ACRM 2018 Longitudinal Data Analysis Workshop

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Practical Session 2: Curvilinear and Higher Order Mixed-Effects Models

This handout is designed to accompany the script you will be working with in the practical session. A copy of the script file, the data, set, and this handout can be found at: https://github.com/keithlohse/LMER_Clinical_Science.

The R code is interspersed with explanations below. All R code is highlighted in grey and color coded to show different functions, arguments, and comments in the code.

First, you will need to open the five packages we will be using for this session using the library function:

```
# Loading the essential libraries.
library("ggplot2"); library("lme4"); library("car"); library("dplyr");
library("lmerTest");
```

If you haven't already installed these packages, you will need to use the install.packages() function first. This can take some time and will require an internet connect.

```
# If these packages are not installed already, run the following code:
install.packages("ggplot2"); install.packages("lme4");
install.packages("car"); install.packages("dplyr");
install.packages("lmerTest");
```

2.1 Data Cleaning and Quality Assurance

One of the first steps is to set the working directory. This is a file-pathway that directs R to the folder in which the various data and script files are stored. Make sure the “data_session2.csv” file is saved in that folder and then use the read.csv() function to read the data into R.

```
##----------------------- Data Cleaning and QA ------------------------------
## Setting the Directory ----------------------------------------------------
getwd()
setwd("C:/Users/u6015231/Box Sync/Collaboration/Al Kozlowski/")
list.files()
# Make sure that the file data_session2.csv is saved in your working
directory.

# Import the .csv file into R.
# We will save this file in the R environment as an object called "DATA".
DATA<-read.csv("./data_session2.csv", header = TRUE, sep=",",
               na.strings=c("NA","NaN"," ",""))

# Use the head() function to check the structure of the data file.
head(DATA)

# Alternately you can also download the data file from the web here:
# DATA <-
read.csv("https://raw.githubusercontent.com/keithlohse/LMER_Clinical_Science/
master/data/data_session2.csv")
# head(DATA)
```

At the end of the first module, you might have noticed that although the linear fit was statistically significant, there was a lot of room for improvement. Especially at the early time points (refer to the figures from Session 1) the linear fit was not very accurate, often over-estimating performance for the different participants. As a first step in fitting curvilinear models we want to plot the data for each participant or (at least) a subset of participants.

It is also important to separate curvilinear from truly nonlinear models. Different people might define these terms slight differently, however we will define curvilinear models as curved, but linear in their parameters. Nonlinear models, conversely, are not necessarily curved, nor are they linear in their parameters. For instance, a cubic model is curvilinear, taking the form:

$$y_{ij} = \beta_0 + \beta_1(Time_{ij}) + \beta_2(Time_{ij}^2) + \beta_3(Time_{ij}^3) + \epsilon_{ij}$$

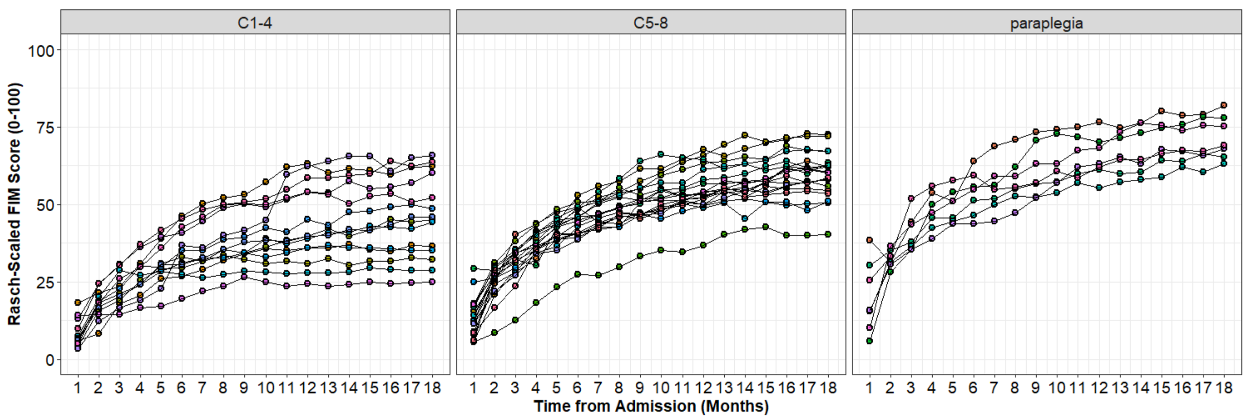
Conversely, the negative exponential model has a similar curve, but does not emerge from a linear combination of the its parameters:

$$y_{ij} = \alpha_i - (\alpha_i - \pi_{0i})e^{-\pi_{1i}(Time_{ij})} + \varepsilon_{ij}$$

We will reserve truly nonlinear models for a later date as fitting these models is often much more complex. However, curvilinear models are very power and, when combined with curvilinear random-effects, can fit unique trajectories for very different individuals. Below, we will walk through visualizing the group-level data and individual linear, quadratic, and cubic models.

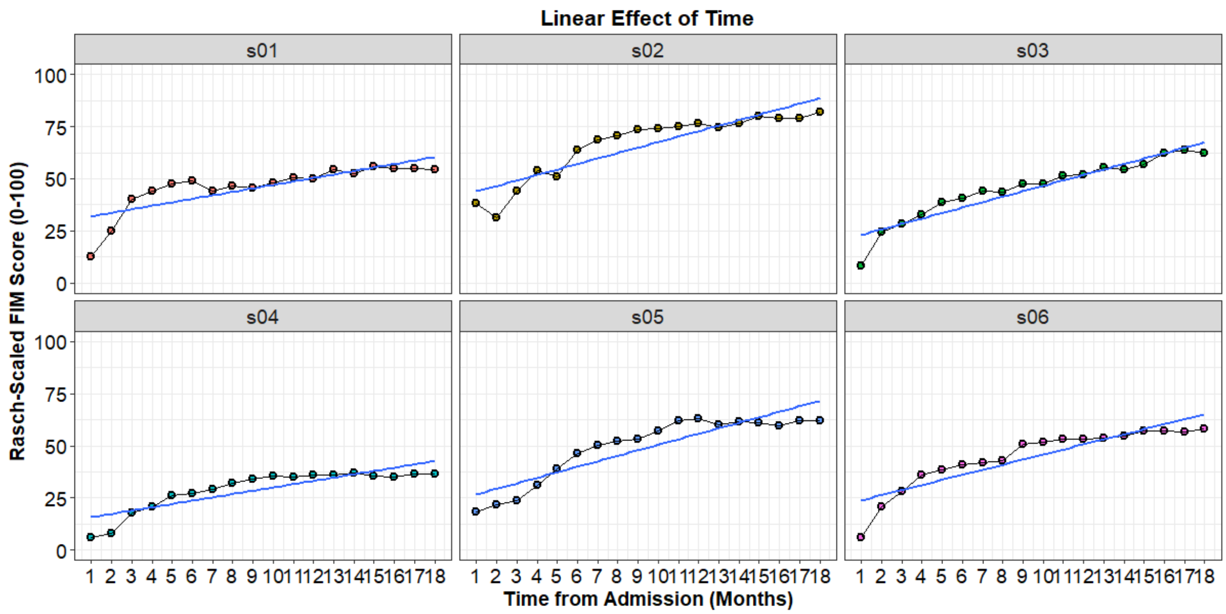
```
## ------------------- Visualizing the Effects of Time ----------------------
```

```
# One of the major questions we address in this module is how to best model
the effects of time. That is, what is the most appropriate "shape" of the
time curve? Is it perfectly straight? Is curved? In this module we will build
from our linear model (that we used in Module 1) to a curvilinear model in
which we add quadratic and cubic components.

## FIM scores by group and time ---------------------------------------------
g1<-ggplot(DATA, aes(x = month, y = rasch_FIM)) +
  geom_point(aes(fill=as.factor(subID)), pch=21, size=2, stroke=1.25) +
  geom_line(aes(group=subID)) +
  facet_wrap(~AIS_grade)
g2<-g1+scale_x_continuous(name = "Time from Admission (Months)",
breaks=c(0:18)) +
  scale_y_continuous(name = "Rasch-Scaled FIM Score (0-100)",limits=c(0,100))
g3 <- g2 + theme_bw() +
  theme(axis.text=element_text(size=14, colour="black"),
        axis.title=element_text(size=14,face="bold")) +
  theme(strip.text.x = element_text(size = 14))+
  theme(legend.position="none")

plot(g3)
```

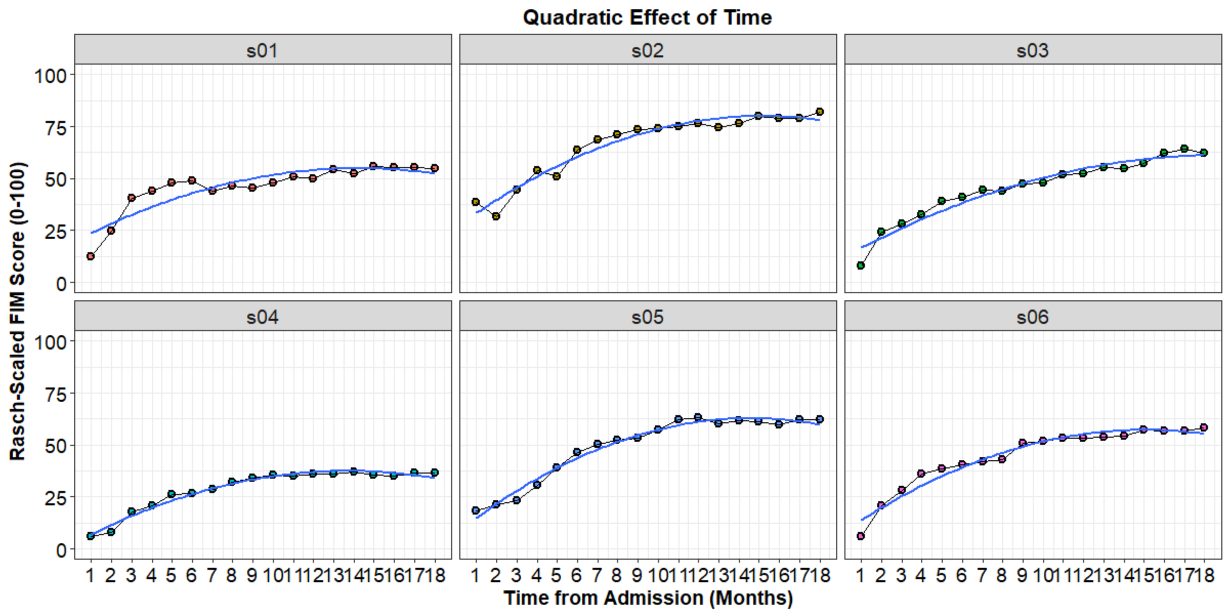



```
# We can see that these patterns are almost certainly not linear:
first6<-DATA[c(1:108),]

g1<-ggplot(first6, aes(x = month, y = rasch_FIM)) +
  geom_point(aes(fill=as.factor(subID)), pch=21, size=2, stroke=1.25) +
  geom_line() +
  stat_smooth(method=lm, se=FALSE)+
  facet_wrap(~subID)
g2<-g1+scale_x_continuous(name = "Time from Admission (Months)",
breaks=c(0:18)) +
  scale_y_continuous(name = "Rasch-Scaled FIM Score (0-
100)",limits=c(0,100))+
  ggtitle("Linear Effect of Time")
g3 <- g2 + theme_bw() +
  theme(plot.title = element_text(size=16, face="bold", hjust=0.5),
        axis.text=element_text(size=14, colour="black"),
        axis.title=element_text(size=14,face="bold"),
        strip.text.x = element_text(size = 14),
        legend.position="none")
```

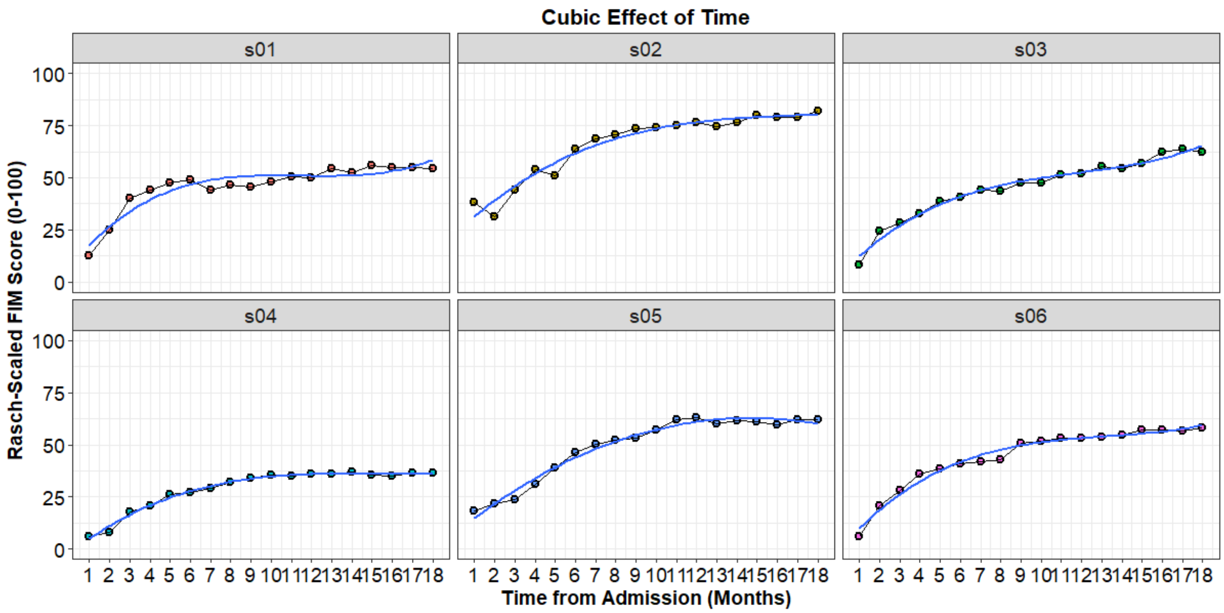
```
plot(g3)
```




```
# Visually, we can test the effect of adding a quadratic effect to the model
g1<-ggplot(first6, aes(x = month, y = rasch_FIM)) +
  geom_point(aes(fill=as.factor(subID)), pch=21, size=2, stroke=1.25) +
  geom_line() +
  stat_smooth(method=lm, formula = y~x+I(x^2), se=FALSE)+
  facet_wrap(~subID)
g2<-g1+scale_x_continuous(name = "Time from Admission (Months)",
breaks=c(0:18)) +
  scale_y_continuous(name = "Rasch-Scaled FIM Score (0-
100)",limits=c(0,100))+
  ggtitle("Quadratic Effect of Time")
g3 <- g2 + theme_bw() +
  theme(plot.title = element_text(size=16, face="bold", hjust=0.5),
        axis.text=element_text(size=14, colour="black"),
        axis.title=element_text(size=14,face="bold"),
        strip.text.x = element_text(size = 14),
        legend.position="none")

plot(g3)
```




```
# Further, we can see the effect of adding a cubic effect to the model
g1<-ggplot(first6, aes(x = month, y = rasch_FIM)) +
  geom_point(aes(fill=as.factor(subID)), pch=21, size=2, stroke=1.25) +
  geom_line() +
  stat_smooth(method=lm, formula = y~x+I(x^2)+I(x^3), se=FALSE)+
  facet_wrap(~subID)
g2<-g1+scale_x_continuous(name = "Time from Admission (Months)",
breaks=c(0:18)) +
  scale_y_continuous(name = "Rasch-Scaled FIM Score (0-
100)",limits=c(0,100))+
  ggtitle("Cubic Effect of Time")
g3 <- g2 + theme_bw() +
  theme(plot.title = element_text(size=16, face="bold", hjust=0.5),
        axis.text=element_text(size=14, colour="black"),
        axis.title=element_text(size=14,face="bold"),
        strip.text.x = element_text(size = 14),
        legend.position="none")

plot(g3)
```



Cubic Effect of Time
s01
s02
s03
s04
s05
s06
Rasch-Scaled FIM Score (0-100)
Time from Admission (Months)

2.2 Comparing Different Effects of Time

In order to quantify what our visualizations show us qualitatively, we need to statistically compare models with linear, quadratic, and cubic effects of time.

```
DATA$year.0_sq<-DATA$year.0^2
DATA$year.0_cu<-DATA$year.0^3


# Linear Effect of Time
time_linear<-lmer(rasch_FIM~
                 # Fixed-effects
                 1+year.0+
                 # Random-effects
                 (1+year.0|subID), data=DATA, REML=FALSE)
summary(time_linear)
```

```
Linear mixed model fit by maximum likelihood t-tests use Satterthwaite
approximations to degrees of
  freedom [lmerMod]
Formula: rasch_FIM ~ 1 + year.0 + (1 + year.0 | subID)
   Data: DATA

     AIC      BIC   logLik deviance df.resid
  4794.8   4822.3  -2391.4   4782.8      714

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.7756 -0.4913  0.1596  0.6602  2.0907

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 subID    (Intercept) 47.83    6.916
          year.0      50.82    7.129    0.37
 Residual             33.77    5.811
Number of obs: 720, groups:  subID, 40

Fixed effects:
            Estimate Std. Error     df t value Pr(>|t|)
(Intercept)   26.589      1.170 40.000   22.73   <2e-16 ***
year.0        25.857      1.233 40.000   20.96   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
       (Intr)
year.0 0.189
```

```
# Quadratic Effect of Time
time_square<-lmer(rasch_FIM~
                     # Fixed-effects
                     1+year.0+year.0_sq+
                     # Random-effects
                     (1+year.0+year.0_sq|subID), data=DATA, REML=FALSE)
summary(time_square)
```

```
Linear mixed model fit by maximum likelihood t-tests use Satterthwaite
approximations to degrees of
  freedom [lmerMod]
Formula: rasch_FIM ~ 1 + year.0 + year.0_sq + (1 + year.0 + year.0_sq |
subID)
   Data: DATA

     AIC      BIC   logLik deviance df.resid
  4039.3   4085.0  -2009.6   4019.3      710

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.4894 -0.5194  0.0454  0.5442  3.0358

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 subID    (Intercept)  46.224   6.799
          year.0      242.048  15.558    0.07
          year.0_sq    44.135   6.643   -0.08 -0.93
 Residual               9.754   3.123
Number of obs: 720, groups:  subID, 40

Fixed effects:
            Estimate Std. Error      df t value Pr(>|t|)
(Intercept)   18.103      1.120  40.000   16.17   <2e-16 ***
year.0        64.044      2.666  40.000   24.03   <2e-16 ***
year.0_sq    -26.956      1.262  40.000  -21.36   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
          (Intr) year.0
year.0    -0.030
year.0_sq  0.049 -0.921
```

```
# Cubic Effect of Time
time_cube<-lmer(rasch_FIM~
                    # Fixed-effects
                    1+year.0+year.0_sq+year.0_cu+
                    # Random-effects
                    (1+year.0+year.0_sq+year.0_cu|subID), data=DATA,
REML=FALSE)
summary(time_cube)
```

```
Linear mixed model fit by maximum likelihood t-tests use Satterthwaite
approximations to degrees of
  freedom [lmerMod]
Formula: rasch_FIM ~ 1 + year.0 + year.0_sq + year.0_cu + (1 + year.0 +
    year.0_sq + year.0_cu | subID)
   Data: DATA

     AIC      BIC   logLik deviance df.resid
  3741.4   3810.1  -1855.7   3711.4      705

Scaled residuals:
    Min      1Q  Median      3Q     Max
```

```
-4.1697 -0.5265 -0.0002  0.5186  3.5018

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 subID    (Intercept)   42.779  6.541
          year.0       635.961 25.218    0.06
          year.0_sq   1179.872 34.349   -0.07 -0.88
          year.0_cu    213.006 14.595    0.10  0.77 -0.98
 Residual                5.481  2.341
Number of obs: 720, groups:  subID, 40

Fixed effects:
            Estimate Std. Error      df t value Pr(>|t|)
(Intercept)   14.977      1.073  40.000  13.954  < 2e-16 ***
year.0        95.033      4.378  39.990  21.706  < 2e-16 ***
year.0_sq    -83.232      6.213  39.990 -13.397 2.22e-16 ***
year.0_cu     26.483      2.698  39.990   9.815 3.30e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
          (Intr) year.0 yr.0_s
year.0    -0.041
year.0_sq  0.027 -0.891
year.0_cu  0.001  0.797 -0.981
```

Now that we have all of these models constructed, we can make a statistical comparison between models using the anova() and the AIC as our index of model fit.

```
anova(time_linear, time_square, time_cube)
```

```
Data: DATA
Models:
object: rasch_FIM ~ 1 + year.0 + (1 + year.0 | subID)
..1: rasch_FIM ~ 1 + year.0 + year.0_sq + (1 + year.0 + year.0_sq |
..1:     subID)
..2: rasch_FIM ~ 1 + year.0 + year.0_sq + year.0_cu + (1 + year.0 +
..2:     year.0_sq + year.0_cu | subID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  6 4794.8 4822.3 -2391.4   4782.8
..1    10 4039.3 4085.0 -2009.6   4019.3 763.58      4  < 2.2e-16 ***
..2    15 3741.4 3810.1 -1855.7   3711.4 307.87      5  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The cubic model provides far and away the best fit, however, not the large degrees of freedom in this model (5 more than the quadratic and 9 more than the linear model!). Having 15 degrees of freedom may not seem like a problem right now, but as we add fixed-effect to our model, we might want to have more "room" in the model to add interactions between fixed effects. As such, it might be interesting to compare our model that has cubic fixed- and random-effects to a model that has only a fixed-effect.

```
# Cubic Fixed-Effect Only
time_cube_fixed<-lmer(rasch_FIM~
                    # Fixed-effects
                    1+year.0+year.0_sq+year.0_cu+
```

```
                    # Random-effects
                    (1+year.0+year.0_sq|subID), data=DATA, REML=FALSE)

anova(time_cube_fixed, time_cube)
# Note that we list time_cube_fixed first as it is the smaller model with
fewer degrees of freedom.
```

```
Data: DATA
Models:
object: rasch_FIM ~ 1 + year.0 + year.0_sq + year.0_cu + (1 + year.0 +
object:     year.0_sq | subID)
..1: rasch_FIM ~ 1 + year.0 + year.0_sq + year.0_cu + (1 + year.0 +
..1:     year.0_sq + year.0_cu | subID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object 11 3795.6 3845.9 -1886.8   3773.6
..1    15 3741.4 3810.1 -1855.7   3711.4 62.181      4  1.009e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on the AIC, it looks like those 4-extra degrees of freedom are worth it and statistically improve our model fit. However, it is worth remembering that you can add fixed- and random-effects for your time variable separately. Occasionally, you might find that a linear random-effect is necessary, but quadratic and cubic random-effects are not, which would imply little variation between individuals in these components. As you build these models, remember that they do not need to be any more complex then necessary and exploring these random-effects of time is an essential first step before moving on to more complicated fixed-effect models.

2.3 Conditional Curvilinear Models

As with our linear models, we are interested in how a person's AIS grade affects not only where they begin, but how they progress through therapy. We will start with our best fitting random-effects model: the cubic model of time:

$$y_{ij} = B_0 + \beta_1(Time_{ij}) + \beta_2(Time_{ij}^2) + \beta_3(Time_{ij}^3) +$$
$$U_0 + U_{1j}(Time_{ij}) + U_{2j}(Time_{ij}^2) + U_{3j}(Time_{ij}^3) + \epsilon_{ij}$$

In the equation, we have highlighted the fixed-effects in blue, the random-effects in red, and random-errors in green. It is important to remember that our AIS grade variable has three levels ("C1-4", "C5-8", and "paraplegia"). To test the effects of this variable, therefore, we need two separate contrasts. By default, R will create these contrasts using "treatment coding" (also referred to as "dummy coding") in which one group serves as a reference against which the other groups are compared. R assigns these levels alphanumerically, so "C1-4" will serve as the reference group. To begin, we will add a main-effect of AIS grade, and the linear interaction of AIS and Time:

$$y_{ij} = B_0 + \beta_1(Time_{ij}) + \beta_2(Time_{ij}^2) + \beta_3(Time_{ij}^3) +$$
$$\beta_4(C5to8_j) + \beta_5(Paraplegia_j) + \beta_6(Time_{ij} \times C5to8_j) + \beta_7(Time_{ij} \times Paraplegia_j) +$$
$$U_0 + U_{1j}(Time_{ij}) + U_{2j}(Time_{ij}^2) + U_{3j}(Time_{ij}^3) + \epsilon_{ij}$$

We have color-coded the main-effect of AIS grade in orange and the linear interactions of AIS and Time in black. Note that AIS grade is only coded by the sub-script "j", because this variable only changes between participants, it does not vary over time.

From this initial model, we will add quadratic and cubic interactions to see which model provides the best explanation of the data.

```
# Effect of AIS Grade on Time
cond_01<-lmer(rasch_FIM~
                  # Fixed-effects
                  1+year.0*AIS_grade+year.0_sq+year.0_cu+
                  # Random-effects
                  (1+year.0+year.0_sq+year.0_cu|subID), data=DATA,
REML=FALSE)
summary(cond_01)


# Effect of AIS Grade on Quadratic Time
cond_02<-lmer(rasch_FIM~
                # Fixed-effects
                1+year.0*AIS_grade+year.0_sq*AIS_grade+year.0_cu+
                # Random-effects
                (1+year.0+year.0_sq+year.0_cu|subID), data=DATA, REML=FALSE)
summary(cond_02)
```

```r
# Effect of AIS Grade on Cubic Time
cond_03<-lmer(rasch_FIM~
                # Fixed-effects
                1+year.0*AIS_grade+year.0_sq*AIS_grade+year.0_cu*AIS_grade+
                # Random-effects
                (1+year.0+year.0_sq+year.0_cu|subID), data=DATA, REML=FALSE)
summary(cond_03)

# Comparing between Models
anova(cond_01, cond_02, cond_03)
```

```
Data: DATA
Models:
object: rasch_FIM ~ 1 + year.0 * AIS_grade + year.0_sq + year.0_cu +
object:     (1 + year.0 + year.0_sq + year.0_cu | subID)
..1: rasch_FIM ~ 1 + year.0 * AIS_grade + year.0_sq * AIS_grade +
..1:     year.0_cu + (1 + year.0 + year.0_sq + year.0_cu | subID)
..2: rasch_FIM ~ 1 + year.0 * AIS_grade + year.0_sq * AIS_grade +
..2:     year.0_cu * AIS_grade + (1 + year.0 + year.0_sq + year.0_cu |
..2:     subID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object 19 3711.5 3798.5 -1836.7   3673.5
..1    21 3714.6 3810.8 -1836.3   3672.6 0.8533      2     0.6527
..2    23 3716.5 3821.8 -1835.3   3670.5 2.1146      2     0.3474
```

Note the large degrees of freedom in these models. Including cubic random-effects and several interactions in the fixed-effects add up quickly! Ultimately, it looks like our first model provides the best explanation of the data based on the AIC:

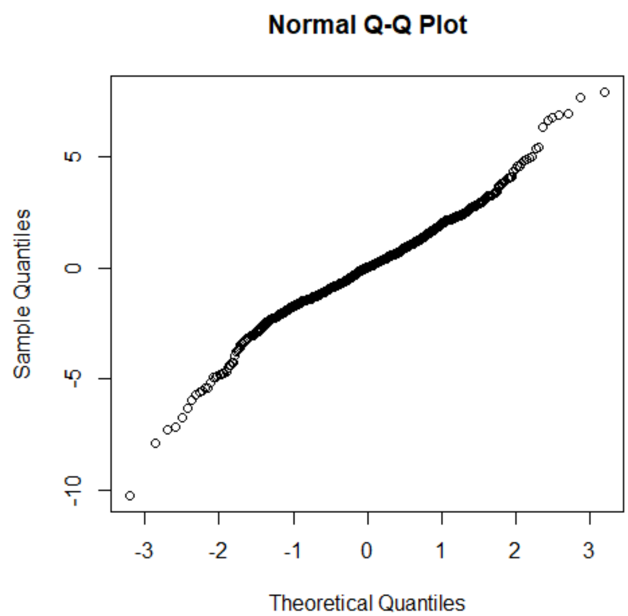
```r
summary(cond_01)
```

```
Linear mixed model fit by maximum likelihood t-tests use Satterthwaite
approximations to degrees of
  freedom [lmerMod]
Formula: rasch_FIM ~ 1 + year.0 * AIS_grade + year.0_sq + year.0_cu +
    (1 + year.0 + year.0_sq + year.0_cu | subID)
   Data: DATA

     AIC       BIC    logLik deviance df.resid
  3711.5    3798.5   -1836.7   3673.5      701

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.3757 -0.5280  0.0089  0.5130  3.3788

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 subID    (Intercept)   21.488  4.636
          year.0       594.874 24.390   -0.33
          year.0_sq   1179.874 34.349    0.21 -0.88
          year.0_cu    213.006 14.595   -0.15  0.77 -0.98
 Residual                5.481  2.341
Number of obs: 720, groups:  subID, 40


Fixed effects:
```

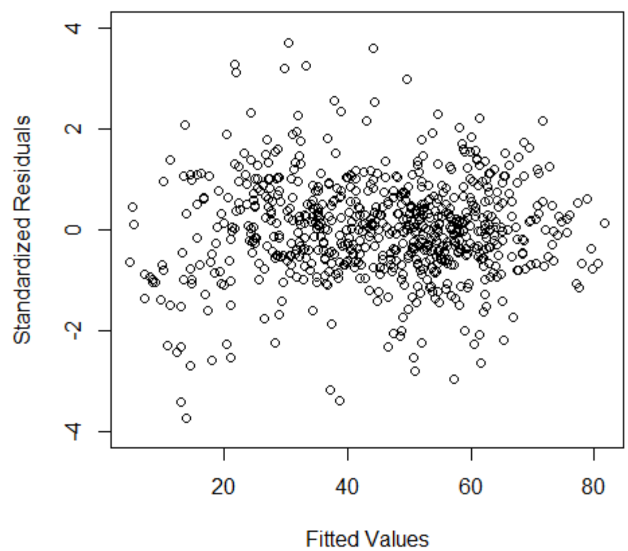
```
                              Estimate Std. Error      df t value Pr(>|t|)
(Intercept)                      9.083      1.260  43.160   7.209 6.28e-09 ***
year.0                          91.906      4.426  45.790  20.765  < 2e-16 ***
AIS_gradeC5-8                    7.022      1.608  40.000   4.367 8.66e-05 ***
AIS_gradeparaplegia             14.619      2.113  40.000   6.918 2.44e-08 ***
year.0_sq                      -83.232      6.213  40.030 -13.397 2.22e-16 ***
year.0_cu                       26.483      2.698  40.020   9.815 3.28e-12 ***
year.0:AIS_gradeC5-8             4.048      1.968  40.000   2.057   0.0462 *
year.0:AIS_gradeparaplegia       6.879      2.586  40.000   2.660   0.0112 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
             (Intr) year.0 AIS_C5 AIS_gr yr.0_s yr.0_c y.0:AIS_C
year.0       -0.281
AIS_grdC5-8 -0.735  0.047
AIS_grdprpl -0.559  0.035  0.438
year.0_sq    0.181 -0.859  0.000  0.000
year.0_cu   -0.144  0.768  0.000  0.000 -0.981
y.0:AIS_C5-  0.134 -0.256 -0.182 -0.080  0.000  0.000
yr.0:AIS_gr  0.102 -0.195 -0.080 -0.182  0.000  0.000  0.438
```

2.4 Checking the Assumptions of our Best Fitting Model

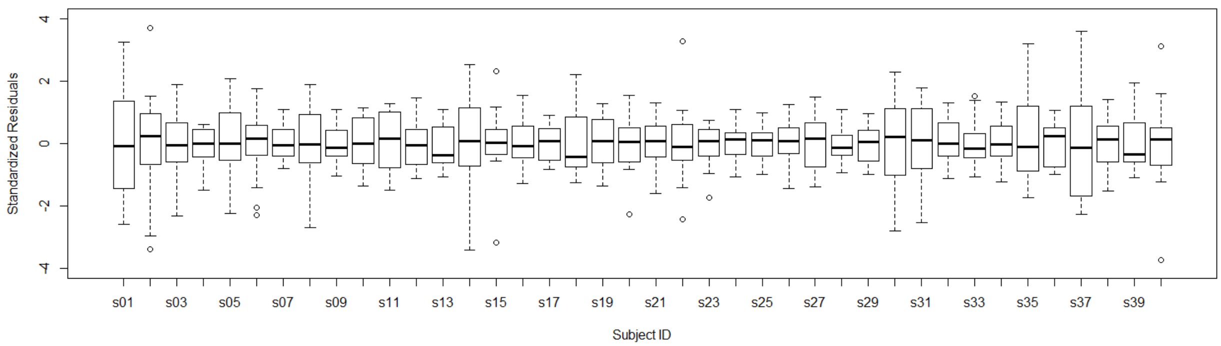
```
## Level 1 Assumptions ----
# Normality
qqnorm(resid(cond_01))
```



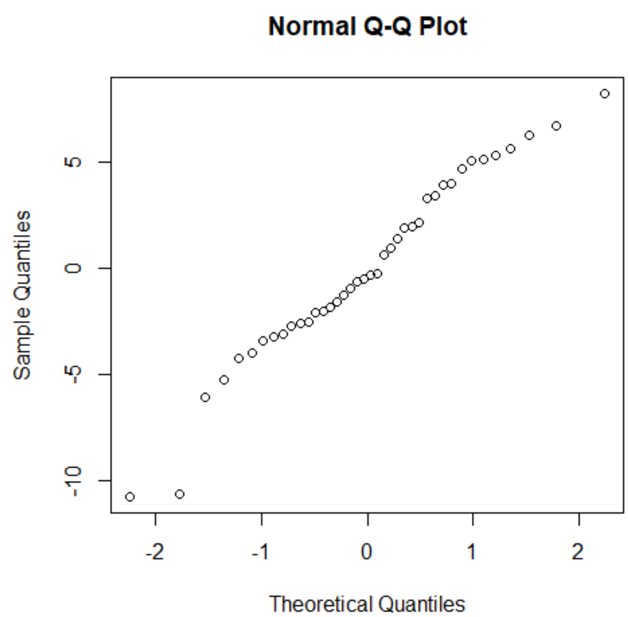

```
# Homoscedasticity
# Comparable Variances at Level 1
x <- fitted(cond_01)
y <- resid(cond_01)/sd(resid(cond_01))
plot(x=x, y=y, xlab = "Fitted Values", ylab="Standardized Residuals",
     ylim=c(-4,4))
```



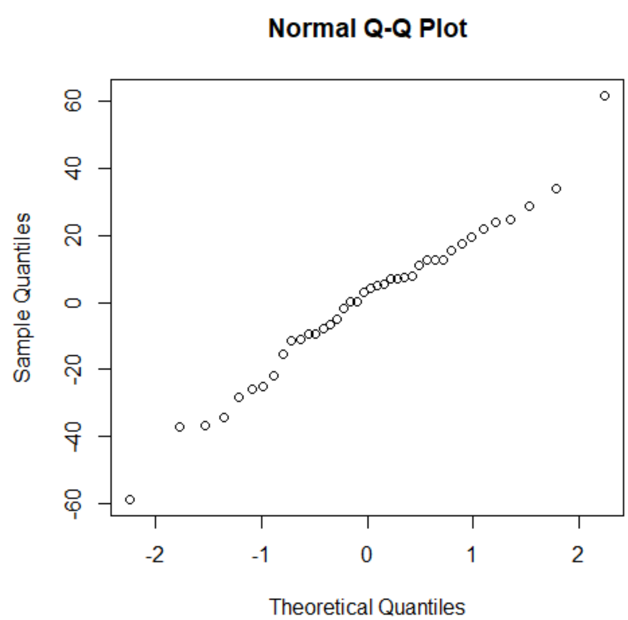
```
# Influential Participants
plot(DATA$subID, resid(cond_01)/sd(resid(cond_01)),
     ylab="Standardized Residuals", xlab="Subject ID",
     ylim=c(-4,4))
```



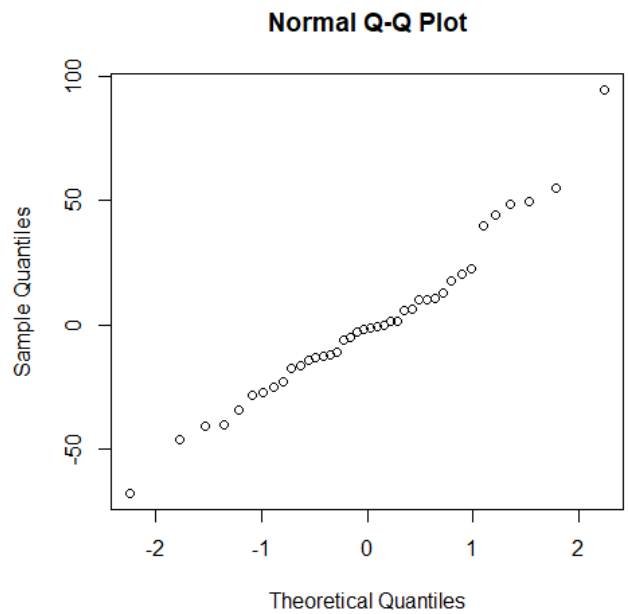

```
## Level 2 Assumptions ----
LVL2<-summarize(group_by(DATA, subID),
                AIS_grade = AIS_grade[1])
head(LVL2)
ranef(cond_01)

LVL2$RE_int<-ranef(cond_01)$subID$`(Intercept)`
LVL2$STD_int<-LVL2$RE_int/sd(LVL2$RE_int)

LVL2$RE_year<-ranef(cond_01)$subID$year.0
LVL2$STD_year<-LVL2$RE_year/sd(LVL2$RE_year)

LVL2$RE_year_sq<-ranef(cond_01)$subID$year.0_sq
LVL2$STD_year_sq<-LVL2$RE_year_sq/sd(LVL2$RE_year_sq)

LVL2$RE_year_cu<-ranef(cond_01)$subID$year.0_cu
LVL2$STD_year_cu<-LVL2$RE_year_cu/sd(LVL2$RE_year_cu)

head(LVL2)

# Normality of Random-Effects
qqnorm(LVL2$RE_int)
```



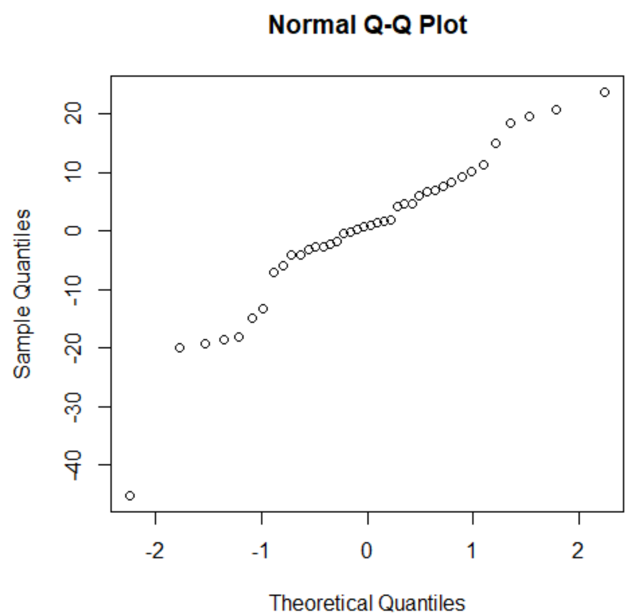

```
qqnorm(LVL2$RE_year)
```



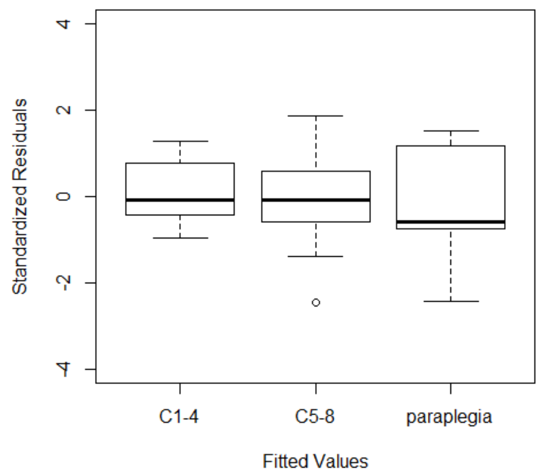
```r
qqnorm(LVL2$RE_year_sq)
```



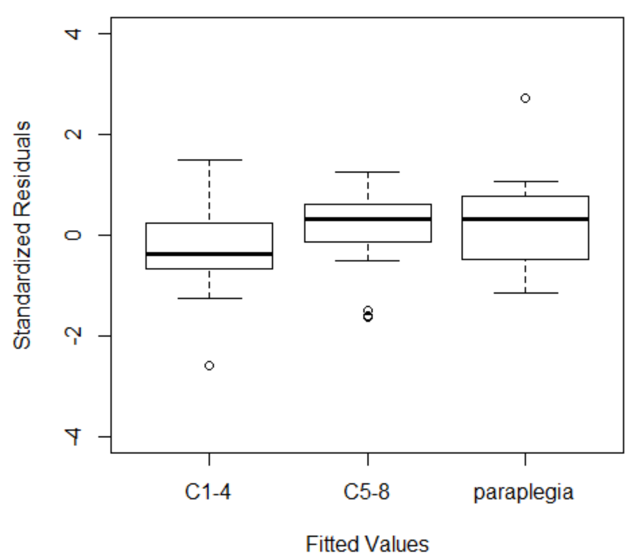

```r
qqnorm(LVL2$RE_year_cu)
```



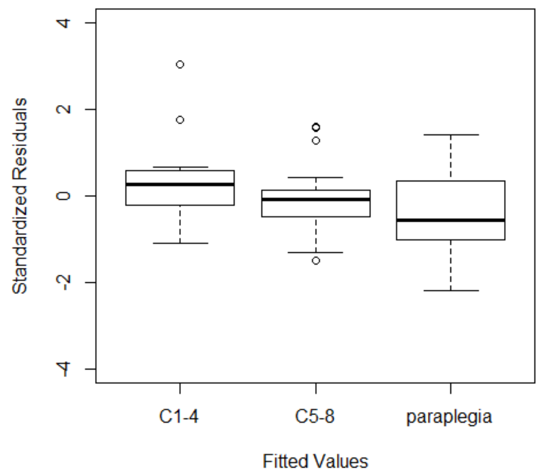
```r
# Homoscedasticity
plot(x=LVL2$AIS_grade, y=LVL2$STD_int,
     xlab = "Fitted Values", ylab="Standardized Residuals",
     ylim=c(-4,4))
```



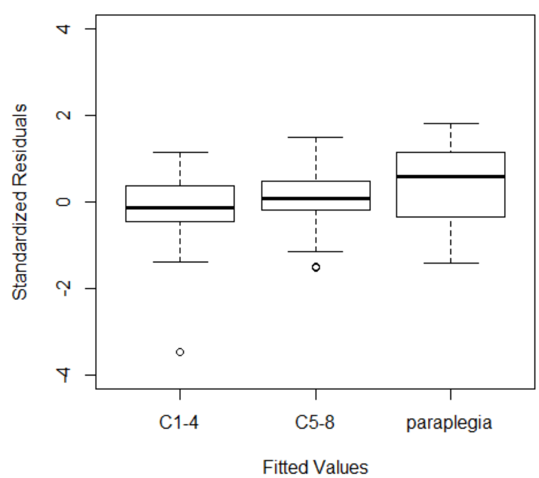

```r
plot(x=LVL2$AIS_grade, y=LVL2$STD_year,
     xlab = "Fitted Values", ylab="Standardized Residuals",
     ylim=c(-4,4))
```



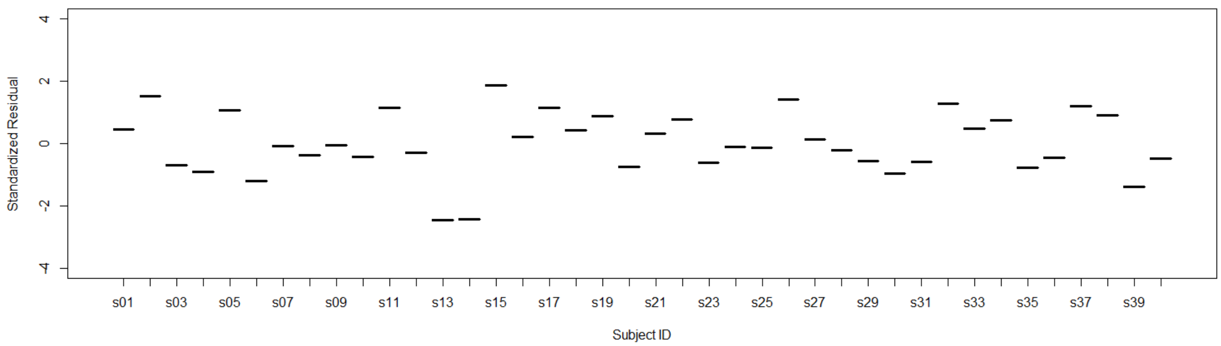
```
plot(x=LVL2$AIS_grade, y=LVL2$STD_year_sq,
     xlab = "Fitted Values", ylab="Standardized Residuals",
     ylim=c(-4,4))
```



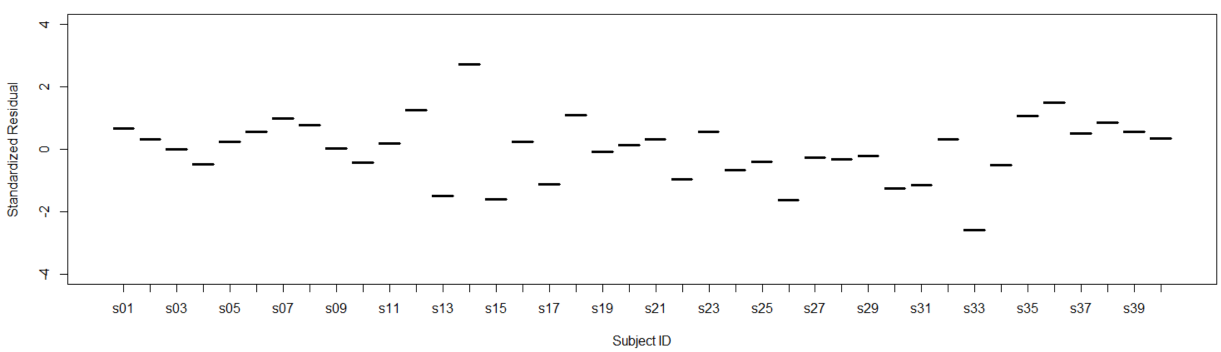

```
plot(x=LVL2$AIS_grade, y=LVL2$STD_year_cu,
     xlab = "Fitted Values", ylab="Standardized Residuals",
     ylim=c(-4,4))
```



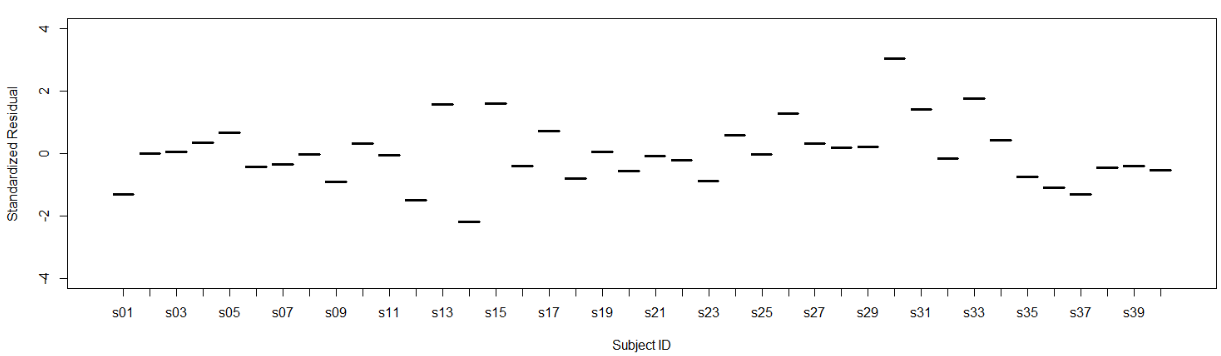
```
# Influential Participants
plot(x=LVL2$subID, y=LVL2$STD_int,
     xlab="Subject ID", ylab="Standardized Residual",
     ylim=c(-4,4))
```




```
plot(x=LVL2$subID, y=LVL2$STD_year,
     xlab="Subject ID", ylab="Standardized Residual",
     ylim=c(-4,4))
```




```
plot(x=LVL2$subID, y=LVL2$STD_year_sq,
     xlab="Subject ID", ylab="Standardized Residual",
     ylim=c(-4,4))
```



```
plot(x=LVL2$subID, y=LVL2$STD_year_cu,
     xlab="Subject ID", ylab="Standardized Residual",
     ylim=c(-4,4))
```