معادله پایستگی جرم

با توجه به اینکه c=1 داریم:

$$\frac{\partial}{\partial t}(\rho-P)+V^{r}\frac{\partial}{\partial r}(\rho-P)+V^{\varphi}\frac{\partial}{\partial \varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)$$
$$+(\rho+P)\Big[\{V^{r}([\Gamma^{r}_{rr}+\Gamma^{\varphi}_{\varphi r}+\Gamma^{z}_{zr}]-\Gamma^{0}_{r0})\}+\{V^{\varphi}([\Gamma^{r}_{r\varphi}+\Gamma^{\varphi}_{\varphi\varphi}+\Gamma^{z}_{z\varphi}]-\Gamma^{0}_{\varphi 0})\}$$
$$+\{V^{r}([\Gamma^{r}_{rz}+\Gamma^{\varphi}_{\varphi z}+\Gamma^{z}_{zz}]-\Gamma^{0}_{z0})\}$$
$$-\{\Gamma^{0}_{rr}V^{r}V^{r}+\Gamma^{0}_{r\varphi}V^{\varphi}V^{r}+\Gamma^{0}_{rz}V^{z}V^{r}+\Gamma^{0}_{\varphi r}V^{r}V^{\varphi}+\Gamma^{0}_{\varphi\varphi}V^{\varphi}V^{\varphi}+\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}$$
$$+\Gamma^{0}_{zr}V^{r}V^{z}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}+\Gamma^{0}_{zz}V^{z}V^{z}\}+\left(\frac{\partial V^{r}}{\partial r}+\frac{\partial V^{\varphi}}{\partial \varphi}+\frac{\partial V^{z}}{\partial z}\right)+(\Gamma^{r}_{r0}+\Gamma^{\varphi}_{\varphi 0}+\Gamma^{z}_{z0})\Big]$$
$$+\frac{1}{(u^{0})^{2}}[-2F_{jk}J^{k}u^{j}u^{0}+F^{0}_{k}J^{k}]+\frac{1}{(u^{0})^{2}}\left[\left(1-\frac{2m}{\sqrt{R^{2}+z^{2}}}\right)\frac{\partial P}{\partial t}+\frac{2maR^{2}}{(R^{2}+z^{2})^{3/2}}\frac{\partial P}{\partial \varphi}\right]$$
$$=0$$

با قرار دادن مقادیر گاما برای قرص به دست می آوریم:

$$V^{R}\frac{\partial}{\partial r}(\rho-P)+(\rho+P)\Big[\{V^{R}([\Gamma^{r}_{rr}+\Gamma^{\varphi}_{\varphi r}+\Gamma^{z}_{zr}]-\Gamma^{0}_{r0})\}-\{\Gamma^{0}_{r\varphi}V^{\varphi}V^{R}+\Gamma^{0}_{\varphi r}V^{R}V^{\varphi}\}$$
$$+\left(\frac{\partial V^{R}}{\partial R}+\frac{\partial V^{\varphi}}{\partial \varphi}\right)\Big]+\frac{1}{(u^{0})^{2}}[-2F_{jk}J^{k}u^{j}u^{0}+F^{0}_{k}J^{k}]=0$$

$$V^{R}\frac{\partial}{\partial r}(\rho-P)+(\rho+P)\left(\frac{\partial V^{R}}{\partial R}+\frac{\partial V^{\varphi}}{\partial \varphi}\right)$$
$$+(\rho+P)[\{V^{R}([\Gamma^{r}_{rr}+\Gamma^{\varphi}_{\varphi r}+\Gamma^{z}_{zr}]-\Gamma^{0}_{r0})\}-\{\Gamma^{0}_{r\varphi}V^{\varphi}V^{R}+\Gamma^{0}_{\varphi r}V^{R}V^{\varphi}\}]$$
$$+\frac{1}{(u^{0})^{2}}[-2\{-B_{\varphi}J^{z}u^{R}u^{0}\}]=0$$

نهایی

با قرار دادن مقادیر گاما برای جت به دست می آوریم:

$$V^{\varphi}\frac{\partial}{\partial \varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial \varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]$$
$$+\frac{1}{(u^{0})^{2}}[-2F_{jk}J^{k}u^{j}u^{0}+F^{0}_{k}J^{k}]=0$$

که این رابطه، معادله پایستگی جرم یا معادله پیوستگی ست.

برای معادله پایستگی تکانه داریم:

$$\left(\rho+\frac{P}{c^{2}}\right)(u^{0})^{2}\Big[cV^{\alpha}_{,0}+V^{\alpha}_{,\gamma}V^{\gamma}+c^{2}\left(\Gamma^{\alpha}_{00}-\Gamma^{0}_{00}\frac{V^{\alpha}}{c}\right)+2cV^{\gamma}\left(\Gamma^{\alpha}_{0\gamma}-\Gamma^{0}_{0\gamma}\frac{V^{\alpha}}{c}\right)$$
$$+V^{k}V^{\gamma}\left(\Gamma^{\alpha}_{\gamma k}-\Gamma^{0}_{\gamma k}\frac{V^{\alpha}}{c}\right)\Big]+\left[\frac{V^{\alpha}}{c}g^{0j}-g^{\alpha j}\right]\frac{\partial P}{\partial x^{j}}+\frac{1}{c}\left(\frac{V^{\alpha}}{c}F^{0}_{k}-F^{\alpha}_{k}\right)J^{k}=0$$

با قرار دادن $\alpha = r, \varphi, z$ معادله را در راستای هر یک از مختصات بدست می آوریم:

- برای $\alpha = R$ معادله پایستگی تکانه شعاعی را داریم: (من متاسفانه از r به جای R استفاده کردم که باید تغییر کند. فقط در بالانویس ها مثل V^r)

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[cV^r_{,0}+V^r_{,\gamma}V^\gamma+c^2\left(\Gamma^r_{00}-\Gamma^0_{00}\frac{V^r}{c}\right)+2cV^\gamma\left(\Gamma^r_{0\gamma}-\Gamma^0_{0\gamma}\frac{V^r}{c}\right)+V^kV^\gamma\left(\Gamma^r_{\gamma k}-\Gamma^0_{\gamma k}\frac{V^r}{c}\right)\right] + \left[\frac{V^r}{c}g^{0j}-g^{rj}\right]\frac{\partial P}{\partial x^j}+\frac{1}{c}\left(\frac{V^r}{c}F^0_k-F^r_k\right)J^k=0$$

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[c\frac{\partial V^r}{\partial t}+\frac{\partial V^r}{\partial r}V^r+\frac{\partial V^r}{\partial \varphi}V^\varphi++\frac{\partial V^r}{\partial z}V^z+c^2\left(\Gamma^r_{00}-\Gamma^0_{00}\frac{V^r}{c}\right)+2cV^r\left(\Gamma^r_{0r}-\Gamma^0_{0r}\frac{V^r}{c}\right)\right.$$
$$+2cV^\varphi\left(\Gamma^r_{0\varphi}-\Gamma^0_{0\varphi}\frac{V^r}{c}\right)+2cV^z\left(\Gamma^r_{0z}-\Gamma^0_{0z}\frac{V^r}{c}\right)+V^tV^r\left(\Gamma^t_{rt}-\Gamma^0_{rt}\frac{V^r}{c}\right)$$
$$+V^tV^\varphi\left(\Gamma^t_{\varphi t}-\Gamma^0_{\varphi t}\frac{V^r}{c}\right)+V^tV^z\left(\Gamma^t_{zt}-\Gamma^0_{zt}\frac{V^r}{c}\right)+V^rV^r\left(\Gamma^r_{rr}-\Gamma^0_{rr}\frac{V^r}{c}\right)$$
$$+V^rV^\varphi\left(\Gamma^r_{\varphi r}-\Gamma^0_{\varphi r}\frac{V^r}{c}\right)+V^rV^z\left(\Gamma^r_{zr}-\Gamma^0_{zr}\frac{V^r}{c}\right)+V^\varphi V^r\left(\Gamma^r_{r\varphi}-\Gamma^0_{r\varphi}\frac{V^r}{c}\right)$$
$$+V^\varphi V^\varphi\left(\Gamma^r_{\varphi\varphi}-\Gamma^0_{\varphi\varphi}\frac{V^r}{c}\right)+V^\varphi V^z\left(\Gamma^r_{z\varphi}-\Gamma^0_{z\varphi}\frac{V^r}{c}\right)+V^zV^r\left(\Gamma^r_{rz}-\Gamma^0_{rz}\frac{V^r}{c}\right)$$
$$\left.+V^zV^\varphi\left(\Gamma^r_{\varphi z}-\Gamma^0_{\varphi z}\frac{V^r}{c}\right)+V^zV^z\left(\Gamma^r_{zz}-\Gamma^0_{zz}\frac{V^r}{c}\right)\right]+\left[\frac{V^r}{c}g^{tt}-g^{rt}\right]\frac{\partial P}{\partial t}+\left[\frac{V^r}{c}g^{tr}-g^{rr}\right]\frac{\partial P}{\partial r}$$
$$+\left[\frac{V^r}{c}g^{02}-g^{r\varphi}\right]\frac{\partial P}{\partial \varphi}+\left[\frac{V^r}{c}g^{03}-g^{rz}\right]\frac{\partial P}{\partial z}+\frac{1}{c}\left(\frac{V^r}{c}F^0_k-F^r_k\right)J^k=0$$

با قرار دادن مقادیر برای قرص به دست می آوریم:

$$(\rho+P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R+\frac{\partial V^R}{\partial \varphi}V^\varphi+\Gamma^R_{tt}+2V^R(-\Gamma^t_{tr}V^R)+2V^\varphi(\Gamma^R_{t\varphi})+V^RV^R(\Gamma^R_{RR})\right.$$
$$\left.+V^RV^\varphi(-\Gamma^t_{\varphi r}V^R)+V^\varphi V^R(-\Gamma^t_{R\varphi}V^R)+V^\varphi V^\varphi(\Gamma^R_{\varphi\varphi})\right]-g^{RR}\frac{\partial P}{\partial R}+(V^RF^t_k-F^R_k)J^k$$
$$=0$$

نهایی

$$(\rho+P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R+\frac{\partial V^R}{\partial \varphi}V^\varphi+\Gamma^R_{tt}+2V^R(-\Gamma^t_{tr}V^R)+2V^\varphi(\Gamma^R_{t\varphi})+V^RV^R(\Gamma^R_{RR})\right.$$
$$\left.+V^RV^\varphi(-\Gamma^t_{\varphi r}V^R)+V^\varphi V^R(-\Gamma^t_{R\varphi}V^R)+V^\varphi V^\varphi(\Gamma^R_{\varphi\varphi})\right]-g^{RR}\frac{\partial P}{\partial R}-(F^R_zJ^z+F^z_RJ^R)$$
$$=0$$

با قرار دادن مقادیر برای جت به دست می آوریم:

$$(\rho+P)(u^0)^2\left[(\Gamma^R_{tt})+2V^\varphi(\Gamma^R_{t\varphi})+V^\varphi V^\varphi(\Gamma^R_{\varphi\varphi})+V^zV^z(\Gamma^R_{zz})\right]-F^R_kJ^k=0$$

معادله پایستگی تکانه در راستای R است.

از آنجا که در تانسور وشکسانی، مولفه $t_{R\varphi}=t_{\varphi R}$ حائز اهمیت است، از جملات عبارت $t^{\nu}_{\mu\,;\nu}$ تنها این جملات باقی می‌ماند :

$$-(g^{i\mu}-u^iu^\mu)t^{\nu}_{\mu\,;\nu}=-[g^{t\varphi}+g^{R\varphi}+g^{\varphi\varphi}+g^{z\varphi}-(u^t+u^R+u^\varphi+u^z)u^\varphi]t^{\nu}_{\mu\,;\nu}$$

$$t^{\nu}_{\mu\,;\nu}=t^{R}_{\varphi\,,R}+\Gamma^{R}_{RR}\,t^{R}_{\varphi}-\Gamma^{\varphi}_{RR}\,t^{R}_{\varphi}+\Gamma^{R}_{R\varphi}\,t^{R}_{\varphi}+t^{R}_{\varphi\,,R}-\Gamma^{\varphi}_{\varphi R}\,t^{R}_{\varphi}+\Gamma^{\varphi}_{zR}\,t^{R}_{\varphi}+t^{R}_{\varphi\,,R}+t^{\varphi}_{R\,,\varphi}-\Gamma^{R}_{RR}\,t^{R}_{\varphi}-\Gamma^{R}_{\varphi R}\,t^{R}_{\varphi}-\Gamma^{\varphi}_{\varphi R}\,t^{R}_{\varphi}-\Gamma^{R}_{zR}\,t^{R}_{\varphi}+t^{\varphi}_{R\,,\varphi}$$

$$\xrightarrow{\frac{\partial}{\partial\varphi}t^{\varphi}_{R}=0}t^{\nu}_{\mu\,;\nu}=2\frac{\partial}{\partial R}t^{R}_{\varphi}-\Gamma^{\varphi}_{RR}\,t^{R}_{\varphi}+\Gamma^{R}_{R\varphi}\,t^{R}_{\varphi}-2\Gamma^{\varphi}_{\varphi R}\,t^{R}_{\varphi}+\Gamma^{\varphi}_{zR}\,t^{R}_{\varphi}-\Gamma^{R}_{\varphi R}\,t^{R}_{\varphi}-\Gamma^{R}_{zR}\,t^{R}_{\varphi}$$

$$\to\ t^{\nu}_{\mu\,;\nu}=2\frac{\partial}{\partial R}t^{R}_{\varphi}+\left(\Gamma^{R}_{R\varphi}-\Gamma^{\varphi}_{RR}\ -2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{\varphi}_{zR}\ -\Gamma^{R}_{\varphi R}\ -\Gamma^{R}_{zR}\ \right)t^{R}_{\varphi}$$

$$for\ disk{:}\ \ t^{\nu}_{\mu\,;\nu}=2\frac{\partial}{\partial R}t^{R}_{\varphi}-2\Gamma^{\varphi}_{\varphi R}t^{R}_{\varphi}=2\left(-\Gamma^{\varphi}_{\varphi R}+\frac{\partial}{\partial R}\right)t^{R}_{\varphi}=2\left(-\Gamma^{\varphi}_{\varphi R}+\frac{\partial}{\partial R}\right)(-\alpha p)$$

$$for\ disk{:}\ \ -(g^{i\mu}-u^iu^\mu)t^{\nu}_{\mu\,;\nu}=-[g^{t\varphi}+g^{\varphi\varphi}-(u^R+u^\varphi)u^\varphi]\left\{2\left(-\Gamma^{\varphi}_{\varphi R}+\frac{\partial}{\partial R}\right)(-\alpha p)\right\}$$

$$for\ jet{:}\ \ t^{\nu}_{\mu\,;\nu}=2\frac{\partial}{\partial R}t^{R}_{\varphi}+\left(\ -2\Gamma^{\varphi}_{\varphi R}\ \ -\Gamma^{R}_{zR}\ \right)t^{R}_{\varphi}$$

$$for\ jet\ as\ \frac{\partial}{\partial R}\equiv 0:\ \ t^{\nu}_{\mu\,;\nu}=-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)$$

$$for\ jet{:}\ \ -(g^{i\mu}-u^iu^\mu)t^{\nu}_{\mu\,;\nu}=-[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi]\left\{-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)\right\}$$

حال معادله پایستگی تکانه سمتی را محاسبه می‌نماییم:

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[cV^{\varphi}_{,0}+V^{\varphi}_{,\gamma}V^{\gamma}+c^2\left(\Gamma^{\varphi}_{00}-\Gamma^{0}_{00}\frac{V^\varphi}{c}\right)+2cV^{\gamma}\left(\Gamma^{\varphi}_{0\gamma}-\Gamma^{0}_{0\gamma}\frac{V^\varphi}{c}\right)+V^kV^\gamma\left(\Gamma^{\varphi}_{\gamma k}-\Gamma^{0}_{\gamma k}\frac{V^\varphi}{c}\right)\right]+\left[\frac{V^\varphi}{c}g^{0j}-g^{\varphi j}\right]\frac{\partial P}{\partial x^j}+\frac{1}{c}\left(\frac{V^\varphi}{c}F^{0}_{k}-F^{\varphi}_{k}(=0)\right)J^k=-(g^{i\mu}-u^iu^\mu)t^{\nu}_{\mu\,;\nu}$$

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[c\frac{\partial V^\varphi}{\partial t}+\frac{\partial V^\varphi}{\partial r}V^r+\frac{\partial V^\varphi}{\partial \varphi}V^\varphi++\frac{\partial V^\varphi}{\partial z}V^z+c^2\left(\Gamma^\varphi_{00}-\Gamma^0_{00}\frac{V^\varphi}{c}\right)+2cV^R\left(\Gamma^\varphi_{tr}-\Gamma^t_{tR}\frac{V^\varphi}{c}\right)\right.$$
$$+2cV^\varphi\left(\Gamma^\varphi_{t\varphi}-\Gamma^t_{t\varphi}\frac{V^\varphi}{c}\right)+2cV^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}\frac{V^\varphi}{c}\right)+V^tV^R\left(\Gamma^\varphi_{Rt}-\Gamma^t_{Rt}\frac{V^\varphi}{c}\right)$$
$$+V^tV^\varphi\left(\Gamma^\varphi_{\varphi t}-\Gamma^t_{\varphi t}\frac{V^\varphi}{c}\right)+V^tV^z\left(\Gamma^\varphi_{zt}-\Gamma^t_{zt}\frac{V^\varphi}{c}\right)+V^RV^R\left(\Gamma^\varphi_{RR}-\Gamma^t_{RR}\frac{V^\varphi}{c}\right)$$
$$+V^RV^\varphi\left(\Gamma^\varphi_{\varphi R}-\Gamma^t_{\varphi R}\frac{V^\varphi}{c}\right)+V^RV^z\left(\Gamma^\varphi_{zR}-\Gamma^t_{zR}\frac{V^\varphi}{c}\right)+V^\varphi V^R\left(\Gamma^\varphi_{R\varphi}-\Gamma^t_{R\varphi}\frac{V^\varphi}{c}\right)$$
$$+V^\varphi V^\varphi\left(\Gamma^\varphi_{\varphi\varphi}-\Gamma^t_{\varphi\varphi}\frac{V^\varphi}{c}\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}\frac{V^\varphi}{c}\right)+V^zV^R\left(\Gamma^\varphi_{Rz}-\Gamma^t_{Rz}\frac{V^\varphi}{c}\right)$$
$$\left.+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}\frac{V^\varphi}{c}\right)+V^zV^z\left(\Gamma^\varphi_{zz}-\Gamma^t_{zz}\frac{V^\varphi}{c}\right)\right]+\left[\frac{V^\varphi}{c}g^{tt}-g^{\varphi t}\right]\frac{\partial P}{\partial t}+\left[\frac{V^\varphi}{c}g^{tR}-g^{\varphi r}\right]\frac{\partial P}{\partial r}$$
$$+\left[\frac{V^\varphi}{c}g^{t\varphi}-g^{\varphi\varphi}\right]\frac{\partial P}{\partial \varphi}+\left[\frac{V^\varphi}{c}g^{tz}-g^{\varphi z}\right]\frac{\partial P}{\partial z}+\frac{1}{c}\left(\frac{V^\varphi}{c}F^0_k-F^\varphi_k(=0)\right)J^k$$
$$=-[g^{t\varphi}+g^{R\varphi}+g^{\varphi\varphi}+g^{z\varphi}-(u^t+u^R+u^\varphi+u^z)u^\varphi]\left\{2\frac{\partial}{\partial R}t^R_\varphi\right.$$
$$\left.+(\Gamma^R_{R\varphi}-\Gamma^\varphi_{RR}\ \ -2\Gamma^\varphi_{\varphi R}\ +\Gamma^\varphi_{zR}\ -\Gamma^R_{\varphi R}\ -\Gamma^R_{zR}\ )t^R_\varphi\right\}$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial z}=V^t=V^z=0\,,c=1\,,t^R_\varphi=-\alpha p$ و گاما برای قرص به دست می آوریم:

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R+\frac{\partial V^\varphi}{\partial \varphi}V^\varphi+2V^R(\Gamma^\varphi_{tr}-\Gamma^t_{tR}V^\varphi)+V^RV^\varphi(\Gamma^\varphi_{\varphi R}-\Gamma^t_{\varphi R}V^\varphi)\right.$$
$$\left.+V^\varphi V^R(\Gamma^\varphi_{R\varphi}-\Gamma^t_{R\varphi}V^\varphi)+V^zV^R(\Gamma^\varphi_{Rz}-\Gamma^t_{Rz}V^\varphi)\right]+(V^\varphi F^t_k-F^\varphi_k(=0))J^k$$
$$-[g^{t\varphi}+g^{\varphi\varphi}-(u^R+u^\varphi)u^\varphi]\left\{2\left(-\Gamma^\varphi_{\varphi R}+\frac{\partial}{\partial R}\right)(-\alpha p)\right\}=0$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial R}=V^t=V^R=0,c=1\,,t^R_\varphi=-\alpha p$ و گاما برای جت به دست می آوریم:

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial t}+\frac{\partial V^\varphi}{\partial \varphi}V^\varphi+\frac{\partial V^\varphi}{\partial z}V^z+2V^z(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi)+V^\varphi V^z(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi)\right.$$
$$\left.+V^zV^\varphi(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi)\right]+(V^\varphi F^0_k-F^\varphi_k)J^k$$
$$-[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi]\left\{-(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ )(-\alpha p)\right\}=0$$

معادله پایستگی تکانه در راستای φ است.

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[c\frac{\partial V^z}{\partial t}+\frac{\partial V^z}{\partial R}V^R+\frac{\partial V^z}{\partial \varphi}V^\varphi+\frac{\partial V^z}{\partial z}V^z+c^2\left(\Gamma^z_{tt}-\Gamma^t_{tt}\frac{V^z}{c}\right)+2cV^R\left(\Gamma^z_{tR}-\Gamma^t_{tR}\frac{V^z}{c}\right)\right.$$
$$+2cV^\varphi\left(\Gamma^z_{t\varphi}-\Gamma^t_{t\varphi}\frac{V^z}{c}\right)+2cV^z\left(\Gamma^z_{tz}-\Gamma^t_{tz}\frac{V^z}{c}\right)+V^tV^R\left(\Gamma^z_{Rt}-\Gamma^0_{Rt}\frac{V^z}{c}\right)$$
$$+V^tV^\varphi\left(\Gamma^z_{\varphi t}-\Gamma^t_{\varphi t}\frac{V^z}{c}\right)+V^tV^z\left(\Gamma^z_{zt}-\Gamma^t_{zt}\frac{V^z}{c}\right)+V^RV^R\left(\Gamma^z_{RR}-\Gamma^t_{RR}\frac{V^z}{c}\right)$$
$$+V^RV^\varphi\left(\Gamma^z_{\varphi R}-\Gamma^t_{\varphi R}\frac{V^z}{c}\right)+V^RV^z\left(\Gamma^z_{zR}-\Gamma^t_{zR}\frac{V^z}{c}\right)+V^\varphi V^R\left(\Gamma^z_{R\varphi}-\Gamma^t_{R\varphi}\frac{V^z}{c}\right)$$
$$+V^\varphi V^\varphi\left(\Gamma^z_{\varphi\varphi}-\Gamma^t_{\varphi\varphi}\frac{V^z}{c}\right)+V^\varphi V^z\left(\Gamma^z_{z\varphi}-\Gamma^0_{z\varphi}\frac{V^z}{c}\right)+V^zV^R\left(\Gamma^z_{Rz}-\Gamma^0_{Rz}\frac{V^z}{c}\right)$$
$$\left.+V^zV^\varphi\left(\Gamma^z_{\varphi z}-\Gamma^t_{\varphi z}\frac{V^z}{c}\right)+V^zV^z\left(\Gamma^z_{zz}-\Gamma^0_{zz}\frac{V^z}{c}\right)\right]+\left[\frac{V^z}{c}g^{tt}-g^{zt}\right]\frac{\partial P}{\partial t}$$
$$+\left[\frac{V^z}{c}g^{tR}-g^{zr}\right]\frac{\partial P}{\partial R}+\left[\frac{V^z}{c}g^{t\varphi}-g^{z\varphi}\right]\frac{\partial P}{\partial \varphi}+\left[\frac{V^z}{c}g^{tz}-g^{zz}\right]\frac{\partial P}{\partial z}$$
$$+\frac{1}{c}\left(\frac{V^z}{c}F^0_k-F^z_k\right)J^k=0$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial z}=V^t=V^z=0, c=1$ و گاما برای قرص به دست می آوریم:

$$-F^z_kJ^k=0 \quad , \quad J^k=0$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial R}=V^t=V^R=0, c=1$ و گاما برای جت به دست می آوریم:

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(V^zF^0_k-F^z_k\right)J^k=0$$

معادله پایستگی تکانه در راستای z است.

حال برای تانسور F داریم:

$$F^t_t=\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_\varphi$$

$$F^t_R=-\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}-2m}E_R-\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_z$$

$$F^t_\varphi=-\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}-2m}E_\varphi$$

$$F^t_z=-\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}-2m}E_z-\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_R$$

$$F_t^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}E_R$$

$$F_R^R = 0$$

$$F_\varphi^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_z$$

$$F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_t^\varphi = -\frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2}+2mR^2}E_\varphi$$

$$F_R^\varphi = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_R + \frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2}+2mR^2}B_z$$

$$F_\varphi^\varphi = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_\varphi$$

$$F_z^\varphi = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_z + \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_R$$

$$F_t^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}E_z$$

$$F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_\varphi^z = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_R$$

$$F_z^z = 0$$

حال با فرض اینکه تمامی مولفه های E برابر با صفر است داریم:

$$F_t^t = 0 \quad , \quad F_R^t = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_z \quad , \quad F_\varphi^t = 0 \quad , \quad F_z^t = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_R$$

$$F_t^R = 0 \quad , \quad F_R^R = 0 \quad , \quad F_\varphi^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_z \quad , \quad F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_t^\varphi = 0 \quad , \quad F_R^\varphi = \frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2}+2mR^2}B_z \quad , \quad F_\varphi^\varphi = 0 \quad , \quad F_z^\varphi = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_R$$

$$F_t^z = 0 \ \ , \ \ F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_\varphi \ \ , \ \ F_\varphi^z = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_R \ \ , \ \ F_z^z = 0$$

که در قرص با توجه به صفر شدن همه مولفه های میدان مغناطیسی جز B_φ ، خواهیم داشت:

$$F_t^t = 0 \ \ , \ \ F_R^t = 0 \ \ , \ \ F_\varphi^t = 0 \ \ , \ \ F_z^t = -0$$

$$F_t^R = 0 \ \ , \ \ F_R^R = 0 \ \ , \ \ F_\varphi^R = 0 \ \ , \ \ F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_\varphi$$

$$F_t^\varphi = 0 \ \ , \ \ F_R^\varphi = 0 \ \ , \ \ F_\varphi^\varphi = 0 \ \ , \ \ F_z^\varphi = 0$$

$$F_t^z = 0 \ \ , \ \ F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_\varphi \ \ , \ \ F_\varphi^z = 0 \ \ , \ \ F_z^z = 0$$

بنابراین برای تانسور انرژی الکترومغناطیسی تنها دو مولفه داریم:

$$F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_\varphi \xRightarrow{z\approx 0} F_z^R = \frac{R}{R+2m} B_\varphi = -F_R^z$$

و در جت با توجه به وجود دو مولفه میدان مغناطیسی B_φ و B_z ، خواهیم داشت:

$$F_t^t = 0 \ \ , \ \ F_R^t = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2} B_z \ \ , \ \ F_\varphi^t = 0 \ \ , \ \ F_z^t = 0$$

$$F_t^R = 0 \ \ , \ \ F_R^R = 0 \ \ , \ \ F_\varphi^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_z \ \ , \ \ F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_\varphi$$

$$F_t^\varphi = 0 \ \ , \ \ F_R^\varphi = \frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2+2mR^2}} B_z \ \ , \ \ F_\varphi^\varphi = 0 \ \ , \ \ F_z^\varphi = 0$$

$$F_t^z = 0 \ \ , \ \ F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2+2m}} B_\varphi \ \ , \ \ F_\varphi^z = 0 \ \ , \ \ F_z^z = 0$$

همچنین در جت با توجه به اینکه $R \ll z$ و فرض $R \equiv 1$ ، خواهیم داشت:

$$F_R^t = -\frac{z^3}{2ma} B_z \quad , \quad F_\varphi^R = -\frac{z}{z+2m} B_z \quad , \quad F_z^R = \frac{z}{z+2m} B_\varphi \quad , \quad F_R^\varphi = \frac{z}{z+2m} B_z \quad , \quad F_R^z = -\frac{z}{z+2m} B_\varphi$$

برای J در قرص داریم:

$$-\frac{4\pi}{c} J^t = 0 \quad \text{و} \quad -\frac{4\pi}{c} J^R = 0 \quad \text{و} \quad -\frac{4\pi}{c} J^\varphi = 0$$

$$\rightarrow \quad -\frac{4\pi}{c} J^z = \frac{\partial}{\partial R}\left(\frac{R^2}{(R+2m)^2} B_\varphi(R)\right) + \left(\Gamma_{tR}^t + \Gamma_{\varphi R}^\varphi + \Gamma_{Rz}^z + \Gamma_{RR}^R\right)\left(\frac{R^2}{(R+2m)^2} B_\varphi\right)$$

همینطور برای J در جت داریم:

$$\frac{4\pi}{c} J^t = \Gamma_{tR}^t \left(\frac{z^4}{2ma(z+2m)}\right) B_z$$

$$\rightarrow \quad \frac{4\pi}{c} J^R = \frac{\partial}{\partial z}\left(\frac{z^2}{(z+2m)^2} B_\varphi(z)\right) + \left(\Gamma_{\varphi z}^\varphi + \Gamma_{zR}^R + \Gamma_{zz}^z + \Gamma_{tz}^t\right) \frac{z^2}{(z+2m)^2} B_\varphi$$

$$\rightarrow \quad \frac{4\pi}{c} J^\varphi = \Gamma_{tR}^\varphi \left(\frac{z^4}{2ma(z+2m)}\right) B_z + \left(\Gamma_{tR}^t + \Gamma_{RR}^R + \Gamma_{R\varphi}^\varphi + \Gamma_{zR}^z\right)\left(\frac{z^2}{(z+2m)^2}\right) B_z$$

$$-\frac{4\pi}{c} J^z = \left(\Gamma_{tR}^t + \Gamma_{\varphi R}^\varphi + \Gamma_{Rz}^z + \Gamma_{RR}^R\right)\left(\frac{z^2}{(z+2m)^2}\right) B_\varphi$$

$$g^{\mu\nu}{}_{jet} = \begin{pmatrix} \frac{z}{z-2m} & 0 & \frac{z^3}{2ma} & 0 \\ 0 & -\frac{z}{z+2m} & 0 & 0 \\ \frac{z^3}{2ma} & 0 & -\frac{z}{z+2m} & 0 \\ 0 & 0 & 0 & -\frac{z}{z+2m} \end{pmatrix}$$

$$g^{\mu\nu}{}_{disk} = \begin{pmatrix} \frac{R}{R-2m} & 0 & \frac{R}{2ma} & 0 \\ 0 & -\frac{R}{R+2m} & 0 & 0 \\ \frac{R}{2ma} & 0 & -\frac{1}{R^2+2mR} & 0 \\ 0 & 0 & 0 & -\frac{R}{R+2m} \end{pmatrix}$$

جمع‌بندی معادلات قرص:

$$V^R\frac{\partial}{\partial R}(\rho-P)+(\rho+P)\left(\frac{\partial V^R}{\partial R}+\frac{\partial V^\varphi}{\partial\varphi}\right)$$
$$+(\rho+P)\left[\left\{V^R\left(\left[\Gamma^R_{RR}+\Gamma^\varphi_{\varphi R}+\Gamma^z_{zR}\right]-\Gamma^t_{Rt}\right)\right\}-\left\{\Gamma^t_{R\varphi}V^\varphi V^R+\Gamma^0_{\varphi R}V^R V^\varphi\right\}\right]$$
$$+\frac{1}{(u^0)^2}\left[-2\left\{-B_\varphi J^z u^R u^0\right\}\right]=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R+\frac{\partial V^R}{\partial\varphi}V^\varphi+\Gamma^R_{tt}+2V^R\left(-\Gamma^t_{tR}V^R\right)+2V^\varphi\left(\Gamma^R_{t\varphi}\right)+V^R V^R\left(\Gamma^R_{RR}\right)\right.$$
$$\left.+V^R V^\varphi\left(-\Gamma^t_{\varphi R}V^R\right)+V^\varphi V^R\left(-\Gamma^t_{R\varphi}V^R\right)+V^\varphi V^\varphi\left(\Gamma^R_{\varphi\varphi}\right)\right]+\frac{R}{R+2m}\frac{\partial P}{\partial R}$$
$$-\frac{R}{R+2m}\left(B_\varphi J^z\right)=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R+\frac{\partial V^\varphi}{\partial\varphi}V^\varphi+2V^R\left(\Gamma^\varphi_{tR}-\Gamma^t_{tR}V^\varphi\right)+V^R V^\varphi\left(\Gamma^\varphi_{\varphi R}-\Gamma^t_{\varphi R}V^\varphi\right)\right.$$
$$\left.+V^\varphi V^R\left(\Gamma^\varphi_{R\varphi}-\Gamma^t_{R\varphi}V^\varphi\right)+V^z V^R\left(\Gamma^\varphi_{Rz}-\Gamma^t_{Rz}V^\varphi\right)\right]$$
$$-\left[\frac{R}{2ma}-\frac{1}{R^2+2mR}-(u^R+u^\varphi)u^\varphi\right]\left\{2\left(-\Gamma^\varphi_{\varphi R}+\frac{\partial}{\partial R}\right)(-\alpha p)\right\}=0$$

$$J^z=-\frac{1}{4\pi}\left\{\frac{\partial}{\partial R}\left(\frac{R^2}{(R+2m)^2}B_\varphi(R)\right)+\left(\Gamma^t_{tR}+\Gamma^\varphi_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\right)\left(\frac{R^2}{(R+2m)^2}B_\varphi\right)\right\}$$

$$
\begin{aligned}
V^{R}\frac{\partial}{\partial R}(\rho-P)+(\rho+P)\frac{\partial V^{R}}{\partial R}
&+(\rho+P)\Big[\Big\{V^{R}\big(\big[\Gamma^{R}_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{zR}\big]-\Gamma^{t}_{Rt}\big)\Big\}-\big\{\Gamma^{t}_{R\varphi}V^{\varphi}V^{R}+\Gamma^{0}_{\varphi R}V^{R}V^{\varphi}\big\}\Big]\\
&+\frac{1}{(u^{0})^{2}}\bigg[-2\bigg\{-B_{\varphi}\bigg[-\frac{R^{2}}{4\pi(R+2m)^{2}}\frac{\partial B_{\varphi}(R)}{\partial R}\\
&-\frac{1}{4\pi}\bigg\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}}+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\bigg(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\bigg)\bigg\}\bigg]u^{R}u^{0}\bigg\}\bigg]\\
&=0
\end{aligned}
$$

$$
\begin{aligned}
(\rho+P)(u^{0})^{2}\bigg[\frac{\partial V^{R}}{\partial R}V^{R}&+\Gamma^{R}_{tt}+2V^{R}\big(-\Gamma^{t}_{tR}V^{R}\big)+2V^{\varphi}\big(\Gamma^{R}_{t\varphi}\big)+V^{R}V^{R}\big(\Gamma^{R}_{RR}\big)+V^{R}V^{\varphi}\big(-\Gamma^{t}_{\varphi R}V^{R}\big)\\
&+V^{\varphi}V^{R}\big(-\Gamma^{t}_{R\varphi}V^{R}\big)+V^{\varphi}V^{\varphi}\big(\Gamma^{R}_{\varphi\varphi}\big)\bigg]+\frac{R}{R+2m}\frac{\partial P}{\partial R}\\
&-\frac{R}{R+2m}\bigg(B_{\varphi}\bigg[-\frac{R^{2}}{4\pi(R+2m)^{2}}\frac{\partial B_{\varphi}(R)}{\partial R}\\
&-\frac{1}{4\pi}\bigg\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}}+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\bigg(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\bigg)\bigg\}\bigg]\bigg)=0
\end{aligned}
$$

$$
\begin{aligned}
(\rho+P)(u^{0})^{2}\bigg[\frac{\partial V^{\varphi}}{\partial R}V^{R}&+2V^{R}\big(\Gamma^{\varphi}_{tR}-\Gamma^{t}_{tR}V^{\varphi}\big)+V^{R}V^{\varphi}\big(\Gamma^{\varphi}_{\varphi R}-\Gamma^{t}_{\varphi R}V^{\varphi}\big)+V^{\varphi}V^{R}\big(\Gamma^{\varphi}_{R\varphi}-\Gamma^{t}_{R\varphi}V^{\varphi}\big)\\
&+V^{z}V^{R}\big(\Gamma^{\varphi}_{Rz}-\Gamma^{t}_{Rz}V^{\varphi}\big)\bigg]\\
&-\bigg[\frac{R}{2ma}-\frac{1}{R^{2}+2mR}-(u^{R}+u^{\varphi})u^{\varphi}\bigg]\bigg\{2\bigg(\Gamma^{\varphi}_{\varphi R}\alpha p-\alpha\frac{\partial p}{\partial R}\bigg)\bigg\}=0
\end{aligned}
$$

$$
\begin{aligned}
J^{z}=-\frac{R^{2}}{4\pi(R+2m)^{2}}\frac{\partial B_{\varphi}(R)}{\partial R}&\\
&-\frac{1}{4\pi}\bigg\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}}+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\bigg(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\bigg)\bigg\}
\end{aligned}
$$

معادلات نهایی قرص:

$$
\begin{aligned}
-(\rho+P)^{-1}\Big[\!\!\Big[V^{R}\frac{\partial}{\partial R}(\rho-P) &+ (\rho+P)\Big[\Big\{V^{R}\big(\big[\Gamma^{R}_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{zR}\big]-\Gamma^{t}_{Rt}\big)\Big\}-\big\{\Gamma^{t}_{R\varphi}V^{\varphi}V^{R}+\Gamma^{0}_{\varphi R}V^{R}V^{\varphi}\big\}\Big] \\
&+\frac{1}{(u^{0})^{2}}\Big[-2\Big\{-B_{\varphi}\Big|-\frac{R^{2}}{4\pi(R+2m)^{2}}\frac{\partial B_{\varphi}(R)}{\partial R} \\
&-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}} \\
&+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\Big(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\Big)\Big\}\Big|\,u^{R}u^{0}\Big\}\Big]\Big]\!\!\Big] = \frac{\partial V^{R}}{\partial R}
\end{aligned}
$$

$$
\begin{aligned}
(\rho+P)(u^{0})^{2}\Big[\big\langle-(\rho+P)^{-1}\Big[\!\!\Big[V^{R}\frac{\partial}{\partial R}(\rho-P)&+(\rho+P)\big[\big\{V^{R}\big(\big[\Gamma^{R}_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{zR}\big]-\Gamma^{t}_{Rt}\big)\big\}-\big\{\Gamma^{t}_{R\varphi}V^{\varphi}V^{R}+\Gamma^{0}_{\varphi R}V^{R}V^{\varphi}\big\}\big] \\
&+\frac{1}{(u^{0})^{2}}\Big[-2\Big\{-B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}}+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\Big(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\Big)\Big\}\Big|\,u^{R}u^{0}\Big\}\Big]\Big]\!\!\Big]\big\rangle V^{R}+\Gamma^{R}_{tt} \\
&+2V^{R}(-\Gamma^{t}_{tR}V^{R})+2V^{\varphi}\big(\Gamma^{R}_{t\varphi}\big)+V^{R}V^{R}(\Gamma^{R}_{RR})+V^{R}V^{\varphi}\big(-\Gamma^{t}_{\varphi R}V^{R}\big)+V^{\varphi}V^{R}\big(-\Gamma^{t}_{R\varphi}V^{R}\big)+V^{\varphi}V^{\varphi}\big(\Gamma^{R}_{\varphi\varphi}\big)\Big]+\frac{R}{R+2m}\frac{\partial P}{\partial R} \\
&-\frac{R}{R+2m}\Big(B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}}+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\Big(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\Big)\Big\}\Big|\Big) \\
&=-\frac{B_{\varphi}R^{3}}{4\pi(R+2m)^{3}}\frac{\partial B_{\varphi}(R)}{\partial R}-\frac{(\rho+P)(u^{0})^{2}V^{R}(\rho+P)^{-1}2B_{\varphi}R^{2}u^{R}u^{0}}{(u^{0})^{2}4\pi(R+2m)^{2}}\frac{\partial B_{\varphi}(R)}{\partial R} \\
&=-\Big[\frac{B_{\varphi}R^{3}}{4\pi(R+2m)^{3}}+\frac{V^{R}2B_{\varphi}R^{2}}{4\pi(R+2m)^{2}}\Big]\frac{\partial B_{\varphi}(R)}{\partial R}
\end{aligned}
$$

$$
\begin{aligned}
-\Big[\frac{B_{\varphi}R^{3}}{4\pi(R+2m)^{3}}+\frac{V^{R}2B_{\varphi}R^{2}u^{R}u^{0}}{4\pi(R+2m)^{2}}\Big]^{-1}\Big\|(\rho& \\
&+P)(u^{0})^{2}\Big[\big\langle-(\rho+P)^{-1}\Big[\!\!\Big[V^{R}\Big(\frac{\partial\rho}{\partial R}-\frac{\partial P}{\partial R}\Big) \\
&+(\rho+P)\Big[\Big\{V^{R}\big(\big[\Gamma^{R}_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{zR}\big]-\Gamma^{t}_{Rt}\big)\Big\}-\big\{\Gamma^{t}_{R\varphi}V^{\varphi}V^{R}+\Gamma^{0}_{\varphi R}V^{R}V^{\varphi}\big\}\Big] \\
&+\frac{1}{(u^{0})^{2}}\Big[-2\Big\{-B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}} \\
&+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\Big(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\Big)\Big\}\Big|\,u^{R}u^{0}\Big\}\Big]\Big]\!\!\Big]\big\rangle V^{R}+\Gamma^{R}_{tt}+2V^{R}(-\Gamma^{t}_{tR}V^{R}) \\
&+2V^{\varphi}\big(\Gamma^{R}_{t\varphi}\big)+V^{R}V^{R}(\Gamma^{R}_{RR})+V^{R}V^{\varphi}\big(-\Gamma^{t}_{\varphi R}V^{R}\big)+V^{\varphi}V^{R}\big(-\Gamma^{t}_{R\varphi}V^{R}\big)+V^{\varphi}V^{\varphi}\big(\Gamma^{R}_{\varphi\varphi}\big)\Big] \\
&+\frac{R}{R+2m}\frac{\partial P}{\partial R} \\
&-\frac{R}{R+2m}\Big(B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^{2}}{(R+2m)^{2}} \\
&+\big(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\big)\Big(\frac{R^{2}}{(R+2m)^{2}}B_{\varphi}\Big)\Big\}\Big|\Big)\Big\| = \frac{\partial B_{\varphi}(R)}{\partial R}
\end{aligned}
$$

$$
\begin{aligned}
-\frac{1}{V^{R}}\big[2V^{R}\big(\Gamma^{\varphi}_{tR}-\Gamma^{t}_{tR}V^{\varphi}\big)&+V^{R}V^{\varphi}\big(\Gamma^{\varphi}_{\varphi R}-\Gamma^{t}_{\varphi R}V^{\varphi}\big)+V^{\varphi}V^{R}\big(\Gamma^{\varphi}_{R\varphi}-\Gamma^{t}_{R\varphi}V^{\varphi}\big)\big] \\
&+\big(V^{R}(\rho+P)(u^{0})^{2}\big)^{-1}\Big[\frac{R}{2ma}-\frac{1}{R^{2}+2mR}-(u^{R}+u^{\varphi})u^{\varphi}\Big]\Big\{2\Big(\Gamma^{\varphi}_{\varphi R}\alpha p-\alpha\frac{\partial p}{\partial R}\Big)\Big\} \\
&=\frac{\partial V^{\varphi}}{\partial R}
\end{aligned}
$$

با قرار دادن مقادیر گاما برای جت به دست می آوریم:

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]$$
$$+\frac{1}{(u^{0})^{2}}\left[-2F_{jk}J^{k}u^{j}u^{0}+F^{0}_{k}J^{k}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-F^{R}_{k}J^{k}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial t}+\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi})+V^{\varphi}V^{z}(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi})\right.$$
$$\left.+V^{z}V^{\varphi}(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi})\right]+(V^{\varphi}F^{0}_{k}-F^{\varphi}_{k})J^{k}$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-(\;2\Gamma^{\varphi}_{\varphi R}\;+\Gamma^{R}_{zR}\;)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}\right.$$
$$\left.+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+(V^{z}F^{0}_{k}-F^{z}_{k})J^{k}=0$$

$$\frac{4\pi}{c}J^{t}=\Gamma^{t}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}$$

$$\frac{4\pi}{c}J^{R}=\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}(z)\right)+(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz})\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}$$

$$\frac{4\pi}{c}J^{\varphi}=\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}+(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR})\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}$$

$$-\frac{4\pi}{c}J^{z}=(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR})\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}$$

$$g^{\mu\nu}{}_{jet}=\begin{pmatrix}\frac{z}{z-2m} & 0 & \frac{z^{3}}{2ma} & 0\\ 0 & -\frac{z}{z+2m} & 0 & 0\\ \frac{z^{3}}{2ma} & 0 & -\frac{z}{z+2m} & 0\\ 0 & 0 & 0 & -\frac{z}{z+2m}\end{pmatrix}$$

$$F^{t}_{R}=-\frac{z^{3}}{2ma}B_{z}\;\;,\;\;F^{R}_{\varphi}=-\frac{z}{z+2m}B_{z}\;\;,\;\;F^{R}_{z}=\frac{z}{z+2m}B_{\varphi}\;\;,\;\;F^{\varphi}_{R}=\frac{z}{z+2m}B_{z}\;\;,\;\;F^{z}_{R}=-\frac{z}{z+2m}B_{\varphi}$$

بنابراین برای معادلات جت داریم:

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]+\frac{1}{(u^{0})^{2}}\left[-2F_{jk}J^{k}u^{j}u^{0}+F^{t}_{R}J^{R}\right]=0$$

$$F_{jk}J^{k}u^{j}u^{0}=B_{\varphi}J^{R}u^{z}u^{0}-B_{\varphi}J^{z}u^{R}u^{0}+B_{z}J^{\varphi}u^{R}u^{0}-B_{z}J^{R}u^{\varphi}u^{0}=u^{0}J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{0}u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]+\frac{1}{(u^{0})^{2}}\left[-2\left[u^{0}J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{0}u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]+F^{t}_{R}J^{R}\right]=0$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]-\frac{2}{u^{0}}\left[J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]+\frac{F^{t}_{R}J^{R}}{(u^{0})^{2}}=0$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]-\frac{2}{u^{0}}\left[J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]-\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}J^{R}=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}\left(\Gamma^{R}_{t\varphi}\right)+V^{\varphi}V^{\varphi}\left(\Gamma^{R}_{\varphi\varphi}\right)+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-F^{R}_{k}J^{k}=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}\left(\Gamma^{R}_{t\varphi}\right)+V^{\varphi}V^{\varphi}\left(\Gamma^{R}_{\varphi\varphi}\right)+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-\left[F^{R}_{\varphi}J^{\varphi}+F^{R}_{z}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}\left(\Gamma^{R}_{t\varphi}\right)+V^{\varphi}V^{\varphi}\left(\Gamma^{R}_{\varphi\varphi}\right)+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-\left[-\frac{z}{z+2m}B_{z}J^{\varphi}+\frac{z}{z+2m}B_{\varphi}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}\left(\Gamma^{R}_{t\varphi}\right)+V^{\varphi}V^{\varphi}\left(\Gamma^{R}_{\varphi\varphi}\right)+V^{z}V^{z}(\Gamma^{R}_{zz})\right]+\frac{z}{z+2m}\left[B_{z}J^{\varphi}-B_{\varphi}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]+\left(V^{\varphi}F^{0}_{k}-F^{\varphi}_{k}\right)J^{k}-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial\varphi}V^\varphi+\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)\right.$$
$$\left.+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]+\left(V^\varphi F^t_R-F^\varphi_R\right)J^R$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi\right]\left\{-\left(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial\varphi}V^\varphi+\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)\right.$$
$$\left.+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]-\left(V^\varphi\frac{z^3}{2ma}B_z+\frac{z}{z+2m}B_z\right)J^R$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi\right]\left\{-\left(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial\varphi}V^\varphi+\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)\right.$$
$$\left.+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]-\left(V^\varphi\frac{z^3}{2ma}+\frac{z}{z+2m}\right)B_zJ^R$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi\right]\left\{-\left(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(V^zF^0_k-F^z_k\right)J^k=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(-V^z\frac{z^3}{2ma}B_z+\frac{z}{z+2m}B_\varphi\right)J^R=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(\frac{z}{z+2m}B_\varphi-V^z\frac{z^3}{2ma}B_z\right)J^R=0$$

$$g^{\mu v}{}_{jet}=\begin{pmatrix}\frac{z}{z-2m} & 0 & \frac{z^3}{2ma} & 0\\ 0 & -\frac{z}{z+2m} & 0 & 0\\ \frac{z^3}{2ma} & 0 & -\frac{z}{z+2m} & 0\\ 0 & 0 & 0 & -\frac{z}{z+2m}\end{pmatrix}$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]$$
$$-\frac{2}{u^{0}}\left[J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]-\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}J^{R}=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]+\frac{z}{z+2m}\left[B_{z}J^{\varphi}-B_{\varphi}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)\right.$$
$$\left.+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]-\left(V^{\varphi}\frac{z^{3}}{2ma}+\frac{z}{z+2m}\right)B_{z}J^{R}$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(2\Gamma^{\varphi}_{\varphi R}+\Gamma^{R}_{zR}\right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}\right.$$
$$\left.+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(\frac{z}{z+2m}B_{\varphi}-V^{z}\frac{z^{3}}{2ma}B_{z}\right)J^{R}=0$$

$$J^{R}=\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]$$

$$J^{\varphi}=\frac{1}{4\pi}\left[\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}+\left(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}\right]$$

$$J^{z}=\frac{-1}{4\pi}\left(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]$$
$$-[J^{R}]\left\{\frac{2}{u^{0}}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}\right\}-\frac{2}{u^{0}}u^{R}\left(B_{z}[J^{\varphi}]-B_{\varphi}[J^{z}]\right)=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]+\frac{z}{z+2m}\left[B_{z}[J^{\varphi}]-B_{\varphi}[J^{z}]\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)\right.$$
$$\left.+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]-\left(V^{\varphi}\frac{z^{3}}{2ma}+\frac{z}{z+2m}\right)B_{z}[J^{R}]$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(2\Gamma^{\varphi}_{\varphi R}+\Gamma^{R}_{zR}\right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}\right.$$
$$\left.+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(\frac{z}{z+2m}B_{\varphi}-V^{z}\frac{z^{3}}{2ma}B_{z}\right)[J^{R}]=0$$

$$
\begin{aligned}
&V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]\\
&\quad-\left[\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]\right]\left\{\frac{2}{u^{0}}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}\right\}\\
&\quad-\frac{2}{u^{0}}u^{R}\left(B_{z}\left[\frac{1}{4\pi}\left[\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}\right.\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}\right]\right]\\
&\quad\left.-B_{\varphi}\left[\frac{-1}{4\pi}\left(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}\right]\right)=0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]\\
&\quad+\frac{z}{z+2m}\left[B_{z}\left[\frac{1}{4\pi}\left[\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}\right.\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}\right]\right]\\
&\quad\left.-B_{\varphi}\left[\frac{-1}{4\pi}\left(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}\right]\right]=0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)\right.\\
&\quad\left.+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]\\
&\quad-\left[\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]\right]\left(V^{\varphi}\frac{z^{3}}{2ma}+\frac{z}{z+2m}\right)B_{z}\\
&\quad+\left[\frac{z}{z+2m}-\frac{z^{3}}{2ma}+(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)\right\}=0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}\right.\\
&\quad\left.+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}\\
&\quad+\left[\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]\right]\left(\frac{z}{z+2m}B_{\varphi}-V^{z}\frac{z^{3}}{2ma}B_{z}\right)=0
\end{aligned}
$$

$$
\begin{aligned}
V^z\frac{\partial}{\partial z}(\rho-P)+(\rho+P)&\left[-\left\{\Gamma^0_{\varphi z}V^zV^\varphi+\Gamma^0_{z\varphi}V^\varphi V^z\right\}+\frac{\partial V^z}{\partial z}\right]\\
&-\left[\frac{1}{4\pi}\left[B_\varphi(z)\frac{\partial}{\partial z}\left(\frac{z^2}{(z+2m)^2}\right)+\frac{z^2}{(z+2m)^2}\frac{\partial B_\varphi(z)}{\partial z}\right.\right.\\
&\left.\left.+\left(\Gamma^\varphi_{\varphi z}+\Gamma^R_{zR}+\Gamma^z_{zz}+\Gamma^t_{tz}\right)\frac{z^2}{(z+2m)^2}B_\varphi\right]\right]\left\{\frac{2}{u^0}\left(B_\varphi u^z-B_z u^\varphi\right)+\frac{z^3}{2ma(u^0)^2}B_z\right\}\\
&-\frac{2}{u^0}u^R\left(B_z\left[\frac{1}{4\pi}\left[\Gamma^\varphi_{tR}\left(\frac{z^4}{2ma(z+2m)}\right)B_z\right.\right.\right.\\
&\left.\left.+\left(\Gamma^t_{tR}+\Gamma^R_{RR}+\Gamma^\varphi_{R\varphi}+\Gamma^z_{zR}\right)\left(\frac{z^2}{(z+2m)^2}\right)B_z\right]\right]\\
&\left.-B_\varphi\left[\frac{-1}{4\pi}\left(\Gamma^t_{tR}+\Gamma^\varphi_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\right)\left(\frac{z^2}{(z+2m)^2}\right)B_\varphi\right]\right)=0
\end{aligned}
$$

$$
\begin{aligned}
(\rho+P)(u^0)^2&\left[\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]\\
&-\left[\frac{1}{4\pi}\left[B_\varphi(z)\frac{\partial}{\partial z}\left(\frac{z^2}{(z+2m)^2}\right)+\frac{z^2}{(z+2m)^2}\frac{\partial B_\varphi(z)}{\partial z}\right.\right.\\
&\left.\left.+\left(\Gamma^\varphi_{\varphi z}+\Gamma^R_{zR}+\Gamma^z_{zz}+\Gamma^t_{tz}\right)\frac{z^2}{(z+2m)^2}B_\varphi\right]\right]\left(V^\varphi\frac{z^3}{2ma}+\frac{z}{z+2m}\right)B_z\\
&+\left[\frac{z}{z+2m}-\frac{z^3}{2ma}+(u^\varphi+u^z)u^\varphi\right]\left\{-\left(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ \right)(-\alpha p)\right\}=0
\end{aligned}
$$

$$
\begin{aligned}
(\rho+P)(u^0)^2&\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.\\
&\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}\\
&+\left[\frac{1}{4\pi}\left[B_\varphi(z)\frac{\partial}{\partial z}\left(\frac{z^2}{(z+2m)^2}\right)+\frac{z^2}{(z+2m)^2}\frac{\partial B_\varphi(z)}{\partial z}\right.\right.\\
&\left.\left.+\left(\Gamma^\varphi_{\varphi z}+\Gamma^R_{zR}+\Gamma^z_{zz}+\Gamma^t_{tz}\right)\frac{z^2}{(z+2m)^2}B_\varphi\right]\right]\left(\frac{z}{z+2m}B_\varphi-V^z\frac{z^3}{2ma}B_z\right)=0
\end{aligned}
$$

$$
\begin{aligned}
&V^z \frac{\partial}{\partial z}(\rho - P) + (\rho + P)\bigg[-\{\Gamma^0_{\varphi z} V^z V^\varphi + \Gamma^0_{z\varphi} V^\varphi V^z\} \\
&+ \langle -(V^z(\rho + P)(u^0)^2)^{-1} \bigg\{(\rho + P)(u^0)^2[\Gamma^z_{tt} + 2V^\varphi \Gamma^z_{t\varphi} - 2V^z V^z \Gamma^t_{tz} + V^\varphi V^\varphi \Gamma^z_{\varphi\varphi} - V^\varphi V^z V^z \Gamma^t_{z\varphi} - V^z V^\varphi V^z \Gamma^t_{\varphi z} + V^z V^z \Gamma^z_{zz}] \\
&- g^{zz}\frac{\partial P}{\partial z} + \bigg|\frac{1}{4\pi}\bigg[B_\varphi(z)\frac{\partial}{\partial z}\bigg(\frac{z^2}{(z+2m)^2}\bigg) + (\Gamma^\varphi_{\varphi z} + \Gamma^R_{zR} + \Gamma^z_{zz} + \Gamma^t_{tz})\frac{z^2}{(z+2m)^2}B_\varphi\bigg]\bigg|\bigg(\frac{z}{z+2m}B_\varphi - V^z\frac{z^3}{2ma}B_z\bigg)\bigg\}\rangle\bigg] \\
&- \bigg|\frac{1}{4\pi}\bigg[B_\varphi(z)\frac{\partial}{\partial z}\bigg(\frac{z^2}{(z+2m)^2}\bigg) + (\Gamma^\varphi_{\varphi z} + \Gamma^R_{zR} + \Gamma^z_{zz} + \Gamma^t_{tz})\frac{z^2}{(z+2m)^2}B_\varphi\bigg]\bigg|\bigg\{\frac{2}{u^0}(B_\varphi u^z - B_z u^\varphi) + \frac{z^3}{2ma(u^0)^2}B_z\bigg\} \\
&- \frac{2}{u^0}u^R\bigg(B_z\bigg|\frac{1}{4\pi}\bigg[\Gamma^\varphi_{tR}\bigg(\frac{z^4}{2ma(z+2m)}\bigg)B_z + (\Gamma^t_{tR} + \Gamma^R_{RR} + \Gamma^\varphi_{R\varphi} + \Gamma^z_{zR})\bigg(\frac{z^2}{(z+2m)^2}\bigg)B_z\bigg]\bigg| \\
&- B_\varphi\bigg|\frac{-1}{4\pi}(\Gamma^t_{tR} + \Gamma^\varphi_{\varphi R} + \Gamma^z_{Rz} + \Gamma^R_{RR})\bigg(\frac{z^2}{(z+2m)^2}\bigg)B_\varphi\bigg|\bigg) \\
&= (\rho + P)(V^z(\rho + P)(u^0)^2)^{-1}\frac{1}{4\pi}\frac{z^2}{(z+2m)^2}\frac{\partial B_\varphi(z)}{\partial z} + \frac{1}{4\pi}\frac{z^2}{(z+2m)^2}\frac{\partial B_\varphi(z)}{\partial z}\bigg\{\frac{2}{u^0}(B_\varphi u^z - B_z u^\varphi) + \frac{z^3}{2ma(u^0)^2}B_z\bigg\} \\
&= \frac{1}{4\pi}\frac{z^2}{(z+2m)^2}\frac{\partial B_\varphi(z)}{\partial z}\bigg](\rho + P)(V^z(\rho + P)(u^0)^2)^{-1} + \bigg\{\frac{2}{u^0}(B_\varphi u^z - B_z u^\varphi) + \frac{z^3}{2ma(u^0)^2}B_z\bigg\}\bigg[
\end{aligned}
$$

$$
\begin{aligned}
&\bigg\{\frac{1}{4\pi}\frac{z^2}{(z+2m)^2}\bigg((\rho + P)(V^z(\rho + P)(u^0)^2)^{-1} \\
&+ \bigg\{\frac{2}{u^0}(B_\varphi u^z - B_z u^\varphi) + \frac{z^3}{2ma(u^0)^2}B_z\bigg\}\bigg)\bigg\}^{-1}\bigg\{V^z\frac{\partial}{\partial z}(\rho - P) \\
&+ (\rho + P)\bigg[-\{\Gamma^0_{\varphi z} V^z V^\varphi + \Gamma^0_{z\varphi} V^\varphi V^z\} \\
&+ \langle -(V^z(\rho + P)(u^0)^2)^{-1}\bigg\{(\rho + P)(u^0)^2[\Gamma^z_{tt} + 2V^\varphi \Gamma^z_{t\varphi} - 2V^z V^z \Gamma^t_{tz} + V^\varphi V^\varphi \Gamma^z_{\varphi\varphi} \\
&- V^\varphi V^z V^z \Gamma^t_{z\varphi} - V^z V^\varphi V^z \Gamma^t_{\varphi z} + V^z V^z \Gamma^z_{zz}] - g^{zz}\frac{\partial P}{\partial z} \\
&+ \bigg|\frac{1}{4\pi}\bigg[B_\varphi(z)\frac{\partial}{\partial z}\bigg(\frac{z^2}{(z+2m)^2}\bigg) \\
&+ (\Gamma^\varphi_{\varphi z} + \Gamma^R_{zR} + \Gamma^z_{zz} + \Gamma^t_{tz})\frac{z^2}{(z+2m)^2}B_\varphi\bigg]\bigg|\bigg(\frac{z}{z+2m}B_\varphi - V^z\frac{z^3}{2ma}B_z\bigg)\bigg\}\rangle\bigg] \\
&- \bigg|\frac{1}{4\pi}\bigg[B_\varphi(z)\frac{\partial}{\partial z}\frac{z^2}{(z+2m)^2}B_\varphi\bigg]\bigg|\bigg\{\frac{2}{u^0}(B_\varphi u^z - B_z u^\varphi) + \frac{z^3}{2ma(u^0)^2}B_z\bigg\} \\
&- \frac{2}{u^0}u^R\bigg(B_z\bigg|\frac{1}{4\pi}\bigg[\Gamma^\varphi_{tR}\bigg(\frac{z^4}{2ma(z+2m)}\bigg)B_z \\
&+ (\Gamma^t_{tR} + \Gamma^R_{RR} + \Gamma^\varphi_{R\varphi} + \Gamma^z_{zR})\bigg(\frac{z^2}{(z+2m)^2}\bigg)B_z\bigg]\bigg| \\
&- B_\varphi\bigg|\frac{-1}{4\pi}(\Gamma^t_{tR} + \Gamma^\varphi_{\varphi R} + \Gamma^z_{Rz} + \Gamma^R_{RR})\bigg(\frac{z^2}{(z+2m)^2}\bigg)B_\varphi\bigg|\bigg)\bigg\} = \frac{\partial B_\varphi(z)}{\partial z}
\end{aligned}
$$

$$
\begin{aligned}
&(-V^z(\rho + P)(u^0)^2)^{-1}(\rho + P)(u^0)^2[2V^z(\Gamma^\varphi_{tz} - \Gamma^t_{tz}V^\varphi) + V^\varphi V^z(\Gamma^\varphi_{z\varphi} - \Gamma^t_{z\varphi}V^\varphi) \\
&+ V^z V^\varphi(\Gamma^\varphi_{\varphi z} - \Gamma^t_{\varphi z}V^\varphi)] \\
&- \bigg|\frac{1}{4\pi}\bigg[B_\varphi(z)\frac{\partial}{\partial z}\bigg(\frac{z^2}{(z+2m)^2}\bigg) + \frac{z^2}{(z+2m)^2}\frac{\partial B_\varphi(z)}{\partial z} \\
&+ (\Gamma^\varphi_{\varphi z} + \Gamma^R_{zR} + \Gamma^z_{zz} + \Gamma^t_{tz})\frac{z^2}{(z+2m)^2}B_\varphi\bigg]\bigg|\bigg(V^\varphi\frac{z^3}{2ma} + \frac{z}{z+2m}\bigg)B_z \\
&+ \bigg|\frac{z}{z+2m} - \frac{z^3}{2ma} + (u^\varphi + u^z)u^\varphi\bigg|\{-(\ 2\Gamma^\varphi_{\varphi R}\ + \Gamma^R_{zR}\ )(-\alpha p)\} = \frac{\partial V^\varphi}{\partial z}
\end{aligned}
$$

$$
\begin{aligned}
-\left(V^{z}(\rho+P)(u^{0})^{2}\right)^{-1}\Bigg\{&(\rho+P)(u^{0})^{2}\left[\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}\right.\\
&\left.-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}\\
&+\left[\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}\right.\right.\\
&\left.\left.+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]\right]\left(\frac{z}{z+2m}B_{\varphi}-V^{z}\frac{z^{3}}{2ma}B_{z}\right)\Bigg\}=\frac{\partial V^{z}}{\partial z}
\end{aligned}
$$