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ORIGINAL PAPER

Compounding: an R package for computing continuous distributions obtained by compounding a continuous and a discrete distribution

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Abstract In this manuscript we introduce R package `Compounding` for dealing with continuous distributions obtained by compounding continuous distributions with discrete distributions. We demonstrate its use by computing values of cumulative distribution function, probability density function, quantile function and hazard rate function, generating random samples from a population with compounding distribution, and computing mean, variance, skewness and kurtosis of a random variable with a compounding distribution. We consider 24 discrete distributions which can be compounded with any continuous distribution implemented in R.

Keywords Compounding · Compounding by a discrete distribution · Zero truncated distributions

1 Introduction

The R package `Compounding` provides functions for calculating main characteristics of the compounding distributions. `Compounding` (version 1.0) comprises of 82

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functions. The main purpose of this manuscript are: (a) to introduce `Compounding` package to R users, (b) to give background necessary for appropriate usage of the algorithms, (c) to demonstrate usage of the package.

Distributions obtained by compounding a parent distribution with a discrete distribution are very common in statistics and in many applied areas. Some situations giving rise to compound distributions are:

- Suppose a device has an unknown number, N, of initial defects of same kind (for example, a number of semiconductors from a defective lot). Suppose X_i's represent their lifetimes and that each defect can be detected only after causing failure. Then the time to the first failure of the device is $X = \min(X_1, X_2, \ldots, X_N)$.
- Suppose a parallel system has N components. Let $X_1, X_2, \ldots, X_N$ denote their lifetimes. The system will fail as soon as any one of the components fails. The system's lifetime is $X = \min(X_1, X_2, \ldots, X_N)$.

The distribution of interest in each case can be obtained by compounding the distributions of X's and N as

$$\begin{aligned} & \mathsf{P}\left[\min\left(X_1, X_2, \ldots, X_N\right) \leq y\right] \\ & \quad = 1 - \sum_{n=0}^{\infty} \mathsf{P}\left(X_1 > y\right) \mathsf{P}\left(X_2 > y\right) \cdots \mathsf{P}\left(X_n > y\right) \mathsf{P}(N = n), \end{aligned}$$

where we have assumed that the X_i are independent random variables. Adamidis and Loukas (1988) introduced the exponential-geometric distribution by compounding the exponential distribution and the geometric distribution with probability mass function (pmf) given by

$$\mathsf{P}(N = n) = \theta(1-\theta)^{n-1}, \quad n \in \mathbb{N}, \ \theta \in (0, 1).$$

Adamidis et al. (2005) assumed that X_i have the Weibull distribution while N has the geometric distribution.

Barreto-Souza and Bakouch (2010) introduced the exponential-Poisson–Lindley distribution by compounding the exponential and Poisson–Lindley distributions. Tahmasbi and Rezaei (2008) introduced the exponential-logarithmic distribution by compounding the exponential distribution and the logarithmic distribution with pmf given by

$$\mathsf{P}(N = n) = \frac{(1-\theta)^n}{-n \log \theta}, \quad n \in \mathbb{N}.$$

Kuş (2007) introduced the exponential-zero-truncated Poisson distribution by compounding the exponential distribution and the zero-truncated Poisson distribution with pmf

$$\mathsf{P}(N = n) = \frac{e^{-\theta}}{1 - e^{-\theta}} \cdot \frac{\theta^n}{n!}, \quad n \in \mathbb{N}.$$

Chahkandi and Ganjali (2009) introduced the exponential-power series distribution by compounding the exponential distribution and the power series distribution with pmf given by

$$\mathsf{P}(N=n) = \frac{a_n \theta^n}{A(\theta)(1-a)}, \quad n \in \mathbb{N}, \ \theta > 0,$$

where $(1-a)A(\theta) = \sum_{z \in \mathbb{N}} a_z \theta^z$ and $a = a_0/A(\theta)$. Morais and Barreto-Souza (2011) take X_i to be Weibull distributed and N to be a power series random variable.

Marshall and Olkin (1997) show that if N is geometric and if X_i's have a common distribution function belonging to the family

$$\alpha \{1 - F(x)\} / [F(x) + \alpha \{1 - F(x)\}],$$

for some distribution function F then the distribution of $\min(X_1, X_2, \ldots, X_N)$ also belongs to the family. This family of distributions is known as Marshall–Olkin family of distributions. Some distributions belonging to Marshall-Olkin family are: Pareto (Alice and Jose 2003), Weibull (Ghitany et al. 2005), Lomax (Ghitany et al. 2007), Marshall-Olkin semi-Burr and Marshall–Olkin Burr (Jayakumar and Mathew 2008), and q-Weibull (Jose et al. 2010). More information about the Marshall-Olkin family of distributions can be found in Nadarajah (2008).

`Compounding` package provides some R programs for computing some properties of the compounding distributions. Many of the considered distributions have never been introduced or surveyed, so these programs can be used as starting points in researching new compounding distributions. We consider the following discrete distributions:

1. geometric with $\mathsf{P}(N=n) = \theta(1-\theta)^n, n \in \mathbb{N} \cup \{0\}; \theta \in (0,1)$,
2. Poisson with $\mathsf{P}(N=n) = e^{-\theta} \frac{\theta^n}{n!}, n \in \mathbb{N} \cup \{0\}; \theta > 0$,
3. binomial with $\mathsf{P}(N=n) = \binom{m}{n} \theta^n (1-\theta)^{m-n}, n = 0, 1, \ldots, m; \theta \in (0,1)$, $m \in \mathbb{N}$,
4. negative binomial with $\mathsf{P}(N=n) = \binom{k+n-1}{k-1}(1-\theta)^n \theta^k, n \in \mathbb{N} \cup \{0\}; \theta \in (0,1), k > 0$,
5. logarithmic with $\mathsf{P}(N=n) = -\frac{(1-\theta)^n}{n \log \theta}, n \in \mathbb{N}; \theta \in (0,1)$,
6. binomial-binomial with $p_1 \in (0,1)$, $p_2 \in (0,1)$, $m \in \mathbb{N}$, $k \in \mathbb{N}$ and

$$\begin{aligned}\mathsf{P}(N=n) \\ = \sum_{j \geq n/k} \binom{m}{j} p_1^j (1-p_1)^{m-j} \binom{kj}{n} p_2^n (1-p_2)^{kj-n}, \quad n = 0, 1, \ldots, mk,\end{aligned}$$

7. binomial-Poisson with $\theta > 0$, $p \in (0,1)$, $k \in \mathbb{N}$ and

$$\mathsf{P}(N=n) = \sum_{j=0}^{k} \binom{k}{j} p^j (1-p)^{k-j} e^{-j\theta} (j\theta)^n / n!, \quad n \in \mathbb{N} \cup \{0\},$$

8. Poisson-binomial with $\theta > 0$, $p \in (0, 1)$, $k \in \mathbb{N}$ and

$$\mathsf{P}(N = n) = \sum_{j=0}^{\infty} \frac{(kj)!p^n(1-p)^{kj-n}}{n!(kj-n)!} \cdot \frac{\theta^j e^{-\theta}}{j!} \quad n \in \mathbb{N} \cup \{0\},$$

9. Neyman type A with $\theta > 0$, $\lambda > 0$ and

$$\mathsf{P}(N = n) = \frac{e^{-\lambda}\theta^n}{n!} \sum_{j=0}^{\infty} \frac{\left(\lambda e^{-\theta}\right)^j j^n}{j!}, \quad n \in \mathbb{N} \cup \{0\},$$

10. Neyman type B with $\theta > 0$, $\lambda > 0$ and

$$\begin{aligned} \mathsf{P}(N = 0) &= e^{\lambda\, {}_1F_1[1;2;-\theta]-\lambda}, \\ \mathsf{P}(N=n+1) &= \frac{\lambda}{n+1} \sum_{j=0}^{n} \frac{\theta^{j+1}(j+1)\, {}_1F_1[j+2;\, j+3;\, -\theta]}{(j+2)!} \mathsf{P}(N = n-j), \\ &\quad n \in \mathbb{N} \cup \{0\}, \end{aligned}$$

where ${}_1F_1[\cdot;\ \cdot;\ \cdot]$ denotes Kummer function of the first kind.

11. Neyman type C with $\theta > 0$, $\lambda > 0$ and

$$\begin{aligned} \mathsf{P}(N = 0) &= e^{\lambda\, {}_1F_1[1;3;-\theta]-\lambda}, \\ \mathsf{P}(N = n+1) &= \frac{2\lambda}{n+1} \sum_{j=0}^{n} \frac{\theta^{j+1}(j+1)\, {}_1F_1[j+2;\, j+4;\, -\theta]}{(j+3)!} \mathsf{P}(N = n-j), \\ &\quad n \in \mathbb{N} \cup \{0\}, \end{aligned}$$

12. Pólya-Aeppli with $\theta > 0$, $p \in (0, 1)$ and

$$\begin{aligned} \mathsf{P}(N = 0) &= e^{-\theta}, \\ \mathsf{P}(N = n) &= e^{-\theta} \left(\frac{\theta(1-p)}{p}\right) p^n\, {}_1F_1\left[1-n;\, 2;\, -\theta(1-p)/p\right], \quad n \in \mathbb{N}, \end{aligned}$$

13. Poisson-Pascal with $\theta > 0$, $p > 0$, $k > 0$ and

$$\begin{aligned} \mathsf{P}(N = 0) &= e^{\theta\left((1+p)^{-k}-1\right)}, \\ \mathsf{P}(N = n) &= \frac{e^{-\theta} p^n}{(1+p)^n n!} \sum_{j=1}^{\infty} \frac{(kj+n-1)!}{(kj-1)!j!} \left(\theta(1+p)^{-k}\right)^j, \quad n \in \mathbb{N}, \end{aligned}$$

14. Pascal-Poisson with $\theta > 0$, $\mu > 0$, $a > 0$ and

$$\mathsf{P}(N=0) = \left(1 + \frac{\mu}{a\theta} - \frac{\mu}{a\theta}e^{-\theta}\right)^{-a},$$
$$\mathsf{P}(N=n) = \left(\frac{a\theta}{a\theta+\mu}\right)^{a} \frac{\theta^n}{\Gamma(a)\Gamma(n+1)} \sum_{j=1}^{\infty} \frac{\Gamma(a+j)}{\Gamma(j+1)} \left(\frac{\mu}{a\theta+\mu}\right)^{j} j^z e^{-j\theta},$$
$$n \in \mathbb{N},$$

15. logarithmic-binomial with $\theta \in (0, 1)$, $p \in (0, 1)$, $k \in \mathbb{N}$ and

$$\mathsf{P}(N=n) = (-\log\theta)^{-1} \sum_{j \geq n/k} \frac{(1-\theta)^j}{j} \binom{kj}{n} p^n (1-p)^{kj-n}, \quad n \in \mathbb{N} \cup \{0\},$$

16. logarithmic-Poisson with $\theta \in (0, 1)$, $\lambda > 0$ and

$$\mathsf{P}(N=n) = (-\log\theta)^{-1} \frac{\lambda^n}{n!} \sum_{j=1}^{\infty} j^{n-1} \left((1-\theta)e^{-\lambda}\right)^{j}, \quad n \in \mathbb{N} \cup \{0\},$$

17. Poisson-Lindley with $\mathsf{P}(N=n) = \theta^2(\theta+2+n)(\theta+1)^{-n-3}$, $n \in \mathbb{N}$; $\theta > 0$,
18. Hyper-Poisson with $\theta > 0$, $\lambda > 0$ and

$$\mathsf{P}(N=n) = \frac{1}{{}_1F_1[1; \lambda; \theta]} \cdot \frac{1}{(\lambda)_n} \cdot \frac{\theta^n}{n!}, \quad z \in \mathbb{N} \cup \{0\},$$

19. Yule with $\mathsf{P}(N=n) = \frac{\theta(\theta!)n!}{(n+\theta+1)!}$, $n \in \mathbb{N} \cup \{0\}$; $\theta > 0$,
20. Waring with $\mathsf{P}(N=n) = \frac{(c-a)(a+n-1)!c!}{c(a-1)!(c+n)!}$, $n \in \mathbb{N} \cup \{0\}$; $c > a > 0$,
21. Katti type H_1 with $\theta > 0$, $a > 0$, $b > 0$ and

$$\mathsf{P}(N=n) = \frac{a \cdots (a+n-1)\theta^n}{(a+b) \cdots (a+b+n-1)z!}$$
$$\times {}_1F_1[a+n; a+b+n; -\theta], \quad n \in \mathbb{N} \cup \{0\},$$

22. Katti type H_2 with $\theta > 0$, $a > 0$, $b > 0$, $k > 0$, $n \in \mathbb{N} \cup \{0\}$ and

$$\mathsf{P}(N=n) = \binom{k+n-1}{n} \frac{\theta^n (a+n-1)!(a+b-1)!}{(a-1)!(a+b+n-1)!}$$
$$\times {}_2F_1[k+n, a+n; a+b+n; -\theta],$$

where ${}_2F_1[\,\cdot;\ \cdot;\ \cdot]$ denotes Gauss hypergeometric function,

23. hypergeometric with

$$\mathsf{P}(N=n)=\binom{mp}{n}\binom{m(1-p)}{k-n}\Big/\binom{m}{k},$$

$n=0,1,\ldots,\min(k,mp);\ m\geq k\in\mathbb{N},\ p\in(0,1)$,

24. Thomas with $\lambda>0,\theta>0$ and

$$\mathsf{P}(N=n)=\frac{e^{-\lambda}}{n!}\sum_{j=1}^{n}\binom{n}{j}\left(\lambda e^{-\theta}\right)^{j}(j\theta)^{n-j},\quad n\in\mathbb{N}\cup\{0\}.$$

More information about these discrete distributions can be found in Johnson et al. (1992). In Sect. 2 we explain the programs and in Sect. 3 we give some examples.

2 Programs

Let N be a discrete random variable with the pmf given by $\mathsf{P}(N=n)$, $n\in\mathbb{N}\cup\{0\}$. Denote by $\Phi(s)=E(s^N)=\sum_{n=0}^{\infty}s^n\mathsf{P}(N=n)$, $|s|\leq 1$, the probability generating function of the random variable N. Then the zero-truncated distribution of the random variable Z obtained by removing zero from N has the pmf given by

$$\mathsf{P}(Z=z)=\mathsf{P}(N=z)/\left(1-\Phi(0)\right),\quad z\in\mathbb{N}.$$

Of course, if N is positive random variable, then $\Phi(0)=0$. Let F be the parent distribution. Then the survival function of the random variable $X=\min(X_1,X_2,\ldots,X_Z)$ obtained by compounding is given by

$$\begin{aligned}\overline{G}(x)&=\mathsf{P}\left(\min\left(X_1,X_2,\ldots,X_Z\right)>x\right)\\&=\sum_{n=1}^{\infty}\mathsf{P}\left(\min\left(X_1,X_2,\ldots,X_n\right)>x\right)\mathsf{P}(Z=n)\\&=(1-\Phi(0))^{-1}\sum_{n=1}^{\infty}\left(\overline{F}(x)\right)^n\mathsf{P}(N=n)\\&=\frac{\Phi\left(\overline{F}(x)\right)-\Phi(0)}{1-\Phi(0)}.\end{aligned}$$

The corresponding cdf and pdf are given by

$$G(x)=\frac{1-\Phi\left(\overline{F}(x)\right)}{1-\Phi(0)},\tag{1}$$

$$g(x)=\frac{f(x)\Phi'\left(\overline{F}(x)\right)}{1-\Phi(0)}.\tag{2}$$

By inverting the Eq. (1), we arrive at the quantile function

$$G^{-1}(p) = F^{-1}\left(1 - \Phi^{-1}\left(1 - p\left(1 - \Phi(0)\right)\right)\right), \quad p \in (0, 1), \tag{3}$$

where F^{-1} is the quantile function of the parent distribution F. The function $G^{-1}(p)$ can be used also for generating random samples. By means of (2), using substitution $u = F(x)$ and properties of cdf, we obtain an expression for the moment of order $k \in \mathbb{N}$

$$\begin{aligned} \mathsf{E}\left(X^k\right) &= \int_a^b x^k g(x)dx = \frac{1}{1-\Phi(0)} \int_a^b x^k \Phi'\left(\overline{F}(x)\right) f(x)dx \\ &= \frac{1}{1-\Phi(0)} \int_0^1 \left(F^{-1}(u)\right)^k \Phi'(1-u)du. \end{aligned} \tag{4}$$

In the following paragraphs we will review new R functions needed for compound distributions.

Since the final distribution depends on the parent and compounding distributions, we will use two string arguments `parent` and `compound`. The argument `parent` represents the parent distribution and the argument `compound` represents the compounding distribution. Possible values for the argument `compound` are given in Table 1. Parameters of the compounding distribution are given in `params`. The parameters of the parent distribution F can be provided as additional parameters `...`. For each discrete distribution we give three functions, `pgf<name>`, `pgfD<name>` and `pgfI<name>`, where `<name>` is the name of a discrete distribution. The function `pgf<name>` calculates values of the pgf. The functions `pgfD<name>` and `pgfI<name>` calculate values of the first derivative and inverse, respectively, of the given pgf. Before invoking any function described here, package `Compounding` should be installed and loaded. It could be done using `require(Compounding)`.

The package `Compounding` provides the following functions:

- The function `pCompound(x, parent, compound, compoundDist, params, ...)` calculates values of the cdf of a random variable X by means of (1).
- The function `dCompound(x, parent, compound, compoundDist, params, ...)` calculates values of the pdf of a random variable X by means of (2).
- The function `hCompound(x, parent, compound, compoundDist, params,...)` calculates values of hazard rate function of random variable X defined by $h(x) = g(x)/(1 - G(x))$.
- The function `qCompound(p,parent,compound,compoundDist, params,...)` calculates values of quantile function of random variable X by means of (3).

Table 1 PGFs of the 24 discrete distributions used to generate continuous distributions

Compounding distribution	PGF	Parameters
"geometric"	$\theta(1-(1-\theta)s)^{-1}$	$\theta \in (0,1)$
"poisson"	$\exp(\theta(s-1))$	$\theta > 0$
"binomial"	$(1-\theta+\theta s)^n$	$\theta \in (0,1), n \in \mathbb{N}$
"negativebinomial"	$\theta^k(1-(1-\theta)s)^{-k}$	$\theta \in (0,1), k > 0$
"logarithmic"	$\log(1-(1-\theta)s)/\log\theta$	$\theta \in (0,1)$
"binomialbinomial"	$(1-p_1+p_1(1-p_2+p_2s)^n)^m$	$p_1, p_2 \in (0,1), m, n \in \mathbb{N}$
"binomialpoisson"	$\left(1-p+pe^{\theta(s-1)}\right)^n$	$\theta > 0, p \in (0,1), n \in \mathbb{N}$
"poissonbinomial"	$e^{\theta((1-p+ps)^n-1)}$	$\theta > 0, p \in (0,1), n \in \mathbb{N}$
"neymantypea"	$\exp(\lambda(\exp(\theta(s-1))-1))$	$\theta > 0, \lambda > 0$
"neymantypeb"	$\exp(\lambda({}_1F_1[1;2;\theta(s-1)]-1))$	$\theta > 0, \lambda > 0$
"neymantypec"	$\exp(\lambda({}_1F_1[1;3;\theta(s-1)]-1))$	$\theta > 0, \lambda > 0$
"polyaaeppli"	$\exp\left(\frac{\theta}{p}\left(\frac{1-p}{1-ps}-1\right)\right)$	$\theta > 0, p \in (0,1)$
"poissonpascal"	$\exp\left(\theta((1+p-ps)^{-k}-1)\right)$	$\theta > 0, p > 0, k > 0$
"pascalpoisson"	$\left(1+\frac{\mu}{a\theta}-\frac{\mu}{a\theta}e^{\theta(s-1)}\right)^{-a}$	$\theta > 0, \mu > 0, a > 0$
"logarithmicbinomial"	$\frac{\log(1-(1-\theta)(1-p+ps)^n)}{\log\theta}$	$\theta, p \in (0,1), n \in \mathbb{N}$
"logarithmicpoisson"	$\frac{\log(1-(1-\theta)\exp(\lambda(s-1)))}{\log\theta}$	$\theta \in (0,1), \lambda > 0$
"poissonlindley"	$\frac{\theta^2(\theta+2-s)}{(\theta+1)(\theta+1-s)^2}$	$\theta > 0$
"hyperpoisson"	$\frac{{}_1F_1[1;\lambda;\theta s]}{{}_1F_1[1;\lambda;\theta]}$	$\theta > 0, \lambda > 0$
"yule"	$\frac{\theta}{\theta+1}\,{}_2F_1[1,1;\theta+2;s]$	$\theta > 0$
"waring"	$\frac{c-a}{c}\,{}_2F_1[1,a;c+1;s]$	$c > a > 0$
"kattitypeh1"	${}_1F_1[a;a+b;\theta(s-1)]$	$\theta, a, b > 0$
"kattitypeh2"	${}_2F_1[k,a;a+b;\theta(s-1)]$	$\theta, a, b, k > 0$
"hypergeometric"	${}_2F_1[-n,-mp;-m;1-s]$	$m \geq n \in \mathbb{N}, p \in (0,1)$
"thomas"	$\exp(\lambda(s\exp(\theta(s-1))-1))$	$\lambda, \theta > 0$

- The function `rCompound(n, parent, compound, compoundDist, params, ...)` generates random sample from a distribution obtained by compounding a parent distribution F and a discrete distribution.
- The function `momentCompound(k, parent, compound, compoundDist, params, ...)` calculates moments of the kth order of random variable X using (2) and (4). This function enables easy computation of the mean, variance, skewness and kurtosis.

The package `Compounding` provides studying discrete distributions whose pgfs are not in closed form are studied. Since their pgfs are given in the form of hypergeometric functions, R package `hypergeo`, introduced by Hankin and Lee (2006), is used.

Distributions not having closed forms for their pgfs also do not have closed forms for their inverse pgfs. Let us consider the equation

$$\Psi(x) \equiv \Phi(x) - s = 0, \tag{5}$$

where $s = 1 - p\,(1 - \Phi(0))$ for $p \in [0, 1]$. Since $\Psi(0) = -(1-p)(1-\Phi(0)) < 0$ and $\Psi(1) = p\,(1 - \Phi(0)) > 0$ the root of the Eq. (5) can be tracked in the interval [0, 1]. As input for numerical solution of (5), we need to provide values of the parameter s using the code

```
scalc<-function(p,compound,params){
  phi <- get(paste(``pgf'',compound,sep=`'''), mode = ``function'')
  s<-matrix(nrow=length(p), ncol=length(params))
  for (i in 1:length(p)) {
    for (j in 1:length(params)){
      s[i,j]<-1-p[i]*(1-phi(0,params[j]))
    }
  }
  return(s)
}
```

In cases when distribution depends on more than one parameter it is very common that `params` is given in the form of matrix. In these cases, function `scalc` will take the form

```
scalc<-function(p,compound,params){
  phi <- get(paste(``pgf'',compound,sep=`'''), mode = ``function'')
  s<-matrix(nrow=dim(params)[1], ncol=length(p))
  for (i in 1:dim(params)[1]) {
    for (j in 1:length(p)){
      s[i,j]<-1-p[j]*(1-phi(0,params[i,]))
    }
  }
  return(s)
}
```

With the parameter s we have enabled use of function `uniroot` with arguments `lower = 0` and `upper = 1` for calculation of the inverse pgf.

For the comprehensive description of the studied package we refer reader to the http://cran.r-project.org/web/packages/Compounding/index.html.

3 Examples

In this section, we illustrate use of the R programs for some interesting published examples.

Example 1 Figure 1 plots the CDF of the Weibull-geometric distribution introduced by Marshall and Olkin (1997). It is assumed that the shape and scale parameters are $a = 2$ and $b = 1$, respectively. Also parameter θ takes values from the set $\{0.1, 0.3, 0.5, 0.7, 0.9\}$.

```
data(compoundDist)
parentD <- ``weibull''
compoundD <- ``geometric''
```

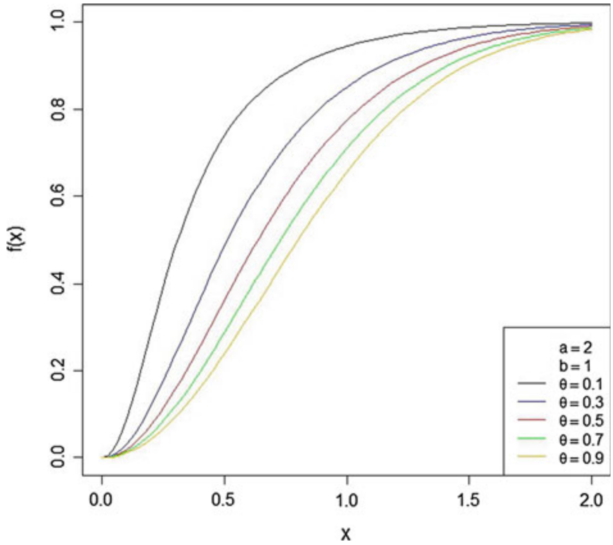

Fig. 1 CDF of the Weibull-geometric distribution

```
curve(pCompound(x, parentD, compoundD, compoundDist,0.1,
shape=2, scale=1), 0, 2, xlab = ''x'', ylab = ''f(x)'',
ylim = c(0, 1),
main=''CDF of Weibull-geometric distribution'')
curve(pCompound(x, parentD, compoundD, compoundDist,0.3,
shape=2, scale=1), 0, 2, xlab = ''x'', col = ''blue'',
add = T)
curve(pCompound(x, parentD, compoundD, compoundDist,0.5,
shape=2, scale=1), 0, 2, xlab = ''x'', col = ''red'',
add = T)
curve(pCompound(x, parentD, compoundD, compoundDist,0.7,
shape=2, scale=1), 0, 2, xlab = ''x'', col = ''green'',
add = T)
curve(pCompound(x, parentD, compoundD, compoundDist,0.9,
shape=2, scale=1), 0, 2, xlab = ''x'', col = ''orange'',
add = T)

legend(''bottomright'', c(expression(a == 2),
expression (b == 1), expression(theta == 0.1),
expression(theta == 0.3), expression(theta == 0.5),
expression(theta == 0.7), expression(theta == 0.9)),
lty = c(0, 0, 1, 1, 1, 1, 1),
col = c(''black'', ''black'', ''black'', ''blue'',
''red'', ''green'', ''orange''))
```

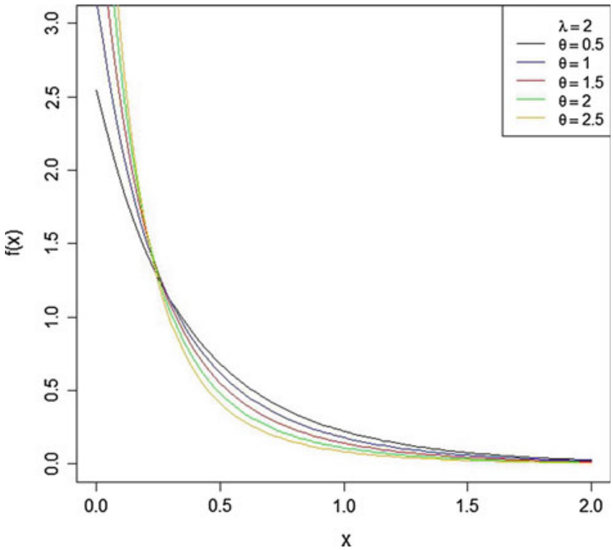

Fig. 2 PDF of the exponential-Poisson distribution

Example 2 Figure 2 plots the PDF of the exponential-Poisson distribution introduced by Kuş (2007). It is assumed that rate parameter $\lambda = 2$. Also parameter θ takes values from the set {0.5, 1, 1.5, 2, 2.5}.

```
data(compoundDist)
parentD <- ``exp''
compoundD <- ``poisson''

curve(dCompound(x, parentD, compoundD, compoundDist,0.5,
rate=2), 0,2, xlab = ``x'', ylab = ``f(x)'', ylim =
c(0,3), main=``PDF of exponential-Poisson distribution'')
curve(dCompound(x, parentD, compoundD, compoundDist,1,
rate=2), 0, 2, xlab = ``x'', col = ``blue'', add = T)
curve(dCompound(x, parentD, compoundD, compoundDist,1.5,
rate=2), 0, 2, xlab = ``x'', col = ``red'', add = T)
curve(dCompound(x, parentD, compoundD, compoundDist,2,
rate=2), 0, 2, xlab = ``x'', col = ``green'', add = T)
curve(dCompound(x, parentD, compoundD, compoundDist,2.5,
rate=2), 0, 2, xlab = ``x'', col = ``orange'', add = T)

legend(``topright'', c(expression(lambda == 2),
expression(theta == 0.5),
expression(theta == 1), expression(theta == 1.5),
expression(theta == 2), expression(theta == 2.5)),
lty = c(0, 1, 1, 1, 1, 1),
col = c(``black'', ``black'', ``blue'', ``red'',
``green'', ``orange''))
```

Example 3 Figure 3 plots the hazard rate function of the exponential-logarithmic distribution introduced by Tahmasbi and Rezaei (2008). It is assumed that parameter θ belongs to the set $\{0.1, 0.3, 0.5, 0.7, 0.9\}$ and the rate parameter λ is equal to 5.

```
data(compoundDist)
parentD <- ''exp''
compoundD <- ''logarithmic''

curve(hCompound(x, parentD, compoundD, compoundDist,0.1,
rate=5), 0, 2, xlab = ''x'', ylab = ''f(x)'', ylim =
c(3, 7), main=''Hazard rate function of the
exponential-logarithmic distribution'')
curve(hCompound(x, parentD, compoundD, compoundDist,0.3,
rate=5), 0, 2, xlab = ''x'', col = ''blue'', add = T)
curve(hCompound(x, parentD, compoundD, compoundDist,0.5,
rate=5), 0, 2, xlab = ''x'', col = ''red'', add = T)
curve(hCompound(x, parentD, compoundD, compoundDist,0.7,
rate=5), 0, 2, xlab = ''x'', col = ''green'', add = T)
curve(hCompound(x, parentD, compoundD, compoundDist,0.9,
rate=5), 0, 2, xlab = ''x'', col = ''orange'', add = T)

legend(''topright'', c(expression(lambda == 5),
expression(theta == 0.1),
expression(theta == 0.3), expression(theta == 0.5),
expression(theta == 0.7), expression(theta == 0.9)),
lty = c(0, 1, 1, 1, 1, 1),
col = c(''black'', ''black'', ''blue'', ''red'',
''green'', ''orange''))
```

Example 4 Let us consider the beta-Yule distribution. In this example, we will compute values of the pgf, as well as its first derivative and inverse. It is assumed that parameter θ belongs to the set $\{2.5, 3.5, 4.5\}$, while both shape parameters are equal to 1. We take the non-centrality parameter to be zero.

Since the pgf of Yule distribution is not in closed form we need to invoke function `scalc`. Let parameter $p \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. We will start by calculating values of the parameter s:

```
p<-seq(0.1,0.9,0.2)
theta<-c(2.5,3.5,4.5)
s<-scalc(p,''yule'',theta)
```

The resulting matrix is

```
          [,1]      [,2]      [,3]
[1,] 0.9714286 0.9777778 0.9818182
[2,] 0.9142857 0.9333333 0.9454545
[3,] 0.8571429 0.8888889 0.9090909
[4,] 0.8000000 0.8444444 0.8727273
```

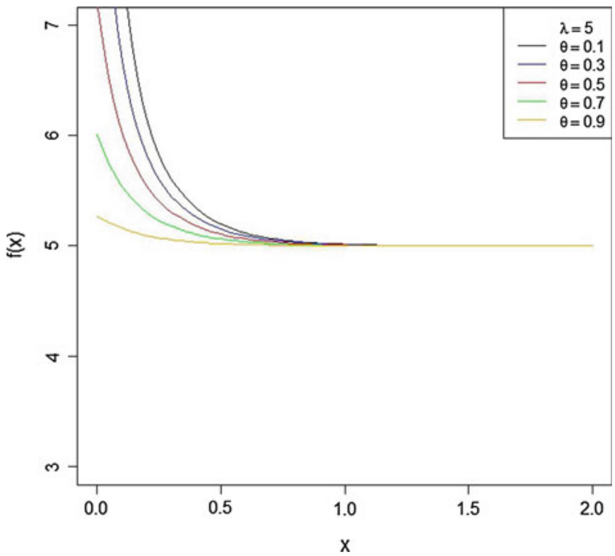

Fig. 3 Hazard rate function of the exponential-logarithmic distribution

```
[5,] 0.7428571 0.8000000 0.8363636
```

Using the code

```
pgfY <- matrix(nrow = dim(s)[1],ncol=length(theta))
for (i in 1:length(theta)) {
for (j in 1:dim(pgfY)[1]) {
pgfY[j,i]<-round(pgfyule(s[j,i],theta[i]),
digits = 3)
}
}
pgfY
```

we obtain the matrix whose elements represent values of the pgf of Yule distribution:

```
      [,1]  [,2]  [,3]
[1,] 0.982 0.991 0.995
[2,] 0.952 0.975 0.985
[3,] 0.926 0.960 0.976
[4,] 0.904 0.947 0.967
[5,] 0.883 0.934 0.958
```

The code

```
pgfDY <- matrix(nrow = dim(s)[1],ncol=length(theta))
for (i in 1:length(theta)) {
for (j in 1:dim(pgfY)[1]) {
pgfDY[j,i]<-round(pgfDyule(s[j,i],theta[i]),
digits = 3)
}
```

```
}
pgfDY
```

is used for calculation of the matrix whose elements represent values of the first derivative of the pgf:

```
      [,1]  [,2]  [,3]
[1,] 0.580 0.379 0.278
[2,] 0.484 0.346 0.264
[3,] 0.423 0.321 0.252
[4,] 0.378 0.300 0.241
[5,] 0.342 0.282 0.231
```

For calculation of the inversion of the pgf, we use

```
pgfIY <- matrix(nrow = dim(s)[1],ncol=length(theta))
for (i in 1:length(theta)) {
for (j in 1:dim(pgfY)[1]) {
pgfIY[j,i]<-round(pgfIyule(s[j,i],theta[i]),
digits = 3)
}
}
pgfIY
```

and resulting matrix is

```
      [,1]  [,2]  [,3]
[1,] 0.952 0.941 0.933
[2,] 0.828 0.798 0.780
[3,] 0.662 0.623 0.599
[4,] 0.447 0.409 0.387
[5,] 0.169 0.150 0.139
```

Example 5 Let us consider the gamma–Poisson–Lindley distribution. Assume that the shape and scale parameters are $a = 5$ and $s = 1$, respectively, Also let parameter $\theta = 2$. The following code generates a sample of size 1,000 and gives theoretical and sample summaries.

```
require(fUtilities)
parentD <- ``gamma''
compoundD <- ``poissonlindley''
sampleSize <- 1000
xSample <- rCompound(sampleSize, parentD, compoundD,
compoundDist,2, shape = 5, scale = 1)
sample <- data.frame(matrix(ncol = 2, nrow = 4))
colnames(sample)[1] <- ``True''
colnames(sample)[2] <- ``Sample''
rownames(sample)[1] <- ``Mean''
rownames(sample)[2] <- ``Variance''
rownames(sample)[3] <- ``Skewness''
rownames(sample)[4] <- ``Kurtosis''
```

```
m<-1
 sample[1, m] <-round(mean(xSample),3)
 sample[2, m] <- round(var(xSample),3)
 sample[3, m] <- round(skewness(xSample),3)
 sample[4, m] <- round(kurtosis(xSample, method =
 ''moment''),3)
 sample[1, m+1] <-round(meanCompound(parentD, compoundD,
 compoundDist,2,  shape = 5,scale = 1),3)
 sample[2, m+1] <- round(varCompound(parentD, compoundD,
 compoundDist,2,  shape = 5,scale = 1),3)
 sample[3, m+1] <- round(skewCompound(parentD, compoundD,
 compoundDist,2,  shape = 5,scale = 1),3)
 sample[4, m+1] <- round(kurtCompound(parentD, compoundD,
 compoundDist,2,  shape = 5, scale = 1),3)
edit(sample)
```

Results are:

```
         True Sample
Mean     4.412  4.412
Variance 4.346  4.359
Skewness 1.209  1.095
Kurtosis 6.001  4.819
```

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References

Adamidis A, Loukas S (1988) A lifetime distribution with decreasing failure rate. Stat Probab Lett 39:35–42
Adamidis A, Dimitrakopoulou T, Loukas S (2005) On an extension of the exponential-geometric distribution. Stat Probab Lett 73:259–269
Alice T, Jose K (2003) Marshall-Olkin Pareto process. Far East J Theor Stat 9:117–132
Barreto-Souza W, Bakouch HS (2010) A new lifetime model with decreasing failure rate. arXiv: 1007.0238v1
Chahkandi M, Ganjali M (2009) On some lifetime distributions with decreasing failure rate. Comput Stat Data Anal 53:4433–4440
Ghitany M, Al-Hussaini E, Al-Jarallah R (2005) Marshall-Olkin extended Weibull distribution and its application to censored data. J Appl Stat 32:1025–1034
Ghitany M, Al-Awadhi F, Alkhalfan L (2007) Marshall-Olkin extended Lomax distribution and its application to censored data. Commun Stat Theory Methods 36:1855–1866
Hankin RKS, Lee A (2006) A new family of non-negative distributions. Aust N Z J Stat 48:67–78
Jayakumar K, Mathew T (2008) On a generalization to Marshall-Olkin scheme and its application to Burr Type XII distribution. Stat Pap 49:421–439
Johnson N, Kotz S, Kemp A (1992) Univariate discrete distributions. Wiley, New York
Jose K, Naik S, Ristić M (2010) Marshall-Olkin q-Weibull distribution and max-min processes. Stat Pap 51:837–851
Kuş C (2007) A new lifetime distribution. Comput Stat Data Anal 51:4497–4509
Marshall A, Olkin I (1997) A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. Biometrika 84:641–652

Morais A, Barreto-Souza W (2011) A compound class of Weibull and power series distributions. Comput Stat Data Anal 55:1410–1425

Nadarajah S (2008) Marshall and Olkin's distributions. Acta Appl Math 103:87–100

Tahmasbi R, Rezaei S (2008) A two-parameter lifetime distribution with decreasing failure rate. Comput Stat Data Anal 52:3889–3901