برای قرص خواهیم داشت:

 $V^{z} = 0$): پایستگی جرم

$$\begin{split} V^R \frac{\partial}{\partial R} \left(R^{3/2} - K R^{5/2} \right) + \left(R^{3/2} + K R^{5/2} \right) \left(\frac{\partial V^R}{\partial R} \right) \\ &= - \left(R^{3/2} + K R^{5/2} \right) \left[\left\{ V^R \left(\left[\varGamma_{RR}^R + \varGamma_{\varphi R}^\varphi + \varGamma_{zR}^z \right] - \varGamma_{r0}^0 \right) \right\} \right. \\ &+ \left. \left\{ V^R \left(\left[\varGamma_{rz}^r + \varGamma_{\varphi z}^\varphi + \varGamma_{zz}^z \right] - \varGamma_{z0}^0 \right) \right\} - \left\{ \varGamma_{R\varphi}^0 V^\varphi V^R + \varGamma_{\varphi R}^0 V^R V^\varphi \right\} \right] \\ &- \frac{1}{(u^0)^2} \left[-2 \left\{ -B_\varphi J^z u^R u^0 \right\} \right] \end{split}$$

یا پستگی تکانه شعاعی:

$$\begin{split} \left(R^{3/2} + KR^{5/2}\right) & (u^0)^2 \frac{\partial V^R}{\partial R} V^R + \left[1 + \frac{2m}{\sqrt{R^2 + z^2}}\right] \frac{\partial (KR^{5/2})}{\partial R} \\ & = - \left(R^{3/2} + KR^{5/2}\right) (u^0)^2 \left[(\Gamma_{00}^R) - 2\Gamma_{0R}^0 V^R V^R + 2V^\varphi \Gamma_{0\varphi}^R + V^t V^R (\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R) \right. \\ & + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^R V^\varphi - \Gamma_{R\varphi}^0 V^R V^\varphi V^R + V^\varphi V^\varphi \Gamma_{\varphi \varphi}^R\right] - \left[B_\varphi J^z\right] \end{split}$$

پایستگی تکانه زاویه ای:

$$\begin{split} \left(R^{3/2} + KR^{5/2}\right) & (u^{0})^{2} \frac{\partial V^{\varphi}}{\partial R} V^{R} \\ & = - \left(R^{3/2} + KR^{5/2}\right) (u^{0})^{2} \left[2V^{R} \left(\Gamma_{tR}^{\varphi} - \Gamma_{tR}^{t} V^{\varphi} \right) + V^{t} V^{R} \left(\Gamma_{Rt}^{\varphi} - \Gamma_{Rt}^{t} V^{\varphi} \right) \right. \\ & + V^{R} V^{\varphi} \left(\Gamma_{\varphi R}^{\varphi} - \Gamma_{\varphi R}^{t} V^{\varphi} \right) + V^{\varphi} V^{R} \left(\Gamma_{R\varphi}^{\varphi} - \Gamma_{R\varphi}^{t} V^{\varphi} \right) \right] \end{split}$$

بایستگی تکانه ارتفاعی:

$$as J^R = 0$$
 , $\frac{\partial}{\partial z} = 0$, $V^z = 0$ \rightarrow $0 = 0$

همینطور برای / در قرص داریم:

$$\begin{split} -\frac{4\pi}{c}J^{z} &= \frac{\partial}{\partial R} \left[\left(\frac{R^{2}}{R^{2} + 2mR + 4m^{2}} \right) B_{\varphi}(R) \right] \\ &+ B_{\varphi} \left(\frac{mR(R^{5} + 2R^{4}m - 2R^{2}a^{2}m)}{R^{2}(R^{6} - 4R^{4}m^{2} + 4R^{2}a^{2}m^{2})} - \frac{m}{R(R + 2m)} \right. \\ &- \frac{m}{R(R + 2m)} \frac{R^{3}m - R^{4} + 2R^{2}m^{2} + 2a^{2}m^{2}}{(R^{4} - 4R^{2}m^{2} + 4a^{2}m^{2})} \left(\frac{R^{2}}{R^{2} + 2mR + 4m^{2}} \right) \end{split}$$

برای قرص خواهیم داشت:

یایستگی جرم ($V^z = 0$):

$$\begin{split} \left(\frac{\partial V^R}{\partial R}\right) &= \left(R^{3/2} + KR^{5/2}\right)^{-1} \left(-V^R \frac{\partial}{\partial R} \left(R^{3/2} - KR^{5/2}\right) \right. \\ &\quad - \left(R^{3/2} + KR^{5/2}\right) \left[\left\{V^R \left(\left[\Gamma_{RR}^R + \Gamma_{\varphi R}^{\varphi} + \Gamma_{ZR}^z \right] - \Gamma_{r0}^0 \right) \right\} + \left\{V^R \left(\left[\Gamma_{rz}^r + \Gamma_{\varphi z}^\varphi + \Gamma_{Zz}^z \right] - \Gamma_{20}^0 \right) \right\} \\ &\quad - \left\{\Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} \right\} \right] - \frac{1}{(u^0)^2} \left[-2 \left\{ -B_{\varphi} J^z u^R u^0 \right\} \right] \right) \\ \left(\frac{\partial V^R}{\partial R}\right) + \frac{1}{(u^0)^2} \frac{B_{\varphi}}{2\pi} \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \frac{\partial B_{\varphi}(R)}{\partial R} \\ &\quad = \left(R^{3/2} + KR^{5/2}\right)^{-1} \left(-V^R \frac{\partial}{\partial R} \left(R^{3/2} - KR^{5/2}\right) \right. \\ &\quad - \left(R^{3/2} + KR^{5/2}\right) \left[\left\{V^R \left(\left[\Gamma_{RR}^R + \Gamma_{\varphi R}^\varphi + \Gamma_{ZR}^z \right] - \Gamma_{r0}^0 \right) \right\} + \left\{V^R \left(\left[\Gamma_{rz}^r + \Gamma_{\varphi z}^\varphi + \Gamma_{Zz}^z \right] - \Gamma_{20}^0 \right) \right\} \\ &\quad - \left\{\Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} \right\} \right] \\ &\quad - \left\{ I_{u^0}^0 \right\} \left[-2 \left\{ -\frac{B_{\varphi}}{4\pi} \left[B_{\varphi} \frac{\partial}{\partial R} \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right. \right. \\ &\quad + B_{\varphi} \left(\frac{mR(R^5 + 2R^4 m - 2R^2 a^2 m)}{R^2(R^6 - 4R^4 m^2 + 4R^2 a^2 m^2)} - \frac{m}{R(R + 2m)} \right. \\ &\quad - \frac{m}{R(R + 2m)} \frac{R^3 m - R^4 + 2R^2 m^2 + 2a^2 m^2}{(R^4 - 4R^2 m^2 + 4a^2 m^2)} \left. \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right] u^R u^0 \right\} \right] \right) \end{split}$$

$$\begin{split} \left(\frac{\partial V^R}{\partial R}\right) &= -\frac{1}{(u^0)^2} \frac{B_{\varphi}}{2\pi} \left(\frac{R^2}{R^2 + 2mR + 4m^2}\right) \left\{ \left(-\frac{B_{\varphi}}{4\pi} \left(1\right) \right) - \left(\frac{R^2}{4\pi} + KR^{\frac{5}{2}}\right) 2V^R u^R u^0\right) \left(\frac{R^2}{R^2 + 2mR + 4m^2}\right) \right\}^{-1} \left\{ \frac{B_{\varphi}}{4\pi} \left(1\right) - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right) 2V^R u^R u^0\right) \left\{ B_{\varphi} \frac{\partial}{\partial R} \left(\frac{R^2}{R^2 + 2mR + 4m^2}\right) \right\} \\ &+ B_{\varphi} \left(\frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right) - \frac{m}{R(R + 2m)} - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{\left(R^4 - 4R^2m^2 + 4a^2m^2\right)} \right) \left(\frac{R^2}{R^2 + 2mR + 4m^2}\right) \right\} \\ &- \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right) \left(u^0\right)^2 \left(\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right)^{-1} \left(-V^R \frac{\partial}{\partial R} \left(R^{\frac{3}{2}} - KR^{\frac{5}{2}}\right) \right) - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right) \left[\left\{V^R \left(\left[I_{RR}^R + \Gamma_{\varphi R}^{\varphi} + \Gamma_{ZR}^Z\right] - \Gamma_{r0}^0\right)\right\} + \left\{V^R \left(\left[\Gamma_{rz}^r + \Gamma_{\varphi z}^{\varphi} + \Gamma_{Zz}^z\right] - \Gamma_{20}^0\right)\right\} - \left\{R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right) \left[\left\{V^R \left(\left[I_{RR}^R + \Gamma_{\varphi R}^{\varphi} + \Gamma_{ZR}^Z\right] - \Gamma_{r0}^0\right)\right\} + \left\{V^R \left(\left[\Gamma_{rz}^r + \Gamma_{\varphi z}^{\varphi} + \Gamma_{Zz}^z\right] - \Gamma_{20}^0\right)\right\} - \left\{R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right) \left[\left\{U^0\right\}^2 \left[\left(\Gamma_{00}^R\right) - 2\Gamma_{00}^0 V^R V^R + 2V^{\varphi} \Gamma_{0\varphi}^R + V^t V^R \left(\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R\right) + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^{\varphi} - \Gamma_{R\varphi}^0 V^R V^{\varphi} V^R + V^{\varphi} V^{\varphi} \Gamma_{\varphi \varphi}^R\right)\right\} - \left\{R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right] \left[\left\{V^R \left(\left[\Gamma_{RR}^R + \Gamma_{\varphi R}^{\varphi} + \Gamma_{ZR}^z\right] - \Gamma_{r0}^0\right)\right\} + \left\{V^R \left(\left[\Gamma_{rz}^r + \Gamma_{\varphi z}^{\varphi} + \Gamma_{ZZ}^z\right] - \Gamma_{z0}^0\right)\right\} - \left\{\Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^R + V^{\varphi} V^R V^{\varphi} V^R + V^{\varphi} V^{\varphi} \Gamma_{\varphi \varphi}^R\right)\right\} - \left\{I_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^R\right\} - \left\{I_{R\varphi}^0 \left(\frac{R^2}{4\pi} \left(\frac{R^2}{4\pi} + \frac{R^2}{4\pi} \right)^2 + \frac{R^2}{4\pi} \left(\frac{R^2}{4\pi} \left(\frac{R^2}{4\pi} + \frac{R^2}{4\pi} \right)^2 + \frac{R^2}{2m^2} \left(\frac{R^2}{2m^2} + \frac{R^2}{2m^2} \right)\right\} - \left\{I_{R\varphi}^0 \left(\frac{R^2}{4\pi} \left(\frac{R^2}{4\pi} + \frac{R^2}{4\pi} \right)^2 + \frac{R^2}{2m^2} + \frac{R^2}{2m^2} \right)\right\} - \frac{R^2}{R^2} \left(\frac{R^2}{2\pi} + \frac{R^2}{2m^2} + \frac{R^2}{2\pi} \left(\frac{R^2}{2\pi} + \frac{R^2}{2\pi} \right)\right)\right\} - \frac{R^2}{R^2} \left(\frac{R^2}{2\pi} + \frac{R^2}{2\pi} + \frac{R^2}{2\pi} \left(\frac{R^2}{2\pi} + \frac{R^2}{2\pi} \right)\right) \left(\frac{R^2}{2\pi} + \frac{R^2}{2\pi} \left(\frac{R^2}{2\pi} + \frac{R^2}{2\pi} \right)\right)$$

$$\begin{split} &\left(1-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)2V^{R}u^{R}u^{0}\right)B_{\varphi}J^{z}\\ &=-\left(R^{\frac{3}{2}}\right.\\ &\left.+KR^{\frac{5}{2}}\right)(u^{0})^{2}\left(\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)^{-1}\left(-V^{R}\frac{\partial}{\partial R}\left(R^{\frac{3}{2}}-KR^{\frac{5}{2}}\right)\right.\\ &\left.-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)\left[\left\{V^{R}\left(\left[\Gamma_{RR}^{R}+\Gamma_{\varphi R}^{\varphi}+\Gamma_{zR}^{z}\right]-\Gamma_{r_{0}}^{r_{0}}\right)\right\}+\left\{V^{R}\left(\left[\Gamma_{rz}^{r}+\Gamma_{\varphi z}^{\varphi}+\Gamma_{zz}^{z}\right]-\Gamma_{z_{0}}^{r_{0}}\right)\right\}\\ &\left.-\left\{\Gamma_{R\varphi}^{0}V^{\varphi}V^{R}+\Gamma_{\varphi R}^{0}V^{R}V^{\varphi}\right\}\right]\right)V^{R}-\left[1+\frac{2m}{\sqrt{R^{2}+z^{2}}}\right]\frac{\partial(KR^{5/2})}{\partial R}\\ &\left.-\left(R^{3/2}+KR^{5/2}\right)(u^{0})^{2}\left[\left(\Gamma_{00}^{0}\right)-2\Gamma_{00}^{0}V^{R}V^{R}+2V^{\varphi}\Gamma_{0\varphi}^{R}+V^{t}V^{R}\left(\Gamma_{Rt}^{t}-\Gamma_{Rt}^{0}V^{R}\right)\right.\\ &\left.+V^{R}V^{R}\Gamma_{RR}^{R}-\Gamma_{\varphi R}^{0}V^{R}V^{R}V^{\varphi}-\Gamma_{R\varphi}^{0}V^{R}V^{\varphi}V^{R}+V^{\varphi}V^{\varphi}\Gamma_{\varphi\varphi}^{R}\right]\\ &\left.-\frac{B_{\varphi}}{4\pi}\left(1-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)2V^{R}u^{R}u^{0}\right)\left(\frac{R^{2}}{R^{2}+2mR+4m^{2}}\right)\frac{\partial B_{\varphi}(R)}{\partial R}\right.\\ &\left.-\frac{B_{\varphi}}{4\pi}\left(1-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)2V^{R}u^{R}u^{\varphi}\right)\left\{B_{\varphi}\frac{\partial}{\partial R}\left(\frac{R^{2}}{R^{2}+2mR+4m^{2}}\right)\right.\\ &\left.+B_{\varphi}\left(\frac{mR(R^{5}+2R^{4}m-2R^{2}a^{2}m)}{R^{2}+4R^{2}a^{2}m^{2}}-\frac{m}{R(R+2m)}\right.\\ &\left.-\frac{m}{R(R+2m)}\frac{R^{3}m-R^{4}+2R^{2}m^{2}+2a^{2}m^{2}}{\left(R^{4}-4R^{2}m^{2}+4a^{2}m^{2}\right)}\right)\left(\frac{R^{2}}{R^{2}+2mR+4m^{2}}\right)\right\}\\ &\left.-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)\left[\left\{V^{R}\left(\left[\Gamma_{RR}^{R}+\Gamma_{\varphi R}^{\varphi}+\Gamma_{zR}^{z}\right]-\Gamma_{r_{0}}^{0}\right)\right\}+\left\{V^{R}\left(\left[\Gamma_{rz}^{r}+\Gamma_{\varphi z}^{\varphi}+\Gamma_{zz}^{z}\right]-\Gamma_{z_{0}}^{0}\right)\right\}\\ &\left.-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)\left[\left\{V^{R}\left(\left[\Gamma_{RR}^{R}+\Gamma_{\varphi R}^{\varphi}+\Gamma_{zR}^{z}\right]-\Gamma_{r_{0}}^{0}\right)\right\}+\left\{V^{R}\left(\left[\Gamma_{rz}^{r}+\Gamma_{\varphi z}^{\varphi}+\Gamma_{zz}^{z}\right]-\Gamma_{z_{0}}^{0}\right)\right\}\\ &\left.-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)\left[\left\{V^{R}\left(\left[\Gamma_{RR}^{R}+\Gamma_{\varphi R}^{\varphi}+\Gamma_{zR}^{z}\right]-\Gamma_{r_{0}}^{0}\right)\right\}+\left\{V^{R}\left(\left[\Gamma_{Rt}^{r}-\Gamma_{\varphi z}^{\varphi}+\Gamma_{zz}^{z}\right]-\Gamma_{z_{0}}^{0}\right)\right\}\\ &\left.-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)\left[\left\{V^{R}\left(\left[\Gamma_{RR}^{R}+\Gamma_{\varphi R}^{\varphi}+\Gamma_{zR}^{z}\right]-\Gamma_{r_{0}}^{\varphi}\right)\right\}+V^{R}V^{R}\left(\Gamma_{Rt}^{r}-\Gamma_{Rt}^{\varphi}V^{R}\right)\right]\right)\right\}\\ &\left.-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)\left[\left\{V^{R}\left(\left[\Gamma_{RR}^{R}+\Gamma_{\varphi R}^{\varphi}+\Gamma_{zR}^{z}\right]-\Gamma_{r_{0}}^{\varphi}\right)\right\}+V^{R}V^{R}\left(\left[\Gamma_{Rt}^{r}-\Gamma_{Rt}^{\varphi}V^{R}\right]\right)\right]\right]\\ &\left.-\left(R^{\frac{3}{2}}+KR^{\frac{5}{2}}\right)\left[\left\{V^{R}\left(\left[\Gamma_{RR}^{R}+\Gamma_{\varphi R}^{\varphi}+\Gamma_{zR}^{z}\right]-\Gamma_{r_{0}}$$

$$\begin{split} \frac{\partial B_{\varphi}(R)}{\partial R} &= \left(-\frac{B_{\varphi}}{4\pi} \left(1 - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) 2V^R u^R u^0 \right) \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right)^{-1} \left\{ \frac{B_{\varphi}}{4\pi} \left(1 \right) \right. \\ & \left. - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) 2V^R u^R u^0 \right) \left\{ B_{\varphi} \frac{\partial}{\partial R} \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right. \\ & \left. + B_{\varphi} \left(\frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right. \\ & \left. - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{\left(R^4 - 4R^2m^2 + 4a^2m^2 \right)} \right) \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right\} \\ & \left. - \left(R^{\frac{3}{2}} \right. \\ & \left. + KR^{\frac{5}{2}} \right) \left(u^0 \right)^2 \left(\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right)^{-1} \left(-V^R \frac{\partial}{\partial R} \left(R^{\frac{3}{2}} - KR^{\frac{5}{2}} \right) \right. \\ & \left. - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) \left[\left\{ V^R \left(\left[\Gamma_{RR}^R + \Gamma_{\varphi R}^{\varphi} + \Gamma_{ZR}^Z \right] - \Gamma_{r0}^0 \right) \right\} + \left\{ V^R \left(\left[\Gamma_{rZ}^r + \Gamma_{\varphi Z}^{\varphi} + \Gamma_{ZZ}^Z \right] - \Gamma_{20}^0 \right) \right\} \right. \\ & \left. - \left(R^{3/2} + KR^{5/2} \right) \left(u^0 \right)^2 \left[\left(\Gamma_{00}^R \right) - 2\Gamma_{0R}^0 V^R V^R + 2V^{\varphi} \Gamma_{0\varphi}^R + V^t V^R \left(\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R \right) \right. \\ & \left. + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^R V^{\varphi} - \Gamma_{R\varphi}^0 V^R V^{\varphi} V^R + V^{\varphi} V^{\varphi} \Gamma_{\varphi \varphi}^R \right] \right\} \end{split}$$

پایستگی تکانه زاویه ای:

$$\begin{split} \left(R^{3/2} + KR^{5/2}\right) & (u^0)^2 \frac{\partial V^{\varphi}}{\partial R} V^R \\ & = - \left(R^{3/2} + KR^{5/2}\right) (u^0)^2 \left[2V^R \left(\Gamma_{tR}^{\varphi} - \Gamma_{tR}^t V^{\varphi}\right) + V^t V^R \left(\Gamma_{Rt}^{\varphi} - \Gamma_{Rt}^t V^{\varphi}\right) \right. \\ & \left. + V^R V^{\varphi} \left(\Gamma_{\varphi R}^{\varphi} - \Gamma_{\varphi R}^t V^{\varphi}\right) + V^{\varphi} V^R \left(\Gamma_{R\varphi}^{\varphi} - \Gamma_{R\varphi}^t V^{\varphi}\right)\right] \\ & \frac{\partial V^{\varphi}}{\partial R} = \left(V^R \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right) (u^0)^2\right)^{-1} \left\{ - \left(R^{3/2} + KR^{5/2}\right) (u^0)^2 \left[2V^R \left(\Gamma_{tR}^{\varphi} - \Gamma_{tR}^t V^{\varphi}\right) + V^t V^R \left(\Gamma_{Rt}^{\varphi} - \Gamma_{R\varphi}^t V^{\varphi}\right) + V^R V^{\varphi} \left(\Gamma_{\varphi R}^{\varphi} - \Gamma_{\varphi R}^t V^{\varphi}\right) + V^{\varphi} V^R \left(\Gamma_{R\varphi}^{\varphi} - \Gamma_{R\varphi}^t V^{\varphi}\right)\right] \right\} \end{split}$$

.....

همینطور برای / در قرص داریم:

$$\begin{split} -\frac{4\pi}{c}J^{z} &= \frac{\partial}{\partial R} \left[\left(\frac{R^{2}}{R^{2} + 2mR + 4m^{2}} \right) B_{\varphi}(R) \right] \\ &+ B_{\varphi} \left(\frac{mR(R^{5} + 2R^{4}m - 2R^{2}a^{2}m)}{R^{2}(R^{6} - 4R^{4}m^{2} + 4R^{2}a^{2}m^{2})} - \frac{m}{R(R + 2m)} \right. \\ &- \frac{m}{R(R + 2m)} \frac{R^{3}m - R^{4} + 2R^{2}m^{2} + 2a^{2}m^{2}}{(R^{4} - 4R^{2}m^{2} + 4a^{2}m^{2})} \left(\frac{R^{2}}{R^{2} + 2mR + 4m^{2}} \right) \end{split}$$

$$\begin{split} \left(\frac{\partial V^R}{\partial R}\right) &= -\frac{1}{(u^0)^2} \frac{B_{\varphi}}{2\pi} \left(\frac{R^2}{R^2 + 2mR + 4m^2}\right) \left\{ \left(-\frac{B_{\varphi}}{4\pi} \left(1\right) \right) - \left(\frac{R^2}{4\pi} \left(1\right) - \left(\frac{R^2}{2} + KR^{\frac{5}{2}}\right) 2V^R u^R u^0\right) \left\{ B_{\varphi} \frac{R}{\partial R} \left(\frac{R^2}{R^2 + 2mR + 4m^2}\right) + B_{\varphi} \left(\frac{mR(R^5 + 2R^4m - 2R^2\alpha^m)}{R^2(R^5 - 4R^4m^2 + 4R^2\alpha^2m^2)} - \frac{m}{R(R + 2m)} - \frac{m}{R(R + 2m)} - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{\left(R^4 - 4R^2m^2 + 4a^2m^2\right)} \right) \left(\frac{R^2}{R^2 + 2mR + 4m^2}\right) \right\} - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}}\right) \left[\left\{ V^R \left(\left[I_{RR}^R + \Gamma_{\varphi R}^{\varphi} + \Gamma_{zR}^Z \right] - \Gamma_{r0}^0 \right) \right\} + \left\{ V^R \left(\left[\Gamma_{rz}^r + \Gamma_{\varphi z}^{\varphi} + \Gamma_{zz}^z \right] - \Gamma_{r0}^0 \right) \right\} - \left\{ \Gamma_{r\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^R \right\} \right] \right) \right\} - \left\{ \Gamma_{r\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^{\varphi} - \Gamma_{R\varphi}^0 V^R V^R + 2V^{\varphi} \Gamma_{\varphi \varphi}^R + V^t V^R \left(\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R \right) + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^R V^{\varphi} - \Gamma_{R\varphi}^0 V^R V^R V^{\varphi} V^R + V^{\varphi} V^{\varphi} \Gamma_{\varphi \varphi}^R \right) \right\} \right\} - \left\{ \Gamma_{r\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^{\varphi} V^R V^{\varphi} V^{\varphi} - \Gamma_{r\varphi}^0 V^R V^{\varphi} V^R + V^{\varphi} V^{\varphi} \Gamma_{\varphi \varphi}^R \right\} - \left\{ \Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^{\varphi} V^R V^{\varphi} V^R + V^{\varphi} V^R V^{\varphi} V^R + V^{\varphi} V^{\varphi} \Gamma_{\varphi \varphi}^R \right\} \right\} - \left\{ \Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^R \right\} - \left\{ \Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^R \right\} - \left\{ \Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^R \right\} \right\} - \left\{ \Gamma_{R\varphi}^0 V^{\varphi} V^R + \Gamma_{\varphi R}^0 V^R V^{\varphi} V^R \right\} - \left\{ R^2 \left(R^2 + 2mR + 4m^2\right) + R_{\varphi} \left(\frac{R^2}{R^2 (R^6 - 4R^4m^2 + 4R^2\alpha^2m^2)} - \frac{m}{R(R + 2m)} - \frac{m}{R(R + 2m)} \right\} \right\} \right\} - \frac{m}{R(R + 2m)} \frac{R^3}{(R^3 - 4R^2m^2 + 4R^2m^2 + 2a^2m^2} + 2a^2m^2}{R(R + 2m)} \left\{ \frac{R^2}{R^2 + 2mR + 4m^2} \right\} \right\} \right\}$$

$$\begin{split} \frac{\partial V^{\varphi}}{\partial R} &= \left(V^R \left(R^{\frac{3}{2}} + K R^{\frac{5}{2}} \right) (u^0)^2 \right)^{-1} \left\{ - \left(R^{3/2} + K R^{5/2} \right) (u^0)^2 \left[2 V^R \left(\Gamma_{tR}^{\varphi} - \Gamma_{tR}^t V^{\varphi} \right) \right. \right. \\ &\left. + V^t V^R \left(\Gamma_{Rt}^{\varphi} - \Gamma_{Rt}^t V^{\varphi} \right) + V^R V^{\varphi} \left(\Gamma_{\varphi R}^{\varphi} - \Gamma_{\varphi R}^t V^{\varphi} \right) + V^{\varphi} V^R \left(\Gamma_{R\varphi}^{\varphi} - \Gamma_{R\varphi}^t V^{\varphi} \right) \right] \right\} \end{split}$$

$$\begin{split} \frac{\partial B_{\varphi}(R)}{\partial R} &= \left(-\frac{B_{\varphi}}{4\pi} \left(1 - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) 2V^R u^R u^0 \right) \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right)^{-1} \left\{ \frac{B_{\varphi}}{4\pi} \left(1 \right) \right. \\ & \left. - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) 2V^R u^R u^0 \right) \left\{ B_{\varphi} \frac{\partial}{\partial R} \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right. \\ & \left. + B_{\varphi} \left(\frac{mR(R^5 + 2R^4m - 2R^2a^2m)}{R^2(R^6 - 4R^4m^2 + 4R^2a^2m^2)} - \frac{m}{R(R + 2m)} \right. \\ & \left. - \frac{m}{R(R + 2m)} \frac{R^3m - R^4 + 2R^2m^2 + 2a^2m^2}{\left(R^4 - 4R^2m^2 + 4a^2m^2 \right)} \right) \left(\frac{R^2}{R^2 + 2mR + 4m^2} \right) \right\} \\ & \left. - \left(R^{\frac{3}{2}} \right. \right. \\ & \left. + KR^{\frac{5}{2}} \right) (u^0)^2 \left(\left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right)^{-1} \left(-V^R \frac{\partial}{\partial R} \left(R^{\frac{3}{2}} - KR^{\frac{5}{2}} \right) \right. \\ & \left. - \left(R^{\frac{3}{2}} + KR^{\frac{5}{2}} \right) \left[\left\{ V^R \left(\left[\Gamma_{RR}^R + \Gamma_{\varphi\varphi}^\theta + \Gamma_{ZR}^Z \right] - \Gamma_{00}^0 \right) \right\} + \left\{ V^R \left(\left[\Gamma_{rz}^r + \Gamma_{\varphi\varphi}^\phi + \Gamma_{ZZ}^z \right] - \Gamma_{20}^0 \right) \right\} \right. \\ & \left. - \left(R^{3/2} + KR^{5/2} \right) (u^0)^2 \left[\left(\Gamma_{00}^R \right) - 2\Gamma_{0R}^0 V^R V^R + 2V^\varphi \Gamma_{0\varphi}^R + V^t V^R \left(\Gamma_{Rt}^t - \Gamma_{Rt}^0 V^R \right) \right. \\ & \left. + V^R V^R \Gamma_{RR}^R - \Gamma_{\varphi R}^0 V^R V^R V^\varphi - \Gamma_{R\varphi}^0 V^R V^\varphi V^R + V^\varphi V^\varphi \Gamma_{\varphi\varphi}^R \right] \right\} \end{split}$$