

## 4.1 Group Actions and Permutation Representations

March 13, 2018

**Exercise 4.1.4.** Let  $S_3$  act on the set  $\Omega$  of ordered pairs:  $\{(i, j) | 1 \leq i, j \leq 3\}$  by  $\sigma((i, j)) = (\sigma(i), \sigma(j))$ . Find the orbits of  $S_3$  on  $\Omega$ . For each  $\sigma \in S_3$  find the cycle decomposition of  $\sigma$  under this action (i.e., find its cycle decomposition when  $\sigma$  is considered as an element of  $S_9$  - first fix a labelling of these nine ordered pairs). For each orbit  $\mathcal{O}$  of  $S_3$  acting on these nine points pick some  $a \in \mathcal{O}$  and find the stabilizer of  $a$  in  $S_3$ .

We first fix an element of  $\Omega$  and find its orbit, let that element be  $(1, 1)$ , then its orbit is  $\{(1, 1), (2, 2), (3, 3)\}$ , leaving us  $\{(1, 2), (2, 3), (3, 2), (2, 1), (1, 3), (3, 1)\}$ , which is the other orbit.

Cycle decompositions are as follows:

(1)  $((1, 1))$   
(12)  $((1, 1)(2, 2))((1, 2)(2, 1))((1, 3)(2, 3))((3, 1)(3, 2))$   
(23)  $((2, 2)(3, 3))((2, 3)(3, 2))((2, 1)(3, 1))((1, 2)(1, 3))$   
(13)  $((1, 1)(3, 3))((1, 3)(3, 1))((1, 2)(3, 2))((2, 1)(2, 3))$   
(123)  $((1, 1)(2, 2)(3, 3))((1, 2)(2, 3)(3, 1))((1, 3)(2, 1)(3, 2))$   
(132)  $((1, 1)(3, 3)(2, 2))((1, 2)(3, 1)(2, 3))((1, 3)(3, 2)(2, 1))$