Assignment 6 Solutions

1

$\neg((\exists x)[A(x)]) \Leftrightarrow (\forall x)[\neg A(x)]$
for $\Rightarrow$:
Assume $\neg(\exists x)[A(x)]$ is true. In words "there is no x for which A(x) is true".
This means that for all x A(x) is false.
In symbols:
$(\forall x)[\neg A(x)]$
for $\Leftarrow$: Assume $(\forall x)[\neg A(x)]$. In works "for all x A(x) is false".
This meansd that there is no for which A(x) is true.
In symbols:
$\neg((\exists x)[A(x)])$

2

There is an even prime bigger than 2.
In symbols: $(\exists x > 2)[E(x) \wedge P(x)]$
To prove that this is false we can try to prove that the negation is true:
$(\forall x > 2)[\neg E(x) \vee \neg P(x)]$
This means that all numbers bigger than two are odd or are not prime. A natural number can be either odd or even.
Even number is a number that is divided by 2, which means it's not prime.
This mean that the statement $(\forall x > 2)[\neg E(x) \vee \neg P(x)]$ is true, which proves that: $(\exists x > 2)[E(x) \wedge P(x)]$ is false

3

a. $(\forall xinP)[S(x) \implies Lp(x)]$
b. $(\exists xinP)[Friend(x) \wedge \neg HaveCar(x)]$
c. $(\exists xinA)[Elephant(x) \wedge \neg LikeMuffins(x)]$
d. $(\forall xinF)[Triangle(x) \implies Isosceles(x)]$
e. $(\exists xinP)[Student(x) \wedge InClass(x) \wedge \neg HereToday(x)]$
f. $(\forall xinP)(\exists yinP)[Loves(x, y)]$
g. The opposite one would be There's someone that loves everyone: $(\exists x)(\forall y)[loves(x, y)]$, so after inverting it:
$(\forall xinP)(\exists yinP)[\neg L(x, y)]$
h. $(\forall x)[Man(x) \wedge Comes(x)] \implies (\forall w)[Woman(w) \implies Leave(w)]$
i. $(\forall x)[Tall(x) \vee Short(x)]$
j. $(\forall x)[Tall(x)] \vee (\forall x)[Short(x)]$
k. $\neg(\forall x \in S)[Precious(x) \wedge Beautiful(x)] \Leftrightarrow (\exists x \in S)[\neg Precious(x) \vee \neg Beautiful(x)]$
l. $(\forall x \in P)[\neg LovesMe(x)]$
m. $(\exists x \in S)[American(x) \wedge Poisonous(x)]$
n. $(\exists x \in A)[Snake(x) \wedge American(x) \wedge Poisonous(x)]$

4

a. $(\exists xinP)[S(x) \wedge \neg Lp(x)]$ - There are some students that do not like pizza.
b. $(\forall xinP)[Friend(x) \implies HaveCar(x)]$ - People that are not my friends have car./None of my friends have car.
c. $(\forall xinA)[Elephant(x) \implies LikeMuffins(x)]$ - All elephans like muffins.
d. $(\exists xinF)[Triangle(x) \wedge \neg Isosceles(x)]$ - There's a triangle that is not isosceles.
e. $(\forall xinP)[\neg(Student(x) \wedge InClass(x)) \vee HereToday(x)] \Leftrightarrow (\forall xinP)[(Student(x) \wedge InClass(x)) \implies HereToday(x)]$ - all the students in the class are here today.
f. $(\exists xinP)(\forall yinP)[\neg Loves(x, y)]$ - There is a person that does not love anyone.
g. $(\exists x)(\forall y)[loves(x, y)]$ - There is a person that loves everyone.
h. $(\forall x)[(Man(x) \wedge Comes(x))] \wedge (\exists x)[(Woman(y) \wedge \neg Leave(y))]$ - A man is coming and there's a woman that won't leave.

i. $(\exists x)[\neg Tall(x) \wedge \neg Short(x)]$ - There is a person that is short and tall.
j. $(\exists x)[\neg Tall(x)] \wedge (\exists x)[\neg Short(x)]$ - There is a person that is short and there is a person that is tall.
k. $(\forall x \in S)[Precious(x) \wedge Beautiful(x)]$ - all stones are precious and beautiful.
l. $(\exists x \in P)[LovesMe(x)]$ - there is a person that loves me.
m. $(\forall x \in S)[\neg American(x) \vee \neg Poisonous(x)] \Leftrightarrow (\forall x \in S)[American(x) \implies \neg Poisonous(x)]$ - All snakes that are american are not poisonous.
n. $(\forall x \in A)[\neg Snake(x) \vee \neg American(x) \vee \neg Poisonous(x)] \Leftrightarrow (\forall x \in A)[Snake(x) \implies \neg American(x) \vee \neg Poisonous(x)]$ - All snakes are not american or not poisonous.

5

a. $(\exists x \in \mathbb{N})[x = 2/3]$ is false, because $2/3 \notin \mathbb{N}$
b. $(\exists x \in \mathbb{Q})[x = + - sqrt2]$ is false because $\sqrt{2} \in \mathbb{R} - \mathbb{Q}$
c. True
d. True
e. $(\forall x)(\exists y)(\forall z)[xy = xz]$ for reals.
when x = 0, the it is true.
when x = 1, should find $(\exists y)(\forall z)[y = z]$, but there is not such y that can be equal to all values of z, so it is False.

f. $(\forall x)(\exists y)(\forall z)[xy = xz]$ for primes
when x = 2, we should find $(\exists y)(\forall z)[2y = 2z]$, but there is not such y that for all z to be $y = z$, so it is False.
g. $(\forall x)[x < 0 \implies (\exists y)(y^2 = x)]$ for real numbers is false, because $(\forall y)[y^2 \geq 0]$
h. $(\forall x)[x < 0 \implies (\exists y)(y^2 = x)]$ for positive real numbers. It is true, because $x < 0$ is false, which means that the implication will always be true

6

a. $(\forall x \in \mathbb{N})[x \neq 2/3]$
b. $(\forall x \in \mathbb{Q})[x \neq + - sqrt2]$
c. $(\exists x)(\forall y)(y \neq x^2)$
d. same as c.
e. $(\exists x)(\forall y)(\exists z)[xy \neq xz]$
f. same as e.
g. $(\exists x)[x < 0 \wedge (\forall y)(y^2 \neq x)]$
h. same as g.

7

a. $(\exists x \in \mathbb{N})(\exists y \in \mathbb{N})(x + y \neq 1)$
b. $(\exists x > 0)(\forall y < 0)(x + y \neq 0)$
c. $(\forall x)(\exists e > 0)(x \leq -e \vee x \geq e)$
d. $(\exists x \in \mathbb{N})(\exists y \in \mathbb{N})(\forall z \in \mathbb{N})[x + y \neq z^2]$

8

In symbols the sentence has the meaning:
$(\forall x)(\exists t)[Fool(x,t)] \vee (\exists x)(\forall t)[Fool(x,t)] \wedge \neg((\forall x)(\forall t)[Fool(x,t)])$
The negated looks like this:
$(\exists x)(\forall t)[\neg Fool(x,t)] \wedge (\forall x)(\exists t)[\neg Fool(x,t)] \vee (\forall x)(\forall t)[Fool(x,t)]$
In words:

There's no one that you can fool all the time and can't fool everyone even once or you can fool everyone all the time.

9

$(\exists \epsilon > 0)(\forall \delta > 0)(\exists x)(|x - a| < \delta \wedge [f(x) - f(a)] \geq \epsilon)$