Assignment 5 Solutions

1

a. $(\exists x \in \mathbb{N})[x^3 = 27]$
b. $(\exists x \in \mathbb{N})[x > 1000000]$
c. Another way of saying it would: There exists a natural number $n > 1$ that is not prime.
Let P(x) be the "not prime property of x"
$(\exists(x > 1 \wedge x \in N))[P(x)]$
$(\exists x \in \mathbb{N})[n > 1 \wedge P(x)]$ - looks better
We know that the set $\mathbb{N}$ starts with $\{1, 2, 3....\}$, so we can rephrase it as:
"There exists a natural number $n \neq 1$ that is not prime."
$(\exists x \in \mathbb{N})[n \neq 1 \wedge P(x)]$

2

a. $(\forall x \in \mathbb{N})[x^3 \neq 28]$
b. $(\forall x \in \mathbb{N})[x > 0]$
c. Let P(x) be the "prime property of x"
$(\exists x \in \mathbb{N})[x > 1 \wedge P(x)]$

3

let P be all people.
a. $(\forall p1 \in P)(\exists p2 \in P)[loves(p1, p2)]$
b. $(\forall p \in P)[tall(p) \vee short(p)]$
c. $(\forall p \in P)[tall(p)] \vee (\forall p \in P)[short(p)]$
d. $(\forall p \in P)[\neg athome(p)]$
e. $(\exists p \in P)[p = John \wedge comes(p)] \implies (\forall p \in P)[woman(p) \implies Leave(p)]$
f. $(\forall p \in P)[man(p) \wedge comes(p)] \implies (\forall p \in P)[woman(p) \implies Leave(p)]$

4

a. $(\exists x \in \mathbb{R})(\forall a \in \mathbb{R})[x^2 + a = 0]$
b. $(\exists x \in \mathbb{R})(\forall a \in \mathbb{R})[a < 0 \implies x^2 + a = 0]$
c. $(\forall x \in \mathbb{R})[rational(x)] \Leftrightarrow (\forall x \in \mathbb{R})(\exists p, q \in \mathbb{N})[x = p/q]$
d. $(\exists x)[irational(x)] \Leftrightarrow (\exists x)(\forall p, q \in \mathbb{N})[x \neq p/q]$
e. Rephrasing to "for each irrational number m there is an irrational number n so $n > m$"
$(\forall m \in \mathbb{R} - \mathbb{Q})[\exists n \in \mathbb{R} - \mathbb{Q}](n > m)] \Leftrightarrow (\forall m)(\exists n)(\forall p, q \in \mathbb{N})[m \neq p/q \wedge n \neq p/q \wedge n > m]$

5

C - all cars
x has property D(x) representing domestic cars
x has property M(x) representing badly made cars
a. $(\forall x \in C)[D(x) \implies M(x)]$
b. $(\forall x \in C)[\neg D(x) \implies M(x)]$
c. $(\forall x \in C)[M(x) \implies D(x)]$
d. $(\exists x \in C)[D(x) \wedge \neg M(x)]$

e. $(\exists x \in C)[\neg D(x) \wedge M(x)]$

6

$\forall m, n \in \mathbb{R}$ we have a x with property Q(x), such that $m < x < n$
$(\forall m, n \in \mathbb{R})(\exists x)[Q(x) \wedge (m < x < n)]$

7

Given:
all people - P
person - p
time - t
person with my default property fool(p,t) $\Leftrightarrow (\exists p \in P)[Viktor]$

Split the sentece into:
You may fool "all the people some of the time", / $(\forall p \in P)(\exists t)[fool(p,t)]$
you can even "fool some of the people al the time", / $(\forall t)(\exists p \in P)[fool(p,t)]$
but you cannot "fool all the people all the time". / $(\forall t)(\forall p \in P)[fool(p,t)]$

$(\forall p \in P)(\exists t)[fool(p,t)] \vee (\forall t)(\exists p \in P)[fool(p,t)] \wedge \neg((\forall t)(\forall p \in P)[fool(p,t)]) \Leftrightarrow$
$(\forall p \in P)(\exists t)[fool(p,t)] \vee (\forall t)(\exists p \in P)[fool(p,t)] \wedge (\exists t)(\exists p \in P)[\neg fool(p,t)])$

8

a driver is involved in an accident every 6 seconds
a driver - d
6 seconds - t
driver has a property accident - A(d, t)
$(\exists d)(\forall t)[A(d,t)]$

9

$(\forall t)(\exists d)[A(d,t)]$