معادله پایستگی جرم

با توجه به اینکه c=1 داریم:

$$\frac{\partial}{\partial t}(\rho-P)+V^r\frac{\partial}{\partial r}(\rho-P)+V^\varphi\frac{\partial}{\partial\varphi}(\rho-P)+V^z\frac{\partial}{\partial z}(\rho-P)$$
$$+(\rho+P)\Bigg[\{V^r([\Gamma^r_{rr}+\Gamma^\varphi_{\varphi r}+\Gamma^z_{zr}]-\Gamma^0_{r0})\}+\{V^\varphi([\Gamma^r_{r\varphi}+\Gamma^\varphi_{\varphi\varphi}+\Gamma^z_{z\varphi}]-\Gamma^0_{\varphi0})\}$$
$$+\{V^r([\Gamma^r_{rz}+\Gamma^\varphi_{\varphi z}+\Gamma^z_{zz}]-\Gamma^0_{z0})\}$$
$$-\{\Gamma^0_{rr}V^rV^r+\Gamma^0_{r\varphi}V^\varphi V^r+\Gamma^0_{rz}V^zV^r+\Gamma^0_{\varphi r}V^rV^\varphi+\Gamma^0_{\varphi\varphi}V^\varphi V^\varphi+\Gamma^0_{\varphi z}V^zV^\varphi$$
$$+\Gamma^0_{zr}V^rV^z+\Gamma^0_{z\varphi}V^\varphi V^z+\Gamma^0_{zz}V^zV^z\}+\left(\frac{\partial V^r}{\partial r}+\frac{\partial V^\varphi}{\partial\varphi}+\frac{\partial V^z}{\partial z}\right)+(\Gamma^r_{r0}+\Gamma^\varphi_{\varphi0}+\Gamma^z_{z0})\Bigg]$$
$$+\frac{1}{(u^0)^2}[-2F_{jk}J^ku^ju^0+F^0_kJ^k]+\frac{1}{(u^0)^2}\left[\left(1-\frac{2m}{\sqrt{R^2+z^2}}\right)\frac{\partial P}{\partial t}+\frac{2maR^2}{(R^2+z^2)^{3/2}}\frac{\partial P}{\partial\varphi}\right]$$
$$=0$$

با قرار دادن مقادیر گاما برای قرص به دست می آوریم:

$$V^R\frac{\partial}{\partial r}(\rho-P)+(\rho+P)\Bigg[\{V^R([\Gamma^r_{rr}+\Gamma^\varphi_{\varphi r}+\Gamma^z_{zr}]-\Gamma^0_{r0})\}-\{\Gamma^0_{r\varphi}V^\varphi V^R+\Gamma^0_{\varphi r}V^RV^\varphi\}$$
$$+\left(\frac{\partial V^R}{\partial R}+\frac{\partial V^\varphi}{\partial\varphi}\right)\Bigg]+\frac{1}{(u^0)^2}[-2F_{jk}J^ku^ju^0+F^0_kJ^k]=0$$

$$V^R\frac{\partial}{\partial r}(\rho-P)+(\rho+P)\left(\frac{\partial V^R}{\partial R}+\frac{\partial V^\varphi}{\partial\varphi}\right)$$
$$+(\rho+P)[\{V^R([\Gamma^r_{rr}+\Gamma^\varphi_{\varphi r}+\Gamma^z_{zr}]-\Gamma^0_{r0})\}-\{\Gamma^0_{r\varphi}V^\varphi V^R+\Gamma^0_{\varphi r}V^RV^\varphi\}]$$
$$+\frac{1}{(u^0)^2}[-2\{-B_\varphi J^zu^Ru^0\}]=0$$

نهایی

با قرار دادن مقادیر گاما برای جت به دست می آوریم:

$$V^\varphi\frac{\partial}{\partial\varphi}(\rho-P)+V^z\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^0_{\varphi z}V^zV^\varphi+\Gamma^0_{z\varphi}V^\varphi V^z\}+\left(\frac{\partial V^\varphi}{\partial\varphi}+\frac{\partial V^z}{\partial z}\right)\right]$$
$$+\frac{1}{(u^0)^2}[-2F_{jk}J^ku^ju^0+F^0_kJ^k]=0$$

که این رابطه، معادله پایستگی جرم یا معادله پیوستگی ست.

برای معادله پایستگی تکانه داریم:

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\Bigg[cV^\alpha_{,0}+V^\alpha_{,\gamma}V^\gamma+c^2\left(\Gamma^\alpha_{00}-\Gamma^0_{00}\frac{V^\alpha}{c}\right)+2cV^\gamma\left(\Gamma^\alpha_{0\gamma}-\Gamma^0_{0\gamma}\frac{V^\alpha}{c}\right)$$
$$+V^kV^\gamma\left(\Gamma^\alpha_{\gamma k}-\Gamma^0_{\gamma k}\frac{V^\alpha}{c}\right)\Bigg]+\left[\frac{V^\alpha}{c}g^{0j}-g^{\alpha j}\right]\frac{\partial P}{\partial x^j}+\frac{1}{c}\left(\frac{V^\alpha}{c}F^0_k-F^\alpha_k\right)J^k=0$$

با قرار دادن $\alpha = r, \varphi, z$ معادله را در راستای هر یک از مختصات بدست می آوریم:

- برای $\alpha = R$ معادله پایستگی تکانه شعاعی را داریم: (من متاسفانه از r به جای R استفاده کردم که باید تغییر کند. فقط در بالانویس ها مثل V^r)

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[cV^r_{,0}+V^r_{,\gamma}V^\gamma+c^2\left(\Gamma^r_{00}-\Gamma^0_{00}\frac{V^r}{c}\right)+2cV^\gamma\left(\Gamma^r_{0\gamma}-\Gamma^0_{0\gamma}\frac{V^r}{c}\right)+V^kV^\gamma\left(\Gamma^r_{\gamma k}-\Gamma^0_{\gamma k}\frac{V^r}{c}\right)\right]+\left[\frac{V^r}{c}g^{0j}-g^{rj}\right]\frac{\partial P}{\partial x^j}+\frac{1}{c}\left(\frac{V^r}{c}F^0_k-F^r_k\right)J^k=0$$

$$\begin{aligned}\left(\rho+\frac{P}{c^2}\right)(u^0)^2\Bigg[&c\frac{\partial V^r}{\partial t}+\frac{\partial V^r}{\partial r}V^r+\frac{\partial V^r}{\partial\varphi}V^\varphi++\frac{\partial V^r}{\partial z}V^z+c^2\left(\Gamma^r_{00}-\Gamma^0_{00}\frac{V^r}{c}\right)+2cV^r\left(\Gamma^r_{0r}-\Gamma^0_{0r}\frac{V^r}{c}\right)\\&+2cV^\varphi\left(\Gamma^r_{0\varphi}-\Gamma^0_{0\varphi}\frac{V^r}{c}\right)+2cV^z\left(\Gamma^r_{0z}-\Gamma^0_{0z}\frac{V^r}{c}\right)+V^tV^r\left(\Gamma^t_{rt}-\Gamma^0_{rt}\frac{V^r}{c}\right)\\&+V^tV^\varphi\left(\Gamma^t_{\varphi t}-\Gamma^0_{\varphi t}\frac{V^r}{c}\right)+V^tV^z\left(\Gamma^t_{zt}-\Gamma^0_{zt}\frac{V^r}{c}\right)+V^rV^r\left(\Gamma^r_{rr}-\Gamma^0_{rr}\frac{V^r}{c}\right)\\&+V^rV^\varphi\left(\Gamma^r_{\varphi r}-\Gamma^0_{\varphi r}\frac{V^r}{c}\right)+V^rV^z\left(\Gamma^r_{zr}-\Gamma^0_{zr}\frac{V^r}{c}\right)+V^\varphi V^r\left(\Gamma^r_{r\varphi}-\Gamma^0_{r\varphi}\frac{V^r}{c}\right)\\&+V^\varphi V^\varphi\left(\Gamma^r_{\varphi\varphi}-\Gamma^0_{\varphi\varphi}\frac{V^r}{c}\right)+V^\varphi V^z\left(\Gamma^r_{z\varphi}-\Gamma^0_{z\varphi}\frac{V^r}{c}\right)+V^zV^r\left(\Gamma^r_{rz}-\Gamma^0_{rz}\frac{V^r}{c}\right)\\&+V^zV^\varphi\left(\Gamma^r_{\varphi z}-\Gamma^0_{\varphi z}\frac{V^r}{c}\right)+V^zV^z\left(\Gamma^r_{zz}-\Gamma^0_{zz}\frac{V^r}{c}\right)\Bigg]+\left[\frac{V^r}{c}g^{tt}-g^{rt}\right]\frac{\partial P}{\partial t}+\left[\frac{V^r}{c}g^{tr}-g^{rr}\right]\frac{\partial P}{\partial r}\\&+\left[\frac{V^r}{c}g^{02}-g^{r\varphi}\right]\frac{\partial P}{\partial\varphi}+\left[\frac{V^r}{c}g^{03}-g^{rz}\right]\frac{\partial P}{\partial z}+\frac{1}{c}\left(\frac{V^r}{c}F^0_k-F^r_k\right)J^k=0\end{aligned}$$

با قرار دادن مقادیر برای قرص به دست می آوریم:

$$\begin{aligned}(\rho+P)(u^0)^2\Bigg[&\frac{\partial V^R}{\partial R}V^R+\frac{\partial V^R}{\partial\varphi}V^\varphi+\Gamma^R_{tt}+2V^R(-\Gamma^t_{tr}V^R)+2V^\varphi(\Gamma^R_{t\varphi})+V^RV^R(\Gamma^R_{RR})\\&+V^RV^\varphi(-\Gamma^t_{\varphi r}V^R)+V^\varphi V^R(-\Gamma^t_{R\varphi}V^R)+V^\varphi V^\varphi(\Gamma^R_{\varphi\varphi})\Bigg]-g^{RR}\frac{\partial P}{\partial R}+(V^RF^t_k-F^R_k)J^k\\&=0\end{aligned}$$

نهایی

$$\begin{aligned}(\rho+P)(u^0)^2\Bigg[&\frac{\partial V^R}{\partial R}V^R+\frac{\partial V^R}{\partial\varphi}V^\varphi+\Gamma^R_{tt}+2V^R(-\Gamma^t_{tr}V^R)+2V^\varphi(\Gamma^R_{t\varphi})+V^RV^R(\Gamma^R_{RR})\\&+V^RV^\varphi(-\Gamma^t_{\varphi r}V^R)+V^\varphi V^R(-\Gamma^t_{R\varphi}V^R)+V^\varphi V^\varphi(\Gamma^R_{\varphi\varphi})\Bigg]-g^{RR}\frac{\partial P}{\partial R}-(F^R_zJ^z+F^z_RJ^R)\\&=0\end{aligned}$$

با قرار دادن مقادیر برای جت به دست می آوریم:

$$(\rho+P)(u^0)^2\left[(\Gamma^R_{tt})+2V^\varphi(\Gamma^R_{t\varphi})+V^\varphi V^\varphi(\Gamma^R_{\varphi\varphi})+V^zV^z(\Gamma^R_{zz})\right]-F^R_kJ^k=0$$

معادله پایستگی تکانه در راستای R است.

از آنجا که در تانسور وشکسانی، مولفه $t_{R\varphi} = t_{\varphi R}$ حائز اهمیت است، از جملات عبارت $t_{\mu\,;\nu}^{\nu}$ تنها این جملات باقی می‌ماند :

$$-(g^{i\mu} - u^i u^\mu) t_{\mu\,;\nu}^{\nu} = -[g^{t\varphi} + g^{R\varphi} + g^{\varphi\varphi} + g^{z\varphi} - (u^t + u^R + u^\varphi + u^z) u^\varphi] t_{\mu\,;\nu}^{\nu}$$

$$t_{\mu\,;\nu}^{\nu} = t_{\varphi\,,R}^{R} + \Gamma_{RR}^{R}\, t_{\varphi}^{R} - \Gamma_{RR}^{\varphi}\, t_{\varphi}^{R} + \Gamma_{R\varphi}^{R}\, t_{\varphi}^{R} + t_{\varphi\,,R}^{R} - \Gamma_{\varphi R}^{\varphi}\, t_{\varphi}^{R} + \Gamma_{zR}^{\varphi}\, t_{\varphi}^{R} + t_{\varphi\,,R}^{R} + t_{R\,,\varphi}^{\varphi} - \Gamma_{RR}^{R}\, t_{\varphi}^{R} - \Gamma_{\varphi R}^{R}\, t_{\varphi}^{R} - \Gamma_{\varphi R}^{\varphi}\, t_{\varphi}^{R} - \Gamma_{zR}^{R}\, t_{\varphi}^{R} + t_{R\,,\varphi}^{\varphi}$$

$$\xrightarrow{\frac{\partial}{\partial\varphi} t_R^\varphi = 0} t_{\mu\,;\nu}^{\nu} = 2\frac{\partial}{\partial R} t_{\varphi}^{R} - \Gamma_{RR}^{\varphi}\, t_{\varphi}^{R} + \Gamma_{R\varphi}^{R}\, t_{\varphi}^{R} - 2\Gamma_{\varphi R}^{\varphi}\, t_{\varphi}^{R} + \Gamma_{zR}^{\varphi}\, t_{\varphi}^{R} - \Gamma_{\varphi R}^{R}\, t_{\varphi}^{R} - \Gamma_{zR}^{R}\, t_{\varphi}^{R}$$

$$\rightarrow\ t_{\mu\,;\nu}^{\nu} = 2\frac{\partial}{\partial R} t_{\varphi}^{R} + (\Gamma_{R\varphi}^{R} - \Gamma_{RR}^{\varphi} - 2\Gamma_{\varphi R}^{\varphi} + \Gamma_{zR}^{\varphi} - \Gamma_{\varphi R}^{R} - \Gamma_{zR}^{R}) t_{\varphi}^{R}$$

$$for\ disk:\ \ t_{\mu\,;\nu}^{\nu} = 2\frac{\partial}{\partial R} t_{\varphi}^{R} - 2\Gamma_{\varphi R}^{\varphi} t_{\varphi}^{R} = 2\left(-\Gamma_{\varphi R}^{\varphi} + \frac{\partial}{\partial R}\right) t_{\varphi}^{R} = 2\left(-\Gamma_{\varphi R}^{\varphi} + \frac{\partial}{\partial R}\right)(-\alpha p)$$

$$for\ disk:\ \ -(g^{i\mu} - u^i u^\mu) t_{\mu\,;\nu}^{\nu} = -[g^{t\varphi} + g^{\varphi\varphi} - (u^R + u^\varphi) u^\varphi]\left\{2\left(-\Gamma_{\varphi R}^{\varphi} + \frac{\partial}{\partial R}\right)(-\alpha p)\right\}$$

$$for\ jet:\ \ t_{\mu\,;\nu}^{\nu} = 2\frac{\partial}{\partial R} t_{\varphi}^{R} + (-2\Gamma_{\varphi R}^{\varphi} - \Gamma_{zR}^{R}) t_{\varphi}^{R}$$

$$for\ jet\ as\ \frac{\partial}{\partial R} \equiv 0:\ \ t_{\mu\,;\nu}^{\nu} = -(2\Gamma_{\varphi R}^{\varphi} + \Gamma_{zR}^{R})(-\alpha p)$$

$$for\ jet:\ \ -(g^{i\mu} - u^i u^\mu) t_{\mu\,;\nu}^{\nu} = -[g^{t\varphi} + g^{\varphi\varphi} - (u^\varphi + u^z) u^\varphi]\left\{-(2\Gamma_{\varphi R}^{\varphi} + \Gamma_{zR}^{R})(-\alpha p)\right\}$$

حال معادله پایستگی تکانه سمتی را محاسبه می‌نماییم:

$$\left(\rho + \frac{P}{c^2}\right)(u^0)^2 \left[c V_{,0}^{\varphi} + V_{,\gamma}^{\varphi} V^\gamma + c^2\left(\Gamma_{00}^{\varphi} - \Gamma_{00}^{0}\frac{V^\varphi}{c}\right) + 2cV^\gamma\left(\Gamma_{0\gamma}^{\varphi} - \Gamma_{0\gamma}^{0}\frac{V^\varphi}{c}\right) + V^k V^\gamma\left(\Gamma_{\gamma k}^{\varphi} - \Gamma_{\gamma k}^{0}\frac{V^\varphi}{c}\right)\right] + \left[\frac{V^\varphi}{c} g^{0j} - g^{\varphi j}\right]\frac{\partial P}{\partial x^j} + \frac{1}{c}\left(\frac{V^\varphi}{c} F_k^0 - F_k^\varphi (=0)\right) J^k = -(g^{i\mu} - u^i u^\mu) t_{\mu\,;\nu}^{\nu}$$

$$
\begin{aligned}
&\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[c\frac{\partial V^\varphi}{\partial t}+\frac{\partial V^\varphi}{\partial r}V^r+\frac{\partial V^\varphi}{\partial \varphi}V^\varphi++\frac{\partial V^\varphi}{\partial z}V^z+c^2\left(\Gamma^\varphi_{00}-\Gamma^0_{00}\frac{V^\varphi}{c}\right)+2cV^R\left(\Gamma^\varphi_{tr}-\Gamma^t_{tR}\frac{V^\varphi}{c}\right)\right.\\
&\quad+2cV^\varphi\left(\Gamma^\varphi_{t\varphi}-\Gamma^t_{t\varphi}\frac{V^\varphi}{c}\right)+2cV^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}\frac{V^\varphi}{c}\right)+V^tV^R\left(\Gamma^\varphi_{Rt}-\Gamma^t_{Rt}\frac{V^\varphi}{c}\right)\\
&\quad+V^tV^\varphi\left(\Gamma^\varphi_{\varphi t}-\Gamma^t_{\varphi t}\frac{V^\varphi}{c}\right)+V^tV^z\left(\Gamma^\varphi_{zt}-\Gamma^t_{zt}\frac{V^\varphi}{c}\right)+V^RV^R\left(\Gamma^\varphi_{RR}-\Gamma^t_{RR}\frac{V^\varphi}{c}\right)\\
&\quad+V^RV^\varphi\left(\Gamma^\varphi_{\varphi R}-\Gamma^t_{\varphi R}\frac{V^\varphi}{c}\right)+V^RV^z\left(\Gamma^\varphi_{zR}-\Gamma^t_{zR}\frac{V^\varphi}{c}\right)+V^\varphi V^R\left(\Gamma^\varphi_{R\varphi}-\Gamma^t_{R\varphi}\frac{V^\varphi}{c}\right)\\
&\quad+V^\varphi V^\varphi\left(\Gamma^\varphi_{\varphi\varphi}-\Gamma^t_{\varphi\varphi}\frac{V^\varphi}{c}\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}\frac{V^\varphi}{c}\right)+V^zV^R\left(\Gamma^\varphi_{Rz}-\Gamma^t_{Rz}\frac{V^\varphi}{c}\right)\\
&\quad\left.+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}\frac{V^\varphi}{c}\right)+V^zV^z\left(\Gamma^\varphi_{zz}-\Gamma^t_{zz}\frac{V^\varphi}{c}\right)\right]+\left[\frac{V^\varphi}{c}g^{tt}-g^{\varphi t}\right]\frac{\partial P}{\partial t}+\left[\frac{V^\varphi}{c}g^{tR}-g^{\varphi r}\right]\frac{\partial P}{\partial r}\\
&\quad+\left[\frac{V^\varphi}{c}g^{t\varphi}-g^{\varphi\varphi}\right]\frac{\partial P}{\partial \varphi}+\left[\frac{V^\varphi}{c}g^{tz}-g^{\varphi z}\right]\frac{\partial P}{\partial z}+\frac{1}{c}\left(\frac{V^\varphi}{c}F^0_k-F^\varphi_k(=0)\right)J^k\\
&\quad=-[g^{t\varphi}+g^{R\varphi}+g^{\varphi\varphi}+g^{z\varphi}-(u^t+u^R+u^\varphi+u^z)u^\varphi]\left\{2\frac{\partial}{\partial R}t^R_\varphi\right.\\
&\quad\left.+(\Gamma^R_{R\varphi}-\Gamma^\varphi_{RR}\ -2\Gamma^\varphi_{\varphi R}\ +\Gamma^\varphi_{zR}\ -\Gamma^R_{\varphi R}\ -\Gamma^R_{zR}\ )t^R_\varphi\right\}
\end{aligned}
$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial z}=V^t=V^z=0\,,c=1\,,t^R_\varphi=-\alpha p$ و گاما برای قرص به دست می آوریم:

$$
\begin{aligned}
&(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R+\frac{\partial V^\varphi}{\partial \varphi}V^\varphi+2V^R(\Gamma^\varphi_{tr}-\Gamma^t_{tR}V^\varphi)+V^RV^\varphi(\Gamma^\varphi_{\varphi R}-\Gamma^t_{\varphi R}V^\varphi)\right.\\
&\quad\left.+V^\varphi V^R(\Gamma^\varphi_{R\varphi}-\Gamma^t_{R\varphi}V^\varphi)+V^zV^R(\Gamma^\varphi_{Rz}-\Gamma^t_{Rz}V^\varphi)\right]+(V^\varphi F^t_k-F^\varphi_k(=0))J^k\\
&\quad-[g^{t\varphi}+g^{\varphi\varphi}-(u^R+u^\varphi)u^\varphi]\left\{2\left(-\Gamma^\varphi_{\varphi R}+\frac{\partial}{\partial R}\right)(-\alpha p)\right\}=0
\end{aligned}
$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial R}=V^t=V^R=0,c=1\,,t^R_\varphi=-\alpha p$ و گاما برای جت به دست می آوریم:

$$
\begin{aligned}
&(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial t}+\frac{\partial V^\varphi}{\partial \varphi}V^\varphi+\frac{\partial V^\varphi}{\partial z}V^z+2V^z(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi)+V^\varphi V^z(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi)\right.\\
&\quad\left.+V^zV^\varphi(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi)\right]+(V^\varphi F^0_k-F^\varphi_k)J^k\\
&\quad-[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi]\left\{-(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ )(-\alpha p)\right\}=0
\end{aligned}
$$

معادله پایستگی تکانه در راستای φ است.

$$\left(\rho+\frac{P}{c^2}\right)(u^0)^2\left[c\frac{\partial V^z}{\partial t}+\frac{\partial V^z}{\partial R}V^R+\frac{\partial V^z}{\partial \varphi}V^\varphi+\frac{\partial V^z}{\partial z}V^z+c^2\left(\Gamma^z_{tt}-\Gamma^t_{tt}\frac{V^z}{c}\right)+2cV^R\left(\Gamma^z_{tR}-\Gamma^t_{tR}\frac{V^z}{c}\right)\right.$$
$$+2cV^\varphi\left(\Gamma^z_{t\varphi}-\Gamma^t_{t\varphi}\frac{V^z}{c}\right)+2cV^z\left(\Gamma^z_{tz}-\Gamma^t_{tz}\frac{V^z}{c}\right)+V^tV^R\left(\Gamma^z_{Rt}-\Gamma^0_{Rt}\frac{V^z}{c}\right)$$
$$+V^tV^\varphi\left(\Gamma^z_{\varphi t}-\Gamma^t_{\varphi t}\frac{V^z}{c}\right)+V^tV^z\left(\Gamma^z_{zt}-\Gamma^t_{zt}\frac{V^z}{c}\right)+V^RV^R\left(\Gamma^z_{RR}-\Gamma^t_{RR}\frac{V^z}{c}\right)$$
$$+V^RV^\varphi\left(\Gamma^z_{\varphi R}-\Gamma^t_{\varphi R}\frac{V^z}{c}\right)+V^RV^z\left(\Gamma^z_{zR}-\Gamma^t_{zR}\frac{V^z}{c}\right)+V^\varphi V^R\left(\Gamma^z_{R\varphi}-\Gamma^t_{R\varphi}\frac{V^z}{c}\right)$$
$$+V^\varphi V^\varphi\left(\Gamma^z_{\varphi\varphi}-\Gamma^t_{\varphi\varphi}\frac{V^z}{c}\right)+V^\varphi V^z\left(\Gamma^z_{z\varphi}-\Gamma^0_{z\varphi}\frac{V^z}{c}\right)+V^zV^R\left(\Gamma^z_{Rz}-\Gamma^0_{Rz}\frac{V^z}{c}\right)$$
$$\left.+V^zV^\varphi\left(\Gamma^z_{\varphi z}-\Gamma^t_{\varphi z}\frac{V^z}{c}\right)+V^zV^z\left(\Gamma^z_{zz}-\Gamma^0_{zz}\frac{V^z}{c}\right)\right]+\left[\frac{V^z}{c}g^{tt}-g^{zt}\right]\frac{\partial P}{\partial t}$$
$$+\left[\frac{V^z}{c}g^{tR}-g^{zr}\right]\frac{\partial P}{\partial R}+\left[\frac{V^z}{c}g^{t\varphi}-g^{z\varphi}\right]\frac{\partial P}{\partial \varphi}+\left[\frac{V^z}{c}g^{tz}-g^{zz}\right]\frac{\partial P}{\partial z}$$
$$+\frac{1}{c}\left(\frac{V^z}{c}F^0_k-F^z_k\right)J^k=0$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial z}=V^t=V^z=0, c=1$ و گاما برای قرص به دست می آوریم:

$$-F^z_kJ^k=0 \quad , \quad J^k=0$$

با قرار دادن مقادیر $\frac{\partial P}{\partial t}=\frac{\partial P}{\partial \varphi}=\frac{\partial P}{\partial R}=V^t=V^R=0, c=1$ و گاما برای جت به دست می آوریم:

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(V^zF^0_k-F^z_k\right)J^k=0$$

معادله پایستگی تکانه در راستای z است.

حال برای تانسور F داریم:

$$F^t_t=\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_\varphi$$

$$F^t_R=-\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}-2m}E_R-\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_z$$

$$F^t_\varphi=-\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}-2m}E_\varphi$$

$$F^t_z=-\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}-2m}E_z-\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_R$$

$$F_t^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}E_R$$

$$F_R^R = 0$$

$$F_\varphi^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_z$$

$$F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_t^\varphi = -\frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2}+2mR^2}E_\varphi$$

$$F_R^\varphi = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_R + \frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2}+2mR^2}B_z$$

$$F_\varphi^\varphi = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_\varphi$$

$$F_z^\varphi = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}E_z + \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_R$$

$$F_t^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}E_z$$

$$F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_\varphi^z = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_R$$

$$F_z^z = 0$$

حال با فرض اینکه تمامی مولفه های E برابر با صفر است داریم:

$$F_t^t = 0 \quad , \quad F_R^t = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_z \quad , \quad F_\varphi^t = 0 \quad , \quad F_z^t = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_R$$

$$F_t^R = 0 \quad , \quad F_R^R = 0 \quad , \quad F_\varphi^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_z \quad , \quad F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_t^\varphi = 0 \quad , \quad F_R^\varphi = \frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2}+2mR^2}B_z \quad , \quad F_\varphi^\varphi = 0 \quad , \quad F_z^\varphi = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_R$$

$$F_t^z = 0 \ \ , \ \ F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi \ \ , \ \ F_\varphi^z = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_R \ \ , \ \ F_z^z = 0$$

که در قرص با توجه به صفر شدن همه مولفه های میدان مغناطیسی جز B_φ ، خواهیم داشت:

$$F_t^t = 0 \ \ , \ \ F_R^t = 0 \ \ , \ \ F_\varphi^t = 0 \ \ , \ \ F_z^t = -0$$

$$F_t^R = 0 \ \ , \ \ F_R^R = 0 \ \ , \ \ F_\varphi^R = 0 \ \ , \ \ F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_t^\varphi = 0 \ \ , \ \ F_R^\varphi = 0 \ \ , \ \ F_\varphi^\varphi = 0 \ \ , \ \ F_z^\varphi = 0$$

$$F_t^z = 0 \ \ , \ \ F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi \ \ , \ \ F_\varphi^z = 0 \ \ , \ \ F_z^z = 0$$

بنابراین برای تانسور انرژی الکترومغناطیسی تنها دو مولفه داریم:

$$F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi \xRightarrow{z\approx 0} F_z^R = \frac{R}{R+2m}B_\varphi = -F_R^z$$

و در جت با توجه به وجود دو مولفه میدان مغناطیسی B_φ و B_z ، خواهیم داشت:

$$F_t^t = 0 \ \ , \ \ F_R^t = -\frac{(R^2+z^2)^{\frac{3}{2}}}{2maR^2}B_z \ \ , \ \ F_\varphi^t = 0 \ \ , \ \ F_z^t = 0$$

$$F_t^R = 0 \ \ , \ \ F_R^R = 0 \ \ , \ \ F_\varphi^R = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_z \ \ , \ \ F_z^R = \frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi$$

$$F_t^\varphi = 0 \ \ , \ \ F_R^\varphi = \frac{\sqrt{R^2+z^2}}{R^2\sqrt{R^2+z^2}+2mR^2}B_z \ \ , \ \ F_\varphi^\varphi = 0 \ \ , \ \ F_z^\varphi = 0$$

$$F_t^z = 0 \ \ , \ \ F_R^z = -\frac{\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}+2m}B_\varphi \ \ , \ \ F_\varphi^z = 0 \ \ , \ \ F_z^z = 0$$

همچنین در جت با توجه به اینکه $R \ll z$ و فرض $R \equiv 1$ ، خواهیم داشت:

$$F_R^t = -\frac{z^3}{2ma}B_z \quad , \quad F_\varphi^R = -\frac{z}{z+2m}B_z \quad , \quad F_z^R = \frac{z}{z+2m}B_\varphi \quad , \quad F_R^\varphi = \frac{z}{z+2m}B_z \quad , \quad F_R^z = -\frac{z}{z+2m}B_\varphi$$

برای J در قرص داریم:

$$-\frac{4\pi}{c}J^t = 0 \quad \text{و} \quad -\frac{4\pi}{c}J^R = 0 \quad \text{و} \quad -\frac{4\pi}{c}J^\varphi = 0$$

$$\rightarrow \quad -\frac{4\pi}{c}J^z = \frac{\partial}{\partial R}\left(\frac{R^2}{(R+2m)^2}B_\varphi(R)\right) + \left(\Gamma_{tR}^t + \Gamma_{\varphi R}^\varphi + \Gamma_{Rz}^z + \Gamma_{RR}^R\right)\left(\frac{R^2}{(R+2m)^2}B_\varphi\right)$$

همینطور برای J در جت داریم:

$$\frac{4\pi}{c}J^t = \Gamma_{tR}^t\left(\frac{z^4}{2ma(z+2m)}\right)B_z$$

$$\rightarrow \quad \frac{4\pi}{c}J^R = \frac{\partial}{\partial z}\left(\frac{z^2}{(z+2m)^2}B_\varphi(z)\right) + \left(\Gamma_{\varphi z}^\varphi + \Gamma_{zR}^R + \Gamma_{zz}^z + \Gamma_{tz}^t\right)\frac{z^2}{(z+2m)^2}B_\varphi$$

$$\rightarrow \quad \frac{4\pi}{c}J^\varphi = \Gamma_{tR}^\varphi\left(\frac{z^4}{2ma(z+2m)}\right)B_z + \left(\Gamma_{tR}^t + \Gamma_{RR}^R + \Gamma_{R\varphi}^\varphi + \Gamma_{zR}^z\right)\left(\frac{z^2}{(z+2m)^2}\right)B_z$$

$$-\frac{4\pi}{c}J^z = \left(\Gamma_{tR}^t + \Gamma_{\varphi R}^\varphi + \Gamma_{Rz}^z + \Gamma_{RR}^R\right)\left(\frac{z^2}{(z+2m)^2}\right)B_\varphi$$

$$g^{\mu\nu}{}_{jet} = \begin{pmatrix} \frac{z}{z-2m} & 0 & \frac{z^3}{2ma} & 0 \\ 0 & -\frac{z}{z+2m} & 0 & 0 \\ \frac{z^3}{2ma} & 0 & -\frac{z}{z+2m} & 0 \\ 0 & 0 & 0 & -\frac{z}{z+2m} \end{pmatrix}$$

$$g^{\mu\nu}{}_{disk} = \begin{pmatrix} \frac{R}{R-2m} & 0 & \frac{R}{2ma} & 0 \\ 0 & -\frac{R}{R+2m} & 0 & 0 \\ \frac{R}{2ma} & 0 & -\frac{1}{R^2+2mR} & 0 \\ 0 & 0 & 0 & -\frac{R}{R+2m} \end{pmatrix}$$

جمع‌بندی معادلات قرص:

$$V^R\frac{\partial}{\partial R}(\rho-P)+(\rho+P)\left(\frac{\partial V^R}{\partial R}+\frac{\partial V^\varphi}{\partial\varphi}\right)$$
$$+(\rho+P)\left[\left\{V^R\left(\left[\Gamma^R_{RR}+\Gamma^\varphi_{\varphi R}+\Gamma^z_{zR}\right]-\Gamma^t_{Rt}\right)\right\}-\left\{\Gamma^t_{R\varphi}V^\varphi V^R+\Gamma^0_{\varphi R}V^R V^\varphi\right\}\right]$$
$$+\frac{1}{(u^0)^2}\left[-2\left\{-B_\varphi J^z u^R u^0\right\}\right]=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R+\frac{\partial V^R}{\partial\varphi}V^\varphi+\Gamma^R_{tt}+2V^R\left(-\Gamma^t_{tR}V^R\right)+2V^\varphi\left(\Gamma^R_{t\varphi}\right)+V^RV^R\left(\Gamma^R_{RR}\right)\right.$$
$$\left.+V^RV^\varphi\left(-\Gamma^t_{\varphi R}V^R\right)+V^\varphi V^R\left(-\Gamma^t_{R\varphi}V^R\right)+V^\varphi V^\varphi\left(\Gamma^R_{\varphi\varphi}\right)\right]+\frac{R}{R+2m}\frac{\partial P}{\partial R}$$
$$-\frac{R}{R+2m}\left(B_\varphi J^z\right)=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R+\frac{\partial V^\varphi}{\partial\varphi}V^\varphi+2V^R\left(\Gamma^\varphi_{tR}-\Gamma^t_{tR}V^\varphi\right)+V^RV^\varphi\left(\Gamma^\varphi_{\varphi R}-\Gamma^t_{\varphi R}V^\varphi\right)\right.$$
$$\left.+V^\varphi V^R\left(\Gamma^\varphi_{R\varphi}-\Gamma^t_{R\varphi}V^\varphi\right)+V^zV^R\left(\Gamma^\varphi_{Rz}-\Gamma^t_{Rz}V^\varphi\right)\right]$$
$$-\left[\frac{R}{2ma}-\frac{1}{R^2+2mR}-(u^R+u^\varphi)u^\varphi\right]\left\{2\left(-\Gamma^\varphi_{\varphi R}+\frac{\partial}{\partial R}\right)(-\alpha p)\right\}=0$$

$$J^z=-\frac{1}{4\pi}\left\{\frac{\partial}{\partial R}\left(\frac{R^2}{(R+2m)^2}B_\varphi(R)\right)+\left(\Gamma^t_{tR}+\Gamma^\varphi_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\right)\left(\frac{R^2}{(R+2m)^2}B_\varphi\right)\right\}$$

$$
\begin{aligned}
&V^R \frac{\partial}{\partial R}(\rho - P) + (\rho + P)\frac{\partial V^R}{\partial R} \\
&\quad + (\rho + P)\left[\left\{V^R\left(\left[\Gamma_{RR}^R + \Gamma_{\varphi R}^\varphi + \Gamma_{zR}^z\right] - \Gamma_{Rt}^t\right)\right\} - \left\{\Gamma_{R\varphi}^t V^\varphi V^R + \Gamma_{\varphi R}^0 V^R V^\varphi\right\}\right] \\
&\quad + \frac{1}{(u^0)^2}\left[-2\left\{-B_\varphi\left|-\frac{R^2}{4\pi(R+2m)^2}\frac{\partial B_\varphi(R)}{\partial R}\right.\right.\right. \\
&\quad \left.\left.\left. - \frac{1}{4\pi}\left\{B_\varphi(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} + \left(\Gamma_{tR}^t + \Gamma_{\varphi R}^\varphi + \Gamma_{Rz}^z + \Gamma_{RR}^R\right)\left(\frac{R^2}{(R+2m)^2}B_\varphi\right)\right\}\right| u^R u^0\right\}\right] \\
&\quad = 0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho + P)(u^0)^2\left[\frac{\partial V^R}{\partial R}V^R + \Gamma_{tt}^R + 2V^R\left(-\Gamma_{tR}^t V^R\right) + 2V^\varphi\left(\Gamma_{t\varphi}^R\right) + V^R V^R\left(\Gamma_{RR}^R\right) + V^R V^\varphi\left(-\Gamma_{\varphi R}^t V^R\right)\right. \\
&\quad \left. + V^\varphi V^R\left(-\Gamma_{R\varphi}^t V^R\right) + V^\varphi V^\varphi\left(\Gamma_{\varphi\varphi}^R\right)\right] + \frac{R}{R+2m}\frac{\partial P}{\partial R} \\
&\quad - \frac{R}{R+2m}\left(B_\varphi\left|-\frac{R^2}{4\pi(R+2m)^2}\frac{\partial B_\varphi(R)}{\partial R}\right.\right. \\
&\quad \left.\left. - \frac{1}{4\pi}\left\{B_\varphi(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} + \left(\Gamma_{tR}^t + \Gamma_{\varphi R}^\varphi + \Gamma_{Rz}^z + \Gamma_{RR}^R\right)\left(\frac{R^2}{(R+2m)^2}B_\varphi\right)\right\}\right|\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho + P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial R}V^R + 2V^R\left(\Gamma_{tR}^\varphi - \Gamma_{tR}^t V^\varphi\right) + V^R V^\varphi\left(\Gamma_{\varphi R}^\varphi - \Gamma_{\varphi R}^t V^\varphi\right) + V^\varphi V^R\left(\Gamma_{R\varphi}^\varphi - \Gamma_{R\varphi}^t V^\varphi\right)\right. \\
&\quad \left. + V^z V^R\left(\Gamma_{Rz}^\varphi - \Gamma_{Rz}^t V^\varphi\right)\right] \\
&\quad - \left[\frac{R}{2ma} - \frac{1}{R^2 + 2mR} - (u^R + u^\varphi)u^\varphi\right]\left\{2\left(\Gamma_{\varphi R}^\varphi \alpha p - \alpha\frac{\partial p}{\partial R}\right)\right\} = 0
\end{aligned}
$$

$$
\begin{aligned}
J^z = &-\frac{R^2}{4\pi(R+2m)^2}\frac{\partial B_\varphi(R)}{\partial R} \\
&\quad - \frac{1}{4\pi}\left\{B_\varphi(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} + \left(\Gamma_{tR}^t + \Gamma_{\varphi R}^\varphi + \Gamma_{Rz}^z + \Gamma_{RR}^R\right)\left(\frac{R^2}{(R+2m)^2}B_\varphi\right)\right\}
\end{aligned}
$$

معادلات نهایی قرص:

$$
\begin{aligned}
-(\rho+P)^{-1} \Big\llbracket & V^R \frac{\partial}{\partial R}(\rho-P) + (\rho+P)\Big[\Big\{V^R\Big(\big[\Gamma^R_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{zR}\big]-\Gamma^t_{Rt}\Big)\Big\} - \big\{\Gamma^t_{R\varphi}V^{\varphi}V^R+\Gamma^0_{\varphi R}V^R V^{\varphi}\big\}\Big] \\
&+ \frac{1}{(u^0)^2}\Big[-2\Big\{-B_{\varphi}\Big|-\frac{R^2}{4\pi(R+2m)^2}\frac{\partial B_{\varphi}(R)}{\partial R} \\
&- \frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} \\
&+ \big(\Gamma^t_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\big)\Big(\frac{R^2}{(R+2m)^2}B_{\varphi}\Big)\Big\}\Big|\, u^R u^0\Big\}\Big]\Big\rrbracket = \frac{\partial V^R}{\partial R}
\end{aligned}
$$

$$
\begin{aligned}
(\rho+P)(u^0)^2\Big[\Big\langle & -(\rho+P)^{-1}\Big\llbracket V^R\frac{\partial}{\partial R}(\rho-P) + (\rho+P)\big[\big\{V^R\big(\big[\Gamma^R_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{zR}\big]-\Gamma^t_{Rt}\big)\big\} - \big\{\Gamma^t_{R\varphi}V^{\varphi}V^R+\Gamma^0_{\varphi R}V^R V^{\varphi}\big\}\big] \\
&+ \frac{1}{(u^0)^2}\Big[-2\Big\{-B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} + \big(\Gamma^t_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\big)\Big(\frac{R^2}{(R+2m)^2}B_{\varphi}\Big)\Big\}\Big|\, u^R u^0\Big\}\Big]\Big\rrbracket\Big\rangle V^R + \Gamma^R_{tt} \\
&+ 2V^R(-\Gamma^t_{tR}V^R) + 2V^{\varphi}\big(\Gamma^R_{t\varphi}\big) + V^R V^R(\Gamma^R_{RR}) + V^R V^{\varphi}\big(-\Gamma^t_{\varphi R}V^R\big) + V^{\varphi}V^R\big(-\Gamma^t_{R\varphi}V^R\big) + V^{\varphi}V^{\varphi}\big(\Gamma^R_{\varphi\varphi}\big)\Big] + \frac{R}{R+2m}\frac{\partial P}{\partial R} \\
&- \frac{R}{R+2m}\Big(B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} + \big(\Gamma^t_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\big)\Big(\frac{R^2}{(R+2m)^2}B_{\varphi}\Big)\Big\}\Big|\Big) \\
&= -\frac{B_{\varphi}R^3}{4\pi(R+2m)^3}\frac{\partial B_{\varphi}(R)}{\partial R} - \frac{(\rho+P)(u^0)^2 V^R(\rho+P)^{-1}2B_{\varphi}R^2}{(u^0)^2 4\pi(R+2m)^2}\frac{\partial B_{\varphi}(R)}{\partial R} \\
&= -\Big[\frac{B_{\varphi}R^3}{4\pi(R+2m)^3} + \frac{V^R 2B_{\varphi}R^2}{4\pi(R+2m)^2}\Big]\frac{\partial B_{\varphi}(R)}{\partial R}
\end{aligned}
$$

$$
\begin{aligned}
-\Big[\frac{B_{\varphi}R^3}{4\pi(R+2m)^3} + \frac{V^R 2B_{\varphi}R^2}{4\pi(R+2m)^2}\Big]^{-1}\Big\| & (\rho \\
&+ P)(u^0)^2\Big[\Big\langle -(\rho+P)^{-1}\Big\llbracket V^R\frac{\partial}{\partial R}(\rho-P) \\
&+ (\rho+P)\Big[\Big\{V^R\Big(\big[\Gamma^R_{RR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{zR}\big]-\Gamma^t_{Rt}\Big)\Big\} - \big\{\Gamma^t_{R\varphi}V^{\varphi}V^R+\Gamma^0_{\varphi R}V^R V^{\varphi}\big\}\Big] \\
&+ \frac{1}{(u^0)^2}\Big[-2\Big\{-B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} \\
&+ \big(\Gamma^t_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\big)\Big(\frac{R^2}{(R+2m)^2}B_{\varphi}\Big)\Big\}\Big|\, u^R u^0\Big\}\Big]\Big\rrbracket\Big\rangle V^R + \Gamma^R_{tt} + 2V^R(-\Gamma^t_{tR}V^R) \\
&+ 2V^{\varphi}\big(\Gamma^R_{t\varphi}\big) + V^R V^R(\Gamma^R_{RR}) + V^R V^{\varphi}\big(-\Gamma^t_{\varphi R}V^R\big) + V^{\varphi}V^R\big(-\Gamma^t_{R\varphi}V^R\big) + V^{\varphi}V^{\varphi}\big(\Gamma^R_{\varphi\varphi}\big)\Big] \\
&+ \frac{R}{R+2m}\frac{\partial P}{\partial R} \\
&- \frac{R}{R+2m}\Big(B_{\varphi}\Big|-\frac{1}{4\pi}\Big\{B_{\varphi}(R)\frac{\partial}{\partial R}\frac{R^2}{(R+2m)^2} \\
&+ \big(\Gamma^t_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^z_{Rz}+\Gamma^R_{RR}\big)\Big(\frac{R^2}{(R+2m)^2}B_{\varphi}\Big)\Big\}\Big|\Big)\Big\| = \frac{\partial B_{\varphi}(R)}{\partial R}
\end{aligned}
$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^{\varphi}}{\partial R}V^R+2V^R\left(\Gamma^{\varphi}_{tR}-\Gamma^{t}_{tR}V^{\varphi}\right)+V^RV^{\varphi}\left(\Gamma^{\varphi}_{\varphi R}-\Gamma^{t}_{\varphi R}V^{\varphi}\right)+V^{\varphi}V^R\left(\Gamma^{\varphi}_{R\varphi}-\Gamma^{t}_{R\varphi}V^{\varphi}\right)\right.$$
$$\left.+V^zV^R\left(\Gamma^{\varphi}_{Rz}-\Gamma^{t}_{Rz}V^{\varphi}\right)\right]$$
$$-\left[\frac{R}{2ma}-\frac{1}{R^2+2mR}-(u^R+u^{\varphi})u^{\varphi}\right]\left\{2\left(\Gamma^{\varphi}_{\varphi R}\alpha p-\alpha\frac{\partial p}{\partial R}\right)\right\}=0$$

با قرار دادن مقادیر گاما برای جت به دست می آوریم:

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]+\frac{1}{(u^{0})^{2}}\left[-2F_{jk}J^{k}u^{j}u^{0}+F^{0}_{k}J^{k}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-F^{R}_{k}J^{k}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial t}+\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi})+V^{\varphi}V^{z}(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi})+V^{z}V^{\varphi}(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi})\right]+(V^{\varphi}F^{0}_{k}-F^{\varphi}_{k})J^{k}-[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}]\left\{-(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ )(-\alpha p)\right\}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+(V^{z}F^{0}_{k}-F^{z}_{k})J^{k}=0$$

$$\frac{4\pi}{c}J^{t}=\Gamma^{t}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}$$

$$\frac{4\pi}{c}J^{R}=\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}(z)\right)+(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz})\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}$$

$$\frac{4\pi}{c}J^{\varphi}=\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}+(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR})\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}$$

$$-\frac{4\pi}{c}J^{z}=(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR})\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}$$

$$g^{\mu\nu}{}_{jet}=\begin{pmatrix}\frac{z}{z-2m} & 0 & \frac{z^{3}}{2ma} & 0\\ 0 & -\frac{z}{z+2m} & 0 & 0\\ \frac{z^{3}}{2ma} & 0 & -\frac{z}{z+2m} & 0\\ 0 & 0 & 0 & -\frac{z}{z+2m}\end{pmatrix}$$

$$F^{t}_{R}=-\frac{z^{3}}{2ma}B_{z}\ \ ,\ \ F^{R}_{\varphi}=-\frac{z}{z+2m}B_{z}\ \ ,\ \ F^{R}_{z}=\frac{z}{z+2m}B_{\varphi}\ \ ,\ \ F^{\varphi}_{R}=\frac{z}{z+2m}B_{z}\ \ ,\ \ F^{z}_{R}=-\frac{z}{z+2m}B_{\varphi}$$

بنابراین برای معادلات جت داریم:

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]+\frac{1}{(u^{0})^{2}}\left[-2F_{jk}J^{k}u^{j}u^{0}+F^{t}_{R}J^{R}\right]=0$$

$$F_{jk}J^{k}u^{j}u^{0}=B_{\varphi}J^{R}u^{z}u^{0}-B_{\varphi}J^{z}u^{R}u^{0}+B_{z}J^{\varphi}u^{R}u^{0}-B_{z}J^{R}u^{\varphi}u^{0}$$
$$=u^{0}J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{0}u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]+\frac{1}{(u^{0})^{2}}\left[-2\left[u^{0}J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{0}u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]+F^{t}_{R}J^{R}\right]=0$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]-\frac{2}{u^{0}}\left[J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]+\frac{F^{t}_{R}J^{R}}{(u^{0})^{2}}=0$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]-\frac{2}{u^{0}}\left[J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]-\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}J^{R}=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-F^{R}_{k}J^{k}=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}\left(\Gamma^{R}_{t\varphi}\right)+V^{\varphi}V^{\varphi}\left(\Gamma^{R}_{\varphi\varphi}\right)+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-\left[F^{R}_{\varphi}J^{\varphi}+F^{R}_{z}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}\left(\Gamma^{R}_{t\varphi}\right)+V^{\varphi}V^{\varphi}\left(\Gamma^{R}_{\varphi\varphi}\right)+V^{z}V^{z}(\Gamma^{R}_{zz})\right]-\left[-\frac{z}{z+2m}B_{z}J^{\varphi}+\frac{z}{z+2m}B_{\varphi}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}\left(\Gamma^{R}_{t\varphi}\right)+V^{\varphi}V^{\varphi}\left(\Gamma^{R}_{\varphi\varphi}\right)+V^{z}V^{z}(\Gamma^{R}_{zz})\right]+\frac{z}{z+2m}\left[B_{z}J^{\varphi}-B_{\varphi}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]+\left(V^{\varphi}F^{0}_{k}-F^{\varphi}_{k}\right)J^{k}-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]+\left(V^{\varphi}F^{t}_{R}-F^{\varphi}_{R}\right)J^{R}-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial\varphi}V^\varphi+\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)\right.$$
$$\left.+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]-\left(V^\varphi\frac{z^3}{2ma}B_z+\frac{z}{z+2m}B_z\right)J^R$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi\right]\left\{-\left(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^\varphi}{\partial\varphi}V^\varphi+\frac{\partial V^\varphi}{\partial z}V^z+2V^z\left(\Gamma^\varphi_{tz}-\Gamma^t_{tz}V^\varphi\right)+V^\varphi V^z\left(\Gamma^\varphi_{z\varphi}-\Gamma^t_{z\varphi}V^\varphi\right)\right.$$
$$\left.+V^zV^\varphi\left(\Gamma^\varphi_{\varphi z}-\Gamma^t_{\varphi z}V^\varphi\right)\right]-\left(V^\varphi\frac{z^3}{2ma}+\frac{z}{z+2m}\right)B_zJ^R$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^\varphi+u^z)u^\varphi\right]\left\{-\left(\ 2\Gamma^\varphi_{\varphi R}\ +\Gamma^R_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(V^zF^0_k-F^z_k\right)J^k=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(-V^z\frac{z^3}{2ma}B_z+\frac{z}{z+2m}B_\varphi\right)J^R=0$$

$$(\rho+P)(u^0)^2\left[\frac{\partial V^z}{\partial z}V^z+\Gamma^z_{tt}+2V^\varphi\Gamma^z_{t\varphi}-2V^zV^z\Gamma^t_{tz}+V^\varphi V^\varphi\Gamma^z_{\varphi\varphi}-V^\varphi V^zV^z\Gamma^t_{z\varphi}-V^zV^\varphi V^z\Gamma^t_{\varphi z}\right.$$
$$\left.+V^zV^z\Gamma^z_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(\frac{z}{z+2m}B_\varphi-V^z\frac{z^3}{2ma}B_z\right)J^R=0$$

$$g^{\mu v}{}_{jet}=\begin{pmatrix}\frac{z}{z-2m} & 0 & \frac{z^3}{2ma} & 0\\ 0 & -\frac{z}{z+2m} & 0 & 0\\ \frac{z^3}{2ma} & 0 & -\frac{z}{z+2m} & 0\\ 0 & 0 & 0 & -\frac{z}{z+2m}\end{pmatrix}$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]$$
$$-\frac{2}{u^{0}}\left[J^{R}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+u^{R}\left(B_{z}J^{\varphi}-B_{\varphi}J^{z}\right)\right]-\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}J^{R}=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]+\frac{z}{z+2m}\left[B_{z}J^{\varphi}-B_{\varphi}J^{z}\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)\right.$$
$$\left.+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]-\left(V^{\varphi}\frac{z^{3}}{2ma}+\frac{z}{z+2m}\right)B_{z}J^{R}$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}\right.$$
$$\left.+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(\frac{z}{z+2m}B_{\varphi}-V^{z}\frac{z^{3}}{2ma}B_{z}\right)J^{R}=0$$

$$J^{R}=\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]$$

$$J^{\varphi}=\frac{1}{4\pi}\left[\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}+\left(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}\right]$$

$$J^{z}=\frac{-1}{4\pi}\left(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}$$

$$V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]$$
$$-[J^{R}]\left\{\frac{2}{u^{0}}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}\right\}-\frac{2}{u^{0}}u^{R}\left(B_{z}[J^{\varphi}]-B_{\varphi}[J^{z}]\right)=0$$

$$(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]+\frac{z}{z+2m}\left[B_{z}[J^{\varphi}]-B_{\varphi}[J^{z}]\right]=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)\right.$$
$$\left.+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]-\left(V^{\varphi}\frac{z^{3}}{2ma}+\frac{z}{z+2m}\right)B_{z}[J^{R}]$$
$$-\left[g^{t\varphi}+g^{\varphi\varphi}-(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(\ 2\Gamma^{\varphi}_{\varphi R}\ +\Gamma^{R}_{zR}\ \right)(-\alpha p)\right\}=0$$

$$(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}\right.$$
$$\left.+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}+\left(\frac{z}{z+2m}B_{\varphi}-V^{z}\frac{z^{3}}{2ma}B_{z}\right)[J^{R}]=0$$

$$
\begin{aligned}
&V^{\varphi}\frac{\partial}{\partial\varphi}(\rho-P)+V^{z}\frac{\partial}{\partial z}(\rho-P)+(\rho+P)\left[-\{\Gamma^{0}_{\varphi z}V^{z}V^{\varphi}+\Gamma^{0}_{z\varphi}V^{\varphi}V^{z}\}+\left(\frac{\partial V^{\varphi}}{\partial\varphi}+\frac{\partial V^{z}}{\partial z}\right)\right]\\
&\quad-\left[\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]\right]\left\{\frac{2}{u^{0}}\left(B_{\varphi}u^{z}-B_{z}u^{\varphi}\right)+\frac{z^{3}}{2ma(u^{0})^{2}}B_{z}\right\}\\
&\quad-\frac{2}{u^{0}}u^{R}\left(B_{z}\left[\frac{1}{4\pi}\left[\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}\right.\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}\right]\right]\\
&\quad\left.-B_{\varphi}\left[\frac{-1}{4\pi}\left(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}\right]\right)=0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho+P)(u^{0})^{2}\left[(\Gamma^{R}_{tt})+2V^{\varphi}(\Gamma^{R}_{t\varphi})+V^{\varphi}V^{\varphi}(\Gamma^{R}_{\varphi\varphi})+V^{z}V^{z}(\Gamma^{R}_{zz})\right]\\
&\quad+\frac{z}{z+2m}\left[B_{z}\left[\frac{1}{4\pi}\left[\Gamma^{\varphi}_{tR}\left(\frac{z^{4}}{2ma(z+2m)}\right)B_{z}\right.\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{t}_{tR}+\Gamma^{R}_{RR}+\Gamma^{\varphi}_{R\varphi}+\Gamma^{z}_{zR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{z}\right]\right]\\
&\quad\left.-B_{\varphi}\left[\frac{-1}{4\pi}\left(\Gamma^{t}_{tR}+\Gamma^{\varphi}_{\varphi R}+\Gamma^{z}_{Rz}+\Gamma^{R}_{RR}\right)\left(\frac{z^{2}}{(z+2m)^{2}}\right)B_{\varphi}\right]\right]=0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{\varphi}}{\partial\varphi}V^{\varphi}+\frac{\partial V^{\varphi}}{\partial z}V^{z}+2V^{z}\left(\Gamma^{\varphi}_{tz}-\Gamma^{t}_{tz}V^{\varphi}\right)+V^{\varphi}V^{z}\left(\Gamma^{\varphi}_{z\varphi}-\Gamma^{t}_{z\varphi}V^{\varphi}\right)\right.\\
&\quad\left.+V^{z}V^{\varphi}\left(\Gamma^{\varphi}_{\varphi z}-\Gamma^{t}_{\varphi z}V^{\varphi}\right)\right]\\
&\quad-\left[\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]\right]\left(V^{\varphi}\frac{z^{3}}{2ma}+\frac{z}{z+2m}\right)B_{z}\\
&\quad+\left[\frac{z}{z+2m}-\frac{z^{3}}{2ma}+(u^{\varphi}+u^{z})u^{\varphi}\right]\left\{-\left(\,2\Gamma^{\varphi}_{\varphi R}\,+\Gamma^{R}_{zR}\,\right)(-\alpha p)\right\}=0
\end{aligned}
$$

$$
\begin{aligned}
&(\rho+P)(u^{0})^{2}\left[\frac{\partial V^{z}}{\partial z}V^{z}+\Gamma^{z}_{tt}+2V^{\varphi}\Gamma^{z}_{t\varphi}-2V^{z}V^{z}\Gamma^{t}_{tz}+V^{\varphi}V^{\varphi}\Gamma^{z}_{\varphi\varphi}-V^{\varphi}V^{z}V^{z}\Gamma^{t}_{z\varphi}-V^{z}V^{\varphi}V^{z}\Gamma^{t}_{\varphi z}\right.\\
&\quad\left.+V^{z}V^{z}\Gamma^{z}_{zz}\right]-g^{zz}\frac{\partial P}{\partial z}\\
&\quad+\left[\frac{1}{4\pi}\left[B_{\varphi}(z)\frac{\partial}{\partial z}\left(\frac{z^{2}}{(z+2m)^{2}}\right)+\frac{z^{2}}{(z+2m)^{2}}\frac{\partial B_{\varphi}(z)}{\partial z}\right.\right.\\
&\quad\left.\left.+\left(\Gamma^{\varphi}_{\varphi z}+\Gamma^{R}_{zR}+\Gamma^{z}_{zz}+\Gamma^{t}_{tz}\right)\frac{z^{2}}{(z+2m)^{2}}B_{\varphi}\right]\right]\left(\frac{z}{z+2m}B_{\varphi}-V^{z}\frac{z^{3}}{2ma}B_{z}\right)=0
\end{aligned}
$$