Classical Math is cool
But I like intuitionistic/constructive
math more

Classical Math discusses Truth

$\vdash \varphi$ means φ is true

$\vdash \varphi \vee \neg \varphi$

either φ is true or $\neg \varphi$ is true

Constructive Math discusses Proof

$\vdash \varphi$ means I can prove φ

$\vdash \varphi \vee \neg \varphi$ is not generally true

It is not the case that for all φ, we have a proof of φ or a proof of $\neg\varphi$

See, Gödel's Incompleteness Thm

Constructive Math is Sound but not Complete

$$\vdash \varphi \Rightarrow \models \varphi$$

$$\models \varphi \not\Rightarrow \vdash \varphi$$

It is complete with respect to a different semantics using Kripke structures

We will not go into this in further detail

Constructive Math "includes" classical math

$$\vdash_i \varphi \Rightarrow \vdash_c \varphi$$

$$\vdash_c \varphi \Rightarrow \vdash_i \neg\neg\varphi$$

- "But how could $\varphi \vee \neg\varphi$ NOT be true???"
- "Don't worry, its not not true"

$$\Gamma \vdash \varphi \subseteq 2^{Prop} \times Prop$$

The right side only has 1 prop

This is really the big change that

differentiates them

$$\frac{}{\Gamma, \varphi \vdash \varphi}\ \text{Axiom} \qquad \frac{\Gamma \vdash \varphi \qquad \Gamma, \varphi \vdash \psi}{\Gamma \vdash \psi}\ \text{Cut}$$

The rest of the rules follow the pattern of "introduction" forms and "elimination" forms. These forms will show up again and again, from usage in ATP, type inference, and more

Connective	Introduction	Elimination
$\land$	$\frac{\Gamma \vdash \varphi \qquad \Gamma \vdash \psi}{}$	$\frac{\Gamma \vdash \varphi \land \psi}{} \qquad \frac{\Gamma \vdash \varphi \land \psi}{}$

	$\Gamma \vdash \varphi \land \psi$	$\Gamma \vdash \varphi \qquad \Gamma \vdash \psi$
$\lor$	$\dfrac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \lor \psi} \qquad \dfrac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \lor \psi}$	$\dfrac{\Gamma \vdash \varphi \lor \psi \quad \Gamma, \varphi \vdash \delta \quad \Gamma, \psi \vdash \delta}{\Gamma \vdash \delta}$
$\rightarrow$	$\dfrac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \rightarrow \psi}$	$\dfrac{\Gamma \vdash \varphi \rightarrow \psi \quad \Gamma \vdash \varphi}{\Gamma \vdash \psi}$
$\top$	$\dfrac{}{\Gamma \vdash \top}$	N/A
$\bot$	N/A	$\dfrac{\Gamma \vdash \bot}{\Gamma \vdash \varphi}$

Note, we've omitted $\neg$. It is much easier, moving forward, to think of $\neg\varphi$ as $\varphi \Rightarrow \perp$.