

مقادیر ثابت و متغیرها:

$$jet: [z] \rightarrow V^z: (V4), \quad V^\varphi: (V3), \quad B_z: (B4), \quad B_\varphi: (B3)$$

$$disk: [r] \rightarrow V^r: (V2), \quad V^\varphi: (V3), \quad B_\varphi: (B3)$$

$$m = 10^9 M_\odot, \quad for \ a: \ 0 < a < 1, \quad for \ K: as \ it \ emits \ energy \rightarrow K < 0 \rightarrow K \approx -0.1$$

$$r_{Schwarzschild} = \frac{2GM}{c^2} \xrightarrow{\frac{2G}{c^2} = 1.48 \times 10^{-27} \text{ m/kg}} r_{Sch} = 1.48 \times 10^{-27} \times 10^9 M_\odot = 1.48 \times 10^{-27} \times 10^9 \times 2 \times 10^{30} \approx 3 \times 10^{12} (m)$$

$$r_{Sch} \approx 3 \times 10^{12} (m) \approx 2 \times 10^4 (AU) \approx 10^{-4} (pc)$$

توقع داریم ابعاد یک سیاه چاله با جرمی در حدود  $10^9 M_\odot$  تقریباً در حدود  $10(AU)$  باشد که شعاع شوارزشیلد آن تقریباً  $10^4(AU)$  به دست می آید.

مقادیر اولیه:

$$jet: z_{init} = 500 AU \rightarrow V^z(z_{init}) = 0.975 c, \quad V^\varphi(z_{init}) = 0.222 c, \quad B_z(z_{init}) = 10^4 G, \quad B_\varphi(z_{init}) = 10^3 G$$

$$disk: r_{init} = 500 AU \approx 7 \times 10^{10} (m) \rightarrow V^r(r_{init}) = 0.12 c, \quad V^\varphi(r_{init}) = 0.918 c, \quad B_\varphi(r_{init}) = 10^4 G$$

$$V^t: (V1) = 0 \quad but: \ u^0 = \gamma$$

ارتباط کمیت ها:

$$u^\alpha = V^\alpha \left( \frac{u^0}{c} \right)$$

$$u^0 = \frac{d(ct)}{d\tau} = \frac{d(c\gamma\tau)}{d\tau} = \gamma c$$

$$u1 = \gamma V1, \quad u2 = \gamma V2, \quad u3 = \gamma V3$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{(V)^2}{c^2}}} \xrightarrow{c^2=1} \gamma = \frac{1}{\sqrt{1 - (V)^2}}$$

$$jet: V^z: (V4), V^\varphi: (V3) \rightarrow \gamma = \frac{1}{\sqrt{1 - (V)^2}} = \gamma = \frac{1}{\sqrt{1 - ((V4)^2 + (V3)^2)}}$$

$$jet: u^0 = \gamma, \quad u^2 = \gamma(V2) = 0, \quad u^3 = \gamma(V3), \quad u^4 = \gamma(V4)$$

$$disk: V^r: (V2), V^\varphi: (V3) \rightarrow \gamma = \frac{1}{\sqrt{1 - (V)^2}} = \gamma = \frac{1}{\sqrt{1 - ((V2)^2 + (V3)^2)}}$$

$$disk: u^0 = \gamma, \quad u^2 = \gamma(V2), \quad u^3 = \gamma(V3), \quad u^4 = \gamma(V4) = 0, \quad (V3) \gg (V2)$$

$$\rho(R, z) = (R^2 + z^2)^{3/4}$$

در این صورت برای قرص داریم  $z = 0$  و در نتیجه

$$\rho(R, z = 0) = (R^2)^{3/4}$$

$$P = K\rho^\gamma, \quad \gamma = \frac{5}{3}, \quad \rho = (R^2)^{3/4} = R^{3/2}$$

$$P = K\rho^\gamma = K\rho^{5/3} = K[(R^2)^{3/4}]^{5/3} = KR^{5/2}$$

در این صورت برای جت داریم  $z \gg R$  و در نتیجه

$$\rho(R, z) = (R^2 + z^2)^{3/4} \cong (z^2)^{3/4}$$

$$P = K\rho^\gamma, \quad \gamma = \frac{5}{3}, \quad \rho = (z^2)^{3/4} = z^{3/2}$$

$$P = K\rho^\gamma = K\rho^{5/3} = K[(z^2)^{3/4}]^{5/3} = Kz^{5/2}$$

برای قرص خواهیم داشت:

پایستگی جرم ( $V^z = 0$ ):

$$\begin{aligned} (V2)D\left(R^{\frac{3}{2}} - KR^{\frac{5}{2}}, R\right) + \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)D(V2[R], R) \\ = -\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)\left(V2(g_{222} + g_{332} + g_{442} - g_{121}) + V2(g_{224} + g_{334} + g_{444} - g_{141})\right. \\ \left. - (g_{123}(V3)(V2) + g_{132}(V2)(V3))\right) + \frac{1}{u_{02}}(2(B3)(J4)(u2)(u0)) \end{aligned}$$

پایستگی تکانه شعاعی:

$$\begin{aligned} \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u_{02})D(V2[R], R)(V2) + \left(1 + \frac{2m}{R}\right)D\left(KR^{\frac{5}{2}}, R\right) \\ = -\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u_{02})\left((g_{211}) - 2(g_{112})(V2)(V2) + 2(V3)(g_{213})\right. \\ + (V1)(V2)((g_{121}) - (g_{121})(V2)) + (V2)(V2)(g_{222}) - (g_{132})(V2)(V2)(V3) \\ \left. - (g_{123})(V2)(V3)(V2) + (V3)(V3)(g_{233})\right) - (B3)(J4) \end{aligned}$$

پایستگی تکانه زاویه ای:

$$\begin{aligned} \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u_{02})D(V3[R], R)(V2) \\ = -\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u_{02})\left(2(V2)((g_{312}) - (g_{112})(V3)) + (V1)(V2)((g_{321}) - (g_{121})(V3))\right. \\ \left. + (V2)(V3)((g_{332}) - (g_{132})(V3)) + (V3)(V2)((g_{323}) - (g_{123})(V3))\right) \end{aligned}$$

پایستگی تکانه ارتفاعی:

$$as J^R = 0, \quad \frac{\partial}{\partial z} = 0, \quad V^z = 0 \rightarrow 0 = 0$$

$$g_{112} = \Gamma(ttr) = \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}$$

$$g_{121} = \Gamma(trt) = \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}$$

$$g_{123} = \Gamma(tr\varphi) = -\frac{maR(3R + 4m)}{R^4 - 4R^2m^2 + 4a^2m^2}$$

$$g_{132} = \Gamma(t\varphi r) = -\frac{maR(3R + 4m)}{R^4 - 4R^2m^2 + 4a^2m^2}$$

$$g_{211} = \Gamma(rtt) = \frac{m}{R(R^2 + 2m)}$$

$$g_{213} = \Gamma(rt\varphi) = -\frac{ma}{R(R + 2m)}$$

$$g_{222} = \Gamma(rrr) = -\frac{m}{R(R+2m)}$$


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$$g_{231} = \Gamma(r\phi t) = -\frac{ma}{R(R+2m)}$$

$$g_{233} = \Gamma(r\phi\phi) = -\frac{R(R+m)}{(R+2m)}$$


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$$g_{244} = \Gamma(rzz) = \frac{m}{R(R+2m)}$$


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$$g_{312} = \Gamma(\phi tr) = \frac{am}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$


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$$g_{321} = \Gamma(\phi rt) = \frac{am}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$

$$g_{323} = \Gamma(\phi r\phi) = -\frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$


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$$g_{332} = \Gamma(\phi\phi r) = -\frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$


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$$g_{424} = \Gamma(zrz) = -\frac{m}{R(R+2m)}$$


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$$g_{442} = \Gamma(zzr) = -\frac{m}{R(R+2m)}$$


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همینطور برای  $J$  در قرص داریم:

$$\begin{aligned} J_4 = & -\frac{c}{4\pi} \left( D \left( \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) B_3[R], R \right) \right. \\ & + B_3 \left( \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R+2m)} \right. \\ & \left. \left. - \frac{m}{R(R+2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} \right) \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) \right) \end{aligned}$$

سپس برای جت خواهیم داشت:

پایستگی جرم:

$$(V4)D\left(z^{\frac{3}{2}} - Kz^{\frac{5}{2}}, z\right) + \left(z^{\frac{3}{2}} + Kz^{\frac{5}{2}}\right)D(V4[z], z) \\ = \left(z^{\frac{3}{2}} + Kz^{\frac{5}{2}}\right)\left((g134)(V4)(V3) + (g143)(V3)(V4)\right) - \frac{1}{(u02)}2(B4)(J2)(u3)(u1)$$

با فرض  $V^\varphi = 0$  و در نتیجه  $u^\varphi = 0$  داریم:

$$(V4)D\left(z^{3/2} - Kz^{5/2}, z\right) + \left(z^{3/2} + Kz^{5/2}\right)D(V4[z], z) = 0$$

پایستگی تکانه شعاعی:

$$\left(1 + \frac{2m}{z}\right)D(Kz^{\frac{5}{2}}, R) \\ = -\left(z^{\frac{3}{2}} + Kz^{\frac{5}{2}}\right)(u02)\left((g211) + 2(V3)(g213) + (V1)(V4)(g141) + (V3)(V3)(g233) \right. \\ \left. + (V4)(V4)(g244)\right) + (B4)(J3)$$

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض  $V^\varphi = 0$  است. در این صورت داریم:

$$(B4)(J3) - \left(z^{\frac{3}{2}} + Kz^{\frac{5}{2}}\right)(u02)\left((g211) + (V1)(V4)(g141) + (V4)(V4)(g244)\right) = \left(1 + \frac{2m}{z}\right)D(Kz^{\frac{5}{2}}, R) \\ = 0, \quad az \frac{\partial z}{\partial R} = 0$$

پایستگی تکانه زاویه ای:

$$(z^{3/2} + Kz^{5/2})(u02)D(V3[z], z)(V4) \\ = -\left(z^{\frac{3}{2}} + Kz^{\frac{5}{2}}\right)(u02)\left(2(V4)\left((g314) - (g114)(V3)\right) + (V1)(V4)\left((g341) - (g141)(V3)\right) \right. \\ \left. + (V3)(V4)\left((g343) - (g143)(V3)\right) + (V4)(V3)\left((g334) - (g134)(V3)\right)\right) - (B4)(J2)$$

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض  $V^\varphi = 0$  است. در این صورت داریم:

$$(z^{3/2} + Kz^{5/2})(u02)[2(V4)(g314) + (V1)(V4)(g341)] + (B4)(J2) = 0$$

پایستگی تکانه ارتفاعی:

$$(z^{3/2} + Kz^{5/2})(u02)D(V4[z], z)(V4) + \left(1 + \frac{2m}{z}\right)D(Kz^{\frac{5}{2}}, z) \\ = -\left(z^{\frac{3}{2}} + Kz^{\frac{5}{2}}\right)(u02)\left((g411) + 2(V3)(g413) - 2(g114)(V4)(V4) + (V1)(V3)(g431) \right. \\ \left. - (V1)(V4)(g141)(V4) + (V3)(V3)(g433) - (g143)(V4)(V3)(V4) - (g134)(V4)(V4)(V3) \right. \\ \left. + (V4)(V4)(g444)\right)$$

با فرض  $V^\varphi = 0$  خواهیم داشت:

$$(z^{3/2} + Kz^{5/2})(u02)D(V4[z], z)(V4) + \left(1 + \frac{2m}{z}\right)D(Kz^{\frac{5}{2}}, z) \\ = -\left(z^{\frac{3}{2}} + Kz^{\frac{5}{2}}\right)(u02)\left((g411) - 2(g114)(V4)(V4) - (V1)(V4)(g141)(V4) + (V4)(V4)(g444)\right)$$

$$g_{112} = \Gamma(ttr) = \frac{m(z + 2z^3 + z^5 + 2m - 2a^2m + 4mz^2 + 4a^2mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g_{114} = \Gamma(ttz) = \frac{mz(z + 2z^3 + z^5 + 2m - 6a^2m + 4mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g_{121} = \Gamma(trt) = \frac{m(z + 2z^3 + z^5 + 2m - 2a^2m + 4mz^2 + 4a^2mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g_{123} = \Gamma(tr\varphi) = -\frac{ma(2z^3 + z - 2z^3 + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g_{132} = \Gamma(t\varphi r) = -\frac{ma(2z^3 + z - 2z^3 + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g_{134} = \Gamma(t\varphi z) = -\frac{maz(3z + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g_{141} = \Gamma(tzt) = \frac{mz(z + 2z^3 + z^5 + 2m - 6a^2m + 4mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g_{143} = \Gamma(tz\varphi) = -\frac{maz(3z + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g_{211} = \Gamma(rtt) = \frac{m}{z^2(z + 2m)}$$

$$g_{213} = \Gamma(rt\varphi) = \frac{2ma}{z^2(z + 2m)}$$

$$g_{222} = \Gamma(rrr) = -\frac{m}{z^2(z + 2m)}$$

$$g_{224} = \Gamma(rrz) = -\frac{m}{z(z + 2m)}$$

$$g_{231} = \Gamma(r\varphi t) = -\frac{-2ma}{z^2(z + 2m)}$$

$$g_{233} = \Gamma(r\varphi\varphi) = -\frac{(z^3 + m + 2mz^2)}{z^2(z + 2m)}$$

$$g_{242} = \Gamma(rzr) = -\frac{m}{z(z + 2m)}$$

$$g_{244} = \Gamma(rzz) = \frac{m}{z^2(z + 2m)}$$

$$g_{312} = \Gamma(\varphi tr) = \frac{am(z - 2z^3 + 4mz^2)}{(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g_{314} = \Gamma(\varphi tz) = \frac{amz(-4m + 3z)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g_{321} = \Gamma(\varphi r t) = \frac{am(z - 2z^3 + 4mz^2)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g_{323} = \Gamma(\varphi r \varphi) = \frac{-(2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 + 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 - z^8)/((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6))}{1}$$

$$g_{332} = \Gamma(\varphi \varphi r) = \frac{-(2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 + 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 - z^8)/((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6))}{1}$$

$$g_{334} = \Gamma(\varphi \varphi z) = -\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g_{341} = \Gamma(\varphi z t) = \frac{amz(-4m + 3z)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g_{343} = \Gamma(\varphi z \varphi) = -\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g_{411} = \Gamma(z t t) = \frac{mz}{(1 + z^2)(z + 2m)}$$

$$g_{413} = \Gamma(z t \varphi) = -\frac{3maz}{(1 + z^2)^2(z + 2m)}$$

$$g_{422} = \Gamma(z r r) = \frac{mz}{(1 + z^2)(z + 2m)}$$

$$g_{424} = \Gamma(z r z) = -\frac{mR}{(1 + z^2)(z + 2m)}$$



$$g_{431} = \Gamma(z \varphi t) = -\frac{3maR^2z}{(1 + z^2)^2(z + 2m)}$$

$$g_{433} = \Gamma(z \varphi \varphi) = \frac{mz}{(1 + z^2)^2(z + 2m)}$$

$$g_{442} = \Gamma(z z r) = -\frac{m}{(1 + z^2)(z + 2m)}$$

$$g_{444} = \Gamma(z z z) = -\frac{mz}{(1 + z^2)(\sqrt{z} + 2m)}$$

همینطور برای  $J$  در جت داریم (تنها آن هایی که استفاده دارند را نوشتیم) :

$$J2 = -\frac{c}{4\pi} B3 \left( D \left( -\frac{z^2}{(z^2 + 2m)^2}, z \right) + \left( -\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)} - \frac{m}{z(z + 2m)} - \frac{mz}{(1 + z^2)(\sqrt{z} + 2m)} \right) \left( -\frac{z^2}{(z^2 + 2m)^2} \right) \right)$$

$$J3 = \frac{c}{4\pi} B4 \left( \frac{am(z - 2z^3 + 4mz^2)}{(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)} \left( \frac{z^4}{2ma(z + 2m)} \right) + \left( -\frac{m}{z^2(z + 2m)} - (2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 + 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 - z^8)/((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)) - \frac{m}{(1 + z^2)(z + 2m)} \right) \left( \frac{z^2}{(z + 2m)^2} \right) \right)$$

و توضیحی کوتاه در مورد  $(u0)^2$  :

$$u02 = \left[ 1 - \frac{2m}{\sqrt{R^2 + z^2}} + 2 \frac{2maR^2}{(R^2 + z^2)^{\frac{3}{2}}} (V3) + \left[ -\left( 1 + \frac{2m}{\sqrt{R^2 + z^2}} \right) (V2)^2 - R^2 \left( 1 + \frac{2m}{\sqrt{R^2 + z^2}} \right) (V3)^2 - \left( 1 + \frac{2m}{\sqrt{R^2 + z^2}} \right) (V4)^2 \right] \right]^{-1}$$

برای قرص:

$$u02 = \left( 1 - \frac{2m}{\sqrt{R^2}} + 2 \frac{2ma}{R} (V3) - \left( 1 + \frac{2m}{R} \right) (V2)^2 - R^2 \left( 1 + \frac{2m}{R} \right) (V3)^2 \right)^{-1}$$

برای جت:

$$u02 = \left( 1 - \frac{2m}{z} + 2 \frac{2ma}{z^3} (V3) - \left( 1 + \frac{2m}{z} \right) (V3)^2 - \left( 1 + \frac{2m}{z} \right) (V4)^2 \right)^{-1}$$

برای جت در صورتی که فرض شود  $V^\varphi = 0$  :

$$u02 = \left( 1 - \frac{2m}{z} - \left( 1 + \frac{2m}{z} \right) (V4)^2 \right)^{-1}$$