CS–E4850 Computer Vision
Week 12

Amirreza Akbari

December 2, 2022

1 Problem 1

2 Problem 2

Given two camera projection matrices $P = [I|O]$ and $P' = [Rt]$, where R is rotation matrix and $t = [t_1, t_2, t_3]^T$ is the translation vector. Therefore

$$\overrightarrow{O'p'} \cdot (\overrightarrow{O'O} \times \overrightarrow{Op}) = 0$$

We have to show that

$$x'^T E x = 0$$

where E is the essential matrix and

$$E = [t]_\times R$$

Given the homogeneous image coordinates of vector p and p', $X = [x, y, 1]^T$ and $X' = [x', y', 1]^T$ we can write:

$$\overrightarrow{O'p'} = X'$$

also from the cam coordinates we have that

$$\overrightarrow{Op} = Rt$$

The translation between camera origins is given by vector t. We can rewrite the following equation:

$$\overrightarrow{O'p'} \cdot (\overrightarrow{O'O} \times \overrightarrow{Op}) = X' \cdot (t \times RX) = 0$$

also we know

$$a \times b = \begin{bmatrix} 0 & -a_x & a_y \\ a_x & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [a]_\times b$$

So we have

$$X' \cdot (t \times RX) = X' \cdot [t]_\times R \cdot X = X' RX = 0$$

3 Problem 3

3.1 Part a

$$
\begin{aligned}
\frac{x_l}{f} = \frac{x'_l}{Z_P} \rightarrow x'_l = \frac{x_l Z_P}{f},\\
\frac{x_r}{f} = \frac{x'_r}{Z_P} \rightarrow x'_r = \frac{x_r Z_P}{f}\\
d = |x_l - x_r|
\end{aligned}
$$

so we will have

$$
\begin{aligned}
x'_l - x'_r = b,\\
\frac{x_l Z_P}{f} - \frac{x_r Z_P}{f} = b,\\
\frac{Z_P}{f}(x_l - x_r) = b\\
\frac{Z_P}{f} d = b\\
Z_p = \frac{bf}{d} = 6\, cm
\end{aligned}
$$

3.2 Part b

We know that

$$
\frac{Z_P}{f} d = b
$$

We want to obtain $d \leq 0.01\, mm$,

$$
\begin{aligned}
d = \frac{bf}{Z_P} \leq 0.01\, mm\\
Z_P \geq \frac{bf}{0.01 mm} = \frac{6\, cm 1\, cm}{0.01\, mm}\\
Z_P \geq 60\, m
\end{aligned}
$$

3.3 Part c

We have

$$
\begin{aligned}
P_l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, P_r = \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\\
Q = (3, 0, 3)
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
x'^T E x = 0,\\
x = P_l Q,\\
x' = P_r Q
\end{aligned}
$$

the image of Q on the image plane of the camera on the left is

$$x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

and the one of the camera on the right is

$$x' = \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix}$$

So

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix}$$

on the image plane of the camera on the right:

$$E^T x' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -6 \\ 0 & 6 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -18 \\ 0 \end{bmatrix}$$

and on the left:

$$Ex = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$