

Write a proof that all positive integers  $\geq 2$  are either prime or can be written as the product of primes.

There are two conditions:

1. Assume  $k$  is a prime, no further work needs to be done.

2. Assume  $k$  is a non-prime  $\in N$

$a$  and  $b \in N$

$k = ab$  where  $1 \leq a, b, < k$

Our Base Case begins at 2.

2 is a prime, and therefore fulfills our first condition.

Thus:

For the purpose of the proof we will assume these statements are true up through  $n$ . Using induction we shall prove it holds true for  $n + 1$ .

If  $n + 1$  is a prime it fulfills the first condition, and therefore the theorem holds true.

If  $n + 1$  is not a prime it can be written as  $(n + 1) = ab$  where  $1 \leq a, b, < (n + 1)$

As both  $a, b < n$ , they fall under the previous proof and the theorem holds true.

Using induction, we can show that  $a = p_1 p_2 p_3 \dots p_h$  and  $b = q_1 q_2 q_3 \dots q_i$  are composed of primes. As  $n = ab$ ,  $n = p_1 q_1 p_2 q_2 \dots p_h q_i$