1. First we will write out a few examples to see if any patterns emerge starting at our base case 8 cents worth of stamps, the smallest amount that can be made up using a three cent stamp and a five cent stamp. Intuitively, there should at least be a pattern that cycles over 15 steps as there might be different combinations.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Total | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 3 cent stamps | 1 | 3 | 0 | 2 | 4 | 1 | 3 | 0 | 2 | 4 | 1 | 3 | 0 | 2 | 4 |
| 5 cent stamps | 1 | 0 | 2 | 1 | 0 | 2 | 1 | 3 | 2 | 1 | 3 | 2 | 4 | 3 | 2 |

* First we can see that there are possible combinations of stamps for all amounts up to twenty-two cents.
* Second, from the examples we can see two different patterns, a cycle of three and a cycle of five. Because there are surjective cycles, we can conclude that indeed, we can produce any amount greater than or equal to eight cents from a combination of three cent and five cent stamps.
* We can choose the cycle of three to minimize use of five cent stamps or the cycle over five to minimize both three cent stamps and overall number of stamps. We will show the later.

Let S(an) = f(an) + t(an) for n ≥ 8

f(an) = f(an–5) + 1;

With base cases: f8 = 1; f9 = 0; f10 = 2; t11 = 1; t12 = 0

t(an) = t(an–5);

With base cases: t8 = 1; t9 = 3; t10 = 0; t11 = 2; t12 = 4