4.1 Group Actions and Permutation Representations

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Exercise 4.1.4. Let S_3 act on the set Ω of ordered pairs: $\{(i,j)|1 \leq i,j \leq 3\}$ by $\sigma((i,j)) = (\sigma(i),\sigma(j))$. Find the orbits of S_3 on Ω. For each $\sigma \in S_3$ find the cycle decomposition of σ under this action (i.e., find its cycle decomposition when σ is considered as an element of S_9 - first fix a labelling of these nine ordered pairs). For each orbit $\mathcal{O}$ of S_3 acting on these nine points pick some $a \in \mathcal{O}$ and find the stabilizer of a in S_3.

We first fix an element of Ω and find its orbit, let that element be $(1,1)$, then its orbit is $\{(1,1),(2,2),(3,3)\}$, leaving us $\{(1,2),(2,3),(3,2),(2,1),(1,3),(3,1)\}$, which is the other orbit.
Cycle decompositions are as follows:
(1) $((1,1))$
(12) $((1,1)(2,2))((1,2)(2,1))((1,3)(2,3))((3,1)(3,2))$
(23) $((2,2)(3,3))((2,3)(3,2))((2,1)(3,1))((1,2)(1,3))$
(13) $((1,1)(3,3))((1,3)(3,1))((1,2)(3,2))((2,1)(2,3))$
(123) $((1,1)(2,2)(3,3))((1,2)(2,3)(3,1))((1,3)(2,1)(3,2))$
(132) $((1,1)(3,3)(2,2))((1,2)(3,1)(2,3))((1,3)(3,2)(2,1))$