Y_1

P. 1.

Weighted Die:

x:	1	2	3	4	5	6	
	$3p$	p	p	p	p	$3p$	$= 1 \rightarrow p = \frac{1}{10}$
P	0.3	0.1	0.1	0.1	0.1	0.3	

Mean $\mu = \sum x.P$ $\quad 0.3 + 0.2 + 0.3 + 0.4 + 0.5 + 1.8 = 3.5$

Variance(σ^2) $= [1^2 \cdot 0.3 + 2^2 * 0.1 + 3^2 * 0.1 + 4^2 * 0.1 + 5^2 \cdot 0.1 + 6^2 \cdot 0.3] - 3.5^2$

$= [0.3 + 0.4 + 0.9 + 1.6 + 2.5 + 10.8] - 12.25$

$= 16.5 - 12.25 = 4.25$

$\boxed{\mu = E[X] = 3.5}$ $\quad \boxed{\sigma^2 = E[(X-\mu)^2] = 4.25}$

Fair Die:

x:	1	2	3	4	5	6
P:	1/6	1/6	1/6	1/6	1/6	1/6

$\mu = E(X) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = 3.5$

$\sigma^2 = E(X - \mu^2)]$ $\quad [\frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6}] - 12.25 = 2.917$

$\boxed{\mu = E(X) = 3.5}$ $\quad \boxed{\sigma^2 = E[(X-\mu)^2] = 2.917}$

i) The means of the two Probability distributions are the same, which means that the sum probability of each x should equal to 1.
However, the Variance of the weighted die is higher then the Variance of the fair die, which mean that the roll Probability of the fair die tends to be closer to the mean 3.5

P2

$$P(x|\omega_A)\cdot P(\omega_A) = P(x|\omega_B)\cdot P(\omega_B)$$

$$\frac{P(x|\omega_A)}{P(x|\omega_B)} = \frac{P(\omega_B)}{P(\omega_A)} = \frac{1/4}{3/4} = \frac{4}{12} = \frac{1}{3}$$

$$\frac{P(x|\omega_A)}{P(x|\omega_B)} = \frac{1}{3} \rightarrow P(x|\omega_B) = 3\,P(x|\omega_A)$$

$$\frac{\frac{1}{\sqrt{2\pi}\,\sigma_A}\cdot e^{\frac{-(x-\mu_A)^2}{2\sigma_A^2}}}{\frac{1}{\sqrt{2\pi}\,\sigma_B}\cdot e^{\frac{-(x-\mu_B)^2}{2\sigma_B^2}}} = \frac{1}{3}$$

$$\frac{\sigma_A}{\sigma_B}\cdot\frac{\sigma_B}{\sigma_A}\cdot e^{-\frac{(x-\mu_A)^2}{2\sigma_A^2}+\frac{(x-\mu_B)^2}{2\sigma_B^2}} = \frac{1}{3}\cdot\frac{\sigma_A}{\sigma_B}$$

$$2\cdot\ \frac{(x-\mu_B)^2}{2\sigma_B^2} - \frac{(x-\mu_A)^2}{2\sigma_A^2}\cdot 2 = \ln\left(\frac{1}{3}\cdot\frac{\sigma_A}{\sigma_B}\right)\cdot 2$$

$$\sigma_A^2(x-\mu_B)^2 - \sigma_B^2(x-\mu_A)^2 = 2\cdot\sigma_A^2\sigma_B^2\cdot\ln\left(\frac{1}{3}\,\frac{\sigma_A}{\sigma_B}\right)$$

$$\sigma_A^2(x^2-2x\mu_B+\mu_B^2) - \sigma_B^2(x^2-2x\mu_A+\mu_A^2) =$$

$$\sigma_A^2x^2 - \sigma_A^2 2x\mu_B + \sigma_A^2\mu_B^2 - \sigma_B^2x^2 + \sigma_B^2 2x\mu_A - \sigma_B^2\mu_A^2$$

P. 2

$$\sigma_A^2(x-\mu_B)^2 - \sigma_B^2(x-\mu_A)^2 - 2\sigma_A^2\sigma_B^2 \ln\left(\frac{1}{3}\frac{\sigma_A}{\sigma_B}\right) = 0$$

$$(\sigma_A^2-\sigma_B^2)x^2 + (-2\sigma_A^2\mu_B + 2\sigma_B^2\mu_A)x + (\sigma_A^2\mu_B^2 - \sigma_B^2\mu_A^2) - \ln\left(\frac{1}{3}\cdot\frac{\sigma_A}{\sigma_B}\right) = 0$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$