5.2 The Fundemental Theorem of Finitely Generated Abelian Groups

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Exercise 5.2.2. In each of the following parts give the lists of invariant factors for all abelian groups of the specified order:

a) order 270.
Answer: $270 = 2 \cdot 3^3 \cdot 5$. Thus the invariant factors are:

$$2 \cdot 3^3 \cdot 5$$

$$2 \cdot 3^2 \cdot 5, 3$$

$$2 \cdot 3 \cdot 5, 3, 3$$

b) order 9801
Answer: $9801 = 3^4 \cdot 11^2$. Thus the invariant factors are:

$$3^4 \cdot 11^2$$

$$3^3 \cdot 11^2, 3$$

$$3^2 \cdot 11^2, 3^2$$

$$3^2 \cdot 11^2, 3, 3$$

$$3 \cdot 11^2, 3, 3, 3$$

$$3^4 \cdot 11, 11$$

$$3^3 \cdot 11, 3 \cdot 11$$

$$3^2 \cdot 11, 3^2 \cdot 11$$

$$3^2 \cdot 11, 3 \cdot 11, 3$$

$$3 \cdot 11, 3 \cdot 11, 3, 3$$

c) Order 320
Answer: $320 = 2^6 \cdot 5$. Thus the invariant factors are:

$$2^6 \cdot 5$$

$$2^5 \cdot 5, 2$$

$$2^4 \cdot 5, 2^2$$
$$2^4 \cdot 5, 2, 2$$
$$2^3 \cdot 5, 2^3$$
$$2^3 \cdot 5, 2^2, 2$$
$$2^3 \cdot 5, 2, 2, 2$$
$$2^2 \cdot 5, 2^2, 2^2$$
$$2^2 \cdot 5, 2^2, 2, 2$$
$$2^2 \cdot 5, 2, 2, 2, 2$$
$$2 \cdot 5, 2, 2, 2, 2, 2$$

Exerise 5.2.3. In each of the following parts give the lists of elementary divisors for all abelian groups of the specified order and then match each list with the corresponding list of invariant factors found in the preceding exercise:

a) order 270
Answer:
$$2, 3^3, 5$$
$$2, 3^2, 5, 3$$
$$2, 3, 5, 3, 3$$

b) order 9801
Answer:
$$3^4, 11^2$$
$$3^3, 11^2, 3$$
$$3^2, 11^2, 3^2$$
$$3^2, 11^2, 3, 3$$
$$3, 11^2, 3, 3, 3$$
$$3^4, 11, 11$$
$$3^3, 11, 3, 11$$
$$3^2, 11, 3^2, 11$$
$$3^2, 11, 3, 11, 3$$
$$3, 11, 3, 11, 3, 3$$

c) order 320
Answer:
$$2^6, 5$$
$$2^5, 5, 2$$
$$2^4, 5, 2^2$$

$$2^4, 5, 2, 2$$
$$2^3, 5, 2^3$$
$$2^3, 5, 2^2, 2$$
$$2^3, 5, 2, 2, 2$$
$$2^2, 5, 2^2, 2^2$$
$$2^2, 5, 2^2, 2, 2$$
$$2^2, 5, 2, 2, 2, 2$$
$$2, 5, 2, 2, 2, 2, 2$$