

برای قرص خواهیم داشت:

پایستگی جرم ( $V^z = 0$ ):

$$\begin{aligned} V^R \frac{\partial}{\partial R} (R^{3/2} - KR^{5/2}) + (R^{3/2} + KR^{5/2}) \left( \frac{\partial V^R}{\partial R} \right) \\ = -(R^{3/2} + KR^{5/2}) \left[ \{V^R ([\Gamma_{RR}^R + \Gamma_{\phi R}^\phi + \Gamma_{zR}^z] - \Gamma_{r0}^0)\} \right. \\ \left. + \{V^R ([\Gamma_{rz}^r + \Gamma_{\phi z}^\phi + \Gamma_{zz}^z] - \Gamma_{z0}^0)\} - \{\Gamma_{R\phi}^0 V^\phi V^R + \Gamma_{\phi R}^0 V^R V^\phi\} \right] \\ - \frac{1}{(u^0)^2} [-2\{-B_\phi J^z u^R u^0\}] \end{aligned}$$

پایستگی تکانه شعاعی:

$$\begin{aligned} (R^{3/2} + KR^{5/2})(u^0)^2 \frac{\partial V^R}{\partial R} V^R + \left[ 1 + \frac{2m}{\sqrt{R^2 + z^2}} \right] \frac{\partial (KR^{5/2})}{\partial R} \\ = -(R^{3/2} + KR^{5/2})(u^0)^2 [\Gamma_{00}^R - 2\Gamma_{0R}^0 V^R V^R + 2V^\phi \Gamma_{0\phi}^R + V^t V^R (\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R) \\ + V^R V^R \Gamma_{RR}^R - \Gamma_{\phi R}^0 V^R V^R V^\phi - \Gamma_{R\phi}^0 V^R V^\phi V^R + V^\phi V^\phi \Gamma_{\phi\phi}^R] - [B_\phi J^z] \end{aligned}$$

پایستگی تکانه زاویه ای:

$$\begin{aligned} (R^{3/2} + KR^{5/2})(u^0)^2 \frac{\partial V^\phi}{\partial R} V^R \\ = -(R^{3/2} + KR^{5/2})(u^0)^2 [2V^R (\Gamma_{tR}^\phi - \Gamma_{tR}^t V^\phi) + V^t V^R (\Gamma_{Rt}^\phi - \Gamma_{Rt}^t V^\phi) \\ + V^R V^\phi (\Gamma_{\phi R}^\phi - \Gamma_{\phi R}^t V^\phi) + V^\phi V^R (\Gamma_{R\phi}^\phi - \Gamma_{R\phi}^t V^\phi)] \end{aligned}$$

پایستگی تکانه ارتفاعی:

$$as J^R = 0, \quad \frac{\partial}{\partial z} = 0, \quad V^z = 0 \rightarrow 0 = 0$$

همینطور برای  $J$  در قرص داریم:

$$\begin{aligned} -\frac{4\pi}{c} J^z = \frac{\partial}{\partial R} \left[ \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) B_\phi(R) \right] \\ + B_\phi \left( \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right. \\ \left. - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} \right) \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) \end{aligned}$$

$$\begin{aligned}
 J^z &= -\frac{c}{4\pi} \left\{ \frac{\partial}{\partial R} \left[ \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) B_\varphi(R) \right] \right. \\
 &\quad + B_\varphi \left( \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right. \\
 &\quad \left. \left. - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} \right) \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) \right\} \\
 J^z &= -\frac{c}{4\pi} \left\{ \left[ \left( \frac{2R(R^2 + 2mR + 4m^2) - R^2(2R + 2m + 4m^2)}{(R^2 + 2mR + 4m^2)^2} \right) B_\varphi(R) \right] \right. \\
 &\quad + \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) \frac{\partial B_\varphi(R)}{\partial R} \\
 &\quad + B_\varphi \left( \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right. \\
 &\quad \left. \left. - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} \right) \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) \right\} \\
 J^z &= -\frac{c}{4\pi} \left\{ \left[ \left( \frac{R}{R^2 + 2mR + 4m^2} \right) B_\varphi(R) \right] + \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) \frac{\partial B_\varphi(R)}{\partial R} \right. \\
 &\quad + B_\varphi \left( \frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right. \\
 &\quad \left. \left. - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} \right) \left( \frac{R^2}{R^2 + 2mR + 4m^2} \right) \right\} \\
 J^z &\approx -\left\{ \frac{B_\varphi(R)}{R} + \frac{\partial B_\varphi(R)}{\partial R} + B_\varphi \left( \frac{1}{R^2} - \frac{m}{R(R + 2m)} - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{(R^4 - 4R^2m^2 + 4a^2m^2)} \right) \right\} \\
 J^z &\approx -\left\{ \frac{B_\varphi(R)}{R} + \frac{\partial B_\varphi(R)}{\partial R} + \frac{B_\varphi}{R^2} \right\} = -\frac{\partial B_\varphi}{\partial R} - B_\varphi \left( \frac{1}{R} + \frac{1}{R^2} \right)
 \end{aligned}$$

پایستگی جرم :

$$\begin{aligned}
 V^R \frac{\partial}{\partial R} (R^{3/2} - KR^{5/2}) + (R^{3/2} + KR^{5/2}) \left( \frac{\partial V^R}{\partial R} \right) \\
 = -(R^{3/2} + KR^{5/2}) \left\{ \left[ V^R ([\Gamma_{RR}^R + \Gamma_{\varphi R}^\varphi + \Gamma_{zR}^z] - \Gamma_{r0}^0) \right] \right. \\
 + \left\{ V^R ([\Gamma_{rz}^r + \Gamma_{\varphi z}^\varphi + \Gamma_{zz}^z] - \Gamma_{z0}^0) \right\} - \left\{ \Gamma_{R\varphi}^0 V^\varphi V^R + \Gamma_{\varphi R}^0 V^R V^\varphi \right\} \\
 \left. - \frac{1}{(u^0)^2} \left[ -2 \left\{ -B_\varphi \left[ -\frac{\partial B_\varphi}{\partial R} - B_\varphi \left( \frac{1}{R} + \frac{1}{R^2} \right) \right] u^R u^0 \right\} \right] \right\} \\
 \frac{\partial V^R}{\partial R} = \frac{1}{4R} \left\{ -(R^{3/2} + KR^{5/2}) \left[ \left\{ V^R ([\Gamma_{RR}^R + \Gamma_{\varphi R}^\varphi + \Gamma_{zR}^z] - \Gamma_{r0}^0) \right\} + \left\{ V^R ([\Gamma_{rz}^r + \Gamma_{\varphi z}^\varphi + \Gamma_{zz}^z] - \Gamma_{z0}^0) \right\} \right. \right. \\
 \left. \left. - \left\{ \Gamma_{R\varphi}^0 V^\varphi V^R + \Gamma_{\varphi R}^0 V^R V^\varphi \right\} - \frac{1}{(u^0)^2} \left[ -2 \left\{ -B_\varphi \left[ -\frac{\partial B_\varphi}{\partial R} - B_\varphi \left( \frac{1}{R} + \frac{1}{R^2} \right) \right] u^R u^0 \right\} \right] \right] \right\}
 \end{aligned}$$

پایستگی تکانه شعاعی:

$$\begin{aligned} (R^{3/2} + KR^{5/2})(u^0)^2 \frac{\partial V^R}{\partial R} V^R + \left[1 + \frac{2m}{\sqrt{R^2 + z^2}}\right] \frac{\partial(KR^{5/2})}{\partial R} \\ = -(R^{3/2} + KR^{5/2})(u^0)^2 [(\Gamma_{00}^R) - 2\Gamma_{0R}^0 V^R V^R + 2V^\varphi \Gamma_{0\varphi}^R + V^t V^R (\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R) \\ + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^R V^\varphi - \Gamma_{R\varphi}^0 V^R V^\varphi V^R + V^\varphi V^\varphi \Gamma_{\varphi\varphi}^R] \\ - \left[ B_\varphi \left[ -\frac{\partial B_\varphi}{\partial R} - B_\varphi \left( \frac{1}{R} + \frac{1}{R^2} \right) \right] \right] \end{aligned}$$

$$\begin{aligned} B_\varphi \left[ -\frac{\partial B_\varphi}{\partial R} - B_\varphi \left( \frac{1}{R} + \frac{1}{R^2} \right) \right] (1 + 2u^R u^0) = \\ = - \left( R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) (u^0)^2 [(\Gamma_{00}^R) - 2\Gamma_{0R}^0 V^R V^R + 2V^\varphi \Gamma_{0\varphi}^R + V^t V^R (\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R) \\ + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^R V^\varphi - \Gamma_{R\varphi}^0 V^R V^\varphi V^R + V^\varphi V^\varphi \Gamma_{\varphi\varphi}^R] \\ - \left( R^{\frac{3}{2}} \right. \\ \left. + KR^{\frac{5}{2}} \right) (u^0)^2 \left( \frac{1}{4R} \left\{ - \left( R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) \left[ \{ V^R ([\Gamma_{RR}^R + \Gamma_{\varphi R}^\varphi + \Gamma_{zR}^z] - \Gamma_{r0}^0) \} \right. \right. \right. \\ \left. \left. + \{ V^R ([\Gamma_{rz}^r + \Gamma_{\varphi z}^\varphi + \Gamma_{zz}^z] - \Gamma_{z0}^0) \} - \{ \Gamma_{R\varphi}^0 V^\varphi V^R + \Gamma_{\varphi R}^0 V^R V^\varphi \} \right] \right\} \Big) V^R \\ \left. - \frac{5R^{3/2}}{2} \left[ 1 + \frac{2m}{R} \right] \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial B_\varphi}{\partial R} = \frac{1}{B_\varphi (1 + 2u^R u^0)} \left\{ \left( R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) (u^0)^2 [(\Gamma_{00}^R) - 2\Gamma_{0R}^0 V^R V^R + 2V^\varphi \Gamma_{0\varphi}^R + V^t V^R (\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R) \right. \right. \\ \left. \left. + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^R V^\varphi - \Gamma_{R\varphi}^0 V^R V^\varphi V^R + V^\varphi V^\varphi \Gamma_{\varphi\varphi}^R \right] \right. \\ \left. + \left( R^{\frac{3}{2}} \right. \right. \\ \left. \left. + KR^{\frac{5}{2}} \right) (u^0)^2 \left( \frac{1}{4R} \left\{ - \left( R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) \left[ \{ V^R ([\Gamma_{RR}^R + \Gamma_{\varphi R}^\varphi + \Gamma_{zR}^z] - \Gamma_{r0}^0) \} \right. \right. \right. \right. \\ \left. \left. + \{ V^R ([\Gamma_{rz}^r + \Gamma_{\varphi z}^\varphi + \Gamma_{zz}^z] - \Gamma_{z0}^0) \} - \{ \Gamma_{R\varphi}^0 V^\varphi V^R + \Gamma_{\varphi R}^0 V^R V^\varphi \} \right] \right\} \Big) V^R \\ \left. + \frac{5R^{3/2}}{2} \left[ 1 + \frac{2m}{R} \right] \right\} - B_\varphi \left( \frac{1}{R} + \frac{1}{R^2} \right) \end{aligned}$$

پایستگی تکانه زاویه ای:

$$\begin{aligned} (R^{3/2} + KR^{5/2})(u^0)^2 \frac{\partial V^\varphi}{\partial R} V^R \\ = -(R^{3/2} + KR^{5/2})(u^0)^2 [2V^R (\Gamma_{tR}^\varphi - \Gamma_{tR}^t V^\varphi) + V^t V^R (\Gamma_{Rt}^\varphi - \Gamma_{Rt}^t V^\varphi) \\ + V^R V^\varphi (\Gamma_{\varphi R}^\varphi - \Gamma_{\varphi R}^t V^\varphi) + V^\varphi V^R (\Gamma_{R\varphi}^\varphi - \Gamma_{R\varphi}^t V^\varphi)] \end{aligned}$$