Suppose $x + \frac{1}{x}$ is an integer. Prove by induction that $x^n + \frac{1}{x^n}$ for every $n \geq 1$

For the Base Case: Let n=1,

$x^n + \frac{1}{x^n}$ will become $x + \frac{1}{x}$, which is exactly the same as $x + \frac{1}{x}$

We will now prove this by induction. Finally, let n=n+1

$x^n + \frac{1}{x^n}$ will become $x^{n+1} + \frac{1}{x^{n+1}}$, which can be rewritten as $(x^n + \frac{1}{x^n}) * (x + \frac{1}{x})$ and further more as $(x + \frac{1}{x})^n * (x + \frac{1}{x})$. Since, n indicates what $(x + \frac{1}{x})$ is taken to the power to, it is equivalent to saying how many times the integer is multiplied to itself, which always results an integer. Even with an extra step (+1), it will still be an integer.

Thus, $x^n + \frac{1}{x^n}$ is considered an integer, for every $n \geq 1$.