Economics with Calculus Notes

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Substitution and Income Effect

> **Definition.** *Substitution Effect* — The change in demand due to the change in the rate of exchange between two goods.

> **Definition.** *Income Effect* — The change in demand due to having more purchasing power.

Suppose a consumer has preferences between two goods that are perfect substitutes. Can you change prices in such a way that the entire demand response is due to the income effect?

The answer is *yes*. To see this, consider red pencils and blue pencils. Suppose red pencils cost 10 cents a piece and blue pencils cost 5 cents a piece, and the consumer spends $1 on pencils. She would then consume 20 blue pencils. If the price of blue pencils drops to 4 cents, she would consume 25 blue pencils, a change which is entirely due to income effect.

Buying and Selling

The Budget Constraint

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

We could just as well express this budget line in terms of net demands as

$$p_1(x_1 - \omega_1) + p_2(x_2 - \omega_2) = 0$$

The Labor Supply Budget Constraint

$$\underbrace{p}_{\text{Price of Consumption}} \overbrace{C}^{\text{Amount of Consumption}} = \underbrace{M}_{\text{Money Income}} + \overbrace{w}^{\text{Wage Rate}} \underbrace{L}_{\text{Amount of Labor Supplied}}$$

This next budget constraint says that the the value of a consumer's consumption plus their leisure has to equal the value of their endowment of consumption and endowment of time.

$$pC + wR = p\bar{C} + w\bar{R}$$

Market Demand

> **Definition.** *Market Demand*— The total demand for a product or service in the market as a whole. Market demand, also called *aggregate demand*, is calculated to determine at what level to set production output for a good or service, and to help to determine optimal pricing levels to maximize sales revenues.

The basic framework is there are n consumers and each consumer has a different income, which we denote m_i. All of them consume goods 1 and 2, and everyone pays the same prices for the goods, denoted p_1 and p_2. Each of the consumers maximize utility of goods 1 and 2. We are using a slightly altered budget constraint,

$$p_1 x_1 + p_2 x_2 = m_i$$

We will focus on the demand for good 1. To find market demand we will use

$$\underbrace{f(p_1, p_2, m_1, \ldots, m_n)}_{\text{Consumer } i's \text{ demand function for good 1}} \qquad = \qquad \overbrace{\sum_{i=1}^{n} f_i(p_1, p_2, m_i)}^{\text{Sum of individual demands over all consumers}}$$

Market Demand Curve

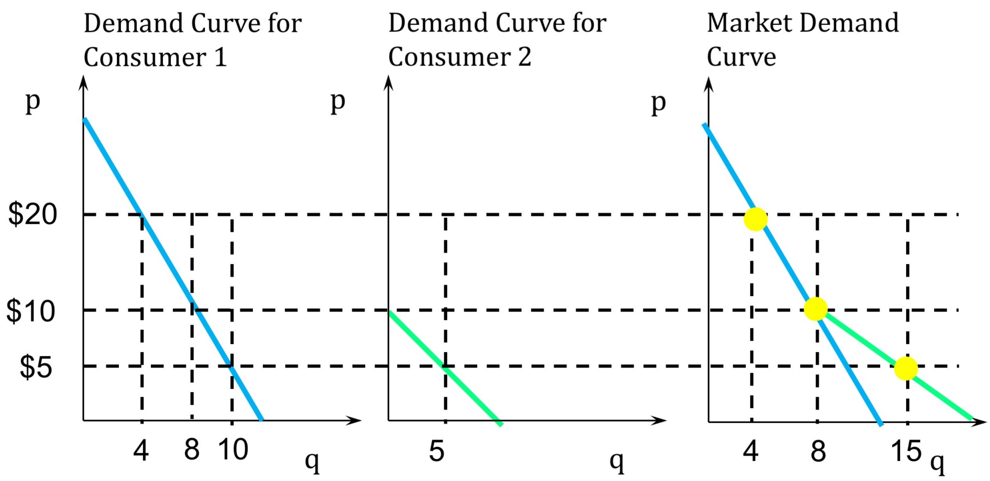
The *market demand curve* is simply the sum of the individual demand curves. We will focus on the relationship between the demand for good 1 and the price of good 1. The market demand curve is given by,

$$D(p) = \sum_{i=1}^{n} D_i(p)$$

In an effort to simplify notation, let's use

$$\begin{aligned} q &= \text{units of good 1 (quantity)} \\ p &= \text{price of good 1 (price)} \\ D_i(p) &= \text{demand curve for individual } i \end{aligned}$$

Adding Horizontally



THE LAW OF DEMAND
If our good is normal, a decrease in price means increase in demand for two reasons.

Reason 1.) Intensive Margin—For consumers with a positive demand, a lower price increases demand.

Reason 2.) Extensive Margin—For consumers with zero demand, a lower price may cause them to enter the market. Take a look at Consumer 2 and imagine a price below $10.

Definition. *Exogenous Factors*
Exogenous, or outside, factors that strike changes are uncontrollable and, most times, unpredictable. For instance, consumer incomes and prices of other goods are exogenous. Changes in exogenous factors will shift the demand curve.

Let's take a look at a few changes graphically.

Figure 1: *Substitute.* Since goods 1 and 2 are substitutes, then we know that increasing the price of good 2 will tend to increase the demand for good whatever its price. Thus, increasing the price of good 2 will tend to shift the aggregate demand curve for good 1 outward.

Figure 2: *Complement.* Similarly, if good 1 and 2 are complements, increasing the price of good 2 will shift the aggregate demand curve for good 1 inward.

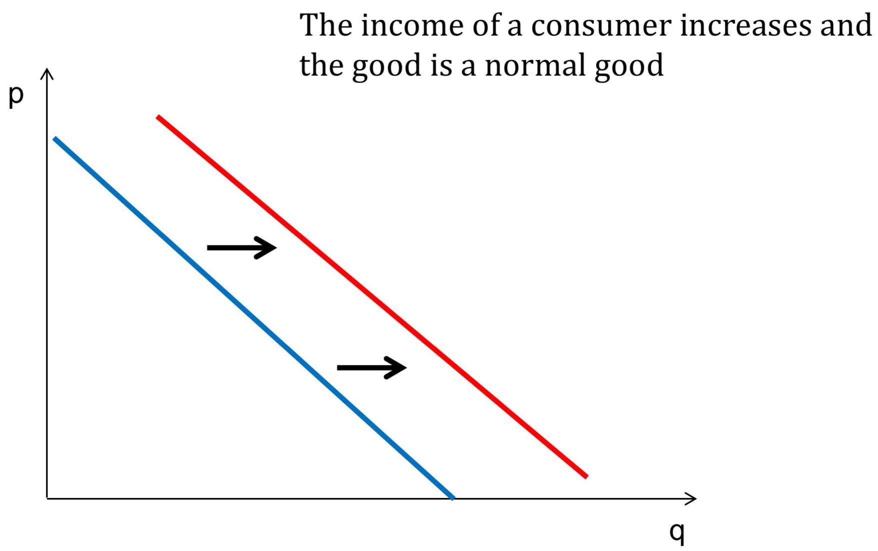


Figure 3: *Normal.* If good 1 is a normal good for an individual, then increasing that individual's money income, would tend to increase that individual's demand and, therefore, shift the aggregate demand curve outward.

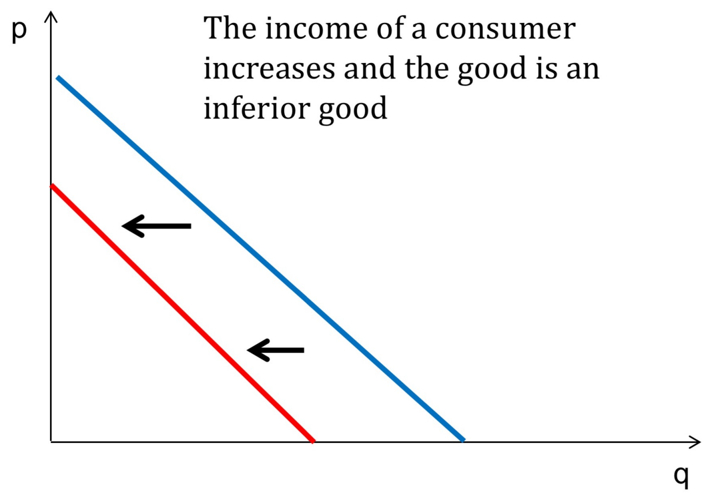


Figure 4: *Inferior.* If good 1 is an inferior good for an individual, then increasing that individual's money income, would tend to decrease that individual's demand and, therefore, shift the aggregate demand curve inward.

Demand Responsiveness to Price

The slope of the demand curve is one measure of responsiveness of demand to own price. It is important to note that the slope depends on unites of measure, while price elasticity does not depend on units.

Price Elasticity

Definition. *Price Elasticity*— Percentage change in demand divided by percentage change in price.

$$\text{Slope of Demand Function } = \frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q/q}{\Delta p/p} = \frac{\Delta q}{\Delta p} \times \frac{p}{q}$$

$$\text{Price Elasticity of Demand } = \varepsilon = \lim_{\Delta p \to 0} \frac{\Delta q}{\Delta p} \times \frac{p}{q} = D'(p)\frac{p}{D(p)}$$

The Elasticity of a Linear Demand Curve

Consider the linear demand curve, $q = a - bp$, depicted in Figure 5. The slope of this demand curve is a constant $-b$. Plugging this into the formula for elasticity we have,

$$\varepsilon = -b\frac{p}{q} = -b\frac{p}{a - bp}$$

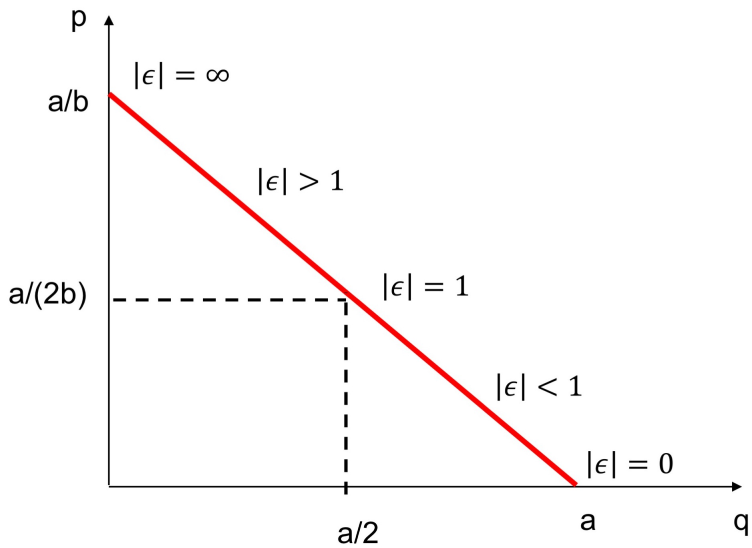


Figure 5: *The Elasticity of a Linear Demand Curve.* Elasticity is infinite at the vertical intercept, one halfway down the curve, and zero at the horizontal intercept.

Classifying Demand

Elastic Demand: $|\varepsilon| > 1$

Inelastic Demand: $|\varepsilon| < 1$

Unit Elastic Demand: $|\varepsilon| = 1$

Elasticity and Revenue

Definition. *Revenue*— The price of a good times the quantity sold of that good.

$$\text{Revenue} = R = p \times q$$

If we let the price change to $p + \Delta p$ and the quantity change to $q + \Delta q$, we have a new revenue of

$$\begin{aligned} R' &= (p + \Delta p)(q + \Delta q) \\ &= \underbrace{pq}_{R} + q\Delta p + p\Delta q + \Delta p \Delta q \end{aligned}$$

Subtracting R from R' we have

$$\Delta R = q\Delta p + p\Delta q + \Delta p \Delta q$$

For small values Δp and Δq, the last term can be safely neglected, leaving us with an expression for the change in revenue,

$$\Delta R = q\Delta p + p\Delta q$$

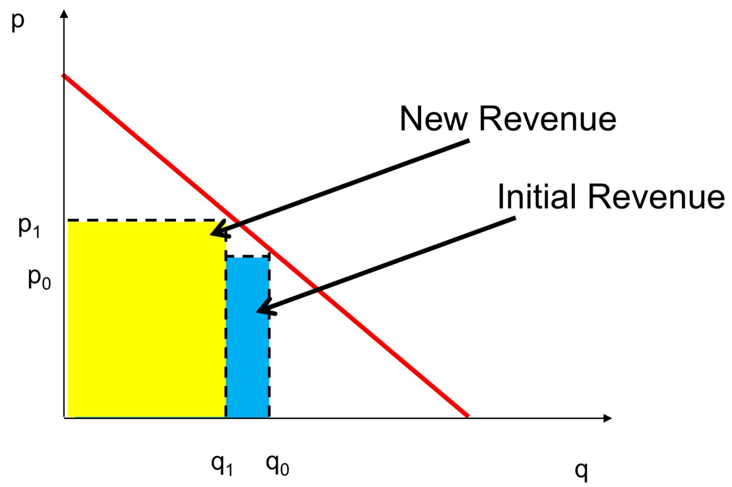


Figure 6: *An Increase in Price.*

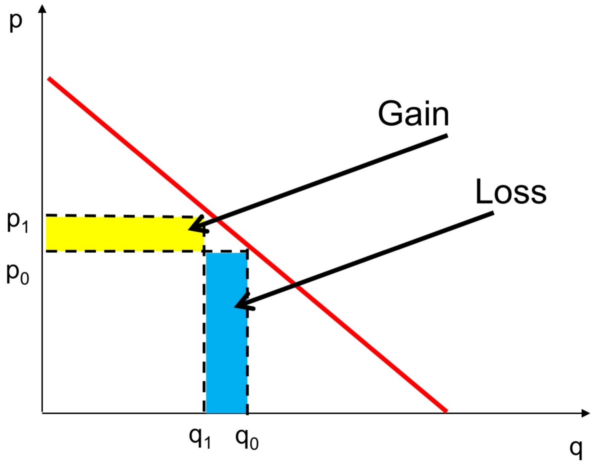


Figure 7: *The Change in Revenue*

The Role of Elasticity

Another way to write the revenue change is

$$\Delta R = p\Delta q + q\Delta p > 0$$

and rearrange this to get

$$-\frac{p}{q} \times \frac{\Delta q}{\Delta p} = |\varepsilon(p)| < 1$$

We can rearrange the formula once more by taking $\Delta R/\Delta p$ and rearranging it as follows

$$\begin{aligned}\frac{\Delta R}{\Delta p} &= q + p\frac{\Delta q}{\Delta p} \\ &= q\left[1 + \frac{p}{q}\frac{\Delta q}{\Delta p}\right] \\ &= q[1 + \varepsilon(p)]\end{aligned}$$

If $-\infty < \varepsilon < -1$ (elastic), $R'(p) < 0$.

If $-1 < \varepsilon < 0$ (inelastic), $R'(p) > 0$.

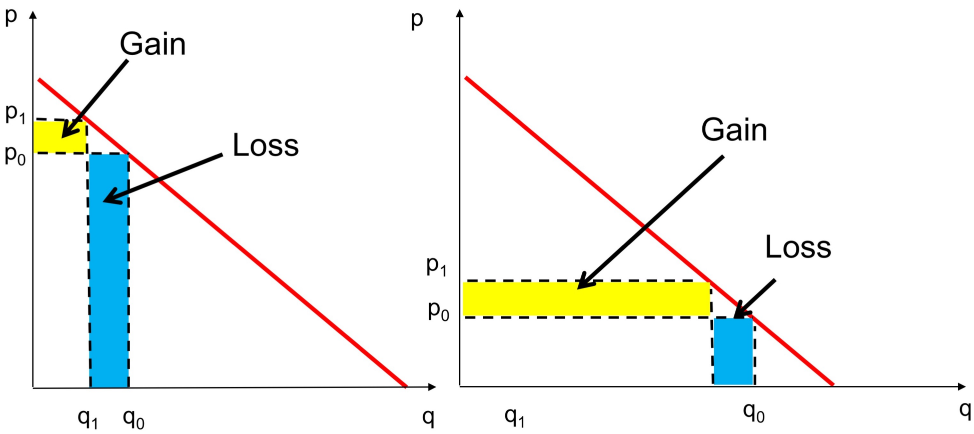


Figure 8: *How Revenue Changes When Price Changes*

A Constant Elasticity Demand Curve

The general formula for a demand with a constant elasticity of ε turns out to be

$$q = Ap^{\varepsilon}$$
$$q' = \varepsilon Ap^{\varepsilon - 1}$$

where A is an arbitrary positive constant and ε, being an elasticity, will typically be negative.

Income Elasticity

Definition. *Income Elasticity of Demand*— Used to describe how the quantity demanded responds to a change in income

$$\text{income elasticity of demand} = \frac{\%\text{ change in quantity}}{\%\text{ change in income}}$$

The Average Income Elasticity

$$p_1 x_1 + p_2 x_2 = m$$

Differentiating with respect to income gives us

$$p_1 \frac{\partial x_1}{\partial m} + p_2 \frac{\partial x_2}{\partial m} = 1$$

Therefore,

$$\underbrace{\frac{p_1 x_1}{m}}_{\text{Expenditure share good 1}} \overbrace{\left(\frac{m}{x_1} \times \frac{\partial x_1}{\partial m}\right)}^{\text{Income elasticity good 1}} + \underbrace{\frac{p_2 x_2}{m}}_{\text{Expenditure share good 2}} \overbrace{\left(\frac{m}{x_2} \times \frac{\partial x_2}{\partial m}\right)}^{\text{Income elasticity good 2}} = 1$$

Practice Problems

1.) Suppose that the demand for a good is given by $D(p) = 10 - p$, where p is the price of the good. What is the drop in consumer surplus from an increase in the price of the good from \$4 per unit to \$6 per unit?

Solution: We are given that $D(p) = 10 - p$, where p is the price of the good. The notation is a little confusing so lets rewrite it to where $D(p) = q$, since that is the equivalent. We now have

$$q = 10 - p \iff p = 10 - q$$

Determining the consumer surplus (CS),

$$\begin{aligned} CS_{p=4} &= \frac{1}{2} \times (6) \times (6) = 18 \\ CS_{p=6} &= \frac{1}{2} \times (4) \times (4) = 8 \\ &18 - 8 = 10 \end{aligned}$$

2.) Edsel consumes 50 bottles of beer per week. His price elasticity of demand for beer is $Ex_b p_b = -0.2$. The price of beer doubles. How many bottles per week does he consume now?

Solution: We are given that Edsel consumes 50 bottles of beer per week, but the price of beer doubles. Let's use our formula and plug in the corresponding values,

$$\begin{aligned} Ex_b p_b &= \frac{\%\Delta x}{\%\Delta p} = \frac{\Delta x / x}{\Delta p / p} = \frac{\Delta x}{\Delta p} \times \frac{p}{x} \\ Ex_b p_b &= \frac{\Delta x}{x} = -0.2 \\ \frac{\Delta x}{50} &= -0.2 \\ \Delta x &= -10 \\ 50 - 10 &= 40 \end{aligned}$$

3.) Stuart and Timothy are the only potential buyers of Compound 1010AA. Stuart's individual demand for the compound is $q = 10–p$, while Timothy's is $q = 30–3p$. What is market demand for Compound 1010AA if the price per unit is \$4?

Solution: We are given that Stuart's individual demand is $q = 10–p$, while Timothy's is $q = 30–3p$. Let's get these in terms of p first.

$$q = 10\text{–}p \overset{\text{Stuart}}{\Longrightarrow} p = 10 - q$$
$$q = 30\text{–}3p \overset{\text{Timothy}}{\Longrightarrow} p = 10 - \frac{q}{3}$$

4.) In general, if the price of a substitute to good X decreases, what changes occur to the equilibrium quantity and price of good X?

5.) P.J. is trying to decide how much to consume in time periods 1 and 2, (c_1, c_2). He will get \$100 in period 1 and \$100 in period 2. He can save and borrow as much as he wants between the two time periods at an interest rate of 20%. If P.J.'s utility function is $U(c_1, c_2) = c_1c_2$, what level of consumption does he choose in period 2 if the inflation rate is also 10%? Assume that $p1$ = \$1.

Solution: We are given $m_1 = \$100, m_2 = \$100, r = 20\%$

$$c_1 + \frac{c_2}{1+\rho} = m_1 + \frac{m_2}{1+\rho}$$

7.) A risk neutral person is offered the choice between a gamble that pays \$1000 with probability 20% and \$100 with probability 80% or a for sure payment of \$200. Which of the following does he prefer?

Solution: A risk neutral would choose highest expected value.

$$\begin{aligned}\mathbf{E}[\text{gamble}] &= 0.2(1000) + 0.8(100)\\ &= 200 + 80 = 280\end{aligned}$$

Therefore, he would take the gamble.

8.) If current and future consumption are both normal goods, an increase in the interest rate will necessarily

Solution: c_1 and c_2 are normal, r increases.
Borrower — Substitution effect of c_1 decreases and c_2 increases. Income effect for c_1 and c_2 decreases.
Saver — Substitution effect of c_1 decreases and c_2 increases. Income effect for c_1 and c_2 increases.
Therefore, this will cause borrowers to borrow less.

9.) Your utility function is $U(W) = \sqrt{W}$. A rich uncle is going to give you 3 stocks in either company A or company B.
Solution: Company A:

$$\begin{aligned}\mathbf{E}[U] &= 0.1U(3 \times 27) + 0.3U(0) + 0.6U(3 \times 3)\\ &= 0.1(\sqrt{81}) + 0.3(\sqrt{0}) + 0.6(\sqrt{9})\\ &= 2.7\end{aligned}$$

10.) Suppose that the demand for a good is given by $D(p) = 1400\text{–}p$, where p is the price of the good. The supply curve for the good is $S(p) = 4p$.

a.) Without a tax, what are the equilibrium quantity and price?

Solution:

Demand: $q = 1400 - p \implies p = 1400 - q$
Supply: $q = 4p \implies p = \dfrac{q}{4}$
We will set $D(p) = S(p)$ and solve to get,

$$\begin{aligned}1400 - q &= \frac{q}{4}\\ 1400 &= \frac{q}{4} + q\\ 1400 &= \frac{5q}{4}\\ q &= 1120\\ p &= 280\end{aligned}$$

b.) What are the equilibrium quantity and price if a \$50 per unit tax is imposed on buyers?

Solution:

$$\begin{aligned}q &= 1400 - p\\ &= 1400 - (p + t)\\ &= 1400 - p - 50\\ &= 1350 - p\\ \implies p &= 1350 - q\end{aligned}$$

Therefore,

$$\begin{aligned}1350 - q &= \frac{q}{4}\\ 1350 &= \frac{5q}{4}\\ q &= 1080\\ p &= 270\end{aligned}$$

Diego has \$6,400. He plans to bet on a soccer game. Team A is a favorite to win. Assume no ties can occur. For \$0.80 he can buy a ticket that will pay \$1 if team A wins and nothing if B wins. For \$0.20 he can buy a ticket that pays \$1 if team B wins and nothing if A wins. Diego thinks the two teams are equally likely to win. He buys tickets so as to maximize the expected value of $\ln W$ (the natural log of his wealth). After he buys his tickets, team A loses a star player and the ticket price moves to \$.50 for either team. Diego buys some new tickets and sells some of his old ones. The game is then played and team A wins. How much wealth does he end up with?

Solution: We are given that $U = \log(W), m = 6400, \pi_A = \frac{1}{2}$, and $\pi_B = \frac{1}{2}$. Plugging in what we know gives us,

$$6400 = 0.8A + 0.2B \tag{1}$$

To find our expected utility, we use

$$\begin{aligned} EU &= \frac{1}{2}U(A) + \frac{1}{2}U(B) \\ &= \frac{1}{2}\ln(A) + \frac{1}{2}\ln(B) \end{aligned}$$

We know that the MRS = Price ratio. Thus,

$$\frac{\frac{1}{2A}}{\frac{1}{2B}} = \frac{B}{A} = 4 \implies B = 4A$$

Plugging back into equation (1), we get

$$\begin{aligned} 6400 &= 0.8A + 0.2(4A) \\ 6400 &= \frac{8}{5}A \\ A &= 4000 \\ B &= 16000 \end{aligned}$$

After team A loses a player and the prices change, he re-optimizes. It is easiest to think about him selling all of his tickets at the new prices and obtaining a new level of income, m. He then buys the optimal level of tickets for team A and team B at the new prices.

$$\begin{aligned} m &= 0.5A + 0.5B \\ 4000(0.5) + 16000(0.5) &= 10000 \\ \frac{B}{A} &= \frac{0.5}{0.5} \implies B = A \\ m &= 0.5A + 0.5(A) \\ m &= A(0.5 + 0.5) \\ A &= 100000 \\ B &= 100000 \end{aligned}$$