معادلات نهایی قرص و جت نیازمند حل عددی

مقادیر ثابت و متغیرها:

$$jet:\ [z]\ \rightarrow V^z:(V4),\qquad V^\varphi:(V3),\qquad B_z:(B4),\qquad B_\varphi:(B3)$$

$$disk:\ [r]\ \rightarrow V^r:(V2),\qquad V^\varphi:(V3),\qquad B_\varphi:(B3)$$

$$m=10^9 M_\odot\ ,\qquad for\ a:\ 0<a<1\ ,\qquad for\ K: as\ it\ emits\ energy \rightarrow K<0 \rightarrow K\approx -0.1$$

$$r_{\text{Schwarzschild}}=\frac{2GM}{c^2}\xrightarrow{\frac{2G}{c^2}=1.48\times10^{-27}\ m/kg} r_{\text{Sch}}=1.48\times10^{-27}\times10^9 M_\odot=1.48\times10^{-27}\times10^9\times2\times10^{30}\approx3\times10^{12}(m)$$

$$r_{\text{Sch}}\approx3\times10^{12}(m)\approx2\times10^4(AU)\approx10^{-4}(pc)$$

توقع داریم ابعاد یک سیاه چاله با جرمی در حدود $10^9 M_\odot$ تقریبا در حدود $10(AU)$ باشد که شعاع شوارتزشیلد آن تقریبا $10^4(AU)$ به دست می آید.

مقادیر اولیه:

$$jet:\ z_{init}=500AU\ \rightarrow V^z(z_{init})=0.975\ c,\qquad V^\varphi(z_{init})=0.222\ c,\qquad B_z(z_{init})=10^4\ G,\qquad B_\varphi(z_{init})=10^3\ G$$

$$disk:\ r_{init}=500AU\approx7\times10^{10}(m)\ \rightarrow V^r(r_{init})=0.12\ c,\qquad V^\varphi(r_{init})=0.918\ c,\qquad B_\varphi(r_{init})=10^4\ G$$

$$V^t:(V1)=0\quad but:\ u^0=\gamma$$

ارتباط کمیت ها:

$$u^\alpha=V^\alpha\left(\frac{u^0}{c}\right)$$

$$u^0=\frac{d(ct)}{d\tau}=\frac{d(c\gamma\tau)}{d\tau}=\gamma c$$

$$u1=\gamma V1\quad ,\quad u2=\gamma V2\quad ,\quad u3=\gamma V3$$

$$\gamma=\frac{1}{\sqrt{1-\frac{(V)^2}{(c)^2}}}\xrightarrow{c^2=1}\gamma=\frac{1}{\sqrt{1-(V)^2}}$$

$$jet: V^z:(V4), V^\varphi:(V3)\ \rightarrow\gamma=\frac{1}{\sqrt{1-(V)^2}}=\gamma=\frac{1}{\sqrt{1-((V4)^2+(V3)^2)}}$$

$$jet:\ u^0=\gamma,\qquad u^2=\gamma(V2)=0\quad ,\quad u^3=\gamma(V3)\quad ,\quad u^4=\gamma(V4)$$

$$disk:\ V^r:(V2), V^\varphi:(V3)\ \rightarrow\gamma=\frac{1}{\sqrt{1-(V)^2}}=\gamma=\frac{1}{\sqrt{1-((V2)^2+(V3)^2)}}$$

$$disk:\ u^0=\gamma,\qquad u^2=\gamma(V2)\quad ,\quad u^3=\gamma(V3)\quad ,\quad u^4=\gamma(V4)=0,\qquad (V3)\gg(V2)$$

$$\rho(R,z)=(R^2+z^2)^{3/4}$$

در این صورت برای **قرص** داریم $z=0$ و در نتیجه

$$\rho(R,z=0)=(R^2)^{3/4}$$

$$P=K\rho^\gamma\ ,\qquad \gamma=\frac{5}{3}\ ,\qquad \rho=(R^2)^{3/4}=R^{3/2}$$

$$P=K\rho^\gamma=K\rho^{\frac{5}{3}}=K[(R^2)^{3/4}]^{\frac{5}{3}}=KR^{5/2}$$

در این صورت برای **جت** داریم $z\gg R$ و در نتیجه

$$\rho(R,z)=(R^2+z^2)^{3/4}\cong(z^2)^{3/4}$$

$$P=K\rho^\gamma\ ,\qquad \gamma=\frac{5}{3}\ ,\qquad \rho=(z^2)^{3/4}=z^{3/2}$$

$$P=K\rho^\gamma=K\rho^{\frac{5}{3}}=K[(z^2)^{3/4}]^{\frac{5}{3}}=Kz^{5/2}$$

برای قرص خواهیم داشت:

پایستگی جرم $(V^z = 0)$:

$$(V2)D\left(R^{\frac{3}{2}} - KR^{\frac{5}{2}}, R\right) + \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)D(V2[R], R)$$
$$= -\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)\Big(V2(g222 + g332 + g442 - g121) + V2(g224 + g334 + g444 - g141)$$
$$- \big(g123(V3)(V2) + g132(V2)(V3)\big)\Big) + \frac{1}{u02}(2(B3)(J4)(u2)(u0))$$

پایستگی تکانه شعاعی:

$$\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u02)D(V2[R], R)(V2) + \left(1 + \frac{2m}{R}\right)D\left(KR^{\frac{5}{2}}, R\right)$$
$$= -\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u02)\Big((g211) - 2(g112)(V2)(V2) + 2(V3)(g213)$$
$$+ (V1)(V2)\big((g121) - (g121)(V2)\big) + (V2)(V2)(g222) - (g132)(V2)(V2)(V3)$$
$$- (g123)(V2)(V3)(V2) + (V3)(V3)(g233)\Big) - (B3)(J4)$$

پایستگی تکانه زاویه ای:

$$\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u02)D(V3[R], R)(V2)$$
$$= -\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)(u02)\Big(2(V2)\big((g312) - (g112)(V3)\big) + (V1)(V2)\big((g321) - (g121)(V3)\big)$$
$$+ (V2)(V3)\big((g332) - (g132)(V3)\big) + (V3)(V2)\big((g323) - (g123)(V3)\big)\Big)$$

پایستگی تکانه ارتفاعی:

$$as\ J^R = 0\ ,\ \frac{\partial}{\partial z} = 0\ \ ,\ \ V^z = 0\ \ \rightarrow\ \ 0 = 0$$

$$g112 = \Gamma(ttr) = \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}$$

$$g121 = \Gamma(trt) = \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)}$$

$$g123 = \Gamma(tr\varphi) = -\frac{maR(3R + 4m)}{R^4 - 4R^2m^2 + 4a^2m^2}$$

$$g132 = \Gamma(t\varphi r) = -\frac{maR(3R + 4m)}{R^4 - 4R^2m^2 + 4a^2m^2}$$

$$g211 = \Gamma(rtt) = \frac{m}{R(R^2 + 2m)}$$

$$g213 = \Gamma(rt\varphi) = -\frac{ma}{R(R + 2m)}$$

$$g222 = \Gamma(rrr) = -\frac{m}{R(R+2m)}$$

$$g231 = \Gamma(r\varphi t) = -\frac{ma}{R(R+2m)}$$

$$g233 = \Gamma(r\varphi\varphi) = -\frac{R(R+m)}{(R+2m)}$$

$$g244 = \Gamma(rzz) = \frac{m}{R(R+2m)}$$

$$g312 = \Gamma(\varphi tr) = \frac{am}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$

$$g321 = \Gamma(\varphi rt) = \frac{am}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$

$$g323 = \Gamma(\varphi r\varphi) = -\frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$

$$g332 = \Gamma(\varphi\varphi r) = -\frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)}$$

$$g424 = \Gamma(zrz) = -\frac{m}{R(R+2m)}$$

$$g442 = \Gamma(zzr) = -\frac{m}{R(R+2m)}$$

همینطور برای J در قرص داریم:

$$J4 = -\frac{c}{4\pi}\left(D\left(\left(\frac{R^2}{R^2+2mR+4m^2}\right)B3[R],R\right) + B3\left(\frac{mR(R^5+2R^4m-2R^2a^2m)}{R^2(R^6-4R^4m^2+4R^2a^2m^2)} - \frac{m}{R(R+2m)} - \frac{m}{R(R+2m)}\frac{R^3m-R^4+2R^2m^2+2a^2m^2}{(R^4-4R^2m^2+4a^2m^2)}\right)\left(\frac{R^2}{R^2+2mR+4m^2}\right)\right)$$

سپس برای جت خواهیم داشت:

پایستگی جرم:

$$(V4)D\left(z^{\frac{3}{2}}-Kz^{\frac{5}{2}},z\right)+\left(z^{\frac{3}{2}}+Kz^{\frac{5}{2}}\right)D(V4[z],z)$$
$$=\left(z^{\frac{3}{2}}+Kz^{\frac{5}{2}}\right)\left((g134)(V4)(V3)+(g143)(V3)(V4)\right)-\frac{1}{(u02)}2(B4)(J2)(u3)(u1)$$

با فرض $V^{\varphi}=0$ و در نتیجه $u^{\varphi}=0$ داریم:

$$(V4)D\left(z^{3/2}-Kz^{\frac{5}{2}},z\right)+\left(z^{3/2}+Kz^{5/2}\right)D(V4[z],z)=0$$

پایستگی تکانه شعاعی:

$$\left(1+\frac{2m}{z}\right)D(Kz^{\frac{5}{2}},R)$$
$$=-\left(z^{\frac{3}{2}}+Kz^{\frac{5}{2}}\right)(u02)\left((g211)+2(V3)(g213)+(V1)(V4)(g141)+(V3)(V3)(g233)\right.$$
$$\left.+(V4)(V4)(g244)\right)+(B4)(J3)$$

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض $V^{\varphi}=0$ است. در این صورت داریم:

$$(B4)(J3)-\left(z^{\frac{3}{2}}+Kz^{\frac{5}{2}}\right)(u02)\left((g211)+(V1)(V4)(g141)+(V4)(V4)(g244)\right)=\left(1+\frac{2m}{z}\right)D(Kz^{\frac{5}{2}},R)$$
$$=0\qquad,\qquad az\,\frac{\partial z}{\partial R}=0$$

پایستگی تکانه زاویه ای:

$$\left(z^{3/2}+Kz^{5/2}\right)(u02)D(V3[z],z)(V4)$$
$$=-\left(z^{\frac{3}{2}}+Kz^{\frac{5}{2}}\right)(u02)\left(2(V4)\left((g314)-(g114)(V3)\right)+(V1)(V4)\left((g341)-(g141)(V3)\right)\right.$$
$$\left.+(V3)(V4)\left((g343)-(g143)(V3)\right)+(V4)(V3)\left((g334)-(g134)(V3)\right)\right)-(B4)(J2)$$

اما در مورد سیال در جت، فرضی که می تواند سبب ساده سازی معادلات شود، فرض $V^{\varphi}=0$ است. در این صورت داریم:

$$\left(z^{3/2}+Kz^{5/2}\right)(u02)[2(V4)(g314)+(V1)(V4)(g341)]+(B4)(J2)=0$$

پایستگی تکانه ارتفاعی:

$$\left(z^{3/2}+Kz^{5/2}\right)(u02)D(V4[z],z)(V4)+\left(1+\frac{2m}{z}\right)D(Kz^{\frac{5}{2}},z)$$
$$=-\left(z^{\frac{3}{2}}+Kz^{\frac{5}{2}}\right)(u02)\left((g411)+2(V3)(g413)-2(g114)(V4)(V4)+(V1)(V3)(g431)\right.$$
$$-(V1)(V4)(g141)(V4)+(V3)(V3)(g433)-(g143)(V4)(V3)(V4)-(g134)(V4)(V4)(V3)$$
$$\left.+(V4)(V4)(g444)\right)$$

با فرض $V^{\varphi}=0$ خواهیم داشت:

$$\left(z^{3/2}+Kz^{5/2}\right)(u02)D(V4[z],z)(V4)+\left(1+\frac{2m}{z}\right)D(Kz^{\frac{5}{2}},z)$$
$$=-\left(z^{\frac{3}{2}}+Kz^{\frac{5}{2}}\right)(u02)\left((g411)-2(g114)(V4)(V4)-(V1)(V4)(g141)(V4)+(V4)(V4)(g444)\right)$$

$$g112 = \Gamma(ttr) = \frac{m(z + 2z^3 + z^5 + 2m - 2a^2m + 4mz^2 + 4a^2mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g114 = \Gamma(ttz) = \frac{mz(z + 2z^3 + z^5 + 2m - 6a^2m + 4mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g121 = \Gamma(trt) = \frac{m(z + 2z^3 + z^5 + 2m - 2a^2m + 4mz^2 + 4a^2mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g123 = \Gamma(tr\varphi) = -\frac{ma(2z^3 + z - 2z^3 + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g132 = \Gamma(t\varphi r) = -\frac{ma(2z^3 + z - 2z^3 + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g134 = \Gamma(t\varphi z) = -\frac{maz(3z + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g141 = \Gamma(tzt) = \frac{mz(z + 2z^3 + z^5 + 2m - 6a^2m + 4mz^2 + 2z^4m)}{z^2(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g143 = \Gamma(tz\varphi) = -\frac{maz(3z + 4m)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g211 = \Gamma(rtt) = \frac{m}{z^2(z + 2m)}$$

$$g213 = \Gamma(rt\varphi) = \frac{2ma}{z^2(z + 2m)}$$

$$g222 = \Gamma(rrr) = -\frac{m}{z^2(z + 2m)}$$

$$g224 = \Gamma(rrz) = -\frac{m}{z(z + 2m)}$$

$$g231 = \Gamma(r\varphi t) = -\frac{-2ma}{z^2(z + 2m)}$$

$$g233 = \Gamma(r\varphi\varphi) = -\frac{(z^3 + m + 2mz^2)}{z^2(z + 2m)}$$

$$g242 = \Gamma(rzr) = -\frac{m}{z(z + 2m)}$$

$$g244 = \Gamma(rzz) = \frac{m}{z^2(z + 2m)}$$

$$g312 = \Gamma(\varphi tr) = \frac{am(z - 2z^3 + 4mz^2)}{(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g314 = \Gamma(\varphi tz) = \frac{amz(-4m + 3z)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g321 = \Gamma(\varphi rt) = \frac{am(z - 2z^3 + 4mz^2)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g323 = \Gamma(\varphi r\varphi) = -(2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 + 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 - z^8)/((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6))$$

$$g332 = \Gamma(\varphi\varphi r) = -(2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 + 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 - z^8)/((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6))$$

$$g334 = \Gamma(\varphi\varphi z) = -\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g341 = \Gamma(\varphi zt) = \frac{amz(-4m + 3z)}{1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6}$$

$$g343 = \Gamma(\varphi z\varphi) = -\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)}$$

$$g411 = \Gamma(ztt) = \frac{mz}{(1 + z^2)(z + 2m)}$$

$$g413 = \Gamma(zt\varphi) = -\frac{3maz}{(1 + z^2)^2(z + 2m)}$$

$$g422 = \Gamma(zrr) = \frac{mz}{(1 + z^2)(z + 2m)}$$

$$g424 = \Gamma(zrz) = -\frac{mR}{(1 + z^2)(z + 2m)}$$

$$g431 = \Gamma(z\varphi t) = -\frac{3maR^2z}{(1 + z^2)^2(z + 2m)}$$

$$g433 = \Gamma(z\varphi\varphi) = \frac{mz}{(1 + z^2)^2(z + 2m)}$$

$$g442 = \Gamma(zzr) = -\frac{m}{(1 + z^2)(z + 2m)}$$

$$g444 = \Gamma(zzz) = -\frac{mz}{(1 + z^2)(\sqrt{z} + 2m)}$$

همینطور برای J در جت داریم (تنها آن هایی که استفاده دارند را نوشتیم) :

$$J2 = -\frac{c}{4\pi}B3\left(D\left(-\frac{z^2}{(z^2+2m)^2}, z\right)\right.$$
$$+\left(-\frac{mz(z+2z^3+z^5-2m+6a^2m-4mz^2-2mz^4)}{(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}-\frac{m}{z(z+2m)}\right.$$
$$\left.\left.-\frac{mz}{(1+z^2)(\sqrt{z}+2m)}\right)\left(-\frac{z^2}{(z^2+2m)^2}\right)\right)$$

$$J3 = \frac{c}{4\pi}B4\left(\frac{am(z-2z^3+4mz^2)}{(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6)}\left(\frac{z^4}{2ma(z+2m)}\right)+\left(-\frac{m}{z^2(z+2m)}\right.\right.$$
$$-(2mz^3+4z^5m+2z^7m-mz-4z^3m-5z^5m-2z^7m-1+2m^2-4z^2+2a^2m^2$$
$$+8m^2z^2-6z^4-4a^2m^2z^2+10m^2z^4-4z^6+4m^2z^6$$
$$-z^8)/((1+z^2)(1-4m^2+3z^2+4a^2m^2-8m^2z^2+3z^4-4m^2z^4+z^6))$$
$$\left.\left.-\frac{m}{(1+z^2)(z+2m)}\right)\left(\frac{z^2}{(z+2m)^2}\right)\right)$$

و توضیحی کوتاه در مورد $(u0)^2$:

$$u02 = \left[1-\frac{2m}{\sqrt{R^2+z^2}}+2\frac{2maR^2}{(R^2+z^2)^{\frac{3}{2}}}(V3)+\left[-\left(1+\frac{2m}{\sqrt{R^2+z^2}}\right)(V2)^2-R^2\left(1+\frac{2m}{\sqrt{R^2+z^2}}\right)(V3)^2-\left(1+\frac{2m}{\sqrt{R^2+z^2}}\right)(V4)^2\right]\right]^{-1}$$

برای قرص:

$$u02 = \left(1-\frac{2m}{\sqrt{R^2}}+2\frac{2ma}{R}(V3)-\left(1+\frac{2m}{R}\right)(V2)^2-R^2\left(1+\frac{2m}{R}\right)(V3)^2\right)^{-1}$$

برای جت:

$$u02 = \left(1-\frac{2m}{z}+2\frac{2ma}{z^3}(V3)-\left(1+\frac{2m}{z}\right)(V3)^2-\left(1+\frac{2m}{z}\right)(V4)^2\right)^{-1}$$

برای جت در صورتی که فرض شود $V^{\varphi}=0$:

$$u02 = \left(1-\frac{2m}{z}-\left(1+\frac{2m}{z}\right)(V4)^2\right)^{-1}$$