از قبل در معادلات پایستگی به دست آوردیم:

پایستگی جرم:

■ $$for\ disk:\quad V^r\frac{\partial}{\partial r}(\rho-P)$$
$$+(\rho+P)\left[\left\{V^r\left(\left[\Gamma_{rr}^r+\Gamma_{\varphi r}^{\varphi}+\Gamma_{zr}^z\right]-\Gamma_{r0}^0\right)\right\}+\left\{V^r\left(\left[\Gamma_{rz}^r+\Gamma_{\varphi z}^{\varphi}+\Gamma_{zz}^z\right]-\Gamma_{z0}^0\right)\right\}\right.$$
$$\left.-\left\{\Gamma_{r\varphi}^0V^{\varphi}V^r+\Gamma_{\varphi r}^0V^rV^{\varphi}+\Gamma_{z\varphi}^0V^{\varphi}V^z\right\}+\left(\frac{\partial V^r}{\partial r}\right)\right]+\frac{1}{(u^0)^2}\left[-2\left\{-B_{\varphi}J^zu^Ru^0\right\}\right]=0$$

א $$for\ jet:\quad V^z\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\left\{\Gamma_{\varphi z}^0V^zV^{\varphi}+\Gamma_{z\varphi}^0V^{\varphi}V^z\right\}+\left(\frac{\partial V^z}{\partial z}\right)\right]$$
$$+\frac{1}{(u^0)^2}\left[-2\left\{-B_zJ^Ru^{\varphi}u^0\right\}\right]=0$$

پایستگی تکانه شعاعی:

■ $$for\ disk:\quad (\rho+P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R+\left(\Gamma_{00}^R\right)-2\Gamma_{0R}^0V^RV^R+2V^{\varphi}\Gamma_{0\varphi}^R+V^tV^R\left(\Gamma_{Rt}^t-\Gamma_{Rt}^0V^R\right)\right.$$
$$\left.+V^RV^R\Gamma_{RR}^R-\Gamma_{\varphi R}^0V^RV^RV^{\varphi}-\Gamma_{R\varphi}^0V^RV^{\varphi}V^R+V^{\varphi}V^{\varphi}\Gamma_{\varphi\varphi}^R\right]+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial R}$$
$$+\left[B_{\varphi}J^z\right]=0$$

א $$for\ jet:\quad (\rho+P)(u^0)^2\left[\left(\Gamma_{00}^R\right)+2V^{\varphi}\Gamma_{0\varphi}^R+V^tV^z\left(\Gamma_{zt}^t\right)+V^{\varphi}V^{\varphi}\Gamma_{\varphi\varphi}^R+V^zV^z\Gamma_{zz}^R\right]$$
$$+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial R}-B_zJ^{\varphi}=0$$

پایستگی تکانه زاویه ای:

■ $$for\ disk:\quad (\rho+P)(u^0)^2\left[\frac{\partial V^{\varphi}}{\partial R}V^R+2V^R\left(\Gamma_{tR}^{\varphi}-\Gamma_{tR}^tV^{\varphi}\right)+V^tV^R\left(\Gamma_{Rt}^{\varphi}-\Gamma_{Rt}^tV^{\varphi}\right)\right.$$
$$\left.+V^RV^{\varphi}\left(\Gamma_{\varphi R}^{\varphi}-\Gamma_{\varphi R}^tV^{\varphi}\right)+V^{\varphi}V^R\left(\Gamma_{R\varphi}^{\varphi}-\Gamma_{R\varphi}^tV^{\varphi}\right)\right]=0$$

א $$for\ jet:\quad (\rho+P)(u^0)^2\left[\frac{\partial V^{\varphi}}{\partial z}V^z+2V^z\left(\Gamma_{tz}^{\varphi}-\Gamma_{tz}^tV^{\varphi}\right)+V^tV^z\left(\Gamma_{zt}^{\varphi}-\Gamma_{zt}^tV^{\varphi}\right)\right.$$
$$\left.+V^{\varphi}V^z\left(\Gamma_{z\varphi}^{\varphi}-\Gamma_{z\varphi}^tV^{\varphi}\right)+V^zV^{\varphi}\left(\Gamma_{\varphi z}^{\varphi}-\Gamma_{\varphi z}^tV^{\varphi}\right)\right]+B_zJ^R=0$$

پایستگی تکانه ارتفاعی:

■ $$for\ disk:\quad (\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial R}V^R+\Gamma_{tt}^z+2V^{\varphi}\Gamma_{t\varphi}^z+V^tV^{\varphi}\Gamma_{\varphi t}^z+V^RV^R\Gamma_{RR}^z+V^{\varphi}V^{\varphi}\Gamma_{\varphi\varphi}^z\right]$$
$$+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial z}-B_{\varphi}J^R=0$$

א $$for\ jet:\quad (\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma_{tt}^z+2V^{\varphi}\Gamma_{t\varphi}^z-2\Gamma_{tz}^tV^zV^z+V^tV^{\varphi}\Gamma_{\varphi t}^z-V^tV^z\Gamma_{zt}^tV^z\right.$$
$$\left.+V^{\varphi}V^{\varphi}\Gamma_{\varphi\varphi}^z-\Gamma_{z\varphi}^tV^zV^{\varphi}V^z-\Gamma_{\varphi z}^tV^zV^zV^{\varphi}+V^zV^z\Gamma_{zz}^z\right]+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial z}=0$$

ابتدا برای قرص، با قرار دادن مقادیر گاما و J خواهیم داشت:

پایستگی جرم:

- $for\ disk$: $V^r \frac{\partial}{\partial r}(\rho - P) + (\rho + P)\left[\{V^r([\Gamma^r_{rr} + \Gamma^{\varphi}_{\varphi r} + \Gamma^z_{zr}] - \Gamma^0_{r0})\} + \{V^r([\Gamma^r_{rz} + \Gamma^{\varphi}_{\varphi z} + \Gamma^z_{zz}] - \Gamma^0_{z0})\} - \{\Gamma^0_{r\varphi}V^{\varphi}V^r + \Gamma^0_{\varphi r}V^rV^{\varphi} + \Gamma^0_{z\varphi}V^{\varphi}V^z\} + \left(\frac{\partial V^r}{\partial r}\right)\right] + \frac{1}{(u^0)^2}[-2\{-B_{\varphi}J^z u^R u^0\}] = 0$

$$V^R \frac{\partial}{\partial R}(\rho - P) + (\rho + P)\left[\left\{V^R\left(\left[-\frac{m}{R(R+2m)} - \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} - \frac{m}{R(R+2m)}\right] - \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}\right)\right\} - \left\{-\frac{maR(3R+4m)}{R^4 - 4R^2m^2 + 4a^2m^2}V^{\varphi}V^R - \frac{maR(3R+4m)}{R^4 - 4R^2m^2 + 4a^2m^2}V^RV^{\varphi}\right\} + \left(\frac{\partial V^R}{\partial R}\right)\right] + \frac{1}{(u^0)^2}[-2\{-B_{\varphi}J^z u^R u^0\}] = 0$$

که J^z را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_{φ} است.

پایستگی تکانه شعاعی:

- $for\ disk$: $(\rho + P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R + (\Gamma^R_{00}) - 2\Gamma^0_{0R}V^RV^R + 2V^{\varphi}\Gamma^R_{0\varphi} + V^tV^R(\Gamma^t_{Rt} - \Gamma^0_{Rt}V^R) + V^RV^R\Gamma^R_{RR} - \Gamma^0_{\varphi R}V^RV^RV^{\varphi} - \Gamma^0_{R\varphi}V^RV^{\varphi}V^R + V^{\varphi}V^{\varphi}\Gamma^R_{\varphi\varphi}\right] + \left[1 + \frac{2m}{\sqrt{R^2 + z^2}}\right]\frac{\partial P}{\partial R} + [B_{\varphi}J^z] = 0$

$$(\rho + P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R + \left(\frac{m}{R(R^2+2m)}\right) - 2\frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}V^RV^R + 2V^{\varphi}(-\frac{ma}{R(R+2m)}) + V^tV^R\left(\frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}\right)(1 - V^R) + V^RV^R\left(-\frac{m}{R(R+2m)}\right) - \left(-\frac{maR(3R+4m)}{R^4 - 4R^2m^2 + 4a^2m^2}\right)V^RV^RV^{\varphi} - (-\frac{maR(3R+4m)}{R^4 - 4R^2m^2 + 4a^2m^2})V^RV^{\varphi}V^R + V^{\varphi}V^{\varphi}\left(-\frac{R(R+m)}{(R+2m)}\right)\right] + \left[1 + \frac{2m}{R}\right]\frac{\partial P}{\partial R} + [B_{\varphi}J^z] = 0$$

پایستگی تکانه زاویه ای:

- $for\ disk$: $$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R+2V^R\left(\Gamma^\varphi_{tR}-\Gamma^t_{tR}V^\varphi\right)+V^tV^R\left(\Gamma^\varphi_{Rt}-\Gamma^t_{Rt}V^\varphi\right)\right.$$ $$\left.+V^RV^\varphi\left(\Gamma^\varphi_{\varphi R}-\Gamma^t_{\varphi R}V^\varphi\right)+V^\varphi V^R\left(\Gamma^\varphi_{R\varphi}-\Gamma^t_{R\varphi}V^\varphi\right)\right]=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R+2V^R\left(\frac{am}{(R^4-4R^2m^2+4a^2m^2)}-\frac{mR(R^5+2R^4m-2R^2a^2m)}{R^2(R^6-4R^4m^2+4R^2a^2m^2)}V^\varphi\right)\right.$$
$$+V^tV^R\left(\frac{am}{(R^4-4R^2m^2+4a^2m^2)}-\frac{mR(R^5+2R^4m-2R^2a^2m)}{R^2(R^6-4R^4m^2+4R^2a^2m^2)}V^\varphi\right)$$
$$+V^RV^\varphi\left(-\frac{R^3m-R^4+2R^2m^2+2a^2m^2}{(R^4-4R^2m^2+4a^2m^2)}+\frac{maR(3R+4m)}{R^4-4R^2m^2+4a^2m^2}V^\varphi\right)$$
$$\left.+V^\varphi V^R\left(-\frac{R^3m-R^4+2R^2m^2+2a^2m^2}{(R^4-4R^2m^2+4a^2m^2)}+\frac{maR(3R+4m)}{R^4-4R^2m^2+4a^2m^2}V^\varphi\right)\right]=0$$

پایستگی تکانه ارتفاعی:

- $for\ disk$: $$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial R}V^R\right]+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial z}-B_\varphi J^R=0$$

$$as\ J^R=0\ ,\ \ \frac{\partial}{\partial z}=0\ \ ,\ \ V^z=0\ :\ \ \ 0=0$$

سپس برای جت، با قرار دادن مقادیر گاما و J خواهیم داشت:

پایستگی جرم:

$$\aleph \quad for\ jet: \quad V^z\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^0_{\varphi z}V^zV^\varphi+\Gamma^0_{z\varphi}V^\varphi V^z\}+\left(\frac{\partial V^z}{\partial z}\right)\right]+\frac{1}{(u^0)^2}[-2\{-B_zJ^Ru^\varphi u^0\}]=0$$

$$V^z\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\left\{\left(-\frac{maz(3z+4m)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}\right)V^zV^\varphi+\left(-\frac{maz(3z+4m)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}\right)V^\varphi V^z\right\}+\left(\frac{\partial V^z}{\partial z}\right)\right]+\frac{1}{(u^0)^2}[-2\{-B_zJ^Ru^\varphi u^0\}]=0$$

که J^R را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_φ است.

با فرض $V^\varphi=0$ و در نتیجه $u^\varphi=0$ داریم:

$$V^z\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left(\frac{\partial V^z}{\partial z}\right)=0$$

پایستگی تکانه شعاعی:

$$\aleph \quad for\ jet: \quad (\rho+P)(u^0)^2\left[(\Gamma^R_{00})+2V^\varphi\Gamma^R_{0\varphi}+V^tV^z(\Gamma^t_{zt})+V^\varphi V^\varphi\Gamma^R_{\varphi\varphi}+V^zV^z\Gamma^R_{zz}\right]+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial R}-B_zJ^\varphi=0$$

$$(\rho+P)(u^0)^2\left[\left(\frac{m}{z^2(z+2m)}\right)+2V^\varphi\left(\frac{2ma}{z^2(z+2m)}\right)+V^tV^z\left(\frac{mz(z+2z^3+z^5+2m-6a^2m+4mz^2+2z^4m)}{z^2(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}\right)+V^\varphi V^\varphi\left(-\frac{(z^3+m+2mz^2)}{z^2(z+2m)}\right)+V^zV^z\left(\frac{m}{z^2(z+2m)}\right)\right]+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial R}-B_zJ^\varphi=0$$

که J^φ را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_z است.

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض $V^\varphi=0$ است. در این صورت داریم:

$$(\rho+P)(u^0)^2\left[\left(\frac{m}{z^2(z+2m)}\right)+V^tV^z\left(\frac{mz(z+2z^3+z^5+2m-6a^2m+4mz^2+2z^4m)}{z^2(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}\right)+V^zV^z\left(\frac{m}{z^2(z+2m)}\right)\right]+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial R}-B_zJ^\varphi=0$$

پایستگی تکانه زاویه ای:

$$\aleph \quad for\ jet:\quad (\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^tV^z\left(\Gamma^\varphi_{zt}-\Gamma^t_{zt}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]+B_zJ^R=0$$

$$\begin{aligned}(\rho+P)(u^0)^2\Bigg[&\frac{\partial V^\varphi}{\partial z}V^z\\
&+2V^z\left(\frac{amz(-4m+3z)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}\right.\\
&\left.-\frac{mz(z+2z^3+z^5+2m-6a^2m+4mz^2+2z^4m)}{z^2(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}V^\varphi\right)\\
&+V^tV^z\left(\frac{amz(-4m+3z)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}\right.\\
&\left.-\frac{mz(z+2z^3+z^5+2m-6a^2m+4mz^2+2z^4m)}{z^2(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}V^\varphi\right)\\
&+V^\varphi V^z\left(-\frac{mz(z+2z^3+z^5-2m+6a^2m-4mz^2-2mz^4)}{(1+z^2)(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}\right.\\
&\left.+\frac{maz(3z+4m)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}V^\varphi\right)\\
&+V^zV^\varphi\left(-\frac{mz(z+2z^3+z^5-2m+6a^2m-4mz^2-2mz^4)}{(1+z^2)(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}\right.\\
&\left.+\frac{maz(3z+4m)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}V^\varphi\right)\Bigg]+B_zJ^R=0\end{aligned}$$

که J^R را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_φ است.

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض $V^\varphi=0$ است. در این صورت داریم:

$$\begin{aligned}(\rho+P)(u^0)^2\Bigg[&2V^z\left(\frac{amz(-4m+3z)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}\right)\\
&+V^tV^z\left(\frac{amz(-4m+3z)}{1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6}\right)\Bigg]+B_zJ^R=0\end{aligned}$$

$$or\ :\qquad (\rho+P)(u^0)^2\left[2V^z\left(\Gamma^\varphi_{tz}\right)+V^tV^z\left(\Gamma^\varphi_{zt}\right)\right]+B_zJ^R=0$$

پایستگی تکانه ارتفاعی:

$$\aleph \quad for\ jet:\quad (\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2\Gamma^t_{tz}V^zV^z+V^tV^\varphi\Gamma^z_{\varphi t}-V^tV^z\Gamma^t_{zt}V^z+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-\Gamma^t_{z\varphi}V^zV^\varphi V^z-\Gamma^t_{\varphi z}V^zV^zV^\varphi+V^zV^z\Gamma^z_{zz}\right]+\left[1+\frac{2m}{z}\right]\frac{\partial P}{\partial z}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2\Gamma^t_{tz}V^zV^z+V^tV^\varphi\Gamma^z_{\varphi t}-V^tV^z\Gamma^t_{zt}V^z+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-\Gamma^t_{z\varphi}V^zV^\varphi V^z-\Gamma^t_{\varphi z}V^zV^zV^\varphi+V^zV^z\Gamma^z_{zz}\right]+\left[1+\frac{2m}{z}\right]\frac{\partial P}{\partial z}=0$$

ابتدا با فرض $V^{\varphi} = 0$ خواهیم داشت:

$$(\rho + P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z + \Gamma_{tt}^z - 2\Gamma_{tz}^t V^z V^z - V^t V^z \Gamma_{zt}^t V^z + V^z V^z \Gamma_{zz}^z\right] + \left[1 + \frac{2m}{z}\right]\frac{\partial P}{\partial z} = 0$$

$$\begin{aligned}(\rho + P)(u^0)^2 \Bigg[\frac{\partial V^z}{\partial z}V^z &+ \frac{mz}{(1+z^2)(z+2m)} \\ &- 2\left(\frac{mz(z + 2z^3 + z^5 + 2m - 6a^2m + 4mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}\right)V^z V^z \\ &- V^t V^z\left(\frac{mz(z + 2z^3 + z^5 + 2m - 6a^2m + 4mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}\right)V^z \\ &- V^z V^z\left(\frac{mz}{(1+z^2)(\sqrt{z} + 2m)}\right)\Bigg] + \left[1 + \frac{2m}{z}\right]\frac{\partial P}{\partial z} = 0\end{aligned}$$

از این پس برای ارتباط بین فشار و چگالی، از رابطه گاز پلی تروپ استفاده می کنیم:

$$P = K\rho^{\gamma} \quad in\ which: \begin{cases}\gamma = \dfrac{5}{3} & for\ relativisticgas \\ \gamma = \dfrac{4}{3} & for\ non\ relativisticgas\end{cases}$$

همچنین با استفاده از مقاله های:

Fendt_2006_ApJ_ The Astrophysical Journal, 651:272–287, 2006 November 1

Ouyed & Pudritz 1997a; Fendt & Cˇ emeljic´ 2002

برای چگالی می توانیم استفاده کنیم از:

$$\rho(R,z) = (R^2 + z^2)^{-3/4}$$

در این صورت برای **قرص** داریم $z = 0$ و در نتیجه

$$\rho(R, z = 0) = (R^2)^{-3/4}$$

$$P = K\rho^{\gamma}\ , \qquad \gamma = \frac{5}{3}\ , \qquad \rho = (R^2)^{-3/4} = R^{-3/2}$$

$$P = K\rho^{\gamma} = K\rho^{-\frac{5}{3}} = K\left[(R^2)^{3/4}\right]^{-\frac{5}{3}} = KR^{-5/2}$$

در این صورت برای **جت** داریم $z \gg R$ و در نتیجه

$$\rho(R,z) = (R^2 + z^2)^{-3/4} \cong (z^2)^{-3/4}$$

$$P = K\rho^{\gamma}\ , \qquad \gamma = \frac{5}{3}\ , \qquad \rho = (z^2)^{-3/4} = z^{-3/2}$$

$$P = K\rho^{\gamma} = K\rho^{\frac{5}{3}} = K\left[(z^2)^{-3/4}\right]^{\frac{5}{3}} = Kz^{-5/2}$$

ابتدا برای قرص خواهیم داشت:

پایستگی جرم ($V^z=0$) :

$$\blacksquare\ \ for\ disk:\quad V^r\frac{\partial}{\partial r}(\rho-P)+(\rho+P)\Big[\{V^r([\Gamma^r_{rr}+\Gamma^{\varphi}_{\varphi r}+\Gamma^z_{zr}]-\Gamma^0_{r0})\}+\{V^r([\Gamma^r_{rz}+\Gamma^{\varphi}_{\varphi z}+\Gamma^z_{zz}]-\Gamma^0_{z0})\}-\{\Gamma^0_{r\varphi}V^{\varphi}V^r+\Gamma^0_{\varphi r}V^rV^{\varphi}\}+\left(\frac{\partial V^r}{\partial r}\right)\Big]+\frac{1}{(u^0)^2}\left[-2\{-B_{\varphi}J^zu^Ru^0\}\right]=0$$

$$V^R\frac{\partial}{\partial R}(\rho-P)+(\rho+P)\left(\frac{\partial V^R}{\partial R}\right)=-(\rho+P)\Big[\{V^R([\Gamma^R_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{zR}]-\Gamma^0_{r0})\}+\{V^R([\Gamma^r_{rz}+\Gamma^{\varphi}_{\varphi z}+\Gamma^z_{zz}]-\Gamma^0_{z0})\}-\{\Gamma^0_{R\varphi}V^{\varphi}V^R+\Gamma^0_{\varphi R}V^RV^{\varphi}\}\Big]-\frac{1}{(u^0)^2}\left[-2\{-B_{\varphi}J^zu^Ru^0\}\right]$$

نهایی

$$V^R\frac{\partial}{\partial R}(R^{-3/2}-KR^{-5/2})+(R^{-3/2}+KR^{-5/2})\left(\frac{\partial V^R}{\partial R}\right)=-(R^{-3/2}+KR^{-5/2})\Big[\{V^R([\Gamma^R_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{zR}]-\Gamma^0_{r0})\}+\{V^R([\Gamma^r_{rz}+\Gamma^{\varphi}_{\varphi z}+\Gamma^z_{zz}]-\Gamma^0_{z0})\}-\{\Gamma^0_{R\varphi}V^{\varphi}V^R+\Gamma^0_{\varphi R}V^RV^{\varphi}\}\Big]-\frac{1}{(u^0)^2}\left[-2\{-B_{\varphi}J^zu^Ru^0\}\right]$$

که J^z را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_{φ} است.

پایستگی تکانه شعاعی:

$$\blacksquare\ \ for\ disk:\quad (\rho+P)(u^0)^2\Big[\frac{\partial V^R}{\partial R}V^R+(\Gamma^R_{00})-2\Gamma^0_{0R}V^RV^R+2V^{\varphi}\Gamma^R_{0\varphi}+V^tV^R(\Gamma^t_{Rt}-\Gamma^0_{Rt}V^R)+V^RV^R\Gamma^R_{RR}-\Gamma^0_{\varphi R}V^RV^RV^{\varphi}-\Gamma^0_{R\varphi}V^RV^{\varphi}V^R+V^{\varphi}V^{\varphi}\Gamma^R_{\varphi\varphi}\Big]+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial R}+[B_{\varphi}J^z]=0$$

$$(\rho+P)(u^0)^2\frac{\partial V^R}{\partial R}V^R+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial R}=-(\rho+P)(u^0)^2[(\Gamma^R_{00})-2\Gamma^0_{0R}V^RV^R+2V^{\varphi}\Gamma^R_{0\varphi}+V^tV^R(\Gamma^t_{Rt}-\Gamma^0_{Rt}V^R)+V^RV^R\Gamma^R_{RR}-\Gamma^0_{\varphi R}V^RV^RV^{\varphi}-\Gamma^0_{R\varphi}V^RV^{\varphi}V^R+V^{\varphi}V^{\varphi}\Gamma^R_{\varphi\varphi}]-[B_{\varphi}J^z]$$

نهایی

$$(R^{-3/2}-KR^{-5/2})(u^0)^2\frac{\partial V^R}{\partial R}V^R+\left[1+\frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial(KR^{-5/2})}{\partial R}=-(R^{-3/2}-KR^{-5/2})(u^0)^2[(\Gamma^R_{00})-2\Gamma^0_{0R}V^RV^R+2V^{\varphi}\Gamma^R_{0\varphi}+V^tV^R(\Gamma^t_{Rt}-\Gamma^0_{Rt}V^R)+V^RV^R\Gamma^R_{RR}-\Gamma^0_{\varphi R}V^RV^RV^{\varphi}-\Gamma^0_{R\varphi}V^RV^{\varphi}V^R+V^{\varphi}V^{\varphi}\Gamma^R_{\varphi\varphi}]-[B_{\varphi}J^z]$$

پایستگی تکانه زاویه ای:

$$\blacksquare \quad for\ disk: \quad (\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R + 2V^R\left(\Gamma^\varphi_{tR} - \Gamma^t_{tR}V^\varphi\right) + V^tV^R\left(\Gamma^\varphi_{Rt} - \Gamma^t_{Rt}V^\varphi\right) + V^RV^\varphi\left(\Gamma^\varphi_{\varphi R} - \Gamma^t_{\varphi R}V^\varphi\right) + V^\varphi V^R\left(\Gamma^\varphi_{R\varphi} - \Gamma^t_{R\varphi}V^\varphi\right)\right] = 0$$

$$(\rho+P)(u^0)^2\frac{\partial V^\varphi}{\partial R}V^R = -(\rho+P)(u^0)^2\left[2V^R\left(\Gamma^\varphi_{tR} - \Gamma^t_{tR}V^\varphi\right) + V^tV^R\left(\Gamma^\varphi_{Rt} - \Gamma^t_{Rt}V^\varphi\right) + V^RV^\varphi\left(\Gamma^\varphi_{\varphi R} - \Gamma^t_{\varphi R}V^\varphi\right) + V^\varphi V^R\left(\Gamma^\varphi_{R\varphi} - \Gamma^t_{R\varphi}V^\varphi\right)\right]$$

$$\left(R^{-3/2} - KR^{-5/2}\right)(u^0)^2\frac{\partial V^\varphi}{\partial R}V^R = -\left(R^{-3/2} - KR^{-5/2}\right)(u^0)^2\left[2V^R\left(\Gamma^\varphi_{tR} - \Gamma^t_{tR}V^\varphi\right) + V^tV^R\left(\Gamma^\varphi_{Rt} - \Gamma^t_{Rt}V^\varphi\right) + V^RV^\varphi\left(\Gamma^\varphi_{\varphi R} - \Gamma^t_{\varphi R}V^\varphi\right) + V^\varphi V^R\left(\Gamma^\varphi_{R\varphi} - \Gamma^t_{R\varphi}V^\varphi\right)\right]$$

نهایی

پایستگی تکانه ارتفاعی:

$$\blacksquare \quad for\ disk: \quad (\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial R}V^R\right] + \left[1 + \frac{2m}{\sqrt{R^2+z^2}}\right]\frac{\partial P}{\partial z} - B_\varphi J^R = 0$$

$$as\ J^R = 0\ ,\ \ \frac{\partial}{\partial z} = 0\ \ ,\ \ V^z = 0\ :\ \ \ 0 = 0$$

نهایی

ضرایب کریستوفل در قرص به این ترتیب است $(1 = t\,, 2 = R\,, 3 = \varphi\,, 4 = z)$:

$$(1,1,2) = \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}$$

$$(1,2,1) = \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}$$

$$(1,2,3) = -\frac{maR(3R + 4m)}{R^4 - 4R^2m^2 + 4a^2m^2}$$

$$(1,3,2) = -\frac{maR(3R + 4m)}{R^4 - 4R^2m^2 + 4a^2m^2}$$

$$(2,1,1) = \frac{m}{R(R^2 + 2m)}$$

$$(2,1,3) = -\frac{ma}{R(R + 2m)}$$

$$(2,2,2) = -\frac{m}{R(R + 2m)}$$

$$(2,3,1) \;=\; -\frac{ma}{R(R+2m)}$$

$$(2,3,3) \;=\; -\frac{R(R+\,m)}{(R+2m)}$$

$$(2,4,4) = \frac{m}{R(R+2m)}$$

$$(3,1,2) = \frac{am}{(R^4 \,-\, 4R^2m^2 \;+\, 4a^2m^2)}$$

$$(3,2,1) \;=\; \frac{am}{(R^4 \,-\, 4R^2m^2 \;+\, 4a^2m^2)}$$

$$(3,2,3) \;=\; -\frac{R^3m \,-R^4 +\, 2R^2m^2 \;+\, 2a^2m^2}{(R^4 \,-\, 4R^2m^2 \;+\, 4a^2m^2)}$$

$$(3,3,2) = -\frac{R^3m \,-R^4 +\, 2R^2m^2 \;+\, 2a^2m^2}{(R^4 \,-\, 4R^2m^2 \;+\, 4a^2m^2)}$$

$$(4,2,4) = \;-\frac{m}{R(R+\,2m)}$$

$$(4,4,2) \;=-\frac{m}{R(R+\,2m)}$$

همینطور برای J در قرص داریم:

$$-\frac{4\pi}{c}J^t = 0 \quad \text{و} \quad -\frac{4\pi}{c}J^R = 0 \quad \text{و} \quad -\frac{4\pi}{c}J^\varphi = 0$$

$$\rightarrow \quad -\frac{4\pi}{c}J^z = \frac{\partial}{\partial R}\left(\frac{R^2}{(R+2m)^2}B_\varphi(R)\right) + \left(\Gamma^t_{tR} + \Gamma^\varphi_{\varphi R} + \Gamma^z_{Rz} + \Gamma^R_{RR}\right)\left(\frac{R^2}{(R+2m)^2}B_\varphi\right)$$

سپس برای جت خواهیم داشت:

پایستگی جرم:

$$\aleph \quad for\ jet: \quad V^z \frac{\partial}{\partial z}(\rho - P) + (\rho + P)\left[-\{\Gamma^0_{\varphi z} V^z V^\varphi + \Gamma^0_{z\varphi} V^\varphi V^z\} + \left(\frac{\partial V^z}{\partial z}\right)\right] + \frac{1}{(u^0)^2}[-2\{-B_z J^R u^\varphi u^0\}] = 0$$

$$V^z \frac{\partial}{\partial z}(\rho - P) + (\rho + P)\left(\frac{\partial V^z}{\partial z}\right) = -(\rho + P)[-\{\Gamma^0_{\varphi z} V^z V^\varphi + \Gamma^0_{z\varphi} V^\varphi V^z\}] - \frac{1}{(u^0)^2}[-2\{-B_z J^R u^\varphi u^0\}]$$

که J^R را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_φ است.

نهایی

$$V^z \frac{\partial}{\partial z}(z^{-3/2} - Kz^{-5/2}) + (z^{-3/2} - Kz^{-5/2})\left(\frac{\partial V^z}{\partial z}\right) = -(z^{-3/2} - Kz^{-5/2})[-\{\Gamma^0_{\varphi z} V^z V^\varphi + \Gamma^0_{z\varphi} V^\varphi V^z\}] - \frac{1}{(u^0)^2}[-2\{-B_z J^R u^\varphi u^0\}]$$

با فرض $V^\varphi = 0$ و در نتیجه $u^\varphi = 0$ داریم:

نهایی

$$V^z \frac{\partial}{\partial z}(z^{-3/2} - Kz^{-5/2}) + (z^{-3/2} - Kz^{-5/2})\left(\frac{\partial V^z}{\partial z}\right) = 0$$

پایستگی تکانه شعاعی:

$$\aleph \quad for\ jet: \quad (\rho + P)(u^0)^2[(\Gamma^R_{00}) + 2V^\varphi \Gamma^R_{0\varphi} + V^t V^z (\Gamma^t_{zt}) + V^\varphi V^\varphi \Gamma^R_{\varphi\varphi} + V^z V^z \Gamma^R_{zz}] + \left[1 + \frac{2m}{\sqrt{R^2 + z^2}}\right]\frac{\partial P}{\partial R} - B_z J^\varphi = 0$$

$$\left[1 + \frac{2m}{\sqrt{R^2 + z^2}}\right]\frac{\partial P}{\partial R} = -(\rho + P)(u^0)^2[(\Gamma^R_{00}) + 2V^\varphi \Gamma^R_{0\varphi} + V^t V^z (\Gamma^t_{zt}) + V^\varphi V^\varphi \Gamma^R_{\varphi\varphi} + V^z V^z \Gamma^R_{zz}] + B_z J^\varphi$$

که J^φ را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_z است.

نهایی

$$\left[1 + \frac{2m}{\sqrt{R^2 + z^2}}\right]\frac{\partial (Kz^{-5/2})}{\partial R} = -(z^{-3/2} - Kz^{-5/2})(u^0)^2[(\Gamma^R_{00}) + 2V^\varphi \Gamma^R_{0\varphi} + V^t V^z (\Gamma^t_{zt}) + V^\varphi V^\varphi \Gamma^R_{\varphi\varphi} + V^z V^z \Gamma^R_{zz}] + B_z J^\varphi$$

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض $V^\varphi = 0$ است. در این صورت داریم:

نهایی

$$\left[1 + \frac{2m}{\sqrt{R^2 + z^2}}\right]\frac{\partial \left(Kz^{\frac{5}{2}}\right)}{\partial R} = -(z^{-3/2} - Kz^{-5/2})(u^0)^2[(\Gamma^R_{00}) + V^t V^z (\Gamma^t_{zt}) + V^z V^z \Gamma^R_{zz}] + B_z J^\varphi$$

پایستگی تکانه زاویه ای:

$$\aleph \quad for\ jet: \quad (\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^tV^z\left(\Gamma^\varphi_{zt}-\Gamma^t_{zt}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]+B_zJ^R=0$$

$$(\rho+P)(u^0)^2\frac{\partial V^\varphi}{\partial z}V^z=-(\rho+P)(u^0)^2\left[2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^tV^z\left(\Gamma^\varphi_{zt}-\Gamma^t_{zt}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]-B_zJ^R$$

که J^R را هم از قبل بدست آوردیم و وابسته به میدان سمتی یا B_φ است.

نهایی

$$\left(z^{-3/2}-Kz^{-5/2}\right)(u^0)^2\frac{\partial V^\varphi}{\partial z}V^z=-\left(z^{-3/2}-Kz^{-5/2}\right)(u^0)^2\left[2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^tV^z\left(\Gamma^\varphi_{zt}-\Gamma^t_{zt}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]-B_zJ^R$$

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض $V^\varphi=0$ است. در این صورت داریم:

نهایی

$$2V^z\Gamma^\varphi_{tz}+V^tV^z\Gamma^\varphi_{zt}+\left(z^{3/2}+Kz^{5/2}\right)^{-1}(u^0)^{-2}B_zJ^R=0$$

$$or: \quad \left(z^{-3/2}-Kz^{-5/2}\right)(u^0)^2\left[2V^z\left(\Gamma^\varphi_{tz}\right)+V^tV^z\left(\Gamma^\varphi_{zt}\right)\right]+B_zJ^R=0$$

پایستگی تکانه ارتفاعی:

$$\aleph \quad for\ jet: \quad (\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2\Gamma^t_{tz}V^zV^z+V^tV^\varphi\Gamma^z_{\varphi t}-V^tV^z\Gamma^t_{zt}V^z+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-\Gamma^t_{z\varphi}V^zV^\varphi V^z-\Gamma^t_{\varphi z}V^zV^zV^\varphi+V^zV^z\Gamma^z_{zz}\right]+\left[1+\frac{2m}{z}\right]\frac{\partial P}{\partial z}=0$$

$$(\rho+P)(u^0)^2\frac{\partial V^z}{\partial z}V^z+\left[1+\frac{2m}{z}\right]\frac{\partial P}{\partial z}=-(\rho+P)(u^0)^2\left[\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2\Gamma^t_{tz}V^zV^z+V^tV^\varphi\Gamma^z_{\varphi t}-V^tV^z\Gamma^t_{zt}V^z+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-\Gamma^t_{z\varphi}V^zV^\varphi V^z-\Gamma^t_{\varphi z}V^zV^zV^\varphi+V^zV^z\Gamma^z_{zz}\right]$$

نهایی

$$\left(z^{-3/2}-Kz^{-5/2}\right)(u^0)^2\frac{\partial V^z}{\partial z}V^z+\left[1+\frac{2m}{z}\right]\frac{\partial\left(Kz^{-5/2}\right)}{\partial z}=-\left(z^{-3/2}-Kz^{-5/2}\right)(u^0)^2\left[\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2\Gamma^t_{tz}V^zV^z+V^tV^\varphi\Gamma^z_{\varphi t}-V^tV^z\Gamma^t_{zt}V^z+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-\Gamma^t_{z\varphi}V^zV^\varphi V^z-\Gamma^t_{\varphi z}V^zV^zV^\varphi+V^zV^z\Gamma^z_{zz}\right]$$

با فرض $V^\varphi=0$ خواهیم داشت:

نهایی

$$\left(z^{-3/2}-Kz^{-5/2}\right)(u^0)^2\frac{\partial V^z}{\partial z}V^z+\left[1+\frac{2m}{z}\right]\frac{\partial\left(Kz^{-5/2}\right)}{\partial z}=-\left(z^{-3/2}-Kz^{-5/2}\right)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}-2\Gamma^t_{tz}V^zV^z-V^tV^z\Gamma^t_{zt}V^z+V^zV^z\Gamma^z_{zz}\right]$$

ضرایب کریستوفل در جت به این ترتیب است $(1 = t\,, 2 = R\,, 3 = \varphi\,, 4 = z)$:

$$(1,1,2) = \frac{m(z + 2z^3 + z^5 \ + \ 2m - 2a^2m + 4mz^2 + 4a^2mz^2 + 2z^4m)}{z^2(1 \ - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6)}$$

$$(1,1,4) = \frac{mz(z \ + \ 2z^3 \ + \ z^5 \ + 2m - \ 6a^2m + \ 4mz^2 \ + \ 2z^4m)}{z^2(1 \ - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6)}$$

$$(1,2,1) = \frac{m(z + 2z^3 + z^5 \ + 2m - \ 2a^2m + \ 4mz^2 + \ 4a^2mz^2 + \ 2z^4m)}{z^2(1 \ - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6)}$$

$$(1,2,3) \ = \ -\frac{ma(2z^3 + z - 2z^3 + \ 4m)}{1 - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6}$$

$$(1,3,2) \ = \ -\frac{ma(2z^3 + z - 2z^3 + \ 4m)}{1 - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6}$$

$$(1,3,4) \ = \ -\frac{maz(3z + 4m)}{1 \ - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6}$$

$$(1,4,1) = \frac{mz(z \ + \ 2z^3 \ + \ z^5 \ + 2m - \ 6a^2m + \ 4mz^2 \ + \ 2z^4m)}{z^2(1 \ - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6)}$$

$$(1,4,3) \ = \ -\frac{maz(3z + 4m)}{1 \ - \ 4m^2 \ + \ 3z^2 \ + \ 4a^2m^2 - \ 8m^2z^2 \ + \ 3z^4 - \ 4m^2z^4 + z^6}$$

$$(2,1,1) = \frac{m}{z^2(z + 2m)}$$

$$(2,1,3) \ = \ \frac{2ma}{z^2(z + 2m)}$$

$$(2,2,2) \ = \ -\frac{m}{z^2(z + 2m)}$$

$$(2,2,4) \ = \ -\frac{m}{z(z + 2m)}$$

$$(2,3,1) \ = \ -\frac{-\ 2ma}{z^2(z + 2m)}$$

$$(2,3,3) \ = \ -\frac{(z^3 + \ m + \ 2mz^2)}{z^2(z + 2m)}$$

$$(2,4,2) \ = \ -\frac{m}{z(z + 2m)}$$

$$(2,4,4) = \frac{m}{z^2(z + 2m)}$$

$$(3,1,2) = \frac{am(z - 2z^3 + 4mz^2)}{(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$(3,1,4) = \frac{amz(-4m + 3z)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$(3,2,1) = \frac{am(z - 2z^3 + 4mz^2)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$\begin{aligned}(3,2,3) = -(&2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 \\ &+ 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 \\ &- z^8)/((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6))\end{aligned}$$

$$\begin{aligned}(3,3,2) = -(&2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 \\ &+ 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 \\ &- z^8)/((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6))\end{aligned}$$

$$(3,3,4) = -\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$(3,4,1) = \frac{amz(-4m + 3z)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$(3,4,2) = 0$$

$$(3,4,3) = -\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$(4,1,1) = \frac{mz}{(1 + z^2)(z + 2m)}$$

$$(4,1,3) = -\frac{3maz}{(1 + z^2)^2(z + 2m)}$$

$$(4,2,2) = \frac{mz}{(1 + z^2)(z + 2m)}$$

$$(4,2,4) = -\frac{mR}{(1 + z^2)(z + 2m)}$$

$$(4,3,1) = -\frac{3maR^2z}{(1 + z^2)^2(z + 2m)}$$

$$(4,3,3) = \frac{mz}{(1 + z^2)^2(z + 2m)}$$

$$(4,4,2) = -\frac{m}{(1 + z^2)(z + 2m)}$$

$$(4,4,4) \;=\; -\frac{mz}{(1+\;z^2)\left(\sqrt{z}+\;2m\right)}$$

همینطور برای J در جت داریم:

$$\frac{4\pi}{c}J^t = \Gamma^t_{tR}\left(\frac{z^4}{2ma(z+2m)}\right)B_z$$

$$\rightarrow \;\frac{4\pi}{c}J^R = \frac{\partial}{\partial z}\left(\frac{z^2}{(z+2m)^2}B_\varphi(z)\right) + \left(\Gamma^\varphi_{\varphi z} + \Gamma^R_{zR} + \Gamma^z_{zz} + \Gamma^t_{tz}\right)\frac{z^2}{(z+2m)^2}B_\varphi$$

$$\rightarrow \;\frac{4\pi}{c}J^\varphi = \Gamma^\varphi_{tR}\left(\frac{z^4}{2ma(z+2m)}\right)B_z + \left(\Gamma^t_{tR} + \Gamma^R_{RR} + \Gamma^\varphi_{R\varphi} + \Gamma^z_{zR}\right)\left(\frac{z^2}{(z+2m)^2}\right)B_z$$

$$-\frac{4\pi}{c}J^z = \left(\Gamma^t_{tR} + \Gamma^\varphi_{\varphi R} + \Gamma^z_{Rz} + \Gamma^R_{RR}\right)\left(\frac{z^2}{(z+2m)^2}\right)B_\varphi$$