

## 5.1 Direct Products

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**Exercise 5.1.1.** Show that the center of a direct product is the direct product of the centers:

$$Z(G_1 \times G_2 \times \cdots G_n) = Z(G_1) \times Z(G_2) \times \cdots Z(G_n)$$

Deduce that a direct product of groups is abelian iff each of the factors is abelian.

**Proof:** It follows trivially from the fact that  $gh = hg$  iff  $g_i h_i = h_i g_i$  for each  $i$  where  $g, h \in G = G_1 \times G_2 \times \cdots G_n$ .

The second part of the proof also follows trivially from the fact that  $G = Z(G)$  iff  $G$  is abelian.

**Exercise 5.1.5.** Exhibit a non-normal subgroup of  $Q_8 \times Z_4$  (note that every subgroup of each factor is normal).

**Example:** Take  $H = \langle (i, x) \rangle$ , we have  $(j, x)(i, x)(-j, x^3) = (-i, x) \notin H$ .