CHW3

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Q1. Optimal evacuation:

The optimization problem we are trying to solve is as follows:

$$\begin{aligned} &\textit{minimize} \sum_{t=1}^{T}(r^T q_t + s^T q_t^2) + \sum_{t=1}^{T-1}(\tilde{r}^T |f_t| + \tilde{s}^T f_t^2) \\ &\textit{subject to} \quad q_{t+1} = Af_t + q_t, \quad t = 1,2,..T-1 \\ &\qquad 0 \le q_t \le Q, \qquad t = 2, \dots, T \\ &\qquad |f_t| \le F, \qquad t = 1,2, \dots .T-1 \end{aligned}$$

With variables $q_2, \dots, q_t$ and$f_1, \dots, f_{t-1}$.After we solve the problem, the optimal evacuation would be 17.

Q2. Optimal circuit design:

To get what we seek, the following optimization problem must be solved:

$$\begin{aligned} &\textit{find} \qquad \theta \\ &\textit{subject to} \sum_{j=1}^{k} \theta_j \log P^{(j)} \le \log P_{spec} \\ &\qquad \sum_{j=1}^{k} \theta_j \log D^{(j)} \le \log P_{spec} \\ &\qquad \sum_{j=1}^{k} \theta_j \log A^{(j)} \le \log P_{spec} \\ &\qquad 1^T\theta = 1, \qquad \theta \ge 0 \end{aligned}$$

Thus we have found a solution that meets the specifications by solving the LP feasibility problem. Θ Is the solution.

Q3. Filling covariance matrix:

Part a: Let's take $S = T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Then we will have:

$$C_{sim} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \det(C_{sim}) = -1$$

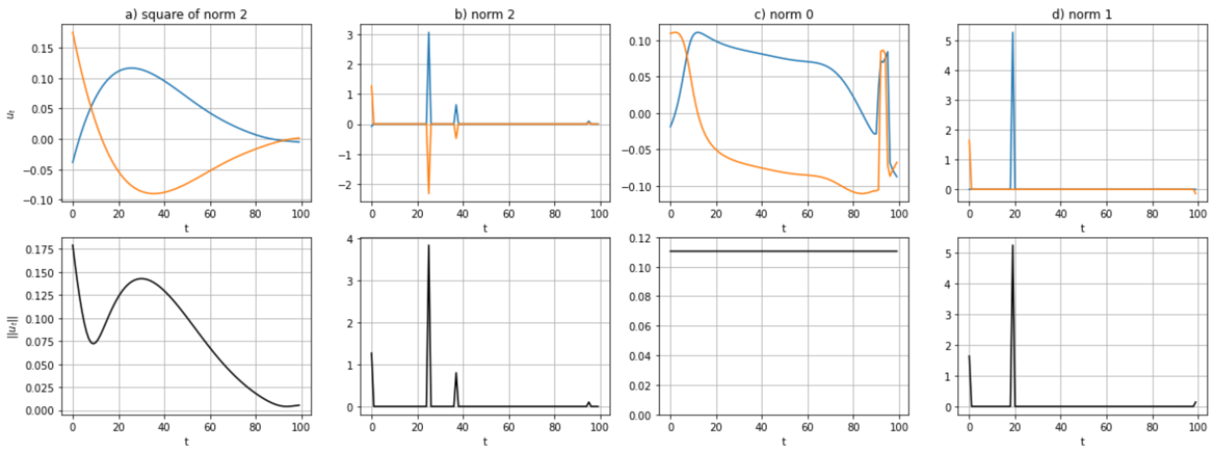
Since C_{sim} is not positive definite (PSD), it cannot be a covariance matrix.

Part b: C_{sim} would be the answer of the problem, if it is PSD, because:

$$||C - C_{sim}||_F^2 = \left||C_{11} - S_{11}|\right|_F^2 + 2\left||C_{12} - S_{12}|\right|_F^2 + ||C_{13}||_F^2 + ||\left|C_{22} - \frac{S_{22} + T_{22}}{2}\right|||_F^2 + 2||C_{23} - T_{23}||_F^2 + ||C_{33} - T_{33}||_F^2$$

$$\rightarrow C_{11} = S_{11},\ C_{12} = S_{12},\ C_{13} = 0,\ C_{22} = \frac{S_{22} + T_{22}}{2},\ C_{23} = T_{23},\quad C_{33} = T_{33}$$

Q4. Control using different objective functions:



Look at the results obtained from code. <u>From left to right</u> we discuss about control inputs:

1: The control inputs are small; but not sparse. This is what we expect with the least squares.

2: The control input is sparse; and when control is nonzero, both components are nonzero.

3: The second norm of the control input is constant over time, the direction of the control input changes over time however.

4: The control input is sparse; the different components are nonzero in different times.

Q5. Portfolio optimization:

The mean worst case risk portfolio problem can be written as a convex optimization problem:

$$minimzie \; -\mu^T w + \gamma t$$

$$subject\ to \;\; w^T \Sigma^{(k)} w \leq t, \quad k = 1, \dots, M$$

$$1^T w = 1$$

Now we apply KKT conditions for optimality:

$$-\mu + v1 + \sum_k 2\lambda_k \Sigma^{(k)} w = 0$$

$$\gamma - \sum_k \lambda_k = 0$$

$$1^T w = 1$$

$$w^T \Sigma^{(k)} w \leq t$$

$$\lambda_k \left(w^T \Sigma^{(k)} w - t \right) = 0, \qquad k = 1,2, \dots M$$

$$\gamma \geq 0$$

In a similar way, the KKT conditions for the problem:

$$maximize \;\; \mu^T w - \sum_{k=1}^{M} \gamma_k w^T \Sigma^{(k)} w$$

$$subject\ to \;\; 1^T w = 1$$

Are

$$-\mu + \alpha 1 + \sum_k 2\lambda_k \Sigma^{(k)} w = 0$$

$$1^T w = 1$$

Where α is a dual variable for the equality constraint.