if the face move out of the camera sight.

2 Problem 2

Solution: This is the equation shown in the Baker and Matthews paper

$$\Delta p = H^{-1} \sum_x [\nabla I \frac{\partial W}{\partial p}][T(x) - I(W(x;\, p))] \tag{1}$$

Therefore we have

$$H\,\Delta p = \sum_x [\nabla I \frac{\partial W}{\partial p}][T(x) - I(W(x;\, p))] \tag{2}$$

We also know that

$$\Delta p = \begin{bmatrix} u \\ v \end{bmatrix}, \frac{\partial W}{\partial p} = \begin{bmatrix} \frac{\partial W_x}{\partial u} & \frac{\partial W_x}{\partial v} \\ \frac{\partial W_y}{\partial u} & \frac{\partial W_y}{\partial v} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

Since H is the $n \times n$ Hessian matrix, we can conclude that

$$\begin{aligned} H &= \sum_x [\nabla I \frac{\partial W}{\partial p}]^t\,[\nabla I \frac{\partial W}{\partial p}] \\ &= \sum_x \frac{\partial W^t}{\partial p} \nabla I^t \nabla I \frac{\partial W}{\partial p} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix} [\frac{\partial I}{\partial x} \quad \frac{\partial I}{\partial y}] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \end{aligned} \tag{4}$$

Thus by re-writing the equation in the Baker and Matthews paper

$$H\,\Delta p = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \Delta p = \sum_x [\nabla I \frac{\partial W}{\partial p}][T(x) - I(W(x;\, p))] \tag{5}$$

$$\rightarrow \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [T(x) - I(W(x;\, p))] \tag{6}$$

Since the Template $T(x)$ is an extracted sub-region of the image at $t = 1$ and $I(x)$ is the image at $t = 2$, we have

$$\begin{aligned} \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} &= \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [-I_t] \\ \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} &= -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} \end{aligned} \tag{7}$$