

برای قرص خواهیم داشت:

پایستگی جرم ($V^z = 0$):

$$\begin{aligned} V^R \frac{\partial}{\partial R} (R^{3/2} - KR^{5/2}) + (R^{3/2} + KR^{5/2}) \left(\frac{\partial V^R}{\partial R} \right) \\ = -(R^{3/2} + KR^{5/2}) \{ V^R ([\Gamma_{RR}^R + \Gamma_{\varphi R}^\varphi + \Gamma_{zR}^z] - \Gamma_{r0}^0) \} + \{ V^R ([\Gamma_{rz}^r + \Gamma_{\varphi z}^\varphi + \Gamma_{zz}^z] - \Gamma_{z0}^0) \} \\ - \{ \Gamma_{R\varphi}^0 V^\varphi V^R + \Gamma_{\varphi R}^0 V^R V^\varphi \} - \frac{1}{(u^0)^2} [-2\{-B_\varphi J^z u^R u^0\}] \end{aligned}$$

پایستگی تکانه شعاعی:

$$\begin{aligned} (R^{3/2} + KR^{5/2})(u^0)^2 \frac{\partial V^R}{\partial R} V^R + \left[1 + \frac{2m}{\sqrt{R^2 + z^2}} \right] \frac{\partial (KR^{5/2})}{\partial R} \\ = -(R^{3/2} + KR^{5/2})(u^0)^2 [(\Gamma_{00}^R) - 2\Gamma_{0R}^0 V^R V^R + 2V^\varphi \Gamma_{0\varphi}^R + V^t V^R (\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R) + V^R V^R \Gamma_{RR}^R \\ - \Gamma_{\varphi R}^0 V^R V^\varphi - \Gamma_{R\varphi}^0 V^R V^\varphi + V^\varphi V^\varphi \Gamma_{\varphi\varphi}^R] - [B_\varphi J^z] \end{aligned}$$

پایستگی تکانه زاویه ای:

$$\begin{aligned} (R^{3/2} + KR^{5/2})(u^0)^2 \frac{\partial V^\varphi}{\partial R} V^R \\ = -(R^{3/2} + KR^{5/2})(u^0)^2 [2V^R (\Gamma_{tR}^\varphi - \Gamma_{tR}^t V^\varphi) + V^t V^R (\Gamma_{Rt}^\varphi - \Gamma_{Rt}^t V^\varphi) + V^R V^\varphi (\Gamma_{\varphi R}^\varphi - \Gamma_{\varphi R}^t V^\varphi) \\ + V^\varphi V^R (\Gamma_{R\varphi}^\varphi - \Gamma_{R\varphi}^t V^\varphi)] \end{aligned}$$

پایستگی تکانه ارتفاعی:

$$as J^R = 0, \quad \frac{\partial}{\partial z} = 0, \quad V^z = 0 \rightarrow 0 = 0$$

برای قرص:

$$(u^0)^2 = \left[1 - \frac{2m}{\sqrt{R^2}} + 2 \frac{2ma}{R} V^\varphi + \left[- \left(1 + \frac{2m}{R} \right) (V^R)^2 - R^2 \left(1 + \frac{2m}{R} \right) (V^\varphi)^2 \right] \right]^{-1}$$

همینطور برای J در قرص داریم:

$$\begin{aligned} -\frac{4\pi}{c} J^z = \frac{\partial}{\partial R} \left[\left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) B_\varphi(R) \right] \\ + B_\varphi \left(\frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right. \\ \left. - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} \right) \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \end{aligned}$$

سپس برای جت خواهیم داشت:

پایستگی جرم:

$$V^z \frac{\partial}{\partial z} (z^{3/2} - Kz^{5/2}) + (z^{3/2} + Kz^{5/2}) \left(\frac{\partial V^z}{\partial z} \right) \\ = -(z^{3/2} + Kz^{5/2}) [-\{\Gamma_{\phi z}^0 V^z V^\phi + \Gamma_{z\phi}^0 V^\phi V^z\}] - \frac{1}{(u^0)^2} [-2\{-B_z J^R u^\phi u^0\}]$$

پایستگی تکانه شعاعی:

$$\left[1 + \frac{2m}{z}\right] \frac{\partial (Kz^{5/2})}{\partial R} = -(z^{3/2} + Kz^{5/2})(u^0)^2 [\Gamma_{00}^R + 2V^\phi \Gamma_{0\phi}^R + V^t V^z (\Gamma_{zt}^t) + V^\phi V^\phi \Gamma_{\phi\phi}^R + V^z V^z \Gamma_{zz}^R] + B_z J^\phi$$

پایستگی تکانه زاویه ای:

$$(z^{3/2} + Kz^{5/2})(u^0)^2 \frac{\partial V^\phi}{\partial z} V^z \\ = -(z^{3/2} + Kz^{5/2})(u^0)^2 [2V^z (\Gamma_{tz}^\phi - \Gamma_{tz}^t V^\phi) + V^t V^z (\Gamma_{zt}^\phi - \Gamma_{zt}^t V^\phi) + V^\phi V^z (\Gamma_{z\phi}^\phi - \Gamma_{z\phi}^t V^\phi) \\ + V^z V^\phi (\Gamma_{\phi z}^\phi - \Gamma_{\phi z}^t V^\phi)] - B_z J^R$$

پایستگی تکانه ارتفاعی:

$$(z^{3/2} + Kz^{5/2})(u^0)^2 \frac{\partial V^z}{\partial z} V^z + \left[1 + \frac{2m}{z}\right] \frac{\partial (Kz^{5/2})}{\partial z} \\ = -(z^{3/2} + Kz^{5/2})(u^0)^2 [\Gamma_{tt}^z + 2V^\phi \Gamma_{t\phi}^z - 2\Gamma_{tz}^t V^z V^z + V^t V^\phi \Gamma_{\phi t}^z - V^t V^z \Gamma_{zt}^t V^z + V^\phi V^\phi \Gamma_{\phi\phi}^z \\ - \Gamma_{z\phi}^t V^z V^\phi V^z - \Gamma_{\phi z}^t V^z V^z V^\phi + V^z V^z \Gamma_{zz}^z]$$

همینطور برای J در جت داریم (تنها آن هایی که استفاده دارند را نوشتیم):

$$-\frac{4\pi}{c} J^R = B_\phi \left\{ \frac{\partial}{\partial z} \left[-\frac{z^2}{(z^2 + 2m)^2} \right] \right. \\ \left. + \left(-\frac{mz(z + 2z^3 + z^5 - 2m + 6a^2m - 4mz^2 - 2mz^4)}{(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)} - \frac{m}{z(z + 2m)} \right. \right. \\ \left. \left. - \frac{mz}{(1 + z^2)(\sqrt{z} + 2m)} \right) \left(-\frac{z^2}{(z^2 + 2m)^2} \right) \right\} \\ -\frac{4\pi}{c} J^\phi = -B_z \left\{ \frac{am(z - 2z^3 + 4mz^2)}{(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)} \left(\frac{z^4}{2ma(z + 2m)} \right) + \left(-\frac{m}{z^2(z + 2m)} \right. \right. \\ \left. \left. - (2mz^3 + 4z^5m + 2z^7m - mz - 4z^3m - 5z^5m - 2z^7m - 1 + 2m^2 - 4z^2 + 2a^2m^2 \right. \right. \\ \left. \left. + 8m^2z^2 - 6z^4 - 4a^2m^2z^2 + 10m^2z^4 - 4z^6 + 4m^2z^6 \right. \right. \\ \left. \left. - z^8) / ((1 + z^2)(1 - 4m^2 + 3z^2 + 4a^2m^2 - 8m^2z^2 + 3z^4 - 4m^2z^4 + z^6)) \right) \right. \\ \left. - \frac{m}{(1 + z^2)(z + 2m)} \right) \left(\frac{z^2}{(z + 2m)^2} \right) \right\}$$

برای جت:

$$(u^0)^2 = \left[1 - \frac{2m}{z} + 2 \frac{2ma}{z^3} V^\phi + \left[-\left(1 + \frac{2m}{z}\right) (V^\phi)^2 - \left(1 + \frac{2m}{z}\right) (V^z)^2 \right] \right]^{-1}$$