

4.1 Group Actions and Permutation Representations

March 14, 2018

Exercise 4.1.4. Let S_3 act on the set Ω of ordered pairs: $\{(i, j) | 1 \leq i, j \leq 3\}$ by $\sigma((i, j)) = (\sigma(i), \sigma(j))$. Find the orbits of S_3 on Ω . For each $\sigma \in S_3$ find the cycle decomposition of σ under this action (i.e., find its cycle decomposition when σ is considered as an element of S_9 - first fix a labelling of these nine ordered pairs). For each orbit \mathcal{O} of S_3 acting on these nine points pick some $a \in \mathcal{O}$ and find the stabilizer of a in S_3 .

We first fix an element of Ω and find its orbit, let that element be $(1, 1)$, then its orbit is $\{(1, 1), (2, 2), (3, 3)\}$, leaving us $\{(1, 2), (2, 3), (3, 2), (2, 1), (1, 3), (3, 1)\}$, which is the other orbit.

Cycle decompositions are as follows:

(1) $((1, 1))$
(12) $((1, 1)(2, 2))((1, 2)(2, 1))((1, 3)(2, 3))((3, 1)(3, 2))$
(23) $((2, 2)(3, 3))((2, 3)(3, 2))((2, 1)(3, 1))((1, 2)(1, 3))$
(13) $((1, 1)(3, 3))((1, 3)(3, 1))((1, 2)(3, 2))((2, 1)(2, 3))$
(123) $((1, 1)(2, 2)(3, 3))((1, 2)(2, 3)(3, 1))((1, 3)(2, 1)(3, 2))$
(132) $((1, 1)(3, 3)(2, 2))((1, 2)(3, 1)(2, 3))((1, 3)(3, 2)(2, 1))$