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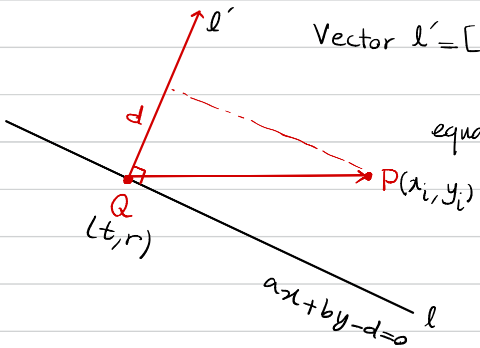
Computer vision – Ex 5

Q1. a) We are looking for distance d. Conside vector l' starting from an arbitrary point Q on line l. Also let l' be perpendicular to l.



Since the slope of line l is $-b/a$

Vector $l' = [a, b]$ has these properties, so distance d is equal to length of orthogonal projection of $\vec{QP}$ on l'.

$$d = \frac{|\vec{QP} \cdot l'|}{\|l'\|}$$

$$\to \|l'\| = \sqrt{a^2+b^2}$$

$$\vec{QP} = [x_i - t, y_i - r] \to \vec{QP} \cdot l' = a(x_i - t) + b(y_i - r)$$

$$d = \frac{|ax_i - at + by_i - br|}{\sqrt{a^2+b^2}} = \frac{|ax_i + by_i - d|}{1} = |ax_i + by_i - d)$$

b) $\text{Min } E = \sum_{i=1}^{n} (ax_i + by_i - d)$

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} \frac{\partial (ax_i+by_i-d)^2}{\partial d} = \sum_{i=1}^{n} -2(ax_i+by_i-d) = 2\left(\sum_{i=1}^{n} d - ax_i - by_i\right)$$

$$= 2\left(nd - a\sum_{i=1}^{n} x_i - b\sum_{i=1}^{n} y_i\right)$$

$$\longrightarrow \frac{\partial E}{\partial d} = 0$$

$$\longrightarrow 2\left(nd - a\sum_{i=1}^{n} x_i - b\sum_{i=1}^{n} y_i\right) = 0$$

$$\longrightarrow d = a\bar{x} + b\bar{y} \text{ , which } \bar{x} = \frac{\sum x_i}{n} \text{ , } \bar{y} = \frac{\sum y_i}{n}$$

c) $E = \sum_{i=1}^{n} (ax_i+by_i-d)^2 = \sum_{i=1}^{n} (ax_i+by_i-a\bar{x}-b\bar{y})^2 = \sum_{i=1}^{n} (a(x_i-\bar{x})+b(y_i-\bar{y}))^2$

$s_i = a(x_i-\bar{x}) + b(y_i-\bar{y})$ $\longrightarrow$

$$= \begin{bmatrix} s_1 & s_2 & s_3 & \cdots & s_n \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$$

$$= \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} x_1-\bar{x} & x_2-\bar{x} & \cdots & x_n-\bar{x} \\ y_1-\bar{y} & y_2-\bar{y} & \cdots & y_n-\bar{y} \end{bmatrix} \underbrace{\begin{bmatrix} x_1-\bar{x} & y_1-\bar{y} \\ x_2-\bar{x} & y_2-\bar{y} \\ \vdots & \vdots \\ x_n-\bar{x} & y_n-\bar{y} \end{bmatrix}}_{U} \begin{bmatrix} a \\ b \end{bmatrix} = [a\ b]U^T U [a\ b]^T$$