

بنابراین برای معادلات جت داریم:

$$g^{\mu\nu}_{jet} = \begin{pmatrix} \frac{z}{z - \gamma m} & \cdot & \frac{z^\gamma}{\gamma m a} & \cdot \\ \cdot & -\frac{z}{z + \gamma m} & \cdot & \cdot \\ \frac{z^\gamma}{\gamma m a} & \cdot & -\frac{z}{z + \gamma m} & \cdot \\ \cdot & \cdot & \cdot & -\frac{z}{z + \gamma m} \end{pmatrix}$$

ضرایب کریستوفل در جت به این ترتیب است $(\gamma = t, \gamma = R, \gamma = \varphi, \xi = z)$:

$$(\gamma, \gamma, \gamma) = \frac{m(z + \gamma z^\gamma + z^\circ + \gamma m - \gamma a^\gamma m + \xi m z^\gamma + \xi a^\gamma m z^\gamma + \gamma z^\xi m)}{z^\gamma (\gamma - \xi m^\gamma + \gamma z^\gamma + \xi a^\gamma m^\gamma - \lambda m^\gamma z^\gamma + \gamma z^\xi - \xi m^\gamma z^\xi + z^\gamma)}$$

$$(\gamma, \gamma, \xi) = \frac{m z (z + \gamma z^\gamma + z^\circ + \gamma m - \gamma a^\gamma m + \xi m z^\gamma + \gamma z^\xi m)}{z^\gamma (\gamma - \xi m^\gamma + \gamma z^\gamma + \xi a^\gamma m^\gamma - \lambda m^\gamma z^\gamma + \gamma z^\xi - \xi m^\gamma z^\xi + z^\gamma)}$$

$$(\gamma, \gamma, \gamma) = \frac{m(z + \gamma z^\gamma + z^\circ + \gamma m - \gamma a^\gamma m + \xi m z^\gamma + \xi a^\gamma m z^\gamma + \gamma z^\xi m)}{z^\gamma (\gamma - \xi m^\gamma + \gamma z^\gamma + \xi a^\gamma m^\gamma - \lambda m^\gamma z^\gamma + \gamma z^\xi - \xi m^\gamma z^\xi + z^\gamma)}$$

$$(\gamma, \gamma, \gamma) = -\frac{m a (\gamma z^\gamma + z - \gamma z^\gamma + \xi m)}{\gamma - \xi m^\gamma + \gamma z^\gamma + \xi a^\gamma m^\gamma - \lambda m^\gamma z^\gamma + \gamma z^\xi - \xi m^\gamma z^\xi + z^\gamma}$$

$$(\gamma, \gamma, \gamma) = -\frac{m a (\gamma z^\gamma + z - \gamma z^\gamma + \xi m)}{\gamma - \xi m^\gamma + \gamma z^\gamma + \xi a^\gamma m^\gamma - \lambda m^\gamma z^\gamma + \gamma z^\xi - \xi m^\gamma z^\xi + z^\gamma}$$

$$(\gamma, \gamma, \xi) = -\frac{m a z (\gamma z + \xi m)}{\gamma - \xi m^\gamma + \gamma z^\gamma + \xi a^\gamma m^\gamma - \lambda m^\gamma z^\gamma + \gamma z^\xi - \xi m^\gamma z^\xi + z^\gamma}$$

$$(\mathfrak{I}, \xi, \mathfrak{I}) = \frac{mz(z + \mathfrak{I}z^{\mathfrak{I}} + z^{\circ} + \mathfrak{I}m - \mathfrak{I}a^{\mathfrak{I}}m + \xi mz^{\mathfrak{I}} + \mathfrak{I}z^{\xi}m)}{z^{\mathfrak{I}}(\mathfrak{I} - \xi m^{\mathfrak{I}} + \mathfrak{I}z^{\mathfrak{I}} + \xi a^{\mathfrak{I}}m^{\mathfrak{I}} - \mathfrak{I}m^{\mathfrak{I}}z^{\mathfrak{I}} + \mathfrak{I}z^{\xi} - \xi m^{\mathfrak{I}}z^{\xi} + z^{\mathfrak{I}})}$$

$$(\mathfrak{I}, \xi, \mathfrak{I}) = -\frac{maz(\mathfrak{I}z + \xi m)}{\mathfrak{I} - \xi m^{\mathfrak{I}} + \mathfrak{I}z^{\mathfrak{I}} + \xi a^{\mathfrak{I}}m^{\mathfrak{I}} - \mathfrak{I}m^{\mathfrak{I}}z^{\mathfrak{I}} + \mathfrak{I}z^{\xi} - \xi m^{\mathfrak{I}}z^{\xi} + z^{\mathfrak{I}}}$$

$$(\mathfrak{I}, \mathfrak{I}, \mathfrak{I}) = \frac{m}{z^{\mathfrak{I}}(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \mathfrak{I}, \mathfrak{I}) = \frac{\mathfrak{I}ma}{z^{\mathfrak{I}}(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \mathfrak{I}, \mathfrak{I}) = -\frac{m}{z^{\mathfrak{I}}(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \mathfrak{I}, \xi) = -\frac{m}{z(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \mathfrak{I}, \mathfrak{I}) = -\frac{-\mathfrak{I}ma}{z^{\mathfrak{I}}(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \mathfrak{I}, \mathfrak{I}) = -\frac{(z^{\mathfrak{I}} + m + \mathfrak{I}mz^{\mathfrak{I}})}{z^{\mathfrak{I}}(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \xi, \mathfrak{I}) = -\frac{m}{z(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \xi, \xi) = \frac{m}{z^{\mathfrak{I}}(z + \mathfrak{I}m)}$$

$$(\mathfrak{I}, \mathfrak{I}, \mathfrak{I}) = \frac{am(z - \mathfrak{I}z^{\mathfrak{I}} + \xi mz^{\mathfrak{I}})}{(\mathfrak{I} - \xi m^{\mathfrak{I}} + \mathfrak{I}z^{\mathfrak{I}} + \xi a^{\mathfrak{I}}m^{\mathfrak{I}} - \mathfrak{I}m^{\mathfrak{I}}z^{\mathfrak{I}} + \mathfrak{I}z^{\xi} - \xi m^{\mathfrak{I}}z^{\xi} + z^{\mathfrak{I}})}$$

$$(\mathfrak{I}, \mathfrak{I}, \xi) = \frac{amz(-\xi m + \mathfrak{I}z)}{\mathfrak{I} - \xi m^{\mathfrak{I}} + \mathfrak{I}z^{\mathfrak{I}} + \xi a^{\mathfrak{I}}m^{\mathfrak{I}} - \mathfrak{I}m^{\mathfrak{I}}z^{\mathfrak{I}} + \mathfrak{I}z^{\xi} - \xi m^{\mathfrak{I}}z^{\xi} + z^{\mathfrak{I}}}$$

$$(\mathfrak{r}, \mathfrak{r}, \mathfrak{l}) = \frac{am(z - \mathfrak{r}z^{\mathfrak{r}} + \xi m z^{\mathfrak{r}})}{\mathfrak{l} - \xi m^{\mathfrak{r}} + \mathfrak{r}z^{\mathfrak{r}} + \xi a^{\mathfrak{r}}m^{\mathfrak{r}} - \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{r}z^{\xi} - \xi m^{\mathfrak{r}}z^{\xi} + z^{\mathfrak{l}}}$$

$$(\mathfrak{r}, \mathfrak{r}, \mathfrak{r}) = -(\mathfrak{r}mz^{\mathfrak{r}} + \xi z^{\circ}m + \mathfrak{r}z^{\mathfrak{v}}m - mz - \xi z^{\mathfrak{r}}m - \circ z^{\circ}m - \mathfrak{r}z^{\mathfrak{v}}m - \mathfrak{l} + \mathfrak{r}m^{\mathfrak{r}} - \xi z^{\mathfrak{r}} + \mathfrak{r}a^{\mathfrak{r}}m^{\mathfrak{r}} + \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} - \mathfrak{r}z^{\xi} - \xi a^{\mathfrak{r}}m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{l} \cdot m^{\mathfrak{r}}z^{\xi} - \xi z^{\mathfrak{l}} + \xi m^{\mathfrak{r}}z^{\mathfrak{l}} - z^{\wedge})$$

$$/((\mathfrak{l} + z^{\mathfrak{r}})(\mathfrak{l} - \xi m^{\mathfrak{r}} + \mathfrak{r}z^{\mathfrak{r}} + \xi a^{\mathfrak{r}}m^{\mathfrak{r}} - \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{r}z^{\xi} - \xi m^{\mathfrak{r}}z^{\xi} + z^{\mathfrak{l}}))$$

$$(\mathfrak{r}, \mathfrak{r}, \mathfrak{r}) = -(\mathfrak{r}mz^{\mathfrak{r}} + \xi z^{\circ}m + \mathfrak{r}z^{\mathfrak{v}}m - mz - \xi z^{\mathfrak{r}}m - \circ z^{\circ}m - \mathfrak{r}z^{\mathfrak{v}}m - \mathfrak{l} + \mathfrak{r}m^{\mathfrak{r}} - \xi z^{\mathfrak{r}} + \mathfrak{r}a^{\mathfrak{r}}m^{\mathfrak{r}} + \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} - \mathfrak{r}z^{\xi} - \xi a^{\mathfrak{r}}m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{l} \cdot m^{\mathfrak{r}}z^{\xi} - \xi z^{\mathfrak{l}} + \xi m^{\mathfrak{r}}z^{\mathfrak{l}} - z^{\wedge})$$

$$/((\mathfrak{l} + z^{\mathfrak{r}})(\mathfrak{l} - \xi m^{\mathfrak{r}} + \mathfrak{r}z^{\mathfrak{r}} + \xi a^{\mathfrak{r}}m^{\mathfrak{r}} - \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{r}z^{\xi} - \xi m^{\mathfrak{r}}z^{\xi} + z^{\mathfrak{l}}))$$

$$(\mathfrak{r}, \mathfrak{r}, \xi) = -\frac{mz(z + \mathfrak{r}z^{\mathfrak{r}} + z^{\circ} - \mathfrak{r}m + \mathfrak{r}a^{\mathfrak{r}}m - \xi m z^{\mathfrak{r}} - \mathfrak{r}m z^{\xi})}{(\mathfrak{l} + z^{\mathfrak{r}})(\mathfrak{l} - \xi m^{\mathfrak{r}} + \mathfrak{r}z^{\mathfrak{r}} + \xi a^{\mathfrak{r}}m^{\mathfrak{r}} - \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{r}z^{\xi} - \xi m^{\mathfrak{r}}z^{\xi} + z^{\mathfrak{l}})}$$

$$(\mathfrak{r}, \xi, \mathfrak{l}) = \frac{amz(-\xi m + \mathfrak{r}z)}{\mathfrak{l} - \xi m^{\mathfrak{r}} + \mathfrak{r}z^{\mathfrak{r}} + \xi a^{\mathfrak{r}}m^{\mathfrak{r}} - \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{r}z^{\xi} - \xi m^{\mathfrak{r}}z^{\xi} + z^{\mathfrak{l}}}$$

$$(\mathfrak{r}, \xi, \mathfrak{r}) = \cdot$$

$$(\mathfrak{r}, \xi, \mathfrak{r}) = -\frac{mz(z + \mathfrak{r}z^{\mathfrak{r}} + z^{\circ} - \mathfrak{r}m + \mathfrak{r}a^{\mathfrak{r}}m - \xi m z^{\mathfrak{r}} - \mathfrak{r}m z^{\xi})}{(\mathfrak{l} + z^{\mathfrak{r}})(\mathfrak{l} - \xi m^{\mathfrak{r}} + \mathfrak{r}z^{\mathfrak{r}} + \xi a^{\mathfrak{r}}m^{\mathfrak{r}} - \lambda m^{\mathfrak{r}}z^{\mathfrak{r}} + \mathfrak{r}z^{\xi} - \xi m^{\mathfrak{r}}z^{\xi} + z^{\mathfrak{l}})}$$

$$(\xi, \mathfrak{l}, \mathfrak{l}) = \frac{mz}{(\mathfrak{l} + z^{\mathfrak{r}})(z + \mathfrak{r}m)}$$

$$(\xi, \mathfrak{l}, \mathfrak{r}) = -\frac{\mathfrak{r}maz}{(\mathfrak{l} + z^{\mathfrak{r}})^{\mathfrak{r}}(z + \mathfrak{r}m)}$$

$$(\xi, \mathfrak{r}, \mathfrak{r}) = \frac{mz}{(\mathfrak{l} + z^{\mathfrak{r}})(z + \mathfrak{r}m)}$$

$$(\xi, \mathfrak{r}, \xi) = -\frac{mR}{(\mathfrak{l} + z^{\mathfrak{r}})(z + \mathfrak{r}m)}$$

$$(\xi, \mathfrak{r}, \mathfrak{l}) = -\frac{\mathfrak{r}maR^{\mathfrak{r}}z}{(\mathfrak{l} + z^{\mathfrak{r}})^{\mathfrak{r}}(z + \mathfrak{r}m)}$$

$$(\xi, \mathfrak{r}, \mathfrak{r}) = \frac{mz}{(\mathfrak{I} + z^{\mathfrak{r}})^{\mathfrak{r}}(z + \mathfrak{r}m)}$$

$$(\xi, \xi, \mathfrak{r}) = -\frac{m}{(\mathfrak{I} + z^{\mathfrak{r}})(z + \mathfrak{r}m)}$$

$$(\xi, \xi, \xi) = -\frac{mz}{(\mathfrak{I} + z^{\mathfrak{r}})(\sqrt{z} + \mathfrak{r}m)}$$

a.

$$\begin{aligned} g^{zz} \frac{\textcolor{green}{\partial P}}{\partial z} - (\rho + P)(u^{\cdot})^{\mathfrak{r}} [\frac{\textcolor{red}{\partial V^z}}{\partial z} V^z] \\ = (\rho + P)(u^{\cdot})^{\mathfrak{r}} \big[\Gamma_{tt}^z + \mathfrak{r} V^{\varphi} \Gamma_{t\varphi}^z - \mathfrak{r} V^z V^z \Gamma_{tz}^t + V^{\varphi} V^{\varphi} \Gamma_{\varphi\varphi}^z - V^{\varphi} V^z V^z \Gamma_{z\varphi}^t \\ - V^z V^{\varphi} V^z \Gamma_{\varphi z}^t + V^z V^z \Gamma_{zz}^z \big] + \left(\frac{z}{z + \mathfrak{r}m} B_{\varphi} - V^z \frac{z^{\mathfrak{r}}}{\mathfrak{r}ma} B_z \right) J^R \end{aligned}$$

b.

$$\begin{aligned} -(\rho + P) \left[\left(\frac{\textcolor{red}{\partial V^z}}{\partial z} \right) \right] \textcolor{teal}{-} V^z \frac{\partial}{\partial z} (\rho - P) + \left[\frac{\mathfrak{I}}{\varepsilon\pi} \left[B_{\varphi}(z) \frac{\partial}{\partial z} \left(\frac{z^{\mathfrak{r}}}{(z+\mathfrak{r}m)^{\mathfrak{r}}} \right) + \frac{z^{\mathfrak{r}}}{(z+\mathfrak{r}m)^{\mathfrak{r}}} \frac{\partial B_{\varphi}(z)}{\partial z} \right] \right] \left\{ \frac{\mathfrak{r}}{u^{\cdot}} (B_{\varphi} u^z - \right. \\ \left. B_z u^{\varphi}) + \frac{z^{\mathfrak{r}}}{\mathfrak{r}ma(u^{\cdot})^{\mathfrak{r}}} B_z \right\} = +(\rho + P) \left[-\{ \Gamma_{\varphi z}^{\cdot} V^z V^{\varphi} + \Gamma_{z\varphi}^{\cdot} V^{\varphi} V^z \} \right] - \\ \frac{\mathfrak{r}}{u^{\cdot}} u^R \left(B_z \left[\frac{\mathfrak{I}}{\varepsilon\pi} \left[\Gamma_{tR}^{\varphi} \left(\frac{z^{\xi}}{\mathfrak{r}ma(z+\mathfrak{r}m)} \right) B_z + (\Gamma_{tR}^t + \Gamma_{RR}^R + \Gamma_{R\varphi}^{\varphi} + \Gamma_{zR}^z) \left(\frac{z^{\mathfrak{r}}}{(z+\mathfrak{r}m)^{\mathfrak{r}}} \right) B_z \right] \right] - B_{\varphi} \left[\frac{-\mathfrak{I}}{\varepsilon\pi} (\Gamma_{tR}^t + \right. \right. \\ \left. \left. \Gamma_{\varphi R}^{\varphi} + \Gamma_{Rz}^z + \Gamma_{RR}^R) \left(\frac{z^{\mathfrak{r}}}{(z+\mathfrak{r}m)^{\mathfrak{r}}} \right) B_{\varphi} \right] \right) \end{aligned}$$

c.

$$\begin{aligned}
& \left[\frac{1}{\xi\pi} \left[B_\varphi(z) \frac{\partial}{\partial z} \left(\frac{z^\mathfrak{r}}{(z + \mathfrak{r}m)^\mathfrak{r}} \right) + \frac{z^\mathfrak{r}}{(z + \mathfrak{r}m)^\mathfrak{r}} \frac{\partial B_\varphi(z)}{\partial z} \right] \right] \left(V^\varphi \frac{z^\mathfrak{r}}{\mathfrak{r}ma} + \frac{z}{z + \mathfrak{r}m} \right) B_z \\
& - (\rho + P)(u^\cdot)^\mathfrak{r} \left[\frac{\partial V^\varphi}{\partial z} V^z \right] \\
& = (\rho + P)(u^\cdot)^\mathfrak{r} \left[+ \mathfrak{r} V^z (\Gamma_{tz}^\varphi - \Gamma_{tz}^t V^\varphi) + V^\varphi V^z (\Gamma_{z\varphi}^\varphi - \Gamma_{z\varphi}^t V^\varphi) \right. \\
& \quad \left. + V^z V^\varphi (\Gamma_{\varphi z}^\varphi - \Gamma_{\varphi z}^t V^\varphi) \right] \\
& - \left[\frac{1}{\xi\pi} \left[(\Gamma_{\varphi z}^\varphi + \Gamma_{zR}^R + \Gamma_{zz}^z + \Gamma_{tz}^t) \frac{z^\mathfrak{r}}{(z + \mathfrak{r}m)^\mathfrak{r}} B_\varphi \right] \right] \left(V^\varphi \frac{z^\mathfrak{r}}{\mathfrak{r}ma} + \frac{z}{z + \mathfrak{r}m} \right) B_z \\
& + \left[\frac{z}{z + \mathfrak{r}m} - \frac{z^\mathfrak{r}}{\mathfrak{r}ma} + (u^\varphi + u^z) u^\varphi \right] \{ - (\mathfrak{r} \Gamma_{\varphi R}^\varphi + \Gamma_{zR}^R) (-\alpha p) \}
\end{aligned}$$

d.

$$\begin{aligned}
& g^{zz} \frac{\partial P}{\partial z} - (\rho + P)(u^\cdot)^\mathfrak{r} \left[\frac{\partial V^z}{\partial z} V^z \right] \\
& + \left[\frac{1}{\xi\pi} \left[B_\varphi(z) \frac{\partial}{\partial z} \left(\frac{z^\mathfrak{r}}{(z + \mathfrak{r}m)^\mathfrak{r}} \right) + \frac{z^\mathfrak{r}}{(z + \mathfrak{r}m)^\mathfrak{r}} \frac{\partial B_\varphi(z)}{\partial z} \right] \right] \left(\frac{z}{z + \mathfrak{r}m} B_\varphi - V^z \frac{z^\mathfrak{r}}{\mathfrak{r}ma} B_z \right) \\
& = (\rho + P)(u^\cdot)^\mathfrak{r} \left[\Gamma_{tt}^z + \mathfrak{r} V^\varphi \Gamma_{t\varphi}^z - \mathfrak{r} V^z V^z \Gamma_{tz}^t + V^\varphi V^\varphi \Gamma_{\varphi\varphi}^z - V^\varphi V^z V^z \Gamma_{z\varphi}^t \right. \\
& \quad \left. - V^z V^\varphi V^z \Gamma_{\varphi z}^t + V^z V^z \Gamma_{zz}^z \right] \\
& + \left[\frac{1}{\xi\pi} \left[(\Gamma_{\varphi z}^\varphi + \Gamma_{zR}^R + \Gamma_{zz}^z + \Gamma_{tz}^t) \frac{z^\mathfrak{r}}{(z + \mathfrak{r}m)^\mathfrak{r}} B_\varphi \right] \right] \left(\frac{z}{z + \mathfrak{r}m} B_\varphi - V^z \frac{z^\mathfrak{r}}{\mathfrak{r}ma} B_z \right)
\end{aligned}$$
